Chinese studies on the Transformation Problem: A Critical Review*

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Abstract

Since the 1980s, Chinese Marxian scholars present several new interpretations and solutions of the transformation problem, which are still unfamiliar to the Western literature. This paper provides an in-depth summary of four major Chinese transformation studies, followed by critical comments focusing on problems regarding methodology, implication and self-consistency. These studies are also compared with the Western literature in order to better understand their significance and limitation.

Keywords: transformation problem; value and price; labor theory of value; Marxian Economics; China

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1 Introduction

Before the 1980s, there are some introductions of transformation appeared in the Chinese literature (Yan & Ma, 2011: 68), but the first wave of systematic studies did not appear until the 1980s. Zhu (1981), Zhu (1983), and Hu (1983) and Hu et al. (1990) were the representatives of this wave of studies. These studies did not propose systemic interpretations or solutions, but mainly devote to review the debate in the Western literature. Since then, the transformation problem had became one of the most debated issues in the Chinese Marxian economics, and the most crucial issue has been whether Marx's two equalities (total values equal total production prices, and total surplus values equal total profits) hold simultaneously or not.

Till the 2000s, Ding (2005), Bai (2006) and Zhang (2004)[1] emerged as three most influential approaches which provide new interpretations or solutions of the transformation problem. Ding (2005) reinterprets the implication of Marx's "modified significance of the cost price" and distinguish the role of variable capital from the role of constant capital in the production-transformation process. Bai (2006) argues that the root of transformation problem is at the existence of "undividable remainders" after redistributing surplus values among sectors. The magnitude of the remainders are small, and in a market production price system Marx's two equalities can both hold. Zhang (2004) claims to solve the transformation problem by using Marx's two equalities as presumptions of modeling. Recently, Yan & Ma (2011) incorporate several new issues, such as the dynamics of profit rate equalization, commercial and banking sectors, and the monopoly stage of capitalism, into the analysis. The critical review of these four studies is the main body of this paper.

Asides from these four, other studies since the 2000s include: Yue (2002) modifies Ding's models and claims that Marx's two equalities can both hold; Based on the modification of Ding's theory, Zhu (2004) presents static and dynamic models with conclusions as Ding (2005); Lü (2004) in a framework similar with the TSSI approach (Freeman & Carchedi, 1996; Kliman, 2007) shows that Marx's two equalities can hold simultaneously; Feng (2008, 2009, 2010) argues that there is no deviation between values and production prices, and the transformation problem is a false question at the first place; Shen (2008) in a two-departments, simple reproduction scheme argues that Marx's two equalities can hold if considering the realization of surplus values. Due to the limit of space, these studies will not be included in this

[1] Their original papers or book sections all publish before 2000.

review.

In terms of their treatments of Marx's two equalities, which is one of the core issues in the literature, the four studies reviewed in this paper can be distinguished into two classes: solution or interpretation. The solution refers to studies which focus on proving that the two equalities can hold simultaneously within the conditions of Marx's original context, through alternative modeling and postulate setting. The interpretation refers to the studies which admit that Marx's two equalities cannot both hold in general. Instead of searching for the traditional type of solution, they put the question in alternative framework and try to show that Marx's two equalities hold (or not hold) for some good reasons, which are still loyal to the basic principles of the labor theory of value. Among these four studies, only Zhang (2004) belongs to the class of solution, all other three are interpretations.

Although these researchers debate a lot against each others, they still share some common features when comparing with the Western counterpart. First, these studies are deeply influenced by the standard approach represented by Bortkiewicz (1949), Sweezy (1942), Seton (1957) and Morishima (1974; 1978) in terms of modes of thinking and modeling, while not without critiques. In contrast, a variety of new approaches stimulated by the controversies around Steedman (1977), including the New Interpretation (Foley, 1982; Duménil, 1983; Lipietz, 1982), TSSI approach (Freeman & Carchedi, 1996; Kliman, 2007), Organic Composition of Capital (OCC) approach (Fine & Saad-Filho, 2004), the probabilistic approach (Farjoun & Machover, 1983), rethinking Marxism approach (Wolff, Roberts & Callari,1982), and Macro-Monetary approach (Moseley, 2000), and so on, find almost no supporter in China. Although these approaches are known to Chinese scholars, only TSSI approach finds a supporter, Lü (2004), till recently.

Second, as most Marxian economists in the west, none of these authors accepts Samuelson-Steedman's argument that labor theory of value is redundant. Even though some of them admit that Marx's method is incomplete or Marx's two equalities do not hold simultaneously, they still try to show the labor theory of value is valid in alternative ways.

Third, they all insist that the production price is just the form of redistribution of value, and the production price is qualitatively the same as value, or being a form of appearance of the latter. For them, the transformation problem is not about how (production) prices can be deduced from values, but how surplus values are distributed among capitalists through the operation of the profit rate equalization.

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In the remaining sections of this paper, the sequence of discussion is: Ding (2005); Bai (2006); Zhang (2004); Yan & Ma (2011). In each section, the main points of each study will be outlined at the beginning. Then I will firstly summarize the theoretical arguments of each studies, and present corresponding models. After presenting arguments and models, critical comments follow. Since most readers of this review may not be able to read Chinese and check the original studies, the summary parts of this review have to be more detailed than those of the normal literature review. Besides, the comments are strictly separated from the summary of arguments and models, in order to provide readers accesses to these studies as honest as possible.

Comments will focus on the methodology, implication and self-consistency of each study. Core elements of their models will be put under scrutiny, while secondary mathematical proofs and formulations is left to the Appendix. Every Chinese study reviewed here contained detailed discussions of existing Western studies. For the limit of space, however, most of them are omitted in this paper, unless they are necessary to understand the innovations of the Chinese studies.

2 Ding, Baojun (2005)

Compared with many mathematics-intensive studies in the literature, Ding's study concentrates instead on analyzing the concepts in Marx's writings. The core of Ding's study is an interpretation of the meaning of the "modified significance of cost price", arguing that the production process needs to be incorporated and the variable capital will reproduce more value than its PoP. He also argues that, although Marx's two equalities can not both hold in general, they can be said to hold in a modified, inter-temporal sense.

2.1 Arguments

2.1.1 Different meanings of the transformation of inputs and outputs

Ding argues that Marx's value transformation process is a process in which produced surplus values are redistributed among sectors, under the condition of profit rate equalization and the formation of prices of production (PoP). Therefore Marx focuses on the value and the production price of output primarily. If inputs were to be incorporated, as some critiques suggest for the need of transforming input values, the process under analysis have to be extended to the value production process, rather than the value transformation process only (Ding, 2005: 99).

In this two processes framework, at the beginning of the new round of production process, the input value has been transformed into PoP, therefore containing value-PoP deviation within it, and the production cost is calculated in terms of PoP. However, because the production process is operated in terms of value, it is still the amount of value contained in the input and the value created by the living labor that will be transferred into the value of new outputs during the new round of production process, rather than the amount of PoP. The production process will still run in terms of value, and then go through the transformation process at the end of the new round of production process.

As Marx (1981: 265) puts it:

> As the price of production of a commodity can diverge from its value, so the cost price of a commodity, in which the price of production of other commodities is involved, can also, stand above or below the portion of its total value that is formed by the value of the means of production going into it. It is necessary to bear in mind this modified significance of the cost price, and therefore to bear in mind too that if the cost price of a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong.

It is interesting to note that, while some critiques read this passage as an evidence of the incompleteness of Marx's transformation process, Ding reads it as a methodological prescription which implies that after transforming the input values into production prices, the deviation between the value and the production price remains inherent in the input commodities, and it is the values that will go into the production process, rather than the production prices.

Based on this reading, Ding sharply criticizes Steedman (1977). Steedman dismisses Marx's transformation method for issuing different exchange values to the outputs used for sale and the same commodities used as purchased inputs.[2] For Ding, Steedman's argument is erroneous for neglecting the value production process

[2] According to Steedman (1977: 43-44), "...while Marx transformed the values of outputs into

and confusing the different meanings of the input PoP and the output PoP. The deviation between output values and output production prices is the result of this round of transformation, while the deviation between input values and input production prices is the result of previous round of transformation. These two deviations and therefore the corresponding production prices will be determined by the rates of profit in two different periods, and generally will not be identical (Ding, 2005: 100). Even if we consider two sequential production processes, in which the output of the first production process, through the transformation process, is purchased as the input of the second production process immediately, the value-PoP deviation in the input remains, and it is still the values of input that counts in the second production process, rather than the production prices.

2.1.2 The difference between the constant capital and the variable capital in value transferring

Closely related with his first argument, Ding further distinguishes the variable capital from the constant capital in the input. Ding points out that, although the variable capital, i.e. the labor power, contains its value-PoP deviation as the constant capital does, the variable capital will always contribute more, or at least the same, amount of value equivalent with its production price into the new product (Ding, 2005: 47). This comes directly from the dual character of labor, and the basic requirement of the capitalist exploitation (Ding, 2005: 59-61).

According to Ding, Marx already provided this methodological prescription in *Theories of Surplus Value*:

> It is clear that what applies to the difference between the cost-price and the value of the commodity as such - as a result of the production process - likewise applies to the commodity insofar as, in the form of constant capital, it becomes an ingredient, a pre-condition, of the production process. Variable capital, whatever difference between value and cost-price it may contain, is replaced by a certain quantity of labour which forms a constituent part of the value of the new commodity, irrespective of whether its price expresses its value correctly or stands above

prices of production he did not so transform either the value of iron used as inputs, or the value of corn advanced as wages. Thus both iron and corn appear to have different exchange values when sold as output from when they are purchased as inputs; but this is nonsensical since sale and purchase are two aspects of the same transaction."

or below the value (Marx & Engels, 1975: 352).

Combining the first and second arguments, the meaning of Marx's "modified significance of the cost price" is reconstructed. For Ding, it is true that after the transformation process the input cost is measured by its price of production. But one part of the input, the constant capital, can transfer only its value into the new product, while the other part, the variable capital, can reproduce the amount of value equal to its price of production, and also creates the surplus value.

2.1.3 The simple reproduction scheme is irrelevant for the transformation problem

Ding (2005: 55-56) argues out that the reproduction scheme, as adopted by Bortkiewicz (1949) and many others, and the n-sectors input-output system after Seton (1957), explicitly or implicitly presumes the balanced reproduction. However, similar with Winternitz (1948) and Meek (1973), Ding argues that the balanced reproduction concerns mainly the realization problem, or the proportions between departments, while the transformation concerns about the redistribution of surplus values among capitals. Besides, in *Capital*, Marx clearly assume full realization when discussing the transformation problem in a multi-sector framework. Therefore, the (simple) reproduction scheme has nothing to do with the transformation problem and should be got rid of.

2.1.4 Choosing the equation of the general rate of profit as the "invariance postulates".

Ding argues that many studies chooses "invariance postulates" in an arbitrary fashion, or simply to add an condition to solve the system of equations, such as Bortkiewicz's "$Z = 1$". By choosing the equation of the general rate of profit as the "invariance postulates", Ding emphasizes that, for Marx, the formation of the general rate of profit, based on the rate of surplus, plays a crucial role in the transformation process. It is by the determination of the general rate of profit that the surplus value transforms to general profit, and the value transforms to the price of production accordingly.

2.2 Models & comments

Based on the above arguments, Ding constructs his models in two steps. first, he expresses Marx own model in a general form. Second, he expands the model to incorporate the transformation of inputs, including the constant capital and the variable capital. While he shows that in Marx's own model the two equalities can both hold, in his expanded model the two equalities can both hold only in a "modified way", corresponding to the "modified significance of the cost price".

2.2.1 Marx's transformation model

According to Ding (2005: 41), Marx's own transformation model can be expressed as follows:

$$\begin{cases} \rho_1(c_1+v_1+s_1)=(1+r)(c_1+v_1) \\ \rho_2(c_2+v_2+s_2)=(1+r)(c_2+v_2) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \rho_n(c_n+v_n+s_n)=(1+r)(c_n+v_n) \\ r=\dfrac{\sum_{i=1}^{n} s_i}{\sum_{i=1}^{n}(c_i+v_i)} \end{cases} \tag{1}$$

where c_i, v_i, s_i and ρ_i denote, respectively, the value of constant capital, the value of variable capital, the surplus value, and the production price-value parameter in the ith sector. r is the (value) rate of profit.

In this model, c_i, v_i and s_i are known, $\rho_1, \rho_2, ..., \rho_n$ and r are unknown, forming an equation system consisted of n+1 equations with n+1 unknow, and therefore contained of an unique set of solution. By this unique set of solution, values can be transformed into prices of production, with the properties that total profit equal total surplus value, and total prices of production equal total values.

