The Role of Consumer Leverage in Financial Crises

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Abstract

Consumer leverage can contribute to financial crises such as the subprime mortgage crisis characterized by increased bankruptcy prospects and tightened credit access. This paper embeds financial frictions in the mortgage contracts of home-buyers within a two-sector economy to show that although households are not production agents and may not start as highly leveraged as financial institutions, their worsening debt levels can generate a lasting financial downturn. Using two seemingly positive disturbances that contributed to the subprime mortgage crisis - increased housing supply and relaxed borrowing conditions - the model demonstrates that the subprime downturn was not a precedent but the natural consequence of financial frictions. The surplus of available housing leads to lower asset prices that in turn reduce the value of the mortgaged houses relative to the loan held. This worsens the leverage of indebted consumers and raises their bankruptcy prospects. A relaxation of borrowing conditions turns credit-constrained households into a potential source of disturbance themselves when market optimism allows them to increase their indebtedness with relatively little downpayment. In both cases, the increased debt, along with higher repayment rates due to the larger default likelihood, impairs household access to credit and plunges mortgage-holders into a lasting recession. Adding credit constraints to the financial sector that provides housing mortgages creates opportunities for risk sharing where banks shift some of the downturn onto indebted consumers in order to hasten their own recovery. This consequence is especially evident in the case of relaxed credit access for banks. Financial institutions repair their debt position relatively fast at the expense of consumers whose borrowing ability is squeezed for a long period despite the fact that they are not the source of this disturbance. The outcome mirrors the recent subprime mortgage crisis characterized by a sharp but brief decline for banks and a protracted recovery for mortgaged households.

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1 Introduction

In 2004, Edmund Andrews, an economic reporter for The New York Times, joined millions of American home-buyers in purchasing a house at the peak of the real estate price bubble (Andrews 2009). The fact that he had regularly reported from the Federal Reserve, covered the Asian financial crisis of 1997, the Russian meltdown of 1998 and the dot-com collapse of 2000, did not prepare him for what was in store at the subprime mortgage party. "I had just come up with almost a half-million dollars, and I had barely lifted a finger. It had been so easy and fast," Andrews said of obtaining his mortgage despite having modest disposable income and putting down very little downpayment. His mortgage was a classic subprime loan. The monthly payment first jumped from $2,500 to $3,700. If he kept the mortgage for two years, the interest rate could jump as high as 11.5 percent, and the monthly payments could ratchet up to $4,500. After his wife lost her job, he fell behind on all payments from the mortgage to the electricity bill. When he finally defaulted, he was far from being the only one. In fact, he had outlived two of his three mortgage lenders. The first one collapsed overnight when the financial markets first froze up in August 2007 and the second one was forced out of the mortgage business by federal regulators.

Andrews' story is just one of the hundreds of recent foreclosure experiences that characterize the mortgaged homeowner's experience with subprime loans. The subprime mortgage crisis was a seemingly unusual one since it was not precipitated by production firms or entrepreneurs and it was not triggered by adverse circumstances. The unraveling of the mortgage market began when two seemingly positive factors - increased housing supply and easy financing conditions - allowed borrowers to excessively leverage with debt to disastrous consequences. At the heart of the downturn were not production agents but heavily indebted consumers and the financial institutions that securitized their mortgages. Nevertheless, the collapse of the highly leveraged mortgage market was sufficient to trigger a significant downturn.

The asset bubble that preceded the subprime mortgage crisis increased housing inventories to record high numbers (Coleman, LaCour-Little and Vandell, 2008). After the bubble burst, the oversupply of real estate reduced both the sale price of real estate and the value of the houses held by homeowners (Duca, Muellbauer and Murphy, 2010; Ellis, 2010). As a result, many mortgage-holders owed more on their housing loan than their residence was worth. Furthermore, at the peak of the asset bubble, many investors eager to tap into mortgage profits underwrote housing loans secured with very little downpayment and with a variable repayment rate, the so-called "subprime" loans (Duca, Muellbauer, and Murphy, 2009a and 2009b). These lax borrowing conditions allowed consumers to hold significant amounts of debt and to be at the mercy of the adjustable interest rate. Once the first signs of trouble raised the repayment rate, their risk premiums rose worsening their indebtedness (Demirgüç-Kunt, Evanoff and Kaufman, 2011; Laeven, Igan and Dell'Ariccia, 2008; Mian and Sufi, 2009). This caused a flurry of short sales and defaults as mortgage-buyers attempted to deleverage or declared bankruptcy. The result was a depressed housing market with even lower asset prices and sharply tightened credit access (Dennis, 2010; Duke, 2012; Madigan, 2012).

The housing market downturn also exported the worsened financial conditions beyond the affected sector and depressed demand for goods in the rest of the economy.

Glick and Lansing (2010) demonstrate that household leverage in the United States and in many industrial countries increased dramatically in the decade prior to 2007. Countries with the largest increases in household leverage underwent the fastest rise in house prices over the same period. These same countries tended to experience the biggest decline in household consumption once house prices started falling. Household leverage growth and the dependence on borrowing also explain a large fraction of the overall consumer default, house price, unemployment, residential investment and durable consumption patterns during the recession (Mian and Sufi, 2010). Kim and Isaac (2010) show that consumer debt can affect macroeconomic dynamics and can contribute to financial fragility as strongly as corporate leverage. The authors demonstrate that looser consumer credit can also make the system more vulnerable to changes in the state of confidence, the interest rate and the saving propensity of renters. Papers like these motivate a microfounded modeling of the consequences of sharp decline in house prices and of improvements in borrowing confidence on the leverage of indebted households.

This paper develops a model of financial frictions in the mortgage market to demonstrate how downturns such as the subprime mortgage crisis occur. It does not attempt to explain the the housing bubble and all aspects of the subsequent collapse. Rather, it focuses on two major factors that contributed to the unraveling of the mortgage market - the excess housing supply that led to a decline in housing prices and the lax financial conditions that permited increased borrowing - on the leverage of mortgage-buying consumers. For this purpose, the paper extends the one-sector model of credit constraints of Bernanke, Gilchrist, Gertler (1999) to two sectors and shifts financial frictions from producers to homeowner consumers. A two-sector model can generate a pro-cyclical risk premium that widens following improvements in the housing market. This risk premium is function of the default risk that in turn depends on the size of the collateral and the value of the mortgage. So a shock that reduces housing prices makes real estate more affordable but also creates a Fisher-type effect where the price decrease worsens the value of the housing collateral of indebted households and increases their leverage. The higher leverage triggers an increase in the adjustable mortgage interest rate and the risk premium widens. A credit relaxation on the other hand impacts the leverage of mortgage-buyers directly but creating the incentive to borrow excessively. Hence, with credit constraints in the consumer sector, both a technological expansion in the housing sector and a financial easing imply a downturn for mortgaged consumers thus bringing the model closer to the subprime crisis.

It order to describe the behavior of leveraged consumers, the paper departs from the traditional modeling of house purchasing decisions such as Kiyotaki and Moore (1997), Aoki, Proudman and Vlieghe (2004) and Iacoviello (2005) in favor of a microfounded model of consumer credit constraints. The housing model of Aoki, Proudman and Vlieghe (2004), which belongs to the Kiyotaki and Moore (1997) collateral constraint type of models, imposes an exogenously determined limit on borrowing depending on the availability of collateral. Such models were a fairly common way of modeling housing loans prior to the financial crisis. However, the emergence of subprime loans requiring little to no collateral and instead have an adjustable repayment rate makes such approaches unsuitable for modeling the subprime recession. To capture better the subprime mortgages, the model developed here avoids exogenously imposed loan-to-value ratios that cap borrowing and limit the resulting volatility. Instead, this paper continues to use the Bernanke, Gilchrist and Gertler (1999) microfounded loan contract to model frictions in the mortgage market. Such a setup imposes no upper restrictions on borrowing but introduces an endogenous adjustable mortgage rate that makes loans costlier as the leverage increases. A subprime loan within this framework could be secured with modest downpayment but at a higher interest rate that adjusts upwards following unfavorable macroeconomic disturbances. This approach allows a robust study of the role of leveraged consumers in the subprime crisis.

Most of the works on financial constraints overlook the role of consumer leverage in the recession since households are generally not as leveraged as financial institutions. Bernanke, Gilchrist and Gertler (1999) and Luk and Vines (2011) show that high firm or financial institutions leverage can be instrumental in precipitating a financial downturn. Similarly, Fernández-Villaverde (2010) and Gertler and Kiyotaki (2010) demonstrate that a disruption to financial intermediation can induce a crisis that affects real economic activity. The majority of models either omits the role of consumer leverage or only attempts to fit models to empirical data. One of the few studies on consumer mortgage decisions, Mian and Sufi (2009), establishes that consumers can experience a worsening in their leverage as a result of deteriorating lending practices but does not study the consequences of this increase in indebtedness for their borrowing and the economy. This paper augments our understanding of credit frictions by considering explicitly the role of household leverage in creating and propagating downturns. It also demonstrates that although households do not start very indebted and do not produce output, changes in their leverage can have lasting consequences for their credit access and for the economy as a whole as mentioned in Kim and Isaac (2010).

At the heart of the financial frictions setup is the inability of credit-constrained consumers to fully finance their housing purchase so they need to borrow external funds from risk-neutral investors. This borrowing is complicated by the presence of an idiosyncratic risk of default on the part of mortgage-buyers that is known to them but is unknown to lenders (Townsend, 1979). Unlike Cúrdia and Woodford (2009) where borrowers can choose whether to default, here bankruptcy is involuntary but depends on factors both endogenous (the size of the mortgage) and exogenous (macroeconomic shocks) to consumer decisions. If credit-constrained households default, investors must pay an auditing fee to assume possession of any remaining assets. Since investors cannot fully diversify away this risk, they charge borrowers a risk premium that would offset the expenses associated with eventual bankruptcy. Investors obtain their funds from the deposits of Ricardian consumers who have significant non-wage income from firm profits and share ownership. In contrast, credit-constrained consumers earn only labor income and have no savings. The economy is completed by two production sectors: one that produces conventional consumption and one that manufactures housing. Housing is a multi-period durable good whose manufacture follows the setup of Iacoviello and Neri (2010). Both production sectors are perfectly competitive and there is no idiosyncratic uncertainty in their returns.

The model demonstrates the effects of two shocks that contributed to precipitating the subprime mortgage crisis: the oversupply of housing and the relaxation of borrowing conditions. The overproduction of housing depresses the price of housing and lowers the return on housing. As a result, the value of the collateral of credit-constrained consumers, primarily composed of their housing equity, decreases by both the price and the return to housing. The drop in their housing equity value is proportionally larger than the fall in the mortgage value which falls only due to the price reduction so the leverage of mortgaged home-buyers increases. The higher leverage signals increased risk of default since lenders that provide loans to credit-constrained consumers are exposed to bigger risks. To compensate for this, investors require a higher return from indebted households so the risk premium rises. The higher risk premium further depresses credit access that can remain lower for a long time while consumers attempt to repair their debt position by gradually improving their real estate holding.

A lower housing price is not the only disturbance that can induce a credit downturn for indebted consumers. A relaxation of credit access improves borrowing conditions so mortgage loans attract a lower risk premium and require a smaller downpayment to secure (Duca, Muellbauer, and Murphy, 2009a and 2009b; Mian and Sufi, 2009). The lax credit access prior to the crisis reduced interest rates and lured many potential homeowners into obtaining mortgages with little collateral. However, the lower collateral implies higher indebtedness for mortgage-buyers that can easily lead to higher probability of default. Borrowers attempt to amass more collateral in order to improve their mortgage position but since they cannot achieve that overnight, their increased bankruptcy prospects prompt investors to raise the repayment interest rate. The higher repayment rate makes improving housing equity even harder for indebted consumers. As a result, households experience a lasting recession with tight credit access.

In an extension to the main model, the paper considers credit-constrained financial institutions that serve as intermediaries in the loan contract between investors and households. Before the financial crisis, the financial sector did not receive much attention in the literature on credit frictions. Papers like Kiyotaki and Moore (1997) and Iacoviello (2005) focus primarily on the demand side of credit and abstract from an active role of financial intermediation. The first models to consider the role of banks are Goodfriend and McCallum (2007) and Christiano, Trabandt and Walentin (2007). Both of these estimate the quantitative importance of the banking sector for central bank policy and for business cycles. Christiano, Motto and Rostagno (2010), Gerali, Neri, Sessa and Signoretti (2009), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2011) and Luk and Vines (2011) develop a financial sector to explain specific precedents of the financial crisis such as excessive volatility, the proliferation of risk accumulation or the popularity of unconventional monetary policy. The existing literature overwhelmingly focuses on credit frictions in only one sector, namely the banking sector, and abstract from constraints in other sectors of the economy, including households. This makes them unsuitable to studying the extent of risk sharing that can occur with more than one type of financially-constrained agents. Only Hirakata, Sudo and Ueda (2009) introduce more than one leveraged sector modeling constraints in the production sector along with the financial sector but they also overlook the role of indebted consumers.

The extension to the main model borrows from the chained contracts model of Hirakata, Sudo and Ueda (2009) but the final borrowers are not entrepreneurs but credit-constrained households. It demonstrates that when there are two types of credit-constrained agents, the chained loan contracts create opportunities for risk sharing. As intermediaries of the two mortgage agreements, financial institutions have the ability to transfer some of the burden of a downturn to final borrowers consumers. Furthermore, banks are often more leveraged so they experience a proportionally larger deterioration of their leverage ratio which is an added impetus for them to unload the leverage fast. The higher initial leverage and the sharper increase necessitate quick deleveraging in order to maintain solvency. That is the primary reason why banks resort to shifting part of the burden of financial tightening onto households. Two characteristics of the financial system allow them to do so. First, credit-constrained consumers to have no recourse to alternative funding so they will participate in the mortgage contract even if the terms deteriorate. Second, the arrangement between banks and mortgage-buying households is such that the return from borrowing to consumers varies with the realization of the idiosyncratic uncertainty going up in times of a downturn. This allows financial institutions to extract a larger share from households to repair their own balance sheet during a deterioration of financial conditions. Consequentially, they can recover relatively fast at the cost of inducing a lasting recessions for mortgaged households.

2 The Consumer Mortgage Contract

2.1 Description of the Consumer Mortgage Contract

This section develops the consumer mortgage contract in a partial equilibrium framework, taking as given the price of the collateral and the risk-free rate of interest. The subsequent section endogenizes these variables as part of a general equilibrium solution. The model developed here extends the one-sector model of financial frictions of Bernanke, Gertler and Gilchrist (1999) and adds credit constraints in the consumer sector to demonstrate the role of household mortgage leverage in contributing to financial crises.

The economy consists of two sectors: one that produces generic consumption and one that manufactures housing (Figure 1). The consumption sector produces its output using sector-specific capital, labor and own technology as inputs. Following Iacoviello and Neri (2010), housing is a durable multi-period good. Housing manufacturers use housing capital, labor and sector-specific technology along with land, a finite resource. Each period only a fraction of housing deteriorates. The rest survives for the subsequent period. In each period, new housing production only replaces the depreciated housing.

This approach to housing as a multi-period good brings a more staggered response to disturbances since newly produced output is only a fraction of total housing on the market. Both final good firms face no risk and they can borrow funds for capital financing at the risk-free rate as part of a fixed share arrangement. In addition to the two final goods, there are capital producing firms that supply final good manufacturers with sector-specific capital. Investment in both types of capital is the output of the consumption sector. The relationship between capital firms and final goods producers is a two-way one in the sense that capital firms buy used capital from goods producers and along with investment transform it into new capital.

In order to motivate credit frictions in the household sector, this model formally separates consumers into two types following Iacoviello and Neri (2010). Ricardian consumers possess no intrinsic risk of default and can borrow at the risk-free rate. They have significant non-wage income in the form of revenue from owning shares in final good firms and from absorbing the profits of capital producers. Their wealth allows them to finance their own housing purchase entirely. Ricardians save any unused income as deposits that are lent to investors and to final good firms. Credit-constrained consumers, on the other hand, receive only wage income so their net worth is not enough to finance their housing acquisition. Therefore, they must obtain external financing from investors.

The housing purchase of credit-constrained consumers is financed by a mortgage contract. The mortgage contract is necessitated by the existence of an agency problem that makes external borrowing for house purchase more expensive than own funds. The reason for this discrepancy is that credit-constrained consumers possess an inherent risk of default that is known to them but unobservable to lenders. In this case, investors cannot perfectly observe the borrower's ability to repay and must pay an auditing cost in order to learn the bankruptcy prospect of the mortgage-buyer. When indebted consumers go bankrupt, investors pay the monitoring fee and take possession of all of the borrower's remaining assets. In the household context, these auditing costs can be interpreted as the costs of legal proceedings to the value the borrower's assets and the administrative costs of selling the house to realize its collateral value. The presence of these financial frictions motivates the need for a loan (mortgage) contract as opposed to a share contract that usually takes place in the absence of idiosyncratic uncertainty. Additionally, the idiosyncratic uncertainty implies that credit-constrained consumers cannot borrow directly from their Ricardian counterparts. The intermediary services of investors are necessary not only to disburse the loan but also to conduct monitoring.

Credit-constrained households obtain a loan from investors who pool the deposits of Ricardian consumers, paying them the risk-free rate, and lend to credit-constrained households at a markup consumer interest rate. The difference between the borrowing and the lending rates, known as the external finance (risk) premium, exists in order to offset the expenses associated with a potential bankruptcy of mortgage-buyers. The greater the default prospect, the more likely the lender will have to pay the auditing cost so the risk premium will be higher. Hence the external finance premium is a function of the default rate, the collateral (net worth) of credit-constrained consumers and the value of the mortgage. Investors do not make any profit on the loan to indebted households. The risk premium is exactly sufficient to offset the costs associated with a

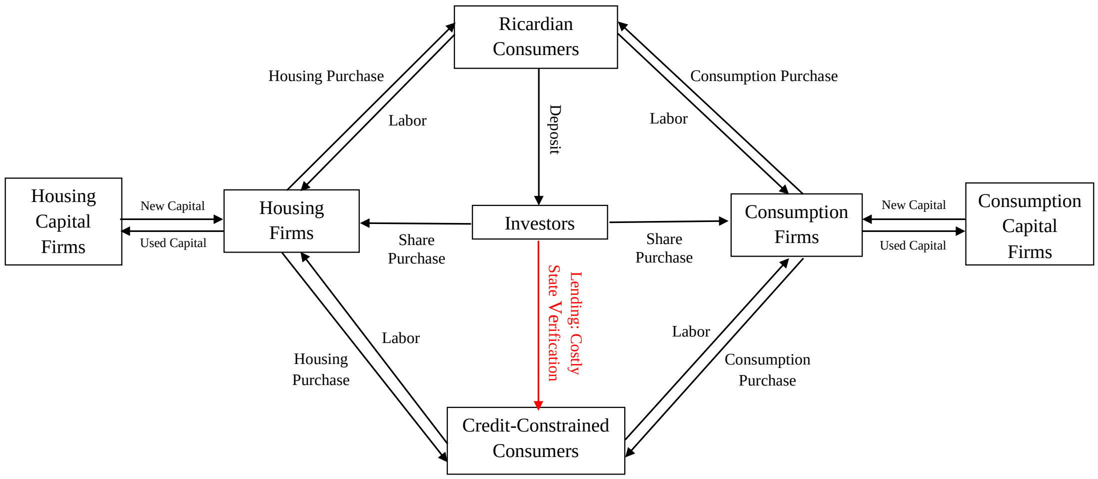


Figure 1: Model with Financial Frictions in the Consumer Sector

potential borrower default.

