Human Capital Accumulation in a Federation

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Abstract

More than half of the variation across U.S. school districts in real K-12 education expenditures per student is due to differences between, rather than within, states. I study the welfare implications of redistribution of education expenditures by the Federal government, using an analytically tractable model of human capital accumulation with heterogeneous agents and endogenous state policies. The net welfare effect of Federal redistribution depends on a trade-off between the positive effect of redistributing resources toward poorer states and the negative effect resulting from misallocation of population across states. Federal redistribution increases welfare in a calibrated version of the model.

Keywords: Human Capital, Education Expenditures, Redistribution, Federal, State and Local Governments, Geographic Mobility.

JEL codes: E24, H7, I2, J6

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1 Introduction

Between one half and three quarters of the variation in primary and secondary education spending per student across school districts in the U.S. is due to differences in average expenditures across U.S. states, as opposed to school districts within a state (Murray et al. (1998), Corcoran et al. (2003), and Section 2 below). A key determinant of the average level of schooling expenditures across states is average state income, with richer states spending more on schooling than poorer ones. While disparities in schooling expenditures that occur within states tend to be offset by state-level redistribution, the Federal government has historically played a much smaller role in education finance. Even relatively poor states such as Mississippi receive only about a fifth of their K-12 education revenue from the Federal government (Hanushek and Lindseth, 2009). While the existence of differences in education expenditures across *states* is not a new phenomenon, it has received relatively little attention in the literature on school finance.[1] The latter has mostly focused on the implications of state-level school finance litigation starting with the 1970's *Serrano* lawsuits in California.[2]

This paper makes two contributions relative to the existing literature (e.g. Fernandez and Rogerson, 1998). First, it proposes a model that simultaneously generates both *within*-state and *between*-states differences in the distribution of education resources.[3] Second, it studies the welfare implications of Federal

[1] The issue of the optimal involvement of the Federal government in K-12 education and its financing is, however, a hotly debated topic in many public policy circles. See, for example, the Koret Task Force on K-12 Education's 2012 report *Choice and Federalism.*

[2] See Silva and Sonstelie (1995), Evans et al. (1997), Murray et al. (1998), Fernandez and Rogerson (1998, 1999), Hoxby (2001) for analysis of the effects of court-ordered education finance reform. Interestingly, Coons et al. (1970, Appendix A, p.465) in the seminal book that provided the theoretical foundation for state-level school finance litigation, draw the reader's attention to the "state-nation analogy to the district-state picture." While they focus on education financing at the state level, they observe that "the variation among states themselves mirrors the pattern of district variation within the states. One of the implications of this is that large-scale federal aid to education is needed if we are to achieve full national equalization."

[3] While I have in mind the financing of primary and secondary education, the ideas developed here are in principle applicable to tertiary education as well.

redistribution of education expenditures taking into account states' policy reactions, movements of population across states, and the dynamic accumulation of human capital across generations. I seek to answer a number of questions related to these two contributions: What economic forces sustain heterogeneity in education expenditures across states despite free geographic mobility of labor? What are the key determinants of between-state differences in expenditures? What are the benefits and costs associated with Federal - as opposed to state - redistribution of education expenditures? How large are the welfare gains from Federal redistribution?

In order to answer these questions, I develop a general equilibrium overlapping generations model of investment in education with heterogeneous agents, heterogeneous states, and state-level voting over redistribution of education expenditures. States are assumed to be characterized by exogenous differences in productivity. Productivity differences are the key determinant of the endogenous dispersion in education expenditures per student across states that emerges in the model's equilibrium. This positive association between income and education expenditures, while intuitive and consistent with the empirical evidence, is not obvious in an economy characterized by free geographic mobility of labor.[4] Labor mobility tends to equalize wages, incomes, and ultimately education expenditures across states. To prevent wage equalization I focus on a different form of congestion, associated with population density, that makes agents indifferent among various states. Thus, although in equilibrium each state has to offer the same utility level to a given agent, it does not have to offer her the same level of income (Roback, 1982).

A key difference between this model and others in this literature (e.g. Fernandez and Rogerson (1997, 1998, 1999, 2003), Bénabou (1996a, 1996b, 2002), etc.) is the fact that I allow for multiple states in addition to multiple school districts within each state. A state in the model is both a *politico-administrative unit* with its own policy toward redistribution of education

[4]The mobility of labor across states is an important point of departure of my analysis relative to the cross-country literature on schooling and cross-country income differences (see e.g. Erosa et al. (2010)).

expenditures as well as an *economic unit* with a distinct labor market. In the literature cited above an individual's wage is independent of her school district of residence as individuals are choosing a school district within a given metropolitan area (or state). The model presented here allows an individual's wage to vary across school districts if they are located in different states. This feature of the model shapes the relevant policy trade-off associated with Federal redistribution. On the one hand, redistributing education resources towards households located in poorer states has the potential of increasing aggregate human capital and welfare due to diminishing returns to inputs in the production of new human capital. On the other hand, redistribution toward poorer states increases the incentives of households to locate there, leading to a lower average productivity of the economy's workforce. This potential for misallocation of population across states induced by Federal policy is a new unique distortion identified in this paper. A sufficient condition under which Federal policy has a positive impact on welfare is that the elasticity of population to wages across states is not too large. A calibrated version of the model yields an increase in welfare stemming from Federal redistribution of education expenditures corresponding to 1.3 percent of consumption.

This paper is related to the literature on investment in human capital and redistributive policies in economies with heterogeneous agents (Glomm and Ravikumar (1992), Boldrin (1992), Fernandez and Rogerson (1997, 1998, 1999, 2003), Bénabou (1996a, 1996b, 2002), Herrington (2013), Heathcote, Storesletten, and Violante (2013)). Relative to this literature I emphasize the geographic - particularly inter-state - dimension of the debate on education financing. The paper is also related to contributions in other neighboring literatures. A number of authors have studied empirically the effects of court-ordered education finance reform in various U.S. states (Evans et al. (1997), Murray et al. (1998), and Hoxby (2001)). There is a small empirical literature on the effects of Federal Title 1 transfers to school districts with a high concentration of students under the poverty line (Gordon (2004) and Cascio et al. (2011)). Within the local public finance literature, reviewed by Epple and Nechyba (2004), Calabrese et al. (2012) find that a centralized provision of public goods is often more efficient than a decentralized one. In the urban literature, Albouy (2009, 2012) uses versions of Roback (1982)'s model to measure the distortions associated with Federal tax and transfer policies.

The rest of the paper is organized as follows. Section 2 presents the two stylized facts that motivate the analysis. Section 3 introduces the benchmark model. Section 4 defines the stationary equilibrium of the model and characterizes its qualitative properties. Section 5 tests some of the model's implications. Section 6 considers the welfare effects of Federal redistribution. Section 7 considers the quantitative version of the model. Section 8 concludes. The paper has two sets of appendices. Appendices A, B, and C provide additional details on the model and on the data. The online technical appendix contains the proofs of all proposition and additional details on the calibration of the model.[5]

2 Background and Stylized Facts

An important characteristic of the U.S. system of financing education is the fact that the Federal government provides a relatively small share of schools' revenue. Figure 1 shows the evolution of the percent contributions of Federal, state and local governments to total primary and secondary education revenues. While the role of the Federal government has increased starting in 1965 with the passage of the Elementary and Secondary Education Act (ESEA), in the last 50 years its share has hovered around 10 percent. The Federal government does not provide unrestricted general aid, but rather funds specialized programs through categorical grants to school districts.[6] For the

[5] I relegated the proofs to the online technical appendix because they are quite lengthy. The technical appendix can be found at: `http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

[6] Funded programs include compensatory education for low-achieving students in low-income districts (Title I of ESEA 1965), special education for students with physical and mental disabilities (Title VI, 1966 amendement to ESEA), bilingual education (Title VII, 1967 amendement to ESEA). The extent of Federal financing varies across states, with the poorer states receiving higher shares of Federal funds. In 2005, for example, New Jersey received 4 percent of its education funds from the Federal government, while Mississipi received 21 percent.

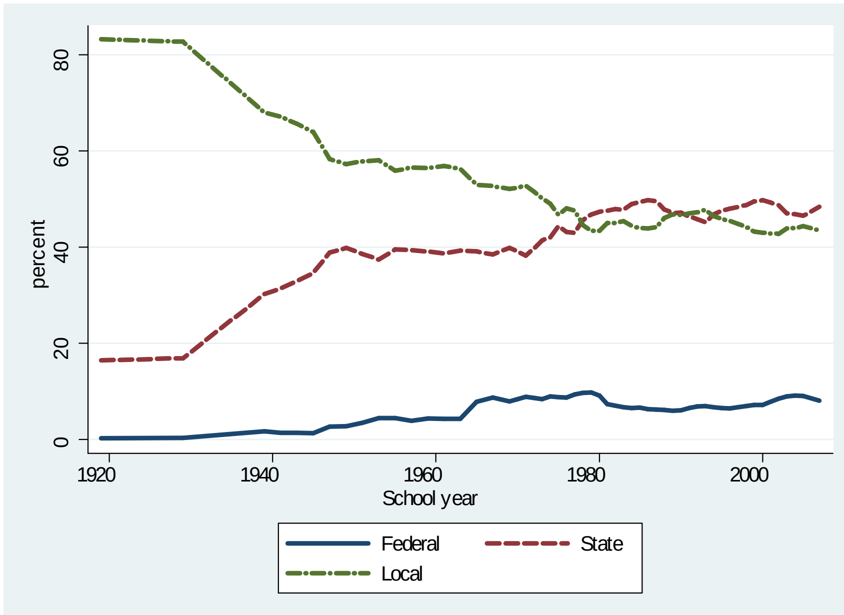

Figure 1: Shares of primary and secondary education revenue in the United States by level of government. Source: National Center for Education Statistics.

U.S. as a whole, state and local governments provide the bulk of financing in almost equal amounts.[7] Education is, on average, the single most important item on state budgets (not including local governments) compared to other expenditures. For example in 2005, it accounted on average for 31 percent of general expenditures by state governments. States provide both general no-strings-attached funds to school districts through a variety of formulas (such as flat and foundation grants) as well as categorical aid for specific programs and goals (such as class-size reduction). The mix of these two types of aid varies by state.[8]

Given this general background, the paper focuses on two related stylized

[7] The shares of funding provided by states and local governments also vary across states. At one extreme of the distribution is Hawaii with 90 percent of funds coming from the state and at the other extreme Nevada with 64 percent of funds raised at the local level (Digest of Education Statistics, 2008, table 172).

