(DRAFT - DO NOT CITE OR CIRCULATE)

on the two prerequisites of balanced growth

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Abstract

In this paper, I argue that the two theoretical prerequisites of balanced growth - purely labor augmenting technical progress and a unitary capital-labor elasticity - are inconsistent with long-run evidence from a simple decomposition of the post-war U.S. economy into heterogeneous industries. My decomposition is motivated by empirical evidence that the degree of substitution between capital and labor varies across industries and so does the capital-output ratio. Embedded into an otherwise standard macroeconomic framework, I show that the assumption of a single representative consumer with homogeneous preferences over industry value added implies that capital and labor have an elasticity of less than unity on aggregate. Thus, balanced growth relies on technical progress to be purely labor augmenting which further implies that technical progress is higher in industries that have a lower capital-labor elasticity and a lower capital-output ratio. In long-run U.S. data, however, I find no evidence that supports such a relation. Yet, I show that the network structure of the post-war U.S. economy can induce balanced growth through demand side heterogeneity in the industries' intermediate input-output linkages. I show that this mechanism can sustain balanced growth for long periods under a variety of empirically plausible parameter assumptions.

JEL: E1, E13, O41
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1 Introduction

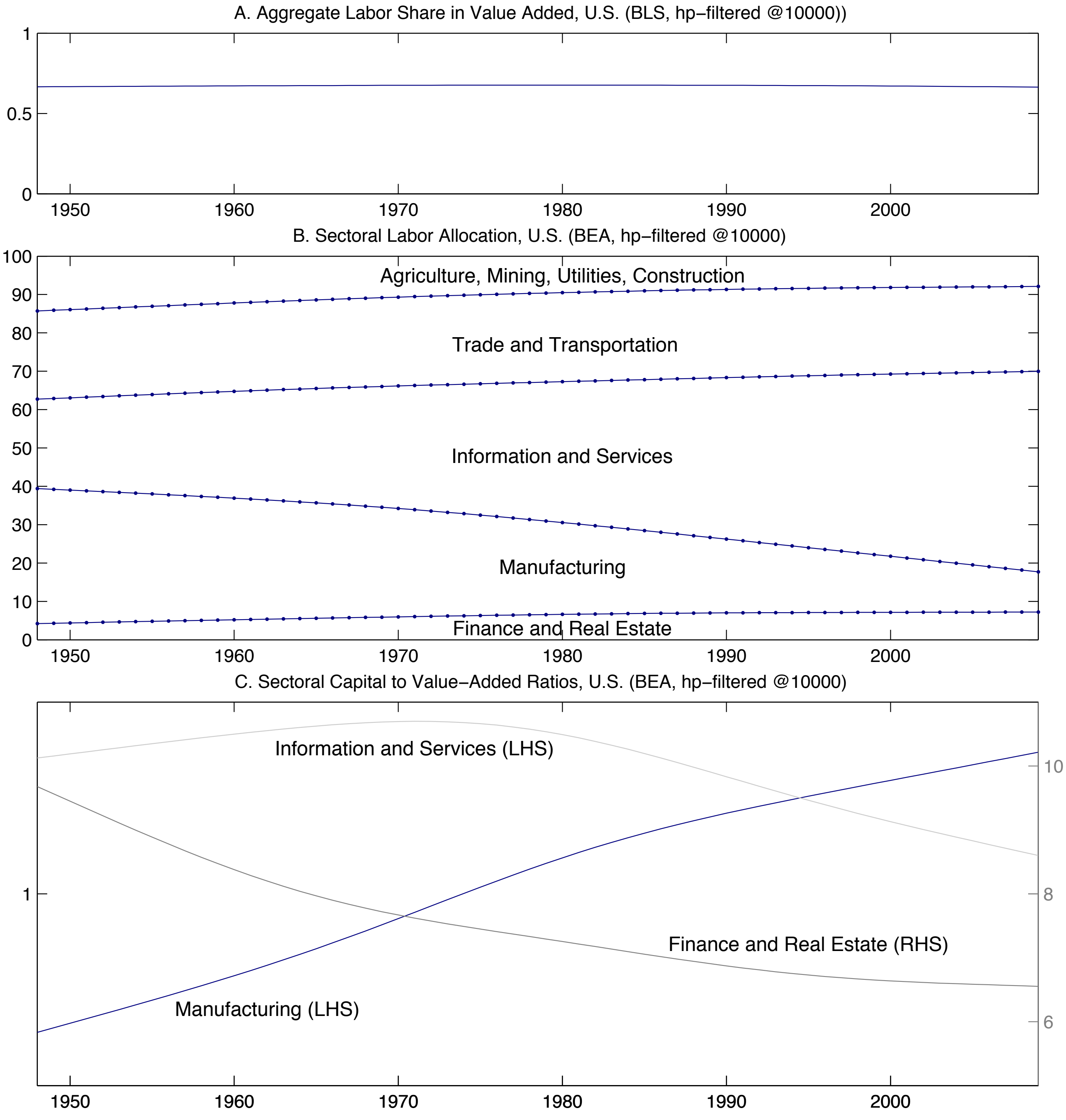
Motivated by the empirical evidence of Figure 1, I conduct a decomposition of the post-war U.S. economy into different industries with a changing employment allocation (Panel B, structural change), heterogeneity in the capital-labor elasticity and heterogeneity in the capital intensity (Panel C, industry heterogeneity, Result 1 of this paper). This decomposition raises a simple question in the light of the evidence of a stable long-run labor share (Panel A, balanced growth).[1] *Why do the aggregate post-war U.S. data look like as if they have been generated by a single representative Cobb-Douglas production technology when, in reality, they are generated by many different micro units that employ heterogeneous production technologies and a continuously changing factor allocation?*

To reconcile the industry heterogeneity with aggregate economic outcomes, the literature on economic growth provides two simple prerequisites of balanced growth: either the aggregate capital-labor elasticity is equal to unity or productivity growth is purely labor augmenting.[2] Using post-war U.S. industry data, I show that the empirical evidence on these two prerequisites is weak at best. In particular, I show that, under the literature standard assumption of a single representative consumer with complementary preferences over industry value added, the decomposition of the post-war U.S. economy into heterogeneous industries implies that capital and labor are complements on aggregate (Result 2 of this paper), i.e. that the elasticity of capital and labor is less than unity. Furthermore, I show that the labor intensity and the capital-labor elasticity are uncorrelated with the allocation of productivity growth across industries (Result 3 of this paper), i.e. technical progress is not biased towards labor. *How can we then reconcile the evidence on industry heterogeneity and structural change with balanced growth?*

To address this question, I show that it is important to acknowledge that the two prerequisites of balanced growth arise in a class of theories that focuses on modeling the overall economy in *value added* terms. Thereby, these theories abstract from the economy's network structure, i.e. the input-output linkages that connect the different industries. I illustrate that in a *gross output* model (a model that incorporates the industries' intermediate input-output linkages) balanced growth, structural change and industry heterogeneity can simultaneously emerge under a variety of empirically plausible parameter assumptions. My

[1] Panel A shows that the labor share in value added is approximately stable. The labor share is based on the BLS (Bureau of Labor Statistics) U.S. private business sector. Some recent studies such as Elsby et al. (2013), Karabarbounis and Neiman (2014) and Piketty and Zucman (2014) among others argue that the U.S. labor share has trend declined since the late 1970s and early 1980s. Bridgman (2014), Gomme and Rupert (2004), Rognlie (2015) and Auerbach and Hassett (2015) among others argue that measurement problems lead to this conclusion. In the BLS' main series that I use - the private business sector's labor share - the labor share is rather stable and declines only by 2 percentage points between 1975 and 2010. These smaller movements and trends can already be observed in the 19th century in France and the U.K. as the evidence of Piketty and Zucman (2014) shows. If these smaller movements should really be interpreted as a departure from the balanced growth path is not yet evident. In this paper I therefore assume that balanced growth is still the long run scenario. See also footnote 1 in Struck (2015).

[2] Uzawa (1961), Kennedy (1964), Samuelson (1965), and Drandakis and Phelps (1966), Jones (2005) and Acemoglu (2002) study value added models and show that technical progress must be purely labor augmenting if production does not take the Cobb-Douglas form.



Notes: Panel A shows the hp-filtered labor share of the U.S. private business sector. The data are drawn from the BLS (U.S. Bureau of Labor Statistics). Panel B shows the hp-filtered allocation of the number of workers across five major industries. The data are in percent of the total labor force and are drawn from the BEA industry data (U.S. Bureau of Economic Analysis). Panel C shows the hp-filtered capital-output ratio of three major industries. The data are drawn from the BEA industry data.

Figure 1: Balanced growth, structural change and industry heterogeneity.

analysis reveals that key to generating balanced growth are demand elasticities outside the value added structure of the post-war U.S. economy.

In my model, I use the U.S. post-war data to break down the heterogeneous industries into two representative sectors: one that relies more on capital and one that relies more on labor. I then use the data to empirically motivate the input-output structure of the model. In the data, I find that i) intermediates maintain a substantial share in gross output (value added plus intermediate inputs) over time (Result 4 of this paper) ii) the labor intensity of intermediates is strongly positively correlated with the labor intensity of value added but industries that are more labor intensive increasingly use the capital intensive intermediate input, while industries that are more capital intensive increasingly use the labor intensive intermediate input over time (Result 5 of this paper). The evidence of *demand side heterogeneity* in input-output linkages across sectors, ii), in combination with the evidence on stable intermediate shares, i), ensures that both intermediate inputs maintain a quantitatively significant share in the gross output of both sectors throughout the entire post-war period.

By maintaining a significant gross output share, the intermediate inputs hint to a mechanism that can sustain balanced growth over long periods of time: when capital becomes abundant and labor remains scarce, the relative price of capital intensive value added falls given that productivity growth is uncorrelated with production characteristics across sectors. This decline in prices leads to an increase in the share of the labor intensive sector in total value added since the strong complementarity of sectoral value added in preferences implies that real value added must grow commensurately across sectors. The economy, thus, quickly becomes more labor intensive and departs from the balanced growth path.

In the gross output model that I study, this shift towards the labor intensive sector does not occur because the fall in relative value added prices is counterbalanced by an increase in the demand for the real value added of the capital intensive sector. This increase comes from the described intermediate structure, i)-ii). As a consequence, relative real value added grows faster in the capital intensive sector. The fast increase in relative real value added stabilizes the share of the capital intensive sector in total value added. Stable value added shares in turn stabilize the labor share in value added. Meanwhile, real gross output still grows commensurately across sectors consistent with the strong complementarity of in consumer preferences.

To compare the gross output and value added models I numerically simulate four different models and evaluate them based on two criteria: their ability to generate the aggregate empirical evidence (balance growth) and the re-allocations of production factors (structural change) from heterogeneity in production technologies. I start off with a baseline value added model with two sectors that are heterogeneous in their labor intensity but have a common capital-labor elasticity of unity. I then study a slightly more advanced value added model with two sectors that also differ in their capital-labor elasticity. In both these models, I show that the value added share of the capital intensive sector declines by almost 10 percentage points during the U.S. post-war period 1950-2010. This decline induces an increase in the labor share by several percentage points. This shift towards the labor intensive sector is reflected in the direction of structural change: both models predict that capital and labor reallocates towards the labor intensive sector. I show that these dynamics also arise in a gross output model in which each sector uses its own output as an intermediate input, i.e. in a model that features i) but not ii).

By contrast, the actual post-war U.S. data indicate that the value added share of the capital intensive sector changes by less than 2 percentage points, that the overall labor share is stable, and, that labor allocates to the more labor intensive sector while that capital allocates to the more capital intensive sector. I show that a gross output model that features the intermediate structure i)-ii) is consistent with these facts. Besides being empirically grounded, a strength of the mechanism is that it is not too knife-edge. In the Appendix I illustrate the mechanism's robustness under different assumptions about the functional form of gross output as well as the bias of technical change.

Related literature. My analysis contributes to many papers in two related literatures. First, it is a contribution to the literature that attempts to reconcile structural change with balanced growth. This literature suggests two types of mechanisms: models that generate factor reallocations from changes in consumer preferences, e.g. Kongsamut et al. (2001) and Foellmi and Zweimüller (2008), and, models that generate structural change from technological differences across industries, e.g. Baumol (1969), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). Recent studies jointly analyze both mechanisms, see e.g. Herrendorf et al. (2013) or Boppart (2014). My paper departs from this literature by focusing on the network structure of the economy in an environment of heterogeneity in industry production technologies. The key takeaway from my analysis is that technological differences across the different industries lead the economy away from the balanced growth path. What restores balanced growth is the heterogeneity in intermediate input-output linkages across the different industries. In modeling industry heterogeneity in production technologies my paper is close to Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado et al. (2014). Both these papers allow for differences in the composition of physical capital and labor across industries. In terms of the underlying research question addressed, my paper is most closely related to Struck (2015) which provides a very different solution approach to the same underlying problem: once measurement problems in inflation are addressed, technical change is labor biassed and balanced growth emerges despite the substantial heterogeneity in production technologies across industries. The mechanism that I present in this paper is a competing explanation.

Second, it is a contribution to the literature that attempts to determine the aggregate degree of substitution between capital and labor. The empirical part of this literature has mostly found capital and labor to be complements, e.g. Antras (2004), Klump et al. (2007), Arrow et al. (1961), Sato (1970) and Lucas (1969). While this literature has usually inferred the elasticity from an aggregate production function, I apply the recent approach of Oberfield and Raval (2014) which is based on an industry decomposition.

My findings highlight the importance of the industry structure that is imposed on the data during the estimation. I find aggregate complementarity between capital and labor when the value added of different industries are complements in preferences. This finding is also confirmed by the numerical simulations I conduct: a value added model of different industries and complementarity in consumer preferences over value added indicates that capital and labor are complements. By contrast, the gross output model with industry structure i)-ii) and complementarity in consumer preferences over gross output indicates a unitary elasticity. The theoretical part of this literature attempts to derive the aggregate elasticity from micro-foundations. Jones (2005), Kortum (1997) and Lagos (2006), for example, present models that follow an idea from Houthakker (1955) in which productivity levels are drawn from Pareto distributions which implies that production takes a Cobb-Douglas form, i.e. the elasticity of substitution between capital and labor is unity. My paper departs from this literature by highlighting the importance of the network structure of the economy in determining the aggregate capital labor elasticity. It shows that this structure contains demand elasticities that crucially affect the degree of substitution between capital and labor.

