Bank Liability Structure

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Abstract

We develop, and solve analytically, a dynamic model of optimal bank liability structure that incorporates bank run, regulatory closure, endogenous default, and endogenous deposit-insurance premium. Value-maximizing banks balance between deposits and debt so that endogenous default coincides with bank closure. Banks' optimal response to regulatory changes often counteracts regulators' objective in reducing bank failures. For example, an optimal response to the introduction of FDIC is to increase leverage by choosing a higher deposit-to-asset ratio and a lower debt-to-asset ratio. We also find that banks' optimal leverage can be substantial even in the absence of material tax benefits.

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1 Introduction

Bank leverage has drawn much attention from regulators and the public after the crises experienced by the banking industry. Regulators around the world have gradually rolled out regulations on bank capital structure, and the shape of bank regulation is still evolving.[1] Banks have been readjusting their capital structure, and academics have been grappling with the questions about the level and composition of capital that banks should hold.[2] Proposals of further regulation are abundant in the literature. There have been arguments for restricting bank leverage to a level similar to non-financial firms.[3] There are also antithetical views on whether banks should hold long-term debt.[4] Some have even proposed cutting the tax rate for banks because it reduces incentives for leverage.[5]

The debate on bank capital regulation calls for a better understanding of bank leverage and the consequence of regulation. Each regulatory mandate typically attempts to fix a particular broken factor observed in bank liability structure.[6] Arguments for a regulatory mandate on the broken factor often implicitly assume that other factors will remain unchanged, ignoring the overall response of banks that optimally adjust various parts of their liability structure. For instance, deposit insurance intends to address bank runs caused by the fear of losing deposits *en masse*. With deposit insurance, however, a bank may find that financing with more deposits increases its value despite being more exposed to the risk of failure. More broadly, it is unclear whether banks' optimal response will undo or significantly diminish the intended effects of a regulatory mandate. It is even possible that a regulation may result in unintended consequences.

To work out banks' optimal responses to regulation, one needs to understand how a bank chooses leverage and liability structure when it maximizes its value. Value maximization is a fiduciary responsibility of bank management: acting in the interest

[1]After the frequent bank runs during the Great Depression, the Banking Act of 1933 created the Federal Deposit Insurance Corporation (FDIC). After the financial crisis during the Great Recession, the Dodd-Frank Act of 2010 brought sweeping regulatory reforms ranging from FDIC deposit insurance to stress tests of banks' capital adequacy. Worldwide regulators agreed on Basel III in 2011 to strengthen restrictions on bank leverage.

[2]See Thakor (2014) for a review of the debate on bank capital.

[3]See the book by Admati and Hellwig (2013).

[4]Bulow and Klemperer (2013) argue that banks should hold no debt, besides equity and securities convertible to equity. The Fed governor, Daniel Tarullo (2013), goes in the opposite direction by arguing for requirement of holding more long-term debt, which he thinks will improve capital structure and resolution of banks.

[5]For example, Fleischer (2013) proposes cutting corporate tax rate for banks to make them safer.

[6]For this reason, Santos (2000) motivates regulation as a policy arising out of market failure.

of its claim-holders.[7] Banks do not maximize social welfare, such as reducing systemic risk or increasing banking services. An analysis of social welfare implications of bank leverage is unquestionably important, but understanding the optimal choice of liability structure by value-maximizing banks is necessary for a proper social welfare analysis of bank regulation.

Banks distinguish themselves from other firms by taking deposits. Deposits are different from other forms of debt partly because banks earn income from the provision of account and liquidity services to depositors.[8] Other important features of deposits are that depositors can run, deposits may be insured, and deposit-taking banks are subject to regulatory closure by charter authorities. Bank run by rational depositors and bank closure by rule-following regulators are reflected in equity holders' endogenous choice of default on its debt obligations in order to maximize equity value. The risk exposure of deposits is also reflected in the insurance premium if the deposits are insured. We develop a dynamic structural model that incorporates these institutional features explicitly.

In our model, which extends the framework pioneered by Merton (1974, 1977) and Leland (1994), we analytically solve for the optimal liability structure for banks that issue deposits, long-term debt, and common equity. The solution offers some new perspectives on bank liability structure. We find it optimal for a value-maximizing bank to choose deposits and debt so that endogenous default coincides exactly either with regulatory bank closure if the bank is FDIC-insured or with bank run if it is uninsured and unregulated. With this optimal choice of liability structure, the distance to default is the same as the distance to regulatory closure or bank run. This optimal structure of liabilities minimizes the debt's protection against deposits and results in a leverage higher than the optimal level for a firm not serving deposits.

The above property of optimal debt has an intuitive economic reason. Because of income from account and liquidity services, deposits are cheaper than debt as financing sources. A bank should generally prefer deposits to debt when balancing the benefits of debt against the potential loss to bankruptcy. Given the amount of deposits, however, debt does not affect bankruptcy risk as long as regulatory closure or bank run happens

[7]Our focus on bank value maximization sets aside the principal-agent problem such as management's conflict of interests with other stake holders. This problem may play a role in bank choices of liability structure. For example, Admati, DeMarzo, Hellwig, and Pfleiderer (2013) discuss how conflict of interests leads bank management to use excessive leverage even if it destroys bank value.

[8]In the literature, account and liquidity services to depositors are also referred to as production of liquidity. The income from these services is sometimes referred to as the liquidity premium of deposits. We refer to it as account service income.

before endogenous default. The bank should therefore take as much debt as possible for availing of the tax benefits but avoid making default happen before regulatory closure or bank run. As a result, the optimal debt sets endogenous default and regulatory closure concurrent in a FDIC-insured bank and makes endogenous default and bank run coincide in an uninsured, unregulated bank.

Another new perspective offered by our model is the optimal response of banks to FDIC insurance. With deposit insurance, a bank takes more deposits and issues less debt than what it would have done without deposit insurance. The optimal mix of deposits and debt ensures that the endogenous default remain coincident with the regulatory closure and avoid protecting the insurer from losses in its insurance obligation. Although the bank holds less debt, the increase in deposits leads to higher overall leverage than what would have been optimal for a comparable uninsured bank. This optimal response from banks prevents insurance program from reducing the expected loss due to bankruptcy. We find that the introduction of FDIC insurance raises optimal leverage even when banks are charged a fair insurance premium.

The endogenous link of FDIC's insurance premium to leverage is also a new perspective offered by our model. On one hand, insurance premium depends on the leverage and liability structure. On the other hand, a bank's decision on leverage and liability structure depends on the premium. We explicitly model this feedback channel, which is crucial in assessing regulatory policies pertaining to bank capital structure.

Our analysis shed light on the regulatory treatment of long-term debt. If such debt is a claim ranked lower than deposits, it is naturally viewed as a source of capital that protects deposits. Reflecting this view, regulators treat certain long-term unsecured debt as Tier 2 regulatory capital. However, if a bank adjusts its liability structure so that optimal default coincides with bank run or regulatory closure, it may not bring additional benefits of deposit protection. Long-term debt offers additional protection of deposits only if it makes endogenous default happen before bank closure. A large body of academic literature debates whether long-term debt provides a market discipline on bankruptcy risk.[9] The optimal choice of debt in bank liability structure is especially relevant to this debate and should not be ignored.

Since banks use much higher leverage than non-financial firms do, corporate tax benefits of debt is particularly important for bank liability structure. Apart from showing that bank leverage is lower in an economy with lower corporate tax benefits, our

[9] Flannery and Serescu (1996) contend that debt price rationally reflects the risk in a bank. Gorton and Santomero (1990), however, opine the opposite.

model shows that it is optimal for banks to shrink more debt than deposits if the tax benefit is lowered. More importantly, the model shows that banks should remain substantially leveraged even when corporate tax benefits approach zero. While a full general equilibrium analysis is needed for a thorough welfare analysis, the optimal response of bank liability structure developed here perhaps lays a stepping stone for the evaluation of the benefits and costs of tax policy reforms in the context of banking.

The road-map for the rest of the paper is as follows. Section 2 develops the model of bank liability structure in alternative regulatory environments. Section 3 characterizes the bank optimal liability structure in each regulatory environment, along with risk-based FDIC premium in the presence of endogenous choice of liability structure. Section 4 illustrates quantitatively the liability structures of banks and compare them with firms that do not serve deposits. Section 5 sheds light on the effects of factors, such as account service income, asset risk, bankruptcy cost, regulatory closure policy, FDIC insurance subsidy, and corporate tax benefits, on bank liability structure. Section 6 relates our work to the literature and discusses potential applications and extensions.

2 Bank Liability Structure

Banks share some common characteristics with non-financial firms: both have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide liquidity services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The banking business of taking deposits and serving accounts is heavily regulated in most countries. In the U.S., a large part of deposit accounts is insured by the FDIC, which charges insurance premium and imposes additional regulations on banks. The model of FDIC deposit insurance has gained popularity outside the U.S., and an increasing number of countries have started to offer deposit insurance.[10] Deposits and the associated services, deposit insurance, and regulations on opening and closure of banks distinguish banking business from other non-financial corporate business and set the capital decision of banks apart from that of other firms.

Firms operate in a market with two frictions: corporate taxes and bankruptcy costs.

[10]The International Association of Deposit Insurers (IADI) was formed on May 6, 2002 to enhance the effectiveness of deposit insurance systems by promoting guidance and international cooperation. As of the end of 2014, IADI represents 79 deposit insurers from 76 countries and areas.

These frictions are crucial for firms in their choice of capital and liability structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed with structural models by Leland (1994). Banks face these frictions too, but they have to simultaneously incorporate other considerations, such as the potential of a run by depositors, FDIC deposit insurance premium, and charter authority's closure of banks, in determining their optimal capital and liability structure. Figure 1 illustrates the liability structure of a typical bank. In Section 2.1, we discuss each part of the structure in detail.

<table>
<tr><th>Asset Side</th><th>Liability Side</th></tr>
<tr><td rowspan="3">Assets: V
Volatility: σ
Cash flow: δ</td><td>Deposits: D
Benefit: deduct tax τ
Benefit: service income η
Cost: bankruptcy α or β
Cost: insurance premium I</td></tr>
<tr><td>Debt: $\mathbf{D_1}$
Benefit: deduct tax τ
Cost: bankruptcy α or β</td></tr>
<tr><td>Tangible equity: $V-(D+D_1)$</td></tr>
<tr><td>Charter value: $\mathbf{F-V}$</td><td>Equity: E</td></tr>
</table>

Bank value: $\mathbf{F=D+D_1+E}$

Figure 1: An Illustration of Bank Liability Structure

2.1 Assets and Liabilities

A typical bank owns a portfolio of risky assets that generate cash flows. The portfolio of assets is valued at V, which is the major part of Figure 1. The asset is risky, and its value follows a stochastic process.[11] The instantaneous cash flow of the assets is δV, where δ is the rate of cash flow. In a non-financial firm, δV is the total earnings, but in a bank, δV represents only the earnings from bank assets such as loans, not including the income from serving deposit accounts. The risk of the asset portfolio is characterized by the volatility of asset value and denoted by σ. Notice that σ is

[11]Following Merton (1974) and Leland (1994), we assume that the stochastic process is a geometric Brownian motion, which is described by equation (20) in Appendix.

also the volatility of asset cash flow. We assume that the portfolio of assets is given exogenously.[12] Following Merton (1974) and Leland (1994), we assume that investors have full information about asset value.[13]

Banks take deposits from households and businesses and provide account and liquidity services to depositors. Deposits, the first part on the liability side in Figure 1, are the most important source of funds for banks to finance their assets. Let D denote the amount of deposits that a bank takes. Deposits are rendered safe through two channels: (1) depositors withdraw their money early enough to ensure that the bank has enough assets to redeem their deposits in full; (2) the bank purchases insurance that guarantees the depositors in full. The first channel may precipitate financial distress costs and potential financial insolvency associated with a bank run. The second channel requires the bank to pay insurance premium. We will discuss bank run and deposit insurance in the next subsection.

If deposits are risk-free, the fair interest rate on deposits is the risk-free rate, denoted by r. Banks typically pay a lower interest rate on deposits. Depositors accept a lower interest rate, say $r-\eta_1$ for $\eta_1 > 0$, because customers receive some liquidity services associated with maintaining accounts and transacting certain normal payments.[14] Banks also charge fees for services such as money transfers, overdrafts, etc. Let η_2 be the banks' fee incomes on each dollar of deposits. A bank's net liability on deposits is $C = (r-\eta_1)D - \eta_2 D$, excluding deposit insurance premium. Let $\eta = \eta_1 + \eta_2$, which is the net income on each dollar of deposits. The net deposit liability is $C = (r-\eta)D$, excluding deposit insurance premium. The parameter η plays a crucial role in our model of banks. It represents a sacrifice to the required rate of return that the households

[12]This assumption rules out interesting issues of endogenous asset substitution. The literature has pointed out that debt may create incentives to substitute assets with higher risk (e.g., Green, 1984, and Harris and Raviv, 1991) and FDIC insurance may also make for such incentive (e.g., Pennacchi, 2006, and Schneidar and Tornell, 2004). Clearly, modeling the endogenous asset choice along with the liability choice is an important direction for future research. Our study may be viewed as an analysis of the optimal liability structure of the bank, which has already optimally chosen its asset portfolio.

[13]In reality, active investors use all available information to assess bank asset value and cash-flows although only accounting values of assets are directly observable in quarterly frequency. The full-information assumption sets aside the disparity between accounting value and intrinsic value. We may therefore interpret V as the fair accounting value. If the assets are of same risk category, we may interpret V as the value of risk-weighted assets.

[14]The Banking Act of 1933, known as the Glass-Steagall Act, prohibited banks from paying interest on demand deposits and gave the Fed the authority to impose ceilings on interest rates paid on time deposits. The prohibition and ceiling of interest rate on deposits were removed after the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Depository Institutions Act of 1983, the latter of which is known as the Garn-St. Germain Act.

are willing to accept for the services provided by the bank. This sacrifice distinguishes deposits from other form of debt. If deposits are risk-free because of deposit insurance, the bank's total liability on deposits is $I + C$, where I is the insurance premium. If deposits are not insured but are risk-free because of depositors' ability to run, we have $I = 0$.