2.2.2 Ding's model: a numerical example and the linear equations system

To complete Marx's transformation process, Ding then transforms values of constant capital and variable capital into prices of production. The results are shown firstly in a numerical example consisted of five sectors (Ding, 2005: 48).

In this numerical example, the first interesting result would be the difference between column (1) and (5). Column (1) represents the total capitals (constant and variable) measured at the prices of production. Column (5) is the value contained in the constant capital in column (1), which then transfers into the value of the new

	(1) Capital measured at the Price of Production	(2) Rate of Surplus Value (%)	(3) Surplus Value, s	(4) Profit Rate, r (%)	(5) Value Transferred from Constant Capital	(6) Reproduced Variable Capital	(7) Output Value (3)+(5) +(6)	(8) Average Profit Rate (%)	(9) Average Profit (8)×(1)	(10) Prices of Production (1)+(9)	(11) Deviation between Prices of Production and Value (10)−(7)
I	80c + 20v	100	20	20	70	20	110	22	22	122	+12
II	70c + 30v	100	30	30	62	30	122	22	22	122	0
III	60c + 40v	100	40	40	54	40	134	22	22	122	-12
IV	85c + 15v	100	15	15	81	15	111	22	22	122	+11
V	95c + 5v	100	5	5	93	5	103	22	22	122	+19
	Total		110		360	110	580		110	610	+30

Table 1: Ding's numerical example (Ding, 2005: 48)

product. As indicated by Ding's first argument above, the inputs in the current period receives their priced of production through the transformation process at the end of last production process, at which capitalists made the purchase of input goods. But the inputs still contain values, which are different from their prices of production[3]. These values, rather than the prices of production, then transfer into new outputs in the current period. In this case, the value of output is calculated by summing up column (3), (5) and (6), rather than the sum of column (1) and (3), as we saw in the Standard Interpretation tradition.

As for the variable capital in column (1), its value (i.e. the value of labor power) might deviate from its price of production as well. However, there is no effect of this deviation on the transformation process. As suggested by Ding's second argument, capitalists will always make workers produce more than (or at least equal as) the production prices of labor power, therefore reproducing the same amount of production price of variable capital. In this way, column (6) equals the production prices of the variable capital in column (1).

After transforming the values of constant capital and variable capital, the value of output, column (7), is given by adding up column (3), (5) and (6). Next, based on the process of profit rate equalization, Ding calculates the prices of production (column (10)) and its deviation from the values (column (11)).

The reader would note immediately that, while the total surplus value equals total profit (column (3) and (9)), the total value does not equal the total price of

[3] In this example, Ding simply assumes that the production prices of constant capitals in column (1) are all higher than their values in column (5).

production (column (7) and (10)). Is this an evidence of flaw of the labor theory of value? Ding argues that it's not, and there are some solid reasons behind the result. He exams this question and elaborates his reasoning further in a general form of equations system as follows.

Based on the equation system (1), Ding further add α_i and β_i to denote the PoP-value transformation parameters, or deviation rates, for the constant capital and the variable capital respectively (Ding, 2005: 51).

$$\begin{cases} \rho_1(c_1+\beta_1 v_1+s_1)=(1+r)(\alpha_1 c_1+\beta_1 v_1) \\ \rho_2(c_2+\beta_2 v_2+s_2)=(1+r)(\alpha_2 c_2+\beta_2 v_2) \\ \dots\dots\dots\dots\dots\dots \\ \rho_n(c_n+\beta_n v_n+s_n)=(1+r)(\alpha_n c_n+\beta_n v_n) \\ r=\frac{\sum_{i=1}^{n} s_i}{\sum_{i=1}^{n}(\alpha_i c_i+\beta_i v_i)} \end{cases} \tag{2}$$

In this system, c_i, v_i, s_i, α_i and β_i are known, ρ_1, ρ_2,...,ρ_n and r are unknown. There are n+1 equations and n+1 unknowns, which is able to determine an unique set of solution to transform values into prices of production.

The right hand side the first nth equations are measured in PoP terms. On the left hand sides, corresponding to Ding's second argument, the value composition of the inputs is $(c_n+\beta_n v_n+s_n)$, rather than $(c_n+v_n+s_n)$. $\beta_n v_n$ here is the reproduced value, rather than PoP, by the variable capital, and ρ_n plays the role of transforming from value to PoP for the whole terms in the bracket. Besides, corresponding to his fourth argument, the transformation parameter ρ's can be solved only through the last equation, Marx's formula of the general rate of profit, which is exactly the theoretical reason and mediation of the transformation process.

After incorporating the transformations of constant and variable capital into the model, the equality between the total profit and the total surplus value can be shown by multiplying r with $\sum_{i=1}^{u}(\alpha_i c_i+\beta_i v_i)$:

$$r\sum_{i=1}^{n}(\alpha_i c_i+\beta_i v_i)=\sum_{i=1}^{n} s_i \tag{3}$$

And the difference between total prices of production and the total values can be shown as:

$$
\begin{aligned}
&(1+r)\sum_{i=1}^{n}(\alpha_i c_i + \beta_i v_i) - \sum_{i=1}^{n}(c_i + \beta_i v_i + s_i) \\
&= \sum_{i=1}^{n}(\alpha_i c_i + \beta_i v_i) + \sum_{i=1}^{n} s_i - \sum_{i=1}^{n}(c_i + \beta_i v_i + s_i) \\
&= \sum_{i=1}^{n}(\alpha_i c_i - c_i)
\end{aligned} \tag{4}
$$

The last line in equation (4) shows that the difference between total prices of production and the total values comes solely from the deviation of the value of constant capital from its price of production.

On the other hand, Ding takes the time period into considerations as follows (Ding, 2005: 48):

$$
\begin{aligned}
&\sum_{i=1}^{n} Q_i^t(k+p) - \sum_{i=1}^{n} Q_i^t(c+\beta v+s) \\
&= \sum_{j=1}^{m} Q_j^{t-1}(k+p) - \sum_{j=1}^{m} Q_j^{t-1}(c+\beta v+s)
\end{aligned} \tag{5}
$$

where $\sum_{i=1}^{n} Q_i^t(k+p)$ denotes the sum of the production prices of outputs of n sectors in the period t, $\sum_{i=1}^{n} Q_i^t(c+\beta v+s)$ denotes the sum of the values of outputs of n sectors in the period t, $\sum_{j=1}^{m} Q_j^{t-1}(k+p)$ demotes the sum of the production prices of the produced constant capital of m sectors in the period $t-1$, and $\sum_{j=1}^{m} Q_j^{t-1}(c+\beta v+s)$ denotes the sum of the sum of value of the produced constant capital of m sectors in the period $t-1$. Equation (5) shows that the difference between the total production prices and the total value in the period t is transmitted from the period $t-1$.

Combining equation (4) and (5), Ding further argues that the magnitude of the transmitted deviation depends on the relative organic composition of capital (OCC) of Department I: If in the period $t-1$ the organic composition of capital of Department I is higher (lower) than the average level, in the period t the total output price of production will be higher (lower) than its value. The deviation would disappear when the organic composition of capital of Department I equals the average level. In fact, when the price of production is higher than the value in the t period, the extra value realized is transmitted from the sectors with lower organic composition of capital in the period $t-1$ through the last round of transformation process.

In this way, Ding believes that if we take the inter-period relationship into considerations, the total price of production still equal the total value. And if we think of Marx's own treatment as an abstract level analysis, two equalities still hold in the more concrete level and in a modified way (Ding, 2005: 50).

2.2.3 Comments

Ding's reinterpretation of the "modified significance of the cost price" is insightful. However, his models are far from clear, and contain few arbitrary assumptions.

First, using his numerical example, Ding (2009: 1233) had calculated the values of α and β in the equation system (2) as: $\alpha_1 = \frac{80}{70}$, $\alpha_2 = \frac{70}{62}$, $\alpha_3 = \frac{60}{54}$, $\alpha_4 = \frac{85}{81}$, $\alpha_5 = \frac{95}{93}$, and $\beta_1 = \frac{20}{18}$, $\beta_2 = \frac{30}{27}$, $\beta_3 = \frac{40}{36}$, $\beta_4 = \frac{15}{14}$, $\beta_5 = \frac{5}{3}$. In these calculations, Ding simply assume the values for each capitals. He also assumes that PoPs of all constant capitals (column (1)) are higher than their values (column (5)), and PoPs of all variable capitals (column (1)) are higher than their values as well, without providing any explanation.

Second, while it's clear that α is computed by denominating the constant variable in the column (1) with column (5), Ding never says how β is computed. Based on his second argument, β should be determined by a variety of forces which affect the value and PoP of the labor power. Since Marx usually assume that labor power is sold on its value, and this special commodity is in fact "produced" through a set of processes which is very different from other commodities, it may need more discussions to clarify how does the labor power get its PoP, and this PoP is different from its value, at the first place. But whatever the formulation may be, the value of variable capital plays no role in Ding's current system, it won't change anything if we set $\beta = 1$.

Third, it is not clear how the two equalities can hold in his inter-period framework. Ding never give clear definitions for each new notations used in equation (5). It seems that m denotes the numbers of sector in Department I (producing constant capital), and $n - m$ denotes the sectors in Department II, and Q_i is the quantity of output in ith sector, and k is the price of production of the transformed input costs, and p is the average profit. If that were the cases, by this setting, Ding implicitly assumes the compositions of capital in each sectors and therefore the deviation rates α_i, β_i and ρ_i are constant over periods. These are strong assumptions and require more elaborations to sustain.

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3 Bai, Baoli (2006)

In several respects, Bai's framework is more closer to the standard framework in the literature than other authors reviewed in this paper. Following the standard approach, he admits that Marx's two equalities can not hold simultaneously. But he argues that the magnitude of value-PoP or surplus value-profit deviation is rather small. Besides, if we incorporate the two equalities into a market production price-profit system, they can hold simultaneously.

3.1 Arguments

3.1.1 The transformation is the process of distributing value and surplus value

The essence of Marx's transformation process is: the value and surplus value in the form of prices of production are distributed among capitals through the working of the average profit rate. In this sense, the average profit (rate) is the form of the surplus value in the distribution, and the production price is the form of the value in the distribution (Bai, 2006: 56-57).

For Bai, this argument is not only to reaffirm the primacy of the value system over the production price system, but also to lay out the foundation for the quantitative relations in the latter arguments, in which the quantities of the total values and surplus values still remain, while being redistributed in different forms.

3.1.2 The existence of the undividable remainders

When we transform the total surplus values and total profits into the total profits and total prices of production, under the regulation of the average profit rate, Marx's two equalities can not hold simultaneously. Between the total value and the total production price, and between the total surplus value and the total profit, there are always some remainders cannot be divided or eliminated. These remainders could be positive or negative (Bai, 2006: 36-38), and would be zero only in some special cases (Bai, 2006: 86-92).

Bai (2006: 38-40) emphasizes that the undividable remainder is not a subjective device simply for dealing with the transformation problem, but an objective entity came from the conditions of distribution in the transformation. These conditions include, first, the regulation of the average profit rate, and, second, the characteristics of the equation system of prices of production (equation (9) or (10)).

3.1.3 The magnitudes of remainders are small

After admitting the existence of the remainder, Bai turns to argue that the magnitudes of the remainders are relatively small. Therefore, although not transforming the input values, the profit rate and the production price in the value system , those used by Marx, are good approximations of the profit rate and the production price in the production price system. The value-term profit rate and production price also reveals the fundamental links between these two systems, which cannot be shown without the concept of value and surplus value. In these senses, Marx's treatment is still valid.

3.1.4 Marx's two equalities hold in a market (production) price system

The remainders will be distributed among sectors through the operation of the market (production) price mechanism. Since market prices will gravitate around the prices of production, and market profit rates fluctuate around the average profit rate, they will differ for the individual commodities in any specific time point. Through the market process, some commodities will received market prices and profits higher than their production prices and the average profit rates, while some commodities receive lower market prices and profits. In this way, the remainders will become parts of the market prices and profits in these sectors.[4] Therefore, Bai argues that, if we consider the market production price-profit system as a whole and connect it to the original value system, the total market prices will equal the total value, and the total market profits will be equal the total surplus values.

