Housing Market Dynamics and Macroprudential Policy*

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Abstract

We performed an analysis to determine how well the introduction of a countercyclical loan-to-value (LTV) ratio can reduce household indebtedness and housing price fluctuations compared to a monetary policy rule augmented with house price inflation. To this end, we construct a New Keynesian model in which a fraction of households borrow against the value of their houses and we introduce news shocks on housing demand. We estimate the model with Canadian data using Bayesian methods. We find that the introduction of news shocks can generate a housing market boom-bust cycle, the bust following an unrealized expectations on housing demand. Our study also suggests that a countercyclical LTV ratio is a useful policy to reduce the spillover from housing market to consumption and to lean against news-driven boom-bust cycles in housing prices and credit generated by expectations of future macroeconomic developments.

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1 Introduction

The correlation between consumption expenditures and house prices over the business cycles is well documented in macroeconomics studies. Indeed, time-series estimates for a variety of countries - including Canada - have shown that the two variables tend to move together. Understanding the dynamics between house prices and the accumulation of household debt is particularly important for policy makers, as it has been established that housing busts preceded by large household debt increases tend to result in deeper recessions (IMF, 2012). The economic fallout resulting from the collapse of the U.S. housing market was more painful and prolonged relative to a standard recession, as households and financial institutions engaged in a long deleveraging process following the crisis. During the same expansionary period, Canada also experienced a significant increase in house prices, residential mortgages and consumer credit.\footnote{\textup{Including Home Equity Lines of Credit, or HELOCs.}} House prices doubled and ratios of house prices to income and house prices to rent increased sharply (IMF, 2013). Mortgage credit expanded by almost 9 percent per year on average between 2000 and 2008, while households debt as a share of disposable income rose from about 100 percent in 2000 to 165 percent in 2013. As a result, mortgage and consumer loans secured by real estate (mostly HELOCS) are estimated to account for 80 percent of household debt and to represent the single largest exposure for Canadian banks with about 35 percent of their assets.

The goal of this paper is twofold. We first investigate the importance of the link between rising house prices and higher consumption expenditures that operates through the loosening of the budget constraint. Specifically, we ask the following questions: Is the statistical evidence suggesting a collateral link between the housing market and the rest of the economy important? Is the correlation between consumption and house prices being more demand- or supply-driven? Can a housing-market boom-bust arise endogenously following an unrealized expectations of a rise in housing demand? To this end, we construct a New Keynesian model in which a fraction of households...
borrow against the value of their houses. We estimate the model with Canadian
data using Bayesian methods. We then assess the model’s ability to capture key
features of consumption and house price data. Secondly, we ask how the introduction
of a countercyclical loan-to-value (LTV) ratio can reduce household indebtedness and
housing price fluctuations compared to a monetary policy rule.

Our paper is related to the business cycle literature on the role of collateral constraints
in the transmission of shocks. A key feature of these models is that collateral effects
are a propagation mechanism rather than a driving force of macro fluctuations. A
non-exhaustive list includes the following three studies. Using U.S. time series data,
Iacoviello and Neri (2010) estimates a New Keynesian model and they study the
sources and consequences of fluctuations in the U.S. housing market. Their results
suggest that slow technological progress in the housing sector explains the upward
trend in real housing prices over the last 40 years and that the housing market spillovers
are non-negligible, concentrated on consumption rather than business investment, and
have become more important over time. Lambertini et al. (2010) analyze housing
market boom-bust cycles driven by changes in households' expectations and find that,
in the presence of nominal rigidities on prices and wages, expectations on both the
conduct of monetary policy and future productivity can generate housing market
boom-bust cycles in accordance with the empirical findings. Finally, Gelain et al.
(2013) find that the introduction of a simple moving-average forecast rule, a deviation
from the rational expectations hypothesis, for a subset of agents and can significantly
magnify the volatility and persistence of house prices and household debt relative to
an otherwise similar model with fully rational expectations.

Our model shares many features with Iacoviello and Neri (2010). At the core of the
model is the borrowers-lenders set-up developed by Kiyotaki and Moore (1997). There
are two types of households differentiated by the degree to which they discount the
future. In equilibrium, one type of household is a lender and the other type a borrower.
Borrowers face a collateral constraint that limits their ability to borrow to a fraction
of the value of their housing assets. Rising house values can therefore improve the
debt capacity of borrowers, allowing them to increase consumption. Households buy and sell housing in a centralized market.

Since our goal is to quantify the links between consumption and house prices in Canada, we estimate the model with Canadian data using Bayesian methods. To this end we extend the model of Iacoviello and Neri (2010) along two important dimensions. First, we introduce multi-period fixed-rate mortgage loans. Considering that the median length of a mortgage contract in Canada is 5 years and the vast majority are at fixed-rate, this feature is potentially crucial to replicate business cycles facts and to study the (in)effectiveness of macroprudential policies, and assuming one-period loans as in Iacoviello and Neri (2010) and subsequent papers is not appropriate.\(^2\) Secondly, we calibrated the share of patient and impatient households to reflect characteristics of wealth and income distribution data (Gelain et al., 2013). By using a calibration that underestimates the share of impatient households, the mortgage debt-to-GDP ratio is also underestimated, and thus results in the underestimation of the amplifier effects of macroprudential policies changes on the broader economy.