Outline. The paper proceeds as follows. Section 2 decomposes the post-war U.S. economy into heterogeneous industries and presents evidence that casts doubt on the two prerequisites of balanced growth. Section 3 analyzes the industries input-output linkages in the post-war U.S. economy. Section 4 presents a model of the network structure of the U.S. economy that incorporates the input-output linkages. Section 5 simulates the model and compares it to alternatives. Section 6 concludes.

2 Empirical Motivation

In this section, I conduct an industry decomposition of the post-war U.S. economy and show that this decomposition conflicts with the two prerequisites of balanced growth. The data I use for this exercise are the Bureau of Economic Analysis' (BEA) National Income and Product Accounts (NIPA) data. For 2-digit North American Industry Classification (NAICS) industries, these data are available from 1948 onwards.

2.1 Data

Consider N different industries. An industry belongs to either the labor intensive sector 1 or to the capital intensive sector 2. Each industry n produces with an industry specific capital-labor elasticity, σ_n, and a capital intensity, α_n. Value added of industry n at time t is given by

$$Y_{\mathbf{VA},n,t} = A_{n,t}[\alpha_n K_{n,t}^{\frac{\sigma_n-1}{\sigma_n}} + (1-\alpha_n) L_{n,t}^{\frac{\sigma_n-1}{\sigma_n}}]^{\frac{\sigma_n}{\sigma_n-1}}, \tag{1}$$

where $L_{n,t}$ denotes the amount of labor and $K_{n,t}$ the amount of capital used in production in industry n at time t. $A_{n,t}$ denotes the industry n's productivity level. The capital-labor elasticity is given by

$$\sigma_n - 1 = \frac{dln(r_{n,t}K_{n,t}/w_{n,t}L_{n,t})}{dln(w_{n,t}/r_{n,t})} \tag{2}$$

where $w_{n,t}$ and $r_{n,t}$ denote the returns to labor and capital, respectively. To back out the industry's capital-labor elasticities from the data (which do not contain information on wages and interest rates) I make two simplifying assumptions. Motivated by the empirical evidence of Figure 1 that shows an approximately stable labor share, I first assume that the aggregate capital-labor elasticity is unity.[3] Second, I assume that the change in relative factor prices, $ln(w_{n,t}/w_{n,t-1}) - ln(r_{n,t}/r_{n,t-1})$, is identical across sectors in the long-run.[4]

Table 1 shows the basic production characteristics of the post-war U.S. industries. It also provides a breakdown of the post-war U.S. private industries into two representative sectors - a capital and labor intensive sector. Columns (1) and (2) show how the value added shares of individual industries change over time. Column (3) shows the industry heterogeneity in labor intensities during 1948-2008. Column (4) shows the industry heterogeneity in capital-labor elasticities. Column (5) shows the industry heterogeneity in productivity growth rates estimated based on the assumption that sectors have a unitary capital-labor elasticity. Column (6) shows the industry heterogeneity in productivity growth rates estimated based on the assumption that production takes the form of Eq. (1).

Result 1 (industry heterogeneity) *In post-war U.S data, industries differ in capital intensity, capital-labor elasticity and productivity growth rates.*

2.2 The goods elasticity of substitution

Let us now assume that a representative consumer has CES preferences over the value added produced by the N industries,

$$Y_{\mathbf{VA},t} = \left(\sum_{n \in N} \gamma_n^{\frac{1}{\epsilon}} Y_{\mathbf{VA},n,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where ϵ denotes the goods elasticity of substitution and γ_n the share of industry n in aggregate value added. To determine the goods elasticity of substitution, ϵ, which I subsequently use to determine the aggregate capital labor elasticity, I run the following regression[5]:

[3] If anything, this assumption works towards finding an aggregate elasticity of unity in Section 2.3 and, thus, against my finding of an elasticity of less than unity. The assumption effectively just sets a reference point and is necessary because information on wages and interest rates are not contained in the data. Industries above this reference point have increasing capital shares and industries below this reference point have decreasing capital shares.

[4] In the long-run factor reallocation frictions do not matter much, so returns equalize across industries.

[5] This is a replica exercise of the one I conduct in the closely related paper Struck (2015) that presents a very different approach to same research question that is addressed here.

Table 1: U.S. value added data from the Bureau of Economic Anlaysis (BEA)

	NAICS	(1) $\frac{P_{\mathbf{VA},n}Y_{\mathbf{VA},n}}{P_{\mathbf{VA}}Y_{\mathbf{VA}}}$ 1948-1958	(2) $\frac{P_{\mathbf{VA},n}Y_{\mathbf{VA},n}}{P_{\mathbf{VA}}Y_{\mathbf{VA}}}$ 1998-2008	(3) $1-\alpha_n$	(4) σ_n	(5) $\frac{A_{n,t}}{A_{n,t-1}}$ (CD)	(6) $\frac{A_{n,t}}{A_{n,t-1}}$ (CES)
Educational services	61	0.42 %	1.05 %	0.60	0.81	0.76	0.76
Health care, social assistance	62	2.06 %	7.23 %	0.83	0.88	0.71	0.71
Management	55	1.71 %	1.93 %	0.88	1.15	1.42	1.40
Administration, waste management	56	0.75 %	3.25 %	0.86	0.60	1.01	1.01
Construction	23	5.10 %	5.37 %	0.86	1.00	0.64	0.63
Arts, entertainment, and recreation	71	0.65 %	1.09 %	0.78	0.69	1.31	1.31
Other services, except government	81	3.20 %	2.85 %	0.88	1.10	1.15	1.14
Professional, scientific, tech. services	54	1.83 %	7.51 %	0.86	1.07	1.00	0.99
Retail trade	44, 45	9.46 %	7.49 %	0.77	1.05	1.74	1.74
Transportation, warehousing	48, 49	5.98 %	3.33 %	0.75	0.71	3.14	3.14
Durable goods	33, 321, 327	17.62 %	8.91 %	0.72	1.27	3.83	3.65
Accommodation, food services	72	2.61 %	3.16 %	0.75	0.78	1.01	1.01
Wholesale trade	42	6.99 %	6.78 %	0.72	1.39	2.67	2.31
Finance and insurance	52	3.54 %	8.36 %	0.64	1.48	0.75	0.59
Information	51	3.41 %	5.61 %	0.53	1.24	2.47	2.25
Nondurable goods	31, 32	13.07 %	6.77 %	0.52	1.27	2.45	2.33
Agriculture, forestry, etc.	11	6.24 %	1.15 %	0.33	1.20	3.23	3.16
Mining	21	2.75 %	1.80 %	0.33	1.46	0.94	0.88
Utilities	22	2.11 %	1.93 %	0.28	1.19	1.97	1.93
Real estate, rental, leasing	53	10.51 %	14.44 %	0.18	0.87	1.49	1.49
labor intensive		58.38 %	59.94 %	0.79	0.89	2.04	2.06
capital intensive		41.62 %	40.06 %	0.39	1.24	1.95	1.79
private industries	11-81	100.0 %	100.0 %	0.63	1.00	1.98	1.98

Notes: the table shows the author's calculations based on the U.S. BEA NIPA data. $P_{\mathbf{VA},n}Y_{\mathbf{VA},n}/P_{\mathbf{VA}}Y_{\mathbf{VA}}$ is the value added share of a sector in total value added. $1-\alpha_n$ is the labor share between 1987-2008 based on BLS data. σ_n is the capital-labor elasticity and is calculated as explained in Section 2.1. $A_{n,t}/A_{n,t-1}$ (CD) are industry total factor productivity estimates based on Cobb-Douglas production functions. $A_{n,t}/A_{n,t-1}$ (CES) are industry total factor productivity estimates based on CES production functions.

$$\ln\left[\frac{\frac{P_{\mathbf{VA},n,t}Y_{\mathbf{VA},n,t}}{P_{\mathbf{VA},t}Y_{\mathbf{VA},t}}}{\frac{P_{\mathbf{VA},n,t-1}Y_{\mathbf{VA},n,t-1}}{P_{\mathbf{VA},t-1}Y_{\mathbf{VA},t-1}}}\right] = \underset{[0.21]}{0.02} + \underset{[5.42]}{0.87^{***}} \ln\left[\frac{\frac{P_{\mathbf{VA},n,t}}{P_{\mathbf{VA},t}}}{\frac{P_{\mathbf{VA},n,t-1}}{P_{\mathbf{VA},t-1}}}\right] + \nu_t, \tag{4}$$

where ν_t is white noise; t-values are in brackets; $P_{\mathbf{VA},n,t}$ is the price index of industry n. To avoid regression results being driven by outliers, I take 10 year averages for the initial and final period. Thus, I set $t = 1998-2008$ and $t-1 = 1948-1958$. Regressing changes in nominal expenditure on changes in prices across industries, I find a regression coefficient of 0.87, implying a low goods elasticity of substitution, $\epsilon = 0.13$.

2.3 The aggregate capital-labor elasticity

To extract the aggregate capital-labor elasticity from the industry data I follow the approach of Oberfield and Raval (2014). Oberfield and Raval (2014) show that the economy's aggregate capital-labor elasticity can be rewritten as a combination of the industries' capital-labor elasticities, the industries' labor intensities, the industries' value added shares and the goods elasticity of substitution:

$$\sigma^{\text{agg}} = (1-\chi)\bar{\sigma}_N + \chi\epsilon \tag{5}$$

where $\chi = \sum_{n\in N} \frac{((1-\alpha_n)-(1-\alpha))^2}{(1-\alpha)\alpha}\phi_n$ and $\bar{\sigma}_N = \sum_{n\in N} \frac{(1-\alpha_n)\alpha_n\phi_n}{\sum_{n'\in N}(1-\alpha_{n'})\alpha_{n'}\phi_n}\sigma_n$; α is the aggregate capital intensity; $\phi_n = (P_{\mathbf{VA},n,t}Y_{\mathbf{VA},n,t})/(P_{\mathbf{VA},t}Y_{\mathbf{VA},t})$ is the share of industry n in total value added. Assuming a goods elasticity of 0.13, the aggregate capital-labor elasticity that I find is 0.90 - below unity and indicting that capital and labor are complements on aggregate. The estimate of an elasticity of less than unity is in line with the estimates of (among many others) Klump et al. (2007), Arrow et al. (1961), Sato (1970), Berndt et al. (2001) and Antras (2004).

Result 2 (failure of prerequisite one) *In post-war U.S data, the assumption of a representative consumer with CES preferences over industry value added implies that capital and labor are complements on aggregate.*

Given the low estimates of the capital-labor elasticity, this elasticity cannot be used in explaining why the aggregate data exhibit balanced growth. The known alternative way to get balanced growth in value added models is to have 'labor biased' technical change, i.e. more productivity growth in industries that are more labor intensive, see e.g., the models of Uzawa (1961), Jones (2005), Acemoglu (2002), Acemoglu and Guerrieri (2008) and Struck (2015).

2.4 The allocation of productivity growth across industries

To obtain an estimate of the theoretically needed allocation of productivity growth that restores balanced growth, I focus on a simple Cobb-Douglas benchmark case, i.e. let's set σ_n to unity in all industries. The next equation is drawn from a standard neoclassical two-sector framework of Acemoglu and Guerrieri (2008). To obtain balanced growth, relative productivity growth needs to equal the relative labor share as Struck (2015) shows:

$$\frac{\dot{A}_1/A_1}{\dot{A}_2/A_2} = \frac{1-\alpha_1}{1-\alpha_2}. \tag{6}$$

Using the data from Table 1 and substituting them into this equation, I find that productivity growth in the labor intensive sector needs to be $(1 - \alpha_1)/(1 - \alpha_2) = 0.79/0.39 \approx 2$ times higher than productivity growth in the capital intensive sector[6]. Table 1 shows that relative productivity growth only amounts to $(2.04^{1/50} - 1)/(1.95^{1/50} - 1) \approx 1.07$ - far below the theoretically required value of 2. Figure 2 confirms these estimates. It plots the relation between the allocation of productivity growth and the production characteristics across the N industries. All four panels of the figure clearly show that there is no tendency for productivity growth to be higher in industries that rely more on labor.

Result 3 (failure of prerequisite two, Struck 2015) *In post-war U.S. data, there is no relation between the allocation of productivity growth, the capital-labor elasticity and the capital-output ratio across industries.*

Given that we neither observe a unitary capital-labor elasticity on aggregate nor much labor augmenting technical progress, why do the post-war U.S. data exhibit approximate balanced growth as shown in Figure 1? In the following section, I present descriptive empirical evidence on the input-output linkages across the U.S post-war industries. I then use this evidence subsequently to motivate a model of the network structure of the U.S. economy in which different industries are connected via input-output linkages. In numerical exercises, I later show that the input-output linkages can restore balanced growth for empirically plausible parameter assumptions.