[4] Here, one might think of Marx's well known assertion that these deviations of market prices from production prices will offset each others as a whole. For example, in a review of major solutions of the transformation problem, Hu (2009: 1212) explicitly interprets Bai's argument in this way. However, while Bai (2006: 40-41) emphasizes that the general law prevails as the dominant tendency "only in a very intricate and approximate way, as an average of perpetual fluctuations which can never be firmly fixed" (Marx, 1981: 261), it is not clear whether Bai (2006) fully accept the above assertion about offsetting. This might reflect a deeper paradox within Bai's study, which will be discussed in this paper after Bai's models are presented.

3.1.5 Marx's two equalities are natural assumptions

Two equalities are the natural presumptions of the transformation process and always hold. There is no need to prove them. Rather, the aim of the study is to analyze the relationships between different variables or parameters under these two presumptions.

3.2 Models & comments

In his models, Bai generally assumes that there is no fixed capital, the whole constant capital will transfer its value completely in each period. And every product will join in the production process of other products, meaning every product is a basic good.

The main steps of Bai's modeling are as followed.

1. Defining the equation system of the prices of production, to show there exists an unique set of solution of the profit rate and the relative production prices.

2. Choosing an "invariant postulate" to pin down the absolute value of production prices. This step will show the existence of the remainders, and the two equalities do not hold generally. Only under some special conditions the remainders would be zero and the two equalities can hold simultaneously.

3. Given the remainders will result at a gap between the value and the production price, Bai goes on to measure how big the gap is.

4. Incorporating the remainders into a market price-profit system to show that the two equalities do hold in this system.

It is worthy to note here that every elements in the following models are all measured by the value or the labor time embodied, rather than the money. The price of production is just a form of value which is modified through the equalization of profit rate. Even the market price in the model remains the market production price, rather than the money price. The money-term profit and price[5] are incomparable with the surplus value and the value (Bai, 2006: 59-60).

3.2.1 The equation system of the prices of production

Bai defines the composition of the prices of production as follows:

[5]Bai (2006: 92-98) incorporates the money and calculates Marx's two equalities. He finds that in the money term the two equalities can hold only in the spacial case in which $r = \frac{\sum S}{\sum(C_c+C_v)}$ and the production price of the money equals its value.

Let W_i^T denotes the value-term production price of total output in ith sector, C_i be the total cost in ith sector, K_i the total amount of capital in ith sector, r the general profit rate, π the profit.

$$W_i^T = C_i + \pi_i = C_i + rK_i, \quad i = 1, 2, ..., n \tag{6}$$

and

$$W_i^T = C_i + rC_i = (1 + r)C_i, \tag{7}$$

in which $K_i = C_i$ if there is no fixed capital. The equation (7) shows the composition of the prices of production.

Then Bai defines the equation system of the prices of production as

$$C_i = X_{i1}w_1^T + X_{i2}w_2^T + ... + X_{in}w_n^T = \sum_{j=1}^{n} X_{ij}w_j^T, \tag{8}$$

where X_{ij} is the amount of jth commodity used in the ith production process, w_j^T is the price of production of the jth commodity. Therefore,

$$W_i^T = (1 + r)\sum_{j=1}^{n} X_{ij}w_j^T, i = 1, 2, ..., n. \tag{9}$$

Equation (9) is the equation system of the prices of production.

Bai defines $w_i^T = \frac{W_i^T}{q_i}$ as the price of production of one unit of commodity, and $a_{ij} = \frac{X_{ij}}{q_i}$ as the $j$the commodity used in the production of one unit of $i$the commodity ,where q_i denotes the amount ith commodity. So the equation (9) can be rewritten as $w_j^T = (1 + r)\sum_{j=1}^{n} a_{ij}w_j^T$, or

$$[(1 + r)\mathbf{A} - \mathbf{I}]\bar{w}^T = 0, \tag{10}$$

where the production price vector $\bar{w}^T = (w_1^T, w_2^T, ..., w_n^T)'$ and the parameter matrix of production expanse $\mathbf{A} = (a_{ij})_{n\times n}$.

In (10), Bai explains, since $\mathbf{A} \geq 0$, and all products are assumed to join in the production of other products directly or indirectly, so $\mathbf{A}$ is undividable. Therefore, according to Frobenius theorem, there exists a unique set of positive, real-number solution for the profit rate $r(\lambda)$ and the relative production prices $\bar{w}^T$, which are in fact determined by $\mathbf{A}$ (Bai, 2006: 65-66).

3.2.2 The existence of the remainders in the general case, and a zero-remainder special case

Next, to pin down the absolute value of the prices of production, we need one more equation. Bai first tests the validity of Marx's average profit rate formula: $r_w = \frac{\sum S}{\sum K} = \frac{\sum S}{\sum C_w}$ (assuming no fixed capital). He finds that this exogenous profit rate can work only when it happens to be equal to the endogenous profit rate based on the $\mathbf{A}$ through Frobenius theorem (Bai, 2006: 34-35). Therefore Bai excludes this choice.

Instead of the exogenous profit rate, Bai points out that one of the following three equations can be chosen: First, total production price equals total value:

$$\sum W^T = \sum W. \tag{11}$$

Second, total (average) profit equals the total surplus value:

$$\sum \pi = \sum S. \tag{12}$$

Third, total cost value equals the total cost price, which is derived from the above two equations:

$$\sum C = \sum C_w. \tag{13}$$

If chooses (11), there will exist a remainder of total profit:

$$d_r = \sum S - \sum \pi. \tag{14}$$

If chooses (12), there will exist a remainder of total value:

$$d_w = \sum W - \sum W^T. \tag{15}$$

If chooses (13), the remainders of the total cost price, $d_c = 0$, and the total profit and total value will coexist at the same time and be equal $d_r = d_w$ (Bai, 2006: 84-85).

By choosing either one of these three, the values of prices of production and the remainders can be solved (Bai, 2006: 67). But in any case, there will be some remainder in the system which cannot be divided or eliminated. A direct implication from the existence of the remainders is that Marx's two equalities do not hold as a general case. This is also the usual result in the traditional literature.

At this moment, it is interesting to note that, although Bai admits that as a general case Marx's two equalities do not hold, he still carefully investigate some special conditions under which the two equalities can hold simultaneously, and gives out a zero-remainder case (Bai, 2006: 86-92).

This special case[6] contains two conditions: (1) All profits (surplus values) are used up in the expanded reproduction; (2) The technical conditions of production process remain unchanged. Under these two conditions, the two profit rates (measured by the value and by the production price) will be equal, and all the remainders will be zero. The mathematical proof is put in the Appendix 1.

For Bai, on the one hand, these special conditions are not some implicit assumptions underlying Marx's writings, as Morishima and Catephores (1978: 173) thought, but just an unrealistic setting produced by modern researchers' modellings. Therefore it is not a good defence of Marx's theory. On the other hand, he clearly does not think that the result in the general case is a failure of the labor theory of value, since he then develops two arguments to defend Marx's theory.

3.2.3 How big/small the remainders are?

To defend the labor theory of value, Bai's first strategy focuses on the magnitudes of the remainders. Let r_w, W_{wi}^T and C_{wi} denote the value-term average profit rate, price of production and cost price respectively. Assume no fixed capital, so the total capital in each department, K will be equal C. So we have

$$r_w = \frac{\sum S}{\sum(C_c + C_v)} \tag{16}$$

$$W_{wi}^T = (1 + r_w)C_{wi} \tag{17}$$

$$C_{wi} = \sum_j X_{ij} w_j \tag{18}$$

First, Bai compares the profit rates in the value and in the production price system (Bai, 2006: 70-71). The profit rate measured by the production price is: $r = \frac{\sum \pi}{\sum C}$. Since $\sum \pi = (\sum S) - d_r$ (from (14)), and let the remainder of total cost $\sum C_w - \sum C = d_c$, so we can rewrite $r = \frac{\sum S - d_r}{\sum C_w - d_c}$. Let $\gamma = \frac{\sum C_w}{\sum C_w - d_c} \frac{\sum S - d_r}{\sum S}$, and given $\sum C_w = \sum(C_c + C_v)$ and (16), therefore, $r = \gamma \frac{\sum S}{\sum C_w} = \gamma \frac{\sum S}{\sum(C_c + C_v)} = \gamma r_w$.

Since d_c and d_r are relatively small numbers comparing with $\sum(C_c + C_v)$ and $\sum S$, so $\gamma \approx 1$ and $r \approx r_w$. The absolute gap of the profit rate $\Delta r = r - r_w = r(1 - \frac{1}{\gamma})$, and the relative gap $\delta_r = \frac{\Delta r}{r} = 1 - \frac{1}{\gamma}$, both would be small, since $\gamma \approx 1$.

[6] Bai points out that this is just one of the cases in which the two equalities can hold.

This shows that the average profit rate r is determined basically by $\frac{\sum S}{\sum(C_c+C_v)}$. The small deviation of r from r_w does not alter the fact that the profit in the production price system still root in the surplus value in the value system (Bai, 2006: 71).

Next, Bai compares the "production prices" in the value and in the production price system (Bai, 2006: 71-73). From equation (17) and (18), $W_{wi}^T = (1+r_w)C_{wi}$. Let $d_{ci} = C_{wi} - C_i$ denote the remainder of the cost price in ith sector, and given $r = \gamma r_w$, then $W_{wi}^T = (1+\frac{r}{\gamma})(C_i + d_{ci}) = \frac{\gamma+r}{\gamma(1+r)}\frac{C_i+d_{ci}}{C_i}(1+r)C_i = \frac{\gamma+r}{\gamma(1+r)}\frac{C_i+d_{ci}}{C_i}W_i^T$. Let $\theta_i = \frac{\gamma+r}{\gamma(1+r)}\frac{C_i+d_{ci}}{C_i}$, so $W_{wi}^T = \theta_i W_i^T$.

Since $\gamma \approx 1$, and d_{ci} is a relatively small number comparing with C_i, so $\theta_i \approx 1$ and $W_{wi}^T \approx W_i^T$. The absolute gap $\Delta W_i^T = W_i^T - W_{wi}^T = (1-\theta_i)W_i^T$, and the relative gap $\delta_{wi} = \frac{\Delta W_i^T}{W_i^T} = 1 - \theta_i$, both would be small, since $\theta_i \approx 1$.

This shows that the production price in the value system is an approximation of the production price in the production price system (Bai, 2006: 73), corresponding to Bai's fourth argument.

3.2.4 Comments

The first question here is where come from the remainders. According to Bai's second arguments and models, the remainder seems to be a simple mathematical fact that given the chosen unit of measurement, there is always some remainders. This is too trivial to be an explanation for the root of transformation problem. In contrast, one of the frequently mentioned cause of deviations is the non-productive consumption by capitalist class, which seeks to explain the cause of deviations more realistically.

Second, how small is enough for a good approximation? In one of his numerical examples, $\Delta r \approx$-11.62%, and $\delta_r \approx$ 21.1%. Are these really small enough? Bai does not provide any plausible criteria.

The second question is: Why we need the method of approximation in the first place? Why not simply adopt the production price system? Bai's answer: it is the value system that reveals the link between the profit and the surplus value (Bai, 2006: 74). However, since we can complete all processes solely in the production price system, the problem of the redundancy of the labor theory of value still cannot be excluded. More seriously, in Bai's models, the general profit rate is determined by $\mathbf{A}$ (equation (10)), which is consisted solely by the prices of production. Although

Bai criticizes Steedman's argument of the redundancy of the labor theory of value, his model seems to be in line with Steedman's argument.

Bai's intention is to argue against Steedman. His strategy is to admit the deviations between total values and total production prices, and between the total surplus values and the total profits, then goes on to show the deviations are relatively small, therefore Marx's method is a valid approximation. However, as Bai shows himself, even the deviations in the numerical example used by Steedman are small as well (Bai, 2006: 168). In this case, Bai's argument is less an efficient defence of Marx than a friendly fire.

3.2.5 The two equalities hold in a market (production) price-profit system

After showing the general case that two equalities do not hold generally, Bai alternatively moves beyond the conventional domain of transformation problem and argues that they will hold if we reconsider them in the market production price-profit system (Bai, 2006: 68).

Let $\sum W_s^T$ denote the total market prices, and $\sum \pi_s$ the total market profits. By equations (14) and (15), the two equalities can be shown as:

$$\begin{cases} \sum W_s^T = \sum W^T + d_w \\ \sum \pi_s = \sum \pi + d_r \end{cases} \Rightarrow \begin{cases} \sum W_s^T = \sum W \\ \sum \pi_s = \sum S \end{cases} \tag{19}$$

where $\sum W_s^T$ is the total market price, $\sum W^T$ is the total price of production, $\sum W$ is the total value, and $\sum \pi_s$ is the total market profit, $\sum \pi$ is the total (average) profit, $\sum S$ is the total surplus value.