[8] See Hanushek and Lindseth (2009) for a critical discussion of the role played by state governments in the funding of public education.

facts about the dispersion in education expenditures per student across school districts and states and the determinants of the cross-state variation.

2.1 Stylized Fact #1: Large Variation in Education Expenditures Between-States

The first stylized fact is that there is a relatively large variation in current education expenditures per student *between* states in the U.S. This variation is "large" relative to the *within-state* variation in current expenditures per students across school districts.

I measure inequality in education expenditures across school districts in the U.S. using the Theil index (see Murray et al, 1998 for details on this index). A useful property of the Theil index is that it can be additively decomposed into a within-state and a between-state component. Table 1 presents, for a selected number of years, the Theil index for nominal education expenditures per student; its within and between components; and the percentage of total inequality accounted for by the between component.

The table shows a very slight reduction in overall expenditures inequality from 1972 to 2009 with variations over the sample period. Indicators of inequality achieve their lowest level in the late 1990s-early 2000s and then increase again. More important for my purposes, the decomposition in Table 1 reveals that the share of inequality attributable to between-states differences in expenditures across school districts is much larger than the within share. Although the relative magnitude of the between-state component has fluctuated over time, it has never fallen below 55 percent (in 1982). In the last year of my data, 2009, differences across states accounted for an all-time high of 76 percent of inequality in nominal education spending per student.[9]

The measure of inequality in education expenditures reported in Table 1 is based on *nominal* data. A price index of education services at the school district level, computed by Taylor (2005), is available for a selected number

[9] For sake of comparison, the Theil index for inequality in average household income across U.S. school districts was 45.3 in the year 2000. The within-state component of household income inequality accounts for about 83 percent of the total dispersion.

Year	1972	1977	1982	1987	1992	1997	2002	2007	2009
Theil index	43.7	37.1	31.0	40.7	40.5	30.6	29.4	39.3	43.2
Within states	13.7	14.4	14.0	12.6	13.4	9.9	8.6	10.1	10.3
Between states	30.0	22.8	17.0	28.2	27.1	20.7	20.8	29.2	32.9
Percent between	69	61	55	69	67	68	71	74	76

Table 1: The Theil index of inequality in nominal current expenditures per student across school districts. To improve readability, the Theil index is multiplied by 1,000. Notice that the within and between-states components add up to the Theil index in the second row of the Table. The row "Percent between" reports the ratio of the between component to the Theil index (times 100). Source: Murray et al. (1998), Corcoran et al. (2003), and author's computations (see Appendix C.1).

of recent years. This index can be used to compute measures of inequality in *real* expenditures.[10] The results are reported in Table 2 for some of the years for which these deflators are available.[11] Adjusting for differences in the price of education across school districts reduces the extent of the between-state variation and increases the extent of the within variation. However, the between-state component accounts for at least 50 percent of the overall variation in real expenditures per student and in 2009 it approaches 64 percent.

In order to gain further insight into the magnitude of between-state differences in education spending per student, the following table provides information on the range of the variation (minimum and maximum expenditures), average and median expenditures, and the coefficient of variation (denoted by CV) for recent years.

[10] Real expenditures are obtained by dividing nominal expenditures by Taylor's price index. The latter is constructed by measuring the cost of hiring a non-teacher worker with the same set of observable characteristics as a representative teacher (e.g. education level, years of experience, sex, etc.) in different geographic areas.

[11] The sample of school districts for these years is the same as in Table 1.

Year	2002		2007		2009	
	Nominal	Real	Nominal	Real	Nominal	Real
Theil index	29.4	22.8	39.3	29.9	43.2	32.5
Within states	8.6	11.1	10.1	12.2	10.3	11.8
Between states	20.8	11.7	29.2	17.7	32.9	20.7
Percent between	71	51	74	59	76	64

Table 2: The Theil index of inequality in real current expenditures per student across school districts. To improve readability, the Theil index is multiplied by 1,000. Notice that the within and between-states components add up to the Theil index in the second row of the Table. The row "Percent between" reports the ratio of the between component to the Theil index (times 100). Source: author's computations using Taylor (2005)'s comparable wage index index (see Appendix C.1).

As the tables makes clear, a state at the top of the distribution spends about 2.5 times more per student in real terms than a state at the bottom.[12]

To summarize, between-state differences in real education expenditures per student are large relative to within-state differences. Moreover, these differences tend to persist over time. In the following subsection I link differences in average education expenditures to differences in average income across states.

[12] Focusing on real rather than nominal expenditures has little impact on the ranking of states: the rank correlation across states between average nominal and real expenditures per student is 0.85 in 2000 and 0.88 in 2005. Finally, the ranking of states in terms of expenditures per student is also fairly stable over time: the rank correlation of nominal average state expenditures per student in 2005 and in 1975 is 0.64 (p-value 0.00). The comparison over time is necessarily based on nominal expenditures due to lack of state-level data on education price indices for years prior to 1997.

Year	Expenditures	Min	Max	Mean	Median	CV
2000	nominal	6,296 (UT)	16,227 (DC)	9,941	9,412	0.20
2000	real	6,751 (UT)	14,707 (VT)	9,941	9,333	0.15
2005	nominal	5,464 (UT)	14,954 (NJ)	9,145	8,301	0.24
2005	real	5,899 (UT)	14,974 (VT)	9,145	8,658	0.19

Table 3: Variation in current expenditures per student across U.S. states (units are 2005 dollars). Real education expenditures are computed using Taylor (2005)'s index. Average real expenditures in each year for the whole U.S. are normalized to equal average nominal expenditures. Data source: author's computations based on National Center for Education Statistics data and Taylor (2005)'s index.

2.2 Stylized Fact #2: Spending per Student Tends to be Higher in Richer States

The second stylized fact is that a state's income is an important determinant of its real education expenditures per student. The model presented in Section 3 builds on this evidence. The income-expenditure correlation can be computed using either cross-state variation in these two variables (Table 4) or within-state variation over time (Table 5).

Table 4 presents estimates of the elasticity of real education expenditures per student to income per student using cross-sectional variation for a selected number of years.

In the cross-section of states, average per student income is positively and significantly correlated with expenditures per student, explaining about 40 percent of the observed variation in real expenditures per student. While the elasticities vary according to the particular year considered, the range is 0.40-0.60. Columns (2), (4), and (6) of Table 4 add a control for state-level public school teachers' unionization rates. The rationale for this is that measured real spending per student might be high in some states not because of a higher quality of K-12 education, but simply because teachers' unions

	(1)	(2)	(3)	(4)	(5)	(6)
year	1997	1997	2000	2000	2005	2005
	dependent variable: *log real education expenditures per student*					
log income per student	0.55*** (0.08)	0.46*** (0.13)	0.52*** (0.09)	0.43*** (0.14)	0.65*** (0.14)	0.60*** (0.18)
teachers' union membership (%)		0.001 (0.002)		0.001 (0.002)		0.001 (0.002)
R^2	0.41	0.43	0.36	0.38	0.36	0.36
number of obs.	51	51	51	51	51	51

Table 4: Cross-sectional regressions of log real education spending per student on log income per student. *** denotes statistical significance at the 1 percent level. Standard errors are in parenthesis. Data sources: State-level yearly education spending and enrollment data from NCES. State-level income data are from the BEA. Public school teachers' union membership data are from the NCES. Regressions are weighted by state-level enrollment. Union membership data refers to the year 1999.

are able to bargain for higher salaries.[13] Controlling for teachers unionization rates slightly reduces the estimated partial correlation between income and expenditures. However, the drop in the correlation is small and the coefficient on income remains highly significant.

Cross-sectional results might be driven by unobserved differences across states that are correlated with per capita income and spending per student. In order to control for unobserved heterogeneity across states, I have computed the elasticity of spending per student to income using state-level panel regressions including state fixed effects. In a panel regression, the effect of

[13]For example, according to the Schools and Staffing Survey of the U.S. Department of Education in 1999 the average unionization rate of public school teachers in the U.S. was 79 percent, with a range between 31 and 99 percent across states. By contrast, private sector unionization was less than 10 percent (Hirsch, 2008). Union membership refers to the year 1999 due to lack of data for each year but is highly persistent over time.

income on education expenditures is identified by the within-state variation in income over time. Table 5 presents the results of the panel regression. In order to exploit a longer time-series of data I start by considering the logarithm of *nominal* expenditures per student as a dependent variable. The use of nominal data is justified if unobserved differences in the prices of education services tend to be constant over time for a given state. In this case the elasticity of expenditures to income per student is 0.66. Focusing on real expenditures data shortens the time period to 1997–2005. The resulting elasticity is now 0.38, at the low end of the range estimated using cross-sectional data.

	(1)	(2)
	dependent variable: log education expenditures per student	
	nominal	*real*
log income per student	0.67***	0.38***
	(0.09)	(0.13)
time fixed effects	yes	yes
state fixed effects	yes	yes
R^2	0.99	0.97
time period	1969-2008	1997-2005
number of obs.	$2,038$	459

Table 5: Panel regression of log nominal and real education spending per student on log income per student. *** denotes statistical significance at the 1 percent level. Standard errors are in parenthesis. Data sources: state-level yearly education spending data from NCES. State-level income data are from the BEA. Regressions are weighted by average enrollment in the state in the sample period.

To summarize, there is evidence that differences in real education expenditures across states are driven, in part, by differences in income. The latter account for about 40 percent of the variation in expenditures in the cross-section. While other factors might also be important, based on this evidence it is plausible to consider a model in which differences in education expenditures are driven by income heterogeneity across states. The next section introduces such a model.

3 Model Economy

The model represents an economy where time lasts forever and individuals' lives last two periods. In the first period the agent is a child and in the second one an adult. Adult agents make labor supply decisions and invest in the human capital of their child. I follow Bénabou (2002) and model the government's education policy as a tax-transfer scheme that redistributes education expenditures towards low income agents. The novel feature of the model is the presence of three geographic/political units. The first is the *school district*, represented by a collection of households with the same human capital level within each state. This definition captures sorting of households by income level and represents an extreme version of sorting by which each school district is internally homogeneous.[14] The second is the *state* (or "location") which is characterized by a given level of productivity and a measure of residents. All agents located in the same state operate in the same labor market. The third is a *Federation* of states, defined as the collection of all states.