We find statistical evidence suggesting an important collateral link between the housing market and the rest of the economy, and this link is mainly driven by demand factors. We also find that the introduction of news shocks can generate a housing market boom-bust cycle, the bust following an unrealized expectations on housing demand.

The second objective of the paper is related to macroprudential policy, more specifically the use of LTV ratios to constrain household borrowing. LTV ratio impose a cap on the size of a mortgage loan relative to the value of a property, thereby requiring a minimum down payment.\(^3\) It is believed that a countercyclical LTV policy can contain boom-bust cycles by controlling both credit and expectations channels and strengthening financial institutions’ resilience: lowering limits on LTV can tighten liquidity constraints of targeted borrowers and thus limits housing demand in targeted

\(^2\)This feature is introduced exogenously. Future research will study why this framework arise endogenously within the mortgage market.

\(^3\)See Appendix C for the recent history of LTV policies in Canada.
segments of the real estate market (and vice versa in the downturn). This has for effect of altering market expectations and speculative incentives that play a key role in bubble dynamics.

Our work is related to a few strands in the literature. First, there are papers that considers either or both the effects of monetary policy of monetary policy and changes in regulatory LTV in a DSGE framework similar to Iacoviello (2005) and Iacoviello and Neri (2010). A non-exhaustive list includes Christensen et al. (2009), Kannan et al. (2012), Justiniano et al. (2013), Lambertini et al. (2013), Gelain et al. (2013) and Gelain et al. (2014). Lambertini et al. (2013) study the potential gains of monetary and macro-prudential policies that lean against house-price and credit cycles and find that, when the implementation of both interest-rate and LTV policies is allowed, heterogeneity in the welfare implications is key in determining the optimal use of policy instruments. Finally, Gelain et al. (2014) find that monetary policy is less effective when contracts are multi-period, but only under fixed rate mortgages or when borrowers cannot be forced to accelerate repayment of their loans.

Our study suggests that higher loan-to-value ratios can amplify housing-market boom-bust cycles by encouraging speculative housing investments by credit-constrained borrowers, but the amplification effect is mainly concentrated via the collateral constraints. However, our study suggests that, in line with preceding dynamic equilibrium models with credit-constrained borrowers considered by Iacoviello (2005) and Kiyotaki et al. (2010), the loan-to-value ratio does not significantly alter aggregate house-price dynamics.

This remainder of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 describes the calibration, discusses estimation issues and our econometric strategy and introduces the data employed. Section 4 discusses the estimation results and the overall performance of the model to describe business cycle characteristics, while Section 5 reports the effect of the introduction of a countercyclical LTV ratio. Section 6 concludes. A detailed description of all data used is presented in the
2 A Model of Irreversible Housing Investment

We start from a standard New Keynesian set-up, extended to incorporate irreversible housing investment and credit frictions, as in Iacoviello (2005) and Iacoviello and Neri (2010). Our economic environment features heterogeneity among economic agents. We consider an economy populated by two types of households, designated as borrowers and lenders. Credit flows are generated by assuming ex-ante heterogeneity in agents’ subjective discount factors. Impatient consumers (borrowers) differ from patient consumers (lenders) in that they discount the future at a faster rate. Hence, in equilibrium, patient agents are net lenders while impatient agents are net borrowers. To prevent borrowing from growing without limit, we assume that borrowers face credit constraints tied to the current value of their collateral. We depart from the usual set-up of one-period loans with variable interest rates by allowing for multi-period loans with fixed interest rates (Gelain et al., 2014; Alpanda et al., 2014; Alpanda and Zubairy, 2014), which is a representation closer to the Canadian context where over 70 percent of the mortgage loans contract have a length of 5 years and have a fixed interest rate.

There are two sectors of production in the economy: consumption and housing. Each variety of consumption goods is produced by a single firm in a monopolistic competitive environment and their prices are set in a staggered fashion à la Calvo (1983). A representative firm produces houses in a perfectly competitive environment. Households supply differentiated labour in a monopolistic competitive environment - their wages being set in a staggered fashion à la Calvo (1983) - and buy goods, deriving their utility from consumption goods and from services provided by their housing stock. Credit flows are generated via perfectly competitive financial intermediaries, which accept deposits from patient households to lend to impatient households. Finally, a central bank conducts monetary policy according to a Taylor-type rule.
2.1 Households