This result, corresponding to his third argument, shows that the remainders will be distributed or allocated among sectors through the fluctuation of market prices, keeping the total market prices equal total values, and the total market profits equal total surplus value (Bai, 2006: 68-69).

According to Bai (2006: 38-39), these relationship can be expressed in the following figures.

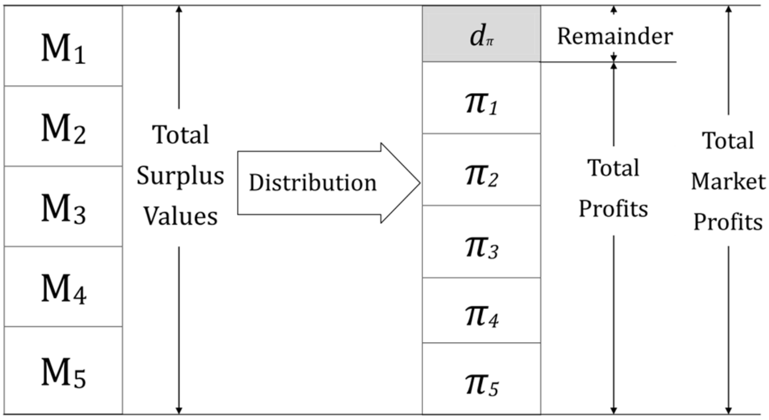

Figure 1: Total Surplus Values - Profits - Remainder

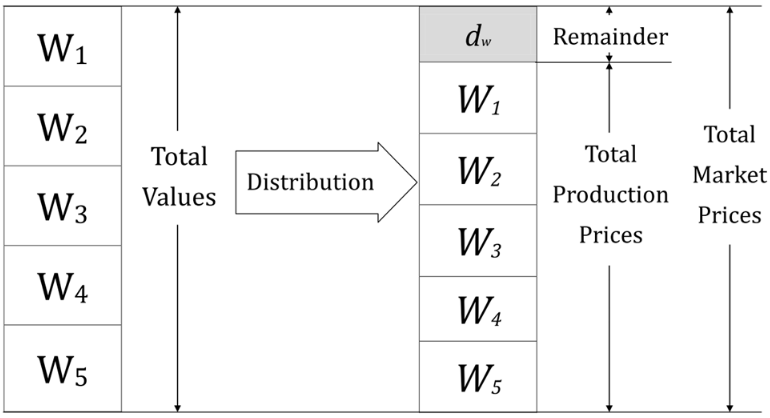

Figure 2: Total Values - Production Prices - Remainder

These figures show that although the two equalities do not hold in the traditional sense, they still hold in a market (production) price-profit system.

3.2.6 Comments

The algebra in this step seems to be problematic. Because, for the two equations in (19) to hold simultaneously, (14) and (15) have to be used together. However, since (14) and (15) came from (11) and (12) respectively, so this means that (11) and (12) are applied at the same time. In this case, $\sum W_s^T = \sum W^T + d_w = \sum W + d_w \neq \sum W$, if $\sum W^T = \sum W$, and $\sum \pi_s = \sum \pi + d_r = \sum S + d_r \neq \sum S$, if $\sum \pi = \sum S$. Or, the two equalities are simply assumed to hold, and there will be no remainder at all.

Even we set aside the mathematical inconsistency, conceptually there seems to be a paradox here. As mentioned in Bai's third argument, the market prices are said to gravitate around the prices of production. In this case, a reasonable questi on is that why the total market price does not equal its center of gravitation, the total production price, but rather the total value? The paradox lies in that, on the one hand, if the deviations of market prices from the production prices offset each others, as the assertion of gravitation implied, then the total market price will equal the total production price. In this case, the total market price will not equal the total value, since there are remainders existing in the deviations of production prices from values, as mentioned in Bai's second argument.

On the other hand, if the deviations do not offset each others, the total market price could equal the total value probably, but then the production price will lose the role as the center of market price gravitation, and the link between the market price and the value, through the mediation of the price of production, becomes questionable. In fact, according to Bai's algebra and the idea that Marx's two equalities would be the natural presumption of transformation, then the market price and profit should equal the value and surplus value, and it is the production price that should gravitate around them.

Here, the underlying issue is that, although it seems to be a promising rescue for the two equalities, Bai's theory, based on the undividable remainder, does not provide a coherent framework to analyze the relationships between values, prices of production and market prices, except the simple statement that the one will fluctuate around the other.

This is particularly regretful since there had been attempts of similar spirits in the literature, that also devote to theoretically refute Steedman's redundancy argument on the one hand, and try to deal with the problem empirically (rather than keeping viewing the transformation problem as a pure logical tack to reconcile the profit rate equalization thesis with the labor theory of value) by verifying the co-movements of values and (market) prices on the other hand. For example, Shaikh (1984) analyze the cause of the deviation between total surplus value and total profit, which is the "leakage" of surplus value from the circuit of capital to the circuit of revenue due to capitalist consumptions, and then shows that the typical deviation between value and price is small by using input-output data. Shaikh's approach is followed by Ochoa (1989), Valle Baeza (1994) and Tsoulfidis & Maniatis (2002). Another group of studies along this line base on the probabilistic approach of Farjoun & Machover (1983). They argue that empirically profit rates are not equalized and therefore there is in fact no transformation at the first place. Nonetheless, it can be shown that values correlate with prices more closely than production prices. So the labor theory of value does work (Cockshott & Cottrell, 1998, 2005; Fröhlich, 2013).

If we put Bai's work in this context, we can see that there seems to be two obstacles preventing Bai from moving forward. First, the lack of realistic explanation of the remainder which is vital for a coherent analysis of the transformation problem. Second, the ambiguity about the relationship between market price (or the "market PoP"), production price and value.

4 Zhang, Zhongren (2004)

Zhang is a Marxian economist grown in China and now works in the University of Shimane in Japan. His study of the transformation problem follows the standard approach, arguing that the task for the transformation analysis is to link the value system and the PoP system by the deviation rates of the PoP from the value (Zhang, 2004a: 139). Although Zhang rejects Samuelson and Steedman's theses that the labor theory of value is redundant, he still term his model as Bortkiewicz-Samuelson-Zhang (BSZ) model, emphasising the similarity among them with respect to the techniques of modeling (Huan & Zhang, 2005). His study provokes a new round of debates in China, since he strongly claim that he had solved the transformation problem, i.e. keeping Marx's two equalities hold after transforming input values into PoPs (Zhang, 2004a: 5). Asides from the two equalities, Zhang (2004: Chapter 7-8) also explores several issues including the dynamic transformation and the historical transformation. The review below will only focus on his "proof" of the two equalities.

4.1 Arguments

4.1.1 Bortkiewicz's two traps

Zhang (2004: 140-142) criticizes Bortkiewicz's two misunderstandings, which became two traps before subsequent researchers. First, by adding a luxury-good sector into a two-departments system, Bortkiewicz confuses the distinction between the "department" and "sector". Zhang clarifies that Marxian "two departments" are not empirically established sectors, but in fact need to be constructed through aggregating respectively the means of production and the means of consumption existed within each sectors[7].

Another misunderstanding concerns about the reproduction scheme adopted by Bortkiewicz. Similar with Ding's third argument mentioned previously, Zhang (2004: 142-143) attacks Bortkiewicz's adoption of the reproduction scheme. However, Zhang's reason is different from Ding. For Zhang, the subject of most transformation studies is static relations between value and PoP of the same period, but the subject of reproduction scheme is the dynamic relation of system across sequential periods. Therefore if the analysis focuses on the static relations as most studies did, the reproduction scheme is not suitable [8].

4.1.2 The distinction between the historical process and historical condition of the transformation.

In the literature the debate about whether there is a "historical" transformation problem is exemplified by the debate between Morishima & Catephores (1978: 178-207) and Meek (1977: 134-145), in which the former gives a negative answer and the later positive.[9]

Zhang (2004a: 211-217) sympathetically criticize Meek for misunderstanding the relationship between logic and history, and proposes to distinguish between three concepts: (1) The logical process of transformation (LP), referring to the static relations between the value and PoP within one period, which has been the main focus of most studies on the transformation problem; (2) The historical process of transformation (HP), which refers to the above mentioned dynamic process of transformation cross several periods[10]; (3) The historical condition of transformation (HC), meaning the "definite degree of capitalist development" mentioned by Marx (Marx, 1981: 277), such as the free competition and the dominance of capitals in the social production system, which are usually assumed to hold in most studies of LP. For Zhang, the so-called historical transformation includes HP and HC, and

[7]By the way, Zhang highly praise Samuelson as the first economist who gives out the correct input-output table for a two-departments system.

[8] From this argument, Zhang goes on to argue that the reproduction scheme can be used as a suitable framework for the analysis of the dynamic transformation problem, and the static relations between the value and the PoP can be viewed as the result of the dynamic process of transformation (Zhang, 2004: 143).

[9] This can be traced back to the long philosophical debate about the method of Marx's Capital. For some comprehensive discussions on Marxian logic-historical issue, see Saad-Filho (2002: chapter 1).

[10]What Zhang says here is in fact the dynamic process of transformation, involving the logical time period rather than the real time.

Meeks failed to distinguishes these two. Both LP and HP have to be based on the fulfilment of HC, but LP can be studied independently from HP and vice versa.

4.1.3 The two equalities are constrained conditions of modelling

Regarding the solution of the transformation problem, Zhang's most controversial argument is: Marx's two equalities can be used as constrained conditions of modelling. As the next section will shows, Zhang (2004a: 144-146) sets up Marx's two equalities within his model, to ensure the model can 'satisfy the two-invariance postulates' (Zhang, 2004a: 146-147; Haun & Zhang, 2005: 29). Facing several critiques against this treatment, Zhang explicitly argue that, according to the common sense in mathematics, Marx's two equalities are constrained conditions for the model building, rather than conclusions or results need to be proved (Zhang, 2004b: 126-127).

4.2 Models & comments

Under standard assumptions[11], let c_i, v_i, m_i and w_i denote the constant capital, variable capital, surplus value, and the total value in ith sector ($i = 1, 2, \dots, n$). $h_i = c_i + v_i$ represents the total amount of capital or cost. $e_i = \frac{m_i}{v_i}$ is the rate of surplus value. $k_i = \frac{c_i}{v_i}$ is the organic composition of capital. $\pi = \frac{m_i}{c_i+v_i}$ is the rate of profit. $H_i = C_i + V_i$, where C_i is the PoP-term constant capital, V_i is the PoP-term variable capital, and H_i is the PoP-term cost. r is the average profit rate. $S_i = rH_i = r(C_i+V_i)$ is the average profit rate in ith sector. $P_i = H_i+S_i = (1+r)H_i$ is the total production price in ith sector.

Let x_i be the deviation rate of the PoP from the value, i.e. $P_i = x_i w_i$ and $C_{ij} = x_i c_{ij}$ ($j = 1, 2, \dots, n$). Let y be the deviation rate of the variable capital v_i, i.e. $V_i = yv_i$. y is exogenous in Zhang's model.

Using the notations above, Zhang establishes firstly the balance equation of the value system as

$$\sum_{j=1}^{n} c_{ij} + v_i + m_i = \sum_{j=1}^{n} c_{ji} + y_i = w_i,$$

[11] Assumptions include: the rates of surplus value are the same in each sectors; OCCs in each sectors do not change; no capital depreciation; the yearly circulation rate is 1 for all capitals; all labor powers are standard, which means homogeneous and socially average (Zhang, 2004a: 140).

and the balance equation of the PoP system as

$$\sum_{j=1}^{n} C_{ij} + V_i + S_i = (1+r)(\sum_{j=1}^{n} C_{ij} + V_i) = \sum_{j=1}^{n} C_{ji} + Y_i = P_i.$$

Then Zhang uses the deviation rates x_i and y to rewrite the PoP system, linking the PoP system with the value system:

$$(1+r)(\sum_{j=1}^{n} x_j c_{ij} + y v_i) = x_i w_i. \tag{20}$$

Since there are $n+2$ unknowns in this system, but only n equations, we need two more equations to solve the system. The first equation Zhang added is Marx's formula of average profit rate:

$$r = \frac{\sum_{i=1}^{n} m_i}{\sum_{i=1}^{n}(c_i + v_i)}. \tag{21}$$

It's clear that this equation, after a simple rearrangement, in fact is equivalent to 'total profit equals total surplus value'.

Next, corresponding to his last argument, Zhang adds

$$\sum_{i=1}^{n} w_i x_i = \sum_{i=1}^{n} w_i, \tag{22}$$

meaning that all deviations of the PoP from the value offset each others, to "ensure" that total PoP equals the total value. Zhang's BSZ model is consisted of these three equations, (20) (21) and (22).