3.1 Timing of Events

The timing of an agent's life is as follows. An agent is born in a state and spends her youth acquiring human capital there. An agent's state of birth has a direct influence on the agent's human capital through its policy of redistribution of education expenditures. At the beginning of her adult period, the agent chooses a state of residence in which to work and raise her child. Upon choosing a location, production, labor supply, consumption, and education investment take place. Redistribution of education expenditures occurs at the end of each period. Federal redistribution policy is exogenous and is

[14] Fernandez and Rogerson (1997, 1999, 2003) have used this approach extensively.

taken as given by the residents of each state when they vote over within-state redistribution. These policies, in conjunction with parental human capital, education investment, and a random shock determine the human capital of the child, who, as an adult, will go through the same sequence of choices. For tractability, I focus on the model's stationary equilibrium.

3.2 Locations, Preferences, and Technology

Formally, the economy is comprised by a continuum of measure one of states S_j, indexed by $j \in [0,1]$ and a continuum of measure one of non-altruistic agents indexed by $i \in [0,1]$ who live for two periods, as a child and as an adult. A school district is defined as a collection of agents with the same human capital and living in the same state (Fernandez and Rogerson, 1997, 1999, 2003). Since I focus on stationary equilibria, in what follows I omit time indices and denote next period variables by a prime symbol.

Children do not have preferences of their own. An agent i living in state j cares about consumption c, time spent working, l, and her child's human capital h'. In addition, in order to generate a well-defined distribution of population across locations, an agent's utility declines with the state's resident population, denoted by n_j.[15] Parental preferences are represented by the following utility function:

$$U_{ij} = \rho \ln c_{ij} - \frac{l_{ij}^{\eta}}{\eta} + (1-\rho)\ln h'_{ij} - \lambda \ln n_j \tag{1}$$

where $\rho \in (0,1)$, $\lambda > 0$ and $\eta > 1$. The logarithmic specification for consumption and the the isoelastic one for labor are borrowed from Bénabou (2002) and are essential to generate closed-form solutions.

An adult agent can freely and costlessly choose the state in which to reside. Upon choosing her residence an agent with human capital h_i who works l_{ij}

[15] Formally, the congestion effect associated with higher population density plays a similar role to the congestion of the locally provided public good in Albouy (2012)'s Rosen-Roback model of Canadian regions. See Section 3.4 for a discussion of this and other modelling assumptions.

units of time earns income $y_{ij} = w_j l_{ij} h_i$, where w_j represents the prevailing wage in the state. The budget constraint of an agent who resides in a state j is

$$y_{ij} = c_{ij} + z_{ij} \tag{2}$$

where z_{ij} is the amount spent on education by an agent in school district i and state j. In what follows, I refer to z as a household's investment in education, to distinguish it from education expenditures received by the agent's child and denoted by e_{ij}. In an economy without government's intervention in education, we would have $z_{ij} = e_{ij}$. Policy interventions by the state and the Federal government introduce a discrepancy between these two variables. Section 3.3 below describes how these policies affect the mapping between z_{ij} and e_{ij}.

Each agent has one child who is born in the state of her parent's residence. The child's human capital h'_{ij} is given by the following Cobb-Douglas production function:

$$h'_{ij} = \xi'_i h_i^{\alpha} e_{ij}^{\beta}, \tag{3}$$

where ξ'_i is an i.i.d. (across generations and households) random shock.[16] The shock follows a lognormal distribution with a unit mean: $\ln \xi'_i \sim N\left(-\sigma_{\xi}^2/2, \sigma_{\xi}^2\right)$. The input h_i in equation (3) reflects the direct effect that a parent's human capital has on the human capital of the child, while e_{ij} represents the effect of education expenditures on the child's human capital. Notice that, given that a school district is composed by a collection of homogeneous households, the input h_i might also reflect district-level peer effects. The exponents α and β on these two inputs are assumed to be such that the accumulation technology exhibits decreasing returns to scale: $\alpha + \beta < 1$.[17]

To summarize, the decision problem of an agent i involves choosing a state

[16] This shock might represent the child's innate ability or simply luck. While here I am assuming that the parent observes ξ'_i at the time of making her decisions, the logarithmic form of utility implies that parental choices would be the same if the parent did not observe this shock and conditioned her choices on its expected value, instead.

[17] It would be straightforward to extend the human capital accumulation technology in equation (3) to allow for exogenous district or state-level productivity differences in the provision of education. I abstract from those to focus on the strong correlation between state-level measures of income per student and real education expenditures documented in Section 2.

of residence S_j, labor supply l_{ij}, consumption c_{ij}, and education investment z_{ij} in order to maximize utility (1) subject to the budget constraint (2), state and Federal governments policies, and the law of motion (3) of her child's human capital.

Production of goods and services in a state j occurs through the constant returns to scale technology:[18]

$$Y_j = A_j L_j$$

where L_j denotes the total supply of efficiency units of labor located in j:

$$L_j = \int_{i \in S_j} l_{ij} h_i di.$$

Total factor productivity, A_j is lognormally distributed across states with mean normalized to one: $\ln A_j \sim N\left(-\sigma_A^2/2, \sigma_A^2\right)$. Competitive firms hire efficiency units of labor in each location in order to maximize profits.

3.3 Education Policy

In order to keep the model analytically tractable, I follow Bénabou (2002)'s approach and model redistribution of education expenditures as an income-dependent subsidy to investment in education. I extend his approach to two layers of government with both Federal and state policies redistributing education expenditures toward low-income agents. The Federal government introduces a first layer of redistribution. After Federal redistribution, education expenditures for a child in household i are given by:

$$e_{ij}^f = \left(\frac{\widetilde{y}^f}{y_{ij}}\right)^{\tau^f} z_{ij}, \tag{4}$$

where the federal education policy is summarized by the couple $\left(\tau^f, \widetilde{y}^f\right)$, with $\tau^f > 0$. Notice that the variable $\widetilde{y}^f$ represents the income level of an agent that

[18] All results go through with decreasing returns at the cost of more complexity in the algebra. They are available from the author upon request.

receives a zero net transfers from the Federal government, or equivalently the break-even level of income such that $e^f_{ij}/z_{ij} = 1$. Higher values of the parameter τ^f are associated with larger proportional transfers e^f_{ij}/z_{ij} of education expenditures toward low income households. While τ^f is a parameter that can be varied exogenously, $\widetilde{y}^f$ is an endogenous variable determined by the budget constraint of the Federal government:

$$\int \int_{i \in S_j} e^f_{ij} di dj = \int \int_{i \in S_j} z_{ij} di dj, \tag{5}$$

where e^f_{ij} depends on $\widetilde{y}^f$ through equation (4). This equation simply states that the aggregate education expenditures post-Federal redistribution (left-hand side of equation 5) have to equal the pre-redistribution aggregate investment in education (right-hand side of equation 5). In other words, the Federal policy is purely redistributive.

The second layer of redistribution occurs at the state level. A state government redistributes expenditures across the school districts located within its boundaries. After state redistribution, education expenditures to which a child from household (or school district) i located in state j is exposed are given by:

$$e_{ij} = \left(\frac{\widetilde{y}^s_j}{y_{ij}} \right)^{\tau^s_j} e^f_{ij}. \tag{6}$$

State j's education policy is summarized by the couple $\left(\tau^s_j, \widetilde{y}^s_j\right)$. The policy variable τ^s_j, assumed to be positive, is the state-level analog of τ^f, and determines the extent of redistribution of education expenditures toward low-income districts within a state. The variable $\widetilde{y}^s_j$ denotes the level of income of an agent that receives a zero net transfer from state j. As in the case of Federal policy, $\widetilde{y}^s_j$ must be consistent with the fact that a state government's policy is purely redistributive:

$$\int_{i \in S_j} e_{ij} di = \int_{i \in S_j} e^f_{ij} di, \tag{7}$$

where the integrals are taken over all agents i who reside in state S_j.

In what follows I treat the extent of Federal redistribution τ^f as a parameter while I endogenize the state-level policy $\left(\tau_j^s, \widetilde{y}_j^s\right)$. State level redistribution is decided after location and production decisions. Following Bénabou (1996b), the decisive agent in the determination of τ_j^s in each state j is assumed to be the agent at the p-th percentile of the distribution of income in the location, with p assumed to be weakly larger than $1/2$ and independent of j. When $p = 1/2$, this reduces to the standard majority voting setting in which the agent with median income is decisive. As shown below, the political process in each state selects $\left(\tau_j^s, \widetilde{y}_j^s\right)$ as a function of τ^f.[19] The policy analysis in Section 6 consists of determining the effects of exogenous changes in τ^f on the equilibrium of the model, taking into account the endogenous reaction of state-level redistribution policies.[20] The following section discusses the main choices made in setting up the model.

3.4 Discussion of Modelling Choices

There are many challenges in constructing an analytically tractable model of human capital investment with multiple governmental units as well as within and between-state heterogeneity in education expenditures. In this section I provide a discussion of the main modeling choices I have made focusing on three aspects of the environment: education policy, congestion costs, and geography.

Education policy. Bénabou (2002)'s formulation of redistributive policies, while analytically convenient, does not feature separate taxes and transfers. There is, however, an important special case in which a tax-transfer interpretation applies. The special case is that of a centralized state system

[19] An alternative to the specification adopted here is to reverse the order of redistribution with state redistribution followed by Federal redistribution. In this alternative situation the multiplicative nature of education subsidies and the logarithmic assumption on utility would eliminate any dependence of τ_j^s on τ^f. Federal redistribution would then be more effective, relative to the economy considered here, because it would not elicit any behavioral offset on the part of state governments.