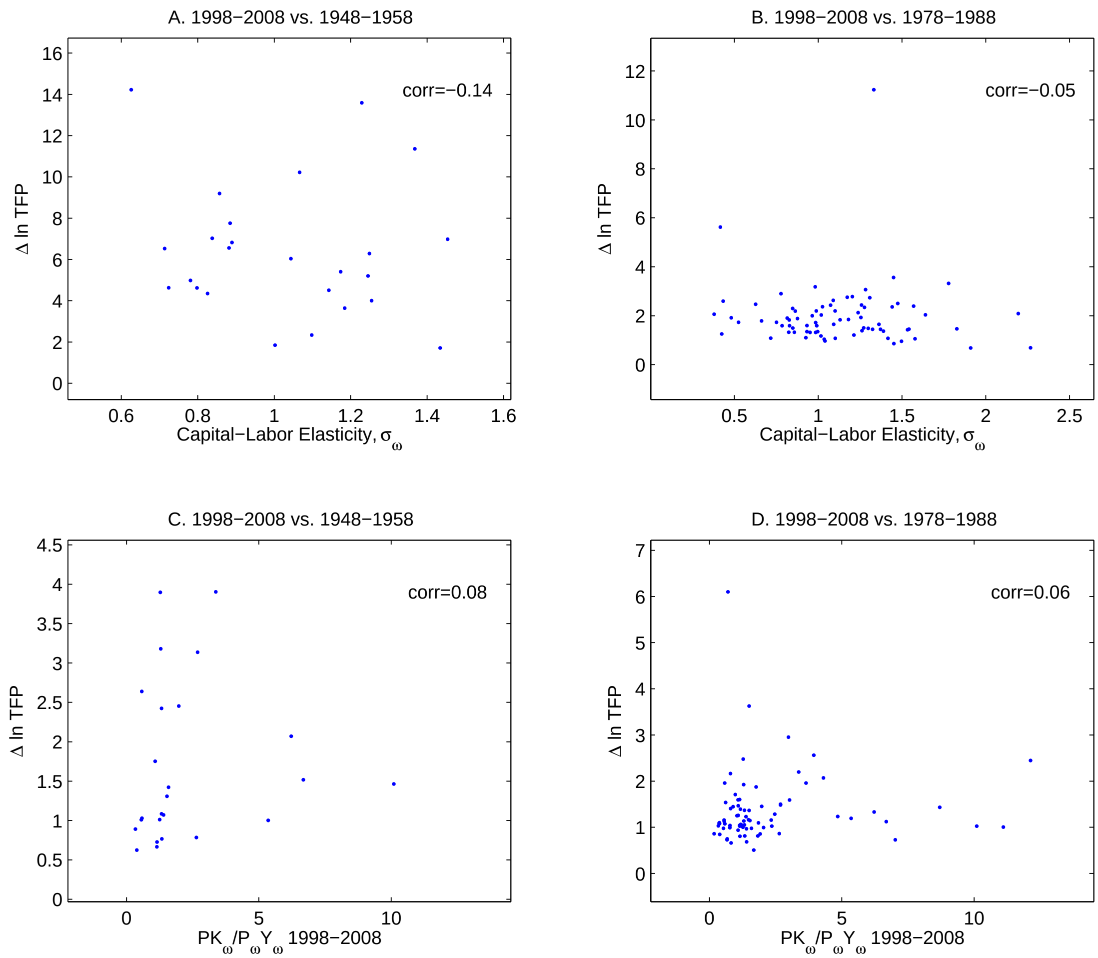
3 Empirical Analysis of the Input-Output structure

In this section I provide descriptive empirical evidence on the input-output structure of the post-war U.S. industries. In particular, I show that i) intermediate inputs maintain a significant share in production during the entire post-war period ii) capital and labor intensive sectors have heterogeneous preferences for the composition of intermediate inputs. In the quantitative analysis below, I show how a model with these features can generate balanced growth in the U.S. post-war environment of structural change and substantial heterogeneity in industry production technologies.

3.1 Data

For the analysis in this section, I rely on the data of Jorgenson et al. (2007). Jorgenson et al. (2007) provides annual data for 35 SIC industries based on the U.S. BEA make and use tables. The data range from 1960 to 2005. Whereas the U.S. BEA NIPA data provide only industry value added information, this dataset provides a breakdown of the industries' input-output linkages. In particular, it provides information to what extend each of the 35 industries uses output from each of the other industries as an

[6] This result comes from a Cobb-Douglas benchmark and is a lower bound as I show in Struck (2015). With a more general CES production function, i.e. a version of Eq. (1) where $\sigma_n \neq 1$, there is no analytical equivalent of Eq. (6). In numerical simulations, however, I show in Struck (2015) that relative productivity growth needs to exceed the theoretical threshold of 2 and needs to be closer to 2.75.



Notes: the figure shows the author's calculations based on the U.S. BEA NIPA data. The figure is taken from Struck (2015). Panel A shows the change in total factor productivity growth 1948-1958 vs. 1998-2008 and the capital labor elasticity. Panel B is a robustness exercise of Panel A with a different period: 1978-1988 vs. 1998-2008. Panel C shows the change in total factor productivity growth 1948-1958 vs. 1998-2008 and the capital-output ratio. Panel D is a robustness exercise of Panel C with a different period: 1978-1988 vs. 1998-2008.

Figure 2: TFP Estimates and Industry Production Characteristics.

input in production. In the analysis, I again focus on the private sector and therefore remove industry 35 'government enterprises' from the dataset.

3.2 The input-output structure

Before I describe the data, I simplify the dataset by reorganizing the 34 different intermediate inputs into two sectors - inputs from capital and inputs from labor intensive sectors. Table 2 shows that the data from Jorgenson et al. (2007) can be classified into two sectors similar to the ones used in Table 1. For the empirical analysis I need to compute the labor share of intermediates and the share of the labor intensive goods in intermediates by industry. To do this, consider nominal gross output in industry n at time t,

$$P_{\mathbf{GO},n,t}Y_{\mathbf{GO},n,t} = r_{n,t}K_{n,t} + w_{n,t}L_{n,t} + P_{\mathbf{GO},1,t}M_{1,n,t} + P_{\mathbf{GO},2,t}M_{2,n,t} + ... + P_{\mathbf{GO},34,t}M_{34,n,t}, \quad (7)$$

where $P_{\mathbf{GO},n,t}$ is the price of gross output, $Y_{\mathbf{GO},n,t}$, of industry n; $M_{i,n,t}$ is the amount of industry i's gross output used as an intermediate input in the production of industry n. I can rearrange this equation as follows:

$$r_{n,t}K_{n,t} + w_{n,t}L_{n,t} = P_{\mathbf{GO},n,t}Y_{\mathbf{GO},n,t} - P_{\mathbf{GO},1,t}M_{1,n,t} - P_{\mathbf{GO},2,t}M_{2,n,t} - .... - P_{\mathbf{GO},34,t}M_{34,n,t}. \quad (8)$$

To introduce a more compact notation, I use a vector notation that contains the industry information: $x_t = [x_{1,t}, x_{2,t}, ..., x_{34,t}]'$, where $x_t \in [r_tK_t, w_tL_t, P_{\mathbf{GO},t}Y_{\mathbf{GO},t}]$. I express the industry structure of the economy in matrix form,

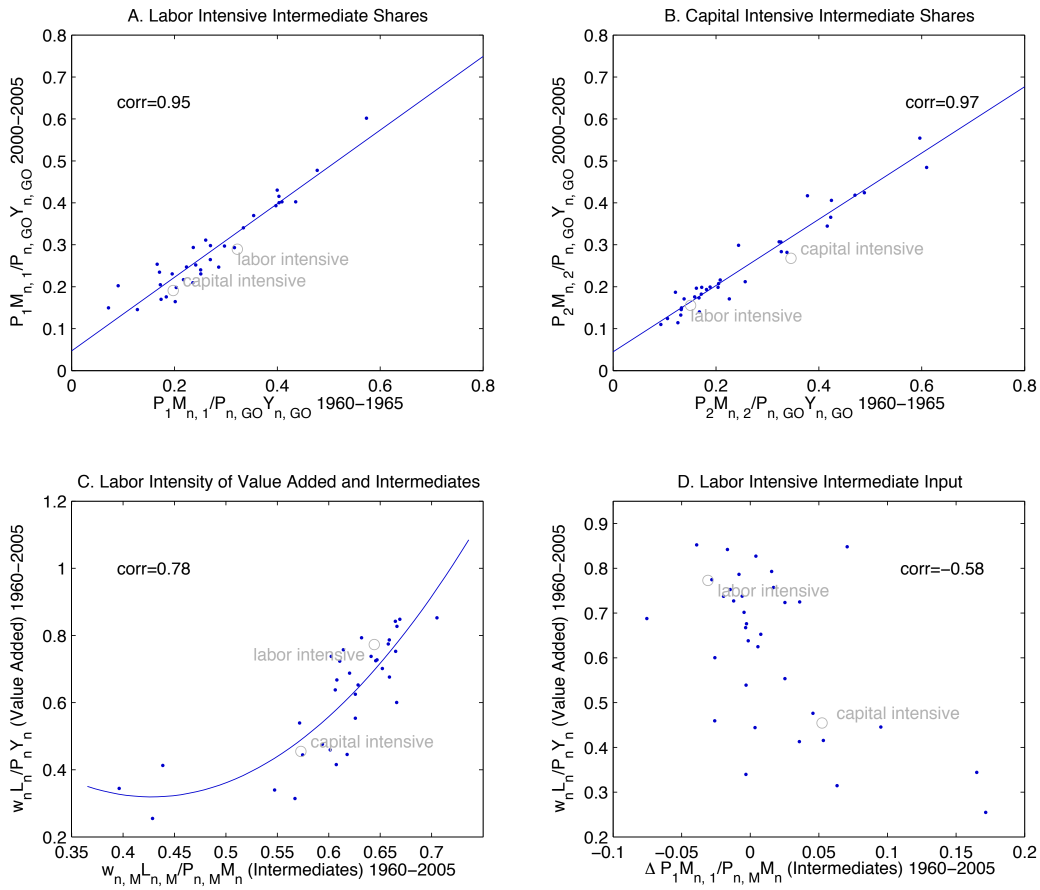
$$r_tK_t + w_tL_t = (\mathbf{I}_{34} - \mathbf{A})P_{\mathbf{GO},t}Y_{\mathbf{GO},t}, \text{ where } \mathbf{A} = \begin{bmatrix} \frac{P_{GO,1,t}M_{1,1,t}}{P_{GO,1,t}Y_{GO,1,t}} & \frac{P_{GO,2,t}M_{2,1,t}}{P_{GO,2,t}Y_{GO,2,t}} & \cdots & \frac{P_{GO,34,t}M_{34,1,t}}{P_{GO,34,t}Y_{GO,34,t}} \\ \frac{P_{GO,1,t}M_{1,2,t}}{P_{GO,1,t}Y_{GO,1,t}} & \frac{P_{GO,2,t}M_{2,2,t}}{P_{GO,2,t}Y_{GO,2,t}} & \cdots & \frac{P_{GO,34,t}M_{34,2,t}}{P_{GO,34,t}Y_{GO,34,t}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{P_{GO,1,t}M_{1,34,t}}{P_{GO,1,t}Y_{GO,1,t}} & \frac{P_{GO,2,t}M_{2,34,t}}{P_{GO,2,t}Y_{GO,2,t}} & \cdots & \frac{P_{GO,34,t}M_{34,34,t}}{P_{GO,34,t}Y_{GO,34,t}} \end{bmatrix}. \quad (9)$$

The labor income in intermediates can then be derived from the previous equation as

$$w_{M,t}L_{M,t} = (\mathbf{I}_{34} - \mathbf{A})^{-1}w_tL_t - w_tL_t. \quad (10)$$

The labor share of intermediates can be derived as $w_{n,M,t}L_{n,M,t}/P_{n,M,t}M_{n,t}$, where $P_{n,M,t}$ is the price of composite intermediates in sector n; $M_{n,t}$ is the total amount of intermediates used by industry n. To back out the shares of the capital and labor intensive goods in intermediate inputs, $\frac{P_{GO,2,t}M_{n,2,t}}{P_{n,M,t}M_{n,t}}$ and $\frac{P_{GO,1,t}M_{n,1,t}}{P_{n,M,t}M_{n,t}}$, I simply add up the inputs from the industries into two the inputs of the two aggregate sectors - the labor and capital intensive goods.

Comparison of the Jorgenson et al. 2007 and BEA NIPA NAICS data. Table 2 contains a summary of the Jorgenson et al. (2007) data. The table shows that the data are roughly comparable to the U.S. BEA NIPA NAICS value added data in terms of the breakdown into the two aggregate sectors. The share of the labor intensive sector in value added is almost constant and rises only slightly from 63.48% to 64.10%. Meanwhile the share of the capital intensive sector drops from 36.52% to 35.90%. The labor shares of the Jorgenson et al. (2007) data are slightly different relative to those in the BEA data in Table 1. The labor intensive sector has a slightly lower labor share of $1 - \alpha_L = 0.77$ (0.79 in the BEA data). The capital intensive sector has a slightly higher labor share of $1 - \alpha_L = 0.45$ (0.39 in the BEA data). Similar to the BEA data capital grows at a slightly higher rate in the capital intensive sector (Column 4) and labor grows at a faster rate in the labor intensive sector (Column 5). The data differ nonetheless in one key dimension. The labor share in the Jorgenson et al. (2007) data declines by several percentage points



Notes: the figure shows the author's calculations based on the data from Jorgenson et al. (2007). Panel A shows that the share of the labor intensive intermediate in gross output is highly correlated over time. Panel B shows that the share of the capital intensive intermediate in gross output is highly correlated over time. Panel C regresses the labor intensity of value added on the labor intensity of intermediates. Section 3.2 describes how the labor intensity of intermediates is calculated. Panel D regresses the labor share in value added on the change in the share of the labor intensive intermediates in total intermediates.

Figure 3: Industry heterogeneity in intermediates and stability of the input-output structure.

over time. This is revealed in Columns 4 and 5 which show that the capital compensation is growing faster than labor compensation in both sectors (19.42 and 21.46 vs. 17.66 and 13.80). This difference is not relevant for the exercise in this section. However, it will matter later when simulating and comparing different models. To simulate the models, I therefore use the BEA value added data as the main dataset and augment this dataset with the information on nominal intermediate shares that are contained in the Jorgenson et al. (2007) data. This means, that the combined dataset remains unaffected by the problem of declining labor shares.

Input-output stability. Panels A and B of Figure 3 show that both capital and labor intensive intermediate inputs have a quantitatively significant share in gross output. The input share of each intermediate varies across industries between 10% to 60%. On average, intermediate inputs account for approximately

50% of gross output. Most importantly, the intermediate input shares in gross output are roughly stable over time. The correlation between initial period and final period input shares is over .90 for both capital and labor intensive intermediate inputs.