Another important source of funding for banks is debt, on which banks do not provide account and payment services. This second part on the liability side in Figure 1. Debt pays coupon until bankruptcy, at which it has a lower priority than deposits in claiming the liquidation value of bank assets. The lower priority potentially protects deposits at bankruptcy. For that reason, certain long-term unsecured debt is treated as Tier 2 capital in bank capital regulation. Debt comes with a cost: its yield contains a credit spread, denoted by s, over the risk-free rate to compensate debt holders for bearing the risk of bankruptcy. The credit spread arises endogenously in our model; it depends on the risk of assets and the leverage of the bank. Thus, a bank's choice of liability structure affects the credit spread which we will solve endogenously along with the value of debt. The liability on debt is $C_1 = (r + s)D_1$, where D_1 is the value of the debt at issuance (face value).

A typical bank is owned by its common equity holders, who garner all the residual value and earnings of the bank after paying the contractual obligations on deposits and debt. The first slice of value that equity owners lay claim to is the asset value exceeding deposits and debt: $V - (D + D_1)$. This slice, also on the liability side in Figure 1, is referred to as tangible equity or book-value of equity. This is the value equity holders would receive if bank assets are liquidated at fair value and all deposits and debt are paid off at par. A larger book-value of equity means a smaller loss for depositors and debt holders after liquidation. Hence, regulators regard it as bank capital of the highest quality, the core Tier 1 capital.

Equity holders are also rewarded by all future earnings of the bank. The present value of future earnings is the bank's charter value, the bottom part on asset side in Figure 1. Part of the earnings is the savings from corporate tax. Since interest expenses are deductible from earnings for tax purposes, the flow of tax saving is $\tau(I + C + C_1)$. The dividend paid to equity holders is the difference between the asset cash flow and the after-tax liability associated with deposits and debt: $\delta V - (1 - \tau)(I + C + C_1)$. Since equity value depends on its dividend, it is affected by the liability structure. In a bank with deposit insurance, the liability structure is characterized by the triplet (I, C, C_1). In an uninsured bank that faces a bank run, the pair (C, C_1) typifies the liability structure.

2.2 Bank Run, FDIC, and Charter Authority

A consequence of borrowing through deposits is the risk that depositors may run, a major challenge commonly faced by banks but not by non-financial firms. As experienced in the crises of the U.S. banking history and theorized by Diamond and Dybvig (1983), depositors may run from a bank if they believe it has difficulty in repaying their deposits promptly upon their demand. When depositors run, the bank will be closed, unless it is recapitalized to stop the run, and its assets will be liquidated. Suppose that the liquidation occurs through bankruptcy courts. Then the costs of liquidation will include dead-weight losses due to liquidation discount and legal expenses. This is a fraction α of the asset value V_a at bankruptcy. The value after bankruptcy is $(1-\alpha)V_a$. Deposits carry with them the risk of a bank run. With full information, it is rational for depositors to run before the bank value drops below $D/(1-\alpha)$. It is also reasonable to assume that depositors may actually wish to run earlier than $D/(1-\alpha)$, worrying about a delay of payments when the bank files for bankruptcy. In our model, depositors run, and the bank is closed, when asset value drops to a level V_a with $V_a \geq D/(1-\alpha)$. Letting $\kappa \geq 1/(1-\alpha)$, the threshold for bank to close due to bank run is $V_a = \kappa D$.

The establishment of the FDIC is to deter bank runs by insuring that deposits (up to a limit) be paid when a bank closes.[15] With FDIC insurance, a bank is closed by its charter authority, which is typically either the bank's state banking commission or the Office of the Comptroller of the Currency (OCC). The charter authority closes a bank if the bank is insolvent or if the bank's capital is deemed to be too low to be sustainable. For example, a bank is categorized by regulators as critically under-capitalized when the total capital that protects deposits drops to a threshold (say, 2% of asset value).[16] The total capital is the sum of Tier 1 and Tier 2 capital. In our model, it is the sum of tangible equity and debt and amounts to $[V-(D+D_1)]+D_1 = V-D$. Let V_a be the threshold when the charter authority closes the bank. Then, $V_a - D = 2\% V_a$ implies $V_a = D/0.98$. In general, charter authority closes a bank when its asset value reaches $V_a = \kappa D$, where $\kappa \geq 1$. The closure rule in our model may also be intuitively interpreted as capital requirement, the minimum capital for a bank to operate, as modeled in Rochet (2008). Under such a capital requirement, charter authorities will shut down the bank, when the capital falls below the capital standards. If the requirement of total

[15] To deter bank runs during the credit crisis of 2007–2009, the FDIC deposit insurance limit was raised from $\$100,000$ to $\$250,000$ on October 3, 2008.

[16] For a review of the rules for the list of critically under-capitalized banks, we refer readers to Shibut, Critchfield and Bohn (2003).

capital is 10%, then $\kappa = 1/0.90$.

The FDIC functions both as a receiver of the closed banks and an insurer of the deposits. As a receiver, the FDIC liquidates the assets of a closed bank in its best effort to pay back the bank's creditors. Suppose the liquidation cost is βV_a, proportional to the asset value V_a when the bank is closed. We allow $\beta \neq \alpha$ because the costs associated with the liquidation by the FDIC may be different from the costs of liquidation through bankruptcy court. Since the FDIC does not go through the lengthy procedure of bankruptcy, $\beta < \alpha$ may potentially be true.[17]

As an insurer, the FDIC pays D to depositors when the bank is closed. The insurance corporation loses $D-(1-\beta)V_a$ if $(1-\beta)V_a < D$ and nothing otherwise. The loss function is $[D-(1-\beta)V_a]^+$, where $[x]^+ = x$ if $x \geq 0$ and $[x]^+ = 0$ if $x < 0$. Since $V_a = \kappa D$, the loss function is positive if $\kappa < 1/(1-\beta)$, in which case the FDIC expects to suffer a loss after bank closure.[18] To cover the loss, the FDIC charges insurance premium on banks. In 2006, Congress passed reforms that permits the FDIC to charge risk-based premium. For deposit insurance assessment purposes, an insured depository institution is placed into one of four risk categories each quarter, depending primarily on the institution's capital level and supervisory evaluation. Hence, a riskier bank pays higher insurance premium than a safer bank does. Recall that I denotes the deposit insurance premium a bank pays.[19]

The economic role of FDIC and charter authority in our model can be explained as follows. Depositors run at the right time to make their deposits risk-free if deposits are not insured. Deposit insurance prevents a bank run and lets the charter authority to close a bank later than the time depositors would have chosen to run were there no deposit insurance. The prevention of a bank run increases the expected life of the bank. The bank pays insurance premium to FDIC in "good states" when it is solvent. Keeping a

[17]Title II of the Dodd-Frank Act is perhaps a reflection of the belief that the cost of FDIC liquidation is lower than the cost of bankruptcy procedures. Title II authorizes the FDIC to receive and liquidate failed large financial institutions in order to avoid lengthy and costly bankruptcy procedures, which are supposed to be harmful for the stability of financial system.

[18]In practice, the FDIC always expects a chance of loss because liquidation cost is uncertain. To keep analysis tractable, we assume a fixed β and $\kappa < 1/(1-\beta)$ so that the FDIC expects a loss.

[19]Until 2010, the FDIC assesses insurance premium based on total deposits. The assessment rate is a such that $I = aD$. There have long been concerns that banks shift deposits out of balance sheet temporally at quarter-ends to lower the assessment base. Since April 2011, the FDIC has changed the assessment base to the difference between the risk-weighted assets and tangible equity, as required by the Dodd-Frank Act (Section 331). If V equals the value of risk-weighted assets, the new assessment base equals $D + D_1$, which implies that assessment rate is b such that $I = b(D + D_1)$. The actual premium calculations may also depend on credit rating and the proportion of long-term debt to deposits. See Federal Deposit Insurance Corporation (2011) for more details.

fixed liability structure, the transfer of payments across the states improves the overall value of the bank by receiving more service income, increasing the tax shields, and reducing the expected cost of default. Section 4.2 will show that the combined actions of the charter authority and the FDIC create additional value for banks, although the expected dead-weight loss associated with bankruptcy increases after banks adjust their liability structures optimally.

Equity holders can choose to default before a bank run or a regulatory closure. Absent a bank run and regulatory closure, there is an optimal point for equity holders to default. The default decision maximizes equity value, given a liability structure. The optimal default of debt is referred to as *endogenous default* and derived by Leland (1994) for firms without deposits. In Section 3.1, we provide the formula of endogenous default in the presence of deposits. Let V_d be the point of endogenous default, i.e., equity holders choose to default if and only if asset value V reaches or drops below V_d, in the absence of a bank run or regulatory closure. Then bankruptcy happens if either the debt is defaulted by equity holders endogenously or the bank is closed due to bank run or by charter authority. In other words, the point of bankruptcy is $V_b = \max\{V_d\,, V_a\}$.

In summary, banks face three types of bankruptcy. The first type is endogenous default chosen by equity holders. In this type of bankruptcy, liquidation of assets has to go through private-sector bankruptcy procedure, and the cost associated with the procedure is αV_d. The second type is a bank run, and bankruptcy cost is αV_a, as it also goes through bankruptcy procedure. The last type is bank closure by charter authority. The cost of closing a bank is βV_a if the FDIC liquidates the assets. In order to keep the formulation simple, we denote the recovery value of assets after bankruptcy by $(1-\phi)V_a$, where ϕ equals α or β, depending on the type of bankruptcy. When bank assets are liquidated after bankruptcy, depositors are paid first, and debt holders are paid the next if there is value left. Consequently, the payoff to debt holders is $\left[(1-\phi)V_b - D\right]^+$.

3 Valuation and Optimization

Table 1 summarizes the exogenous parameters in the model and the assumptions on them. In the table, service income is positive but with a rate smaller than the risk-free rate: $0 < \eta < r$. Corporate tax is present: $0 < \tau < 1$. The bankruptcy and FDIC liquidation are both costly: $0 < \alpha < 1$ and $0 < \beta < 1$. These assumptions are not only realistic but also the requisite mathematical conditions to carry out valuation and optimization.

Parameter	Notation	Allowed range
Asset volatility	σ	$(0,\infty)$
Asset cash flow	δ	$[0,\infty)$
Asset value	V	$(0,\infty)$
Riskless interest rate	r	$(0,\infty)$
Bank service income	η	$(0,r)$
Corporate tax benefit	τ	$(0,1)$
Court bankruptcy cost	α	$(0,1)$
FDIC liquidation cost	β	$(0,1)$
Regulatory closure	κ	$[1,\infty)$
Insurance subsidy	$1-\omega$	$[0,1]$

Table 1: Exogenous parameters are pre-specified, not determined by either valuation or optimization in the model. The allowed ranges are assumptions of the model.

3.1 Bank Valuation and Insurance Premium

Since deposits are either withdrawn with full value or insured by FDIC, the value of deposits is its par value D. Because the net liability on deposits is $C=(r-\eta)D$, the value of deposits is related to its liability by $D=C/(r-\eta)$.

The values of debt and equity are affected by the risk of bankruptcy. The Arrow-Debreu state price of bankruptcy plays a key role in bank valuation. Consider a security that pays \$1 when bankruptcy occurs, and pays nothing otherwise. The price of this security is the state price of bankruptcy. The state price is $P_b=[V_b/V]^{\lambda}$, where λ is the positive root of a quadratic equation, which is given by (22) in Appendix. The quadratic equation implies that λ is an increasing function of r and a decreasing function of δ and σ. If the cash flow of assets is zero, i.e., $\delta=0$, we have $\lambda=2r/\sigma^2$, which is proportional to r and inversely proportional to σ^2. The state price P_b is a solution to Merton's (1974) pricing restriction, which is equation (21) in Appendix.

Bank value depends on its liability structure (I,C,C_1) because the liabilities affect bankruptcy boundary and its state price. The following theorem, derived in Appendix A.1, summarizes the relation between bank value and liability structure.

Theorem 1 *Given a liability structure (I,C,C_1), the boundary of bank run or regulatory closure and the boundary of endogenous default are, respectively,*

$$V_a=\kappa C/(r-\eta) \tag{1}$$

$$V_d=(1-\tau)[\lambda/(1+\lambda)](I+C+C_1)/r\,. \tag{2}$$

The bankruptcy boundary is $V_b = \max\{V_a, V_d\}$. *The equity, debt, and bank values are, respectively,*

$$D_1 = (1 - P_b)C_1/r + P_b\big[(1-\phi)V_b - D\big]^+ \tag{3}$$

$$E = V - (1-\tau)(1-P_b)(I + C + C_1)/r - P_b V_b \tag{4}$$

$$\begin{aligned} F = {} & V - P_b \min\{\phi V_b\,,\, V_b - D\} \\ & +\ (1-P_b)\big[C\eta/(r-\eta) + \tau(I + C + C_1) - I\big]/r\,. \end{aligned} \tag{5}$$

In equation (3), debt value is the sum of the expected value of coupons before bankruptcy and expected value of recovery at bankruptcy. In equation (4), equity value is the residual asset value after subtracting the expected after-tax liabilities on insurance, deposits, and debt and the expected value of bankruptcy loss. In equation (5), which is for bank value, the first term is asset value, the second term reflects the value of expected loss from bankruptcy, and the last term shows the value of expected service income and tax benefit after paying insurance premium.

Theorem 1 shows the role of service income and deposit insurance in bank valuation. Along with tax savings on debt, account service income (η) increases bank value as shown by the last term on the right-hand side of equation (5). The ability of a bank to attract deposits at a rate lower than the risk-free rate comes at a price: the bank has to close and incur bankruptcy cost if depositors run or if the charter authority closes the bank. For a bank with deposit insurance, insurance premium reduces bank value, which is evidenced by the last term of the equation. Although the liability structure in the theorem includes insurance premium, these formulas in the theorem apply to banks without deposit insurance if we set $I = 0$.

We obtain the endogenous credit spread of debt from Theorem 1. The endogenous credit spread is $s = C_1/D_1 - r$, where D_1 is a function of C_1 as given in equation (3). The credit spread takes the probability of bankruptcy into account through state price P_b; at the same time, the state price is affected by the liability structure. Insurance premium affects the credit spread even though I does not appear in equation (3) explicitly. The premium affects the endogenous default boundary in equation (2), which in turn affects bankruptcy boundary V_b and its state price. The last two affect the credit spread directly.