[20] It is feasible, but cumbersome, to endogenize the choice of τ^f. The problem is that the policy space becomes two-dimensional and standard median voter results do not apply. See Nechyba (1997) for an analysis of voting in such setting that borrows from Shepsle (1979)'s structure-induced equilibrium.

with no Federal redistribution ($\tau_j^s = 1$ and $\tau^f = 0$). This situation can be interpreted as one in which each state imposes a proportional income tax to finance constant expenditures per pupil across income levels.[21] A second important feature of Bénabou (2002)'s formulation is that state and Federal policies affect the tax price of education expenditures, or the resource cost for a school district of marginally increasing its purchases of education services. The incentive properties of this scheme resemble most closely those of power equalizing education finance schemes adopted by a number of U.S. states (Fernandez and Rogerson, 2003).[22]

Congestion costs. One of the main objectives of the model is to give rise to the positive cross-state correlation between education expenditures and income documented in Section 2.2 while allowing for free mobility of labor across states. In a version of this model without congestion costs ($\lambda = 0$) in which states have (exogenously) different productivity and labor demand curves are downward sloping, free labor mobility would tend to equalize wages across locations. In turn, equalization of wages leads to equalization of education expenditures and human capital levels in the long-run. It follows that, in order to generate persistent differences in education expenditures across states, there has to be a mechanism different from the adjustment of wages to make agents indifferent across different locations. In this model, the mechanism is the disutility an agent experiences from living in a "crowded" location. In Appendix B.4 I show how the presence of n in the utility function of an agent can be interpreted as the reduced-form version of a more general model in which each state is also characterized by a housing market. In this case the

[21] The proof of this equivalence result is available upon request. Centralized systems are observed in some states, such as California.

[22] In a power equalizing scheme a school district chooses a tax rate that does not apply to its own tax base, but rather to a tax base specified by the state. Thus, according to such scheme, the tax price faced by a district is equal to the ratio of its tax base (income in this case) and the tax base specified by the state, with poorer districts facing tax prices less than one and richer districts tax prices larger than one. Transfers from the Federal government to school districts under Title I of ESEA are independent of local effort (subject to a lower bound; see Gordon (2004)) and so, to a first approximation, are lump-sum in nature. While analyzing lump-sum transfers in this model is feasible, doing so would sacrifice its analytical tractability.

negative effect of n on utility reflects the pressure of a larger population on local housing prices.

Geography. There are three geographic units in the model: school districts, states, and a Federation of states. My approach to modelling school districts borrows from Fernandez and Rogerson (1997 and 1999) who assume that each income type resides in a separate local community within a state. I model a state as both a political and an economic entity. The former view is clearly more accurate than the latter as states play an important role in education financing but metropolitan areas more accurately delimit local labor (and housing) markets and commuting zones. Therefore, one might, in principle, want to distinguish between labor markets and states.[23] The average economic characteristics of a *state*, such as its average income, degree of congestion, etc., could then be obtained by aggregating across all metropolitan areas within its boundaries. For simplicity, I do not explicitly pursue this approach which would require allowing for multiple *heterogeneous* metropolitan areas embedded within the same state. In this respect the approach taken here is consistent with that followed by other papers in the education finance literature that focus on equalization of education expenditures by state governments (see e.g. Fernandez and Rogerson, 1997 and 1999). In these models agents are implicitly assumed to be mobile across local districts within a single metropolitan area because their income does not depend on the local community in which they choose to reside. Thus, when the state government equalizes education funding within its boundary, it simply equalizes funding across the local communities that are part of the only metropolitan area. This literature has abstracted from modeling multiple coexisting states and their interactions through a Federal redistribution policy. This is the objective of my paper.

[23] Notice that other papers in labor and public finance also identify a state or a Census region with a labor market (see e.g. Kennan and Walker (2011) and Albouy (2012)).

4 Stationary Equilibrium

In this section I define and characterize the stationary equilibrium of the economy described in Section 3.

Definition [Stationary equilibrium]. Given a Federal redistribution policy that satisfies the budget constraint of the Federal government, a stationary equilibrium is represented by a wage, a measure of residents, and a redistribution policy for each state; consumption, education expenditures and labor supply choices for each agent; hiring decisions for each firm such that: i) firms optimize; ii) state-level policies are determined by the agent in the p-th percentile of the state's income distribution, taking as given the geographic distribution of population, and the labor supply and education expenditure choices of the agents; iii) agents choose consumption and education spending to maximize their utility subject to the budget constraint in their state of residence, taking as given redistribution policies; iv) adult agents are indifferent among all possible locations of residence; v) the within-state distribution of human capital and the between-state distribution of population are constant over time; vi) all agents reside in some location.

The main properties of the stationary equilibrium are summarized in the following proposition.

Proposition 1 [Stationary equilibrium]. There exists a stationary equilibrium for this economy with the following properties.
(i) Wage and consumer choices. The equilibrium wage in state j is $w_j^* = A_j$ and the choices of a consumer i with income $y_{ij} = w_j^* h_i l^*$ are given by:

$$c_{ij}^* = \frac{\rho}{\rho + (1-\rho)\beta} y_{ij} \tag{8}$$

$$z_{ij}^* = \frac{(1-\rho)\beta}{\rho + (1-\rho)\beta} y_{ij} \tag{9}$$

$$l^* = \left(\rho + (1-\rho)\beta\left(1 - \tau^{s*} - \tau^f\right)\right)^{\frac{1}{\eta}}. \tag{10}$$

(ii) State's population. It is given by

$$n_j^* = A_j^{\vartheta} \exp\left\{\frac{\sigma_A^2}{2}\vartheta\left(1-\vartheta\right)\right\}, \tag{11}$$

where the composite parameter $\vartheta > 0$ is formally defined in Appendix A.
(iii) Distribution of human capital within a state. It is lognormal $h_{ij} \sim \ln N\left(m_j^*, (\Delta^*)^2\right)$ where:

$$m_j^* = \widetilde{m} + \frac{\beta\left(1-\tau^f\right)}{1-\alpha-\beta\left(1-\tau^f\right)}\ln A_j, \tag{12}$$

$\widetilde{m}$ is defined in Appendix A, and Δ^* is the unique positive solution to the quadratic equation:

$$\Delta^2 - \left(\alpha\Delta + \beta\Phi^{-1}(p)\right)^2 - \sigma_\xi^2 = 0.$$

Φ denotes the cumulative distribution function of the standard normal distribution.
(iv) Redistribution of education expenditures. The education subsidy received by an agent living in state j with rank q in the within-state income distribution is:

$$\frac{e_{ij}^*(q)}{z_{ij}^*(q)} = \underbrace{\left(\frac{\widetilde{y}^f}{\widetilde{y}_j^{s*}}\right)^{\tau^f}}_{\text{state-specific component}} \times \underbrace{\exp\left\{\left(\tau^{s*}+\tau^f\right)\Delta^*\left[\Delta^*\left(1-\tau^f-0.5\tau^{s*}\right)-\Phi^{-1}(q)\right]\right\}}_{\text{individual-specific component}}, \tag{13}$$

where state j's redistributive policy $\left(\tau^{s*}, \widetilde{y}_j^{s*}\right)$ is such that:

$$\begin{aligned} \tau^{s*} &= 1-\tau^f-\frac{\Phi^{-1}(p)}{\Delta^*}, \\ \widetilde{y}_j^{s*} &= A_j l^* \exp\left\{m_j^* + \left(1-\tau^f-\frac{\tau^{s*}}{2}\right)(\Delta^*)^2\right\}, \end{aligned} \tag{14}$$

and $\widetilde{y}^f$ is the (endogenous) break-even level of income for Federal redistribution whose expression is given in Appendix A.

Proof. See technical appendix:
`http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

In what follows, I comment on the main features of this stationary equilibrium.[24]

Production and consumption choices. Part (i) of Proposition 1 describes the decisions of firms and consumers in each location. The wage is equal to the marginal product of labor in each state. The logarithmic structure of utility implies that each consumer allocates the same constant fraction of income to consumption c and education investment z. Federal and state redistribution of education expenditures are distortionary and reduce labor supply, so l^* is decreasing in $\tau^{s*} + \tau^f$ in equation 10.

States' population. Part (ii) describes the equilibrium level of population in state j. States with higher productivity have higher population levels ($\vartheta > 0$). However, congestion effects prevent population from being concentrated in the state with the highest productivity. Thus, in equilibrium the distribution of population is non-degenerate. It follows a lognormal distribution, just like productivity. The composite parameter ϑ is important because it determines the elasticity of states' population to productivity and, therefore, wages. It is worthwhile to emphasize the dependence of ϑ on two structural parameters, τ^f and λ.[25] A more redistributive policy by the Federal government (i.e. a higher value of τ^f) reduces ϑ because it diminishes the attractiveness of locations with relative high total factor productivity. The parameter λ determines the sensitivity of congestion costs to an increase in population. As λ increases, this sensitivity increases, and agglomeration of individuals in more productive locations declines, i.e., ϑ decreases. In the limit, as $\lambda \to \infty$, $\vartheta \to 0$ and all states have the same population, $n = 1$. As discussed in Section 6.2, the parameter λ plays an important role in determining the magnitude of the welfare benefits associated with Federal redistribution.

Human capital distribution. Part (iii) of Proposition 1 shows the sta-

[24] The stationary equilibrium of the economy is unique within the class of lognormal distributions for human capital in each location.

[25] See equation (A1) in Appendix A for a formal definition of ϑ.

tionary distribution of human capital in a state. As equation (12) illustrates, locations with higher total factor productivity are characterized by higher average human capital, as long as $\tau^f < 1$. Federal redistribution (a higher value of the parameter τ^f) reduces the sensitivity of a state's average human capital to its productivity.

Redistribution of education expenditures. Part (iv) of Proposition 1 characterizes the key features of the equilibrium subsidy to education spending by the Federal government and the states. Recall that the Federal policy (τ^f) is exogenous, although subject to the budget constraint in equation (5), while each state's policy (τ_j^s) is chosen by the state's decisive agent (ranked p in the within-state distribution of income) after production, consumption, labor supply and mobility choices. Thus, this agent chooses the state's policy to maximize the post-redistribution expenditures $e_{ij}^*(p)$ he receives, taking as given the Federal policy. The resulting subsidy received by an agent, given by equation (13), depends only on her state of residence j and on her rank q in the state's income distribution. Consider first the dependence on state of residence j. The first multiplicative term on the right-hand side of equation (13) shows that states with lower break-even levels of income $\widetilde{y}_j^{s*}$ benefit from Federal redistribution because they tend to have lower productivity and income. At the individual level, as an agent's rank q in the within-state distribution of income increases, the subsidy she receives declines, as it appears from the second multiplicative term on the right-hand side of equation (13). Notice that a more redistributive Federal policy (i.e. a higher value of τ^f) does not affect the magnitude of the subsidy received by an agent of rank q relative to the subsidy received by another agent living in the same state. The reason for this result is that as τ^f increases, τ^{s*} declines on a one-for-one basis, keeping the within-state distribution of education expenditures unchanged. As a consequence, the extent of cross-sectional inequality in human capital and income within each state, as measured by Δ^*, does not depend on Federal policy. For the same reason, the Federal component of redistribution τ^f has no effect on intergenerational mobility, as measured by the correlation between the log human capital of a parent and of her child.[26]

5 The Model at Work: Stylized Facts and Empirical Implications

In Section 5.1, I analyze the model's implications for the two stylized facts that motivate the paper. Then, in Section 5.2, I derive and empirically test some of its additional implications.