Result 4 (intermediate stability) *Both intermediate inputs maintain quantitatively significant shares in nominal gross output.*

Heterogeneity in the IO structure. Table 2 shows that the share of capital intensive intermediate inputs is higher in the capital intensive sector and lower in the labor intensive sector (Columns 6 and 7). By contrast, the share of labor intensive intermediate inputs is higher in the labor intensive sector and lower in the capital intensive sector (Columns 8 and 9). Over time the share of capital intensive intermediate inputs in total intermediate inputs increases from 31.87% to 34.96% in the labor intensive sector. Meanwhile, the share of labor intensive intermediate inputs in total intermediate inputs increases from 36.36% to 41.59% in the capital intensive sector. Figure 3 visualizes this heterogeneity. Panel C shows that industries with a higher labor share in value added use more labor intensive intermediate inputs (correlation 0.78). Panel D shows that industries with a higher labor share in value added use a decreasing share of labor intensive intermediate inputs (correlation -0.58). Motivated by this evidence, in the following section, I build a model of two-sided heterogeneity, i.e. heterogeneity in production technologies and heterogeneity in preferences over these technolgoies. Simulating this model subsequently, I illustrate that the input-output structure can induce balanced growth in a gross output model with substantial industry heterogeneity for long periods.

Result 5 (demand side heterogeneity) *More labor intensive industries use more labor intensive intermediate inputs. More capital intensive industries use more capital intensive intermediate inputs. Over time, more labor intensive industries shift to more capital intensive intermediate inputs while more capital intensive industries shift to more labor intensive intermediate inputs.*

4 Model

Because of the theoretical complications that adding an input-output structure to a standard value added model introduces, I present the model and highlight the main assumptions that I make in this section. In the subsequent section I solve the model numerically and derive the main results of this paper.

4.1 A gross output model with two-sided heterogeneity

Consider an economy in which two sectors produce two goods $n = 1, 2$. Good 1 is labor intensive and good 2 is capital intensive. Gross output of sector n, $Y_{\mathbf{GO},n,t}$, is produced using four different inputs: capital, $K_{n,t}$, labor, $L_{n,t}$, and the intermediate inputs from the two sectors, $M_{n,1,t}$ and $M_{n,2,t}$. The output of the

Table 2: U.S. input-ouptut data from Jorgenson et al. (2007)

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		value added					intermediate inputs			
							capital intensive		labor intensive	
Industry	Jorgenson	$\frac{P_n Y_n}{PY}$ 1960-1965	$\frac{P_n Y_n}{PY}$ 2000-2005	$1 - \alpha_n$	$\frac{r_t K_t}{r_{t-1} K_{t-1}}$	$\frac{w_t L_t}{w_{t-1} L_{t-1}}$	$\frac{P_K M_{n,K}}{P_{n,M} M_n}$ 1960-1965	$\frac{P_K M_{n,K}}{P_{n,M} M_n}$ 2000-2005	$\frac{P_L M_{n,L}}{P_{n,M} M_n}$ 1960-1965	$\frac{P_L M_{n,L}}{P_{n,M} M_n}$ 2000-2005
Personal and business services	34	12.89 %	28.48 %	0.79	30.61	42.17	41.59 %	40.04 %	58.41 %	59.96 %
Wholesale and retail trade	32	17.77 %	15.30 %	0.76	22.73	13.60	47.91 %	46.22 %	52.09 %	53.78 %
Transportation, warehousing	28	5.39 %	3.30 %	0.69	12.62	10.25	30.98 %	38.53 %	69.02 %	61.47 %
Construction	6	8.99 %	5.65 %	0.84	18.64	10.23	24.81 %	26.48 %	75.19 %	73.52 %
Durable goods	11,12,19-27	18.43 %	11.37 %	0.73	12.35	10.57	26.03 %	26.79 %	73.97 %	73.21 %
Finance, insurance, real estate	33	11.26 %	17.34 %	0.34	27.97	26.71	57.41 %	57.73 %	42.59 %	42.27 %
Information	29	2.41 %	2.98 %	0.45	25.63	18.39	50.11 %	40.59 %	49.89 %	59.41 %
Nondurable goods	7-10, 13-18	13.16 %	9.02 %	0.62	17.57	9.87	63.06 %	60.37 %	36.94 %	39.63 %
Agriculture, forestry, etc.	1	3.98 %	2.34 %	0.54	13.85	8.31	67.40 %	67.71 %	32.60 %	32.29 %
Mining	2,3,4,5	3.03 %	1.45 %	0.32	10.03	6.15	71.53 %	59.66 %	28.47 %	40.34 %
Utilities	30,31	2.69 %	2.76 %	0.32	19.35	16.31	71.31 %	55.46 %	28.69 %	44.54 %
labor intensive		63.48 %	64.10 %	0.77	19.42	17.66	31.87 %	34.96 %	68.13 %	65.04 %
capital intensive		36.52 %	35.90 %	0.45	21.46	13.80	63.64 %	58.41 %	36.36 %	41.59 %
private industries		100.0 %	100.0 %	0.66	20.57	16.61	45.63 %	43.68 %	54.37 %	56.32 %

Notes: the panels show the author's calculations based on the data drawn from Jorgenson et al. (2007). $P_n Y_n / PY$ denotes industry n's value added share in the total value added of all private industries. $1 - \alpha_n$ denotes the average labor share of industry n during 1960-2005. $r_t K_t / (r_{t-1} K_{t-1})$ denotes the change in the capital income. $w_t L_t / w_{t-1} L_{t-1}$ denotes the change in the labor income. $P_K M_{n,K} / (P_{n,M} M_n)$ is the share of the capital intensive intermediate in total intermediates in industry n. $P_L M_{n,L} / (P_{n,M} M_n)$ is the share of the labor intensive intermediate in total intermediates in industry n.

two sectors can be combined to form a final good that can be used for consumption and investment.

4.2 Sectors

The representative producer in the perfectly competitive sector n maximizes profits

$$\pi_{n,t} = P_{\mathbf{GO},n,t} Y_{\mathbf{GO},n,t} - r_{n,t} K_{n,t} - w_{n,t} L_{n,t} - P_{\mathbf{GO},1,t} M_{n,1,t} - P_{\mathbf{GO},2,t} M_{n,2,t} \tag{11}$$

where (as above) $r_{n,t}$, $w_{n,t}$ and $P_{\mathbf{GO},n,t}$ denote the prices of capital, labor and gross output of sector n at time t, respectively. Sector value added is given by

$$P_{\mathbf{VA},n,t} Y_{\mathbf{VA},n,t} = r_{n,t} K_{n,t} + w_{n,t} L_{n,t}, \tag{12}$$

where (as above) $P_{\mathbf{VA},n,t}$ denotes the price of value added and $Y_{\mathbf{VA},n,t}$ real value added in sector n at time t. Total expenditure on intermediate inputs in sector n is given by

$$P_{M,n,t} M_{n,t} = P_{\mathbf{GO},1,t} M_{n,1,t} + P_{\mathbf{GO},2,t} M_{n,2,t}, \tag{13}$$

where $P_{M,n,t}$ denotes the price of total intermediate inputs and $M_{n,t}$ the real total intermediate inputs in sector n at time t. Sector value added and intermediate inputs are combined in both sectors with the same Cobb-Douglas production technology to mimic the relatively stable share of intermediate inputs in gross output:

$$Y_{\mathbf{GO},n,t} = Y_{\mathbf{VA},n,t}^{\nu} M_{n,t}^{1-\nu}. \tag{14}$$

where (as above) $A_{n,t}$ denotes sector n's productivity. The assumption of a Cobb-Douglas technology that combines value added and intermediate inputs is admittedly simplifying. It is meant to capture the empirical evidence that intermediates maintain a substantial share in nominal gross output over time (Result 4). In Appendix A I relax this assumption of a Cobb-Douglas functional form between intermediates and value added and show that the main results of this paper are robust to using a more flexible functional form.

Assumption 1 (Intermediate Stability). *The share of intermediate inputs in gross output is stable, i.e. the elasticity of intermediates and value added equals unity.*

I now introduce sector heterogeneity in production technologies to build in the empirical evidence of Result 1. Value added is given by

$$Y_{\mathbf{VA},1,t} = A_{1,t} \left[\alpha_1 K_{1,t}^{\frac{\sigma_1 - 1}{\sigma_1}} + (1-\alpha_1) L_{1,t}^{\frac{\sigma_1 - 1}{\sigma_1}} \right]^{\frac{\sigma_1}{\sigma_1 - 1}}, \quad Y_{\mathbf{VA},2,t} = A_{2,t} \left[\alpha_2 K_{2,t}^{\frac{\sigma_2 - 1}{\sigma_2}} + (1-\alpha_2) L_{2,t}^{\frac{\sigma_2 - 1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_2 - 1}}, \tag{15}$$

where the degree of substitution between capital and labor, σ_n, as well as the capital intensity, α_n, differs across sectors. The heterogeneity across sectors resembles the one presented in Table 1 in that the capital intensive sector becomes more capital intensive and the labor intensive sector becomes more labor intensive over time (Result 1).

Assumption 2 (Industry Side Heterogeneity). *The capital intensity of sector 1 is lower than the capital intensity of sector 2, i.e.* $\alpha_1 < \alpha_2$*. Capital and labor are complements in sector* 1*, i.e.* $\sigma_1 < 1$ *and substitutes in sector* 2*, i.e.* $\sigma_2 > 1$*.*

In Appendix A I also relax this assumption of a CES functional form of value added and total factor productivity growth. In this Appendix, I show that the main results are robust to using a more flexible functional form that also allows for biased technical change. Intermediate inputs are combined with a CES function as follows:

$$M_{1,t} = \left[\alpha_{M,1} M_{1,1,t}^{\frac{\sigma_{M,1}-1}{\sigma_{M,1}}} + (1-\alpha_{M,1}) M_{1,2,t}^{\frac{\sigma_{M,1}-1}{\sigma_{M,1}}}\right]^{\frac{\sigma_{M,1}}{\sigma_{M,1}-1}},$$

$$M_{2,t} = \left[\alpha_{M,2} M_{2,2,t}^{\frac{\sigma_{M,2}-1}{\sigma_{M,2}}} + (1-\alpha_{M,2}) M_{2,1,t}^{\frac{\sigma_{M,2}-1}{\sigma_{M,2}}}\right]^{\frac{\sigma_{M,2}}{\sigma_{M,2}-1}}, \quad (16)$$

where $\sigma_{M,n}$ denotes the degree of substitution between capital and labor intensive intermediates; $\alpha_{M,n}$ denotes the share of the capital intensive intermediate input. I now introduce sector heterogeneity in intermediate inputs to build in the empirical evidence of heterogeneity in input-output linkages (Result 5).

Assumption 3 (Demand Side Heterogeneity). *The intermediate goods,* $M_{1,n,t}$ *and* $M_{2,n,t}$*, are substitutes in sector* 1*, i.e.* $\sigma_{M,1} > 1$ *and complements in sector* 2*, i.e.* $\sigma_{M,2} < 1$*. The use of capital intensive intermediate inputs in sector 1 is lower than the use of capital intensive intermediate inputs in sector 2, i.e.* $\alpha_{M,1} < \alpha_{M,2}$*.*

To highlight the importance of this assumption, in the simulations in Section 5.2, I run also a model in which each industry uses only its own output as an intermediate input, i.e. a model where $\alpha_{M,1} = \alpha_{M,2} = 1$. In this model, I show that the economy departs quickly from the balanced growth path, which shows that this assumption is necessary in addition to Assumption 1 to generate balanced growth. Again, in Appendix A I also relax the restrictive assumption of a CES functional form that combines different intermediates and show that the main results of this paper are robust to using a more flexible functional form here.

4.3 Consumers

Labor, L, grows at an exogenously given rate m,

$$L_t = e^{mt} L_0. \tag{17}$$

Workers are allocated across the two sectors, $L_t = L_{1,t} + L_{2,t}$. All workers have identical preferences over a final good that can be used for both consumption and investment,

$$C_t = \left[\gamma^{\frac{1}{\epsilon}} C_{1,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}} C_{2,t}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \quad \text{and} \quad I_t = \left[\gamma^{\frac{1}{\epsilon}} J_{1,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}} J_{2,t}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{18}$$

where C_t and I_t denote aggregate consumption and investment. $C_{n,t}$ and $J_{n,t}$ denote the use of good n in aggregate consumption and investment. $\gamma \in [0,1]$ the share of the capital intensive good in the final good. $\epsilon < 1$ is the goods elasticity and is assumed to exhibit complementarity motivated by the empirical evidence from the regression of Eq. (4). Capital in sector n can be accumulated over time according to

$$K_{n,t+1} = (1-\delta) K_{n,t} + I_{n,t}, \tag{19}$$

where $\delta \in (0,1)$ denotes the depreciation rate. Aggregate investment is the sum of investment in the capital intensive and in the labor intensive sector, i.e. $I_t = I_{1,t} + I_{2,t}$. The representative consumer maximizes lifetime utility given by

$$U_t = \sum_{j=0}^{\infty} \beta^t e^{mt} \frac{c_{t+j}^{1-\phi} - 1}{1-\phi}, \tag{20}$$

where $c_t = C_t / L_t$ denotes the per-capita level of real consumption; β denotes the time preference parameter; θ is the inverse of the intertemporal elasticity of substitution. The representative consumer faces the following budget constraint:

$$r_{1,t} K_{1,t} + w_{1,t} L_{1,t} + r_{2,t} K_{2,t} + w_{2,t} L_{2,t} = P_t C_t + P_t I_t \tag{21}$$

where P_t denotes the price level of the aggregate consumption and investment good. Aggregate capital is the sum of capital in the two sectors, $K_t = K_{1,t} + K_{2,t}$. The no-ponzi condition therefore takes the form of

$$\lim_{t\to\infty} \left\{ K_t \times \exp\left[- \int_{s=0}^{t} (r(s) - m) ds \right] \right\} \geq 0. \tag{22}$$

Maximization of Eq. (20) subject to (21) with respect to c_t yields

$$c_t^{-\phi} = c_{t+1}^{-\phi} \beta (r_{t+1} + 1 - \delta), \tag{23}$$

which describes the intertemporal trade-off that the consumer faces.