Households \( i \in \{P, I \} \), respectively patient and impatient households, derive in period \( t \) strictly increasing utility from consumption goods \( c_{i,t} \) and from services provided by their housing stock \( h_{i,t} \). They supply labour and derive a strictly decreasing utility from hours worked in the consumption sector \( n_{i,t}^{c} \), and hours worked in the housing sector \( n_{i,t}^{h} \). They maximize their expected lifetime utility:

\[
E_{0} \sum_{t=0}^{\infty} \beta_{i}^{t} \epsilon_{i}^{h} U \left(c_{i,t}, h_{i,t}, n_{i,t}^{c}, n_{i,t}^{h} \right),
\]

where \( E_{0} \) is the mathematical expectation operator given the time 0 information set, \( \beta_{i} \in (0, 1) \) is the subjective discount factor and \( \epsilon_{i}^{h} \) represents an exogenous process on discount rates that affects the intertemporal substitution of households.\(^{4}\) The functional form of \( U \) is

\[
U(\bullet) = \log(x_{i,t}) - \frac{\epsilon_{i}^{n}}{1 + \eta_{i}} \left( (n_{i,t}^{c})^{\theta_{i}^{n} + 1} + (n_{i,t}^{h})^{\theta_{i}^{n} + 1} \right)^{\theta_{i}^{n}(1 + \eta_{i})},
\]

where \( \epsilon_{i}^{n} \) is an exogenous process on labour supplies, and \( \theta_{i}^{n} \) and \( \eta_{i} \) are, respectively, the strictly positive intratemporal elasticity of substitution between sectoral labour supplies and the inverse of Frish elasticity of substitution. This specification of disutility of labour follows Horvath (2000). When \( \theta_{i}^{n} \to \infty \), hours worked in each sector tends to be perfect substitutes as agents devote all of their time to the sector paying the highest wage and all sectors pay the same hourly wage at the margin. The final good \( x_{i,t} \) is defined as a CES composite of consumption (non-durable) goods \( c_{i,t} \) and housing stock \( h_{i,t} \)

\[
x_{i,t} = \left[(1 - \epsilon_{i}^{h}) c_{i,t}^{\frac{\theta_{i}^{n} - 1}{\theta_{i}^{h} - 1}} + (\epsilon_{i}^{h}) h_{i,t}^{\frac{\theta_{i}^{n}}{\theta_{i}^{h} - 1}} \right]^{\frac{\theta_{i}^{n}}{\theta_{i}^{n} - 1}},
\]

\(^{4}\)The process \( \epsilon_{i}^{h} \) has been identified to be an important driver of consumption fluctuations in recent DSGE literature (Smets and Wouters, 2007; Justiniano et al., 2010).
where $\epsilon_i^h$ is an exogenous process on the preference for services provided by the housing stock (i.e. housing demand shock) and $\theta_i^x$ is the strictly positive intratemporal elasticity of substitution between consumption goods and the services provided by the housing stock. When $\theta_i^x \to \infty$, both goods tend to be perfect substitutes whereas they tend to be perfect complements when $\theta_i^x \to 0$.

**Labour**  
Labour decisions are made by a central authority within the households, which supplies, in a monopolistic competitive environment, differentiated labour $n_{i,l,t}^j$ in a continuum of labour markets $l \in [0, 1]$ in sector $j \in \{c, h\}$. By assuming that all households will act as a representative household, this setup avoids the need to assume separability of preferences and the existence of insurance for labour market. This feature is already reflected in the notation since there are no subscript for the continuum of households of each type $i$.

Both sectors are, in terms of notation, the same. Given the wage charged in each labour market $l$ of sector $j$, the central authority supplies labour to satisfy the demand given by

$$n_{i,l,t}^j = \left( \frac{w_{i,l,t}^j}{w_{i,t}^j} \right)^{-\theta n^j} n_{i,t}^{j,d},$$

where $w_{i,l,t}^j = W_{i,l,t}^j P_t$ and $w_{i,t}^j = W_{i,t}^j P_t$ are the real wages. $W_{i,l,t}^j$ denotes the nominal wage charged by the central authority in the labour market $l$ in sector $j$ for agents of type $i$, $W_{i,t}^j$ is the nominal wage index, $n_{i,t}^{j,d}$ is a measure of aggregate labour demand by firms and $\theta n^j$ is the wage-elasticity of demand. In each labour market, the central authority takes $W_{i,t}^j$ and $n_{i,t}^{j,d}$ as given. In addition, the total numbers of hours allocated to the different labour markets must satisfy the resource constraint in each sector

$$n_{i,t}^j = \int_0^1 (n_{i,l,t}^j) \, dl.$$  

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5By assuming that all households will act as a representative household, this setup avoids the need to assume separability of preferences and the existence of insurance for labour market. This feature is already reflected in the notation since there are no subscript for the continuum of households of each type $i$.

6The formal derivation of the labour demand function is presented in the Section 2.2.
Combining this restriction with equation (3) yields the aggregated labour supply in each sector $j$:

$$n_{i,t}^j = n_{i,t}^{j,d} \int_0^1 \left( \frac{w_{i,t}^j}{w_{i,t}^j} \right)^{-\theta n^j} dl. \quad (5)$$