5.1 Stylized Facts

The model introduced in Section 3 is consistent with the two stylized facts presented in Section 2. The first stylized fact is the existence of a (quantitatively important) between-state dimension of inequality in education expenditures. The following proposition summarizes the model's implications for inequality in education expenditures within and between states.

Proposition 2 [Theil index decomposition]. In the stationary equilibrium of the model, the Theil index of inequality in education expenditures, denoted by T^e, can be decomposed into a within-state and a between-state component as follows:

$$T^e = \underbrace{\frac{\left(\Phi^{-1}(p)\right)^2}{2}}_{\text{within-state}} + \underbrace{\frac{\sigma_A^2}{2}\left(\frac{(1-\alpha)\left(1-\tau^f\right)}{1-\alpha-\beta\left(1-\tau^f\right)}\right)^2}_{\text{between-state}}. \tag{15}$$

[26] On a more technical note, the model can be solved in closed-form because the decisive voter in each state chooses the same value of the policy variable τ^s. The fact that the distribution of human capital within each state is lognormal with the same variance implies that the decisive voters in all states, although differing in terms of their human capital, face the same incentives to redistribute. The fact that τ^{s*} is constant across states implies that they all choose the same degree of progressivity in redistributing education expenditures. As a consequence, there is no equilibrium selection of agents across states based on their human capital level. This, in turn, sustains the equilibrium with homogeneous variance of log human capital and income across states.

Proof. See technical appendix:
`http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

Notice that this decomposition is the model-counterpart of the empirical analysis in Section 2.1, Tables 1 and 2, with school-district level data. The expression in equation (15) clarifies the determinants of the distribution of expenditures across individuals and locations. Within-state inequality in education expenditures in the model is only a function of the identity p of the decisive voter. Median voters with higher relative incomes choose less redistribution of education expenditures leading to higher inequality.[27] The between-state component is strictly positive as long as locations have different productivity ($\sigma_A^2 > 0$) and Federal redistribution is not perfect ($\tau^f < 1$). The between-state term declines as τ^f increases and Federal redistribution becomes more pronounced. Between-state expenditures inequality increases with the parameters β and α because differences in expenditures are amplified and perpetuated over time through their effect on states' human capital accumulation.

The second stylized fact, discussed in Section 2.2, is the existence of a positive correlation between average state spending per student and average income per student. The following proposition summarizes the model's implications for this correlation.

Proposition 3 [Income-expenditures correlation]. In the stationary equilibrium of the model, the cross-sectional correlation between log average state spending per student and log average state income per student is equal to $\left(1 - \tau^f\right)$.

Proof. See technical appendix:
`http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

Notice that spending per student and income are jointly determined in the model's equilibrium. A higher income per student leads to higher spending and higher spending, over time, increases the state's human capital and income.

[27] Recall that $p \geq 1/2$, so that when $p = 1/2$ within-state redistribution is full ($\Phi^{-1}(0.5) = 0$) and there is no within-state inequality.

Absent Federal redistribution, the homotheticity of preferences would imply a unit correlation between these two variables (expressed in logs). Federal redistribution of education expenditures lowers this correlation below one, so that schooling expenditures represent a higher share of income in a relatively poor state than in a richer one.

Propositions 2 and 3 show that the model is qualitatively consistent with the broad stylized facts that motivated the analysis. In the next subsection I derive and evaluate empirically some of its additional implications.

5.2 Additional Implications

I focus on two additional implications of the model.

Implication 1. The *within-state* Theil index of education expenditures inequality is not systematically correlated with the average level of income per student in the state.

Proposition 1 actually shows that the within-state Theil index is the same for each state and equal to the first term on the right-hand side of equation (15). In reality, empirical Theil indices of within-state inequality display some cross-state variation. This is to be expected given that the model has focused on only one, although important, source of heterogeneity across states - productivity - and has abstracted from many others. In light of this consideration, I focus on the weaker but still informative implication that heterogeneity in the level of productivity across states does not translate into differences in the extent of inequality of education expenditures across school districts within states. To test this hypothesis, I run separate cross-sectional regressions of the state-level Theil index on log average income per student by year. The results are reported in Table 6 for all the years and states for which I am able to construct the Theil index of real education spending per student, or 2000–2008. In each year, the coefficient on income per student is always statistically insignificant at conventional levels. Hence, within-state inequality in education expenditures does not seem to vary systematically with a state's income per student. This result is consistent with the model's prediction.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year	2000	2001	2002	2003	2004	2006	2007	2008
	dependent variable: *Theil index of within-state inequality in real education expenditures*							
log income per student	0.002 (0.006)	-0.002 (0.022)	-0.003 (0.025)	-0.004 (0.032)	-0.005 (0.037)	-0.005 (0.0037)	-0.006 (0.006)	-0.004 (0.034)
R^2	0.01	0.01	0.01	0.03	0.04	0.03	0.05	0.02
number of obs.	46	46	46	46	46	46	46	46

Table 6: Cross-sectional regressions of state-level Theil index on log income per student. Regressions are weighted by state-level enrollment. Standard errors in parenthesis. Following Murray et al (1998), I did not compute Theil indices for the following states: Hawaii, Washington DC, Montana, Vermont and Alaska.

Implication 2. States with higher income and education expenditures per student have higher average human capital.

The model predicts the following cross-state relationship between the log of average human capital and the log of average income:

$$\ln E_j\left[h_{ij}\right] = \frac{\beta\left(1-\tau^f\right)}{1-\alpha} \ln E_j[y_{ij}] + \text{constant}.$$

The correlation between these two variables is positive as long as $\beta > 0$, or education expenditures play a role in the formation of human capital. Proposition 3 then implies that, since income and education expenditures are positively correlated across states, the correlation between the log of average human capital and education expenditures is also positive.

In order to test Implication 2 of the model I need a state-level measure of human capital. I focus on state-level average math and writing test scores for 4th and 8th graders on the National Assessment of Education Progress

(NAEP).[28] The math test scores refer to 2003 while the writing ones to 2002. Notice that I focus on measures of human capital of individuals who are not yet part of the workforce in order to avoid capturing the straightforward positive association between a state's workforce average years of schooling and its average income (as would be implied by a Mincer equation, for example).

Table 7 reports cross-sectional rank correlations between NAEP average test scores and states' average income and real expenditures per student. All

	NAEP Mathematics (2003)		NAEP Writing (2002)	
	4th grade	8th grade	4th grade	8th grade
income per student	0.36^{***}	0.32^{**}	0.62^{***}	0.47^{***}
real education expenditures per student	0.40^{***}	0.42^{***}	0.32^{**}	0.32^{**}
number of observations	51	51	44	42

Table 7: Cross-sectional rank correlations of state-level NAEP test scores and income and real education expenditures per student. ** significant at 5% level; *** significant at 1% level. Note that the writing test data are not available for all states.

correlations are positive and statistically different from zero at conventional levels, supporting the second implication of the model.

6 Welfare Effects of Federal Redistribution

In this section I use the model to evaluate the welfare effects of redistribution of education expenditures. In Section 6.1, I abstract from cross-state heterogeneity in productivity in order to connect the analysis to the existing literature (i.e., Bénabou (2002) and Fernandez and Rogerson (1997)). Then, in Section 6.2, I consider the full model and evaluate the human capital and

[28] See Appendix C.2 for a description of these data.

welfare effects of varying Federal redistribution, as indexed by the parameter τ^f. The results associated with Federal redistribution represent the novel contribution of this paper.

6.1 Redistribution in the Economy with Homogeneous Locations

In order to relate my analysis to the rest of the literature (i.e. Bénabou (2002) and Fernandez and Rogerson (1997)), I first consider the one-location version of the model. This is achieved by eliminating differences in productivity across states, i.e. imposing $\sigma_A^2 = 0$. In this case, the distinction between state and Federal redistribution is irrelevant, so denote $\tau^s + \tau^f$ by τ^*. I evaluate the impact of redistribution on the economy's average log human capital and a measure of welfare.[29] The welfare measure is the proportional increase in an agent's consumption in the economy without redistribution ($\tau^* = 0$) that is needed to yield the same expected utility as in an economy with a given positive level of redistribution ($\tau^* > 0$).[30]

The following proposition summarizes the results of the version of the model with homogeneous states.

Proposition 4 [Within-state redistribution with homogeneous states]. Suppose that $\sigma_A^2 = 0$. Then, if the elasticity of labor supply $1/\eta$ is not too large, average log human capital and welfare are maximized at some positive level of redistribution $\tau^* \in (0, 1)$.

Proof. See technical appendix:
`http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

Higher levels of redistribution of education expenditures, as indexed by τ^*, have opposite effects on human capital. On the one hand, as τ^* grows labor supply l^* and income decline, and so does average human capital. On the other hand, redistribution can increase average human capital by shifting ex-

[29] I focus on average log human capital instead of average human capital because the former is the relevant variable to compute expected utility and the benchmark welfare measure.

[30] Appendix B.1 provides a formal definition of this welfare measure.

penditures away from richer households with lower marginal returns towards poorer ones with higher returns. Thus, as long as the elasticity of labor supply is not too high, average human capital and income may be larger in the economy with some redistribution than in the economy without any redistribution. This is a point made by Bénabou (2002) and others.

Differently from these authors, in what follows I focus on a different policy experiment, namely varying τ^f. This requires considering the version of the economy with heterogeneous locations.

6.2 Federal Redistribution in the Economy with Heterogeneous Locations

Consider now the economy with heterogeneous locations, i.e., $\sigma_A^2 > 0$. The policy experiment I undertake is to exogenously modify the extent of Federal redistribution, as indexed by τ^f, and evaluate its effect on average log human capital and welfare. Notice that, as τ^f varies, the sum of τ^f and τ^{s*}, denoted by τ^*, stays constant because of the offsetting actions of state governments (equation 14 in Proposition 1). The welfare measure is the proportional increase in an agent's consumption in the economy without Federal redistribution ($\tau^f = 0$) that is needed to yield the same expected utility as in an economy with a given positive level of Federal redistribution ($\tau^f > 0$).[31] The following proposition summarizes the results of the version of the model with heterogeneous states.