2.2 The Loan Contract of Credit-Constrained Households

The mortgage loan contract begins when at time t, credit-constrained household i obtains a mortgage from investors to purchase housing at price p_t for use at $t+1$. To improve the model tractability, the paper assumes that mortgage contacts last one period only and are renegotiated every period. This is not a significant departure from reality since it captures the adjustable interest rate phenomenon of the subprime downturn. In this sense, this loan contract can be thought as a multi-period mortgage with an adjustable rate. A multi-period mortgage with a fixed rate would not be properly microfounded since it will not update according to the most recent information on the value of the housing stock and the default risk and hence would not provide a fair return to lenders.

The quantity of housing purchased is denoted $H_{i,t}^C$ with the subscript denoting the household that obtains it and the period in which the housing is purchased and the superscript indicating the type of consumer that purchases it (C denotes credit-constrained households and R refers to Ricardian consumers). Each credit-constrained household faces an idiosyncratic shock ω_i^C to their return to housing. In each period, homeowner i draws from a distribution of ω_i^C where ω_i^C follows a log-normal distribution with mean one and variance $\left(\sigma^C\right)^2$. The variance $\left(\sigma^C\right)^2$ can also be thought as a measurement of the volatility of the default probability ω_i^C - a lower variance would imply safer loan returns. The ex-post gross return on housing is $\omega_i^C R^C$, where ω_i^C is an idiosyncratic disturbance to household i's return and R^C is the ex-post aggregate return to housing for credit-constrained consumers (the markup consumer interest rate paid on the mortgage). The idiosyncratic shock ω_i^C is also independent across time and across credit-constrained households. Lenders can diversify away the idiosyncratic risk of the borrowers by investing in a lot of borrowers.

This setup follows the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999). This approach assumes that there is costly state verification on the lender's part that motivates financial frictions. If investors want to observe the idiosyncratic shock ω_i^C for a specific homeowner, they need to pay a monitoring cost which is a fraction μ^C of the homeowner's total housing stock. Due to the monitoring cost, it would be too costly to monitor every contract. Therefore, there is a cutoff value of the shock $\bar{\omega}_i^C$ above which investors do not monitor and below which they do. Above the cutoff, the credit-constrained homeowner gets a return sufficient to pay investors a fixed rate of interest Z^C and keeps what is left for themselves; investors have no incentive to monitor the homeowner' true return. Below the cutoff, credit-constrained households are bankrupt, and the lenders pay the monitoring cost and take their entire wealth.

The investors who make loans to credit-constrained households are risk-neutral and the opportunity cost of their funds is the risk-free interest rate. They will agree to lend to credit-constrained households only if they can break even on their investment. This implies that at the time of the loan repayment, the interest rate R^C charged to borrowers must be such that the expected gross return on the loan must equal the opportunity cost of lending:

$$\bar{\omega}_i^C R^C p H_i^C = Z_i^C (p H_i^C - N_i^C)$$

The left hand side is the gross revenue from housing to credit-constrained homeowner i whose risk $\bar{\omega}_i^C$ is just at the cutoff point. The right hand side is the amount the consumer needs to repay the investors: the total value of the loan pH_i^C minus the net worth of the borrower N_i^C. The household is just able to repay the loan at the contractual rate.

The idiosyncratic shock is continuous and has a cumulative distribution function $F^C(\bar{\omega}_i^C)$. To be perfectly insured against the idiosyncratic shocks of households, each investor signs contracts with an infinite number of households. The solution of the loan contract defines how the investment revenue is split between the lenders (investors) and the borrowers (credit-constrained households). Each lender gets a gross share $\Gamma^C(\bar{\omega}_i^C)$ that is the sum of the share (before monitoring) in case the household defaults $\int_0^{\bar{\omega}_i^C} \bar{\omega}_i^C dF^C(\bar{\omega}_i^C)$ and the fixed interest payment the lender gets if the debtor does not default with probability $(1 - F^C(\bar{\omega}_i^C))$. Let the defaulting share $\int_0^{\bar{\omega}_i^C} \bar{\omega}_i^C dF^C(\bar{\omega}_i^C)$ be denoted $G^C(\bar{\omega}_i^C)$, then the gross share $\Gamma^C(\bar{\omega}_i^C)$ is:

$$\Gamma^C(\bar{\omega}_i^C) = \int_0^{\bar{\omega}_i^C} \bar{\omega}_i^C dF^C(\bar{\omega}_i^C) + (1 - F^C(\bar{\omega}_i^C))\bar{\omega}_i^C = G^C(\bar{\omega}_i^C) + (1 - F^C(\bar{\omega}_i^C))\bar{\omega}_i^C$$

The borrower receives the remaining share $(1 - \Gamma^C(\bar{\omega}_i^C))$. After taking into account the monitoring cost $\mu^C G(\bar{\omega}_i^C)$, the net share that goes to the lender is:

$$\Gamma^C(\bar{\omega}_i^C) - \mu^C G^C(\bar{\omega}_i^C)$$

By assumption, the investors make a zero profit on their loans to credit-constrained households. Investors can diversity away the loan risk so their opportunity cost is the risk-free rate R_t. Hence the zero-profit condition for lending to a particular borrower at time t to be repaid at time $t+1$ is:

$$E_t\left(\left(\Gamma^C(\bar{\omega}^C_{i,t+1})-\mu^C G^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t}=R_t\left(p_t H^C_{i,t}-N^C_{i,t}\right) \tag{1}$$

The left hand side is the net return on the loan to investors (the gross return $E_t\Gamma^C(\bar{\omega}^C_{i,t+1})$ minus the monitoring cost $\mu^C E_t G^C(\bar{\omega}^C_{i,t+1})$) at the time of the repayment. The right side is the opportunity cost of lending the funds (the value of the housing purchase $p_t H^C_{i,t}$ minus the consumer's net worth $N^C_{i,t}$) valued elsewhere at the risk-free rate R_t that prevails at the time of the loan agreement.

Since the price of housing, the risk-free interest rate and the homeowner's net worth $N^C_{i,t}$ are predetermined, at time t the credit-constrained household gets to choose a pair of $(E_t\bar{\omega}^C_{i,t+1},\ H^C_{i,t})$ according to the zero-profit condition. Given the other variables, this is equivalent to the lenders offering a schedule of loans $E_t B^C_{i,t+1}$ and a non-default interest rate R_t to the household.

In this partial equilibrium setting, the credit-constrained homeowner i is faced with a menu of loan $E_t B^C_{i,t+1}$ and interest rate $E_t R^C_{t+1}$ where both the loan and the interest rate are related by the participation constraint of the investors. At time t, borrowers choose the optimal pair of housing and the expected cutoff risk $(H^C_{i,t},\ E_t\bar{\omega}^C_{i,t+1})$ to maximize their expected share of the loan at time $t+1$ of the loan repayment:

$$max\ E_t\left(\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t} \tag{2}$$

subject to the zero profit condition of investors (1).

Households optimize their mortgage from the perspective of time t in terms of $E_t R^C_{t+1}$ but repay it at time $t+1$ at the realized rate R^C_{t+1}. The mortgage is state-contingent so its return depends on the realization of the ex-post return on housing R^C_{t+1}. It is at this rate that the loan contract is repaid, rather than at the expected return on housing $E_t R^C_{t+1}$. The expected return on housing $E_t R^C_{t+1}$ is all that is known at the time the contract is negotiated but it does not account for unexpected shocks that can affect the repayment ability of credit-constrained consumers at the mortgage maturity date. Hence to ensure a fair return, investors negotiate the loan contact at $E_t R^C_{t+1}$ but require repayment at the realized ex-post return R^C_{t+1} that is formed only after all disturbances have occurred.

Credit-constrained consumers maximize their share of the loan subject to the participation constraint of investors:

$$\begin{aligned} L^C_t &= E_t\left(\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t}+ \\ &+\lambda_t\left[E_t\left(\left(\Gamma^C(\bar{\omega}^C_{i,t+1})-\mu^C G^C(\bar{\omega}^C_{i,t+1}\right)R^C_{t+1}\right)p_t H^C_{i,t}-R_t\left(p_t H^C_{i,t}-N^C_{i,t}\right)\right] \end{aligned}$$

Solving the maximization problem gives the external finance (risk) premium defined as the relationship between the risk-free rate and the markup consumer interest rate $\frac{E_t R^C_{t+1}}{R_t}$ as a function of the cutoff default risk $E_t\bar{\omega}^C_{i,t+1}$:

$$\frac{E_t R^C_{t+1}}{R_t}=E_t\left(\frac{\frac{\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{i,t+1})\right)}}{\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)+\frac{\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{i,t+1})\right)}\left(\Gamma^C(\bar{\omega}^C_{i,t+1})-\mu^C G^C(\bar{\omega}^C_{i,t+1})\right)}\right) \tag{3}$$

This equation implies that unless the monitoring cost μ^C is zero, investors would require the aggregate return to the their investment $E_t R^C_{t+1}$ to be larger than the risk-free rate R_t.

An interesting feature of the risk premium $\frac{E_t R^C_{t+1}}{R_t}$ is that it does not depend on the amount of the loan since the monitoring fee μ^C is scale independent. It depends only on the cutoff risk $E_t \bar{\omega}^C_{i,t+1}$. This implies that a higher cutoff risk will adjust the consumer repayment rate $E_t R^C_{t+1}$ upward for a given risk-free rate. Similarly, the interest rates ratio is proportional to the variance $\left(\sigma^C\right)^2$ (the volatility parameter) of $E_t \bar{\omega}^C_{i,t+1}$ so a lower risk volatility would reduce the gap between the consumer rate and risk-free interest rate.

Since every credit-constrained consumer solves the same loan contract, they choose the same expected cutoff risk $E_t \bar{\omega}^C_{i,t+1}$ to maximize their share of the loan. Each household is essentially a representative home-buyer so equation (3) holds for every mortgage-buyer. Hence it is possible to aggregate across all credit-constrained households so that this equation is valid for the whole economy. The ratio of the realized return to housing to the expected return on the right hand side is determined macroeconomically, and given that every borrower chooses the same expected cutoff default risk, then the aggregate cutoff risk is $E_t \bar{\omega}^C_{i,t+1} = E_t \bar{\omega}^C_{t+1}$ Hence equation (3) holds on the aggregate level and determines the macroeconomic cutoff risk $E_t \bar{\omega}^C_{t+1}$:

$$\frac{E_t R^C_{t+1}}{R_t} = E_t \left(\frac{\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1}) - \mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}}{\left(1 - \Gamma^C(\bar{\omega}^C_{t+1})\right) + \frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1}) - \mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)} \left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1})\right)} \right) \tag{4}$$

Similarly, the aggregate amount of net worth held by indebted households in the economy is $N^C_t = \sum N^C_{i,t}$ and the aggregate amount of housing purchased by credit-constrained consumers is $H^C_t = \sum H^C_{i,t}$.

The first order condition (4) and the participation constraint (1) hold at any point in time in which there have not been any unanticipated shocks. However, even after a shock occurs at time $t+1$, the borrowed share $p_t H^C_t - N^C_t$ is already fixed so investors must continue to receive the risk-free rate R_t. Taken together, these two equations determine the realized cutoff value $\bar{\omega}^C_{t+1}$ that adjusts after an unexpected disturbance at $t+1$. Hence although consumers optimize using the expected participation constraint of lenders, the model utilizes the realized participation constraint that incorporates the realized default risk along with the expected first order condition:

$$\left(\Gamma^C(\bar{\omega}^C_t) - \mu^C G^C(\bar{\omega}^C_t)\right) R^C_t p_{t-1} H^C_{t-1} = R_{t-1}\left(p_{t-1} H^C_{t-1} - N^C_{t-1}\right) \tag{5}$$

A positive productivity shock reduces the price of housing p_t. This causes the net worth of indebted households to decrease. To satisfy their participation constraint, the investors will either reduce lending or raise the interest rate they charge to the households or both which implies that the participation constraint contracts. However, in the general equilibrium context, the supply of housing is predetermined. To maintain equilibrium with the original level of housing, the first-order condition moves to a higher level. This means that the ratio of the realized return to housing to the risk-free rate increases raising the external finance premium. Observing this, households choose a higher cutoff risk $\bar{\omega}^C$ and an unchanged profit-maximizing level of housing. This is not the long-run equilibrium. Over time, consumers reduce their housing stock and return to the previous profit-maximizing level of $\bar{\omega}^C$.

In order to complete the partial equilibrium setting, it is necessary to determine the evolution of the credit-constrained households' net worth. In any given period, the equity of the credit-constrained households, V_t^C, is the remaining share of the mortgage after replaying back investors:

$$V_t^C = \left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C \tag{6}$$

Consumers can spend this dividend income on new housing. When house prices fall - and therefore the equity of the households V_t^C - households face the following decision problem. If they decrease housing demand today, current household utility would fall. But, if demand were kept constant, net worth would decrease, increasing the future external finance premium. Thus households face a trade-off between current housing purchase and future borrowing.

It is also necessary to make sure that credit-constrained consumers do not eventually grow out of their financial constraints. This paper assumes that every period a constant fraction $1-\nu^C$ of households retire. When they retire, they spend their remaining equity on consumption. The retirement consumption $C_{E,t}^C$ of credit-constrained consumers is:

$$C_{E,t}^C = \left(1 - \nu^C\right) \left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C \tag{7}$$

Over time, the number of credit-constrained consumers decreases but their individual net worth increases as they cycle through many periods of mortgage contracts. Hence macroeconomically, the value of the net worth remains the same.

Credit-constrained consumers need to get started on their net worth with some income not devoted to purchasing consumption goods and housing. This is equivalent to establishing a savings account dedicated to the initial mortgage downpayment. The model assumes they provide one unit of labor inelastically to the production of housing that generates a wage w_t^C. This labor supply is solely for the purposes of starting their net worth accumulation and is weighted heavily so that it does not distort the overall labor supply. Having determined the period equity and the start-up net worth of credit-constrained consumers, is it easy to describe the evolution of their net worth. The evolution of the credit-constrained households' net worth is the sum of equity of non-retiring households plus their income from work:

$$N_t^C = \nu^C V_t^C + w_t^C \tag{8}$$

2.3 The Share Contracts of the Consumption and Housing Sectors

The contracts between investors on one side and producers of consumption and of housing on the other hand are share contracts. As equity holders of final goods firms in both sectors, investors finance their capital purchase and absorb their profits and losses. No monitoring takes place. The manufacturers and investors split the revenue according to the shares of their investments, regardless of the idiosyncratic shocks to consumption producers ω^F and to housing producers ω^H. To diversify away from the firm-specific idiosyncratic risk, each consumer will invest in an infinite number of firms.

Investors finance the capital purchase $x_t^F K_t^F$ of consumption firms, which occurs one period prior to production. The aggregate return to capital purchased by the consumption firms is the risk-free rate R_t. In the following period, the share of consumption firms' revenue that goes to investors is Γ_t^F.

$$\Gamma_t^F R_t x_t^F K_t^F = \frac{x_t^F K_t^F - N_t^F}{x_t^F K_t^F} \int_0^\infty \omega^F R_t x_t^F K_t^F dF(\omega^F)$$

Since lenders can diversify over an infinite number of food producers and there is no upper cutoff amount of ω^F, $\int_0^\infty \omega^F dF(\omega^F) = 1$ This implies that the share is independent of the idiosyncratic shock ω^F:

$$\Gamma_t^F = \frac{x_t^F K_t^F - N_t^F}{x_t^F K_t^F} \tag{9}$$

Firms in the consumption sector accrue profits which they split with investors according to their share contract. In any given period, the equity of consumption firms is their part of the share agreement in the previous period:

$$V_t^F = \left(1 - \Gamma_t^F\right) R_{t-1} x_{t-1}^F K_{t-1}^F = R_{t-1} N_{t-1}^F \tag{10}$$

In order to prevent consumption firms from growing out of the financial constraints, the model assumes that at the end of every period, a constant fraction $1 - \nu^F$ exits the market. Exiting firms consume immediately their remaining equity. In this case consumption producers' consumption $C_{E,t}^F$ on exit is:

$$C_{E,t}^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F \tag{11}$$

Similarly to consumers, firms need to start off with some net worth so this setup assumes they provide inelastically one unit of labor to the production of their output for which they receive a wage w_t^F. The labor contribution of consumption producers is significantly discounted to avoid crowding out regular household labor supply. The evolution of firms' net worth is the sum of equity of surviving firms plus their income from work:

$$N_t^F = \nu^F V_t^F + w_t^F \tag{12}$$

Investors also finance the capital purchase $x_t^H K_t^H$ of housing firms. Similarly to the consumption sector, there is no idiosyncratic risk in the housing sector and firms can borrow at the risk-free rate. The share purchase in the housing sector Γ_t^H equals:

$$\Gamma_t^H = \frac{x_t^H K_t^H - N_t^H}{x_t^H K_t^H} \tag{13}$$

Housing producers also accrue profits which they split with investors according to their share contract. In any given period, the equity of housing firms is:

$$V_t^H = \left(1 - \Gamma_t^H\right) R_{t-1} x_{t-1}^H K_{t-1}^H = R_{t-1} N_{t-1}^H \tag{14}$$

Housing producers' consumption $C_{E,t}^H$ on exit is:

$$C_{E,t}^H = \left(1 - \nu^H\right) R_{t-1} N_{t-1}^H \tag{15}$$

Housing firms also provide inelastically one unit of labor to the production of their output for which they receive a wage w_t^H. The labor contribution of housing producers is also discounted. The evolution of firms' net worth is the sum of equity of surviving firms plus their income from work:

$$N_t^H = \nu^H V_t^H + w_t^H \tag{16}$$

3 The Complete Model with Consumer Mortgage

This section embeds the partial equilibrium of the loan contract derived in the previous section into a general equilibrium framework that endogenizes the risk-free rate R_t and the price of housing p_t. The economy consists of two production sectors: consumption and housing. Capital producers supply sector-specific capital to both types of final good firms. Households consume both consumption and housing.

3.1 Consumption Capital Sector

Firms that produce capital K_t^F for the consumption sector own technology that converts final goods into capital. They purchase depreciated capital from final good firms and make investments to produce new capital. The investment I_t^F is consumption. The newly produced capital is sold back to consumption producing firms.