A comparison our model of banks and the model of firms in Leland (1994) shows the connection and distinction between banks and non-financial firms. If we set $C = I = 0$ but $C_1 > 0$, the formulas in Theorem 1 reduce to those in Leland (1994) for a firm with only equity and unprotected debt. If we set $I = \eta = C_1 = 0$ but $C > 0$, the formulas in Theorem 1 coincide with Leland's for firms with only equity and debt protected at level κD. Leland's seminal capital structure theory is about non-financial firms, which is not applicable to banks that take deposits and earn service income, that may pay deposit insurance premium and face the risk of bank run or regulatory closure. Our model extends Leland's to banks and offers a consistent framework for understanding the similarities and differences between banks and other firms.

While the deposit insurance premium is exogenously given in Theorem 1, it should endogenously depend on the amount of deposits under insurance and the risk involved. In principle, an insurance corporation should charge each bank a fair insurance premium. A fair premium makes the insurance contract worth zero to each party of the contract. The next theorem, derived in Appendix A.2, characterizes the fair insurance premium.

Theorem 2 *Given D dollars of deposits, the fair insurance premium is*

$$I^\circ = r[1-(1-\beta)\kappa]^+ DP_a/(1-P_a), \tag{6}$$

where $P_a = [\kappa D/V]^\lambda$ is the state price of bank closure.

An alternative way to write the insurance pricing equation is

$$(1-P_a)(I^\circ/r) = P_a[1-(1-\beta)\kappa]^+ D, \tag{7}$$

which says that the expected present value of insurance premium paid to the insurance corporation equals the expected present value of the insurance obligations at bank closure. If $\kappa < 1/(1-\beta)$, the fair premium I° is positive. It converges to zero as κ rises to $1/(1-\beta)$. If $\kappa \geq 1/(1-\beta)$, the fair premium is zero because the bank will be closed with enough asset value to cover the deposits in full.

The fair insurance premium I° increases with D. If deposits increase, not only the insurance premium increases, the assessment rate of insurance premium, which is the premium on each dollar of deposits, also increases. By Theorem 2, the assessment rate is

$$h \equiv I^\circ/D = r[1-(1-\beta)\kappa]^+ P_a/(1-P_a). \tag{8}$$

The rate is increasing with D because P_a is bigger for a larger D. The positive relation between h and D makes sense because an expansion of deposits exposes the insurance corporation to a bigger risk.

Some academics have argued that the FDIC does not charge enough insurance premium to cover its risk exposure.[20] A premium lower than the fair rate provides subsidized insurance to banks. To allow for subsidized insurance premium, we assume that the FDIC insurance premium is $I = \omega I^{\circ}$, where $\omega = 1$ represents a fair premium and $\omega < 1$ represents a subsidized premium. Relating to the net cash outflow on deposits by $D = C/(r-\eta)$, we have $I = iC$, where

$$i = \omega[1-(1-\beta)\kappa]^{+}[r/(r-\eta)]P_a/(1-P_a). \tag{9}$$

If the FDIC subsidizes deposit insurance, it increases the bank value because the bank pays lower insurance premium for enjoying the risk-free value of deposits. Even with the subsidy, the assessment rate and the total premium a bank pays still endogenously depends on the amount of deposits and the bank's risk profile.

With endogenous insurance premium, a liability structure is characterized by the pair (C, C_1) because C determines I. Imposing $I = iC$ in the bank value formula (5), we obtain

$$\begin{aligned} F = V \;+\; (1-P_b)[\eta/(r-\eta)+\tau-(1-\tau)i]C/r \;+\; (1-P_b)\tau C_1/r \\ -\, P_b \min\{\phi V_b\, ,\, V_b - D\}. \end{aligned} \tag{10}$$

On the right-hand side of equation (10), the second term is the value of tax deduction and account service income, netted off against the insurance premium. The third term is the value of tax benefits to the bank for its interest expense on debt. The last term is the expected value of bankruptcy loss, for which bankruptcy cost ϕ takes the value of α or β, depending on the type of bankruptcy.

3.2 Optimal Liability Structure

Now we proceed to examine how a value-maximizing bank chooses its liability structure. We first consider an uninsured bank, which is neither under FDIC deposit insurance nor subject to regulatory closure. The bank takes into account the possibility that depositors may run in order to protect their deposits. The uninsured bank is important for understanding the inherent difference between banks and other firms because we want to know whether the properties of bank liability structure are driven by banking business per se or government regulation, or both.

The uninsured bank serves as a benchmark for evaluating the effects of regulatory mandates such as FDIC insurance and charter authority's closure of troubled banks.

[20]See Duffie, Jarrow, Purnanandam, and Yang (2003) for evidence of FDIC insurance subsidy. On the other hand, one may argue that a lower premium is necessary to compensate the insured banks for the costs of reporting requirements and tight regulation.

The benchmark is useful for examining how a bank might arrange its liability structure differently in alternative regulatory environments. The theory presented in this section will allow us gain insights into the optimal response of banks to the FDIC. The optimally-responding banks adjust their leverage, liability structure, and default decisions, relative to the benchmark in which banks make their choices of liability structure unfettered by any government interventions.

As pointed out earlier, a liability structure of an uninsured bank is described by the pair (C, C_1). An optimal liability structure is the deposit liability C^* and debt liability C_1^* that maximize bank value. The next theorem, derived in Appendix A.3, provides a characterization of the optimal liability structure for an uninsured bank, which is not subject to regulatory closure.

Theorem 3 *Suppose* $0 < \eta < r$*,* $0 < \tau < 1$*, and* $\kappa \geq 1/(1-\alpha)$*. The optimal liability structure of an uninsured bank is unique. In the optimal liability structure,* $V_a^* = V_d^*$*, and the state price of bankruptcy is*

$$P_b^* = \frac{1}{1+\lambda} \cdot \frac{\eta(1-\tau)\lambda + r\tau(1+\lambda)\kappa}{\eta(1-\tau)\lambda + r\tau(1+\lambda)\kappa + r(1-\tau)\alpha\lambda\kappa}. \tag{11}$$

The optimal deposit and debt liabilities are

$$C^* = (r-\eta)VP_b^{*1/\lambda}/\kappa \tag{12}$$

$$C_1^* = rVP_b^{*1/\lambda}\left[\frac{1+\lambda}{(1-\tau)\lambda} - \frac{r-\eta}{r\kappa}\right]. \tag{13}$$

Equation (11) gives the exact formula of the optimal state price P_b^*; it is an elementary algebraic function of the following exogenous parameters: r, σ, δ, τ, η, and α.

The theorem characterizes the optimal liability structure of a bank that faces corporate tax, bears the risk of costly bankruptcy, and takes deposits to earn service income. Combining Theorem 3 with Theorem 1, we obtain analytical solutions, which we omit to save space, for deposit value D^*, debt value D_1^*, equity value E^*, bank value F^*, bankruptcy boundary V_b^*, and credit spread s^* in the optimal capital structure of the uninsured bank. The ratio of debt liability to deposit liability is a characterization of the liability structure and is thus referred to as *liability ratio*. By the theorem, the optimal liability ratio is $x^* = C_1^*/C^* = r(1+\lambda)/[\lambda(1-\tau)(r-\eta)]$.

The theorem states that the optimal amount of debt makes the endogenous default boundary coincide with the bank-run boundary. Deposits attract a discount in the deposit rate as well as service fees, in addition to tax savings. The cost of taking deposits is the expected loss due to a bank run. In contrast, debt brings tax savings but produces no account services or fee income; its cost is the expected loss due to bankruptcy. Therefore, at the margin, the bank should use deposits, not debt, to balance the benefits of leverage with the loss to bankruptcy. With this balance, the bank should take as much debt as possible for availing the tax benefits but should avoid the expected bankruptcy cost resulting from endogenous default. To avoid the expected cost associated with endogenous default, the bank should not set the endogenous default boundary above the bank-run boundary. As a result, the optimal debt should make default occur at exactly the same point at which bank run takes place.

The services associated with deposits are important for Theorem 3. If $\eta = 0$ while $\tau \in (0,1)$ and $\kappa \geq 1/(1-\alpha)$, the optimal liability structure is not unique. If we set $\eta = 0$ in formulas (11)–(13), we obtain the optimal structure with the maximum deposits, but every C with $0 \leq C \leq C^*$ leads to the same maximum bank value. The optimal capital structure is not unique for a bank with $\eta = 0$ because deposits and debt have the same tax benefits and bankruptcy costs in the absence of account service.[21] However, setting $\eta = 0$ and $C = 0$ in Theorem 3 gives the unique optimal liability structure for a firm that takes no deposits. This corresponds to the unique optimal capital structure of a firm as modeled by Leland (1994). Obviously, a firm holding no deposits and providing no account service is not a bank. Hence, a structural model without considering deposits and bank services is not appropriate for understanding banks' optimal liability structure and leverage.

Corporate tax plays an important role in Theorem 3. If $\eta \in (0, r)$ and $\kappa \geq 1/(1-\alpha)$ but $\tau = 0$, the bank can maximize its value without issuing debt. In the absence of tax benefits, increasing debt from zero does not alter bank value, and thus the optimal level of debt is indeterminate. However, if a bank finances its operations only by taking deposits, then there is a unique optimal level of deposits when tax rate is zero, as stated in the theorem below.

Theorem 4 *Suppose $\tau = 0$, $0 < \eta < r$, and $\kappa \geq 1/(1-\alpha)$. The optimal liability structure is unique for an uninsured bank that finances its operations by deposits and equity. In the optimal liability structure, the bank-run boundary is higher than the endogenous default*

[21]Without service income, deposits in our model bears resemblance to the secured debt in Leland's (1994) because deposits are protected by bank run. Leland considers the optimal capital structure of firms that take either secured or unsecured debt but not the optimal mix of the two. He analytically solves for the optimal capital structure of firms that take unsecured debt, but for a firm that takes secured debt, he solves for the optimal structure numerically.

boundary: $V_a^* > V_d^*$*. The state price of bankruptcy is*

$$P_a^* = \frac{1}{1+\lambda} \cdot \frac{\eta}{\eta + r(\kappa - 1)}, \tag{14}$$

and the optimal deposit liability is

$$C^* = (r - \eta) V P_a^{*1/\lambda} / \kappa \,. \tag{15}$$

The proof of this theorem is shown in Appendix A.4. This optimal liability structure in the absence of taxes is especially relevant to banks as the motive to issue deposits arises from liquidity provision, a feature that is unique to banks. Our result in Theorem 4 may help explain why banks in the 19th century used substantial leverage in the form of deposits when there were no tax advantages, as documented by Calomiris and Carlson (2015). Theorem 4 shows that leverage is desired by banks despite the absence of corporate taxes benefits.

Moreover, the inequality $V_a^* > V_d^*$ in Theorem 4 explains why equity holders never chose to default before bank run as observed in history. A bank described in this theorem provides account and liquidity services on deposits but receives no tax benefit. For this reason, tax benefit is typically ignored in the literature that focuses on bank account services. Models of banks without tax benefit and debt are nevertheless largely inconsistent with the typical bank capital structure in today's world.

When there is no corporate tax, debt is however indeterministic in optimal liability structure. The indeterminacy suggests that a model ignoring tax savings may not be appropriate for understanding modern bank liability structure that includes debt. The empirical relevance of taxes in bank capital structure has been recently examined by Schepens (2014) who exploits a recent change in the tax code in Belgium, which permitted tax advantages to equity for the first time (thereby reducing the tax discrimination between bank debt and bank equity). He found that the tax changes led the banks to issue more equity and improved their capital ratios. This evidence shows that banks do respond to tax changes by altering their liability structure

Theorem 3 ignores FDIC deposit insurance and regulatory closure, but it serves as a useful benchmark so that we can examine the effects of deposit insurance and bank regulation. For an FDIC insured bank, the optimal liability structure is a pair of C^* and C_1^* that maximizes the bank value in equation (10) subject to equation (9). The value-maximizing bank in our framework is fully aware that any decision pertaining to leverage and liability structure has a consequence on the FDIC insurance premium. The bank should therefore be mindful of the channel in its choice of leverage and liability structure. The endogenous determination of FDIC premium and liability structure captures the feedback channel from FDIC to the banks and vice versa.

The next theorem, derived in Appendix A.5, characterizes the conditions for a liability structure to be optimal in a bank under FDIC insurance.

Theorem 5 *Suppose* $0 < \eta < r$, $0 < \tau < 1$, $1 < \kappa < 1/(1-\beta)$, *and the FDIC insurance premium is* $I = \omega I^\circ$, *where* I° *is defined in Theorem 2. A liability structure with* $V_d < V_a$ *is never optimal for an FDIC-insured bank. There exists* $\kappa^* \in [1, 1/(1-\beta))$ *such that for all* $\kappa \in (\kappa^*, 1/(1-\beta))$, *the optimal structure is unique and satisfies* $V_d^* = V_a^*$. *In such optimal structure, the state price of bankruptcy is*

$$P_b^* = \frac{1}{1+\lambda} \cdot \frac{\eta(1-\tau)\lambda + r\tau\kappa(1+\lambda)}{\eta(1-\tau)\lambda + r\tau\kappa(1+\lambda) + r(1-\tau)\lambda\{\kappa - 1 + \omega[1-(1-\beta)\kappa]^+\}}. \tag{16}$$

The optimal deposit and debt liabilities are

$$C^* = (r-\eta)VP_b^{*1/\lambda}/\kappa \tag{17}$$

$$C_1^* = rVP_b^{*1/\lambda}\left[\frac{1+\lambda}{(1-\tau)\lambda} - \frac{r-\eta}{r\kappa} - \omega[1/\kappa - (1-\beta)]^+ \frac{P_b^*}{1-P_b^*}\right]. \tag{18}$$

Formula (16) shows that the optimal state price P_b^* is a function of the following exogenous parameters: r, σ, δ, τ, η, β, κ, and ω. Combining this theorem with Theorems 1 and 2, we obtain analytical solutions, which we omit to save space, for the deposits value D^*, debt value D_1^*, equity value E^*, bank value F^*, bankruptcy boundary V_b^*, credit spread s^*, and insurance premium I^* in the optimal capital structure of the FDIC insured bank.

Theorem 5 shows that it is optimal for banks to leverage so that the endogenous default of debt is as late as the closure of banks. FDIC insurance and regulatory closure do not directly affect how a bank optimally chooses debt, but they indirectly affect the choice through closure boundary. The optimal debt still maximizes tax benefit and minimizes its protection of deposits. In particular, Theorem 5 shows that zero debt $C_1 = 0$ is not optimal for a bank as long as the insurance premium is not too high. The economic intuition is simple. When the insurance premium is not too high, it is optimal to use a positive level of deposits. By the optimal condition that the bank closure boundary must equal the default boundary, the bank should issue a positive amount of debt to meet the condition. It is also easy to reason analytically. If $C_1 = 0$, it follows from Theorem 1 that

$$V_a = \kappa C/(r-\eta), \quad V_d = (1-\tau)[\lambda/(1+\lambda)](1+i)C/r. \tag{19}$$

If i is not too large, the above equations imply $V_a > V_d$, indicating that the structure is suboptimal by Theorem 5.