Proposition 5 [Federal redistribution with heterogeneous states]. Consider the case $\sigma_A^2 > 0$. Then, if λ is sufficiently large, average log human capital and welfare are maximized at strictly positive, although generally different, levels $\tau^f > 0$ of Federal redistribution.

Proof: See technical appendix:
`http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

The intuition behind this result is as follows. Federal redistribution gives rise to a number of opposing forces whose net impact on human capital and

[31] Appendix B.2 provides a formal definition of this welfare measure.

welfare is, in principle, ambiguous. Consider the effects on human capital. As τ^f increases there are two effects on human capital. First, the between-state component of inequality in education expenditures declines (Proposition 2). This redistribution from high to low spending states tends to increase the simple average of log human capital across states due to diminishing returns to education expenditures in the production function of new human capital. This effect is the between-state analog of the one discussed in Section 6.1 with reference to redistribution from high to low spending *households* within a state. The second effect of Federal redistribution on average human capital is due to the redistribution of population away from high productivity locations and towards low productivity ones. This effect is unambiguously negative. As low productivity locations are characterized by lower average human capital than high productivity ones, redistribution of population towards the former tends to reduce the overall average. This second effect disappears as the parameter λ increases. Thus, Federal redistribution of education expenditures leads to higher average (log) human capital if the elasticity of population to wages across states is not too large.[32]

Consider now the impact of Federal redistribution on welfare. Since average (log) human capital is an important component of expected utility in the model, the previous discussion of the effects of Federal redistribution on human capital applies here as well. The effect of Federal redistribution on welfare depends on two additional factors as well. First, Federal redistribution distorts the allocation of labor towards less productive states and through this channel it tends to reduce aggregate productivity, output, and ultimately welfare. Second, recall that agents' utility depends directly (and negatively) on a location's population density. Federal redistribution reduces population density in high productivity locations and increases it in low productivity ones. This flattening of the productivity-density gradient results in a welfare gain because the relationship between utility and population density in equation (1) is convex. It is straightforward to show that the last two effects (the one

[32] Recall that as $\lambda \to \infty$, the elasticity ϑ of population to wages tends to zero and all locations are populated by a measure one of agents.

associated with the productivity distortion and the direct effect of density) disappear as the disutility associated with higher density increases (i.e., λ becomes larger) because in this case a location's population becomes independent of its productivity and density is equalized across states.

In summary, if the elasticity of population to wages is not too large (i.e., λ is relatively large so that $\vartheta \rightarrow 0$), welfare is higher in the economy with some Federal redistribution ($\tau^f > 0$) than in the economy without ($\tau^f = 0$). A similar result holds for average (log) human capital.

The welfare measure employed in Proposition 5 is based on a comparison of the expected utilities of living in two different economies, one with $\tau^f = 0$ and the other with $\tau^f > 0$. As discussed in Bénabou (2002), a comparison of expected utilities places a positive welfare value on pure redistribution of consumption across agents, even when average consumption in the economy may not change. In order to show that the welfare gains associated with Federal redistribution are not driven by pure redistribution of consumption, I follow Bénabou (2002) and consider an alternative welfare measure. The latter is defined as the proportional gain in consumption in the economy with $\tau^f = 0$ that makes the utility of living in such economy, evaluated at the *average value* of its arguments, equal to its counterpart in the economy with $\tau^f > 0$.[33] Notice that, while the measure in Proposition 5 equalizes the expectation of two utility functions, the alternative proposed here equalizes the utilities of the expectations of their arguments. This alternative welfare measure records a gain only if Federal redistribution induces an increase in average consumption, or an increase in average human capital, or a reduction in average population density. Pure redistribution of consumption from one set of agents to another does not, per se, lead to welfare gains. The welfare effect of Federal redistribution, according to this alternative measure, is summarized in Proposition 6.

Proposition 6 [Alternative welfare measure]. Consider the case $\sigma_A^2 > 0$. Then, if λ is sufficiently large and returns to scale in human capital accu-

[33] See Appendix B.3 for a formal definition of this alternative measure.

mulation satisfy the following condition:

$$\alpha + 3\beta < 1, \tag{16}$$

the alternative welfare measure is maximized at a strictly positive level $\tau^f > 0$ of Federal redistribution.

Proof: See technical appendix:
`http://www.pitt.edu/~coen/research/technical_appendix_federal.pdf`.

Relative to Proposition 5, the conditions to obtain a welfare gain from Federal redistribution become more stringent. There are no new economic effects relative to those discussed in relation to Proposition 5, except that their analytical expressions are now different because they measure the log of an average rather than the average of a log. The condition in equation (16) guarantees that Federal redistribution of education expenditures increases average human capital, rather than average *log* human capital. Redistribution is more likely to increase average human capital when accumulation quickly runs into diminishing returns.[34]

7 Quantitative Assessment

In order to establish the welfare-maximizing degree of Federal redistribution and to further illustrate the effects discussed in the previous section I now turn to a numerical version of the model.

In order to compute the welfare effects I need to assign values to the following parameters $\left(\beta, \alpha, \rho, \lambda, p, \sigma_A, \tau^f, \sigma_\xi\right)$.[35] I follow Bénabou (2002), and set the human capital production function parameters to $\alpha = 0.35$ and $\beta = 0.25$. Given these two parameters, the other ones can be calibrated in separate

[34] It is worthwhile to point out that condition (16) is only sufficient and not necessary for a welfare gain to occur. In fact in the calibrated version of the model discussed in Section 7, condition (16) fails to hold and yet the alternative welfare measure registers a gain for a strictly positive $\tau^f > 0$ (see Figure 3).

[35] The welfare effects of Federal redistribution are independent of the elasticity of labor supply η.

blocks. First, the weight on consumption ρ is set to 0.82 to match the share of K-12 education in aggregate consumption in the year 2000. Second, to pin down p, σ_A, and τ^f, I target the following three moments. The first one is the standard deviation of the log of per capita GDP across U.S. states in 2002. The second one is the aggregate Theil index of real education expenditures for 2002. Third, I impose that half of the inequality in real expenditures is within-states (see Section 2.1). The resulting three parameters are $p = 0.56$, $\sigma_A = 0.18$, and $\tau^f = 0.38$. The standard deviation of idiosyncratic shocks σ_ξ is set to match the average (across states) within-state standard deviation of log income (Δ^* in the model) in the 2000 Census. The implied value of σ_ξ is 0.95.[36] Last, the population mobility parameter λ is set to obtain an elasticity of population to wages (ϑ in the model) equal to 0.5, as estimated by Kennan and Walker (2011). Given the other parameters, this procedure yields a value of $\lambda = 1.71$. Table 8 summarizes the calibrated parameters and the targeted moments.

Parameter	Value	Moment	Value
α	0.35	n.a. (parameter set a-priori)	n.a.
β	0.25	n.a. (parameter set a-priori)	n.a.
ρ	0.82	K-12 exp share in consumption (1)	0.051
p	0.56	Theil index of education spending (2)	0.023
σ_A	0.18	std dev of log per capita income (3)	0.243
τ^f	0.38	fraction of spending inequality between states (4)	0.500
λ	1.71	elasticity of population to wage difference (5)	0.500
σ_ξ	0.95	average std. dev. of log income within states (6)	1.030

Table 8: The model's parameters and the moments targeted in the calibration. Data sources: moment (1) is from the Economic Report of the President (2012); (2) is from Table 2; (3) is from the BEA Regional Economic Accounts; (4) is from Table 2; (5) is from Kennan and Walker (2011); (6) is from the 2000 Census.

The calibrated model is used to analyze the consequences of varying τ^f away from its calibrated value of 0.38. Notice that, as τ^f varies, τ^* stays constant (see Proposition 1).

Figure 2 traces out the benchmark measure of welfare (see Proposition 5)

[36] The calibration of σ_ξ, p and τ^f implies a value of τ^{s*} equal to 0.47.

as a function of τ^f in the range $[0, \tau^*]$.

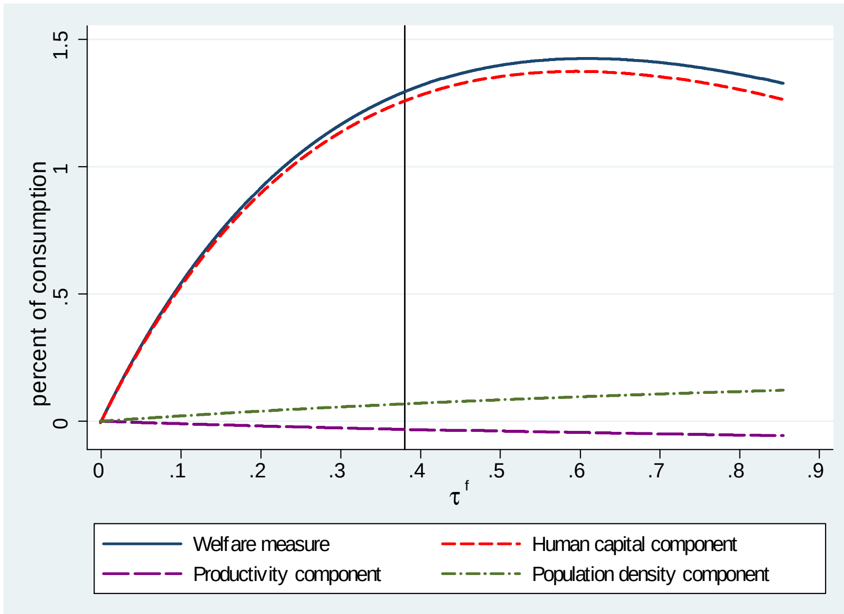

Figure 2: Welfare effect of Federal redistribution and its components. Vertical line marks benchmark $\tau^f = 0.38$.

The vertical line marks the calibrated value of $\tau^f = 0.38$. The welfare gain associated with this policy relative to $\tau^f = 0$ is about 1.3 percent of consumption. The peak welfare gain is reached at τ^f close to 0.61, but the additional welfare gain relative to the benchmark economy is relatively small: 1.4 rather than 1.3 percent of consumption.

The figure also represents the three additive components of the welfare gain discussed in Section 6.2. The first component is the change in average (log) human capital; the second component is the effect associated with the misallocation of population; the third component is the effect associated with the impact of population density on utility. The largest (in absolute value) of these three components is the one associated with the change in average log human capital induced by Federal redistribution. The second and third are quite small compared to the first.