There are standard quadratic adjustment costs to producing capital. The capital adjustment costs for the consumption capital are:

$$K_t^F = (1-\delta) K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right) K_{t-1}^F \tag{17}$$

The function J is such that $J' > 0$ and $J" < 0$. New capital is produced within the period and sold to final good producing firms at the price x_t^F. The optimal condition for investment is:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1 \tag{18}$$

3.2 Consumption Producers

Firms in the consumption sector use capital K_t^F, labor L_t^F and sector-specific technology A_t^F to produce their output. Consumption producing firms buy capital one period in advance. They borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. In order to do so, these firms issue claims to Ricardian consumers at the prevailing price of consumption capital x_t^F. At the end of each production period, they sell the remaining capital back to capital producing firms. The production function of consumption firms is:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_t^F)^{(1-\alpha_F)} \tag{19}$$

According to the share purchase setup, consumption firms supply inelastically one unit of labor to their own production in order to start the accumulation of their net worth. Factoring in this labor supply in the production function, the total labor supply in the consumption sector is:

$$L_t^F = (L_{F,t}^F)^{\Omega_F} (L_{H,t}^F)^{(1-\Omega_F)} = (L_{H,t}^F)^{(1-\Omega_F)} \tag{20}$$

where $L_{H,t}^F$ is the regular labor supply by both types of consumers and $L_{F,t}^F$ is the labor supply by consumption producers.

Recasting the production function only in terms of household labor yields:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)} \tag{21}$$

The firms in the sector are perfectly competitive so they maximize profits subject to input costs. The first-order conditions for optimal capital and labor are:

$$w_t = (1-\alpha_F)(1-\Omega_F)\left(\frac{Y_t^F}{L_{H,t}^F}\right) \tag{22}$$

$$R_t = \frac{\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F}{x_t^F} \tag{23}$$

The wage consumption firms receive for their labor supply is:

$$w_t^F = (1-\alpha_F)\,\Omega_F Y_t^F \tag{24}$$

3.3 Housing Capital Sector

Firms that produce capital K_t^H for the housing sector own technology that converts goods into capital. They purchase depreciated capital from final goods firms in the same sector and obtain investments to produce new capital. The investment I_t^H is consumption. The newly produced capital is sold back to housing producers.

Housing capital is subject to the same adjustment costs as consumption capital. The housing capital production equation is:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right) K_{t-1}^H \tag{25}$$

New capital is produced within the period and sold to final good producing firms at the price x_t^H. The optimal level of investment in housing capital:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1 \tag{26}$$

3.4 Housing Producers

Housing is a multi-period good that survives for more than one period, unlike consumption which is not durable beyond the period in which it is produced. The production of housing follows closely Iacoviello and Neri (2010). Housing producers use capital K_t^H, labor L_t^H, land X_t and sector-specific technology A_t^H to produce new houses. Housing firms also buy capital one period earlier. In this version, housing firms have no aggregate uncertainty so they can borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. In order to do so, these firms issue claims to Ricardian consumers at the prevailing price of housing capital x_t^H. At the end of each production period, they sell the remaining capital back to housing firms. The production function of housing firms is:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (X_{t-1})^{\varepsilon} (L_t^H)^{(1-\alpha_H-\varepsilon)} \tag{27}$$

The amount of land is fixed and normalized to one. Furthermore, both the share purchase setup and the loan contract of credit-constrained consumers assumed that housing producers and indebted households supply inelastically one unit of labor in order to start the accumulation of their respective net worth. Factoring in that labor supply, the total labor supply in the production of housing by origin is:

$$L_t^H = (L_{F,t}^H)^{\Omega_H} (L_{CC,t}^H)^{\Omega_C} (L_{H,t}^H)^{(1-\Omega_H-\Omega_C)} = (L_{H,t}^H)^{(1-\Omega_H-\Omega_C)} \tag{28}$$

Where $L_{H,t}^H$ is the labor supply of both types of consumers for the purpose of financing their regular consumption, $L_{CC,t}^H$ is the labor supply of credit-constrained consumers in order to start their net worth accumulation and $L_{F,t}^H$ is the labor supply of housing producers.

Recasting the production function only in terms of household labor and factoring in the fixed supply of land yields:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)} \tag{29}$$

The price of housing is p_t. Housing firms in the sector are perfectly competitive so they maximize profits subject to input costs obtaining the following optimal conditions for housing capital and labor:

$$w_t = (1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right) \tag{30}$$

$$R_t = \frac{\alpha_H \frac{p_{t+1} Y_{t+1}^H}{K_t^H} + (1-\delta) x_{t+1}^H}{x_t^H} \tag{31}$$

The wage housing firms receive for their labor supply is:

$$w_t^H = (1 - \alpha_H - \varepsilon)\, \Omega_H p_t Y_t^H \tag{32}$$

The wage credit-constrained consumers receive for the purpose of starting their net worth is:

$$w_t^C = (1 - \alpha_H - \varepsilon)\, \Omega_C p_t Y_t^H \tag{33}$$

3.5 Consumers

Both Ricardian and credit-constrained consumers have the same preferences. Households choose consumption C_t^i, housing H_t^i and labor L_t^i subject to their respective budget constraints. Here the superscript i denotes the type of consumers: R for Ricardian and C for credit-constrained. Housing is purchased one period in advance. This approach matches empirical reality better where acquiring a house involves transactional delays that involve search time, time spent with real estate agents and time to process escrow, payment and home insurance. Furthermore, there are financial motivations for the advance purchase. Since credit-constrained consumers purchase housing in advance of using it and they do not have non-wage income like Ricardians, they need the mortgage arrangement to facilitate the housing acquisition. Furthermore, the mortgage is intertemporal where consumers optimize their expected share of the loan at the time of the housing purchase but repay it only in the subsequent period after using the housing. As a result, they are exposed to unexpected shocks at the time of repayment so the mortgage is an inherently risky undertaking.

Each household seeks to maximize its lifetime expected utility:

$$U = E_t \sum_{t=0}^{\infty} \beta^t U(C_t^i, H_{t-1}^i, L_t^i) \tag{34}$$

The period utility of each household is given by:

$$U(C_t^i,\, H_{t-1}^i,\, L_t^i) = log(C_t^i) + \kappa log(H_{t-1}^i) - \gamma \frac{L_t^{i(1+\varphi)}}{1+\varphi} \tag{35}$$

The period utility function is separable in consumption C_t^i, housing H_{t-1}^i and labor L_t^i. Housing is purchased one period in advance and consumed the following period. At the end of the period, the remaining housing minus depreciation is sold back on the market. Following Iacoviello and Neri (2010), housing enters the utility function additively, rather than as part of a consumption aggregator in order to demonstrate its direct effect on consumer decisions. The additive nature of the utility function also facilitates housing to be be purchased both directly (by Ricardians) and via a mortgage (by credit-constrained consumers). There is a taste parameter κ that reflects the relative preference for consumption and housing.

3.5.1 Ricardian Consumers

Ricardian consumers purchase consumption and housing. Each period, they lend an amount B_t at the risk-free rate. Their lending covers the mortgage loan to credit-constrained households as well as finances the capital purchase of consumption firms and housing producers through their respective share arrangements. Ricardian consumers also absorb the profits Π_t of both capital sectors. The budget constraint of Ricardian households is:

$$C_t^R + p_t H_t^R + B_t = w_t L_t^R + (1-\delta) p_t H_{t-1}^R + R_{t-1} B_{t-1} + \Pi_t \tag{36}$$

Ricardian consumers maximize their utility function subject to this budget constraint. The left hand side reflects their consumption and housing purchase as well as their lending, while the right hand side represents their income from wages and from reselling the non-depreciated housing from the previous period as well as their returns from lending and from capital firms profits. The Lagrangian for Ricardian consumers yields three first-order conditions for consumption, housing and leisure. The first-order condition for the consumption-labor tradeoff is fairly standard:

$$\gamma (L_t^R)^\varphi C_t^R = w_t \tag{37}$$

The relationship between consumption and housing reflects the fact that housing is purchased one period in advance so the tradeoff between housing and consumption depends on both the current and on the future price of housing as well as on the intertemporal consumption substitution and the depreciation rate of housing:

$$\left(\frac{p_t}{\beta} E_t \left(\frac{C_{t+1}^R}{C_t^R} \right) - (1-\delta) p_{t+1} \right) H_t^R = \kappa E_t C_{t+1}^R \tag{38}$$

Ricardian consumers also have a standard Euler equation:

$$E_t \left(\frac{C_{t+1}^R}{C_t^R} \right) = \beta R_t \tag{39}$$

Combing the last two equations yields a simpler expression for the consumption-housing substitution:

$$\left(R_t p_t - (1-\delta) p_{t+1} \right) H_t^R = \kappa E_t C_{t+1}^R \tag{40}$$

3.5.2 Credit-Constrained Consumers

Just like Ricardian households, credit-constrained households also consume both consumption and housing. They earn income only from labor and do not own any shares. Since their income is not sufficient to allow them to purchase housing in full, they must obtain a mortgage from investors. Their mortgage is subject to an idiosyncratic risk of default so their borrowing is not riskless. Due to this probability of bankruptcy, they cannot borrow at the risk-free rate R_t and can do so only at the consumer interest rate R_t^C. As a result, buying a house is costly for them. Credit-constrained households also cannot optimize intertemporally their purchase of consumption since they have no access to risk-free financing. Hence their consumption needs must be met solely with their wages after their mortgage is repaid. The budget equation for credit-constrained households is:

$$C_t^C + R_t^C p_{t-1} H_{t-1}^C = w_t L_t^C + (1-\delta) p_t H_{t-1}^C \tag{41}$$

The left hand side reflects their consumption purchase as well as their housing mortgage, while the right hand side represents their income from wages and returns from reselling the non-depreciated housing from the previous period. Credit-constrained consumers maximize their utility function subject to this budget constraint. The optimization problem yields two first-order conditions for housing and leisure. The first-order condition for consumption-labor tradeoff is identical to that of Ricardian households:

$$\gamma(L_t^C)^{\varphi}C_t^C = w_t \tag{42}$$

The relationship between consumption and housing however depends on the consumer interest rate R_t^C instead of the risk-free rate R_t:

$$\left(R_t^C p_{t-1} - (1-\delta)p_t\right) H_{t-1}^C = \kappa C_t^C \tag{43}$$

The first-order condition for housing-consumption tradeoff for credit-constrained consumers is lagged, unlike that for Ricardian households which is forward-looking. This is due to the fact that the return rate at which credit-constrained consumers repay their mortgage is state-contingent and depends on the realization of the ex-post return on housing R_t^C that incorporates all shocks at the time of the repayment. The realized return on housing R_t^C depends on the past purchasing price p_{t-1} and the current selling price p_t of housing. It does not depend on the expected future price of housing which is unknown at the time of the loan contract. Hence equation (43) is a function of the ex-post return on housing R_t^C, while the corresponding equation for Ricardian households (40) is not lagged since Ricardians borrow at the risk-free rate which is not state-contingent.

Finally, credit-constrained consumers cannot optimize intertemporally their consumption purchase since they cannot borrow at the risk-free rate R_t. Their demand for consumption must be met by their income once all housing loans are repaid:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C \tag{44}$$

Any disturbances that can increase the mortgage repayment, would crowd out regular consumption and the resulting diminished demand might impact negatively the consumption sector.

3.6 Market Clearing

Market clearing requires that the output of consumption must cover household consumption, consumption by conventional good producers on exit and consumption by credit-constrained consumers on retirement as well as investment in the two sectors:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + I_t^F + I_t^H \tag{45}$$

Housing is a multi-period good and each period a fraction δ of the housing available on the market depreciates. The remaining non-depreciated housing along with new production constitutes the available housing in the subsequent period. In each period, the sum of new production and leftover housing must meet the housing needs of consumers as well as the consumption of housing firms on exit and the monitoring costs of the mortgage contract:

$$p_t Y_t^H + (1-\delta) p_t (H_{t-1}^R + H_{t-1}^C) = p_t (H_t^R + H_t^C) + C_{E,t}^H + \mu^C G^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{46}$$

The labor that both types of households supply equals the demand by housing and consumption firms:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C \tag{47}$$

Finally, Ricardian consumer lending must equal the loan to credit-constrained households as well as the share purchase of consumption firms and of housing firms:

$$B_t = p_t H_t^C - N_t^C + x_t^F K_t^F - N_t^F + x_t^H K_t^H - N_t^H \tag{48}$$

All the equations for this model are in Appendix A.

3.7 Model Calibration

The calibrated model is quarterly so four periods correspond to a year. The model parameters are chosen to be as close as possible to standard literature values while reflecting a number of key model features. The parameters governing the loan contract are calibrated to match those in Bernanke, Gertler and Gilchrist (1999) and satisfy several steady state conditions:

1. The steady state rate of the external risk premium is 0.5%. (Bernanke, Gertler and Gilchrist, 1999).

2. The steady state housing firm leverage, i.e. value of housing capital stock to net worth ratio is $\frac{x^H K^H}{N^H} = 2$ (Bernanke, Gertler and Gilchrist, 1999).

3. The failure rate of housing manufacturers $F^H(\bar{\omega}^H)$ (i.e. the number of firms that exit the market each period) is 2% (Bernanke, Gertler and Gilchrist, 1999).

The remaining model parameters are chosen to be as close as possible to a representative two-sector economy. (Table 1). Both sectors are identical in their production function. Since housing and consumption enter the utility function additively, the parameter κ determines the consumption-housing tradeoff for consumers taking as given the interest rate and the depreciation rate of housing. Housing depreciates at the same rate as capital, δ. For the temporary volatility shock, the persistence parameter ϱ has a value of 0.95, a conventional choice for the frequency of the model (Fernández-Villaverde, 2010).

Table 1: General Parameters in the Consumer Mortgage Contract Economy

Parameter	α_F	α_H	ε	β	γ	φ	κ	J"	δ	Ω_i
Value	0.35	0.25	0.10	0.99	0.03	0.33	0.41	-10	0.0025	0.01

Solving the first-order condition of credit-constrained consumers, the participation constraint of investors and the cumulative distribution function $F^C(\bar{\omega}^C)$ in the steady state gives the parameters μ^C, ν^C , ν^H, ν^F, the steady state value of the cutoff risk $\bar{\omega}^C$ and the standard deviation σ^C. The credit contract parameters are given in Table 2.

Table 2: Loan Contract Parameters in the Consumer Mortgage Contract Economy

Parameter	ν^F	ν^H	ν^C	μ^C
Value	0.99	0.99	0.98	0.06

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The steady state values of the model variables are in Table 3.

Table 3: Steady State Variables in the Consumer Mortgage Contract Economy

Variable	Y^F	Y^H	C^R	C^C	H^R	H^C	L^R	L^C	I^F	I^H
Value	100	10	32.8	30.2	210	169	15.5	19.9	24.9	3.28
Variable	K^F	K^H	L^F	L^H	p	w	R	R^C	N^F	C_E^F
Value	997	131	29.9	5.48	1.84	2.15	1.01	1.015	498	5.68
Variable			N^H	C_E^H	N^C	C_E^C	$\bar{\omega}^C$	σ^C		
Value			65.7	0.78	156	3.10	0.5	0.31		
Variable	Γ^C	G^C	Γ_ω^C	G_ω^C	Γ_σ^C	G_σ^C	$\Gamma_{\omega\omega}^C$	$G_{\omega\omega}^C$	$\Gamma_{\omega\sigma}^C$	$G_{\omega\sigma}^C$
Value	0.50	0.01	0.98	-0.02	-0.02	0.16	-0.31	2.02	-0.36	1.89

Appendix B outlines the log-linearization of the complete model. The derivations of the log-normal distribution of $\bar{\omega}^C$ are in Appendix E.

4 The Consumer Mortgage Model Results

This section demonstrates the effect of two disturbances that were the major contributors to the subprime mortgage crisis - the oversupply of housing depressed the return on housing and the financial easing permitted credit-constrained consumers to amass too much debt. The oversupply of housing is modeled as a 10% technological innovation in the production of housing. This stylistically simple setup allows generating excess supply of houses without unnecessarily complicating the model with discussions of the preceding housing bubble. Nevertheless, this approach successfully captures the housing inventory buildup that amassed during the boom. This disturbance is simulated as a permanent shock since the excess housing availability began at the peak of the asset bubble and continued during the subsequent recession as the housing demand shrank (Duke, 2012; Madigan, 2012).

The relaxation of borrowing conditions is introduced as a 10% reduction in the volatility of the default risk that made loans seem safer to lenders. The easing permitted borrowers to secure loans with less collateral and as a result to increase their debt holdings. The reduction in financial volatility is a temporary shock corresponding to the brief period of lax lending practices that prevailed on the cusp of the subprime crisis. There are empirical and theoretical underpinnings for the temporary nature of the disturbance. Empirically, the lax access to credit was quickly reversed as the proliferation of defaults signaled the beginning of an economic downturn (Dennis, 2010; Duke, 2012). Theoretically, the model assumes that lenders update the loan terms after observing the borrower risk. A permanent shock would be at odds with the microfounded nature of the loan contract and would imply that the loan terms remain relaxed despite the worsened bankruptcy prospects of borrowers following a leverage buildup. Taken together, the two disturbances demonstrate that positive shocks can have negative consequences for credit-constrained consumer borrowers and that indebted agents can experience a downturn even if they are not production firms or financial institutions.

4.1 The Housing Oversupply

A permanent expansion of the housing stock depresses the price of all houses on the market including the value of the housing collateral held by indebted consumers. Since the value of their housing stock falls relative to their mortgage, households enter a recession characterized by a worsening in their leverage and an increased likelihood of default. Seeing this, lenders require a higher risk premium restricting household access to house financing for a prolonged period of time.

The oversupply of housing affects the leverage of indebted consumers by reducing the value of the mortgage collateral (Figure 2). A permanent positive technological shock in the production of housing raises the supply of houses (Figure 2.2) and creates excess real estate inventory. In the absence of concurrent changes in housing demand, the equilibrium price of housing decreases (Figure 2.17) to accommodate the extra supply. The lower price of housing, however, applies not only to newly produced houses, but also to the existing housing stock that includes the housing equity held by credit-constrained consumers. The housing equity of borrowers consists of their share of housing bought with the loan contract after repaying investors. It depends both on the price of housing and on the realized return on housing that is the mortgage borrowing rate R_t^C. The unanticipated fall in the housing price reduces not only the value of the housing stock but also the return to housing in the period in which the shock occurs (Figure 2.20), i.e. the adjustable rate of the subprime-like loans decreases. Hence the lower price of housing affects the net worth of credit-constrained consumers both directly through the consumer share of real estate holdings and indirectly via the ex-post return on housing R_t^C. In contrast, the value of the loan diminishes only by the drop in the housing price. As a result, the net worth of indebted households (Figure 2.21) falls proportionally more than the value of the mortgage so the leverage of borrowers increases (Figure 2.24). Higher leverage signals increased probability of default for credit-constrained consumers (Figure 2.23).

The aggregate net worth of credit-constrained consumers falls not only because the realized return of housing is low relative to the expected return evaluated one period before, but also because a higher cutoff default risk increases bankruptcy prospects and reduces the share of the mortgage returns that goes to credit-constrained consumers in the loan contract. The investors are now exposed to bigger risks of default on the part of indebted households. To compensate for the rise in the monitoring costs, the investors require a higher return from borrowers, forcing up the external finance premium (Figure 2.29), i.e. the adjustable mortgage rate increases. Consequentially, the falling equilibrium price of housing generates a pro-cyclical risk premium. In both cases, the shock reduces the equilibrium price of housing. Borrowing consumers see the value of their housing collateral fall so they experience a worsening of their bankruptcy prospects.