In theory, if asset volatility σ and liquidation cost β are high enough, it is possible for a liability structure with $V_d > V_a$ to be optimal for some low κ close to 1. We have confirmed this possibility by both mathematical derivations and numerical optimization. If σ and β are very high and κ is very low, the fair insurance premium rate i may be very high, making deposits too expensive as source of funds compared to debt. When that happens, reducing deposits to have $V_a < V_d$ may be optimal. Preventing i from being too high is the reason for κ to be higher than a threshold κ^* in the theorem. Nevertheless, for all the asset volatility and liquidation cost we consider in later sections, we find $\kappa^* = 1$. That is, $V_a^* = V_d^*$ and the formulas in Theorem 5 hold for all $\kappa \in (1, 1/(1-\beta))$.

Theorem 5 incorporates the endogenous insurance premium in optimal liability structure. Besides considering the tradeoff between tax benefits, account service income, regulatory closure, and bankruptcy costs, banks take the cost of deposit insurance into account. If we set $\kappa \geq 1/(1-\alpha)$ and assume that $\alpha \geq \beta$, the insurance premium becomes zero. Consequently, the formulas in this theorem reduce to those in Theorem 3. With a general κ and a positive ω in this theorem, the assessment rate ωh of insurance premium is an increasing function of D, and thus i increases with C. Therefore, under the assumption of Theorem 5, banks consider both the increase in insurance premium caused directly by the expansion of deposits as well as the increase caused indirectly through the rise of assessment rate. In Section 5.2, we will show the impact of endogenous insurance premium on banks' optimal choice of capital structure.

4 A Quantitative Illustration

The model just developed paves a way to characterize quantitatively the optimal bank liability structure. As discussed earlier, we consider two types of banks. The first type operates without deposit insurance and regulatory closure. Since deposits in these banks are not insured, they face the risk of a bank run. In practice, many non-U.S. banks face the risk of a bank run because their governments do not provide deposit insurance.[22] Some U.S. banks are also not covered by FDIC deposit insurance. In

[22] In September 2007, Northern Rock, a U.K. Bank, experienced a run on its deposits, and had to be nationalized in 2008. See Shin (2008) for a cogent analysis of the Northern Rock bank run. Bank runs

theory, an uninsured bank serves as a counterfactual for the second type of banks, which are covered by FDIC insurance and subject to regulatory closure. The second type is the majority of banks in the U.S.

4.1 A Practical Range of Exogenous Parameters

While the theoretical range of the exogenous parameters are as wide as those listed in Table 1, the practical range of the parameters should be much narrower. In the numerical illustration and analysis of comparative statics, we choose a range that are practically conceivable and interesting.

As our model inherits the major advantage of structural models that coherently connects the risk of debt and equity to the risk of assets, the risk comes from asset volatility (σ), which is one of the most important parameters to affect the leverage and liability structure. Since asset volatility is not directly observable, investors infer asset volatility from accounting data and market prices. Moody's KMV provides estimates of asset volatility for a large number of companies across a wide range of industries.

In panel A of Figure 2, we present the average, median, and the 10/90-percentiles of Moody's estimates of bank asset volatilities. As a comparison, in panel B we present Moody's estimates of manufacturing-firm asset volatilities. The figure shows a difference between the assets held by banks and those owned by manufacturing firms: bank assets have much lower volatility. The average asset volatility is around 10% for banks, whereas it is 40 $\sim$ 50% for manufacturing firms. Although bank asset volatility fluctuates over time, the median is around 5% for 2001–2012. The 90 percentile of bank asset volatilities is well below 15% for 2001–2007, and it stays below 25% even for the period of 2007–2012. In view of these stylized facts, we let $\sigma \in [0.03, 0.20]$ in our study of comparative statics.

Another parameter of bank assets is its rate of cash flow (δ). If the assets contain only commercial and consumer loans, the cash flow are interest and principal payments of the loans. In the numerical illustration and comparative statics, we set the cash-flow rate to 8%, which is the average mortgage rate in the U.S. during 1984–2013. Correspondingly, we also set the risk-free rate to the average Federal funds rate during the same period; this gives $r = 5\%$. We choose this period because we would like to

have happened in Europe and Asia even after the recent financial crisis. In 2010, depositors "ran" from two Swedish banks, Swedbank and SEB, and a Chinese bank, Jiangsu Sheyang Rural Commercial Bank. In 2013, depositors ran from Cypriots banks and forced the country to close its banks for many days. In 2014, depositors ran from two Bulgarian banks, Corporate Commercial Bank and First Investment Bank.

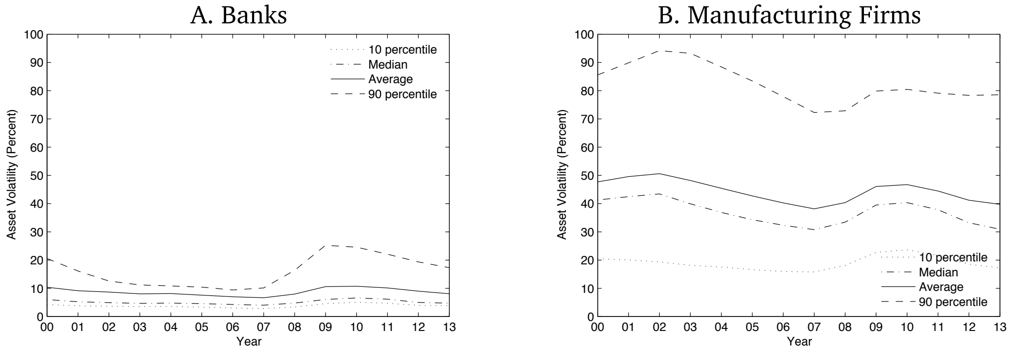

Figure 2: Plots of the average, median, and 10/90-percentiles of asset volatilities of banks (panel A) and manufacturing firms (panel B) from 2000 to 2013. Moody's KMV Investor Service provided the estimates of asset volatilities.

make our numbers broadly comparable to the aggregate balance sheet data of FDIC-insured commercial banks and savings institutions. The balance sheet data for this period are provided by the FDIC, and both the mortgage rate and Federal funds rate data are obtained from Table H.15 from the Federal Reserve.

The income from deposit services is important in bank liability structure. The net income from deposit services should be determined in the competitive market of deposits. In a perfect competitive market with free entry, the net income would be driven to zero, or it should just cover the insurance premium if a bank has deposit insurance. At least in the U.S., new entry of banks into the market is regulated by charter authorities.[23] Without free entry, deposit rents arise from the market power enjoyed by the bank (De Nicolo and Rurk Ariss, 2010). The profitability should depend on the amount of deposits and the bank. Thus, parameter η may differ across banks and should be a function of D. We do not explicitly model the equilibrium of deposits or the demand function of deposits, $D(\eta)$, in order to keep the model tractable and to focus on the choice of liability. We instead assume η to be a constant but allow it to be different across banks. A range of values, $\eta \in [0.02\,,\,0.04]$, are examined in our analysis of comparative statics.

Since a benefit of leverage is the tax deductibility of interest expenses on debt, corporate tax rate is an important parameter in capital structure. The statutory corporate tax rate in the U.S. ranges up to 35%. The U.S. Department of Treasury (2007) reports that the effective marginal tax rate on investment in business varies substantially

[23] Founders of a new bank have to show their integrity and ability to manage the bank. In addition, the regulators demand evidence of need for a new bank before granting a charter. Peltzman (1965) documents the restriction on entry of commercial banking. Jayaratne and Strahan (1998) examines the effects of entry restrictions on bank efficiency.

by business sectors. The academic literature suggests that the effective corporate tax rate is around 10% for non-financial firms (Graham, 2000) but can be more important for banks (Heckemeyer and Mooij, 2013). In our analysis of comparative statics, we consider a wide range, $\tau \in [0.01, 0.20]$.

Bankruptcy cost counters the benefit of leverage, but the task of measuring it has always been a challenge. A well-known reference is the study of Altman (1984), which examines a sample of 19 industrial firms which went bankrupt over the period of 1970–1978. The estimated bankruptcy cost is 19.7% of the firm's asset value just prior to its bankruptcy. Bris, Welch and Zhu (2006), however, show that bankruptcy cost varies across firms and ranges between 0% and 20% of firm assets. Banks experienced higher bankruptcy costs. Based on 791 FDIC-regulated commercial banks failed during 1982–1988 (the Savings and Loan Crisis), James (1991) estimates that bankruptcy cost is 30% of a failed bank's assets. Based on 325 insured depository institutions failed during 2008–2010 (the Great Recession), Flannary (2011) estimates that bankruptcy cost is about 27% of a failed bank's assets. In light of these estimates, we choose 27% as the mid-point of the range. The range we consider is $[0.17, 0.37]$.

The other exogenous parameters are as follows. For an unregulated bank, we assume bank run happens at the point when the bank has exactly enough assets to repay deposits after liquidation in bankruptcy procedures, i.e, $\kappa = 1/(1-\alpha)$. For an FDIC insured bank, state banking regulatory agency closes it when it is unable to meet its obligations to depositors. The parameter κ for bank closure should thus be at least 1. When a bank's total capital is less than 2% of its assets, the FDIC classifies it as "critically undercapitalized," and the charter authority typically closes the bank. Given these institutional arrangements, we set the regulatory closure rule as $\kappa = 1/(1-0.02) \approx 102\%$. For insurance subsidy $(1-\omega)$, we examine a wide range from zero to 40%.

4.2 An Example of Endogenous Liability Structure

The optimal liability structure is characterized by a set of ratios endogenously determined by bank management.[24] The first endogenous variable of our interest is the ratio of deposits to assets, D/V. The next is the ratio of debt to assets, D_1/V. The leverage of the bank is measured by the ratio of tangible equity to assets, which is referred to as Tier 1 ratio in bank regulation. Since the tangible equity is $V - D - D_1$, the ratio

[24] Even though the amount of deposits is partially determined by the supply in the markets, banks have control of the ratios because they can adjust the level of assets to achieve their desired capital structure.

of tangible equity to assets is $(V - D - D_1)/V$. Higher leverage corresponds to lower tangible equity.[25] The amount of deposits determines the boundary of bank run or regulatory closure, whereas the total leverage affects the boundary of endogenous default. The two boundaries relative to asset value, V_a/V and V_d/V, determine the distance to bankruptcy. The higher of the two is the bankruptcy boundary V_b/V, which influences the expected value of bankruptcy loss, $P_b \phi V_b/V$, another endogenous variable of our interest. The possibility of bankruptcy causes the bank to pay a credit spread, s, on debt. The credit spread is observable in the market. For banks with deposit insurance, the insurance premium I is endogenously determined by the liability structure. The charter value of the bank, which is the difference between bank value and asset value, is the objective function. We measure charter value as percent of asset value: $(F - V)/V$.

Table 2 presents optimal liability structure and related endogenous variables for both FDIC-insured and uninsured banks, as well as for a non-financial firm that does not take deposits. The parameters used for generating the optimal liability structure are chosen from the ranges discussed in Section 4.1. We later vary these parameters to examine the effect of each.

Endogenous optimal ratio	Mathematical definition	FDIC-insured bank	Uninsured bank	Unregulated firm
Deposit	D/V	45.30	32.22	0.00
Debt	D_1/V	46.48	52.07	60.03
Tangible equity	$(V - D - D_1)/V$	8.23	15.71	39.97
Closure boundary	V_a/V	46.20	44.14	0.00
Default boundary	V_d/V	46.20	44.14	35.28
Bankruptcy loss	$P_b \phi V_b/V$	3.89	3.47	1.98
Credit spread	s	2.27	2.05	0.75
Insurance premium	I/V	0.24	0.00	0.00
Charter value	$(F - V)/V$	24.50	19.42	6.23

Table 2: The optimal liability structures in FDIC-insured and uninsured banks and an unregulated firm that does not take deposits. All values are reported in percentage points. In the calculation of endogenous variables, we set $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.9$, and $\alpha = \beta = 0.27$. For FDIC-insured banks, $\kappa = 1.02$, but $\kappa = 1/(1 - \alpha)$ for uninsured banks.

Table 2 reveals some distinctive characteristics of bank optimal liability structure. The most striking is the high level of leverage for FDIC-insured banks, which is typical in banks but not common in non-financial firms. In the FDIC-insured bank, the optimal

[25] It is useful to point out that tangible equity of a bank can be negative in practice. In its December 2011 filing, last time as a bank holding company, the U.S. operations of Deutsche Bank had total assets of \$355 billion and Tier 1 capital of negative \$5.68 billion.

	Average	Median	StDev	Maximum	Minimum
Demand/Savings Deposits	45%	44%	5%	59%	39%
Other Liabilities	47%	49%	6%	53%	30%
Total Liabilities	92%	92%	2%	95%	89%
Total Equity	8%	8%	2%	11%	5%

Table 3: Statistics of the aggregate liability structure of all FDIC-insured commercial banks and savings institutions from 1984 to 2013. Source: The Federal Deposit Insurance Corporation.

deposit-to-asset ratio is 45.30%, and the optimal debt-to-asset ratio is 46.48%. This liability structure leaves tangible equity to be only 8.23% of asset value. Although this liability structure is for illustrating our theory, it is broadly comparable to the average liability structure of FDIC-insured banks. In Table 3, we present the statistics of liability structure of all FDIC-insured commercial banks and savings institutions from 1984 to 2013.[26]

Consideration of uninsured banks helps answer an important question: are banks fundamentally different from non-financial firms regardless of FDIC insurance and regulatory closure? This question is important for understanding the effects of deposit insurance and regulatory closure, which will be discussed in the next subsection. Table 2 shows that high level of leverage is optimal for a bank even without deposit insurance. With the baseline values of exogenous parameters, the sum of deposits and debt amounts to about 84.29% of the asset value, leaving tangible equity to be only 15.71% of the asset value. This means the FDIC insurance is not the sole reason for high leverage of banks.