Figure 3 compares the benchmark welfare measure with the alternative welfare measure discussed in Section 6.2 (see Proposition 6). The adoption of the alternative welfare measure reduces the optimal τ^f to about 0.55 and the welfare gain to 0.9 percent of consumption relative to the case $\tau^f = 0$. Thus, the quantitative results are not overly sensitive to the specific welfare index in use.

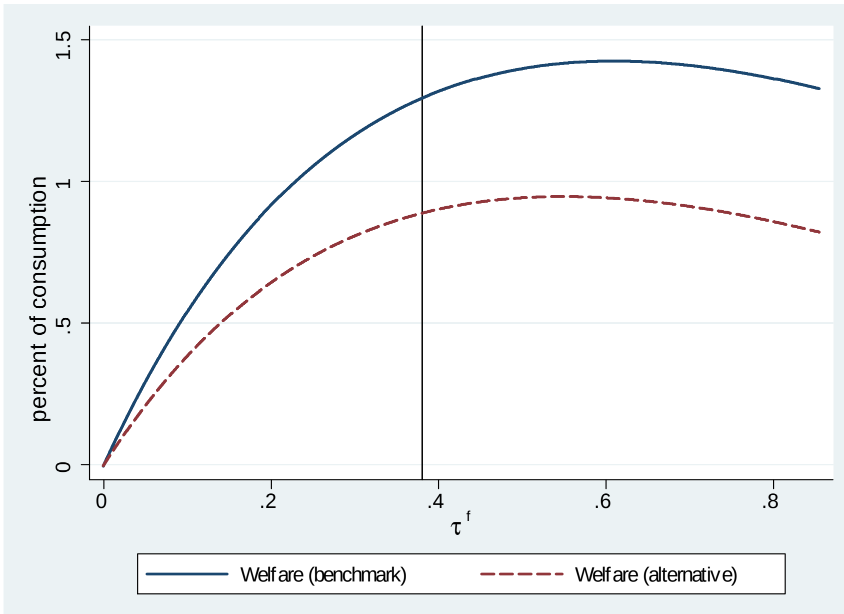

Figure 3: Benchmark (solid line) and alternative (dash line) welfare measures. The vertical line marks the benchmark $\tau^f = 0.38$.

Table 9 summarizes the main quantitative results of this section and performs sensitivity analysis relative to the utility parameter λ, shown in the first column of the table. The latter plays a central role in the analysis because it determines the elasticity of population to wages (second column of Table 9) across labor markets.[37] Recall that in the benchmark calibration $\lambda = 1.71$.[38] The third column reports the welfare gain of setting $\tau^f = 0.38$ (the benchmark value of $\tau^f = 0.38$) relative to $\tau^f = 0$. The fourth and fifth columns of the table show the welfare-maximizing level of Federal redistribution τ^f and the

[37] For example, Albouy (2012) calibrates a somewhat higher elasticity of population to wages than in my benchmark.

[38] Notice that the parameter λ can be recalibrated to match a higher elasticity of population to wages without changing any of the other parameters.

associated welfare gain (again relative to the case $\tau^f = 0$). Columns six and seven of Table 9 present the implications of the welfare-maximizing level of redistribution for the overall Theil index of inequality in expenditures and the between-state component of inequality.

Consider the benchmark calibration ($\lambda = 1.71$) of the model first. The welfare benefits of Federal redistribution ($\tau^f = 0.38$) are quantitatively very similar to the ones obtained if the population was completely immobile ($\lambda \to \infty$). There are no large additional gains from the welfare-maximizing level of Federal redistribution relative to the benchmark $\tau^f = 0.38$. However, welfare-maximizing Federal redistribution leads to a reduction in the Theil index of inequality in education expenditures by one-third relative to the benchmark economy. The associated between-state component of expenditures inequality is now only about 23 percent of the total, against 50 percent in the benchmark calibration of the model.

As the elasticity of population to wages increases relative to the benchmark (i.e., the parameter λ decreases), states with higher productivity tend to absorb more population in equilibrium. Hence, the net welfare benefit of Federal redistribution and the optimal τ^f decline. However, notice how the benchmark calibrated value of $\tau^f = 0.38$ would still be approximately welfare-maximizing for an elasticity of population to wages equal to 20, a relatively large number.[39]

To summarize, the quantitative analysis of this section has established the following points. First, for a reasonable parameterization of the model, Federal redistribution of education expenditures leads to a welfare gain of about 1.3 percent of consumption. Moreover, this welfare gain is relatively robust to an alternative welfare criterion that does not place a value in reductions of inequality per se. These two points are the quantitative counterparts of the theoretical analysis of Section 6. Second, there appear to be small additional welfare gains from maximizing the degree of Federal redistribution in the model relative to the benchmark calibration (see Table 9). Last, the parameter λ

[39] It is possible to show that the welfare gains (or losses) from Federal redistribution increase linearly with σ_A^2 while the *optimal* degree of Federal redistribution τ^f is independent of σ_A^2 because the latter scales up proportionately both the benefits and costs of Federal redistribution.

(1) λ	(2) Elasticity population to wage	(3) Welfare gain (%) $\tau^f = 0.38$	(4) Welfare maximizing τ^{f^*}	(5) Welfare gain (%) τ^{f^*}	(6) Implied Theil index	(7) Implied between-state inequality (%)
∞	0	1.31	0.62	1.44	0.015	22
1.71*	1/2	1.29	0.61	1.42	0.015	23
0.86	1	1.28	0.60	1.41	0.015	24
0.09	10	1.05	0.50	1.09	0.018	36
0.04	20	0.80	0.40	0.80	0.021	47
0.02	50	0.04	0.18	0.23	0.035	67

Table 9: Sensitivity analysis of welfare gains from Federal redistribution of education expenditures. Notes. The parameter λ is calibrated to match the elasticity of population to wages in column (2). The symbol * denotes the benchmark calibrated value for λ. Column (3) represents the benchmark measure of welfare (comparing $\tau^f = 0.38$ with $\tau^f = 0$). Columns (4) and (5) report the welfare-maximizing τ^f and the associated welfare measure. Columns (6) and (7) report the Theil index of inequality in expenditures and its between-state component when τ^f takes its welfare-maximizing value.

plays a quantitatively important role in determining the magnitude of the welfare gain from Federal redistribution of education expenditures.

8 Summary and Conclusions

This paper studies the process of human capital accumulation in an heterogeneous agents economy characterized by multiple states within a federation. Previous literature has mostly focused on the implications of school finance reforms - such as those that have occurred in the U.S. in the last 40 years - involving an increase in states' role at the expense of local funding. This paper takes this analysis further by drawing a distinction between state and Federal financing. This focus is motivated by the evidence that at least half, and possibly more, of the differences in education spending per student in the U.S. are between, rather than within, states.

In conclusion and as a summary of the paper's results, it is worthwhile to return to the questions that motivated my analysis. What economic forces sustain heterogeneity in education expenditures across states despite free geographic mobility of labor? In my model, states are assumed to be characterized by different levels of productivity. This heterogeneity is not, in general, sufficient to generate equilibrium dispersion in education expenditures across states. With free labor mobility and endogenous wages, all states would have the same wage and education expenditures in the long-run. I have shown that it is necessary to rely on an additional force - different from equalization of wages - to support an equilibrium with free mobility of labor and heterogeneity in education spending per student across states. In my model such as force is a direct utility cost associated with higher population density. Differences in housing prices represent a natural interpretation of this cost.

What are the benefits and costs associated with Federal - as opposed to state - redistribution of education expenditures? Federal redistribution allows low income states to invest more in human capital than they would otherwise do. Diminishing returns to human capital accumulation suggest that this redistribution can lead to higher average human capital for the overall economy. However, Federal redistribution is also distortionary in that it reduces the incentives to migrate out of states with low productivity towards richer ones.

How large are the welfare gains from Federal redistribution? According to the calibrated model, the overall gains of current levels of Federal redistribution are of the order of one percent of consumption. There are relatively small additional welfare gains from setting Federal redistribution optimally relative to existing levels. The welfare gain is not due to pure redistribution of consumption across agents, but rather to the increase in economy-wide output that such policy entails. The elasticity of population to wages across states is an important parameter in determining the magnitude of the welfare gains.

I conclude the paper offering a number of possible extensions and more general comments. In order to keep the model tractable I have abstracted from some potentially important issues. The first one is competition among states for population and human capital in the politico-economic process that determines the extent of state redistribution. The standard argument is that aggressive redistribution leads to a race to the bottom among localities, providing a further motivation in favor of a Federal role. Second, while the logarithmic specification of utility keeps the model tractable, it also implies that individuals spend a constant fraction of income on education, independently of the degree of redistribution. Empirical evidence (Bergstrom et al (1982)) shows that the demand for education services is relatively inelastic, suggesting that redistributive policies that affect the tax price of education should lead to an increase in effort by richer districts and a decline by poorer ones. The net effect is, in principle, ambiguous and numerical analysis may be used to quantify it. Third, for tractability reasons, the model does not allow for a private schooling option. It appears that Federal redistribution, by shifting education resources towards poorer states, would increase incentives to enroll into private schools in richer states and reduce them in poorer ones. Last, an important area of future research is to complement the analysis of the effects of specific Federal policies with research on the *design* of Federal policies with desirable properties. I leave these extensions to future research.