The worsened indebtedness levels prompt mortgaged households to attempt to repair their net worth but the higher risk premium implies that borrowers can increase their housing stock very gradually. The inability of credit-constrained consumers to easily afford housing after the deterioration of their leverage is the reason housing output makes a dip from its initially high levels (Figure 2.2). The innovation in the housing sector also affects demand for consumption so output decreases (Figure 2.1). Due to the lower price of housing, Ricardian consumers switch away from consumption towards real estate (Figure 2.3). Indebted households initially enjoy a lower repayment rate on their existing loan so they can initially afford slightly more consumption (Figure 2.5). This rise, however, is a token amount so it is not sufficient to compensate for the reduced demand by Ricardians. Furthermore, the higher consumption demand by credit-constrained households is short-lived. The negative effects of the worsening leverage and the widening risk premium soon catch up with them and credit-constrained consumers have to switch away from consumption and toward repairing their net worth (Figure 2.21) and their borrowing abilities. Over time, indebted consumers gradually increase their housing stock (Figure 2.6) which translates into improving net worth (Figure 2.21) and decreasing default prospects (Figure 2.23).

The leverage and probability of default of credit-constrained consumers eventually returns to pre-shock levels owing to their diverting resources away from consumption toward increasing their housing stock (Figures 2.21 and 2.23). However, the recovery of household leverage is a protracted process. Their improvement is relatively slow and takes indebted households about 100 quarters (25 years) to return to their pre-crisis financial position. The simulations demonstrate that the interim period is characterized by a subprime-like recession with higher leverage and increased bankruptcy prospects for credit-constrained households.

4.2 The Temporary Financial Relaxation

Temporarily easing access to house financing for credit-constrained consumers allows them to obtain a mortgage with less downpayment. As a result, their debt holding are higher implying a rise in their bankruptcy prospects. Since the external finance premium reflects the solvency of borrowers, it rises with the increased risk. A higher risk premium tightens the previously relaxed access to credit plunging consumers into a recession despite the short-lived nature of the initial shock.

Leading up to the subprime mortgage crisis, the housing market was characterized by exceptionally favorable borrowing conditions with low interest rates and little downpayment requirements. In the model, this lax lending atmosphere is captured by a negative temporary shock on $\left(\sigma^C\right)^2$, the variance of the consumer default risk ω^C. A lower variance implies narrower swings in the bankruptcy prospects ceteris paribus and signals safer loans (Figure 3). For borrowers, it can translate into a smaller risk premium on loans, less required collateral, or both. The reduction in volatility is modeled as an autoregressive temporary disturbance with moderate persistence to reflect that this phenomenon was relatively short-lived and to demonstrate the worsening of refinancing possibilities as the initial shock dies off. A permanent credit easing, on the other hand, would imply that borrowing terms remain permanently favorable although consumers have raised their leverage, a contradiction to the microfounded nature of the model. The temporary shock reflects that relaxed borrowing conditions prompt mortgagers to raise their leverage leading to a deterioration of their debt position as credit access tightens subsequently.

Reduced financial volatility implies safer loan contracts that require a smaller downpayment and that attract a reduced risk premium (Figure 3.29). The lower risk premium is passed on to credit-constrained consumers as a decreased mortgage interest rate R_t^C (Figure 3.20). Faced with a more favorable borrowing rate, indebted households increase their housing demand (Figure 3.6), driving marginally up the price of housing (Figure 3.17). Housing output also increases being demand driven by consumers (Figure 3.2). The higher housing price raises the value of their housing stock and positively influences their net worth immediately after the shock. However, the downward effect of the reduced consumer rate is quantitatively larger than the upward influence of the housing price so the net worth of borrowers eventually diminishes below pre-shock levels (Figure 3.21). The falling net worth, coupled with the subsequent credit tightening as the temporary shock wears out, worsen the leverage of credit-constrained households (Figure 3.24). Higher leverage signals increased probability of bankruptcy despite the improved financial conditions (Figure 3.23).

Seeing their indebtedness increase, credit-constrained consumers attempt to deleverage. They reduce their housing demand after the initial increase in an attempt to improve their net worth (Figures 3.6 and 3.21). The lower demand for housing reduces housing production shortly after the initial increase (Figure 3.2). It takes 100 quarters (25 years) of gradual improvement for their debt position to recover and for their bankruptcy prospects to return to pre-shock levels (Figures 3.23 and 3.24). The recovery is staggered since the higher refinancing rate restricts the ability of debtors to easily improve their housing stock which in turn maintains a higher refinancing rate. Overall, the effects of reduced default risk volatility are far from beneficial to indebted consumers. The initial benefits of relaxed borrowing conditions are quickly overshadowed by the tightening of credit access as the shock wears out. The result is rising consumer leverage and increased bankruptcy prospects. Even a temporary easing of lending has long-lasting consequences for the debt position of credit-constrained households. The volatility shock, just as the production shock, shows that positive disturbances can have negative effects on the indebtedness of households since they permit them to increase their leverage. It also demonstrates that non-production agents, such as consumers, can be the source of financial disturbances when they are leveraged.

Figure 2: Housing Oversupply in Economy with Credit-Constrained Consumers

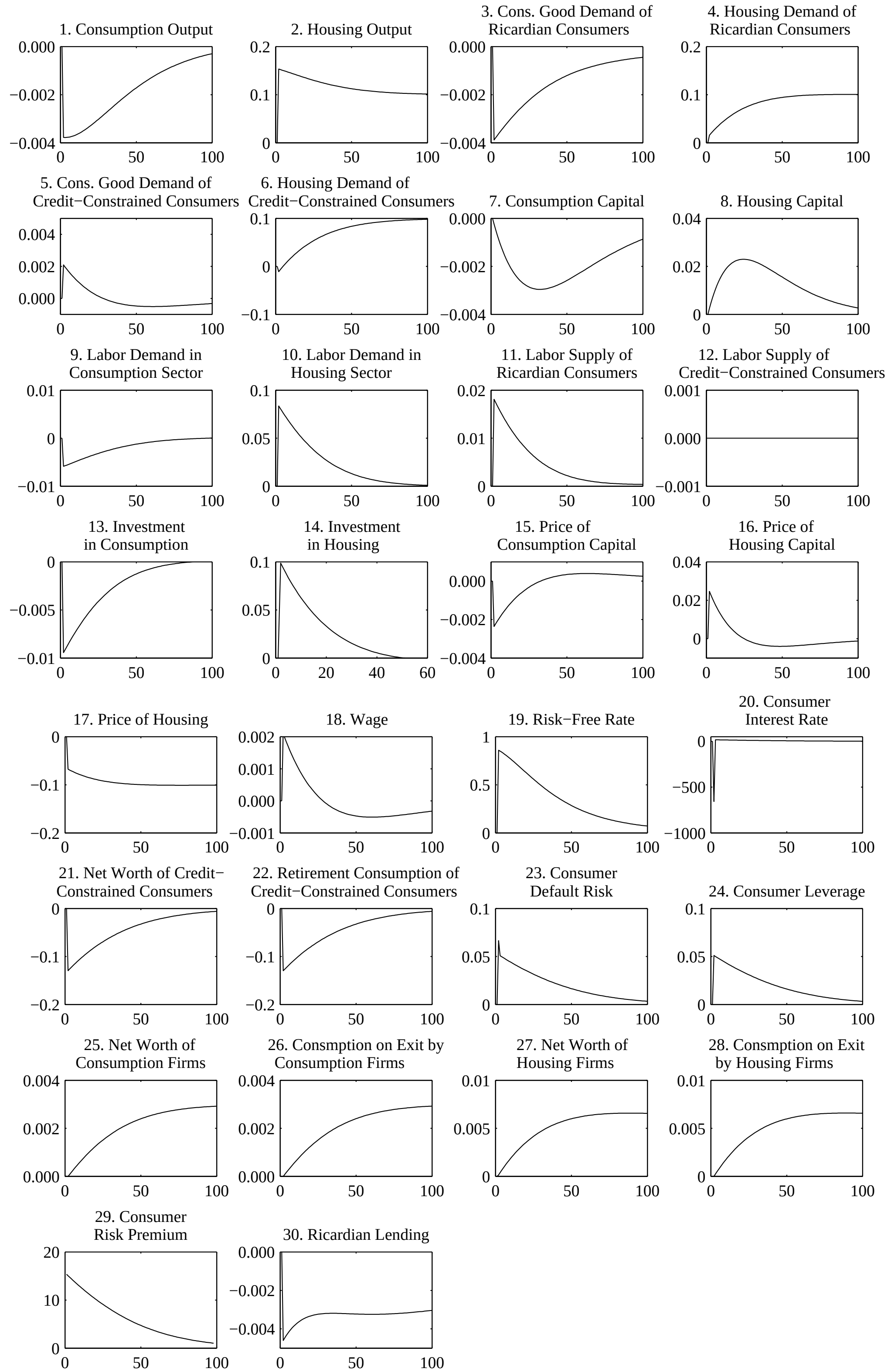
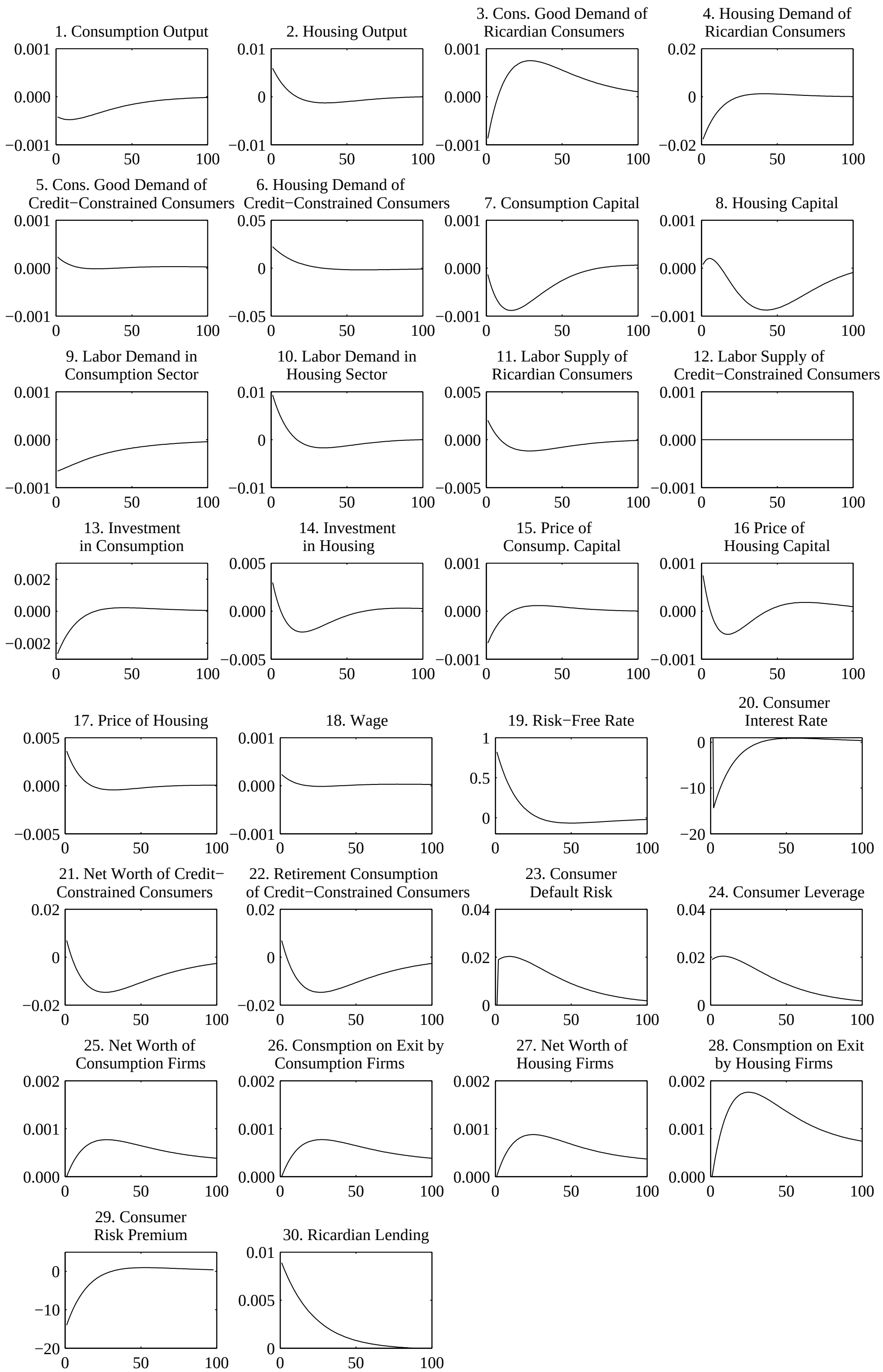


Figure 3: Reduced Volatility in Economy with Credit-Constrained Consumers



5 The Chained Loan Contracts

5.1 Description of the Chained Loan Contracts

The model in the previous sections considered credit frictions in the loan contract between investors and credit-constrained consumers. This extension adds financial institutions to the mortgage contract. The model assumes that financial institutions act as intermediaries that borrow funds from investors and in turn lend to credit-constrained households. Both banks and borrowing consumers are subject to idiosyncratic uncertainty and participate in loan contracts. Taken together, the two transactions constitute two joined mortgage contracts similar to the ones in Hirakata, Sudo and Ueda (2009).

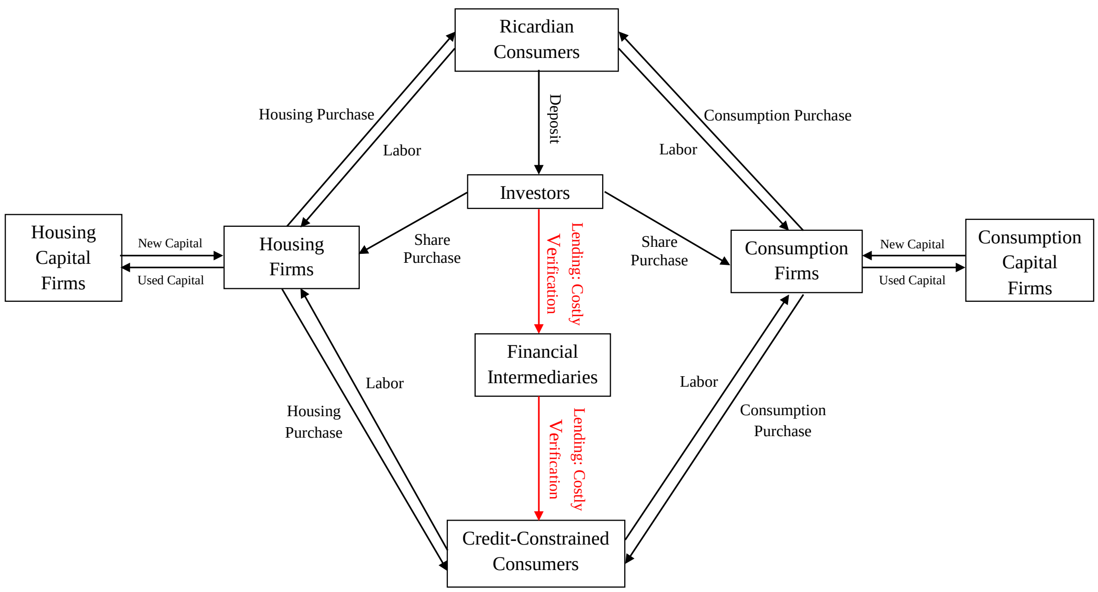


Figure 4: Model with Financial Frictions in the Consumer and Financial Sectors

In this extension, banks borrow from investors and in turn lend to households. Unlike investors whose sole role in the model is to facilitate the lending of Ricardian consumers and who do not possess intrinsic risk, in this version banks have an inherent probability of default just as credit-constrained consumers do but their risk is separate from that of borrowers (Figure 4). The distinction between investors and banks is not purely a model complication but mirrors real life financial markets. Investors can be characterized as representing "safe" mutual funds institutions that possess no idiosyncratic risk, while financial institutions correspond to investment banks that are highly leveraged. The emergence of such highly leveraged banks may be traced back to the desire for larger profits from riskier investments which safe financial agents such as the investors in this model would be unwilling to finance directly.

The presence of idiosyncratic uncertainty in the returns of banks and households necessitates two loan contracts that provide state-contingent returns to investors. At the beginning of the financial intermediation, investors lend to financial institutions in a loan arrangement that ensures them a fair return regardless of the realization of the idiosyncratic shock of banks. At the tail end of the financial intermediation, credit-constrained consumers borrow from financial institutions as part of another loan contract that ensures their participation. Banks are the intermediaries that participate in both loan contracts so they optimize both arrangements at the same time. This is also consistent with empirical observations of real life banks. Hence the two loan contracts must be chained in the sense that they are linked in the same optimization problem. The remainder of the paper explores the role of leveraged financial institutions in magnifying the effect of exogenous disturbances.

5.2 The Chained Loan Contracts

Financial institutions (i.e banks) borrow from investors one period in advance and lend to mortgage-buying consumers to finance their housing purchase. Both banks and credit-constrained consumers are borrowing-constrained in the sense that they each have a distinct probability of default that is known to them but unknown to other participants in the financial market who know only the distribution of the bankruptcy likelihood. Hence there are financial frictions both in the contract between investors and financial institutions and in the contract between banks and credit-constrained households. The presence of credit constraints necessitates that both arrangements are loan contracts where the returns are contingent on the realization of the two idiosyncratic shocks. Since financial institutions are intermediaries and participate in both contacts, the contracts are chained so that the returns from the first contract can provide sufficient funds to cover the lending in the second one.

In this version, there is an idiosyncratic shock ω^B associated with lending to financial institutions and a distinct idiosyncratic shock ω^C associated with lending to credit-constrained consumers. Similar to the financial institutions' loan contract, investors have to pay an auditing fee μ^B to learn the realization of ω^B. This makes lending to banks risky so financial institutions have to pay a premium on external funds. Financial institutions on their part, must pay a monitoring cost μ^C to learn the realization of the consumer default probability ω^C.