**Patient (Lenders)** Patient households ($i = P$) have a higher propensity to save (i.e. $\beta_P > \beta_I$). In equilibrium, they supply loans to impatient households ($i = I$) via their deposit $d_t$ at financial intermediaries and accumulate housing and capital stock. Since lenders are the owners of the banks and firms in both sectors, they receive dividends $f_{i,c}^t$, $f_{i,h}^t$ and $f_{i,fi}^t$ from the consumption and housing sectors and from financial intermediaries, respectively. They maximize their expected lifetime utility (1) subject to their budget constraint in real terms (in units of consumption goods)

$$c_{P,t} + q_{i,t}^h h_{P,t} + \sum_{j \in \{c,h\}} q_{i,t}^j i_{t}^j + q_{i,t}^d l_t + b_{P,t} + d_t = \sum_{j \in \{c,h\}} t_{i}^j u_{i,t}^j k_{t-1}^j + (q_{i,t}^l + r_{i,t}^l) l_{t-1} + \quad (6)$$

$$\sum_{j \in \{c,h\}} n_{P,t}^{j,d} \int_0^1 w_{P,t}^j \left( \frac{w_{P,t}^j}{w_{P,t}^j} \right)^{-\theta n^j} dl + \frac{R_{t-1} b_{P,t-1}}{\pi_t^c} + \sum_{j \in \{c,h,fi\}} f_{i,t}^j + \frac{1}{\phi_m^m} \sum_{s=1}^{\phi_m^m} d_{t-s}^{d,s} - R_{t-s}^d \frac{d_{t-s}^{d,s} - \pi_t^{c,v}}{\Pi_{v=m-s}^{t-v}} ,$$

the law of motion for capital in sector $j$

$$k_{i,t}^j = \left( 1 - \delta_t^k \right) k_{i,t-1}^j + z_{i,t}^j i_{i,t}^j \left[ 1 - \frac{\phi_m^m}{2} \left( \frac{i_{i,t}^j}{i_{i,t-1}^j} - 1 \right) \right] , \quad (7)$$

and the law of motion for housing stock

$$h_{P,t} = \left( 1 - \delta_t^h \right) h_{P,t-1} + i_{i,t}^h , \quad (8)$$

where $q_{i,t}^h$ is the real price of housing, $i_{i,t}^h$ is the investment in housing stock and $\delta_t^h$ is its fixed depreciation rate. With $j \in \{c,h\}$, $k_{i,t}^j$ is the stock of capital specific to sector $j$, $i_{i,t}^k$ its investment level, $r_{i,t}^k$ its real rental rate, $u_{i,t}^k$ its variable capacity utilization
rate and $\delta_{k_j}^t$ its variable depreciation rate. Lenders invest $i_{k_j}^t$ in $k_j$ with $q_{t}^j$ being the real price of investment in sector $j$. Lenders owns all the land stock $l_t$, which has a real price $q_{t}^l$ and a real rental rate $r_{t}^l$. The stock of land is exogenous and evolves with an autoregressive process. $\pi_{t}^C$ is the gross inflation rate in the consumption sector and $R_{t}$ is the gross nominal interest rate on risk-less one-period bonds $b_{P,t-1}$. Lenders’ savings take the form of a long-term deposit at the financial intermediaries at the constant interest rate $R_{d}^t$. The deposit length is $\phi^m$ periods and, at each period, the lenders receive a share $\frac{1}{\phi^m}$ of the principal as a reimbursement of the deposit and a fixed return on investment $(R_{d}^t - 1)$ on the principal not reimbursed at the last period. Finally, $\phi^{ke}$ and $\phi^{kh}$ are the adjustment cost parameters. The technology transforming investment goods into capital goods is subject to a transitory exogenous process denoted $z_{t}^k$.

Lenders can control the intensity at which the capital stock is utilized. The effective amount of capital services supplied to firms in the consumption and housing sectors in period $t$ are given by $u_{t}^{k_c} k_{t-1}^c$ and $u_{t}^{k_h} k_{t-1}^h$, respectively. We assume that increasing the intensity of capital utilization entails a cost in the form of a faster rate of depreciation. Specifically, we assume that depreciation rates $\delta_{t}^{k_j}$ are an increasing and convex function of the rate of capacity utilization

$$\delta_{t}^{k_j} = \delta_{0}^{k_j} + \delta_{1}^{k_j} (u_{t}^{k_j} - 1) + \frac{\delta_{2}^{k_j}}{2} (u_{t}^{k_j} - 1)^2,$$

with $\delta_{0}^{k_j}, \delta_{1}^{k_j}, \delta_{2}^{k_j} > 0$ (as in Schmitt-Grohe and Uribe (2012)).

**Impatient (Borrowers)** The impatient households ($i = I$) do not accumulate physical capital nor hold any equity, and have access to multi-period fixed-rate mortgage loans with fixed (linear) principal payments, so that in each period a borrowers have to pay interest on the outstanding debt and repay the amount of principal due. They

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7. This type of process has been identified by Justiniano et al. (2010) as an important source of aggregate economic fluctuations.
8. See Appendix B for the Lagrangian and the complete set of first-order necessary conditions.
maximize their expected lifetime utility (1) subject to a budget constraint

\[ c_{I,t} + q_i h_{I,t} + b_{I,t} + \frac{1}{\phi^m} \sum_{s=1}^{\phi^m} \frac{m_{t-s}}{\prod_{v=-s}^{v} \pi_{I+v}} + \sum_{s=1}^{\phi^m} \left( R_{I,t}^{m} - 1 \right) \left( \frac{\phi^{m-s}+1}{\phi^m} \right) \frac{m_{t-s}}{\prod_{v=-s}^{v} \pi_{I+v}} = \]