High leverage in uninsured banks is likely related to a fundamental factor that distinguishes banks from other firms: taking deposits to provide account and liquidity services, besides earning cash flows from low-volatility assets such as loans. Without the insurance and regulation of FDIC, the optimal liability structure of the uninsured bank in Table 2 consists of substantial amount of deposits, which is 32.22 percent of asset value, although is less than the comparable insured bank. By contrast, the uninsured bank takes up more debt, which is 52.07 percent of the asset value.

A noticeable characteristic of the optimal liability structures in Table 2 is the signifi-

[26] Since our model considers only two kinds of leverage: deposits that can run and earn service/liquidity premium and debt that cannot run or earn service/liquidity premium, Table 3 distinguishes only demand and savings deposits from the rest of bank liabilities. The actual liability structure of banks are far more complex than the structure presented in the table. The complexity of full array of bank leverage is beyond the scope of this paper.

cance of debt.[27] With the baseline parameter values, the optimal debt is 46.48% of the asset value in the insured bank and 52.07% in the uninsured bank in Table 2. As noted in Theorem 3, banks optimally set debt to a level such that default boundary is exactly the same as bank-run boundary. With this strategy, banks maximize tax deduction without further increasing the probability of bankruptcy. In Table 2, the endogenous default boundary is exactly same as the closure or run boundary of the bank.

In order to better understand the nature of bank liability structure, Table 2 presents in the last column the optimal leverage of a firm that does not take deposits. The calculation of optimal leverage in the firm uses the same parameters as those used for banks. Without taking deposits, the firm drastically reduces leverage. The tangible equity in the firm is nearly 40 percent of asset value, much higher than the tangible equity in either insured or uninsured banks. Debt in the firm is slightly higher than debt in the comparable bank. This level of firm leverage is still rather high, but this is due to the low volatility for bank assets in Table 2. Recall that Figure 2 shows that the median asset volatility of manufacturing firms ranges from 30% to 50%. If we increase asset volatility to 30%, the optimal debt-to-asset ratio in the firm reduces to 45%, and the tangible equity ratio rises to 55%, which is typical for non-financial firms.

A major objective of FDIC is to reduce the probability of bank failure. For the baseline parameters, the optimal closure boundary of the FDIC-insured bank is 46.20% of asset value, as shown in Table 2. This boundary is slightly higher than the 44.14% bank-run boundary of the comparable uninsured bank in the table. The higher closure boundary leads to a larger expected bankruptcy loss. In the table, the expected bankruptcy loss of the FDIC-insured bank is 3.89% of asset value, slightly larger than the 3.47% in the comparable uninsured bank. The larger expected bankruptcy loss results in 22-bps increase in credit spread. In contrast, the credit spread for the debt in the firm that does not take deposits is only 0.75%. The higher closure boundary, expected bankruptcy loss, and credit spread of insured banks are not necessarily consonant with the mandates of FDIC. This outcome is a result of the bank's optimal response when it counteracts the intended impact of FDIC by taking more deposits.

FDIC deposit insurance plays a part in driving banks to high leverage. In Table 2, the tangible equity ratio in the insured bank is 8.23%, substantially lower than the 15.71% ratio in the comparable uninsured bank. With deposit insurance, the bank

[27] In practice, debt has always been an important source of funding for banks. Avdjiev, Katasheva and Bogdanova (2013) report that during the 2009–2013 period alone, banks around the world have issued $4.1 trillion of unsecured long-term debt.

holds more deposits and less debt, but its total leverage is higher. Intuitively, deposit insurance, accompanied by regulatory closure, allows the insured bank to take more deposits and keep the closure boundary from becoming too high. Thus, a major benefit banks receive from the FDIC is that it prevents a sharp increase in the probability of bank run or closure when they increase deposits. With this benefit, the banks use more leverage.

Although driving up bank leverage, the FDIC brings two benefits to the banking industry, according to our model. The first benefit is to allow banks to provide more deposit services. In Table 1, the amount of deposits in the FDIC-insured bank is 45.30% of asset value, 13.08 percentage points higher than the deposits in the comparable uninsured bank. The second benefit is to increase the value of banks. The charter value of the FDIC-insured bank in Table 2 is 24.50% of asset value, about 5 percentage points higher than the value of the comparable uninsured bank. Only a small part of the increase in charter value is due to the subsidy of FDIC insurance. If we assume that the FDIC charges fair insurance premium ($\omega = 1$), the charter value would still be about 24% of asset value. The FDIC increases bank value because it allows the bank better take advantage of the income from serving deposits.

5 Comparative Statics

Bank optimal liability structure depends on the characteristics of bank business. Three important characteristics are deposit service income, asset volatility, and bankruptcy cost. In the case of FDIC insured banks, the bankruptcy cost is the FDIC liquidation cost. In Section 5.1, we examine the effects of these three factors by considering various values of η, σ, and β.

Keeping bank business characteristics fixed, the FDIC policy on insurance and regulatory closure also affect bank optimal liability structure. The policy have changed multiple times and may change further in the future. Optimal responses of bank liability structure to policy changes are especially important in understanding the effects of FDIC and other regulations because the responses may counteract the intended objectives of the regulators. In Section 5.2, we re-calculate bank optimal liability structure under various assumptions about bank closure policy (κ) and insurance subsidy (ω).

Another well-known reason for leverage in all firms, not just in banks, is tax deduction of interest expenses. Although the tax benefit of leverage is not a unique reason for banks to be different from other companies, corporate tax is more important for bank liability structure because banks use more leverage than non-financial firms do. Observing the importance of tax benefit to banks, several recent papers have attempted to measure empirically the link between leverage and taxes. These papers include Heckemeyer and de Mooij (2013), Keen (2011), Schepens (2013), and Schandlbauer (2013). In Section 5.3, we look at bank optimal liability structure under alternative hypothetical rates of tax benefit for interest expenses.

5.1 Effects of Bank Business Characteristics

Service income appears to be a driver of bank leverage, as shown in panel A of Table 4. The tangible equity is lower in a bank with higher service income. When the service income rate changes from 2 percent to 4 percent, the tangible equity-to-asset ratio decreases from 16.52% to 0.55%. The optimal leverage increases with the service income because not only the optimal amount of deposits goes up and the optimal debt also goes up. It is interesting to notice that an increase in service income rate does not have a substitution effects between deposits and debt. They both went up because the additional deposits raise the closure boundary, giving more room for debt. As a result, the optimal liability structure consists of more deposits and debt if the service income is higher.

The effects of account service income on bank leverage is consistent with DeAngelo and Stulz (2013), who suggest that premium on liquidity production is a reason for high leverage in banks that take deposits. The banks in their paper do not have FDIC insurance and regulation. The presence of FDIC, however, does not alter the effect of service income on optimal leverage. In panel B of Table 4, we present the optimal liability structure in an uninsured bank and find similar relation between leverage and service income. A major distinction of our analysis from DeAngelo and Stulz's is that their banks are financed by only deposits besides equity. Our model predicts that the optimal debt is also positively related to the service income.

It is important to observe that the credit spread is positively related with the account service income rate. This is true for both FDIC-insured and uninsured banks. The bank regulations during 1930s through 1970s prohibited banks from paying interests on demand and savings deposits and limited bank competition. It was thought that making deposit service more profitable would reduce the probability of bank failure. This way of thinking ignores bank's optimal response to profitability of deposit service.

A. FDIC-Insured Banks

Endogenous optimal ratio	Account service income rate (η)				
	2.00	2.50	3.00	3.50	4.00
Deposit	43.52	44.50	45.30	45.96	46.53
Debt	39.96	43.23	46.48	49.71	52.93
Tangible equity	16.52	12.27	8.23	4.33	0.55
Closure boundary	44.39	45.39	46.20	46.88	47.46
Default boundary	44.39	45.39	46.20	46.88	47.46
Bankruptcy loss	3.52	3.72	3.89	4.04	4.16
Credit spread	2.08	2.18	2.27	2.34	2.40
Insurance premium	0.21	0.22	0.24	0.25	0.26

B. Uninsured Banks

Endogenous optimal ratio	Account service income rate (η)				
	2.00	2.50	3.00	3.50	4.00
Deposit	30.86	31.60	32.22	32.75	33.22
Debt	46.67	49.41	52.07	54.68	57.26
Tangible equity	22.46	19.00	15.71	12.56	9.52
Bank-run boundary	42.28	43.29	44.14	44.87	45.50
Default boundary	42.28	43.29	44.14	44.87	45.50
Bankruptcy loss	3.11	3.30	3.47	3.62	3.74
Credit spread	1.88	1.97	2.05	2.13	2.19

Table 4: Effects of account service income on bank optimal structure. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When η varies, the other parameters are fixed at $\sigma = 0.05$, $\tau = 0.15$, $\omega = 0.9$, and $\alpha = \beta = 0.27$. For FDIC-insured banks, $\kappa = 1.02$, but $\kappa = 1/(1-\alpha)$ for uninsured banks.

Table 4 shows that when deposit service income is higher, bank's optimal response is to take more deposits, as well as debt. This raises the closure boundary, which increases the probability of bank failure. As a result, expected bankruptcy loss is higher, and credit spread is higher, opposite to what the early regulators thought. The comparative statics of account service income effects demonstrates the importance of incorporating banks' optimal response.

Bank asset volatility is typically low, as noted previously, and low asset volatility appears to be an important driver of bank leverage, as shown in panel A of Table 5. If volatility is as low as 3%, the optimal tangible equity is only 6.75%. By contrast, for an asset volatility of 20%, the tangible equity ratio is 23.45%. A bank with 20% asset volatility takes much less deposits, which is 35% of asset value. In view of the inverse relationship between volatility and leverage, a manufacturing company with asset volatility higher than 30% or 40% is unlikely to use the level of leverage that banks use. Thus, low asset volatility is crucial for banks to use high leverage. When a high leverage consists of large amounts of deposits, it allows a bank to engage in

A. FDIC-Insured Banks

Endogenous optimal ratio	Asset volatility (σ) 3.00	4.00	5.00	10.00	20.00
Deposit	46.28	45.84	45.30	41.90	35.00
Debt	46.96	46.74	46.48	44.82	41.54
Tangible equity	6.75	7.42	8.23	13.29	23.45
Closure boundary	47.21	46.75	46.20	42.74	35.70
Default boundary	47.21	46.75	46.20	42.74	35.70
Bankruptcy loss	3.82	3.85	3.89	4.10	4.38
Credit spread	2.14	2.20	2.27	2.76	4.17
Insurance premium	0.23	0.23	0.24	0.27	0.34

B. Uninsured Banks

Endogenous optimal ratio	Asset volatility (σ) 3.00	4.00	5.00	10.00	20.00
Deposit	32.99	32.64	32.22	29.56	24.18
Debt	52.67	52.40	52.07	50.01	45.78
Tangible equity	14.34	14.96	15.71	20.43	30.03
Bank-run boundary	45.20	44.72	44.14	40.49	33.13
Default boundary	45.20	44.72	44.14	40.49	33.13
Bankruptcy loss	3.41	3.44	3.47	3.64	3.84
Credit spread	1.94	1.99	2.05	2.49	3.76

Table 5: Effects of asset volatility on bank optimal liability structure. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When σ varies, the other parameters are fixed at $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.9$, and $\alpha = \beta = 0.27$. For FDIC-insured banks, $\kappa = 1.02$, but $\kappa = 1/(1-\alpha)$ for uninsured banks.

service on deposits as its major business. Therefore, low asset volatility is important for banking business.

Without the FDIC insurance and regulation, asset volatility is still an important a driver of leverage. As shown in Panel B of Table 5, the optimal leverage is sensitive to asset volatility in uninsured banks. If asset volatility is as low as 3%, the tangible equity goes down to 14.34%. If asset volatility is 20%, the tangible equity ratio goes up to 30.03%. The effect of asset volatility on the composition of leverage, as shown in panel B, is similar to the effect in panel A, although the leverage in uninsured banks is generally lower. As asset volatility increases, the proportion of deposits decreases and the proportion of debt increases. The reduction of deposits is larger than the increase of debt, leading to lower leverage in the bank. Thus, the FDIC insurance does not diminish the role of asset volatility in bank leverage.

In the above analysis, we assume that FDIC's liquidation costs of assets is same as the cost of liquidation under bankruptcy procedures. That is, $\beta = \alpha$. This is inconsistent with the views that the FDIC can lower liquidation costs. As noted in Section 2.2,

A. FDIC-Insured Banks

Endogenous optimal ratio	Cost of FDIC Liquidation (β) 17.00	22.00	27.00	32.00	37.00
Deposit	47.87	46.54	45.30	44.14	43.05
Debt	48.24	47.34	46.48	45.64	44.84
Tangible equity	3.90	6.12	8.23	10.22	12.11
Closure boundary	48.83	47.47	46.20	45.02	43.91
Default boundary	48.83	47.47	46.20	45.02	43.91
Bankruptcy loss	2.81	3.39	3.89	4.32	4.69
Credit spread	2.56	2.41	2.27	2.14	2.03
Insurance premium	0.17	0.21	0.24	0.26	0.28

B. Uninsured Banks

Endogenous optimal ratio	Cost of Bankruptcy Procedure (α) 17.00	22.00	27.00	32.00	37.00
Deposit	39.66	35.83	32.22	28.81	25.60
Debt	52.14	52.18	52.07	51.83	51.46
Tangible equity	91.80	88.01	84.29	80.64	77.06
Bank-run boundary	47.78	45.94	44.14	42.37	40.64
Default boundary	47.78	45.94	44.14	42.37	40.64
Bankruptcy loss	2.67	3.13	3.47	3.71	3.86
Credit spread	2.44	2.24	2.05	1.88	1.73

Table 6: Effects of liquidation or bankruptcy costs on bank optimal liability structure. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When β varies in panel A, the other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\kappa = 1.02$, $\omega = 0.9$, and $\alpha = 0.27$. When α varies in panel B, the other relevant parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, and κ always equals $1/(1-\alpha)$.

the U.S. lawmakers believe that the FDIC can implement a more efficient and orderly liquidation that protects bank value and thus grant the "orderly liquidation authority" to the FDIC by Title II of the Dodd-Frank Act. On the other hand, some suspect the FDIC has no incentive to maximize liquidation value and thus may result in larger loss of asset value (James, 1991). The views of liquidation cost may further affect the leverage of banks. Panel A of Table 6 demonstrates the effects of FDIC liquidation cost on bank optimal liability structure. We vary the FDIC liquidation cost β in a range from 17% to 37% but keep the private sector bankruptcy cost α at 27%. Both deposits and debt vary inversely with FDIC liquidation cost. The tangible equity ratio is lower if the FDIC liquidation costs less. The higher leverage has a consequence in credit spread. Without considering the optimal response of banks, one would expect credit spread to be lower if liquidation cost is lower. To the contrary, credit spread goes up slightly. The apparently counterintuitive change of credit spread is due to the optimal increase of leverage.