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A Definition of some variables in Proposition 1

In this section I report the expression for the endogenous variables $(\vartheta, \widetilde{m}, \widetilde{y}^f)$ that appear in Proposition 1. The first one has the following form:

$$\vartheta \equiv \frac{\rho}{\lambda} + \frac{(1-\rho)\beta\left(1-\tau^f\right)}{\lambda}\left(1 + \frac{\beta\tau^{s*}}{1-\alpha-\beta\left(1-\tau^f\right)}\right). \tag{A1}$$

The second one:

$$\begin{aligned} \widetilde{m} \quad \equiv \quad & \frac{1}{1-\alpha-\beta}\left(\beta \ln \frac{(1-\rho)\,\beta l^*}{\rho+(1-\rho)\,\beta} - \frac{\sigma_\xi^2}{2}\right) + \frac{\beta}{1-\alpha-\beta}\frac{\sigma_A^2}{2}Q \\ & + \frac{\beta}{1-\alpha-\beta}\frac{(\Delta^*)^2}{2}\left(1-\left(1-\tau^{s*}-\tau^f\right)^2\right), \end{aligned} \tag{A2}$$

where the expression for Q is:

$$Q = \frac{(1-\alpha)\,\tau^f}{1-\alpha-\beta\left(1-\tau^f\right)}\left[2\vartheta + \frac{\left(1-\tau^f\right)(\beta+1-\alpha)}{1-\alpha-\beta\left(1-\tau^f\right)}\right], \tag{A3}$$

and where ϑ is defined in equation (A1). Notice that when $\tau^f = 0$, $Q = 0$, and when $\tau^f > 0$ we have $Q > 0$. The last one is:

$$\widetilde{y}^f = l^* \exp\left\{(\Delta^*)^2\left(1-\frac{\tau^f}{2}\right) + \widetilde{m} + \frac{\sigma_A^2}{2}\frac{Q}{\tau^f}\right\}.$$

B Welfare Measures

In what follows I define the welfare measures used in Propositions 4, 5, and 6.

B.1 Proposition 4: Economy with Homogeneous Locations

The welfare measure for the economy with homogeneous locations ($\sigma_A^2 = 0$) is defined as the proportional increase in consumption $\widetilde{T}$ in the economy with

$\tau^* = 0$ that makes an agent indifferent, in expectation, between this economy and one with $\tau^* > 0$. Formally, $\widetilde{T}$ satisfies the following equation:

$$\begin{aligned} & \rho E\left[\ln c; \tau^*, \sigma_A^2 = 0\right] + (1-\rho) E\left[\ln h; \tau^*, \sigma_A^2 = 0\right] \\ = \;& \rho E\left[\ln c\left(1+\widetilde{T}\right); \tau^* = 0, \sigma_A^2 = 0\right] + (1-\rho) E\left[\ln h; \tau^* = 0, \sigma_A^2 = 0\right], \end{aligned}$$

where I have taken into account that in this economy $n = 1$ in all locations. After the appropriate substitutions I obtain:

$$\ln\left(1+\widetilde{T}(\tau^*)\right) = \frac{1}{\rho}\frac{1-\alpha}{1-\alpha-\beta}\ln\frac{l^*(\tau^*)}{l^*(0)} + \frac{\beta}{1-\alpha-\beta}\frac{\sigma_\xi^2}{2}\frac{1-(1-\tau^*)^2}{1-(\alpha+\beta(1-\tau^*))^2},$$

where the ratio of labor supplies in the two economies is:

$$\ln\frac{l^*(\tau^*)}{l^*(0)} = \frac{1}{\eta}\ln\left(\frac{\rho+(1-\rho)\beta(1-\tau^*)}{\rho+(1-\rho)\beta}\right).$$

B.2 Proposition 5: Economy with Heterogeneous Locations

The welfare measure for the economy with heterogeneous locations, T, is formally defined as the proportional increase in consumption T such that expected utility is the same in the economy with $\tau^f > 0$ and in the economy with $\tau^f = 0$:

$$\begin{aligned} & \rho E\left[\ln c; \tau^f\right] + (1-\rho) E\left[\ln h; \tau^f\right] - \lambda E\left[\ln n; \tau^f\right] \\ = \;& \rho E\left[\ln c(1+T); \tau^f = 0\right] + (1-\rho) E\left[\ln h; \tau^f = 0\right] - \lambda E\left[\ln n; \tau^f = 0\right], \end{aligned}$$

where $E\left[\ln c; \tau^f\right]$ is the average log consumption in the economy with $\tau^f > 0$ and similarly for the other terms. Taking into account that:

$$E\left[\ln c; \tau^f\right] = E\left[\ln A; \tau^f\right] + E\left[\ln h; \tau^f\right] + l^*,$$

it follows that T takes the following form:

$$\ln(1+T) = \frac{1}{\rho}\left(E\left[\ln h;\tau^f\right] - E\left[\ln h;\tau^f=0\right]\right) + \\ + E\left[\ln A;\tau^f\right] - E\left[\ln A;\tau^f=0\right] - \frac{\lambda}{\rho}\left(E\left[\ln n;\tau^f\right] - E\left[\ln n;\tau^f=0\right]\right),$$

where an expression for each term is derived as a function of the structural parameters of the model in the Technical Appendix.

B.3 Proposition 6: Alternative Welfare Measure

The alternative welfare measure $\widehat{T}$ is formally defined as the proportional increase $\widehat{T}$ in average consumption such that the utility of average consumption, human capital and density is the same in the economy with $\tau^f > 0$ and in the economy with $\tau^f = 0$:

$$\begin{aligned} & \rho \ln E\left[c;\tau^f\right] + (1-\rho)\ln E\left[h;\tau^f\right] - \lambda \ln E\left[n;\tau^f\right] \\ = \; & \rho \ln E\left[c\left(1+\widehat{T}\right);\tau^f=0\right] + (1-\rho)\ln E\left[h;\tau^f=0\right] - \lambda \ln E\left[n;\tau^f=0\right] \end{aligned}$$

where $E\left[c;\tau^f\right]$ is the average consumption in the economy with Federal redistribution τ^f and similarly for the other variables. It follows that $\widehat{T}$ takes the following form:

$$\begin{aligned} \ln\left(1+\widehat{T}\right) = \; & \frac{1}{\rho}\left(\ln E\left[h;\tau^f\right] - \ln E\left[h;\tau^f=0\right]\right) + \\ & + \ln E\left[A;\tau^f\right] - \ln E\left[A;\tau^f=0\right] - \frac{\lambda}{\rho}\left(\ln E\left[n;\tau^f\right] - \ln E\left[n;\tau^f=0\right]\right), \end{aligned}$$

where an expression for each term is derived as a function of the structural parameters of the model in the Technical Appendix.

B.4 Version of the Model with Housing

In the version of the model with housing (see e.g. Roback (1982)) the utility function is

$$U = \widehat{\rho}\left(\ln c + \phi \ln x\right) - \frac{l^{\eta}}{\eta} + (1 - \widehat{\rho}) \ln h' \tag{A4}$$

where $\widehat{\rho} \in (0, 1)$, $\phi > 0$, $\eta > 1$, and housing consumption is denoted by x. The budget constraint of an agent is:

$$y_{ij} = c_{ij} + p_j x_{ij} + z_{ij}$$

where p_j is the rental price of housing in state j. The optimal demand for housing services by household i is given by:

$$x_{ij} = \frac{\phi}{p_j} c_{ij}. \tag{A5}$$

Housing services in state j are produced by combining land, whose supply is normalized to one in each location, and units of the homogeneous final good. Let the production function for housing services in location j be given by:

$$X_j = K_j^{\psi}$$

with $\psi < 1$. The representative firm supplying housing services chooses K to maximize profits:

$$\max_K \left\{p_j K^{\psi} - K\right\}$$

leading to the first-order condition:

$$\psi p_j K_j^{\psi - 1} = 1.$$

Taking into account the definition of X_j this leads to the following inverse supply function for housing services:

$$p_j = \frac{1}{\psi} X_j^{1/\psi - 1}. \tag{A6}$$

Equilibrium in the housing market requires that the supply of housing equals the demand for it. Replacing the latter equation into equation (A6) and solving for p_j yields the following expression for the price of housing as a function of population n_j and average income $\overline{y}_j$ in state j:

$$p_j = \frac{1}{\psi^{\psi}} \left(\frac{\phi \widehat{\rho}}{\widehat{\rho}\,(1+\phi) + (1-\widehat{\rho})\,\beta} n_j \overline{y}_j \right)^{1-\psi}, \tag{A7}$$

It is then simple to show that, replacing equations (A5) and (A7) in the term $\ln x$ in equation (A4), yields an indirect utility function that depends negatively on $\ln n_j$. Average income $\overline{y}_j = A_j l \exp\left(m_j + \Delta^2/2\right)$ depends on the same variables that already appear in the indirect utility of the benchmark model (see Technical Appendix, equation TS6). Hence, the indirect utility function of the model with housing depends on the same variables and through the same functional relations (essentially logarithms and summations) as the benchmark indirect utility function.

C Data

The school district level data and the state-level data used in the paper are available at:
`https://sites.google.com/site/danicpiraniweb/Home/publication`. In what follows I provide information on the sample selection criteria applied to the school district data and on the sources of the state-level data.

C.1 School-District Level Data

Table 1 contains statistics and data on current education expenditures from three sources. First, the Theil coefficients for the years 1972–1992 are taken from Table 2 in Murray et al. (1998). In turn, these indices are based on a sample of about 10,000 unified regular operating school districts from 46 states (districts from Alaska, Hawaii, DC, Montana and Vermont are not in their sample). They drop from their sample districts that, in a given year, either exceed 150 percent of expenditures of the district in the 95th percentile of the expenditures distribution, or that fall short of 50 percent of expenditures of the district in the 5th percentile of the same distribution. The current education expenditures data for school districts are taken from the Census of Governments School System Finance (F-33) File. Corcoran et al. (2003) extend the Murray et al. (1998) data sample and computations of the Gini and Theil indices to the year 1997. The statistics for 1997 in my Table 1 are from Corcoran et al. (2003, Table 1). Finally, I have computed the statistics in my Table 1 for the years 2002, 2007 and 2009 using school district-level current expenditure data. I constructed the sample of school districts using exactly the same criteria as Murray et al. (1998) listed above. Table 2 considers real education expenditures as well as nominal. The columns titled "Real" contain Theil indices for the same sample of school districts as the "Nominal" column. In order to compute real expenditures data, I divide the nominal expenditure data by Taylor (2005)'s comparable wage index, which is available at the school district and state level for a selected number of years. Taylor's data is obtained from `http://nces.ed.gov/edfin/adjustments.asp`.

C.2 State-Level Data Sources

Public school education expenditures and enrollment data are from the National Center for Education Statistics' Digest of Education Statistics:
`http://nces.ed.gov/programs/digest/`

Data on personal income are from the Bureau of Economic Analysis' Regional Economic Accounts: `http://www.bea.gov/regional/index.htm`

Unionization data for public school teachers are from the National Center for Education Statistics' website:
`http://nces.ed.gov/surveys/sass/tables/sass0708_043_t1s.asp`

The states' rankings in the National Assessment of Educational Progress writing and math tests were downloaded from the National Center for Education Statistics' website:
`http://nces.ed.gov/nationsreportcard/naepdata/dataset.aspx`