\[ \sum_{j \in \{c,h\}} n_{I,t,d} \int_{0}^{1} w_{j,i,t} \left( \frac{w_{j,i,t}}{w_{j,i,t}} \right)^{-\theta^{m}} dl + \frac{R_{t-1}b_{I,t-1}}{\pi_{I}^{c}} + m_{t}, \quad (10) \]

to their law of motion for housing stock

\[ h_{I,t} = (1 - \delta^h) h_{I,t-1} + i_{I,t}^{h}, \quad (11) \]

and a borrowing constraint. Private borrowing is subject to an endogenous limit. At any time \( t \), borrowers agree to borrow no more than a share \( \omega \) (Kiyotaki and Moore, 1997; Monacelli, 2009) of the current value of their housing stock:

\[ M_t \geq -\omega q_i^h h_{i,t}, \quad (12) \]

where

\[ M_t = \sum_{s=0}^{\phi^m} \left( \frac{\phi^{m-s}+1}{\phi^m} \right) \frac{m_{t-s}}{\prod_{v=-s}^{v} \pi_{I+v}}, \quad (13) \]

is the total mortgage debt.\(^9\) The model reflects the fact that mortgage debt is reoptimized only for the share of contracts that reach their end and have to refinance.\(^10\)

**Wages** We introduce wage stickiness in the model by assuming that, in each period, the central authority within the households \( i \) cannot set the nominal wage optimally for a share \( \xi^{w_j} \in (0, 1) \) of labour markets chosen randomly. The first-order necessary conditions for \( w^{j}_{i,t} \) is

\[ w^{j}_{i,t} = \begin{cases} 
\tilde{w}^{j}_{i,t} & \text{if set optimally in } t \\
\frac{w^{j}_{i,t-1}(\pi^{c})^{w_j}}{\pi_{i}^{c}} & \text{otherwise}
\end{cases} \quad (14) \]

\(^9\)We use the convention that \( m < 0 \) is a debt.

\(^{10}\)See Appendix B for the Lagrangian and the complete set of first-order necessary conditions.
In equations (14), \( \tilde{w}_{i,j,t} \) denotes the real wage prevailing in the \( (1 - \xi w^j) \) labour markets in which the central authority can set wages optimally in sector \( j \) in period \( t \). Because the labour demand curve faced by the union is identical across all labour markets, and because the cost of supplying labour is the same for all markets, one can assume that wage rate \( \tilde{w}_{i,j,t} \) is identical for all industries within a given sector (but not necessarily across sectors). In labour market in which the wage rate is re-optimized in period \( t \), the real wage is set to equate the expected future average marginal revenue to the average marginal cost of supplying labour, with \( \frac{\theta^{\nu^j}}{\theta^{\mu^j} - 1} \) being the markup of wages over the marginal cost of labour that would prevail in the absence of wage stickiness and trend inflation. In the \( \xi w^j \) labour markets that cannot set wages optimally, the wages are imperfectly indexed at rate \( t^{\nu^j} \) to the steady-state inflation.\(^{11}\)

2.2 Firms

2.2.1 Consumption sector

Final good producers In each period \( t \), perfectly competitive final consumption goods producers purchase differentiated intermediate goods \( m \in [0, 1] \) to assemble final goods \( y^c_t \) via the Dixit-Stiglitz aggregator

\[
y^c_t = \left[ \int_0^1 \left( y^c_{m,t} \frac{\theta^c}{\theta^c - 1} \right) dm \right] \frac{\theta^c}{\theta^c - 1},
\]

where parameter \( \theta^c \) denotes the intratemporal elasticity of substitution across varieties of intermediate differentiated goods\(^{12}\) and \( y^c_{m,t} \) is the demand for goods of variety \( m \).

When maximizing their profits, final goods producers take as given the prices of intermediate goods and the aggregate price index. For any given level of final consumption

\(^{11}\)Part of the optimization problem (i.e. the Lagrangian) that is relevant for this purpose along with the first-order necessary condition can be found in Appendix B.

\(^{12}\)When \( \theta^c \to 0 \), intermediate goods are perfect complements, whereas they are perfect substitutes when \( \theta^c \to \infty \).
goods produced, they must solve the expenditure-minimizing problem

$$\min \left\{ y_{m,t} \right\} \int_{0}^{1} P_{m,t}^{c} y_{m,t}^{c} dm$$

subject to aggregation constraint (15), where $P_{m,t}^{c}$ denotes the price of the intermediate consumption good $m$ at time $t$. The demand for goods of variety $m$ is then given by

$$y_{m,t}^{c} = \left( \frac{P_{m,t}^{c}}{P_{t}^{c}} \right)^{-\theta^{c}} y_{t}^{c},$$

where $P_{t}^{c}$ is the nominal price index defined as

$$P_{t}^{c} = \left[ \int_{0}^{1} \left( P_{m,t}^{c} \right)^{1-\theta^{c}} dm \right]^{1-\frac{1}{\theta^{c}}}.$$  \hfill (17)