Since the individual optimization problem of each bank can be aggregated to hold for the whole economy in the same way as before, the chained contracts setup proceeds directly on the aggregate level. Banks borrow funds from investors and in turn lend to credit-constrained households. Every period, they choose the optimal pair of cutoff risk $\bar{\omega}^B$ and housing H^C to maximize their next period expected share $1 - \Gamma^B(\bar{\omega}^B_{t+1})$ of the total value of the contact that consists of the housing stock $p_t H^C_t$ minus the net worth of credit-constrained consumers N^C_t:

$$max\ E_t \left(\left(1 - \Gamma^B(\bar{\omega}^B_{t+1})\right) R^B_{t+1} \right) \left(p_t H^C_t - N^C_t \right) \tag{49}$$

Banks lend to households at a state contingent markup rate $E_t R^C_{t+1}$ that is different from the rate $E_t R^B_{t+1}$ at which they repay their loans. The difference accounts for the distinct probabilities of default of both types of indebted agents. The expected earnings of financial institutions from lending to credit-constrained consumers equal the share $E_t\Gamma^C(\bar{\omega}^C_{t+1})$ they will receive tomorrow from the loan made today to households minus the auditing fee on insolvent consumer loans $\mu^C E_t G^C(\bar{\omega}^C_{t+1})$:

$$E_t\left(\left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1})\right) R^C_{t+1}\right) p_t H^C_t$$

The earnings of banks must equal the opportunity cost of lending to credit-constrained consumers, which is the value of the loan to banks at their markup interest rate $E_t R^B_{t+1}$. Hence the gross return on the banks' loan to credit-constrained consumers is:

$$E_t\left(\left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1})\right) R^C_{t+1}\right) p_t H^C_t = E_t R^B_{t+1}\left(p_t H^C_t - N^C_t\right) \tag{50}$$

The left hand side is the banks' share of the loan to borrowing households after monitoring and the right hand side is the gross return (the value of the housing purchase $p_t H^C_t$ minus the consumers' net worth N^C_t valued at the bank interest rate $E_t R^B_{t+1}$) from the housing purchase to financial institutions.

Credit-constrained households will participate in the chained loan contracts only if their participation constraint is met. Instead of taking part in the chained loan contracts, credit-constrained consumers can purchase housing using their own net worth N^C_t. In this alternative case, the ex-post return to their investments equals $E_t R^C_{t+1} N^C_t$. Hence credit-constrained consumers will participate in the chained contracts only if their share of the loan is at least equal to the value of their net worth:

$$E_t\left(\left(1 - \Gamma^C(\bar{\omega}^C_{t+1})\right) R^C_{t+1}\right) p_t H^C_t \geq E_t R^C_{t+1} N^C_t \tag{51}$$

The first part of the chained contracts consists of investors lending to banks. Financial institutions split the gross profit from their loan to credit-constrained consumers with investors. This contract has the same costly state verification structure as the single loan contract. Banks own the net worth N^B_t and invest the amount $p_t H^C_t - N^C_t$ in the loan to credit-constrained consumers. They borrow the rest $p_t H^C_t - N^C_t - N^B_t$ from investors and repay the loan using their profits from lending to credit-constrained households. Investors receive a net share of the loan to banks $E_t\left(\Gamma^B(\bar{\omega}^B_{t+1}) - \mu^B G^B(\bar{\omega}^B_{t+1})\right)$ that includes the monitoring fees paid on failed banks. Financial institutions are subject to an idiosyncratic bankruptcy shock ω^B and their ex-post gross return on the loans to credit-constrained consumers is $\omega^B R^B$. Investors would participate in the chained contracts only if they get a fair return on their lending to financial institutions. Like in the single contract model, the lender's share of the profit in the contract with financial institutions $E_t\left(\Gamma^B(\bar{\omega}^B_{t+1}) R^B_{t+1}\right)\left(p_t H^C_t - N^C_t\right)$ after paying the monitoring fee $\mu^B E_t G^B(\bar{\omega}^B_{t+1})$ must at least equal the opportunity cost of the investors' funds $R_t\left(p_t H^C_t - N^C_t - N^B_t\right)$ valued at the risk-free rate R_t. Hence the zero profit participation constraint for investors must specify the amount of funds that banks borrow from investors $pH^C - N^C - N^B$, the cut-off value of the idiosyncratic shock $\bar{\omega}^B$ and the return rate of the loan to non-defaulting banks R^B. Since banks borrow at time t and repay the funds at $t+1$, the participation constraint of investors is:

$$E_t\left(\left(\Gamma^B(\bar{\omega}^B_{t+1}) - \mu^B G^B(\bar{\omega}^B_{t+1})\right) R^B_{t+1}\right)\left(p_t H^C_t - N^C_t\right) \geq R_t\left(p_t H^C_t - N^C_t - N^B_t\right) \tag{52}$$

Lenders sign contracts with a lot of banks, to diversify away the idiosyncratic risk of financial institutions.

Substituting equation (50) into (52) and noting that both equations (51) and (52) bind at the optimum, eliminates the bank interest rate $E_t R^B_{t+1}$ and reduces the conditions that financial institutions must satisfy to two:

$$E_t \left(\left(\Gamma^B(\bar{\omega}^B_{t+1}) - \mu^B G^B(\bar{\omega}^B_{t+1}) \right) \left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1}) \right) R^C_{t+1} \right) p_t H^C_t = \\ = R_t \left(p_t H^C_t - N^C_t - N^B_t \right) \tag{53}$$

$$E_t \left(1 - \Gamma^C(\bar{\omega}^C_{t+1}) \right) p_t H^C_t = N^C_t \tag{54}$$

In the previous loan contract, credit-constrained consumers maximized the arrangement. The superior knowledge of borrowers about their own possibility of default allowed them to push lenders to their participation constraint and extract maximum returns from the loan. The chained loan contracts, however, are linked by the presence of financial institutions. Hence it is the intermediaries (financial institutions) which maximize their profits subject to satisfying the participation constraints of both credit-constrained consumers and of investors. In this case banks, which are borrowers in one part of the contract and lenders in the other part, optimize the two sides of the contract. The reason for banks maximizing their profits is twofold. First, it is empirical fact that financial institutions are often the ones which dictate both the lending and the borrowing terms. The reason for that may be informational asymmetry. Banks use monitoring technology to collect information on borrowers and lenders that would reduce the agency cost associated with lending. This process is costly and location-specific (a bank would not be willing to lend outside of its geographic and sectoral area of expertise). On the other hand, investors, who are geographically dispersed and consumers who lack the means find the cost of this monitoring technology prohibitive. Second, financial institutions, as the intermediaries that participate in both parts of the loan contract, have a transactional advantage. It is easier for them to optimize the two loan contracts together rather than optimizing only their borrowing arrangements with investors and leaving the second loan contract to consumers. The subsequent analysis demonstrates that the results from the chained contracts model are not analytically different from those of two separate loan arrangements.

In the chained contracts model, the presence of two sectors with idiosyncratic uncertainty has important implications for risk sharing between the indebted banks and households. As intermediaries in the two mortgage contracts, financial institutions are the first to experience a deterioration in their balance sheet. However, since they also maximize the chained contracts, they can shift some of the burden of the increased risk of default onto credit-constrained consumers. As end participants in the mortgage arrangement, households have no control over the leverage distribution between indebted sectors. In times of a downturn characterized by worsened leverage for financial institutions, they can improve their own position by forcing consumers to bear a share of the deteriorating leverage. Banks achieve this by extracting a larger return from the loan contract with households in times when the default probabilities of both borrowers are higher. The participation constraint of households, unlike the share agreement in the single loan contract of financial institutions, depends on changes in bankruptcy prospects. The increased bankruptcy prospects allow financial institutions to demand a higher repayment thus shifting the participation constraint (54) of consumers to the left. This can work because, in contrast to the share contract, consumers in the chained mortgage contracts have no recourse to funds outside of the borrowing arrangement due to their idiosyncratic uncertainty. Hence credit-constrained consumers have no choice but to bear some of the deterioration in indebtedness. Oftentimes, this affords banks a speedier recovery at the expense of indebted households who are faced with a protracted recession.

The chained loan optimization problem involves banks simultaneously maximizing both loan arrangements. Let for simplicity of expression $\Gamma^B(\bar{\omega}_t^B) - \mu^B G^B(\bar{\omega}_t^B) = \Psi^B(\bar{\omega}_t^B)$ and $\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C) = \Psi^C(\bar{\omega}_t^C)$. Banks choose the optimal level of $\bar{\omega}^C$, $\bar{\omega}^B$ and H^C by solving the following Lagrangian:

$$\begin{aligned} L_t^B \quad = \quad & E_t\left(\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right)p_t H_t^C + \\ & +\lambda_{1,t}\left[E_t\left(\Psi^B(\bar{\omega}_{t+1}^B)\Psi^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right)p_t H_t^C - R_t\left(p_t H_t^C - N_t^C - N_t^B\right)\right] \\ & +\lambda_{2,t}\left[E_t\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)p_t H_t^C - N_t^C\right] \end{aligned}$$

The first order conditions yield two equations for the Lagrange multipliers:

$$\lambda_{1,t} = E_t\left(\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\Psi_\omega^B(\bar{\omega}_{t+1}^B)}\right) \tag{55}$$

$$\lambda_{2,t} = E_t\left[\left[\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right) + \frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)\Psi^B(\bar{\omega}_{t+1}^B)}{\Psi_\omega^B(\bar{\omega}_{t+1}^B)}\right] R_{t+1}^C \frac{\Psi_\omega^C(\bar{\omega}_{t+1}^C)}{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}\right] \tag{56}$$

Here $\lambda_{1,t}$ is the marginal value of the internal funds to financial institutions, and $\lambda_{2,t}$ is the marginal increase in the profits of financial institutions per unit increase in the net worth of credit-constrained consumers.

Substituting both into the first order condition for H_t^C and rearranging gives the consumer risk premium $\frac{E_t R_{t+1}^C}{R_t}$:

$$\begin{aligned} \frac{E_t R_{t+1}^C}{R_t} \quad = \quad & E_t\left(\frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)\Psi_\omega^C(\bar{\omega}_{t+1}^C) + \Gamma_\omega^C(\bar{\omega}_{t+1}^C)\Psi^C(\bar{\omega}_{t+1}^C)}\right) \times \\ & E_t\left(\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi_\omega^B(\bar{\omega}_{t+1}^B) + \Gamma_\omega^B(\bar{\omega}_{t+1}^B)\Psi^B(\bar{\omega}_{t+1}^B)}\right) \end{aligned} \tag{57}$$

Equation (57) along with the two participation constraints determines the realized default probabilities $\bar{\omega}^B$ and $\bar{\omega}^C$. Even after a shock occurs, investors continue to receive the risk-free rate R_t so the model implements the realized participation constraints:

$$\begin{aligned} & \left(\Gamma^B(\bar{\omega}_t^B) - \mu^B G^B(\bar{\omega}_t^B)\right)\left(\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C = \\ & = R_{t-1}\left(p_{t-1} H_{t-1}^C - N_{t-1}^C - N_{t-1}^B\right) \end{aligned} \tag{58}$$

$$\left(1-\Gamma^C(\bar{\omega}_t^C)\right) p_{t-1} H_{t-1}^C = N_{t-1}^C \tag{59}$$

along with the first order condition that contains the expectation of what will occur at $t+1$.

The period equity of credit-constrained consumers remains the same as in the consumer mortgage contract. The period equity of financial institutions is amended to include the return to banks $\Psi^C(\bar{\omega}_t^C)$ from lending to households:

$$\begin{aligned} V_t^B &= \left(1-\Gamma^B(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1}H_{t-1}^C - N_{t-1}^C\right) = \\ &= \left(1-\Gamma^B(\bar{\omega}_t^B)\right) \Psi^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \end{aligned} \tag{60}$$

The consumption on retirement of borrowing households remains the same as in the consumer mortgage contract. However, the consumption of banks $C_{E,t}^B$ on exit becomes:

$$C_{E,t}^B = \left(1-\nu^B\right)\left(1-\Gamma^B(\bar{\omega}_t^B)\right) \Psi^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{61}$$

6 The Complete Model with Chained Loan Contracts

6.1 Complete Model Changes

The production sectors and consumer behavior remain the same. The introduction of financial institutions necessitates some changes to accommodate their presence. These involve the labor supply of banks in the housing sector and the market clearing conditions for consumption and housing. The production functions in both sectors, the capital development and the goods demand by consumers remain unchanged.

The labor supply equation in the housing sector must account for the unit labor supply by financial institutions denoted $L_{B,t}^H$. Factoring in that labor supply, the total labor supply in the production of housing by origin is:

$$L_t^H = (L_{F,t}^H)^{\Omega_H}(L_{CC,t}^H)^{\Omega_C}(L_{B,t}^H)^{\Omega_B}(L_{H,t}^H)^{(1-\Omega_H-\Omega_C-\Omega_B)} = (L_{H,t}^H)^{(1-\Omega_H-\Omega_C-\Omega_B)} \tag{62}$$

where $L_{F,t}^H$ is the labor supply by housing firms, $L_{CC,t}^H$ the labor supply by credit-constrained consumers dedicated to starting their net worth, $L_{B,t}^H$. The labor supply by financial institutions and $L_{H,t}^H$ is regular household labor supply.

The production function for housing becomes:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C-\Omega_B)} \tag{63}$$

The optimal condition for housing capital remains the same while that for labor becomes:

$$w_t = (1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right) \tag{64}$$

The market clearing condition for consumption includes the consumption of financial institutions on exit $C_{E,t}^B$ in addition to that of consumers $C_{E,t}^C$:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + C_{E,t}^B + I_t^F + I_t^H \tag{65}$$

The market clearing condition for housing is augmented by the monitoring fee paid by investors on their loan to banks $\mu^B G^B(\bar{\omega}_t^B) R_t^B \left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right)$ which is in addition to the one investors paid on their lending to credit-constrained consumers:

$$\begin{aligned} p_t Y_t^H + (1-\delta) p_t (H_{t-1}^R + H_{t-1}^C) &= p_t (H_t^R + H_t^C) + C_{E,t}^H + \\ &+ \left[\mu^C G^C(\bar{\omega}_t^C) + \mu^B G^B(\bar{\omega}_t^B) \Psi^C(\bar{\omega}_t^C)\right] R_t^C p_{t-1} H_{t-1}^C \end{aligned} \quad (66)$$

A complete list of the model equations is in Appendix C.

6.2 Model Calibration

The parameters that govern the general equilibrium for the chained contracts model are the same as those in the consumer mortgage contract. The chained contracts model satisfies the same steady state requirements as the respective single loan contracts for consumers and for financial institutions:

1. The steady state rate of the external consumer risk premium is 0.5% (Bernanke, Gertler and Gilchrist, 1999).

2. The steady state consumer leverage, i.e. value of housing stock to net worth ratio is $\frac{pH^C}{N^C} = 2$ (Bernanke, Gertler and Gilchrist, 1999).

3. The steady state leverage of financial institutions is $\frac{pH^C - N^C}{N^B} = 5$ (Hirakata, Sudo and Ueda, 2009).

4. The failure rate of both credit-constrained consumers $F^C(\bar{\omega}^C)$ and financial institutions $F^B(\bar{\omega}^B)$ is 2% (Bernanke, Gertler and Gilchrist, 1999).

The loan parameters that satisfy the chained contracts model are in Table 4.

Table 4: Loan Contract Parameters in the Chained Contracts Economy

Parameter	ν^F	ν^H	ν^C	ν^B	μ^C	μ^B
Value	0.99	0.99	0.98	0.97	0.02	0.03

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The steady state values of the model variables are in Table 5.

Table 5: Steady State Variables in the Chained Contracts Economy

Variable	Y^F	Y^H	C^R	C^C	H^R	H^C	L^R	L^C	I^F	I^H
Value	100	10	32.5	29.9	210	169	15.6	19.9	24.9	3.35
Variable	K^F	K^H	L^F	L^H	R^C	R^B	N^F	C_E^F	N^H	C_E^H
Value	997	134	30.0	5.53	1.015	1.0147	498	5.68	67.0	0.80
Variable	N^C	C_E^C	N^B	C_E^B	$\bar{\omega}^C$	σ^C	Γ^C	G^C	Γ_ω^C	G_ω^C
Value	159	2.53	31.9	1.11	0.5	0.31	0.50	0.01	0.98	0.15
Variable	Γ_σ^C	G_σ^C	$\Gamma_{\omega\omega}^C$	$G_{\omega\omega}^C$	$\Gamma_{\omega\sigma}^C$	$G_{\omega\sigma}^C$	$\bar{\omega}^B$	σ^B	Γ^B	G^B
Value	-0.02	0.16	-0.31	2.02	-0.36	1.89	0.79	0.11	0.79	0.02
Variable	Γ_ω^B	G_ω^B	Γ_σ^B	G_σ^B	$\Gamma_{\omega\omega}^B$	$G_{\omega\omega}^B$	$\Gamma_{\omega\sigma}^B$	$G_{\omega\sigma}^B$	p	w
Value	0.98	0.45	-0.04	0.74	-0.57	10.8	-0.97	14.4	1.88	2.15
Variable	Ψ^B	Ψ_ω^B	Ψ_σ^B	$\Psi_{\omega\omega}^B$	$\Psi_{\omega\sigma}^B$	Ψ^C	Ψ_ω^C	Ψ_σ^C	$\Psi_{\omega\omega}^C$	$\Psi_{\omega\sigma}^C$
Value	0.79	0.97	-0.05	-0.83	-1.32	0.50	0.98	-0.02	-0.36	-0.41

The log-linearization of the additional equations is in Appendix D.

7 The Chained Loan Contracts Results

This section compares the effect of an oversupply of housing and of a reduction in financial volatility in a chained contracts economy with both single contract versions where either credit-constrained consumers or financial institutions are the subject to credit constraints.

7.1 The Housing Oversupply

Previously consumers experienced a downturn following an increase in housing inventory that reduced the market price of housing. Chained loan contracts provide more than a single loan contract when they are the sole indebted borrowers. As a result, a disturbance that reduces the value of the housing stock generates a smaller improvement in their leverage than in the share contract version but nevertheless puts them in a more favorable position than before. When consumers participate in the chained loan contracts, intermediary financial institutions transfer some of their leverage worsening onto households in order to repair their own debt position faster.

This version again simulates a permanent technological improvement in the production of housing. However, since both types of agents are credit-constrained the combined leverage in the economy is larger contributing to more volatility. The presence of credit frictions in both the financial sector and the household sector does not change the directional impact of the housing innovation shock on households and on producers but creates opportunities for risk sharing driven by the larger joint leverage (Figure 5). The oversupply of housing still triggers a reduction in the housing price that raises consumer demand for housing (Figures 5.4 and 5.6). The return to housing decreases and the value of the housing stock held as collateral falls (Figures 5.17 and 5.18). However, the chained model version yields qualitatively and quantitatively different effects on the leverage of indebted agents. Credit-constrained consumers experience a mild improvement in their debt position while leveraged financial institutions enter a considerable recession (Figures 5.25 and 5.26). The diverging experiences can be attributed to the relative position in the chained contracts of both borrowers and their initial leverage ratios. As optimizers of the chained contracts, banks must meet the participation constraint of both investors and households so they absorb most of the adverse effects of the shock. Their leverage deteriorates by as much as in their single loan contract.

Consumers know that banks will honor their participation constraint so they are relatively shielded from the adverse effects of the falling housing price on their equity. Their net worth still diminishes but by less than in the single loan contract when credit-constrained consumers had to meet the participation constraint of investors. Overall, the decrease in the net worth of credit-constrained consumers (Figure 5.19) is proportionally less than the fall in the value of their housing stock so households experience a small improvement in their leverage position. The reduced leverage, however, is sufficient to imply lower bankruptcy prospects despite the higher risk premium (Figure 5.23). Hence the chained contracts model creates opportunities for risk sharing between indebted agents. Consumers are not as insulated from the downturn despite the presence of financial intermediaries.