4.4 Equilibrium

A competitive equilibrium is a combination of quantities c_t, $C_{n,t}$, $I_{n,t}$, I_t, $J_{n,t}$, $K_{n,t}$, $L_{n,t}$, $Y_{\mathbf{GO},n,t}$, $M_{n,1,t}$, $M_{n,2,t}$, $M_{n,t}$ and prices $w_{n,t}$, $r_{n,t}$, $P_{\mathbf{GO},n,t}$, $P_{M,n,t}$, P_t for $n = 1,2$ given the levels of productivity, $A_{n,t}$, and labor, L_t, such that

1. $K_{n,t}, L_{n,t}, M_{n,1,t}, M_{n,2,t}$ solve the maximization problem of the representative producer in the perfectly competitive sector n

$$\max_{K_{n,t},L_{n,t},M_{n,1,t},M_{n,2,t}} P_{\mathbf{GO},n,t}Y_{\mathbf{GO},n,t} - r_{n,t}K_{n,t} - w_{n,t}L_{n,t} - P_{\mathbf{GO},1,t}M_{n,1,t} - P_{\mathbf{GO},2,t}M_{n,2,t}$$

   taking $P_{\mathbf{GO},n,t}$ as given.

2. $M_{n,1,t}$ and $M_{n,2,t}$ solve producer n's intermediate input problem

$$\max_{M_{n,1,t},M_{n,2,t}} M_{n,t}$$

   subject to:

   (i) $M_{n,t} - \left[\alpha_{M,n}M_{n,n,t}^{1-\frac{1}{\sigma_{M,n}}} + (1-\alpha_{M,n})M_{n,\neg n,t}^{1-\frac{1}{\sigma_{M,n}}}\right]^{\frac{\sigma_{M,n}}{\sigma_{M,n}-1}} = 0$

   (ii) $P_{M,n,t}M_{n,t} - P_{\mathbf{GO},1,t}M_{n,1,t} - P_{\mathbf{GO},2,t}M_{n,2,t} = 0$

3. c_{t+j}, $L_{n,t+j}$, $K_{n,t+j+1}$, $I_{n,t+j}$ solve the representative consumer's intertemporal maximization problem

$$\max_{c_{t+j},L_{n,t+j},K_{n,t+j+1},I_{n,t+j}} \sum_{j=0}^{\infty} \beta^t e^{mt} \frac{c_{t+j}^{1-\phi} - 1}{1-\phi}$$

   subject to:

   (i) $r_{1,t+j}K_{1,t+j} + w_{1,t+j}L_{1,t+j} + r_{2,t+j}K_{2,t+j} + w_{2,t+j}L_{2,t+j} - P_{t+j}C_{t+j} - P_{t+j}I_{t+j} = 0$

   (ii) $K_{n,t+j+1} - (1-\delta)K_{n,t+j} - I_{n,t+j} = 0$

   (iii) $L_{t+j} - L_{2,t+j} - L_{1,t+j} = 0$

4. $C_{n,t}$ and $J_{n,t}$ solve the representative consumer's intratemporal maximization problem

$$\max_{C_{n,t},J_{n,t}} C_t + I_t$$

subject to:

(i) $C_t - [\gamma^{\frac{1}{\epsilon}}[C_{1,t}]^{1-\frac{1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}}[C_{2,t}]^{1-\frac{1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} = 0$

(ii) $I_t - [\gamma^{\frac{1}{\epsilon}}[J_{1,t}]^{1-\frac{1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}}[J_{2,t}]^{1-\frac{1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} = 0$

(iii) $P_t I_t + P_t C_t - \sum_{n=1,2} P_{\mathbf{GO},n,t} C_{n,t} - \sum_{n=1,2} P_{\mathbf{GO},n,t} J_{n,t} = 0$

5. Markets clear

$$P_{M,n,t} M_{n,t} - P_{\mathbf{GO},1,t} M_{n,1,t} - P_{\mathbf{GO},2,t} M_{n,2,t} = 0$$

$$P_{\mathbf{GO},1,t} Y_{\mathbf{GO},1,t} + P_{\mathbf{GO},2,t} Y_{\mathbf{GO},2,t} - P_{M,1,t} M_{1,t} - P_{M,2,t} M_{2,t} - P_t C_t - P_t I_t = 0$$

$$I_t - I_{1,t} - I_{2,t} = 0$$

$$K_t - K_{1,t} - K_{2,t} = 0$$

$$L_t - L_{2,t} - L_{1,t} = 0$$

$$P_t I_t = \sum_{n=1,2} P_{\mathbf{GO},n,t} J_{n,t}$$

$$P_t C_t = \sum_{n=1,2} P_{\mathbf{GO},n,t} C_{n,t}$$

To illustrate the main dynamics, I simulate this model in the following section.

5 Quantitative Analysis

In this section, I establish the main results of this paper: i) a standard value added model of heterogeneous industries does not achieve balanced growth based on an empirically grounded parameterization from the post-war U.S. data ii) adding an empirically motivated input-output structure to an otherwise standard value added model can restore balanced growth.

5.1 Data

To simulate the model, I first estimate the parameters of the model from a combined dataset. I use the BEA NAICS industry data as the main dataset and augment this dataset with the information on the shares of intermediates from the data of Jorgenson et al. (2007). I construct value added prices as a weighted average of input prices. I then construct gross output prices as a weighted average of value added and intermediate input prices. For the elasticities and shares of capital and labor, I use the previous estimates from Section 2 that are based on the BEA NAICS industry data. From the estimates of Section 2 I also use the productivity growth rates, $\xi_n = A_{n,t}/A_{n,t-1} - 1$. For the share of value added in gross output, η, I use the 1960-2005 private industry average. To back out the elasticities of intermediates in the two sectors, I use an equation similar to Eq. (4). For the shares of intermediate I use the 1960-1970 average. All elasticities and shares are backed out from normalized CES functions.

5.2 Simulations

Parameters. To establish the main results, I run four different models. Table 3 summarizes the parameter choices. All four models share a common set of parameters: a representative consumer has CES preferences over the output of different sectors[7]. The output of the different sectors are complements, i.e. $\epsilon = 0.13$. This choice is motivated by the empirical evidence of regression (4). The initial value added share of the capital intensive sector 2 is set to $\gamma = 0.42$ to reflect the share of the capital intensive sector in aggregate output of Table 1 at the beginning of the sample period. For simplification, I assume that the preferences for the composition of the aggregate investment good are identical to the preferences of the aggregate consumption good. Other parameters that are common across all four models are: population growth is set to $m = 0.0182$ - the 1948-2008 U.S. average annual rate. The time discount factor is set to $\beta = 0.97$. The depreciation rate of capital is set to $\delta = 0.1$. The initial level of the aggregate labor supply is normalized to $L = 1$. I set $\phi = 2$. On the industry side, the four models differ in the following way:

1. **Cobb-Douglas Valued-added.** Output of sector n is given by: $Y_{\mathbf{VA},n,t} = A_{n,t} K_{n,t}^{\alpha_n} L_{n,t}^{1-\alpha_n}$. In this model there are two sectors of heterogeneous production technologies. There are no input-output linkages across sectors. The capital intensive sector 2 has a capital share of $\alpha_2 = 0.61$. The labor intensive sector 1 has a capital share of $\alpha_1 = 0.21$. For simplicity, the capital labor elasticity in each sector is set to equal unity, i.e. $\sigma_1 = \sigma_2 = 1.00$. The productivity growth rates are taken from Table 1. The initial productivity level of the labor intensive industry is normalized to unity. The productivity of the capital intensive sector is set such that the initial price level equals unity in both sectors.

2. **CES Valued-added.** Output of sector n is given by: $Y_{\mathbf{VA},n,t} = A_{n,t}[\alpha_n K_{n,t}^{\frac{\sigma_n-1}{\sigma_n}} + (1-\alpha_n) L_{n,t}^{\frac{\sigma_n-1}{\sigma_n}}]^{\frac{\sigma_n}{\sigma_n-1}}$. In this model there are two sectors of heterogeneous production technologies. There are no input-output linkages across sectors. The capital intensive sector 2 has a capital share of $\alpha_2 = 0.61$. The labor intensive sector 1 has a capital share of $\alpha_1 = 0.21$. The capital-labor-elasticities are chosen based on the post-war U.S. industry estimates of Table 1. The elasticity of the capital intensive sector is set to $\sigma_2 = 1.24$. The elasticity of the labor intensive sector is set to $\sigma_1 = 0.89$. The productivity growth rates are also taken from Table 1. The initial productivity level of the labor intensive industry is set to unity. The productivity of the capital intensive sector is set such that the initial price level equals unity in both industries.

3. **CES Gross Output (one-sided heterogeneity).** Output of sector n is given by: $Y_{\mathbf{GO},n,t} = \left[A_{n,t}[\alpha_n K_{n,t}^{\frac{\sigma_n-1}{\sigma_n}} + (1-\alpha_n) L_{n,t}^{\frac{\sigma_n-1}{\sigma_n}}]^{\frac{\sigma_n}{\sigma_n-1}}\right]^{\nu} M_{n,n,t}^{1-\nu}$. In this model I make the same parameter choices as in the CES valued added model before. In addition, I add an input-output structure to this model. The share of intermediate inputs in both industries is set to $\nu = 0.56$ - the 1960-2005 average share of intermediates in gross output. For simplicity, I let each sector use only its own output as an intermediate input. Hence I set $\alpha_{M,1} = \alpha_{M,2} = 1$. This parameter choice is not motivated

[7] In the two-value added models the consumer's preferences are over value added. In the two gross output models the consumer has CES preferences over gross output.

Table 3: Simulation parameters and results

Parameters

Preferences	$\beta = 0.97$, $\phi = 2$, $\epsilon = 0.13$, $\delta = 0.10$, $m = 0.018$, $\gamma = 0.42$
Cobb-Douglas Value Added	$\alpha_2 = 0.61$, $\alpha_1 = 0.21$, $\sigma_2 = 1.00$, $\sigma_1 = 1.00$, $\xi_2 = 0.0134$, $\xi_1 = 0.0144$
CES Value Added	$\alpha_2 = 0.61$, $\alpha_1 = 0.21$, $\sigma_2 = 1.24$, $\sigma_1 = 0.89$, $\xi_2 = 0.0117$, $\xi_1 = 0.0146$
CES Gross Output (one-sided)	$\alpha_2 = 0.61$, $\alpha_1 = 0.21$, $\sigma_2 = 1.24$, $\sigma_1 = 0.89$, $\xi_2 = 0.0117$, $\xi_1 = 0.0146$ $\nu = 0.56$, $\alpha_{M,2} = 1$, $\alpha_{M,1} = 1$
CES Gross Output (two-sided)	$\alpha_2 = 0.61$, $\alpha_1 = 0.21$, $\sigma_2 = 1.24$, $\sigma_1 = 0.89$, $\xi_2 = 0.0117$, $\xi_1 = 0.0146$ $\nu = 0.56$, $\sigma_{M,2} = 0.17$, $\sigma_{M,1} = 1.58$, $\alpha_{M,2} = 0.64$, $\alpha_{M,1} = 0.68$

Results

	industry | period	$\frac{P_{n,\mathbf{VA}}Y_{n,\mathbf{VA}}}{P_{\mathbf{VA}}Y_{\mathbf{VA}}}$ (t-1)	$\frac{P_{n,\mathbf{VA}}Y_{n,\mathbf{VA}}}{P_{\mathbf{VA}}Y_{\mathbf{VA}}}$ (t)	$\frac{PK_n}{PK}$ (t-1)	$\frac{PK_n}{PK}$ (t)	$\frac{L_n}{L}$ (t-1)	$\frac{L_n}{L}$ (t)	$\frac{P_{n,K}M_{n,K}}{P_{n,M}M_n}$ (t-1)	$\frac{P_{n,K}M_{n,K}}{P_{n,M}M_n}$ (t)	$\frac{P_{n,L}M_{n,L}}{P_{n,M}M_n}$ (t-1)	$\frac{P_{n,L}M_{n,L}}{P_{n,M}M_n}$ (t)
actual U.S. data	labor intensive	0.58	0.6	0.27	0.24	0.7	0.83	0.32	0.35	0.68	0.65
	capital intensive	0.42	0.4	0.73	0.76	0.3	0.17	0.64	0.58	0.36	0.42
Cobb-Douglas VA	labor intensive	0.58	0.66	0.33	0.41	0.74	0.8	-	-	-	-
	capital intensive	0.42	0.34	0.68	0.59	0.26	0.2	-	-	-	-
CES VA	labor intensive	0.58	0.68	0.28	0.34	0.8	0.88	-	-	-	-
	capital intensive	0.42	0.32	0.72	0.66	0.2	0.12	-	-	-	-
CES GO (one-sided)	labor intensive	0.58	0.65	0.34	0.37	0.75	0.83	-	-	-	-
	capital intensive	0.42	0.35	0.66	0.63	0.25	0.17	-	-	-	-
CES GO (two-sided)	labor intensive	0.58	0.6	0.37	0.34	0.72	0.78	0.32	0.36	0.68	0.64
	capital intensive	0.42	0.4	0.63	0.66	0.28	0.22	0.54	0.5	0.46	0.5

Notes: the table shows the parameter choices and the results from the numerical simulations of the four models described in Section 5.2. $P_{n,\mathbf{VA}}Y_{n,\mathbf{VA}}/(P_{\mathbf{VA}}Y_{\mathbf{VA}})$ denotes the share of sector n's value added in the economy's total value added. PK_n/PK denotes the amount of capital allocated to sector n as a fraction in total capital. L_n/L denotes the amount of labor allocated to sector n as a fraction in total labor. $P_2M_{n,2}/(P_{n,M}M_n)$ denotes the share of the capital intensive intermediate input in total intermediate inputs in sector n. $P_1M_{n,1}/(P_{n,M}M_n)$ denotes the share of the labor intensive intermediate input in total intermediate inputs in sector n. t is 1998-2008. $t-1$ is 1948-1958.

empirically. Yet, this model serves as an interlace between the value added models and the next model. Its purpose is to highlight the relative importance of Assumptions 1 and 3.