**Intermediate good producers**  Each variety of intermediate goods in the consumption sector is produced by a single firm $m$ evolving in a monopolistic competitive environment. The production function for each of these firm $m \in [0, 1]$ is

$$y_{m,t}^{c} = z_{t}^{c} \left( k_{m,t}^{c} \right)^{\gamma^{c}} \left( n_{P,m,t}^{c,d} \right)^{\alpha^{c}} \left( n_{I,m,t}^{c,d} \right)^{1-\alpha^{c}} \right)^{1-\gamma^{c}},$$

where $y_{m,t}^{c}$ is its total production, $n_{P,m,t}^{c,d}$ and $n_{I,m,t}^{c,d}$ are the number of hours of work demanded by the firm for both type of workers, $z_{t}^{c}$ is the sector-wide total factor productivity and $k_{m,t}^{c}$ is the capital stock rented by the firm. Also, $\gamma^{c}$ is the capital share of income and $\alpha^{c}$ is the lenders’ share of labour income. The nominal profits (i.e. dividends) of the firm are denoted by

$$F_{m,t}^{c} = P_{m,t}^{c} y_{m,t}^{c} - R_{t}^{k} k_{m,t}^{c} - W_{t}^{c} n_{P,m,t}^{c} - W_{t}^{c} n_{I,m,t}^{c}.$$  \hfill (19)
We assume that the firm must satisfy the aggregated demand for good \( m \) at posted price 
\[
y_{m,t}^c = \left( \frac{P_{m,t}^c}{P_t^c} \right)^{-\theta^c} y_t^c. \tag{20}
\]
The firm’s objective is a static problem of profit maximization 
\[
\max_{\{k_{m,t}^c, n_{P,m,t}^c, n_{I,m,t}^c\}} F_{m,t}^c \tag{21}
\]
subject to demand function (20). Real wages and the real rental rate of capital are then given by 
\[
\begin{align*}
    r_{k,t}^c &= m c_t^z_t \gamma^c \left( k_{m,t}^c \right)^{\gamma^c-1} \left( \left( \frac{n_{P,m,t}^c}{n_{I,m,t}^c} \right)^{\alpha^c} \left( n_{I,m,t}^c \right)^{1-\alpha^c} \right)^{1-\gamma^c}, \\
    w_{P,t}^c &= m c_t^z_t \left( 1 - \gamma^c \right) \alpha^c \left( k_{m,t}^c \right)^{\gamma^c} \left( \left( \frac{n_{P,m,t}^c}{n_{I,m,t}^c} \right)^{\alpha^c} \left( n_{I,m,t}^c \right)^{1-\alpha^c} \right)^{(-\gamma^c)} \left( \frac{n_{P,m,t}^c}{n_{I,m,t}^c} \right)^{\alpha^c-1}, \tag{22}
\end{align*}
\]
and 
\[
\begin{align*}
    w_{I,t}^c &= m c_t^z_t \left( 1 - \gamma^c \right) \left( 1 - \alpha^c \right) \left( k_{m,t}^c \right)^{\gamma^c} \left( \left( \frac{n_{P,m,t}^c}{n_{I,m,t}^c} \right)^{\alpha^c} \left( n_{I,m,t}^c \right)^{1-\alpha^c} \right)^{(-\gamma^c)} \left( \frac{n_{P,m,t}^c}{n_{I,m,t}^c} \right)^{\alpha^c}, \tag{23}
\end{align*}
\]
where \( mc_t \) is the firm’s real marginal cost. From the optimality conditions, all firms \( m \) face the same prices of factors, and since they have access to the same technology, marginal cost is equal across all firms at every period \( t \).

Firms are able to reoptimize their prices as in Calvo (1983) and Yun (1996). Specifically, each firm faces a price rigidity with a non-zero probability \( \xi_{t}^{p} \) of being unable to adjust its nominal price in a given period. These firms are able to imperfectly index their price at rate \( \iota_{t}^{p} \) to the steady-state inflation. The reoptimization probability is independently and identically distributed across firms and over time. Firms maximize the expected present value of their real dividends. Therefore, in setting their price in period \( t \), firms take into account the fact that they may have to wait some time
until they are able to reoptimize their price. In particular, the probability of not reoptimizing between dates \( t \) and \( t + s \) is \( (\xi^p)^s \). Since all reoptimizing firms face the same problem, they will all choose \( \bar{p}_t^c \) as the real optimal price. Optimizing firms set nominal prices so that average future expected marginal revenues equate average future expected marginal costs.\(^{13}\)

Given that the opportunity to reoptimize prices arrives probabilistically for each firm in each period, the aggregate price index (17) can be written in this recursive form:

\[
1 = (1 - \xi^p) (\bar{p}_t^c)^{1-\theta^c} + \xi^p \left( \frac{(\pi^c)^h}{\pi_t^c} \right)^{1-\theta^c}.
\]  

(25)

### 2.2.2 Housing sector

A representative firm produces houses in a perfectly competitive environment. Its production function is

\[
y_t^h = z_t^h \left( u_t^{k^h} k_{t-1}^h \right)^{\gamma_h} \left( n_t^{h,d} \right)^{\alpha_h} \left( n_t^{I,I} \right)^{1-\alpha^h} \left( 1 - \gamma^h - \gamma^l \right),
\]  