Financial institutions, on the other hand, experience considerable deterioration in their leverage. Although the combined worsening of consumer and bank net worth is larger in the chained contracts model than in the single contract version, banks shift some of the burden of the downturn onto consumers. This is due to the fact that as maximizers of the chained contracts, financial institutions can force consumers to the break-even point of their participation constraint and extract a higher return from them. Despite this, they still must satisfy the participation constraints of both lenders (investors) and final borrowers (credit-constrained consumers) so their ability to share risk is limited. As a result, the net worth of banks bears most of the devaluation in the housing equity and their leverage ratio deteriorates accordingly (Figures 5.20 and 5.26). Unlike in the single loan contract where financial institutions are exposed only to their own idiosyncratic risk, here they are exposed both to their own risk of the default and that of credit-constrained households. Hence their bankruptcy prospects deteriorate more than in the single contract (Figure 5.24). However, due to the risk sharing, banks see a smaller rise in their risk premium. This implies that despite the dual risks in the chained contracts, the worsening in the leverage position of financial institutions is not proportionally larger (Figure 5.26). The opportunity for risk sharing allows banks to dampen their recession at the expense of mortgaged households.

Comparing the subsequent recovery of the optimizing agents in the two versions, it is clear that financial institutions return to their pre-shock position faster than consumers. Banks start with a higher leverage which necessitates a speedy recovery in order to maintain their solvency. Furthermore, they are the ones that maximize the loan contracts so they can recover faster than mortgaged households, who have no say in the loan arrangement and no recourse to outside funds (Figures 5.26 and 5.25). The leverage of financial institutions in the chained contracts version takes about 60 quarters (15 years) to return to pre-shock levels while that of credit-constrained consumers in the single contract remains above its steady state value for almost 100 quarters (25 years). The default risk mirrors the respective leverage evolution of the two credit-constrained agents (Figures 5.24 and 5.23).

The single loan contract models and the chained contracts version offer insight into the contrasting experiences of credit-constrained households. In the single consumer mortgage model they maximize the loan contract, and in the chained contracts extension they are final borrowers whose participation constraint must be satisfied by financial institutions. As maximizers in the first version of the model, indebted households bear fully the negative consequences of the oversupply of houses, while in this case they are relatively shielded by the binding of their participation condition or the share agreement with financial institutions. In the chained contracts version, they are part of the joint mortgage contract where they are exposed to bankruptcy prospects but the binding of their participation constraint still guarantees them a degree of protection from the recession, albeit on a smaller scare than in the financial institutions' loan contract. The chained contracts model is more favorable toward credit-constrained consumers than the single mortgage version. However, it is not preferable since the worsening of the debt position of banks is quantitatively larger making the economy as a whole more volatile. Its only redeeming quality is that it results in a relatively short-lived downturn compared to the single loan contract economy since financial institutions use their role as optimizers to repair their leverage position relatively fast. What used to be a moderate but protracted credit crunch in the consumer loan setup becomes a steep but short-lived financial downturn in the chained loans version. Nevertheless, in both scenarios the falling housing price can act as a vehicle of shock propagation that turns a positive innovation in the housing sector into the main cause of debt worsening for credit-constrained agents that characterizes a downturn such as the subprime mortgage crisis.

The models described so far, although, theoretical in nature, have important policy implications. The three versions of credit frictions demonstrate the importance of sound financial regulation. In order to protect consumers from an unduly high burden of risk sharing from the financial system, policymakers should consider regulating the maximum permissible leverage ratio for borrowers. However, mandating a debt-to-income cap only for indebted households may be of limited usefulness when banks are also leveraged. As intermediaries in the mortgage contract, financial institutions usually enter the loan arrangement with much less equity and experience a more volatile leverage than households. Their excessive indebtedness in the chained contracts version causes a far larger drag on economic performance than that of consumers in the single mortgage model so it is important that there are sufficient loan loss provisions and reserve requirements in place to guarantee the liquidity of the financial system in a downturn.

7.2 The Temporary Financial Relaxation

In the chained contracts setting, both consumers and financial institutions can experience a credit easing. Relaxing borrowing access for households spreads the resulting leverage worsening across both types of credit-constrained agents but the combined downturn is deeper than that of households in their sole borrowing arrangement due to the larger combined leverage. However, a financial relaxation for banks implies stronger risk sharing that is more tilted toward consumers. Banks struggle to repair their debt positions after the credit easing in their own sector so they shift a large share of the downturn onto households. Credit-constrained consumers suffer the negative consequences of a reduction in bank volatility with the same magnitude as following a decrease in their own volatility, although in this case they are not the originators of the disturbance. While the downturn is sharp but brief for banks, consumers are drawn into a prolonged recession.

Unlike the single contract model, the chained contracts setup allows to study both versions of a relaxation in borrowing conditions - one that targets credit-constrained consumers and one that pertains to financial institutions - within the same model. A reduction in the volatility of consumer borrowing in the chained contracts model yields qualitatively different results than the same disturbance in the single contract version (Figure 6). The decreased consumer volatility in the chained contracts version reduces not only the household risk premium but also the risk premium of banks since the two loan contracts are linked in the same maximization problem (Figures 6.27 and 6.28). The reduction in risk premium is passed from consumers to financial institutions. As a result, households experience a smaller reduction in the external finance premium than in the consumer loan contract.

The improvements in consumer credit access also affect financial institutions which, as intermediaries in the loan contract, need less downpayment to secure funding from investors on behalf of consumers (Figure 6.20). Less net worth implies higher indebtedness both for financial institutions and for households. However, as optimizing agents in the chained loan contract, financial institutions experience most of the consequences of the collateral reduction on their own debt position (Figure 6.26). Their indebtedness rises more than that of consumers but less than the corresponding increase in the single loan contract version. The presence of two chained loan contracts allows them to share the downturn with consumers. The leverage of credit-constrained consumers rises as well but by only half as much as in their own mortgage arrangement described (Figure 6.25). Overall, the risk sharing in the chained loan contract dampens the effect of the shock, spreading it over both types of indebted agents while at the same time concentrating a larger share in the intermediary sector that optimizes the dual contracts. The default probability of both agents also increases but by less than before (Figures 6.23 and 6.24).

Taken together, the worsening in the leverage of both credit-constrained consumers and banks is larger than that of only consumers in the single loan contract. This is due to the higher initial indebtedness of banks which reflects on both agents. Despite the large combined downturn in the chained contracts model, the debt position of financial institutions recovers relatively fast and about 70 quarters (17.5 years) after the shock, their leverage ratio and bankruptcy prospects return to pre-shock levels (Figure 6.26). Bank recovery, however, is at the expense of the revival of the borrowing ability of households. As maximizers of the chained contracts, financial institutions can push consumers to their participation constraint so most of the subsequent improvements in housing equity benefit them at the cost of delaying the recovery of mortgaged households. The indebtedness of credit-constrained consumers unwinds slowly and remains above its pre-shock level for close to 100 quarters (25 years) (Figure 6.25). Risk sharing implies that the net worth of banks and their leverage position improve relatively fast, while consumers are faced with a protracted recovery.

A model with relaxed credit access for financial institutions produces the deepest recession of all scenarios discussed so far (Figure 7). Following a reduction in their own volatility, financial institutions see a smaller increase in their leverage than when they are the only borrowers since some of the increase is transferred onto consumers (Figures 7.25 and 7.26). Unlike the single loan contract version where the arrangement between consumers and financial institutions is a share one, here households participate in the chained contract creating opportunities for risk sharing. Consumers have no ability to borrow outside of the loan contract due to their idiosyncratic uncertainty so banks can afford to transfer some of the downturn onto them without turning them away from the mortgage purchase. Hence households are forced to absorb some of the deterioration in the balance sheets of banks although they are not the source of the disturbance. Following a shock to the volatility of banks, the leverage of mortgaged households rises by as much as when their own volatility is reduced in the same chained contracts setting (Figure 7.25).

The leverage of banks, however, rises more than in the previous case with consumer credit easing. Since the default probability of financial institutions is initially much higher than that of households, a reduction in their own volatility leads to a more significant tightening of their financial conditions. In order to mitigate this threat to their own solvency, financial institutions shift a considerable share of the burden onto households. Despite the risk sharing, the higher pre-shock indebtedness of financial institutions implies that their leverage worsens slightly more than when they are faced with a reduction in consumer volatility. As a result, the combined worsening of financial positions of both credit-constrained agents is the most severe of all considered scenarios. The subsequent recovery again benefits financial institutions more than credit-constrained consumers. As optimizers of the chained loan contract, financial institutions can repair their debt position faster at the expense of consumers who experience a smaller leverage worsening and despite not being the source of the disturbance, undergo a slower recovery.

When it comes to relaxing credit access, the single consumer loan contract produces the sharpest deterioration in the leverage of credit-constrained households since there is no risk sharing. The two chained contract simulations demonstrate that consumers share some of the burden of higher indebtedness with financial institutions so their leverage does not deteriorate as much as before. Nevertheless, their subsequent recovery could be more protracted since banks can use their optimizing position to extract more equity from consumers in order to improve their own debt position. This is especially evident in the case with a decrease in financial volatility. Since consumers are not the source of the disturbance, they should be relatively protected by their participation constraint. However, the deterioration in the balance sheets of banks is so considerable that financial institutions resort to extracting as much equity from households as possible in order to repair their own debt position. Credit-constrained consumers suffer the negative consequences of a reduction in bank volatility with the same magnitude as following a decrease in their own volatility. The financial troubles of banks draw them into a prolonged downturn and their leverage remains above its steady state value for more than 100 quarters (25 years). The last scenario describing a reduction in bank volatility in a chained contracts setup offers the closest cautionary tale to the subprime recession. Leading up to the subprime crisis, the balance sheets of banks were deemed sufficiently low risk and fairly stable allowing them to accumulate excessive debt. The resulting downturn was passed on to consumers who experienced significant worsening in their debt positions that led to a prolonged tightening of credit access. Nevertheless, all three scenarios in the single and chained contracts context demonstrate the perils of higher leverage that a temporary relaxation in borrowing conditions can create.

The policy recommendations for the scenario with decreased financial volatility point in the same direction as these for housing oversupply. In order to safeguard against such sudden relaxations in borrowing conditions that reflect animal spirits and are not supported by macroeconomic fundamentals, it is important to have loan-to-value requirements and debt-to-income caps in place for mortgage-buyers. Both would ensure that homeowners would not raise their leverage significantly or secure loans with little downpayment. Like in the case with housing oversupply, macroprudential regulations should be aimed at financial institutions as well to ensure that they hold sufficient reserves to weather unexpected downturns that would reduce the incentive to shift the burden of recessions onto consumers. The scenario with a reduction in bank volatility also implies that in times of bank distress due to excessive leverage, consumers should not be forced to share the burden of repairing the balance sheets of financial institutions and may need alternative sources of credit.

Figure 5: Housing Oversupply in a Chained Contracts Model

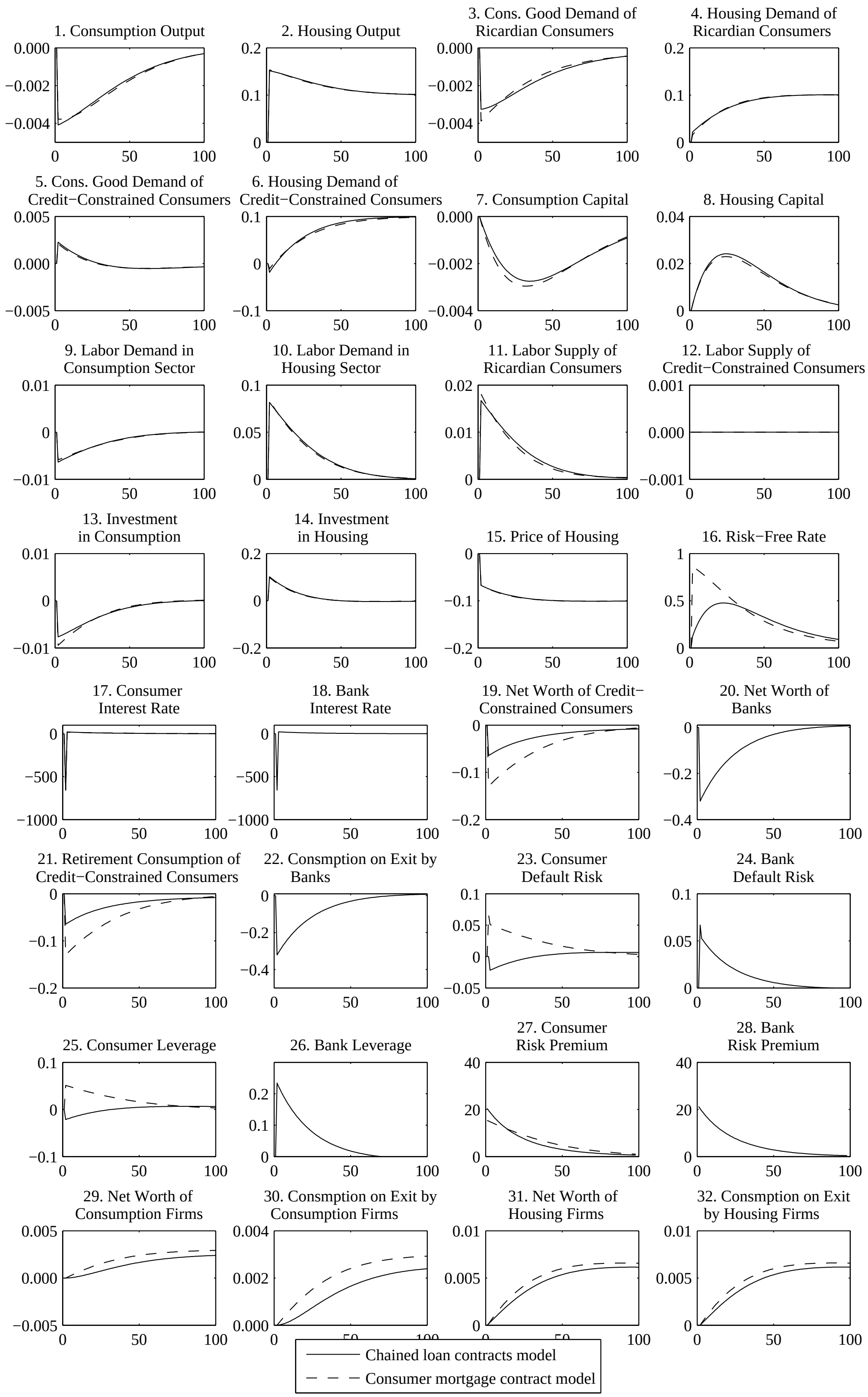


Figure 6: Reduced Consumer Volatility in a Chained Contracts Model

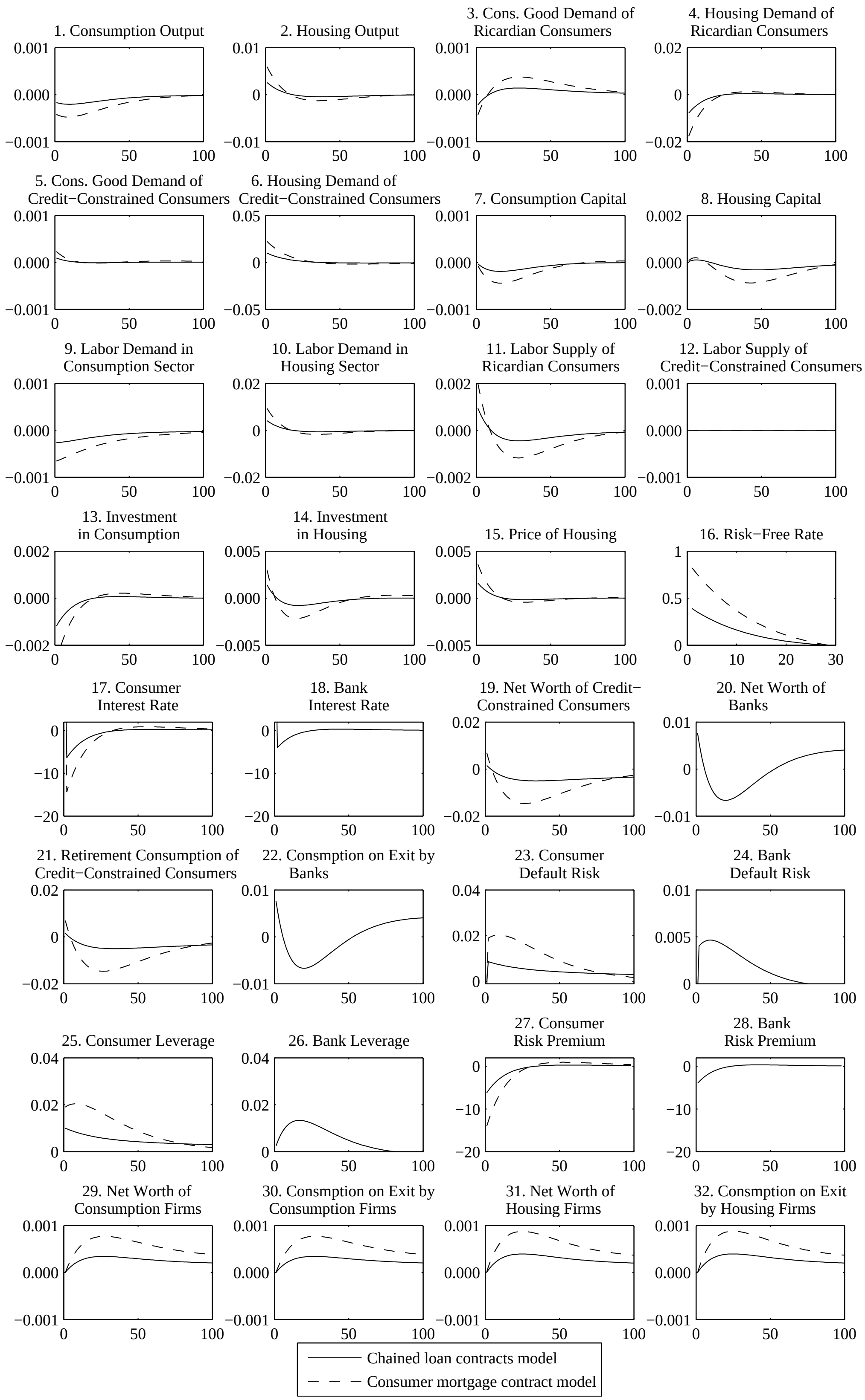
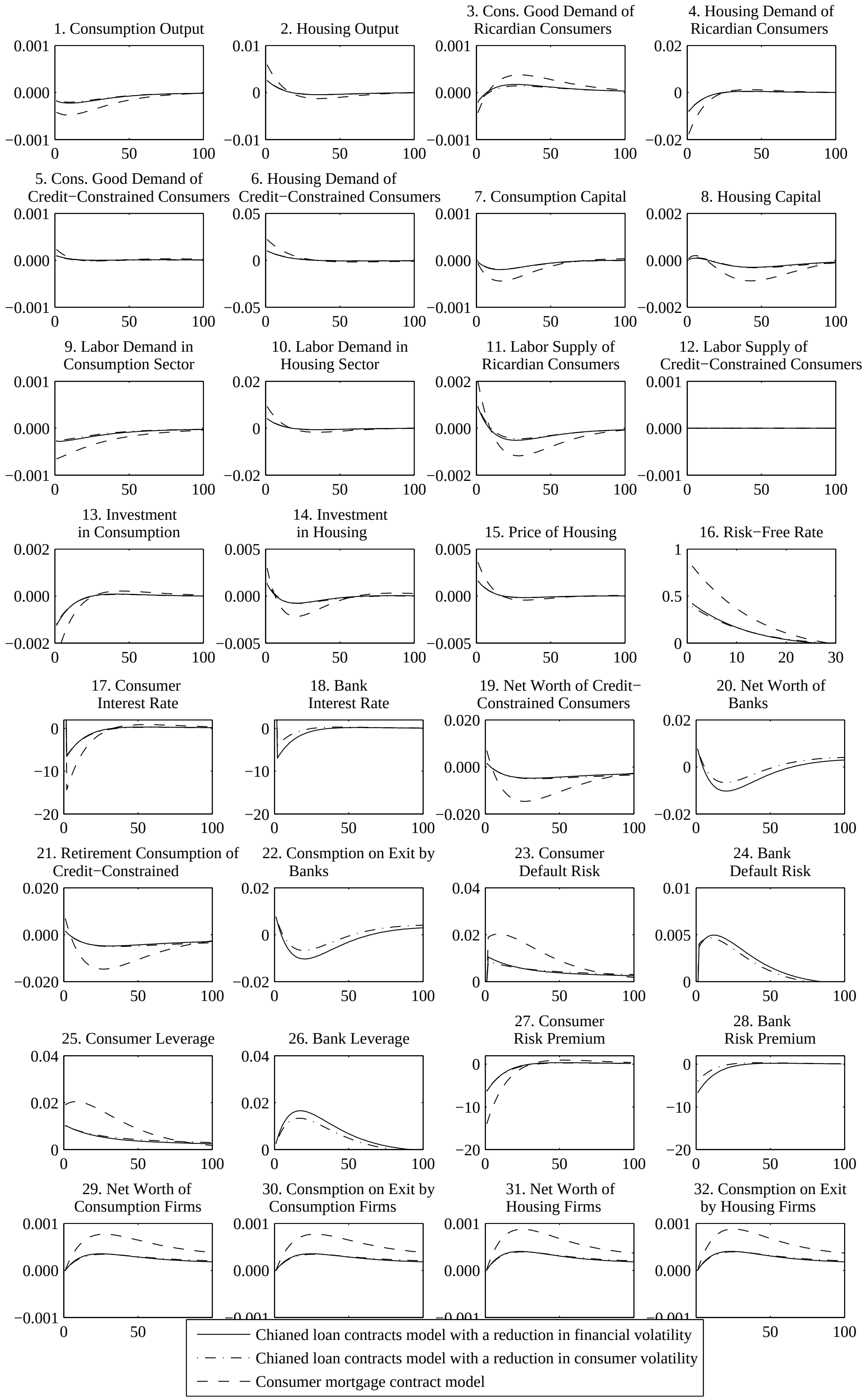