After presenting the model, Zhang goes on to show that Marx's two equalities hold in this model (Zhang, 2004: 146-7). The equation (22) has ensured that the total PoP equals the total value since $\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} w_i x_i = \sum_{i=1}^{n} w_i$. By summing up the first n equations, we get $(1+r)\sum_{i=1}^{n}(\sum_{j=1}^{n} c_{ij}x_j + v_i y) = \sum_{i=1}^{n} w_i x_i = \sum_{i=1}^{n} w_i$, and therefore, $\sum_{i=1}^{n}(\sum_{j=1}^{n} c_{ij}x_j + v_i y) = \frac{1}{1+r}\sum_{i=1}^{n} w_i$. On the other hand, the total profit measured by the PoP is $\sum_{i=1}^{n} S_i = r\sum_{i=1}^{n}(\sum_{j=1}^{n} c_{ij}x_j + v_i y)$. Then by combining these equations, we obtain $\sum S_i = \frac{r}{1+r}\sum w_i = \frac{\frac{\sum m_i}{\sum(c_i+v_i)}}{1+\frac{\sum m_i}{\sum(c_i+v_i)}}\sum w_i = \sum m_i$, i.e. the total profit equals the total surplus value.

In addition to Marx's two equalities, Zhang gives out a mathematical proof for the existence of the unique and positive solutions to the model, to ensure the model's validity in the economic sense (Haun & Zhang, 2005).[12]

[12] Haun & Zhang (2005) is written in English and published in a Japanese journal.

4.2.1 Comments

As mentioned previously, Zhang's solution stimulates lots of debates in China. The most controversial part is, of course, the validity of using Marx's two equalities as postulates or conditions of modelling. Ding & Li (2005) fiercely criticize Zhang for illegitimately assuming the result that needs to be proven, and therefore assuming all difficulties away. On the other hand, Zhang's argument wins supports from Meng (2005) and, indirectly, Yu (2009).

Unfortunately, Zhang's defence, by resorting to the common sense in mathematics (Zhang, 2004b: 126-127), is unconvincing. It would be a circular reasoning if the holding of the two equations is used as the condition or hypothesis, because the goal claimed by Zhang at the beginning of his analysis is exactly to show that the two equalities can hold simultaneously.

As an indirect supporter[13], Yu (2009: 69) does not mention Zhang's works explicitly, but he does argue that the two equalities can be used as conditions of modelling, because in the framework of Marx's value theory, total PoPs and total values are just different forms of the same matter, so are total profits and total surplus values, and therefore are bound to hold simultaneously. However, Yu also admits that generally the two equalities do not hold simultaneously because the capitalist class have unproductive consumptions.

How can these two contradictory statements be reconciled? Yu's way is to assume the gap resulted by the unproductive consumption to be zero (Yu, 2009: 69), without explaining why this gap can be viewed as "a matter of indifference" (Marx, 1981: 265) with respect to the issue of the holding of the two equations. In this sense, Yu's way is, at best, analogous to the special cases set by many in the literature, such as Bortkiewicz-Sweezy (Sweezy, 1942: 115-123), Seton's special-assumptions models (Seton, 1957: 154-6), Morishima (1978: 160-166)[14], or Bai's zero-remainder case (Bai, 2006: 86-92), as shown in the appendix 1. Instead of sustaining Yu's argument, these special cases and their assumptions seem more in line with the understanding that, in their system, Marx's two equalities can both hold only under some strict assumptions.

[13] Meng (2005) support Zhang's method explicitly, but he simply repeat Zhang's words without adding any new argument, so here we discuss Yu (2009) only.

[14] As indicated by Yen & Ma (2011: 105), the two equalities can both hold in Morishima's model because he assumes that the ratios of surplus outputs to the total outputs in all sectors will converge, which is similar with Bortkiewicz-Sweezy's "$Z = 1$" assumption. Morishima just employ the iterative method to avoid assuming directly.

5 Yan and Ma (2011)

Yan & Ma (2011) firstly clarify a series of assumptions made by Marx and build a model with transformed inputs accordingly, in which Marx's two equalities do hold. However, one of the key assumption, the value-production price deviations within inputs will offset each others, is said to be problematic. After removing this assumption, total values do not equal total production prices.

While this sounds disappointed, the remaining part of their book shows no intention of finding the 'correct solution'. Instead, they are more interested in analyzing the features of the system after relaxing original assumptions. Particularly, their analysis goes in two directions. First, they analyze the transformation problem in a dynamic framework in which different sectoral rates of profit converge gradually through the mobilities of capitals.

Second, they extend the 'historical' dimension of transformation problem in two ways which are rarely connected with the transformation problem in the literature: 1. analyzing the transformation problem in a system with industrial, commercial and banking sectors; 2. extending the analysis to the monopoly and international monopoly stages of capitalism.

5.1 Arguments

5.1.1 Only under a series of strict assumptions made by Marx, the two equalities can hold.

Yan & Ma (2011: 50-60) at first clarify a series of strict assumptions on which Marx bases his analysis of the transformation process, and then show in a static model with transformed inputs that under these assumptions Marx's two equalities do hold. Specifically, Yan & Ma (2011: 50-52) identify four strict assumptions made by Marx:

(a) The free competition based on free flows of capital and labor within and among sectors.

(b) In all sectors the conditions of production, including the scale of capital, organic composition of capital, and rate of valorization, are kept constant. Based on this assumption, the total amount of value, the average organic composition of capital, and the average rate of surplus value all remain constant.

(c) The rates of surplus value in all sectors are equal.

(d) The total prices of production of inputs as a whole equal their total values. Marx is aware of the deviations of production prices from values existed in the constant capital and the variable capital. However, he assumes that, in terms of the aggregate, "whenever too much surplus value goes into one commodity, too little goes into another, and that the divergences from value that obtain in the production prices of commodities therefore cancel each other out" (Marx, 1981: 261).

(e) In addition to these four assumptions, Yan & Ma also point out the particularity of the variable capital in contrast to the constant capital, similar with Ding (2005). However, while both refer to the same passage, their interpretations are different. In *Theories of Surplus Value* Marx states that, for the variable capital, the deviation of its production price from its value will be "replaced by a certain quantity of labour which forms a constituent part of the value of the new commodity" (Marx & Engels, 1975: 352). For Yan & Ma (2011: 59), this means that the value-PoP deviation within the variable capital will be compensated by the surplus value-profit deviation within the same variable capital. In this case, if the production price of the variable capital is higher (lower) than its value, the production price of the surplus value will be lower (higher) than its value correspondingly. In other words, the total value created by the same amount of living labor will remain constant. This can be called as the assumption (e). This implicit assumption, called "the invariant equation" (Yan & Ma, 2011: 100), is particularly important and remains in their various models.

By these assumptions, especially by the assumption (d) and (e), Yan & Ma shows that Marx's two equalities will always hold. The model corresponding to this argument, the static model with transformed inputs, is presented in Appendix 2.

5.1.2 The value-production price deviations within inputs will not offset each others, so Marx's two equalities generally do not hold simultaneously.

After modeling under assumptions from (a) to (e), however, Yan & Ma argue that the assumption (d), the value-production price deviations will offset each others at the aggregate level, is in fact problematic. Because, referring to Duménil (1983: 441, 450) and Shaikh (1981: 285-92), they argue that in the economy there exists a non-production class which will consume some surplus values without producing or replacing them, so generally not every outputs are used as inputs in the reproduction. In this case, there is always a gap between the total value and the total production price (Yan & Ma, 2011: 97-98).[15]

Without trying to find the 'correct solution' to keep Marx's two equalities, Yan & Ma turn to analyze, first, the features of a dynamic system of transformation, and, second, the features of a system with various types of capital, and different stages of capitalism.

5.1.3 From the static to the dynamic analysis

According to Yan & Ma, Marx is aware of the dynamic nature of the transformation process, and spends many pages in *Capital* Vol. 3 to analyze the formation of the average profit rate through the mobilities of capitals based on differential initial profit rates and the changing organic compositions of capitals. But Marx still analyze his numerical example as a static one, setting aside the impacts of the dynamics of the transformation on the result of the transformation. However, they argue that the static treatment is inappropriate, since the dynamics of transformation might lead the production price system to deviate from the value system, by changing the organic composition of the social capital (Yan & Ma, 2011: 90-91).

For Yan & Ma, most studies in the literature are static. Although some studies, like Morishima (1976) and Kliman & McGlone (1999), incorporate inter-temporal settings, but remain static in the sense that, either the conditions of production are still hold constant, or the analysis still focus on the final or equilibrium results (profit rates equal in all sectors), rather than trying to capture the dynamics and their determinants during the transformation process.

They propose a switch of focus from the static toward the dynamic and realistic analysis, which consists of two parts (Yan & Ma, 2011: 95-96). The first part is, through competition and supply-demand interactions, the way the PoP deviates from the value, such that profit rates of different sectors would converge to the average profit rate. Besides, the dynamic analysis should also study the impacts of the changes of various variables, like the profit rate, on the transformation results. To incorporate these elements, the above assumptions (b) and (c) should be relaxed. The second part is, after the formations of the equalized profit rate and the production price system, the quantitative and qualitative relationships between the values and prices of production.

[15] Their readings of Shaikh and Duménil seem inaccurate. This problem will be discussed in the comments section later.

5.1.4 Two extensions of analysis

The final part of Yan & Ma devotes to two extensions which are beyond the traditional realm of analysis in the literature.

The first extension is to incorporate the commercial and banking capitals in the static transformation analysis (Yan & Ma, 2011: 150-153). It is not a new idea that the transformation process should incorporate the distribution of surplus value among not only the industrial capitals, but also the commercial capital, banking capital and the landlord ((((MECW, vol.43, p.20)))). For example, Baumol (1974) had raised this issue. But to the best of my knowledge, Yan & Ma's model seems to be the first attempt to formally deal with this issue in the literature. They call the PoP after incorporating the commercial and banking sectors as the 'mature form of PoP' (Yan & Ma, 2011: 150), and the transformation here starts from the PoP to the mature form of PoP.

The second extension is to extend Marx's analysis of the competitive stage to the monopoly and the international monopoly stages (Yan & Ma, 2011: 153-160). Yan & Ma clearly support Meek's thesis and argue that the process in which the value transforms to the PoP occurs in the long historical transition from the stage of simple commodity production to the stage of capitalist free competition (Yan & Ma, 2011: 89). Further, after the competitive stage of capitalism, the free mobilities of capital and labor would be modified by various institutions in subsequent historical periods, especially the monopoly and international monopoly stages. So the transformation process would be modified accordingly. Therefore, the above assumptions (a) and (c), and especially the profit rate equalization assumption, need to be changed.[16].

5.2 Models & comments

Corresponding to the above arguments, Yan & Ma (2011) establish several models in their book, including one static model with transformed input values, three static models without transforming input values, a dynamic model without transforming the input values, a dynamic model with transformed inputs, a static model incorporating the industrial, financial and commercial capitals, a static model with monopoly sectors, and a static model with international monopoly sectors.

[16] There are fierce debates concerning whether profit rates are equalized or not in the 'monopoly' stage of capitalism (for example, Moudud et al. ed. (2012)), but Yan & Ma say no word regarding these debates.

For the limit of space, only two models, the dynamic model with transformed inputs, and the static model incorporating the industrial, financial and commercial capitals, will be discussed in this paper. For the static model with transformed input values which corresponds to their first argument, please see the Appendix 2.

5.2.1 The dynamic model with transformed inputs

This model puts the transformation problem into a dynamic process of profit rates equalization, and tries to capture the effects on the transformation results brought by the changes in variables during this process (Yan & Ma, 2011: 116).

Denote c_i^t constant capital, v_i^t variable capital, m_i^t surplus value, ε_i^t rate of surplus value, k_i^t rate of change in the amount of capital, p_i^t realized value of one unit commodity (or production price), r_i^t profit rate, q_i^t amount of outputs, C_i^t total capital outlay, all in ith sector, tth period. Besides, $\bar{r}^t$ is the average profit rate in period t, and $\bar{r}$ is the final average profit rate.

Yan & Ma (2011: 137-150) employ the physical quantity method to construct the dynamic model in which inputs are measured by the prices of production. To simplify the settings, they adopt following technical assumptions: (1) There are n sectors producing different products, with the same physical type of constant capital (i.e. exogenous technologies); (2) The physical amount of aggregate constant capital does not change; (3) The amount of output depends solely on the physical amount of constant capital, with the property of constant return to scale, and the initial amount of outputs in all sectors are unity ($q_i^0 = 1$); (4) No fixed capital, $C_i^t = c_i^t + v_i^t$; (5) The turn-over times are the same among sectors, so the differences in initial profit rates depend on the organic composition of capital and the rate of surplus value.