(26)

where \( y_t^h \) is the total production for the housing sector, \( n_{t,t}^{h,d} \) are the number of hours of work demanded by the firm for both types of workers, \( z_t^h \) is the sector-wide total factor productivity, \( u_t^{k^h} k_{t-1}^h \) is the capital stock rented, and \( l_{t-1} \) is the land stock rented.\(^{14}\) Also, \( \gamma^h \) is the capital share of income, \( \alpha^h \) is the lenders’ share of labour income and \( \gamma^l \) is the land share of income.\(^{15}\) The nominal profits (i.e. dividends) of the firm are denoted by

\[
F_t^h = Q_t^h y_t^h - R_t^{k^h} u_t^{k^h} k_{t-1}^h - R_t^l l_{t-1} - W_t^{n^h} n_{t,t}^{h,d} - W_t^{n^h} n_{t,t}^{h,d}.
\]  

(27)

\(^{13}\)Part of the optimization problem (i.e. the Lagrangian) that is relevant for this purpose along with the first-order necessary condition can be found in Appendix B.

\(^{14}\)Since we assume a representative firm, to simplify the notation, we have already included the fact that the firm rents all the available capital in the sector, represented by \( u_t^{k^h} k_{t-1}^h \), and all the available land, represented by \( l_{t-1} \).

\(^{15}\)In the calibration, we will assume that \( \alpha^h = \alpha^c = \alpha \)
The firm’s objective is a static problem of profit maximization

\[
\max \left\{ u^h k^h_{t-1}, n^{h,d}_{P,t}, n^{h,d}_{I,t} \right\} F^h_t
\]  (28)

subject to (26). Real wages and real rental rates of capital and land are then given by

\[
r^h_t = q_t^h z_t^h \gamma^h \left( u^h k^h_{t-1} \right)^{\gamma - 1} \left( \left( n^{h,d}_{P,t} \right)^{\alpha^h} \left( n^{h,d}_{I,t} \right)^{1-\alpha^h} \right)^{1-\gamma^h - \gamma^l} ,
\]  (29)

\[
r^l_t = q_t^l z_t^l \gamma^l \left( u^l k^l_{t-1} \right)^{\gamma - 1} \left( \left( n^{h,d}_{P,t} \right)^{\alpha^h} \left( n^{h,d}_{I,t} \right)^{1-\alpha^h} \right)^{1-\gamma^h - \gamma^l} ,
\]  (30)

\[
w^h_{P,t} = q_t^h z_t^h \left( 1 - \gamma^h - \gamma^l \right) \alpha^h \left( u^h k^h_{t-1} \right)^{\gamma^h} \left( n^{h,d}_{P,t} \right)^{\alpha^h} \left( n^{h,d}_{I,t} \right)^{1-\alpha^h} \gamma^l_{t-1} \times \left( \left( n^{h,d}_{P,t} \right)^{\alpha^h} \right)^{\gamma^l} \left( n^{h,d}_{I,t} \right)^{1-\alpha^h} ,
\]  (31)

and

\[
w^h_{I,t} = q_t^h z_t^h \left( 1 - \gamma^h - \gamma^l \right) \left( 1 - \alpha^h \right) \left( u^h k^h_{t-1} \right)^{\gamma^h} \left( n^{h,d}_{P,t} \right)^{\alpha^h} \left( n^{h,d}_{I,t} \right)^{1-\alpha^h} \gamma^l_{t-1} \times \left( \left( n^{h,d}_{P,t} \right)^{\alpha^h} \right)^{\gamma^l} \left( n^{h,d}_{I,t} \right)^{1-\alpha^h} \gamma^l_{t-1} ,
\]  (32)

### 2.2.3 Labour input

The labour input used by the firms in a given sector, denoted by \( n^{l,d}_{i,t} \), is assumed to be a composite made of a continuum of differentiated labour services \( n^{l,d}_{i,t,m} \). In the case of the consumption sector, we need integrate labour demand over all intermediate firms \( m \in [0, 1] \), which yields

\[
n^{c,d}_{i,t} = \int_0^1 \left( n^{c,d}_{i,t,m} \right) dm,
\]  (33)
where $i$ and $l$ have the same meaning as before. The aggregated labour demand for agents of type $i$ in sector $j$ is given by

$$n_{i,j,t}^{d} = \left[ \int_{0}^{1} \left( n_{i,j,t}^{d} \right) ^{\frac{\theta_{nj}}{\delta_{nj}} - 1} \frac{d}{\delta_{nj}} dl \right] ^{\frac{1}{\theta_{nj} - 1}}. \quad (34)$$

By solving the cost minimization problem

$$\min_{\{n_{P,l,t}^{j,d}, n_{I,l,t}^{j,d}\}} \int_{0}^{1} W_{j,t}^{P,l,t} n_{P,l,t}^{j,d} dl + \int_{0}^{1} W_{I,t}^{j,I,l,t} n_{I,l,t}^{j,d} dl,$$

the optimal demand for labour type $i$ in labour market $l$ at time $t$ is

$$n_{i,l,t}^{j,d} = \left( \frac{W_{i,t}^{j,I,l,t}}{W_{i,t}^{j}} \right)^{-\theta_{nj}} n_{i,t}^{j,d}, \quad (36)$$

where the nominal wage index is given by

$$W_{i,t}^{j} = \left[ \int_{0}^{1} \left( W_{i,t}^{j} \right)^{1-\theta_{nj}} dl \right] ^{1-\theta_{nj}}. \quad (37)$$