Figure 7: Reduced Bank Volatility in a Chained Contracts Model



8 Conclusion

The subprime mortgage crisis dashed the hopes of many for home ownership and set o a deep recession. Loan applicants saw their financing prospects reduced for years ahead as suddenly prudent banks struggled to improve their balance sheet positions. Many financial institutions were brought to the brink of collapse or saved by the too big to fail policy only to push through their recovery at the expense of credit-squeezed consumers. The consequences of excessive leverage and excessive lending as subprime loans had many calling for more stringent supervision of borrowing transactions. An improved regulation framework cannot emerge without understanding how the subprime mortgage crisis happened to be.

The straightforward setup of financial frictions in the household and financial sectors in this paper updates the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999) to capture some of the causes and consequences of the subprime recession. The model establishes that the subprime crisis is not a unique occurrence; on the contrary, a financial downturn can be triggered relatively easily when there are credit frictions associated with lending. Using a two-sector economy, this model demonstrates that a positive housing supply shock can have negative repercussions for mortgage-buyers since it reduces the value of the good used as collateral in the mortgage contract. A lower value of the collateral implies higher leverage of indebted home-buyers, increased default prospects and a higher risk premium. Such a pro-cyclical risk premium that occurs in a two-sector economy is in stark contrast to Bernanke, Gilchrist and Gertler (1999) where one-sector model shocks can generate only an anti-cyclical external finance premium.

An improvement in the borrowing conditions also causes adverse consequences for leveraged agents and demonstrates that the credit-constrained sector can be the source of a crippling downturn even if it is not a production sector. A reduction in the riskiness of the financial market can impact indebted consumers directly by allowing them to excessively leverage using relatively little collateral. When as a result of the over-leveraging, the lending rate adjusts upwards, these households end up with subprime mortgages that they cannot service and are more likely to become bankrupt. Finally, as the chained contacts extension demonstrated, there are opportunities for risk sharing between banks and consumers when financial institutions also participate in the mortgage contract. Banks may shift some of the consequences of the downturn onto credit-constrained consumers dragging them into a protracted recession. The scenario with relaxed credit access for banks captures fairly well the consequences of the subprime mortgage crisis with a sharp but brief decline for banks and a protracted recovery for mortgaged-households.

While the presence of idiosyncratic default probabilities implies that financial frictions cannot be easily eliminated, the consequences of excessive leverage hint at the type of macroprudential regulation needed to safeguard a sound financial market (Dimova, Kongsamut and Vandenbussche, 2014). Debt-to-income caps and loan-to-value requirements for consumers would ensure that households obtain sensible mortgages that they can service even in a downturn. Reserve requirements and loan loss provisions for nancial institutions would guarantee sufficient liquidity in times of distress. Taken together, these measures would prevent a potential overleverage that can cripple credit access for a prolonged time and can spill out of the mortgage market and distress the whole economy.

The model, while aptly demonstrating the role of leverage in triggering a financial downturn, could be enriched further to offer a deeper understanding of credit mechanisms. When the crisis started, many banks attempted to salvage their debt position by recalling loans to other financial institutions, rather than merely bearing out the increasing bankruptcy risk as described in this paper. The model can be augmented by adding inter-bank relationships and loan networks that may trigger a domino-like effect of rising default risk. Furthermore, the potential role of a bailout policy could be explored by adding government to the existing setup. It may also be a cautionary tale to consider the possibility of endogenous steady state leverage ratio for both banks and consumers. As history leading to the crisis demonstrates, leverage limits were poorly regulated and enforced prior to the subprime mortgage crisis allowing instead financial institutions to reach dangerously high debt to equity ratios. Finally, an important contribution could be to model the role of heterogeneous consumer expectations. It may be especially interesting to consider to what extent departures from the representative agent theory could explain the collective failure of agents to foresee the subprime mortgage crisis.

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Appendix A
Complete Consumer Mortgage Contract Model

The participation constraint for investors:

$$\left(\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C = R_{t-1}\left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right)$$

The first order condition of credit-constrained households' maximization:

$$\frac{E_t R_{t+1}^C}{R_t} = E_t\left(\frac{\frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(\Gamma_\omega^C(\bar{\omega}_{t+1}^C) - \mu^C G_\omega^C(\bar{\omega}_{t+1}^C)\right)}}{\left(1 - \Gamma^C(\bar{\omega}_{t+1}^C)\right) + \frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(\Gamma_\omega^C(\bar{\omega}_{t+1}^C) - \mu^C G_\omega^C(\bar{\omega}_{t+1}^C)\right)}\left(\Gamma^C(\bar{\omega}_{t+1}^C) - \mu^C G^C(\bar{\omega}_{t+1}^C)\right)}\right)$$

The consumption of credit-constrained households on retirement:

$$C_{E,t}^C = \left(1 - \nu^C\right)\left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$N_t^C = \nu^C\left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C + (1 - \alpha_H - \varepsilon)\Omega_C p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{E,t}^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$N_t^F = \nu^F R_{t-1} N_{t-1}^F + (1 - \alpha_F)\Omega_F Y_t^F$$

The consumption on exit by housing firms:

$$C_{E,t}^H = \left(1 - \nu^H\right) R_{t-1} N_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$N_t^H = \nu^H R_{t-1} N_{t-1}^H + (1 - \alpha_H - \varepsilon)\Omega_H p_t Y_t^H$$

The consumption capital accumulation equation:

$$K_t^F = (1 - \delta) K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right) K_{t-1}^F$$

Investment demand by consumption capital firms:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1$$

The consumption production function:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)}$$

The labor demand by consumption firms:

$$w_t = (1 - \alpha_F)(1 - \Omega_F)\left(\frac{Y_t^F}{L_{H,t}^F}\right)$$

The capital demand by consumption firms:

$$R_t = \frac{\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F}{x_t^F}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)}$$

The labor demand by housing firms:

$$w_t = (1-\Omega_H-\Omega_C)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing firms:

$$R_t = \frac{\alpha_H \frac{p_{t+1} Y_{t+1}^H}{K_t^H} + (1-\delta)x_{t+1}^H}{x_t^H}$$

The labor supply of Ricardian consumers:

$$\gamma (L_t^R)^{\varphi} C_t^R = w_t$$

The consumption housing tradeoff of Ricardian consumers:

$$\left(R_t p_t - (1-\delta)p_{t+1}\right) H_t^R = \kappa E_t C_{t+1}^R$$

The Euler equation of Ricardian consumers:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t$$

The labor supply of credit-constrained consumers:

$$\gamma (L_t^C)^{\varphi} C_t^C = w_t$$

The consumption-housing tradeoff of credit-constrained consumers:

$$\left(R_t^C p_{t-1} - (1-\delta)p_t\right) H_{t-1}^C = \kappa C_t^C$$

The consumption purchase equation of credit-constrained consumers:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C$$

Market clearing in the consumption sector:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + I_t^F + I_t^H$$

Market clearing in the housing sector:

$$p_t Y_t^H + (1-\delta)p_t(H_{t-1}^R + H_{t-1}^C) = p_t(H_t^R + H_t^C) + C_{E,t}^H + \mu^C G^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C$$

Market clearing in the labor market:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C$$

Appendix B
Log-Linearization of Consumer Mortgage Contract Model

This appendix derives the log-linearized form of the model equations as used in the simulations. The steady state value for each variable appears without time subscript and hats denote the percentage deviation of the variable from its steady state such that $\hat{Z}_t = \frac{Z_t - Z}{Z}$.

The participation constraint for investors:

$$\begin{aligned} \hat{R}_t^C &= \hat{R}_{t-1} + \frac{N^C}{pH^C - N^C}\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C - \hat{N}_{t-1}^C\right) - \\ & -\frac{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}{\Gamma^C(\bar{\omega}^C) - \mu^C G^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C) - \mu^C G_\sigma^C(\bar{\omega}^C)}{\Gamma^C(\bar{\omega}^C) - \mu^C G^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C \end{aligned}$$

The first-order condition of credit-constrained households' maximization:

$$\begin{aligned} 0 &= \frac{\Gamma_\omega^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}(E_t\hat{R}_{t+1}^C - \hat{R}_t) - \\ & -(1-\Gamma^C(\bar{\omega}^C))\frac{R^C}{R}\left(\frac{\Gamma_{\omega\omega}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\Gamma_{\omega\omega}^C(\bar{\omega}^C) - \mu^C G_{\omega\omega}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}\right)\bar{\omega}^C E_t\hat{\bar{\omega}}_{t+1}^C + \\ & +\frac{R^C}{R}\left(-\Gamma_\sigma^C(\bar{\omega}^C) + \frac{\Gamma_\omega^C(\bar{\omega}^C)\left(\Gamma_\sigma^C(\bar{\omega}^C) - \mu^C G_\sigma^C(\bar{\omega}^C)\right)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}\right)\sigma^C\hat{\sigma}_t^C - \\ & -\frac{R^C}{R}\left((1-\Gamma^C(\bar{\omega}^C))\left(\frac{\Gamma_{\omega\sigma}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\Gamma_{\omega\sigma}^C(\bar{\omega}^C) - \mu^C G_{\omega\sigma}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}\right)\right)\sigma^C\hat{\sigma}_t^C \end{aligned}$$

The consumption of credit-constrained households on retirement:

$$\hat{C}_{E,t}^C = \hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$\begin{aligned} \hat{N}_t^C &= \nu^C(1-\Gamma^C(\bar{\omega}^C))\frac{R^C pH^C}{N^C}(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C) - \nu^C\frac{R^C pH^C}{N^C}\Gamma_\omega^C(\bar{\omega}^C)\bar{\omega}^C\hat{\bar{\omega}}_t^C - \\ & -\nu^C\frac{R^C pH^C}{N^C}\Gamma_\sigma^C(\bar{\omega}^C)\sigma^C\hat{\sigma}_{t-1}^C + (1-\alpha_H-\varepsilon)\Omega_C\frac{pY^H}{N^C}\left(\hat{p}_t + \hat{Y}_t^H\right) \end{aligned}$$

The consumption on exit by consumption firms:

$$\hat{C}_{E,t}^F = \hat{R}_{t-1} + \hat{N}_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$\hat{N}_t^F = \nu^F R(\hat{R}_{t-1} + \hat{N}_{t-1}^F) + (1-\alpha_F)\Omega_F\frac{Y^F}{N^F}\hat{Y}_t^F$$

The consumption on exit by housing firms:

$$\hat{C}_{E,t}^H = \hat{R}_{t-1} + \hat{N}_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$\hat{N}_t^H = \nu^H R(\hat{R}_{t-1} + \hat{N}_{t-1}^H) + (1-\alpha_H-\varepsilon)\Omega_H\frac{pY^H}{N^H}\left(\hat{p}_t + \hat{Y}_t^H\right)$$

The consumption capital accumulation equation:

$$\hat{K}_t^F = \hat{K}_{t-1}^F + \delta\left(\hat{I}_t^F - \hat{K}_{t-1}^F\right)$$

Investment demand by consumption capital firms:

$$\hat{x}_t^F = J"\delta\left(\hat{K}_{t-1}^F - \hat{I}_t^F\right)$$

The consumption production function:

$$\hat{Y}_t^F = \hat{A}_t^F + \alpha_F\hat{K}_{t-1}^F + (1-\alpha_F)(1-\Omega_F)\hat{L}_{H,t}^F$$

The labor demand by consumption firms:

$$\hat{w}_t = \hat{Y}_t^F - \hat{L}_{H,t}^F$$

The capital demand by consumption firms:

$$\hat{R}_t + \hat{x}_t^F = \frac{R-(1-\delta)}{R}(\hat{Y}_{t+1}^F - \hat{K}_t^F) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^F$$

The housing capital accumulation equation:

$$\hat{K}_t^H = \hat{K}_{t-1}^H + \delta\left(\hat{I}_t^H - \hat{K}_{t-1}^H\right)$$

Investment demand by housing capital firms:

$$\hat{x}_t^H = J"\delta\left(\hat{K}_{t-1}^H - \hat{I}_t^H\right)$$

The housing production function:

$$\hat{Y}_t^H = \hat{A}_t^H + \alpha_H\hat{K}_{t-1}^H + (1-\Omega_H-\Omega_C)(1-\alpha_H-\varepsilon)\hat{L}_{H,t}^H$$

The labor demand by housing firms:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_t^H - \hat{L}_{H,t}^H$$

The capital demand by housing firms:

$$\hat{R}_t + \hat{x}_t^H = \frac{R-(1-\delta)}{R}(\hat{p}_{t+1} + \hat{Y}_{t+1}^H - \hat{K}_t^H) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^H$$

The labor supply of Ricardian consumers:

$$\varphi\hat{L}_t^R + \hat{C}_t^R = \hat{w}_t$$

The consumption housing tradeoff of Ricardian consumers:

$$\hat{C}_{t+1}^R = \hat{H}_t^R + \hat{p}_{t+1} + \frac{R}{(R-1+\delta)}\left(\hat{R}_t + \hat{p}_t - \hat{p}_{t+1}\right)$$

The Euler equation of Ricardian consumers:

$$E_t\hat{C}_{t+1}^R - \hat{C}_t^R = \hat{R}_t$$

The labor supply of credit-constrained consumers:

$$\varphi\hat{L}_t^C + \hat{C}_t^C = \hat{w}_t$$

The consumption housing tradeoff of credit-constrained consumers:

$$\hat{C}_t^C = \hat{H}_{t-1}^C + \hat{p}_t + \frac{R^C}{(R^C - 1 + \delta)}\left(\hat{R}_t^C + \hat{p}_{t-1} - \hat{p}_t\right)$$

The consumption purchase equation of credit-constrained consumers:

$$\hat{C}_t^C = \frac{wL^C}{C^C}(\hat{w}_t + \hat{L}_t^C) + \frac{(1-\delta)pH^C}{C^C}\hat{p}_t - \frac{R^C pH^C}{C^C}(\hat{R}_t^C + \hat{p}_{t-1}) + \frac{(1-\delta-R^C)pH^C}{C^C}\hat{H}_{t-1}^C$$

Market clearing in the consumption sector:

$$\hat{Y}_t^F = \frac{C^R}{Y^F}\hat{C}_t^R + \frac{C^C}{Y^F}\hat{C}_t^C + \frac{C_E^F}{Y^F}\hat{C}_{E,t}^F + \frac{C_E^C}{Y^F}\hat{C}_{E,t}^C + \frac{I^F}{Y^F}\hat{I}_t^F + \frac{I^H}{Y^F}\hat{I}_t^H$$

Market clearing in the housing sector:

$$\begin{aligned}\hat{Y}_t^H \quad = \quad & \frac{\delta(H^R + H^C) - Y^H}{Y^H}\hat{p}_t + \frac{H^R}{Y^H}\hat{H}_t^R + \frac{H^C}{Y^H}\hat{H}_t^C - \frac{(1-\delta)}{Y^H}(H^R\hat{H}_{t-1}^R + H^C\hat{H}_{t-1}^C) + \\ & + \frac{C_E^H}{pY^H}\hat{C}_{E,t}^H + \mu^C G^C(\bar{\omega}^C)\frac{R^C H^C}{Y^H}\left(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C\right) + \\ & + \mu^C G_\omega^C \bar{\omega}^C \hat{\bar{\omega}}_t^C \frac{R^C H^C}{Y^H} + \mu^C G_\sigma^C \sigma^C \hat{\sigma}_{t-1}^C \frac{R^C H^C}{Y^H}\end{aligned}$$

Market clearing in the labor market:

$$L_H^F \hat{L}_{H,t}^F + L_H^H \hat{L}_{H,t}^H = L^R \hat{L}_t^R + L^C \hat{L}_t^C$$

Appendix C
Complete Chained Contracts Model

The participation constraint for investors:

$$\Psi^B(\bar{\omega}_t^B)\Psi^C(\bar{\omega}_t^C)R_t^C p_{t-1}H_{t-1}^C = R_{t-1}\left(p_{t-1}H_{t-1}^C - N_{t-1}^C - N_{t-1}^B\right)$$