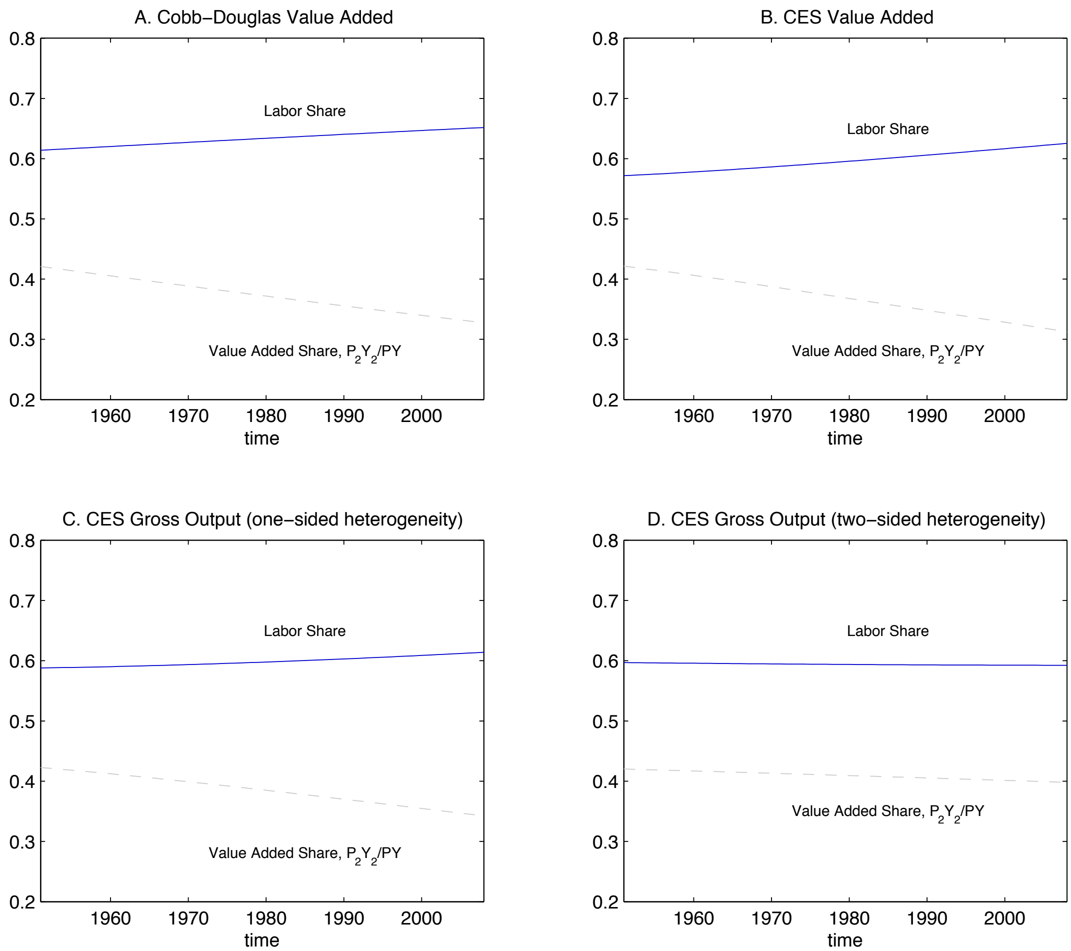
4. **CES Gross Output (two-sided heterogeneity).** Output of sector n is given by: $Y_{\mathbf{GO},n,t} = \left[A_{n,t}[\alpha_n K_{n,t}^{\frac{\sigma_n-1}{\sigma_n}} + (1-\alpha_n)L_{n,t}^{\frac{\sigma_n-1}{\sigma_n}}]^{\frac{\sigma_n}{\sigma_n-1}}\right]^{\nu}\left[[\alpha_{M,n}M_{n,n,t}^{\frac{\sigma_{M,n}-1}{\sigma_{M,n}}} + (1-\alpha_{M,n})M_{n,\neg n,t}^{\frac{\sigma_{M,n}-1}{\sigma_{M,n}}}]^{\frac{\sigma_{M,n}}{\sigma_{M,n}-1}}\right]^{1-\nu}$. In this model I make the same assumptions as in the two previous models about the value added side of the economy. In contrast to the CES Gross Output (one-sided heterogeneity) model, however, I motivate the parameter choices on the input-output structure of the model empirically. The elasticity of substitution between intermediate inputs in the capital intensive sector is $\sigma_{M,2} = 0.17$. In the labor intensive sector, this elasticity is set to $\sigma_{M,1} = 1.58$. The shares of the intermediate goods are then set to $\alpha_{M,2} = 0.64$ and $\alpha_{M,1} = 0.68$ to closely resemble the initial intermediate input shares in gross output of Table 3.

How do I simulate the models? Motivated by the long-run empirical evidence of balanced growth, I initially start from a steady state. This steady state is given by the exogenous parameter choices as well as the normalized levels of productivity and labor. I then conduct a global search that maximizes the discounted utility of the consumer under the first order conditions as constraints over a 60 year period.

Results. I now evaluate the performance of these four models based on two criteria: i) their ability to generate balanced growth ii) their ability to generate the direction of factor reallocations within the economy. Three Figures and the lower half of Table 3 summarize the results.

Figure 4 shows how the four models compare in terms of sectoral value added shares and the labor share in value added. Figure 5 shows how the four models compare in terms of the factor allocation they generate within the economy. Figure 6 shows how the four models perform in terms of generating stable long-term interest rates as well as equal and stable growth rates of capital and value added. The lower half of Table 3 shows how the economy's composition of inputs behaves across models and in comparison to the actual U.S. data.

Figure 4 shows that the labor share and the share of the capital intensive sector in value added are strongly trending in the two value added models (models 1 and 2) and in the gross output model with one-sided heterogeneity (model 3). By contrast, in the gross output model with two-sided heterogeneity (model 4), the labor share is approximately stable. Meanwhile, the share of the capital intensive sector in value added only marginally declines over time as seen in the actual U.S. data of Table 1. The predictions of model 4 are, thus, in line with the empirical evidence on the labor share of Figure 1 and the evidence on the value added composition presented in Table 1. By contrast, the predictions of the models 1, 2 and 3 are not in line with these data.

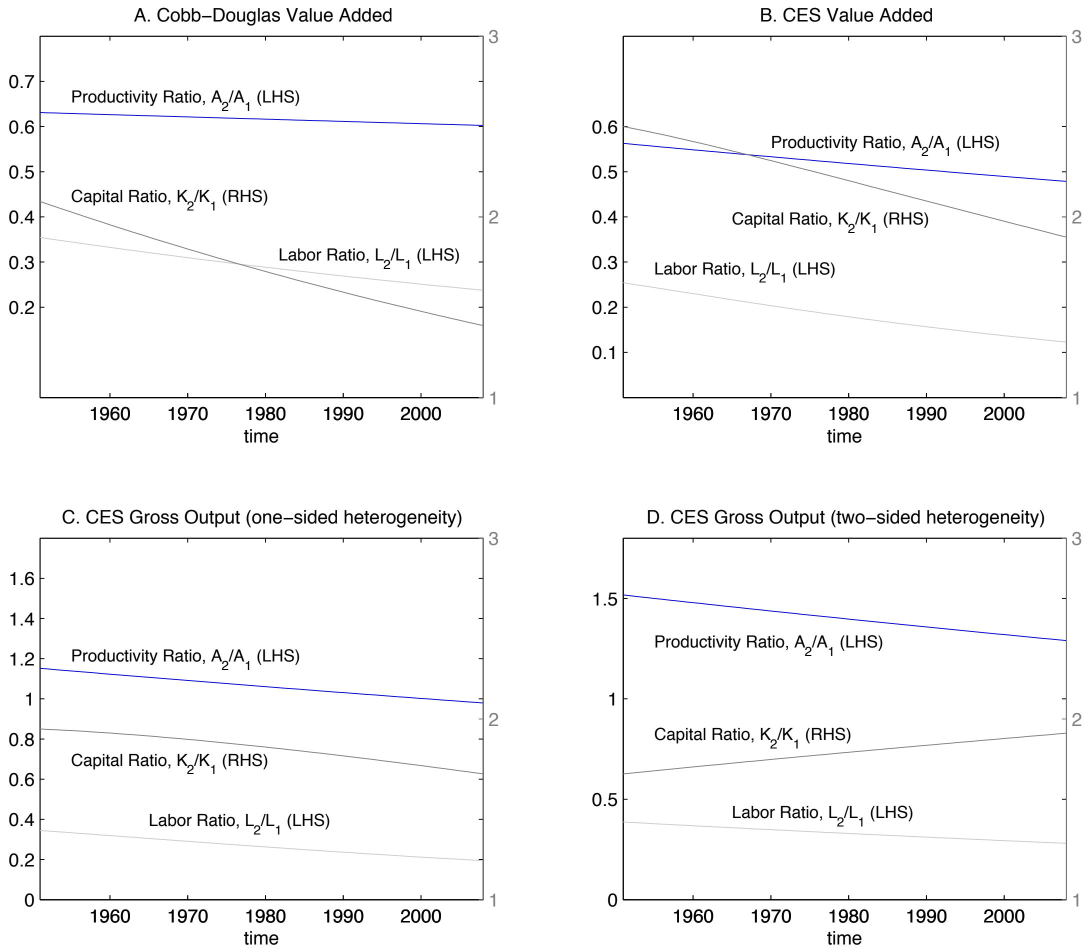


Notes: the figure shows the labor share in value added and the value added share of the capital intensive sector in the economy's total value added. Panel A plots the simulation results of model 1. Panel B plots the simulation results of model 2. Panel C plots the simulation results of model 3. Panel D plots the simulation results of model 4. The models and the underlying parameter choices are described in Section 5.2 and in Table 3.

Figure 4: The labor share and the composition of value added.

Figure 5 shows how the allocation of factors changes over time. Both value added models and the first gross output model predict that the relative amounts of both capital and labor increase over time in the labor intensive industry. The last gross output model predicts that the relative amount of capital increases in the capital intensive industries and that the relative amount of labor increases in the labor intensive industries. Table 3 compares these predictions to the actual U.S. data. The comparison shows that only the last gross output model (model 4) correctly tracks the direction of movements of capital and labor within the economy. Relative productivity growth in all four models is exogenously given and therefore not of particular interest here.

Figure 6 shows that the long-term interest rate, as well as the growth rates of output and capital are stable

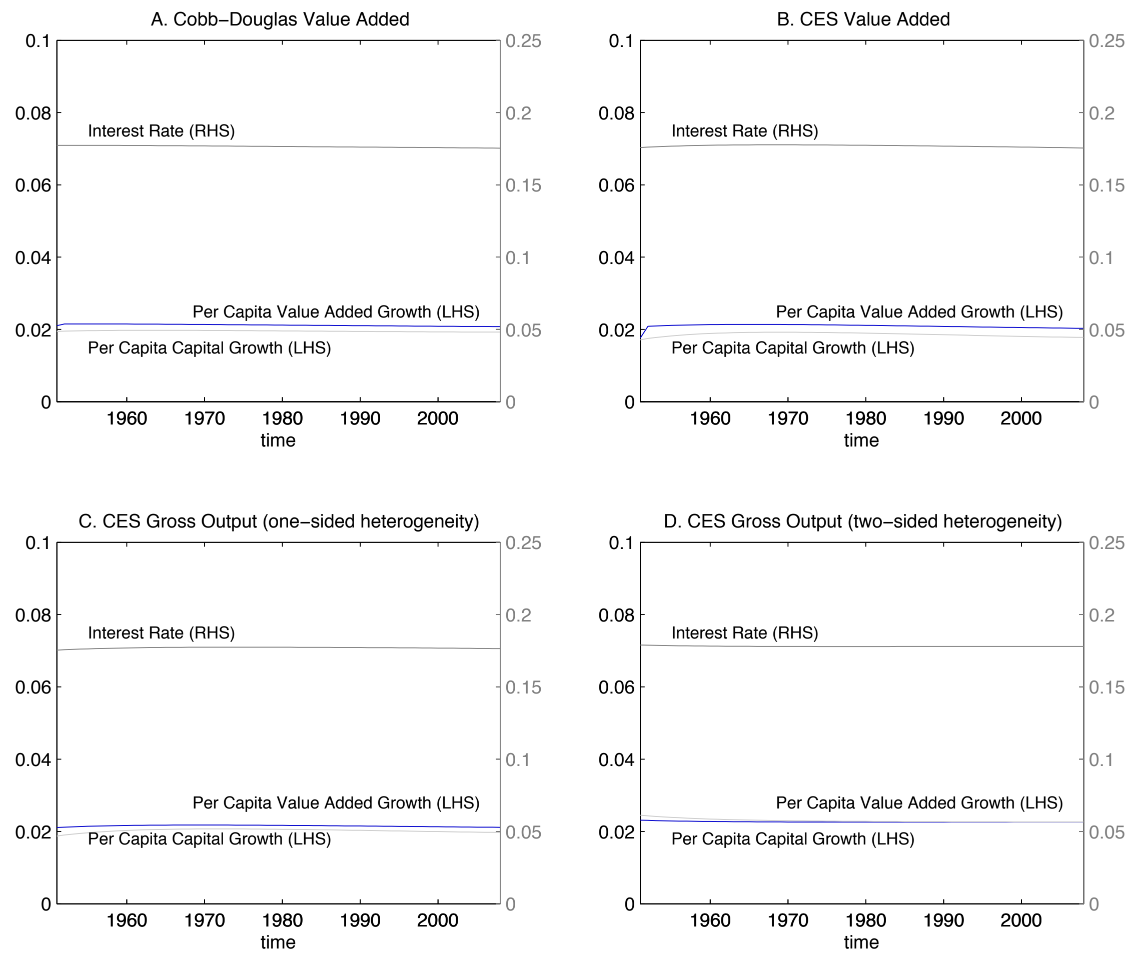


Notes: the figure shows the allocation of productivity, capital and labor of sector 2 relative to sector 1. Panel A plots the simulation results of model 1. Panel B plots the simulation results of model 2. Panel C plots the simulation results of model 3. Panel D plots the simulation results of model 4. The models and the underlying parameter choices are described in Section 5.2 and in Table 3.