In an uninsured bank, when the liability structure optimally responds to the cost of bankruptcy procedure, the relation between bankruptcy boundary and bankruptcy cost is different from what we expect in fixed liability structure. Mathematically, $V_a = D/(1-\alpha)$ is an increasing function of α if we fix D at a constant, but $V_a^* = D^*/(1-\alpha)$ is a decreasing function of α because the optimal amount of deposits D^* also depends on α. Panel B of Table 6 demonstrates that the bank-run boundary is lowered from 47.78% to 40.64% of asset value when α rises from 17% to 37%. As a result of the optimal response, the credit spread in the optimal structure drops from 244 to 173 bps, as the bank cuts deposits down from 39.66% to 25.60% of asset value. Without considering the bank's optimal response, one would expect the credit spread to rise when bankruptcy cost goes up. Meanwhile, the bank adjusts debt only slightly down from 52.14% to 51.46%. The irresponsiveness of debt to the increase of bankruptcy cost would have been difficult to understand without considering banks' optimal responses.

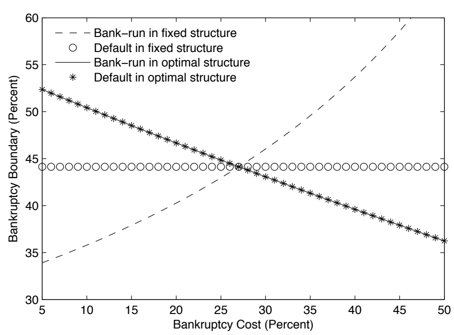

Figure 3: Bankruptcy boundaries of uninsured banks. A bank first optimizes its liability structure for $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, and $\alpha = 0.27$. When bankruptcy cost α varies in the range from 5% to 50%, the capital structure is either kept fixed or re-optimized for new α. The panel plots the bank-run boundary V_a/V and default boundary V_d/V for each value of α in the fixed or re-optimized liability structure.

This relation between bank-run boundary and bankruptcy cost is illustrated in Figure 3. In the figure, we first assume that the uninsured bank optimizes its liability structure and then keeps it fixed when we alter bankruptcy cost α. Since the liability structure is fixed, the default boundary (as marked by circles) is independent of α in the figure. Nevertheless, a bank's optimal response completely changes the relation between the bank-run boundary and bankruptcy cost. When the bank optimally responds to the increase in bankruptcy cost, it reduces deposits, resulting a inverse relation between V_a^* and α (plotted as the solid line). Since the endogenous default boundary is the same as the bank-run boundary, the default boundary also decreases as α increases.

5.2 Effects of Closure Rule and Insurance Subsidy

We first consider the potential effects of changing the regulatory closure rule. In fact, the Dodd-Frank Act requires bank regulators to consider increasing the threshold for banks to be closed. In the meantime, Basel III and new rules in the U.S. increased the capital requirement for banks to operate. Recall that the closure rule in our model may be interpreted as a capital requirement, which is the minimum capital for a bank to operate, as modeled in Rochet (2008). Then, an increase in κ can be interpreted as a raise in capital requirement.

In Table 7, we vary κ from 100% to 130%. Tightening the closure rule reduces bank leverage. As κ changes from 100% to 130%, optimal tangible equity increases from 7.66% of asset value to 14.50%. The reduction of leverage is due to the drop in deposits: the deposit-to-asset ratio drops from 46.35% to 34.23%. Therefore, tightening of closure rule causes banks to reduce deposit services. Instead, it forces banks to increase debt from 45.99% to 51.27% of the asset value.

Endogenous	Regulatory Closure Rule (κ)				
optimal ratio	100.00	102.00	110.00	120.00	130.00
Deposit	46.35	45.30	41.52	37.55	34.23
Debt	45.99	46.48	48.18	49.90	51.27
Tangible equity	7.66	8.23	10.30	12.55	14.50
Closure boundary	46.35	46.20	45.67	45.06	44.50
Default boundary	46.35	46.20	45.67	45.06	44.50
Bankruptcy loss	3.92	3.89	3.78	3.65	3.54
Credit spread	2.28	2.27	2.21	2.15	2.09
Insurance premium	0.26	0.24	0.16	0.09	0.03

Table 7: Effects of regulatory closure rule on bank optimal liability structure. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When κ varies, the other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.9$, and $\alpha = \beta = 0.27$.

Perhaps a striking observation in Table 7 is that the closure boundary in optimal liability structure of a bank actually decreases if the charter authority tightens the closure rule. As κ increases from 102% to 110%, the closure boundary gets lower, dropping from 46.20% to 45.67% of asset value. This reverse relation between the closure boundary and closure rule is due to the optimal response of bank liability structure. For a higher κ, banks take fewer deposits. The drop of deposits is so large that it entirely counteracts the increase of κ.

The influence of optimal response of an FDIC-insured bank on the relation between the closure boundary and the closure rule can be seen in Figure 4. In the figure, an insured bank first optimizes its liability structure for $\kappa = 102\%$, and then κ changes. If the bank does not adjust the liability structure for the changes of κ, the closure boundary, $V_a = \kappa D$, should be a linear function of κ plotted as dashed line in the figure,

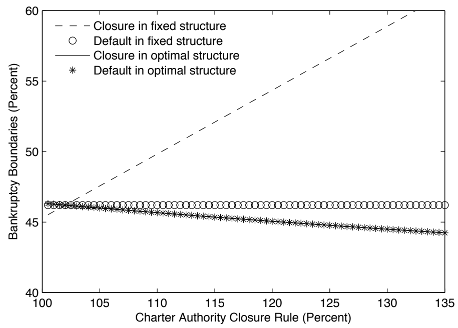

Figure 4: Bankruptcy boundaries of FDIC-insured banks. A bank first optimizes its liability structure for $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.9$, $\alpha = \beta = 0.27$, and $\kappa = 1.02$. When the closure rule κ varies in the range from 100% to 135%, the capital structure is either kept fixed or re-optimized for new κ. The panel plots the closure boundary V_a/V and default boundary V_d for each value of κ in the fixed or re-optimized liability structure.

and the default boundary V_d should be independent of κ, as marked by circles in the figure. If the bank optimally responds to the change of κ, however, the relation between closure boundary and κ is completely different; the closure boundary decreases as κ increases. The optimal closure boundary $V_a^* = \kappa D^*$ is not a linear function of κ anymore because the bank optimally reduces deposits in response to the increase of κ. The endogenous default boundary V_d^* is related to κ the same way as V_a^* because $V_d^* = V_a^*$ for all κ, as shown in the figure.

Endogenous	Insurance Subsidy $(1-\omega)$				
optimal ratio	0.00	10.00	20.00	30.00	40.00
Deposit	44.64	45.30	45.98	46.68	47.42
Debt	46.01	46.48	46.95	47.44	47.94
Tangible equity	9.35	8.23	7.07	5.88	4.65
Closure boundary	45.54	46.20	46.90	47.62	48.37
Default boundary	45.54	46.20	46.90	47.62	48.37
Bankruptcy loss	3.75	3.89	4.04	4.20	4.36
Credit spread	2.20	2.27	2.34	2.42	2.51
Insurance premium	0.25	0.24	0.22	0.20	0.18

Table 8: Effects of insurance on bank optimal liability structure. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When $1-\omega$ varies, other exogenous parameters are fixed: $\sigma = 0.05$, $\eta = 0.03$, $\tau = 0.15$, $\kappa = 1.02$, and $\alpha = \beta = 0.27$.

In the preceding analysis, we assume that the subsidy in FDIC insurance premium is $1-\omega = 10\%$. Under this assumption, the insurance premium of the FDIC-insured bank in Table 2 is 24 bps on assets. In Table 8, we consider various magnitudes of the subsidy, ranging from zero to 40%, and look at its impact on the optimal liability of insured banks. Clearly, FDIC subsidy encourages bank leverage. As the subsidy increases from zero to 40%, tangible equity ratio drops from 9.35% to 4.65%. More interestingly, the subsidy not only increases the optimal deposit-to-asset ratio but also increases the optimal debt ratio. While the deposit ratio goes up from 44.64% to

47.42%, the debt ratio rises from 46.01% to 47.94%. In the meantime, both closure boundary and credit spread are higher. As expected, though, the insurance premium drops as the insurance is more subsidized. These effects of FDIC subsidy appear to support the FDIC reforms in its effort to charge banks fair insurance premium.

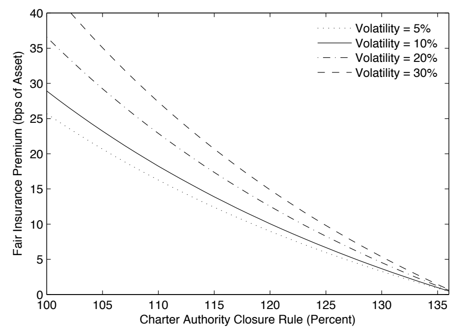

Figure 5: Insurance premium, closure rule, and volatility. An insured bank optimizes its liability structure for each κ, ranging from 100% to 135%, and each σ, taking values of 5%, 10%, 15%, and 25%. The figure plots the fair insurance premium in the optimal liability structure for various values of κ and σ. The other exogenous parameters are fixed at $\eta = 0.03$, $\tau = 0.15$, $\omega = 0.9$, and $\alpha = \beta = 0.27$.

The fair insurance premium depends on how risky the bank assets are and how early the charter authority closes the bank, besides the liability structure of the bank. Figure 5 shows how the insurance premium for the optimal structure depends on its two important factors: the closure rule and asset volatility. The range of insurance premium in the figure is broadly in line with the assessment rates published by the FDIC. The Federal Deposit Insurance Corporation (2011) reports that the initial base assessment rates are 12–16, 22, 32, 45 bps for banks in four risk categories. The insurance premium is a cost to the bank, but the insured bank is able to benefit from FDIC insurance because of its optimal response, which in turn reduces the effect of FDIC on the bank's probability of bankruptcy. The endogenous relation between insurance premium and liability structure has been overlooked in the literature.

5.3 Effects of Corporate Tax Benefits

Our model offers a coherent framework for the link between tax rate and leverage. According to Goldstein et al. (2001), the asset value V in Leland's (1994) model, and also in our model, is the value of an all-equity firm that owns the assets and faces corporate tax. In other words, V is the after-tax value of assets. If the before-tax value of assets is V^*, then the after-tax value is $V = (1 - \tau)V^*$. If the government lowers corporate tax rate from τ to τ', the after-tax value of assets should be higher. Let V' be the after-tax value under corporate tax τ'. Then, the after-tax values of assets in the two tax regimes are related by $V' = V(1 - \tau')/(1 - \tau)$. Incorporating the effect of a tax change on asset value is necessary when analyzing the relation between tax rate and capital structure, as Goldstein et al. point out. Otherwise, one would erroneously conclude that government can increase firm value by raising corporate tax rate.

Table 9 shows the optimal response of bank liability structure to changes of corporate tax policy. If corporate tax rate is lowered from 15% to 10%, the bank reduces the ratios of both deposit and debt to assets. The ratio of deposit to asset drops by less than 1%, from 45.30% to 44.40%. By contrast, the ratio of debt to assets reduces by nearly 3.4 percentage points. Since debt does not bring service income, the reduction of debt is larger than the reduction of deposits. Overall, banks use less leverage in a regime of lower tax rate. In Table 9, tangible equity is up by nearly 4.5 percentage points when the rate of tax benefit lowers from 15% to 10%. Because the leverage drops, the expected bankruptcy loss and the credit spread both drop when corporate tax rate is lowered.

Although the association of lower leverage with lower tax rate appears to support those who argue for lowering corporate tax rate in order to stabilize banks, we need to be cautious about this policy proposal. If lowering corporate tax rate leads to a loss in tax revenue, then it may be an expensive policy change for the public to achieve a greater stability of banking industry. An alternative is to lower corporate tax rate just for banks as suggested by Fleischer (2013). Lowering tax for banks, not for other firms, will make banking a subsidized business, begging the question of fairness of corporate tax policy and the question of distortions in the economy. These important issues are beyond the scope of this paper, but the link between the tax rate and bank leverage in our model lays a stepping stone for a welfare analysis of the benefit and cost of tax policy reforms. As noted earlier, Schepens (2014), who exploits a recent tax code change permitting some tax advantages to equity, shows that banks do respond to changes in the tax code by altering their liability structure. Collectively, the current research suggests that a welfare analysis of tax policy must not ignore banks' optimal responses.

The effect of tax on bank leverage is limited. The first column of Table 9 shows the optimal liability structure of the bank that enjoys only 1% tax benefit on debt liability. Even with such low tax benefit, it is still optimal for the bank to use substantial leverage. The ratio of deposits to assets is still above 42%, and the ratio of debt to assets is about 37%. With these levels of deposits and debt, the bank's optimal tangible equity is about 20%. In fact, as the tax rate approaches to zero, the optimal deposit level is still about 42% of asset value, and the optimal debt-to-asset ratio approaches a limit above 36%. These numerical results indicate the limitation of tax policy as incentives for banks to

A. FDIC-Insured Banks

Endogenous	Alternative corporate tax benefits				
optimal ratio	1.00	5.00	10.00	15.00	20.00
Deposit	42.54	43.41	44.40	45.30	46.11
Debt	37.24	39.63	42.87	46.48	50.50
Tangible equity	20.22	16.96	12.72	8.23	3.38
Closure boundary	43.39	44.28	45.29	46.20	47.03
Default boundary	43.39	44.28	45.29	46.20	47.03
Bankruptcy loss	3.32	3.50	3.70	3.89	4.07
Credit spread	1.98	2.07	2.17	2.27	2.36
Insurance premium	0.19	0.21	0.22	0.24	0.25

B. Uninsured Banks

Endogenous	Alternative corporate tax benefits				
optimal ratio	1.00	5.00	10.00	15.00	20.00
Deposit	29.14	30.15	31.25	32.22	33.07
Debt	41.53	44.36	48.08	52.07	56.43
Tangible equity	29.33	25.49	20.67	15.71	10.50
Bank-run boundary	39.91	41.30	42.81	44.14	45.31
Default boundary	39.91	41.30	42.81	44.14	45.31
Bankruptcy loss	2.70	2.94	3.21	3.47	3.70
Credit spread	1.67	1.79	1.93	2.05	2.17

Table 9: Effects of corporate tax policy. The definitions of endogenous variables are given in the second column of Table 2. All values are reported in percentage points. When τ varies, other exogenous parameters are fixed at $\sigma = 0.05$, $\eta = 0.03$, $\omega = 0.9$, and $\alpha = \beta = 0.27$. For FDIC-insured banks, $\kappa = 1.02$, but $\kappa = 1/(1-\alpha)$ for uninsured banks.

lower leverage.