In this model, the aggregate means of production does not change, and inputs are measured in terms of production prices, so the total amount of capital in each period is:

$$C_i^t = \mathbf{p}^{t-1}(\mathbf{a}_i^t + \mathbf{b}l_i^t), \text{ and } C_i^0 = \mathbf{w}(\mathbf{a}_i^0 + \mathbf{b}l_i^0), \tag{23}$$

where C_i^t is the total amount of capital of ith sector in tth period, measured by the prices of production or the realized unit commodity values. $\mathbf{p}^t = (p_1^t, p_2^t, ..., p_n^t)$ is the vector of the realized value, or the production price, in tth period. $\mathbf{w} = (w_1, w_2, ..., w_n)$ is the value of one unit commodity. $\mathbf{a}_i^t + \mathbf{b}l_i^t$ is the vector of inputs in tth period, in which $\mathbf{a}_i^t = (a_{1i}^t, a_{2i}^t, ..., a_{ni}^t)'$ is the vector of the means of production, $\mathbf{b}l_i^t$ is the vector of the means of subsistence, $\mathbf{b} = (b_1, b_2, ..., b_n)$ is the vector of unit wage goods, l_i^t is the amount of living labor input of ith sector in tth period.

Since k_i^t is the rate of change of the amount of capital in ith sector tth period, $k_i^t > 0$ means capital inflow, and $k_i^t < 0$ means capital outflow, depending on the difference between sectoral profit rate and the average profit rate:

$$k_i^t = k(r_i^t - \bar{r}^t), k'(\cdot) > 0 \text{ and } k(0) = 0, \text{ where } r_i^t = \frac{p_i^t q_i^t - C_i^t}{C_i^t} \tag{24}$$

is the profit rate of ith sector in tth period, and $\bar{r}^t = \frac{\sum_i m_i^t}{\sum_i C_i^t}$ is the weighted average profit rate.

The amount of capital in each period, including the means of production and the living labor, depends on the amount of capital and the capital mobility in the last period. In which the amount of means of production can be expressed as:

$$\mathbf{A}^t = \mathbf{A}^{t-1}(\mathbf{K}^{t-1} + \mathbf{I}), \tag{25}$$

where $\mathbf{A}^t$ is the matrix of the means of production in tth period, and $\mathbf{K}^t$ is the matrix of capital mobility in tth period:

$$\mathbf{A}^t = \begin{pmatrix} a_{11}^t & a_{12}^t & \dots & a_{1n}^t \\ a_{21}^t & a_{22}^t & \dots & a_{2n}^t \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}^t & a_{n2}^t & \dots & a_{nn}^t \end{pmatrix}, \text{ and } \mathbf{K}^t = \begin{pmatrix} k_1^t & 0 & \dots & 0 \\ 0 & k_2^t & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & k_n^t \end{pmatrix}.$$

Since technologies are exogenous in this model, so, similarly, the amount of living labor depends on the capital mobility:

$$\mathbf{l}^t = \mathbf{l}^{t-1}\mathbf{K}^{t-1}, \tag{26}$$

where $\mathbf{l}^t = (l_1^t, l_2^t, \dots, l_n^t)$.

By the assumption (2), the total amount of aggregate constant capital will not be affected by the capital mobility, so k_i^t would satisfies

$$\mathbf{A}^t\mathbf{K}^t\mathbf{i} = 0, \tag{27}$$

where $\mathbf{i} = (1, 1, \dots, 1)'$.

From the assumption (3),

$$q_i^t = q_i^{t-1}(1 + k_i^{t-1}). \tag{28}$$

In this model the demand is set constant, so the realized value of one unit commodity would be:

$$p_i^t = p(q_i^t), \text{ and } p' < 0, p_i^0 = w_i, p_i^\infty = p_i, \tag{29}$$

where w_i is the realized value of unit commodity of ith sector, and p_i is the production price of unit commodity of ith sector. Given the initial amount of the means of production and the living labor, w_i is determined by $\mathbf{w} = \mathbf{A}^0 + \mathbf{l}^0$, where $\mathbf{w} = (w_1, w_2, w_3, \ldots, w_n)$.

Here, Yan & Ma argue that the amount of the realized value (production price) of unity commodity p_i^t bases on the amount of the value of unit commodity, in a similar way as the assumption (e) in their first argument. In other words, the function $p(\cdot)$ must satisfy the condition that the total value created by the same amount of living labor remains constant:

$$\mathbf{l}^t\mathbf{i} = \mathbf{p}^t\mathbf{q}^t - \mathbf{p}^{t-1}\mathbf{A}^t\mathbf{i}. \tag{30}$$

Combined equations from (23) to (29), we get the dynamic model of transformation with transformed inputs:

$$\begin{cases} C_i^t = \mathbf{p}^{t-1}(\mathbf{a}_i^t + \mathbf{b}l_i^t) \\ \mathbf{A}^t\mathbf{K}^t\mathbf{i} = 0 \\ \mathbf{A}^t = \mathbf{A}^{t-1}(\mathbf{K}^{t-1} + \mathbf{I}) \\ \mathbf{l}^t = \mathbf{l}^{t-1}\mathbf{K}^{t-1} \\ k_i^t = k(r_i^t - \bar{r}^t), \text{ and } k' > 0, k(0) = 0 \\ r_i^t = \frac{p_i^t q_i^t - C_i^t}{C_i^t} \\ q_i^t = q_i^{t-1}(1 + k_i^{t-1}) \\ p_i^t = p(q_i^t), \text{ and } p' < 0, p_i^0 = w, p_i^\infty = p_i \end{cases}$$

Readers will notice that in this model the cost price, including the constant and variable capital, is measured in terms of production prices of the last period, which is similar with TSSI approach. However, since Yan & Ma construct both simultaneous and temporal models, they do not completely reject the theoretical validity of the simultaneous setting as TSSI approach does. For them (Yan & Ma, 2011: 99), the choice between simultaneity and temporality simply concerns about different technical focuses between the dynamic and static analyses, rather than different methodological/theoretical principles.

The dynamic analysis tries to capture the changes of variables in the process of the transformation, especially how the the profit rates would converge gradually to the final average profit rate and make the value transformed into the final production price. As the system converges to the equilibrium, the difference in prices of production in this and next periods (but not the gap between total PoP and value) would be infinite small and negligible. The resulted profit rate and production price then become the focus of the static analysis. Therefore, Yen & Ma believe that the simultaneous determination of the production prices of inputs and outputs is a natural and reasonable setting for the static analysis, while the temporal setting is natural for the dynamic analysis.

Based on the system of equations, the dynamic equation of output is derived by substituting k_i^t into q_i^t:

$$q_i^t = q_i^{t-1}[1 + k(r_i^{t-1} - \bar{r}^{t-1})]. \tag{31}$$

The dynamic equation of the production price is derived by substituting q_i^t into p_i^t:

$$p_i^t = p(q_i^{t-1}[1 + k(r_i^{t-1} - \bar{r}^{t-1})]), \tag{32}$$

which shows that the realized value of one unit commodity depends on the capital flow in the last period, and therefore depends on the difference between the sectoral profit rate and the average profit rate. Given the constant demand, the realized value of one unit commodity decreases as the capital inflow ($p' < 0$) and converges to the production price p_i.

The dynamic equation of profit rate is derived by substituting (23), (31) and (32) into (r_i^t):

$$r_i^t = \frac{p(q_i^{t-1}[1 + k(r_i^{t-1} - \bar{r}^{t-1})])q_i^{t-1}}{\mathbf{p}^{t-1}(\mathbf{a}_i^t + \mathbf{b}l_i^t)} - 1. \tag{33}$$

From this dynamic equation, if the sectoral profit rate r_i^{t-1} larger than the average profit rate $\bar{r}^{t-1}$, capitals will flow into this sector and increase the output in period t. The increasing output will make the production price decrease ($p' < 0$), and therefore the profit rate in period t will decrease. Conversely, if $r_i^{t-1} < \bar{r}^{t-1}$, r_i^t will increase. This dynamic process will continue until the sectoral profit rates converge to the average profit rate.

5.2.2 The numerical simulation and dynamic analysis

Corresponding to their third argument and to show the results of the dynamics of transformation clearly, Yan & Ma's next step is to establish an example of numerical simulation (Yan & Ma, 2011: 142-146). For the method of the simulation setting, please see the Appendix 3. We present the results as the Table 2, which shows profit rates of three sectors converge as capitals flow out from low-profit-rate sectors into high-profit-rate sectors.

	Sector 1			Sector 2			Sector 3		
Period	k_i^t	p_i^t	r_i^t	k_i^t	p_i^t	r_i^t	k_i^t	p_i^t	r_i^t
0	-12.81	4.00	14.29	10.84	4.00	37.93	15.76	3.00	42.86
1	-3.53	4.44	26.88	1.43	3.82	31.86	4.64	2.84	35.08
2	-1.17	4.78	30.84	0.01	3.90	32.02	2.03	2.85	34.04
3	0.59	5.05	31.34	-0.75	3.99	30.00	0.04	2.89	30.79
4	0.90	5.21	29.99	-0.67	4.09	28.42	-0.62	2.94	28.46
5	0.81	5.30	28.46	-0.47	4.16	27.18	-0.74	2.98	26.91
6	0.57	5.34	27.24	-0.28	4.21	26.39	-0.61	3.01	26.06
7	0.35	5.35	26.43	-0.14	4.23	25.94	-0.42	3.03	25.66
8	0.19	5.35	25.96	-0.06	4.24	25.72	-0.25	3.04	25.53
9	0.08	5.35	25.73	-0.01	4.24	25.63	-0.13	3.04	25.52
10	0.02	5.34	25.64	0.01	4.24	25.62	-0.05	3.04	25.56
11	0.00	5.33	25.62	0.01	4.24	25.64	-0.01	3.04	25.61
12	-0.01	5.33	25.64	0.01	4.24	25.66	0.01	3.04	25.66
13	-0.01	5.33	25.66	0.01	4.24	25.69	0.01	3.04	25.69
14	-0.01	5.33	25.69	0.01	4.23	25.70	0.01	3.04	25.71
15	-0.01	5.33	25.70	0.00	4.23	25.71	0.01	3.04	25.72
16	0.00	5.33	25.71	0.00	4.23	25.72	0.01	3.04	25.72
17	0.00	5.33	25.72	0.00	4.23	25.72	0.00	3.04	25.72
18	0.00	5.33	25.72	0.00	4.23	25.72	0.00	3.04	25.72
19	0.00	5.33	25.72	0.00	4.23	25.72	0.00	3.04	25.72

Table 2: Numerical simulation of profit rates equalization process (Yan & Ma, 2011: 144-145)

Then, after setting up and computing this dynamic process of profit rate equalization, the crucial question remains: what happens to the transformation results? To compare with the traditional studies, Yan & Ma further analyze whether Marx's two equalities hold after finishing this dynamic process.

At the end of the dynamic process, the total surplus value can be expressed as $M = \mathbf{li} - \mathbf{pbli} = (\mathbf{1} - \mathbf{pb})\mathbf{li}$, and the total profit can be expressed as $\Pi = \mathbf{pq} - \mathbf{p}(\mathbf{A} + \mathbf{bl})\mathbf{i} = (\mathbf{1} - \mathbf{pb})\mathbf{li}$, which shows that $\Pi = M$, i.e. total surplus value equals total profit.

On the other hand, the total value after transformation is $W = \mathbf{wq} = \mathbf{wAi} + \mathbf{li}$, and the total production price is $P = \mathbf{pq} = \mathbf{pAi} + \mathbf{li}$. The gap between them is

$P - W = (\mathbf{p} - \mathbf{w})\mathbf{Ai}$. Therefore, the question is: Will $P - W$ be 0, so to make Marx's two equalities both hold?