The participation constraint for credit-constrained consumers:

$$N_{t-1}^C = \left(1-\Gamma^C(\bar{\omega}_t^C)\right)p_{t-1}H_{t-1}^C$$

The first order condition of financial institutions' maximization:

$$\begin{aligned}\frac{E_t R_{t+1}^C}{R_t} &= E_t\left(\frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)\Psi_\omega^C(\bar{\omega}_{t+1}^C)+\Gamma_\omega^C(\bar{\omega}_{t+1}^C)\Psi^C(\bar{\omega}_{t+1}^C)}\right)\times \\ & \quad E_t\left(\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi_\omega^B(\bar{\omega}_{t+1}^B)+\Gamma_\omega^B(\bar{\omega}_{t+1}^B)\Psi^B(\bar{\omega}_{t+1}^B)}\right)\end{aligned}$$

The consumption on exit by banks:

$$C_{E,t}^B = \left(1-\nu^B\right)\left(1-\Gamma^B(\bar{\omega}_t^B)\right)\Psi^C(\bar{\omega}_t^C)R_t^C p_{t-1}H_{t-1}^C$$

The law of motion of the net worth of banks:

$$N_t^B = \nu^B\left(1-\Gamma^B(\bar{\omega}_t^B)\right)\Psi^C(\bar{\omega}_t^C)R_t^C p_{t-1}H_{t-1}^C + (1-\alpha_H-\varepsilon)\Omega_B p_t Y_t^H$$

The consumption of credit-constrained households on retirement:

$$C_{E,t}^C = \left(1-\nu^C\right)\left(1-\Gamma^C(\bar{\omega}_t^C)\right)R_t^C p_{t-1}H_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$N_t^C = \nu^C\left(1-\Gamma^C(\bar{\omega}_t^C)\right)R_t^C p_{t-1}H_{t-1}^C + (1-\alpha_H-\varepsilon)\Omega_C p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{E,t}^F = \left(1-\nu^F\right)R_{t-1}N_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$N_t^F = \nu^F R_{t-1}N_{t-1}^F + (1-\alpha_F)\Omega_F Y_t^F$$

The consumption on exit by housing firms:

$$C_t^H = \left(1-\nu^H\right)R_{t-1}N_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$N_t^H = \nu^H R_{t-1}N_{t-1}^H + (1-\alpha_H-\varepsilon)\Omega_H p_t Y_t^H$$

The consumption capital accumulation equation:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F$$

Investment demand by consumption capital firms:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1$$

The consumption production function:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)}$$

The labor demand by consumption firms:

$$w_t = (1-\alpha_F)(1-\Omega_F)\left(\frac{Y_t^F}{L_{H,t}^F}\right)$$

The capital demand by consumption firms:

$$R_t = \frac{\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta) x_{t+1}^F}{x_t^F}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta) K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right) K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C-\Omega_B)}$$

The labor demand by housing firms:

$$w_t = (1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing firms:

$$R_t = \frac{\alpha_H \frac{p_{t+1} Y_{t+1}^H}{K_t^H} + (1-\delta) x_{t+1}^H}{x_t^H}$$

The labor supply of Ricardian consumers:

$$\gamma (L_t^R)^{\varphi} C_t^R = w_t$$

The consumption housing tradeoff of Ricardian consumers:

$$\left(R_t p_t - (1-\delta) p_{t+1}\right) H_t^R = \kappa E_t C_{t+1}^R$$

The Euler equation of Ricardian consumers:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t$$

The labor supply of credit-constrained consumers:

$$\gamma (L_t^C)^{\varphi} C_t^C = w_t$$

The consumption-housing tradeoff of credit-constrained consumers:

$$\left(R_t^C p_{t-1} - (1-\delta) p_t\right) H_{t-1}^C = \kappa C_t^C$$

The consumption purchase equation of credit-constrained consumers:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C$$

Market clearing in the consumption sector:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + C_{E,t}^B + I_t^F + I_t^H$$

Market clearing in the housing sector:

$$\begin{aligned} p_t Y_t^H + (1-\delta)p_t(H_{t-1}^R + H_{t-1}^C) &= p_t(H_t^R + H_t^C) + C_{E,t}^H + \\ &+ \left[\mu^C G^C(\bar{\omega}_t^C) + \mu^B G^B(\bar{\omega}_t^B)\Psi^C(\bar{\omega}_t^C)\right] R_t^C p_{t-1} H_{t-1}^C \end{aligned}$$

Market clearing in the labor market:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C$$

Appendix D
Log-Linearization of the Chained Contracts Model

The participation constraint for investors:

$$\begin{aligned}\hat{R}_t^C &= \hat{R}_{t-1} + \frac{1}{pH^C - N^C - N^B}\left(\left(N^C + N^B\right)\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C\right) - N^C\hat{N}_{t-1}^C - N^B\hat{N}_{t-1}^B\right) - \\ &\quad -\frac{\Psi_\omega^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Psi_\omega^B(\bar{\omega}^B)}{\Psi^B(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_t^B - \frac{\Psi_\sigma^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C - \frac{\Psi_\sigma^B(\bar{\omega}^B)}{\Psi^B(\bar{\omega}^B)}\sigma^B\hat{\sigma}_{t-1}^B\end{aligned}$$

The first-order condition of financial institutions' maximization of the chained contracts:

$$\begin{aligned}&E_t\hat{R}_{t+1}^C - \hat{R}_t = \\ &= \left[\frac{\Gamma_{\omega\omega}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_{\omega\omega}^C(\bar{\omega}^C) + \Gamma_{\omega\omega}^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_\omega^C(\bar{\omega}^C) + \Gamma_\omega^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}\right]\bar{\omega}^C E_t\hat{\bar{\omega}}_{t+1}^C + \\ &+ \left[\frac{\Gamma_{\omega\sigma}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_{\omega\sigma}^C(\bar{\omega}^C) - \Gamma_\sigma^C(\bar{\omega}^C)\Psi_\omega^C(\bar{\omega}^C) + \Gamma_\omega^C(\bar{\omega}^C)\Psi_\sigma^C(\bar{\omega}^C) + \Gamma_{\omega\sigma}^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_\omega^C(\bar{\omega}^C) + \Gamma_\omega^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}\right]\sigma^C\hat{\sigma}_t^C + \\ &+ \left[\frac{\Gamma_{\omega\omega}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B)} - \frac{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_{\omega\omega}^B(\bar{\omega}^B) + \Gamma_{\omega\omega}^B(\bar{\omega}^B)\Psi^B(\bar{\omega}^B)}{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_\omega^B(\bar{\omega}^B) + \Gamma_\omega^B(\bar{\omega}^B)\Psi^C(\bar{\omega}^B)}\right]\bar{\omega}^B E_t\hat{\bar{\omega}}_{t+1}^B + \\ &+ \left[\frac{\Gamma_{\omega\sigma}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B)} - \frac{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_{\omega\sigma}^B(\bar{\omega}^B) - \Gamma_\sigma^B(\bar{\omega}^B)\Psi_\omega^B(\bar{\omega}^B) + \Gamma_\omega^B(\bar{\omega}^B)\Psi_\sigma^B(\bar{\omega}^B) + \Gamma_{\omega\sigma}^B(\bar{\omega}^B)\Psi^B(\bar{\omega}^B)}{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_\omega^B(\bar{\omega}^B) + \Gamma_\omega^B(\bar{\omega}^B)\Psi^B(\bar{\omega}^B)}\right]\sigma^B\hat{\sigma}_t^B\end{aligned}$$

The financial institutions' interest rate:

$$\hat{R}_t^B = \hat{R}_t^C + \frac{N^C}{pH^C - N^C}\left(\hat{N}_{t-1}^C - \hat{p}_{t-1} - \hat{H}_{t-1}^C\right) + \frac{\Psi_\omega^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C + \frac{\Psi_\sigma^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The participation constraint of credit-constrained consumers:

$$\hat{N}_{t-1}^C = \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The consumption on exit of financial institutions:

$$\hat{C}_{E,t}^B = \hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^B(\bar{\omega}^B)}{1-\Gamma^B(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_t^B - \frac{\Gamma_\sigma^B(\bar{\omega}^B)}{1-\Gamma^B(\bar{\omega}^B)}\sigma^B\hat{\sigma}_{t-1}^B +$$

$$+\frac{\Psi_\omega^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C + \frac{\Psi_\sigma^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The law of motion of the net worth of financial institutions:

$$\begin{aligned}\hat{N}_t^B &= \nu^B(1-\Gamma^B(\bar{\omega}^B))\Psi^C(\bar{\omega}^C)\frac{R^C pH^C}{N^B}(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C) + (1-\alpha_H-\varepsilon)\Omega_H\frac{pY^H}{N^B}\left(\hat{p}_t + \hat{Y}_t^H\right) + \\ &\quad -\nu^B\frac{R^C pH^C}{N^B}\Psi^C(\bar{\omega}^C)\left(\Gamma_\omega^B(\bar{\omega}^B)\bar{\omega}^B\hat{\bar{\omega}}_t^B + \Gamma_\sigma^B(\bar{\omega}^B)\sigma^B\hat{\sigma}_{t-1}^B\right) + \\ &\quad +\nu^B\frac{R^C pH^C}{N^B}\left(1-\Gamma^B(\bar{\omega}^B)\right)\left(\Psi_\omega^C\bar{\omega}^C\hat{\bar{\omega}}_t^C + \Psi_\sigma^C\sigma^C\hat{\sigma}_{t-1}^C\right)\end{aligned}$$

The consumption of credit-constrained households on retirement:

$$\hat{C}_{E,t}^C = \hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$\begin{aligned}\hat{N}_t^C &= \nu^C(1-\Gamma^C(\bar{\omega}^C))\frac{R^C pH^C}{N^C}(\hat{R}_t^C+\hat{p}_{t-1}+\hat{H}_{t-1}^C)-\nu^C\frac{R^C pH^C}{N^C}\Gamma_\omega^C(\bar{\omega}^C)\bar{\omega}^C\hat{\bar{\omega}}_t^C - \\ &\quad -\nu^C\frac{R^C pH^C}{N^C}\Gamma_\sigma^C(\bar{\omega}^C)\sigma^C\hat{\sigma}_{t-1}^C+(1-\alpha_H-\varepsilon)\Omega_C\frac{pY^H}{N^C}\left(\hat{p}_t+\hat{Y}_t^H\right)\end{aligned}$$

The consumption on exit by consumption firms:

$$\hat{C}_{E,t}^F = \hat{R}_{t-1}+\hat{N}_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$\hat{N}_t^F = \nu^F R(\hat{R}_{t-1}+\hat{N}_{t-1}^F)+(1-\alpha_F)\Omega_F\frac{Y^F}{N^F}\hat{Y}_t^F$$

The consumption on exit by housing firms:

$$\hat{C}_{E,t}^H = \hat{R}_{t-1}+\hat{N}_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$\hat{N}_t^H = \nu^H R(\hat{R}_{t-1}+\hat{N}_{t-1}^H)+(1-\alpha_H-\varepsilon)\Omega_H\frac{pY^H}{N^H}\left(\hat{p}_t+\hat{Y}_t^H\right)$$

The consumption capital accumulation equation:

$$\hat{K}_t^F = \hat{K}_{t-1}^F+\delta\left(\hat{I}_t^F-\hat{K}_{t-1}^F\right)$$

Investment demand by consumption capital firms:

$$\hat{x}_t^F = J\text{''}\delta\left(\hat{K}_{t-1}^F-\hat{I}_t^F\right)$$

The consumption production function:

$$\hat{Y}_t^F = \hat{A}_t^F+\alpha_F\hat{K}_{t-1}^F+(1-\alpha_F)(1-\Omega_F)\hat{L}_{H,t}^F$$

The labor demand by consumption firms:

$$\hat{w}_t = \hat{Y}_t^F-\hat{L}_{H,t}^F$$

The capital demand by consumption firms:

$$\hat{R}_t+\hat{x}_t^F = \frac{R-(1-\delta)}{R}(\hat{Y}_{t+1}^F-\hat{K}_t^F)+\frac{(1-\delta)}{R}\hat{x}_{t+1}^F$$

The housing capital accumulation equation:

$$\hat{K}_t^H = \hat{K}_{t-1}^H+\delta\left(\hat{I}_t^H-\hat{K}_{t-1}^H\right)$$

Investment demand by housing capital firms:

$$\hat{x}_t^H = J\text{''}\delta\left(\hat{K}_{t-1}^H-\hat{I}_t^H\right)$$

The housing production function:

$$\hat{Y}_t^H = \hat{A}_t^H+\alpha_H\hat{K}_{t-1}^H+(1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\hat{L}_{H,t}^H$$

The labor demand by housing firms:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_t^H - \hat{L}_{H,t}^H$$

The capital demand by housing firms:

$$\hat{R}_t + \hat{x}_t^H = \frac{R-(1-\delta)}{R}(\hat{p}_{t+1} + \hat{Y}_{t+1}^H - \hat{K}_t^H) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^H$$

The labor supply of Ricardian consumers:

$$\varphi \hat{L}_t^R + \hat{C}_t^R = \hat{w}_t$$

The consumption housing tradeoff of Ricardian consumers:

$$\hat{C}_{t+1}^R = \hat{H}_t^R + \hat{p}_{t+1} + \frac{R}{(R-1+\delta)}\left(\hat{R}_t + \hat{p}_t - \hat{p}_{t+1}\right)$$

The Euler equation of Ricardian consumers:

$$E_t\hat{C}_{t+1}^R - \hat{C}_t^R = \hat{R}_t$$

The labor supply of credit-constrained consumers:

$$\varphi \hat{L}_t^C + \hat{C}_t^C = \hat{w}_t$$

The consumption housing tradeoff of credit-constrained consumers:

$$\hat{C}_t^C = \hat{H}_{t-1}^C + \hat{p}_t + \frac{R^C}{(R^C-1+\delta)}\left(\hat{R}_t^C + \hat{p}_{t-1} - \hat{p}_t\right)$$

The consumption purchase equation of credit-constrained consumers:

$$\hat{C}_t^C = \frac{wL^C}{C^C}(\hat{w}_t + \hat{L}_t^C) + \frac{(1-\delta)pH^C}{C^C}\hat{p}_t - \frac{R^C pH^C}{C^C}(\hat{R}_t^C + \hat{p}_{t-1}) + \frac{(1-\delta-R^C)pH^C}{C^C}\hat{H}_{t-1}^C$$

Market clearing in the consumption sector:

$$\hat{Y}_t^F = \frac{C^R}{Y^F}\hat{C}_t^R + \frac{C^C}{Y^F}\hat{C}_t^C + \frac{C_E^F}{Y^F}\hat{C}_{E,t}^F + \frac{C_E^C}{Y^F}\hat{C}_{E,t}^C + \frac{C_E^B}{Y^F}\hat{C}_{E,t}^B + \frac{I^F}{Y^F}\hat{I}_t^F + \frac{I^H}{Y^F}\hat{I}_t^H$$

Market clearing in the housing sector:

$$\begin{aligned}
\hat{Y}_t^H \quad = \quad & \frac{\delta(H^R+H^C)-Y^H}{Y^H}\hat{p}_t + \frac{H^R}{Y^H}\hat{H}_t^R + \frac{H^C}{Y^H}\hat{H}_t^C - \frac{(1-\delta)}{Y^H}(H^R\hat{H}_{t-1}^R + H^C\hat{H}_{t-1}^C) + \\
& + \frac{C_E^H}{pY^H}\hat{C}_{E,t}^H + \left[\mu^C G^C(\bar{\omega}^C) + \mu^B G^B(\bar{\omega}^B)\Psi^C(\bar{\omega}^C)\right]\frac{R^C H^C}{Y^H}\left(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C\right) + \\
& + \frac{R^C H^C}{Y^H}\left[\mu^C\left(G_\omega^C(\bar{\omega}^C)\bar{\omega}^C\hat{\bar{\omega}}_t^C + G_\sigma^C(\bar{\omega}^C)\sigma^C\hat{\sigma}_{t-1}^C\right) + \mu^B G(\bar{\omega}^B)\left(\Psi_\omega^C\bar{\omega}^C\hat{\bar{\omega}}_t^C + \Psi_\sigma^C\sigma^C\hat{\sigma}_{t-1}^C\right)\right] + \\
& + \mu^B\Psi(\bar{\omega}^C)\frac{R^C H^C}{Y^H}\left[G_\omega^B\bar{\omega}^B\hat{\bar{\omega}}_t^B + G_\sigma^B\sigma^B\hat{\sigma}_{t-1}^B\right]
\end{aligned}$$

Market clearing in the labor market:

$$L_H^F\hat{L}_{H,t}^F + L_H^H\hat{L}_{H,t}^H = L^R\hat{L}_t^R + L^C\hat{L}_t^C$$

Appendix E
Derivatives of the Log-Normal Distribution

The distribution of $\bar{\omega}$ is log-normal such that $ln\bar{\omega} \sim N\left(\frac{1}{2}\sigma^2,\ \sigma^2\right)$ follows the Normal distribution and where σ^2 is the variance of the cutoff idiosyncratic risk. The cumulative distribution function $F(\bar{\omega})$, the investors' share of the loan contract from non-defaulting borrowers $\Gamma(\bar{\omega})$ and from defaulting debtors $G(\bar{\omega})$ are defined as:

$$F(\bar{\omega}) = \Phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)$$

$$G(\bar{\omega}) = \Phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)$$

$$\Gamma(\bar{\omega}) = \Phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) + \left[1 - \Phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\right]\bar{\omega}$$

The derivatives of the distribution terms, namely F_ω, F_σ, Γ_ω, Γ_σ, G_ω, G_σ, $\Gamma_{\omega\omega}$, $\Gamma_{\omega\sigma}$, $G_{\omega\omega}$ and $G_{\omega\sigma}$ are derived below. The derivatives use the normal distribution property that $\phi'(x) = -x\phi(x)$ where Φ and ϕ are the cumulative distribution function and the probability distribution function of the standard normal distribution.

$$F_\omega = \frac{1}{\sigma\bar{\omega}}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$F_\sigma = -\frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$G_\omega = \frac{1}{\sigma\bar{\omega}}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$G_\sigma = -\frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$\Gamma_\omega = 1 - F(\bar{\omega}) = 1 - \Phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$\begin{aligned}\Gamma_\sigma &= G_\sigma - \bar{\omega}F_\sigma = \\ &= -\frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega}^C - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) + \frac{\bar{\omega}}{\sigma}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\end{aligned}$$

$$\Gamma_{\omega\omega} = -F_\omega = -\frac{1}{\sigma\bar{\omega}}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) < 0$$

$$\Gamma_{\omega\sigma} = -F_\sigma = \frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) < 0$$

$$
\begin{aligned}
G_{\omega\omega} &= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)+\left(\frac{1}{\sigma\bar{\omega}}\right)^2\phi'\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)= \\
&= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)-\left(\frac{1}{\sigma\bar{\omega}}\right)\phi\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)
\end{aligned}
$$

$$
\begin{aligned}
G_{\omega\sigma} &= -\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)-\frac{1}{\sigma\bar{\omega}}\phi'\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma^2}\right)= \\
&= -\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)+\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega}-\frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega}+\frac{1}{2}\sigma^2}{\sigma}\right)
\end{aligned}
$$