Calomiris and Carlson (2015) report that tangible equity ratio in the national banks in the 1890s, when there was no corporate tax or FDIC insurance, ranges from 8 to 76 percent, and most of the leverage in those banks are in the form of deposits. Since the parameters used in Table 9 are based on relatively recent historical observations, it is difficult to compare the numbers in the table to the empirical results reported by Calomiris and Carlson. Nonetheless, our model's implication that substantial leverage is optimal for value-maximizing banks in the absence of substantial tax benefit of debt is qualitatively consistent with their empirical findings.

6 Conclusion

Our theory of bank liability structure explicitly models an array of factors that determine the optimal leverage ratios and liability composition of banks. The factors include tax deductibility of interest expenses, service income from deposits, FDIC insurance, and the risk of bank closure. Since our model is structural and solved analytically, it provides a convenient setting for quantifying bank capital structure and designing practical capital strategies.[28] A salient feature of our model is the interaction between deposits and debt: a bank takes as much debt as possible to benefit from tax deduction but not as much to offer extra protection for deposits. Optimal liability structure of any bank is likely to have the features derived in this paper because the choice of leverage ratios and liability composition should be made relative to the optimal choice of bank assets.[29]

The literature pertinent to our theory falls under three categories: bank run, bank leverage, and capital structure theory. The bank run literature was pioneered by Diamond and Dybvig (1984), who construct a model in which bank run emerges as an equilibrium. The literature has subsequently been extended significantly by Allen and Gale (1998) and others. In our model, depositors at uninsured banks can run in order to make their claims risk-free. There is no panic risk in our full-information model as depositors observe asset value and can implement an optimal withdrawal strategy without loss. The model does not have a situation in which a fraction of the depositors withdraw deposits late and suffer losses. Nevertheless, bankruptcy costs arising from a bank run force banks to decide the right amount of deposits and debt to hold.[30] The FDIC enables the economy to avoid the equilibrium with bank run and replaces it with an equilibrium with deposit insurance and regulatory closure.

Our theory contributes to the literature of bank leverage. A number of papers have studied the reason for high leverage of banks, going back to Buser, Chen and Kane (1981), who conceptually, not quantitatively, discuss banks that optimize deposits in the presence of FDIC. Song and Thakor (2007) examine banks' choice between deposits and non-deposits as financing sources in a discrete two-period model, not a structural model. Focusing on the liquidity service on deposits, excluding debt and assuming that banks can hedge asset risk to zero, DeAngelo and Stulz (2013) provide a rationale for bank leverage by positing that households and firms value a hedge against liquidity

[28]Models with a few time steps and discrete asset value are common in banking studies. Those models are useful for certain conceptual issues, but not for the issues investigated in this paper or for practical applications.

[29]As in Leland (1994), our paper does not have a complete model of capital structure that includes the optimal choice of assets.

[30]Auh and Sundaresan (2013) consider repo run in a similar setting, in which repo investors have full information. In their model, the repo borrower chooses the right level of unsecured debt, taking cognizance of a run by repo cash lenders. But they do not model FDIC insurance, charter authorities' closure decisions, and the resulting feedback effects, which are central to our analysis.

shocks and are willing to pay a premium. Garnall and Strebulaev (2013) posit that high leverage of banks arises from low volatility of bank assets due to diversification. Their banks pay no premium on deposit insurance and have an exogenously-given mix of deposits and long-term debt. Allen and Carletti (2013) focus on banks holding only deposits and equity. They assume deposits are cheaper than equity as a financing source because they are traded in segmented markets. Our work goes beyond these insights to theorize the endogenous decision of leverage, which combines deposits and debt optimally, for banks that face the risk of bank-run or regulatory closure and pay fair premium if they have FDIC insurance.

Our model is an extension to the structural framework of Merton (1974, 1977) and Leland (1994). Unprotected long-term debt and secured debt are considered by Leland (1994), who interprets secured debt as rolling short-term debt. Leland considers each class of debt separately, whereas we model the endogenous choice of deposits and long-term debt simultaneously. Rochet (2008) applies Merton's model and Leland's concept of endogenous default to the monitoring and regulation problem of banks. Unlike our model, Rochet's analysis takes bank deposit or debt as given, setting aside the question of optimal response of bank liability structure to the changes of regulatory environment. Harding, Liang and Ross (2009) set out a structural model for banks whose leverage consists of only deposits, treating banks as firms in Leland (1994) and ignoring the special features of banks. We model the endogenous bank decision on the choice of deposits and debt as well as the the overall bank leverage. Hugonnier and Morellec (2015) consider a dynamic model with costly equity issuance to motivate a role for liquidity reserves. But their paper abstracts from issues of endogenous FDIC insurance premium, its effects on optimal liability structure, and the interactions between the run boundary of depositors and the optimal default of debt. These issues are central to our paper.

In addition, we explicitly model deposit insurance and closure policy of charter authorities. In a setting with these detailed institutional features, we derive the endogenous FDIC insurance premium, taking into account the optimal liability structure. While a number of papers, notably Merton (1977), Ronn and Verma (1986) have derived risk-adjusted FDIC insurance policies, our work extends their insights to the endogenous decisions on default and closure boundaries when banks optimally respond to FDIC insurance.

The theory of bank liability structure has important implications to bank regulation. Since the financial crisis of 2007–2009, regulators have decided to raise capital requirement for banks. Basel III lifts the required equity ratio from 4%, which was set in Basel II, to 7% for all banks and to nearly 10% for large banks that are designated as systemically important.[31] As we have noted in footnote 1, European and U.S. regulators have laid out higher capital requirements for banks, and academics have proposed raising equity ratio to as high as 20%. Two important questions for regulators are: how will banks adjust their liability structure in response to higher equity ratio requirement, and what are the potential unintended consequences? It is important to know whether a value-maximizing bank cuts down deposits or debt, or both, when reducing leverage to meet a capital requirement.[32] Cutting back deposits reduces banking service, and lowering debt shrinks an important funding source for banks. If we interpret capital requirement as the minimum capital for a bank to operate without being closed, our results show that banks respond to an increase in capital requirement by lowering deposits and increasing debt.

The model of bank liability structure provides a tool for studying other securities held by or proposed to banks. Perhaps the most controversial security since the crisis of 2007–2009 is contingent capital, which converts to equity when the bank becomes under-capitalized, as proposed by Flannery (2009). Sundaresan and Wang (2010) analyze the intricate issues in designing contingent capital. Albul, Dwight and Tchistyi (2010) and Chen, Glasserman and Nouri (2012) add contingent capital to capital structure of firms but not to the capital structure of banks. Extension of our model to include reverse convertible debt as in Sundaresan and Wang (2014) shed light on the interactions of reverse convertible debt with deposits, debt, bank run, and the FDIC. These interactions are critical for figuring out whether convertible debt helps stabilize banks.

Our model can be extended to a setting in which banks dynamically change the liability structure as well as its composition of assets. This is especially important as banks, which are able to borrow below the risk-free rate, can dynamically finance the purchases of risk-free assets by issuing deposits and potentially avoid a bank run. In reality, this prospect is much less likely as the supply of deposits is finite and in a depressed economy, the risk-free rate itself can be very low, limiting the ability of banks to gain from this channel. Goldstein et al. (2001) have developed a dynamic framework for a corporate borrower. In their model, firms with a single debt consider the opportunity of issuing additional debt when they optimize today's capital structure. In contrast,

[31] Hirtle (2011a, 2011b) explains the rationale for these capital requirements.

[32] Subramanian and Yang (2013) consider the question of prudential regulation in a structural model, in which firms issue only perpetual debt and do not take deposits, as in Leland's (1994a) model.

an important issue for banks is the ability to reduce leverage when it becomes poorly capitalized after losses. Another important issue for banks is their dynamic adjustment of their asset structure in response to changes of risk.[33] Extension of our model to a dynamic setting will be a useful, although challenging, project that warrants further research.

A Appendix

Following Merton (1974) and Leland (1994), we assume bank asset value follows a geometric Brownian motion. In the risk-neutral probability measure, the stochastic process of asset value is

$$dV = (r-\delta)V\,dt + \sigma V\,dW\,. \tag{20}$$

where r is the risk-free interest rate, δ is the rate of cash flow, σ is the volatility of asset value, and W is a Wiener process.[34] Following Leland (1994), V denotes the after-tax value of assets, and thus δV is the after-tax cash flow.[35]

For a given bankruptcy boundary V_b, consider a security that pays one dollar if and only if V hits V_b for the first time. The price of this security, denoted by P_b, is referred to as the state price of V_b. According to Merton (1974), P_b satisfies a differential equation:

$$\frac{1}{2}\sigma^2 V^2 P_b'' + (r-\delta)V P_b' - r P_b = 0\,, \tag{21}$$

where P_b' and P_b'' are the first and second partial derivatives of P_b with respect to asset value V. The general solution to the equation is $P_b = a_1 V^{-\lambda} + a_2 V^{-\lambda'}$, where $\lambda > 0$ and $\lambda' < 0$ are the two roots of quadratic equation

$$\frac{1}{2}\sigma^2 \lambda(1+\lambda) - (r-\delta)\lambda - r = 0\,. \tag{22}$$

The boundary conditions are $P_b(V_b) = 1$ and $\lim_{V\to\infty} P_b(V) = 0$. The conditions imply $a_2 = 0$ and $a_1 = V_b^{\lambda}$, which give $P_b = (V_b/V)^{\lambda}$.

[33] Adrian and Shin (2010) document that financial intermediaries adjust balance sheets to their forecast of risk.

[34] Notice that cash flow δV also follows a geometric Brownian motion with volatility σ. One may start with the assumption that asset cash flow follows a geometric Brownian motion with volatility σ and then show that asset value follows process (20).

[35] Alternatively, one may specify the before-tax value of the assets, V^*, as in Goldstein et al. (2001). Then, δV^* is earnings before interests and tax (EBIT), and the after-tax asset value is $V = (1-\tau)V^*$. Recovery value of bankruptcy is then $(1-\phi)(1-\tau)V_b^*$, which is equivalent to $(1-\phi)V$. All the results in this paper can be derived and presented accordingly.

Equity holders earn dividend, $\delta V - (1-\tau)(I + C + C_1)$, until bankruptcy, for given insurance premium I, deposit liability C, and debt liability C_1. The pricing equation of equity value before bankruptcy is

$$\frac{1}{2}\sigma^2 V^2 E'' + (r-\delta)VE' - rE + \delta V - (1-\tau)(I + C + C_1) = 0\,, \tag{23}$$

where E' and E'' are partial derivatives respect to asset value V. We assume $I \geq 0$ in general, but setting $I = 0$ gives the valuation without FDIC. There are two boundary conditions. Since bankruptcy wipes out equity, we have $E(V_b) = 0$. If $V \to \infty$, bankruptcy is remote, and $E(V)$ approximately equals to $V - (1-\tau)(I + C + C_1)/r$.

The pricing equation of debt D_1 is

$$\frac{1}{2}\sigma^2 V^2 D_1'' + (r-\delta)VD_1' - rD_1 + C_1 = 0\,, \tag{24}$$

where D_1' and D_1'' are partial derivatives with respect to V. There are also two boundary conditions. Debt holder receives $\big[(1-\phi)V_b - D\big]^+$ at bankruptcy. If $V \to \infty$, the debt almost risk-free, and D_1 approaches C_1/r.

Theorem 1 in Section 3.1 presents the solutions to equations (23) and (24) and their boundary conditions. The solutions can be derived similarly to those in Leland (1994) and Goldstein et al. (2001). Section A.1 provides the details.

To simplify the derivation of optimal liability structure, we introduce the following notations:

$$x = C_1/C, \quad c = C/(rV), \quad v_a = rV_a/C, \quad v_d = rV_d/C, \quad v_b = rV_b/C \tag{25}$$

$$\iota = \eta/(r-\eta), \quad \theta = (1-\tau)\lambda/(1+\lambda)\,. \tag{26}$$

We refer to x as the liability ratio and c as the deposit liability scaled by asset. The state price of bankruptcy is then simplified to $P_b = (v_b c)^\lambda$. By Theorem 1, the scaled boundaries are

$$v_a = \kappa(1+\iota) \quad v_d = \theta(1+i+x) \quad v_b = \max\{\kappa(1+\iota)\,,\, \theta(1+i+x)\}\,. \tag{27}$$

Notice that $V_a < V_d$ if and only if $v_a < v_d$. Furthermore, equation (9) can be written as a function of c

$$i = \omega[1-(1-\beta)\kappa]^+(1+\iota)(v_a c)^\lambda/[1-(v_a c)^\lambda]\,. \tag{28}$$

We can also express the bank value in Theorem 1 as a ratio to asset value:

$$\begin{aligned} f(x,c) = F/V = 1 &+ [1-(v_b c)^\lambda][\iota + \tau(1+i+x) - i]c \\ &- (v_b c)^\lambda \min\{\phi v_b\,,\, v_b - (1+\iota)\}c\,. \end{aligned} \tag{29}$$

Choosing (C, C_1) to maximize bank value F is equivalent to choosing the duplet (x, c) to maximize f. Once we obtain the optimal (x^*, c^*), the optimal liabilities (C^*, C_1^*) can be obtained easily as $C^* = c^* r V$ and $C_1^* = x^* C^*$.

A.1 Proof of Theorem 1

The general solution to pricing equation (23) is

$$E = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + V - (1-\tau)(I + C + C_1)/r \tag{30}$$

where $\lambda > 0$ and $\lambda' < 0$ are the two solutions to equation (22), and a_1 and a_2 are arbitrary constants. The boundary conditions of E imply $a_2 = 0$ and $a_1 = -[V_b - (1-\tau)(I + C + C_1)/r]V_b^{\lambda}$, which give equation (4).