As stated in their second argument, since the means of production $\mathbf{Ai}$ is just part of the total output, its total value would be different from the total PoP it received through the transformation and the redistribution of surplus value. For the whole economy, the gap between the total value and total production price is determined by the value-PoP difference within the means of production, since there is unproductive consumption (Yan & Ma, 2011: 103). If every means of production expended keeps a fixed proportion with the total commodity produced, i.e. $\mathbf{Ai} = \alpha\mathbf{q}$, α is a scalar, and $0 < \alpha < 1$. In other words, the OCCs in all sectors are identical. Then the gap between the total PoP and the total value, $P - W = \mathbf{pq} - \mathbf{wq} = (\mathbf{p} - \mathbf{w})\mathbf{Ai} = (\mathbf{p} - \mathbf{w})\alpha\mathbf{q}$, can be rearranged as $(1 - \alpha)(\mathbf{pq} - \mathbf{wq}) = 0$. Given $0 < \alpha < 1$, we get $\mathbf{pq} = \mathbf{wq}$ and therefore $P = W$ (Yan & Ma, 2011: 148-149). However, Yan & Ma argue that this special case does not hold generally, unless there is no unproductive consumption and all outputs are used as inputs, i.e. $\mathbf{Ai} = \mathbf{q}$ (Yan & Ma, 2011: 149)[17]. Numerically, they calculate the total value after the dynamic process is: $W = 11.44$, and the total PoP is: $P = 12.88$, while the value of the means of production is: $\mathbf{wAi} = 6$ and the PoP of the means of production is $\mathbf{pAi} = 7.44$. So the gap between total value and total PoP equals the gap between the value and PoP of the means of production.

Another comparison in their analysis is the relative magnitudes of the total value after the dynamic process *vs.* of the total surplus value before the process, and of the total PoP after the process *vs.* of the total value before the process. In their system, both the total profit and production price at the end are larger than the total surplus value and value at the beginning. The reason for this feature is that the total amount of living labor will increase as the capital flow into the low-OCC sector from the high-OCC sector. (Yan & Ma, 2011: 147-148).

5.2.3 Comments

Yan & Ma (2011) is the most comprehensive and in-depth study of the transformation problem in the Chinese literature. But compared with the literature, Yan & Ma's dynamic model is rather unusual in terms of the aim and implication

[17] It has to be noted here that this last condition is transmitted from Yan & Ma (2011: 103), where the same proposition appears in an equivalent model: As $\mathbf{Aq} = \alpha\mathbf{q}$, $W - P = \mathbf{wq} - \mathbf{pq} = \alpha(\mathbf{wq} - \mathbf{pq})$, and the condition of no consumption is expressed as $\mathbf{Aq} = \mathbf{q}$.

of the analysis. From 1970s, there are several attempts in employing the dynamic framework, such as Morishima (1974; 1978: 147-177), Shaikh (1977) and the TSSI's temporal approach (Kliman & McGlone, 1999). For these studies, the aim of building the dynamic model is to show that Marx's two equalities can both hold in dynamic frameworks[18] and therefore to "solve" the transformation problem. From the standpoint of defending Marxian theory, they all argue that the dynamic framework is the better way to capture what Marx really mean. But for Yan & Ma, such goal has been declared as infeasible at the first place. According to them, the reason why the goal can be reached in Morishima or TSSI framework is only because some additional conditions are imposed (Yan & Ma: 67, 105). Based on this understanding, the dynamic and static frameworks are treated as equivalent techniques, the choice between them depends on the focus of the researcher.

On the one hand, this may bring some fresh air into the ongoing debates, however, one the other hand, this may lead some readers to question the purpose of their sophisticated dynamic analysis. Since the dynamic results are basically the same as the static one in terms of the transformation and Marx's two equalities, the value-PoP deviation doesn't decrease, then, why bother? The main value added of their study seems to be the analysis of the dynamic process of profit rate equalization, which they argued is necessary for a truly dynamic analysis of the transformation problem, and the pedagogical comparisons between various approaches.

Besides, there are some caveats within their model. As shown above, they argue that OCCs in all sectors will not be identical generally, and the value-PoP deviation in the means of production remains, *unless* there is no non-production consumption. Related with their second argument, the reasoning looks like: the existence of non-production consumption $\rightarrow$ OCCs vary/value-PoP deviation in the means of production $\rightarrow$ total value doesn't equal total PoP. But, first, even if there was no non-production consumption, there can still be various OCCs among sectors. Regarding their mathematical formulation, when there is no non-productive consumption, expressed as $\mathbf{Ai} = \mathbf{q}$ or $\mathbf{Aq} = \mathbf{q}$, i.e. $\alpha = 1$, we get $(1 - \alpha)(\mathbf{pq} - \mathbf{wq}) = 0 \times (\mathbf{pq} - \mathbf{wq}) = 0$. This does not ensure that $\mathbf{pq} - \mathbf{wq} = 0$, and therefore does not ensure that $P = W$.

Second, it's not very clear what's the connection between the existence of non-production consumption and the value-PoP deviation in the means of production,

[18] Or at least the value-PoP gap within one of the equality will converge to zero dynamically, as in the case of Shaikh (1977).

such that the total value-total PoP gap is resulted.

Although they refer to Shaikh (1981) and Duménil (1983), their readings seem inaccurate. For Shaikh (1981: 285-92) the consequence of the existence of the unproductive consumption is the gap between the total surplus value and the total profit, rather than the gap between the total value and the total PoP. On the other hand, Duménil (1983) concludes that "Marx is erroneously assuming that the value/prices of production discrepancies in the workers' consumption basket will compensate exactly" (Duménil, 1983: 450), which contradicts more with Yan & Ma's assumption (e), rather than (d), in their first argument.

Third, one reason that the gap between the total value and total PoP is determined by the value-PoP deviation in the means of production, rather than the value-PoP deviation in the variable capital, is because they assume that the value-PoP deviation within the variable capital will be compensated by the surplus value-profit deviation within the same variable capital, as stated in their first argument, assumption (e). Compared with Ding (2005), their choices of "the invariant equation" seem to be the same. But Ding reasonably recourse to the exploitation for the variable capital (and the absence of exploitation for the constant capital), while Yan & Ma's explanation is far from clear, and their reasoning of non-production consumption seems related to some disproportionality in the system.

5.2.4 The static model with the industrial, commercial and banking capitals

This model is built on the following assumptions (Yan & Ma, 2011: 150-151). (a) The profit of the commercial sector comes solely from the difference between the purchase price and the selling price. The profit of the banking sector comes solely from the difference between the interest rate of lending and of saving. (b) Under the condition of free competition, the values of inputs have transformed into PoPs, and all capitals in the industrial sector obtain the average profit rate.

In this model, there are n capitals in the industrial sector producing n commodities, and n firms in the commercial sector and n banks in the banking sector. The product of the ith industrial capital is distributed and sold by the ith commercial firm. The ith back gives loans to the ith industrial capital and then the ith commercial firm $((i = 1, 2, \dots, n))$. The cost price, average profit, and capital outlay of the n capitals in the industrial sector are k_i, π_i and C_i^p respectively. C_i^b and C_i^f are the amounts of capital outlay of the ith commercial firm and the ith bank. C_i^{pf}

denotes the capital the ith industrial firm borrowed from the ith bank. C_i^{bf} denotes the capital the ith commercial firm borrowed from the ith bank. Assume $C_i^{pf} \leq C_i^p$, and $C_i^{bf} \leq C_i^b$. r^o and p_i^o denote the average profit rate and the PoP before incorporating the commercial and banking sectors into the system. After incorporating the commercial and banking sectors, r denotes the new average rate of profit, p_i denotes the mature PoP of the $i$the product. i_c and i_d denote the interest rates of saving and lending respectively.

After incorporating the commercial and banking sectors, the new average profit rate is determined by:

$$r = \frac{\sum_{i=1}^{n} \pi_i}{\sum_{i=1}^{n} C_i^p + \sum C_i^b}. \tag{34}$$

In the meantime, the mature PoP of the commodity produced by the ith industrial firm and sold by the ith commercial firm is:

$$p_i = k_i + C_i^p r + C_i^b r, \tag{35}$$

where $C_i^p r = \pi_i^p$ is the average profit obtained by the industrial sector, and $C_i^b r = \pi_i^b$ is the average profit obtained by the commercial sector. The PoP of outputs of the industrial sector is $p_i^p = k_i + C_i^p r$, and the commercial sector obtains the average profit through $p_i - p_i^p$, i.e. the difference between the purchasing price and the selling price.

For the banking sector, the average profit is:

$$\pi_i^f = C_i^f r = (i_d - i_c) C_i^{pf} + (i_d - i_c) C_i^{bf}. \tag{36}$$

In this equation, the profit of the banking sector depends on the average profit rate, which is determined by the industrial and commercial sectors. And the difference of saving and lending interest $(i_d - i_c)$ can be determined endogenously as

$$i_d - i_c = \frac{C_i^f r}{C_i^{pf} + C_i^{bf}}, \tag{37}$$

which is satisfied for all i, meaning the lending ratios for each banking sectors are all the same. This also reflects the result of the free competition among banks.

With the above settings, Yan & Ma (2011: 151-153) analyze successively the relations between total profits, total PoP and the average profit rate before and after the formation of the mature PoP.

Equations ()() and () show that, after incorporating the commercial and banking sectors, the profit created by the industrial sector has to be distributed among these three sectors. The average profit of the industrial sector is: $\pi_i^p = C_i^p r$, but the net profit is:

$$\tilde{\pi}_i^b = C_i^p r - i_d C_i^{pf}, \tag{38}$$

since the industrial sector has to pay interests $i_d C_i^{pf}$ to the banking sector. Similarly, the net profit of the commercial sector is:

$$\tilde{\pi}_i^b = C_i^b r - i_d C_i^{bf}, \tag{39}$$

where $i_d C_i^{bf}$ is the interest paid to the banking sector.

The total profit can be expressed by summing up equation () () and ():

$$\sum_{i=1}^{n}(\pi_i^f + \tilde{\pi}_i^p + \tilde{\pi}_i^b) = r\sum_{i=1}^{n} C_i^p + r\sum_{i=1}^{n} C_i^b - i_c(\sum_{i=1}^{n} C_i^{pf} + \sum_{i=1}^{n} C_i^{bf}), \tag{40}$$

where $i_c(\sum_{i=1}^n C_i^{pf} + \sum_{i=1}^n C_i^{bf})$ is the interest paid to the saver by the banking sector. This shows that, after incorporating the commercial and banking sector, the total profit $\sum_{i=1}^n \pi_i$ will be redistributed among the three sectors, plus the amount of interests paid to the saver.

Next, the PoP of the ith commodity before incorporating the commercial and banking sectors is:

$$p_i^o = k_i + C_i^p r^o = k_i + \pi_i, \text{ and } \sum_{i=1}^{n} p_i^o = \sum_{i=1}^{n}(k_i + \pi_i). \tag{41}$$

After incorporating the commercial sector, the total PoP becomes:

$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} k_i + r\sum_{i=1}^{n}(C_i^p + C_i^b) = \sum_{i=1}^{n} k_i + \sum_{i=1}^{n} \pi_i, \tag{42}$$

which shows the total PoP does not change.

However, since the ratios between the amounts of capitals of each commercial sectors and the corresponding industrial sectors will not keep fixed generally, i.e. $p_i^o \neq p_i$, which means that the PoP of each commodity will not be the same after incorporating the commercial and banking sectors. Therefore, while the total PoP remains the same, the PoP of each commodity will change.

Finally, the average profit rate before incorporating the commercial and banking sectors is:

$$r^o = \frac{\sum_{i=1}^n \pi_i}{\sum_{i=1}^n C_i^p}. \tag{43}$$

Compared with the profit rate after incorporating the commercial and banking sectors shown in the equation (), we see $r < r^o$. This shows that the average profit rate decreases because the non-productive sectors, the commercial and banking sectors, now also join in the distribution of total profit.

5.2.5 Comments

...to be written...

6 Conclusion

...to be written...

Appendix 1: Bai's proof of the zero-remainder case

To obtain the two equalities simultaneously, we need $d_r = d_w = d_c = 0$.

The two special conditions in Bai's zero-remainder case are:

1. All profit (surplus values) are used up in the expanded reproduction (SC1);

2. The technical conditions of production process remain unchanged (SC2).

Let $X_j = \sum_{i=1}^{n} X_{ij}$, where X_j denotes the the amount of jth input commodity in the production system.

From SC2, during the process of expanded reproduction, the ratios among various commodities in this period remain the same as the previous period, such that $\frac{\Delta X_i}{\Delta X_j} = \frac{X_i}{X_j}$. Rearranging and multiplying the right-hand side by w_j (the value of jth commodity), we obtain $\frac{\Delta X_i}{X_i} = \frac{\Delta X_j}{X_j} = \frac{w_j \Delta X_j}{w_j X_j}$.