Figure 5: The allocation of capital and labor across sectors.

over time. All four models generate this feature of the data, see e.g. Kaldor (1957) among others. There is, however, one important difference between the models 1,2 and 3 and the last gross output model. Both value added models and the first gross output model predict that value added grows at a higher rate than capital. This difference in growth rates is reflected in the fact that the overall economy becomes more labor intensive as the increasing labor share of models 1-3 shows in Figure 4. By contrast, in the last gross output model, capital and value added grow at approximately the same rate which implies a stable labor share. Thus, again, only model 4 is in line with the empirical evidence on the long-term real interest rate and the growth rates of value added and capital.

The lower half of Table 3 shows how the composition of intermediates changes over time. In the actual



Notes: the figure shows the real interest rate, the growth rate of the economy's real value added and of the growth rate of the capital stock. Panel A plots the simulation results of model 1. Panel B plots the simulation results of model 2. Panel C plots the simulation results of model 3. Panel D plots the simulation results of model 4. The models and the underlying parameter choices are described in Section 5.2 and in Table 3.

Figure 6: Long-term interest rates, value added and capital growth.

U.S. data, the share of the labor intensive intermediate input grows in the capital intensive sector while the share of the capital intensive intermediate input grows in the labor intensive sector. Models 1-3 do not predict any movements here. This is since the first two models only model the value added side of the economy and it is because the third model does not allow for a composition of intermediates by assumption. Model 4 makes a predictions about the composition. Since labor is the scarce factor in the economy, the price of labor increases over time relative to the price of capital. This means that the labor intensive intermediate input must gain share when the intermediate inputs are complements and it loses share when the intermediate inputs are substitutes. The intermediate inputs are complements in the capital intensive sector and substitutes in the labor intensive sector. Thereby, the shares of intermediate inputs move in opposite directions just like in the actual U.S. data as Table 3 shows.

To summarize, evaluating the value added and gross output models based on the two criteria balanced growth and factor composition gives a clear picture: only the last gross output model is able to qualitatively replicate balanced growth (approximately constant interest rates, constant and identical capital and value added growth, constant labor share) and the direction of the composition of factors (more capital allocates to the more capital intensive sector while more labor allocates to the more labor intensive sector) over time. By contrast, both value added models and model 3 fail to generate these two key characteristics of the post-war U.S. economy. Before I explain the mechanism behind the last gross output model, I briefly discuss the quantitative shortcomings of model 4.

Model 4 tracks the overall direction of the actual data very well, but it cannot fully account for the quantitative movements. This is not surprising given that we have to simultaneously match a large number of features of the data. The quantitative strength of model 4 is to i) generate the overall fall in the share of the capital intensive sector in value added from 42% to 40% ii) generate the 3 percentage point increase in the share of capital used by the capital intensive sector. The shortcomings are as follows: i) while the data show a decline in the share of labor allocated to the capital intensive sector from 30% to 17%, the last models generates a decline only of about 6 percentage points ii) the share of capital intensive intermediate input in the capital intensive sector falls from 64% to 58% in the actual data, but only by 4 percentage points in the model; the share of capital intensive intermediate input in the capital intensive sector increases from 32% to 35% in the actual data, but by 4 percentage points in the model. Again, a better quantitative replication of the empirical evidence is not the goal of this paper, but could be reached by a more precise configuration of the parameters. Appendix A estimates the production parameters more precisely with the same qualitative results but is quantitatively closer to the data.

5.3 Mechanism

To understand the mechanism that induces balanced growth in the last gross output model (models 4), let me first break down the final use C, I, M_1 and M_2 into the value added components of the two sectors. To do this, I use a two-step method. First, I determine the value added content of gross output of each sector by sector:

$$P_{\mathbf{GO},t} Y_{\mathbf{GO},t} = (\mathbf{I}_2 - \mathbf{A})^{-1} P_{\mathbf{VA},t} Y_{\mathbf{VA},t}, \text{ where } \mathbf{A} = \begin{bmatrix} \frac{P_{GO,1,t} M_{1,1,t}}{P_{GO,1,t} Y_{GO,1,t}} & \frac{P_{GO,2,t} M_{2,1,t}}{P_{GO,2,t} Y_{GO,2,t}} \\ \frac{P_{GO,1,t} M_{1,2,t}}{P_{GO,1,t} Y_{GO,1,t}} & \frac{P_{GO,2,t} M_{2,2,t}}{P_{GO,2,t} Y_{GO,2,t}} \end{bmatrix}. \tag{24}$$

Second, I determine the gross output content of the final goods C, I, M_1 and M_2 by sector. Finally, I combine the information on the value added content in gross output and the gross output content in final goods to determine the value added content in final goods. Table 4 shows how the value added content in final goods changes over time. The table shows the values of the capital intensive sector 2 relative to the labor intensive sector 1. The initial relative values are normalized to unity.

Table 4: Growth of Real Value Added Components (initial values normalized to 1)

	$P_{2,\mathbf{VA}}/P_{1,\mathbf{VA}}$ $t-1 \rightarrow t$	$Y_{2,\mathbf{VA}}/Y_{1,\mathbf{VA}}$ $t-1 \rightarrow t$	$P_{2,\mathbf{GO}}/P_{1,\mathbf{GO}}$ $t-1 \rightarrow t$	$Y_{2,\mathbf{GO}}/Y_{1,\mathbf{GO}}$ $t-1 \rightarrow t$
Cobb-Douglas VA	1.00 → 0.67	1.00 → 1.05	-	-
CES VA	1.00 → 0.63	1.00 → 1.06	-	-
CES GO (one-sided)	1.00 → 0.67	1.00 → 1.11	1.00 → 0.71	1.00 → 1.05
CES GO (two-sided)	1.00 → 0.70	1.00 → 1.32	1.00 → 0.84	1.00 → 1.10
	$Y_{2,\mathbf{VA}}/Y_{1,\mathbf{VA}}$ Consumption, C $t-1 \rightarrow t$	$Y_{2,\mathbf{VA}}/Y_{1,\mathbf{VA}}$ Investment, I $t-1 \rightarrow t$	$Y_{2,\mathbf{VA}}/Y_{1,\mathbf{VA}}$ Intermediates, M_2 $t-1 \rightarrow t$	$Y_{2,\mathbf{VA}}/Y_{1,\mathbf{VA}}$ Intermediates, M_1 $t-1 \rightarrow t$
Cobb-Douglas VA	1.00 → 1.05	1.00 → 1.05	-	-
CES VA	1.00 → 1.06	1.00 → 1.06	-	-
CES GO (one-sided)	1.00 → 1.05	1.00 → 1.05	-	-
CES GO (two-sided)	1.00 → 1.32	1.00 → 1.32	1.00 → 1.30	1.00 → 1.59

Notes: the table shows a breakdown of the simulation results in terms of changes in the relative real value added prices, $P_{2,\mathbf{VA}}/P_{1,\mathbf{VA}}$, relative real value added $Y_{2,\mathbf{VA}}/Y_{1,\mathbf{VA}}$, relative gross output prices $P_{2,\mathbf{GO}}/P_{1,\mathbf{GO}}$ and relative real gross output $Y_{2,\mathbf{GO}}/Y_{1,\mathbf{GO}}$. It further shows how relative real value added changes within consumption, investment, intermediates of sector 2 and intermediates of sector 1. A desciption of how the real value added components of C, I, M_1 and M_2 are backed out is contained in Section 5.3. t is 1998-2008. $t-1$ is 1948-1958.

Why can the last gross output model, not the other models, match the key features of the data? The failure of the value added models to replicate the empirical evidence goes back to the failure of the two prerequisites of balanced growth to hold. Uzawa (1961) shows that in a neoclassical value added model, we need either one the two prerequisites to sustain balanced growth. I already showed above that the two prerequisites are inconsistent with long-run evidence from an industry decomposition of the U.S. post-war data. The simulation results of the two value added models reflect this failure: as Table 4 shows, in the two value added models the relative price of value added of the capital intensive sector 2 falls from 1 to 0.67 (0.63) in model 1 (model 2). The fall in the model 3 is similar: from 1 to 0.67. This means that the price of the labor intensive good increases relative to the price of the capital intensive good over time.

Because the two value added goods are complements in aggregate consumption and investment, resources are dynamically reallocated to make real value added grow commensurately across sectors. Table 4 shows that real value added grows approximately commensurately across sectors: the real value added of sector 2 relative to sector 1 increases only marginally from 1 to 1.05 (1.06, 1.11) in model 1 (model 2, model 3). Given the fall in relative prices, $P_{\mathbf{VA},2,t}/P_{\mathbf{VA},1,t}$, that is caused by similar productivity growth rates across the heterogeneous sectors, and, given the constancy of relative real value added, $Y_{\mathbf{VA},2,t}/Y_{\mathbf{VA},1,t}$, the labor intensive sector 1 quickly gains share in total value added as the ratio (25) illustrates. The labor share therefore increases over time which drives us away from the balanced growth path.

$$\frac{P_{\mathbf{VA},2,t}Y_{\mathbf{VA},2,t}}{P_{\mathbf{VA},1,t}Y_{\mathbf{VA},1,t}} \tag{25}$$

By contrast, the last gross output model can sustain balanced growth for long periods. Why? Table 4 shows that relative real value added, $Y_{\mathbf{VA},2,t}/Y_{\mathbf{VA},1,t}$, is *not* (approximately) constant as in the other three models. Instead, relative real value added grows from 1 to 1.32. This increase comes from a higher use of capital intensive real value added components as intermediate inputs. Table 4 shows that the increase in the use of real capital intensive value added in the intermediate input of the labor intensive sector 1 is from 1 to 1.30 in model 4. It is from 1 to 1.59 in the intermediate input of the capital intensive sector 2. The increase in demand for capital intensive value added is quietly driven by two elasticities outside the value added structure: first, it is due to the model structure that links value added and intermediate components determined by Eq. (14). This structure (the unitary elasticity) ensures that intermediate inputs maintain a significant share in nominal gross output. Second, it is due to the model specific intermediate structure Eqs. (16). This structure (with the parameterization of Table 3) ensures in model 4 that both intermediate inputs maintain a significant share in gross output in both sectors. The increase in demand for capital intensive value added that results from these two elasticities then counterbalances the relative price movement and therefore stabilizes the share of capital intensive value added as the ratio 25 illustrates. Furthermore, because of the increase in demand for capital intensive value added components, more capital allocates to the capital intensive sector (Figure 5). The model therefore can also track the direction of the dynamic factor allocation correctly.

What helps the mechanism is that complementary preferences are over gross output not over value added. The presence of both goods as an intermediate input with quantitatively significant share in nominal gross output of each sector implies that relative gross output prices fall only from 1 to 0.84 in model 4.[8] Because of the small decline in relative gross output prices in model 4, the complementarity of sectoral gross output in consumption and investment does not require a massive shift towards the labor intensive sector.

[8]Example: sector 2's value added price falls relative to sector 1's valued added price. Because sector 2 uses value added from sector 1 and sector 1 uses value added from sector 2 as an input, the relative price movement in value added prices only partially translates into a relative price movement in gross output prices.

6 Conclusion

This paper shows that the two theoretical prerequisites of balanced growth - purely labor augmenting technical progress and a unitary capital-labor elasticity - are inconsistent with evidence from a simple decomposition of the U.S. post-war economy into different industries of heterogeneous production technologies. First, I show that this decomposition implies that capital and labor have an elasticity of less than unity when a single representative consumer has complementary preferences over industry value added. Second, I show that this decomposition reveals that there is no relation between the capital intensity, the capital-labor elasticity and the allocation of productivity growth. *How can we reconcile the evidence on industry heterogeneity with balanced growth and structural change?*

In my analysis, I underline that the two prerequisites arise in a class of theories that focuses on modeling the economy in value added terms. Thereby, these models deemphasize the importance of the network structure of the economy that connects the different industries by their use of output as an intermediate input. I employ a gross output model that takes into account the input-output linkages across the industries and show that balanced growth, industry heterogeneity and structural change can simultaneously emerge in such a framework under empirically plausible parameter assumptions. Key to understanding the mechanism is the empirical observation that intermediate inputs from both capital and labor intensive industries maintain quantitatively significant shares in gross output throughout the entire U.S. post-war period.

In the gross output model that I study, this stability is the result of a combination of two demand elasticities. First, a preference that keeps intermediate inputs a substantial part of gross output. Second, a preferences that induces slow shifts in some industries to capital intensive intermediate inputs while others shift to labor intensive intermediate inputs. When the price of capital falls relative to the price of labor these two elasticities quietly raise the demand for capital intensive value added. Real capital intensive value added thus grows faster than labor intensive value added. This increase in demand stabilizes the share of capital in total value added. By contrast, in standard value added models of the economy, there is no force that offsets the relative price movement as the real output of capital and labor intensive sectors grows commensurately due to the strong complementarity of different sectors in final use. Therefore, the share of capital intensive goods in value added declines fast in these models.

Using numerical procedures, I show that this decline amounts to nearly 10 percentage points during the post-war period in standard value added models. Meanwhile, both capital and labor shift towards the labor intensive industries in these models. By contrast, in the actual data, capital shifts to the capital intensive industries and labor shifts to the labor intensive industries. The share of the capital intensive industries in value added declines by only 2 percentage points. In the gross output model that I study, the increase in demand for capital intensive real value added offsets the price effect and, thus, allows these models to replicate these main features of the data.