If $V_b = V_a = \kappa D$, then $C = (r-\eta)D$ gives equation (1). To prove V_d in equation (2) is the endogenous default boundary, we need to show that V_b maximizes the equity value when $V_b = V_d$. Differentiation of equation (4) with respect to V_b leads to

$$\partial E/\partial V_b = [(1+\lambda)/V_b](V_b/V)^{\lambda}\left(V_d - V_b\right). \tag{31}$$

Since the above is positive if $V_b < V_d$ and negative if $V_b > V_d$, we know $V_b = V_d$ maximizes the equity value. Notice that V_d is independent of V. Equity holders choose to default before bank-run or regulatory closure if V drops to V_d before V_a. The bank fails before default if V drops to V_a first. Therefore, the bankruptcy boundary is $V_b = \max\{V_a, V_d\}$.

The general form of solution to pricing equation (24) is

$$D_1 = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + C_1/r, \tag{32}$$

where a_1 and a_2 can be any constants. The boundary conditions of D_1 imply $a_2 = 0$ and $a_1 = \left\{[(1-\phi)V_b - D]^+ - C_1/r\right\}V_b^{\lambda}$, which give equation (3).

Bank value is $F = D + D_1 + E$. We obtain equation (5) by substituting equations (3) and (4), and using $D = C/(r-\eta)$.

A.2 Proof of Theorem 2

Let Q be the value of deposit insurance to banks. Its pricing equation is

$$\frac{1}{2}\sigma^2 V^2 Q'' + (r-\delta)VQ' - rQ - I = 0, \tag{33}$$

where Q' and Q'' denote the first and second partial derivatives of Q with respect to V. The general solution to the equation is $Q(V) = -I/r + a_1 V + a_2 V^{-\lambda}$, where a_1 and a_2 can be any constants. The boundary conditions of the value of the insurance product are $\lim_{V\to\infty} Q = -I/r$ and $Q(V_a) = [D-(1-\beta)V_a]^+$, where $V_a = \kappa D$. They imply $a_1 = 0$ and $-I/r + a_2 V_a^{-\lambda} = [D-(1-\beta)V_a]^+$, which give $Q(V) = -(1-P_a)I/r + [D-(1-\beta)V_a]^+ P_a$, where $P_a = [V_a/V]^\lambda$. The insurance premium I° is fair iff $Q(V) = 0$. It follows that $(1-P_a)I^\circ = r[D-(1-\beta)V_a]^+ P_a$. We obtain equation (6) by substituting $V_a = \kappa D$ and factoring D out.

A.3 Proof of Theorem 3

Since $i = 0$, $\phi = \alpha$, and $\kappa \geq 1/(1-\alpha)$, we have $v_b = \max\{(1+\iota)\kappa\, ,\, \theta(1+x)\}$. It follows that $\phi v_b \leq v_b - (1+\iota)$ and simplifies equation (29) to

$$f = 1 + \Big\{\iota + \tau(1+x) - (v_b c)^\lambda [\iota + \tau(1+x) + \alpha v_b]\Big\} c \tag{34}$$

Then, the first-order condition for c to be optimal is

$$\iota + \tau(1+x) - (1+\lambda)\Big[\iota + \tau(1+x) + \alpha v_b\Big](v_b c)^\lambda = 0\,. \tag{35}$$

Let $x^* = (1+\iota)\kappa/\theta - 1$. Notice that $v_b = \theta(1+x)$ if $x \geq x^*$, and $v_b = (1+\iota)\kappa$ otherwise. If $x < x^*$, then $f'_x(x,c) = \tau[1-(v_b c)^\lambda]c > 0$, which means f increases with x. If $x > x^*$, then

$$f'_x = \Big\{\tau - \big[\lambda\iota/(1+x) + (1+\lambda)(\tau + \alpha\theta)\big](v_b c)^\lambda\Big\} c\,. \tag{36}$$

Let c_x be the c that satisfies condition (35) for given $x > x^*$. Imposing the condition in equation (36) gives

$$f'_x(x, c_x) = -\frac{\iota}{1+x}[1-(v_b c_x)^\lambda]c_x < 0\,. \tag{37}$$

Thus, f decreases with x for $x > x^*$ if c is always kept optimal relative to x.

Therefore, $x^* = (1+\iota)\kappa/\theta - 1$ is the optimal point for x. At the optimal point, $v_b^* = v_d^* = v_a = (1+\iota)\kappa$. We can solve $(v_b^* c^*)^\lambda$ from equation (35). Substituting out $1+x^* = (1+\iota)/\theta$ and $v_b^* = (1+\iota)\kappa$, we obtain $(v_b^* c^*)^\lambda = \pi$, where π

$$\pi = \frac{1}{1+\lambda} \cdot \frac{\iota\theta + \tau(1+\iota)\kappa}{\iota\theta + \tau(1+\iota)\kappa + (1+\iota)\kappa\alpha\theta}\,, \tag{38}$$

From $(v_b^* c^*)^\lambda = \pi$, we obtain $c^* = \pi^{1/\lambda}/[(1+\iota)\kappa]$. We arrive at equations (12)–(13) after replacing ι, θ, and the scaled variables in (25) by the original parameters and variable.

A.4 Proof of Theorem 4

Setting $\tau = 0$, $I = 0$, and $C_1 = 0$ in Theorem 1, we obtain $V_a = \kappa C/(r-\eta)$, $V_d = [\lambda/(1+\lambda)]C/r$. Observing that $\lambda/(1+\lambda) < 1$ and $\kappa[r/(r-\eta)] > 1$, we know $V_b = \max\{V_a, V_d\} = V_a$. Also from Theorem 1 we obtain the equity value as $E = V - (1 - P_a)C/r - P_a V_a$, where $P_a = [V_a/V]^\lambda$. Thus, the bank value is

$$F = D + E = V + [\eta/(r-\eta)]C/r + (1-v_a)P_a C/r\,, \tag{39}$$

where $v_a = \kappa r/(r-\eta)$. Dividing the above by V and letting $c = C/(rV)$ and $f = F/V$, we have

$$f = 1 + [\eta/(r-\eta)]c + (1-v_a)(v_a c)^\lambda c\,. \tag{40}$$

The first order condition for choosing c to maximize f is

$$f'_c = \eta/(r-\eta) + (1+\lambda)(1-v_a)(v_a c)^\lambda = 0\,, \tag{41}$$

which gives

$$P_a^* = (v_b c)^\lambda = \frac{1}{1+\lambda}\frac{\eta/(r-\eta)}{v_a - 1}\,. \tag{42}$$

Then, we obtain equation (14) by substituting $v_a = \kappa r/(r-\eta)$. We obtain equation (15) by solving C^* from $P_a^* = [(v_a C^*/r)/V]^\lambda$.

A.5 Proof of Theorem 5

We first show that $v_d < v_a$ is not optimal. If $v_d < v_a$, then $\theta(1+i+x) < (1+\iota)\kappa$, $v_b = v_a = (1+\iota)\kappa$, and

$$f(x,c) = 1 + c\Big\{[\iota - i + \tau(1+i+x)][1-(v_a c)^\lambda] - (\kappa-1)(1+\iota)(v_a c)^\lambda\Big\}\,. \tag{43}$$

We then obtain $f'_x(x,c) = \tau[1-(v_a c)^\lambda]c > 0$, which implies the current x is not optimal.

It follows from equation (28) that $i'_c = \partial i/\partial c = \lambda i c^{-1}/[1-(v_a c)^\lambda]$. Since $\kappa < 1/(1-\beta)$, both i and i'_c are positive. We also have $c i'_c[1-(v_b c)^\lambda] \le \lambda i$ because $v_b \ge v_a$. The equality, $c i'_c[1-(v_b c)^\lambda] = \lambda i$ holds when $v_b = v_a$. Both i and i'_c converge to zero when κ rises to $1/(1-\beta)$ while other parameters and variables are fixed. Thus, given any $v > v_a$, there exists $\kappa^* \in [1, 1/(1-\beta))$ such that $\kappa \in (\kappa^*, 1/(1-\beta))$ implies $i + c i'_c < \iota$ for all c in $[0, 1/v]$.

If $v_d > v_a$, we have $\theta(1+i+x) > (1+\iota)\kappa$, $v_b = \theta(1+i+x)$, and $\phi = \alpha$. Notice that

$$\min\{\phi v_b\,,\,v_b-(1+\iota)\}=\begin{cases} v_b-(1+\iota) & \text{if} \quad v_a<v_b\le(1+\iota)/(1-\alpha)\\ \alpha v_b & \text{if} \quad v_a<v_b>(1+\iota)/(1-\alpha).\end{cases} \tag{44}$$

If $v_a < v_d \le (1+\iota)/(1-\alpha)$, equations (44) and (29) give

$$f_x'(x,c)=c\Big[\tau-(\tau+\lambda)(v_dc)^\lambda+\lambda(v_dc)^\lambda\frac{1+i}{1+i+x}\Big] \tag{45}$$

$$\begin{aligned} f_c'(x,c)&=1+\iota-(1+i+ci_c')\big[1-(v_dc)^\lambda\big]\\ &\quad+\ (1+i+x+ci_c')\Big\{\tau-(\tau+\lambda)(v_dc)^\lambda+\lambda(v_dc)^\lambda\frac{1+i}{1+i+x}\Big\}. \end{aligned} \tag{46}$$

Let c_x be the optimal c for given x, then equation (46) implies

$$\tau-(\tau+\lambda)(v_dc_x)^\lambda+\lambda(v_dc_x)^\lambda\frac{1+i}{1+i+x}=-\frac{1+\iota-(1+i+c_xi_c')[1-(v_dc_x)^\lambda]}{1+i+x+c_xi_c'}. \tag{47}$$

For $\kappa\in(\kappa^*,1/(1-\beta))$, we have $i+c_xi_c'<\iota$, which implies that the numerator is positive. Substitution of the above into equation (45) shows $f_x'(x,c_x)<0$. Thus, $v_a<v_d<(1+\iota)/(1-\alpha)$ is not optimal because lowering x and v_d increases $f(x,c_x)$.

If $v_b\ge(1+\iota)/(1-\alpha)$, equation (44) and (29) give

$$f_x'(x,c)=c\Big\{\tau-\big[(1+\lambda)(\tau+\alpha\theta)+\lambda(\iota-i)/(1+i+x)\big](v_dc)^\lambda\Big\}. \tag{48}$$

$$\begin{aligned} f_c'(x,c)&=(\iota-i-ci_c')[1-(v_dc)^\lambda]\\ &\quad+(1+i+ci_c'+x)\cdot\Big\{\tau-\Big[(1+\lambda)(\tau+\alpha\theta)+\frac{\lambda(\iota-i)}{1+i+x}\Big](v_dc)^\lambda\Big\}. \end{aligned} \tag{49}$$

Let c_x be the optimal c relative to x. Then, equation (49) implies

$$\tau-(1+\lambda)(\tau+\alpha\theta)+\frac{\lambda(\iota-i)}{1+i+x}=-\frac{[\iota-i-c_xi_c'][1-(v_dc_x)^\lambda]}{1+i+x+c_xi_c'}. \tag{50}$$

For $\kappa>\kappa^*$, we have $i+ci_c'\le\iota$. Then, the numerator is positive. Substitution of the above into equation (48) shows $f_x'(x,c_x)<0$, which means the current x is not optimal because lowering x increases $f(x,c_x)$.

The above two cases show that there exists κ^* such that for $\kappa^*<\kappa<1/(1-\beta)$, we have $f_x'<0$ for all x satisfying $\theta(1+i+x)>(1+\iota)\kappa$, if c is kept to be optimal relative to x. Therefore, $\theta(1+i+x)>(1+\iota)\kappa$ cannot be optimal because reducing x adds value to the bank. Consequently, the optimal x^* and c^* must satisfy $\theta(1+i^*+x^*)=(1+\iota)\kappa$, which implies $v_d^*=(1+\iota)\kappa$ and thus $v_b^*=v_d^*=v_a$.

With $v_a^*=v_d^*=v_b^*$, the state price of bankruptcy is: $\pi=(v_a^*c^*)^\lambda=(v_b^*c^*)^\lambda$. This equation implies $v_a^*=\pi^{1/\lambda}/c^*$. In view of equation (27), we have $(1+\iota)\kappa=\pi^{1/\lambda}/c^*$. It follows that $c^*=\pi^{1/\lambda}/[(1+\iota)\kappa]$, $i^*=(1+\iota)[1-(1-\beta)\kappa]^+\pi/(1-\pi)$, and $x^*=$

$(1+\iota)\{\kappa/\theta - \omega[1-(1-\beta)\kappa]^{+}\pi/(1-\pi)\} - 1$.

Let $x_c = v_a/\theta - (1+i)$ for any $c \in [1, 1/v_a]$ and $g(c) = f(x_c, c)$. It follows from equation (43) that

$$g(c) = 1 + c\big\{[\iota - i + \tau v_a/\theta][1-(v_a c)^{\lambda}] - (\kappa-1)(1+\iota)(v_a c)^{\lambda}\big\}. \tag{51}$$

This function is differentiable in c, and

$$\begin{aligned} g'(c) = {} & [\iota - i + \tau v_a/\theta]\big[1-(1+\lambda)(v_a c)^{\lambda}\big] \\ & - (\kappa-1)(1+\iota)(1+\lambda)(v_a c)^{\lambda} - c i'_c\big[1-(v_a c)^{\lambda}\big]. \end{aligned} \tag{52}$$

With equation (28) and the formula of i'_c, the above simplifies to

$$\begin{aligned} g'(c) = {} & \iota + \tau v_a/\theta - \big\{\iota + \tau v_a/\theta + (\kappa-1)(1+\iota) \\ & + \omega[1-(1-\beta)\kappa]^{+}(1+\iota)\big\}(1+\lambda)(v_a c)^{\lambda} \end{aligned} \tag{53}$$

If (x^*, c^*) is a maximum, c^* must maximizes $g(c)$, and thus $g'(c^*) = 0$. Letting $\pi = (v_a c^*)^{\lambda}$ and setting equation (53) to zero, we obtain

$$\pi = \frac{1}{1+\lambda} \cdot \frac{\iota\theta + \tau(1+\iota)\kappa}{\iota\theta + \tau(1+\iota)\kappa + (1+\iota)\{\kappa - 1 + \omega[1-(1-\beta)\kappa]^{+}\}\theta}. \tag{54}$$

Finally, we complete the proof by substituting the original parameters into (54) and the original variables into the formulas for c^*, i^*, and x^*.

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