Let $\omega = \frac{\Delta X_j}{X_j}$, then $\omega_j = \frac{\Delta X_j}{X_j} = \frac{\sum_j (w_j \Delta X_j)}{\sum_j (w_j X_j)}$, we get $\sum(w_j X_j) = \sum C_w = \sum(C_c + C_v)$.

And from SC1, we have $\sum(w_j \Delta X_j) = \sum S$. Therefore, $\omega_j = \frac{\Delta X_j}{X_j} = \frac{\sum S}{\sum(C_c + C_v)}$.

Under SC1 and SC2, the total amount of various products less the products used up during the production process is the surplus products, which would equals ΔX_j. So $\omega_j = \frac{\Delta X_j}{X_j}$ equals the rate of surplus product of jth commodity, and the rate of surplus product of every product is the same. Denotes this general rate of surplus product as ω, $\omega = \frac{\sum S}{\sum(C_c + C_v)}$.

Let $\bar{q} = (q_1, q_2, q_3, ...q_n)'$ be the ratios of various products during the production process. We obtain $[(1+\omega)\mathbf{A}' - \mathbf{I}]\bar{q} = 0$, in which $(1+\omega)^{-1}$ is the Frobuis root of $\mathbf{A}'$. Since $\mathbf{A}'$ and $\mathbf{A}$ have the same Frobuis root, so $(1+\omega)^{-1}$ is the Frobuis root of $\mathbf{A}$ as well.

Since $[(1+\omega)\mathbf{A}' - \mathbf{I}]\bar{w}^T = 0$, and $(1+r)^{-1}$ is also the Frobuis root of $\mathbf{A}$, so we know that $r = \omega$. Therefore, $r = r_w = \frac{\sum S}{\sum(C_c + C_v)}$.

This can be rewritten as $|[(1+r_w)\mathbf{A} - \mathbf{I}]| = 0$, which is a special case and will not exist in real economy generally.

Given this result, since $r = \frac{\sum \pi}{\sum C}$, we have $\frac{\sum \pi}{\sum C} = \frac{\sum S}{\sum(C_c + C_v)}$. By ???theorem, $\frac{\sum \pi}{\sum C + \sum \pi} = \frac{\sum S}{\sum(C_c + C_v) + \sum S}$, which is $\frac{\sum \pi}{\sum W^T} = \frac{\sum S}{\sum W}$ by definitions.

When $\sum \pi = \sum S$, we can obtain simultaneously $\sum W^T = \sum W$ and $\sum C = \sum C_w$, which means that $d_w = d_r = d_c = 0$. Therefore, the two equalities can be obtained simultaneously under SC1 and SC2.

According to Bai (2006: 153), these two conditions lie at the core of Morishima's solution. In this sense, Morishima's solution is a special case as well.

Appendix 2: Yan & Ma's static model with transformed inputs

Following the assumption from (a) to (d), the static model is constructed as followed (Yan & Ma, 2011: 58-60). Let C_i, c_i, v_i, m_i, w_i, w_i, p_i denote respectively the total capital outlay, constant capital, variable capital, surplus value, value ,and production price, in ith sector, all measured by the value. $\tilde{C}_i$, $\tilde{c}_i$, and $\tilde{v}_i$ denote respectively the capital outlay, constant capital and variable capital, measured by the production price. r is the average profit rate. The cost price of ith sector is $(c_i + v_i)$. Since the existence of fixed capital, the capital outlay will be larger than the cost price: $C_i \geq c_i + v_i$. Under these settings, Marx's model with transformed inputs can be expressed as:

$$\begin{cases} \tilde{c}_1 + \tilde{v}_1 + \tilde{C}_1 \times r = p_1 \\ \tilde{c}_2 + \tilde{v}_2 + \tilde{C}_2 \times r = p_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \tilde{c}_n + \tilde{v}_n + \tilde{C}_n \times r = p_n \\ r = \dfrac{\sum \tilde{m}_i}{\sum \tilde{C}_i} \end{cases}$$

where $\tilde{m}_i$ is the amount of surplus value in ith sector under the condition that the variable capital is measured by the price of production. In this model $\tilde{C}_i$, $\tilde{c}_i$, $\tilde{v}_i$, $\tilde{m}_i$ are known, and r, p_i are unknown. With n+1 equations and n+1 unknown, the system can determine an unique set of solution.

In this model, the assumption (e) that the deviation within the variable capital will be compensated by the deviation within surplus value means $\tilde{v}_i + \tilde{m}_i = v_i + m_i$. And since this applies to the variable capital only, so $\tilde{c}_i \neq c_i$ presumably, although this is not explained explicitly in their texts. Meanwhile, they argue that this difference between the variable capital and the constant capital exists only at the level of individual sectors. At the aggregate level, in contrast, Marx's assumption (d) indicates that the value-production price deviations within inputs (including constant and variable capitals) of each sectors, will offset each others. Therefore, $\sum \tilde{c}_i = \sum c_i$ and $\sum \tilde{v}_i = \sum v_i$, so $\sum(\tilde{c}_i + \tilde{v}_i) = \sum(c_i + v_i)$.

Yan & Ma go on to show that Marx's two equalities do hold in this model. First, the total surplus value is $M = \sum m_i$, and the total profit is $\Pi = \sum \pi_i = r \sum \tilde{C}_i$. Since we know $\tilde{v}_i + \tilde{m}_i = v_i + m_i$ and $\sum \tilde{v}_i = \sum v_i$, so $\sum \tilde{m}_i = \sum m_i$. From the last equation of the model, we get $r\sum\tilde{C}_i = \sum\tilde{m}_i = \sum m_i$. Therefore $M = \Pi$, the total surplus value equals the total profit.

Second, the total value is $W = \sum(c_i+v_i+m_i)$, and the total price of production $P = \sum p_i$. By summing up the n equations in the model, we have $\sum p_i = \sum(\tilde{c}_i + \tilde{v}_i + \tilde{C}_i \times r) = \sum(\tilde{c}_i+\tilde{v}_i) + r\sum\tilde{C}_i = \sum(\tilde{c}_i+\tilde{v}_i+\tilde{m}_i)$, given $r\sum\tilde{C}_i = \sum\tilde{m}_i$. Since we knew $\sum(\tilde{c}_i+\tilde{v}_i) = \sum(c_i+v_i)$ and $\sum\tilde{m}_i = \sum m_i$, so $\sum(\tilde{c}_i+\tilde{v}_i+\tilde{m}_i) = \sum(c_i+v_i+m_i)$, which means $P = W$, the total price of production equals the total value.

Comments

By the assumption (d) and (e), the total values of the constant capital, the variable capital, and the surplus value all equal their production prices respectively. In this case, the total value will equal the total production price naturally. Combined with the formula of the general profit rate, there should be no surprise that Marx's two equalities would hold. Therefore they don't call this result as a solution or anything innovative.

In this respect, Yan & Ma's treatment might be thought to be similar with the TSSI approach, which claims to be more loyal to Marx's labor theory of value (Yan & Ma, 2011: 67). However, while in TSSI the system relies on a MELT and the direct link between the value and the market price, in Yan & Ma the key role lies at the function of the variable capital, with strictly distinguishing the production price from the market price.

Appendix 3: Yan & Ma's simulation method

To simulate numerically, Yan & Ma need to specify explicitly the functional form in the equation (24) and (29).

For the rate of change in the amount of capital (the rate of capital mobility) k_i^t, assume

$$k_i^t = r_i^t - \bar{r}^t, \tag{44}$$

which must satisfies the condition $\mathbf{A}^t\mathbf{K}^t\mathbf{i} = a_{i1}^t k_1^t + a_{i2}^t k_2^t + \ldots + a_{in}^t k_n^t = 0$ (from equation (27)). By substituting k_i^t into this condition, the average profit rate can be calculated as a weighted average profit rate:

$$\bar{r}^t = \frac{a_{i1}^t r_1^t + a_{i2}^t r_2^t + \ldots a_{in}^t r_n^t}{a_{i1}^t + a_{i2}^t + \ldots + a_{in}^t} = \frac{\sum_j a_{ij}^t r_j^t}{\sum_j a_{ij}^t}. \tag{45}$$

For the realized value (production price) of unit commodity p_i^t, assume

$$p_i^t = \frac{\mathbf{p}^{t-1}\mathbf{a}_i^0}{q_i^t} + l_i^0. \tag{46}$$

This can be shown to satisfy $\frac{dp_i^t}{dq_i^t} < 0$ and $p_i^0 = w_i$, and also the condition that the total value created by the same amount of living labor remains constant, i.e. equation (30) (Yan & Ma, 2011: 142-143).

Specify the function of sectoral profit rate by substituting equation (44) and (46) into equation (33):

$$r_i^t = \frac{\dfrac{\mathbf{p}^{t-1}\mathbf{a}_i^0}{1 + r_i^{t-1} - \bar{r}^{t-1}} + l_i^{t-1}}{\mathbf{p}^{t-1}(\mathbf{a}_i^{t-1} + \mathbf{b}l_i^{t-1})} - 1. \tag{47}$$

Instead of proving formally the sectoral profit rates will converge, Yan & Ma (2011: 143) employ the method of numerical simulation to show an example of the convergence. The numerical simulation of the dynamic transformation process goes as follows. In a 3-sectors model ($n = 3$), assume the amount of means of production $\mathbf{a}_1^0 = (0.5, 0.25, 0)'$, $\mathbf{a}_2^0 = (0.3, 0.15, 0)'$, and $\mathbf{a}_3^0 = (0.2, 0.1, 0)'$, the amount of living labor $\mathbf{l}^0 = (1, 2.2, 1.8)$, and the unit wage goods $\mathbf{b} = (0, 0.05, 0.1)'$. From equation (29), the value of unit commodity is

$$w_i = \frac{\mathbf{w}\mathbf{a}_i^t + l_i^t}{q_i^t} = \mathbf{w}\mathbf{a}_i^0 + l_i^0. \tag{48}$$

So we can calculate the value of unit commodity in each sector based on the above initial values:

$$\begin{cases} w_1 = 0.5w_1 + 0.25w_2 + 0w_3 + 1 \\ w_2 = 0.3w_1 + 0.15w_2 + 0w_3 + 2.2 \\ w_3 = 0.2w_1 + 0.1w_2 + 0w_3 + 1.8 \end{cases}$$

and obtain $\mathbf{w} = (4, 4, 3)$ by solving this linear system of equations.

Based on these conditions, we can get the profit rates in the period 0 for each sectors according to equations (45):

$$r_1^0 = \frac{w_1 - \mathbf{w}(\mathbf{a}_1^0 + \mathbf{b}l_1^0)}{\mathbf{w}(a_1^0 + \mathbf{b}l_1^0)} = \frac{4 - 3.5}{3.5} = 14.29\%$$

$$r_2^0 = \frac{w_2 - \mathbf{w}(\mathbf{a}_2^0 + \mathbf{b}l_2^0)}{\mathbf{w}(a_2^0 + \mathbf{b}l_2^0)} = \frac{4 - 2.9}{2.9} = 37.93\%$$

$$r_3^0 = \frac{w_3 - \mathbf{w}(\mathbf{a}_3^0 + \mathbf{b}l_3^0)}{\mathbf{w}(a_3^0 + \mathbf{b}l_3^0)} = \frac{3 - 2.1}{2.1} = 42.86\%$$

Then we can obtain the average profit rate in the period 0 from equation (45): $\bar{r}^0 = 27.09\%$.

After obtaining the initial average profit rate, they apply the initial values to the equation (47) to compute the dynamic equation of the profit rate in each sector:

$$r_1^t = \frac{\dfrac{\mathbf{p}^{t-1}\mathbf{a}_1^0}{1 + r_1^{t-1} - \bar{r}^{t-1}} + 1}{\mathbf{p}^{t-1}(\mathbf{a}_1^{t-1} + \mathbf{b}l_1^{t-1})} - 1 \tag{49}$$

$$r_2^t = \frac{\dfrac{\mathbf{p}^{t-1}\mathbf{a}_2^0}{1 + r_2^{t-1} - \bar{r}^{t-1}} + 2.2}{\mathbf{p}^{t-1}(\mathbf{a}_2^{t-1} + \mathbf{b}l_2^{t-1})} - 1 \tag{50}$$

$$r_3^t = \frac{\dfrac{\mathbf{p}^{t-1}\mathbf{a}_3^0}{1 + r_3^{t-1} - \bar{r}^{t-1}} + 1.8}{\mathbf{p}^{t-1}(\mathbf{a}_3^{t-1} + \mathbf{b}l_3^{t-1})} - 1 \tag{51}$$

Based on these dynamic equations and the initial average profit rate, they calculate the dynamic process as the Table 2.

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