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A Robustness: Factor Substitutability and Biased Technical Change

A potential weakness of my approach is that the assumptions of a simple functional form and of total factor productivity growth may drive the results. To test the robustness of my results under alternative assumptions, in this section, I employ a translog production function as in Kim (1992), Jorgenson (2000) and Jin and Jorgenson (2010) to model the input-output structure in the two sectors $n = 1, 2$. This functional form is more general and allows for pairwise elasticities between the four factors as well as a comprehensive bias of technical change. Gross output is given by

$$lnY_{\mathbf{GO},n,t} = \alpha_{0,n} + \sum_i \alpha_{i,n} lnX_{i,n,t} + \frac{1}{2}\sum_i \sum_j \beta_{i,j,n} lnX_{i,n,t} lnX_{j,n,t} + \sum_i \gamma_{i,n} lnX_{i,n,t} T_t + \delta_{T,n} T_t, \tag{A.1}$$

where $\alpha_{0,n}$, $\alpha_{i,n}$, $\beta_{j,i,n}$, $\gamma_{i,n}$ and $\delta_{T,n}$ are time-invariant parameters. T_t is the time index in period t. $X_{i,n,t}$ and $X_{j,n,t}$ denote the inputs i and j in sector n at time t. The share elasticities are symmetric, i.e. $\beta_{i,j,n} = \beta_{j,i,n}$. I assume that the production function is homogeneous of degree 1, thereby imposing the following restrictions: $\sum_i \alpha_{i,n} = 1$ for each sector n, $\sum_i \gamma_{i,n} = 0$ and $\sum_i \beta_{i,j,n}$ for each j. Maximizing profits, still given by (11), and assuming that production now takes the translog form of Eq. (A.1) yields an expression for the share of each input j in gross output

$$\frac{\partial ln\pi_n}{\partial lnX_{j,n}} = 0 \quad \Leftrightarrow \quad v_{j,n,t} = \frac{q_{j,n,t} X_{j,n,t}}{P_{\mathbf{GO},n,t} Y_{\mathbf{GO},n,t}} = \alpha_{j,n} + \sum_i \beta_{j,i,n} lnX_{i,n,t} + \gamma_{j,n} T_t \tag{A.2}$$

where $q_{j,n,t}$ denotes the price of input j in industry n at time t. Relative to the CES setup in the main text, this setup relaxes both the assumption that technical progress is factor neutral as imposed by Eq. (15) and that production takes a simple functional form as implied by Eqs. (14), (15) and (16). Instead, the flexible functional form of translog allows for both biased technical change and a more realistic substitution structure.

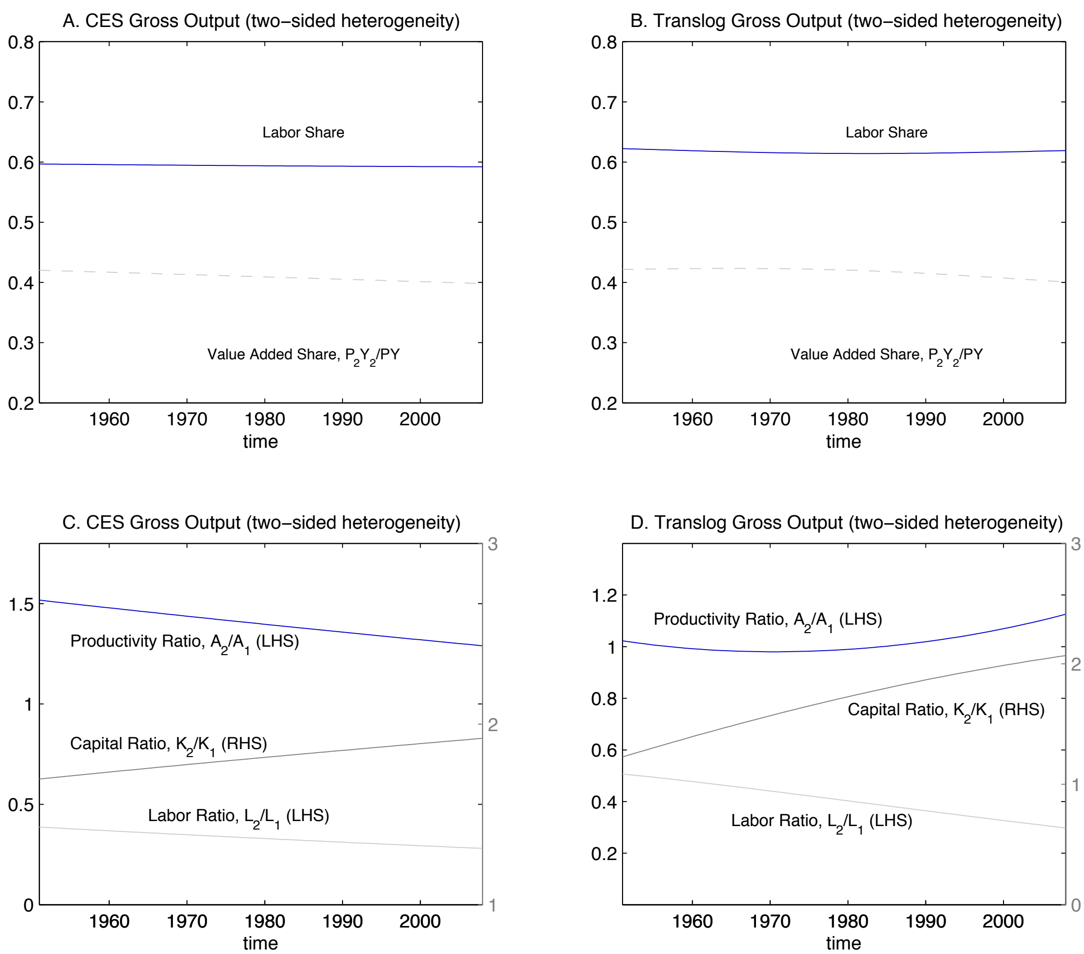
Estimation of parameters. To estimate the parameters of this model, I follow the approach of Wooldridge (2002) and estimate a system with cross equation restrictions. For each sector I estimate a system of equations that consists of Eq. (A.1) and four Eqs. (A.2), one for each production factor. In addition I impose the above mentioned constraints and concavity on the production function. To impose concavity, I effectively impose negative semidefiniteness on the matrix of $\beta_{j,i,n}$ elasticities:

$$\mathbf{B}_n = \begin{bmatrix} \beta_{K,K,n} & \beta_{K,L,n} & \beta_{K,M_1,n} & \beta_{K,M_2,n} \\ \beta_{K,L,n} & \beta_{L,L,n} & \beta_{L,M_1,n} & \beta_{L,M_2,n} \\ \beta_{K,M_1,n} & \beta_{L,M_1,n} & \beta_{M_1,M_1,n} & \beta_{M_1,M_2,n} \\ \beta_{K,M_2,n} & \beta_{L,M_2,n} & \beta_{M_1,M_2,n} & \beta_{M_2,M_2,n} \end{bmatrix}. \tag{A.3}$$

For the estimation, I use 10 observations from the beginning and 10 observations from the end of the sample period to capture the long-run changes in the data.

Quantitative analysis. The upper half of Table 5 contains the parameter estimates. I now compare the simulation results from model 4 to the translog model. To simulate the translog model, I use the same procedure as above. There is one difference though. Because the level of productivity is a part of the parameters, I cannot normalize productivity initially. The gross output prices are, thus, not equal across sectors initially. To match the initial value added shares, I therefore need to set $\gamma = 0.30$.

Results. The simulation results are contained in Table 5 and in Figure 7. The two top panels of the figure show that the labor share remains approximately stable and that the value added share of the capital intensive sector only declines by 2 percentage points. The translog model, thus, makes similar qualitative predictions as model 4 when it comes to balanced growth. The two bottom panels of the figure show that this model also predicts the correct direction of structural change: more capital allocates to the more capital intensive sector and more labor allocates to the more labor intensive sector. The table further confirms that the direction of shifts within the intermediates in the two sectors is accurate. The model, thus, confirms that the restrictive and simplifying assumptions about the functional forms and the bias of technical change do not drive the results in the CES model with two-sided heterogeneityQuantitatively, a strength of the translog model is to be close to the shifts and levels within the intermediates. For example, 5 shows that the CES model with two-sided heterogeneity has difficulties matching the levels of intermediates in the capital intensive sector. In this sector the capital intensive intermediate declines from 0.54 to 0.50. In the actual data, this decline is from 0.64 to 0.58. In the translog model, the decline is from 0.63 to 0.56 - closer to the level of the actual data. The labor reallocation is also closer to the actual data. While the CES model with two-sided heterogeneity predicts a labor reallocation of 6 percentage points, the translog model predicts an allocation of 9 percentage points. The actual data indicate it is rather 13 percentage points. A weakness of the translog model is the allocation of capital. While the actual data indicate the allocation amounts to 3 percentage points (from 0.27 to 0.23), the translog model predicts a decline from 0.43 to 0.34 - 9 percentage points - in the labor intensive industry.



Notes: Panels A and B show the labor share in value added and the value added share of the capital intensive sector in the economy's total value added. Panels C and D show the allocation of productivity, capital and labor of sector 2 relative to sector 1. Panels A and C plot the simulation results from model 4. Panels B and D plot the simulation results from the translog model. The models and the underlying parameter choices are described in Sections 5.2 and A and in Table 5.

Figure 7: Labor share, output composition and factor allocation.

Table 5: Simulation parameters and results

Parameters

Preferences	$\beta = 0.97$, $\phi = 2$, $\epsilon = 0.13$, $\delta = 0.10$, $m = 0.018$
CES Gross Output (two-sided)	$\alpha_2 = 0.61$, $\alpha_1 = 0.21$, $\sigma_2 = 1.24$, $\sigma_1 = 0.89$, $\xi_2 = 0.0117$, $\xi_1 = 0.0146$ $\nu = 0.56$, $\sigma_{M,2} = 0.17$, $\sigma_{M,1} = 1.58$, $\alpha_{M,2} = 0.64$, $\alpha_{M,1} = 0.68$
Translog Gross Output (two-sided)	$\alpha_{0,2} = -0.39$, $\alpha_{K,2} = 0.24$, $\alpha_{L,2} = 0.26$, $\alpha_{M_2,2} = 0.32$, $\alpha_{M_1,2} = 0.18$ $\alpha_{0,1} = -0.42$, $\alpha_{K,1} = 0.15$, $\alpha_{L,1} = 0.38$, $\alpha_{M_1,1} = 0.33$, $\alpha_{M_2,1} = 0.15$ $\beta_{K,K,2} = -5.28e-07$, $\beta_{L,L,2} = -8.68e-07$, $\beta_{M_2,M_2,2} = -5.31e-05$, $\beta_{M_1,M_1,2} = -7.95e-05$ $\beta_{K,K,1} = -8.42e-08$, $\beta_{L,L,1} = -6.14e-09$, $\beta_{M_1,M_1,1} = -1.15e-07$, $\beta_{M_2,M_2,1} = -3.24e-07$ $\beta_{K,L,2} = -6.58e-07$, $\beta_{K,M_2,2} = -5.17e-06$, $\beta_{K,M_1,2} = 6.35e-06$, $\beta_{L,M_2,2} = -6.67e-06$ $\beta_{L,M_1,2} = 8.19e-06$, $\beta_{M_1,M_2,2} = 6.49e-05$ $\beta_{K,L,1} = -6.56e-09$, $\beta_{K,M_1,1} = -6.09e-08$, $\beta_{K,M_2,1} = 1.52e-07$, $\beta_{L,M_1,1} = 7.77e-09$ $\beta_{L,M_2,1} = 4.94e-09$, $\beta_{M_1,M_2,1} = 1.68e-07$ $\gamma_{K,2} = 0.0025$, $\gamma_{L,2} = -2.57e-04$, $\gamma_{M_1,2} = -0.0020$, $\gamma_{M_2,2} = -2.83e-04$ $\gamma_{K,1} = -5.80e-04$, $\gamma_{L,1} = 0.0016$, $\gamma_{M_1,1} = -0.0010$, $\gamma_{M_2,1} = -1.05e-05$ $\delta_{T,2} = 0.0092$, $\delta_{T,1} = 0.0099$, $\gamma = 0.30$

Results

	industry | period	$\frac{P_{n,\mathbf{VA}}Y_{n,\mathbf{VA}}}{P_{\mathbf{VA}}Y_{\mathbf{VA}}}$ (t-1)	$\frac{P_{n,\mathbf{VA}}Y_{n,\mathbf{VA}}}{P_{\mathbf{VA}}Y_{\mathbf{VA}}}$ (t)	$\frac{PK_n}{PK}$ (t-1)	$\frac{PK_n}{PK}$ (t)	$\frac{L_n}{L}$ (t-1)	$\frac{L_n}{L}$ (t)	$\frac{P_{n,2}M_{n,2}}{P_{n,M}M_n}$ (t-1)	$\frac{P_{n,2}M_{n,2}}{P_{n,M}M_n}$ (t)	$\frac{P_{n,1}M_{n,1}}{P_{n,M}M_n}$ (t-1)	$\frac{P_{n,1}M_{n,1}}{P_{n,M}M_n}$ (t)
actual U.S. data	labor intensive	0.58	0.60	0.27	0.24	0.7	0.83	0.32	0.35	0.68	0.65
	capital intensive	0.42	0.40	0.73	0.76	0.3	0.17	0.64	0.58	0.36	0.42
CES GO (two-sided)	labor intensive	0.58	0.60	0.37	0.34	0.72	0.78	0.32	0.36	0.68	0.64
	capital intensive	0.42	0.40	0.63	0.66	0.28	0.22	0.54	0.50	0.46	0.50
Translog GO (two-sided)	labor intensive	0.58	0.60	0.43	0.34	0.67	0.76	0.32	0.36	0.68	0.64
	capital intensive	0.42	0.40	0.57	0.66	0.33	0.24	0.63	0.56	0.37	0.44

Notes: the table shows the parameter choices and the results from the numerical simulations of the four models described in Section 5.2. $P_{n,\mathbf{VA}}Y_{n,\mathbf{VA}}/(P_{\mathbf{VA}}Y_{\mathbf{VA}})$ denotes the share of sector n's value added in the economy's total value added. PK_n/PK denotes the amount of capital allocated to sector n as a fraction in total capital. L_n/L denotes the amount of labor allocated to sector n as a fraction in total labor. $P_KM_{n,K}/(P_{n,M}M_n)$ denotes the share of the capital intensive intermediate input in total intermediate inputs in sector n. $P_LM_{n,L}/(P_{n,M}M_n)$ denotes the share of the labor intensive intermediate input in total intermediate inputs in sector n. t is 1998-2008. $t-1$ is 1948-1958.