Given that the opportunity to reoptimize wages arrives probabilistically for each household in each period, the aggregate wage index (37) for agent $i$ in sector $j$ can be written in this recursive form:

$$\left( w_{i,t}^{j} \right)^{1-\theta_{nj}} = \left( 1 - \xi^{w_{i,t}^{j}} \right) \left( w_{i,t-1}^{j} \right)^{1-\theta_{nj}} + \xi^{w_{i,t}^{j}} \left( \frac{w_{i,t-1}^{j} \left( \pi_{c}^{i} \right)^{c_{w_{i,t}^{j}}}}{\pi_{t}^{i}} \right)^{1-\theta_{nj}}. \quad (38)$$

### 2.3 Financial Intermediaries

We assume that households use financial intermediaries because they cannot borrow and lend with each other directly. Financial intermediaries accept deposit $d_{t}$ from
lenders at the cost $R_t^d$ and lend to borrowers $m_t$ at rate $R_t^m$. The spread between rates on loans and deposits reflects a time-varying intermediation cost and it is assumed to be a deadweight loss to the economy. Financial intermediaries are assumed to be perfectly competitive. To maximize the expected present value of their real dividends, financial intermediaries must solve

$$\max_{\{d_t, m_t\}} E_t \sum_{s=0}^{\infty} (\beta_P)^s \left[ \frac{\lambda_{P,t+s}^c}{\lambda_{P,t}^c} \frac{P_{t+s}}{P_t} \right] f_t^{f_i}$$

subject the balance sheet (in real terms)

$$d_t + \frac{1}{\phi^m} \sum_{s=1}^{\phi^m} \frac{m_{t-s}}{\prod_{v=-s}^{0} \pi_{t+v}^c} + \sum_{s=1}^{\phi^m} (R_{t-s}^m - 1) \left( \frac{\phi^m - s + 1}{\phi^m} \right) \frac{m_{t-s}}{\prod_{v=-s}^{0} \pi_{t+v}^c} =$$

$$m_t + \frac{1}{\phi^m} \sum_{s=1}^{\phi^m} \frac{d_{t-s}}{\prod_{v=-s}^{0} \pi_{t+v}^c} + \sum_{s=1}^{\phi^m} (R_{t-s}^d - 1) \left( \frac{\phi^m - s + 1}{\phi^m} \right) \frac{d_{t-s}}{\prod_{v=-s}^{0} \pi_{t+v}^c} + f_t^{f_i} + \epsilon_t^{R_i} m_t. \quad (39)$$

As documented by Campbell (2013), in many countries (as in Canada in particular), the vast majority of housing loans are long-term fixed-rate mortgages. We then incorporate a simple form of this type of contract with one type of mortgage being available, with principal being reimburse linearly over $\phi^m$ periods. The deposit is also of the same form.\(^{16}\)

By taking the first-order necessary conditions for $d_t$ and $m_t$ yields the solution for $R_t^m$

$$R_t^m = \frac{1 + \epsilon_t^{R_i} - \frac{1}{\phi^m} \sum_{s=0}^{\phi^m} \beta_P^s E_t \left[ \frac{\lambda_{P,t+s}^c}{\lambda_{P,t}^c} \frac{1}{\prod_{v=1}^{t} \pi_{t+v}^c} \right]}{\sum_{s=0}^{\phi^m} \frac{\phi^m - s + 1}{\phi^m} \beta_P^s E_t \left[ \frac{\lambda_{P,t+s}^c}{\lambda_{P,t}^c} \frac{1}{\prod_{v=1}^{t} \pi_{t+v}^c} \right]}.$$

\(^{16}\)This hypothesis is not central for our results. We tried one-period variable-rate deposit and it yielded similar results.
2.4 Monetary policy

The central bank implements a Taylor-type (Taylor, 1993) monetary policy rule with interest smoothing:

\[
\frac{R_t}{R_{ss}} = \left( \frac{R_{t-1}}{R_{ss}} \right)^{\rho_r} \left( \frac{\pi^c_t}{\epsilon_t^c} \right)^{(1-\rho_r)\rho_{\pi^c}} \left( \frac{Y_t}{Y_{ss}} \right)^{(1-\rho_r)\rho_y} \exp \left( \epsilon_R^t \right).
\] (40)

The monetary authority adjusts the nominal gross interest rate \( R_t \) from its steady-state value in response to deviations of inflation \( \pi^c_t \) from its target, deviations of the GDP \( (Y_t) \) from its steady state value, and the i.i.d. monetary policy innovation \( \epsilon_R^t \) with variance \( \sigma_{\epsilon_R^t}^2 \). \( \rho_r \), \( \rho_{\pi^c} \) and \( \rho_y \) are the persistence parameter, and the inflation and output response parameters, respectively. The central bank’s target, \( \epsilon_{\pi^c}^t \), is assumed to be an exogenous time varying process subject to shocks, as in Smets and Wouters (2003) and Adolfson et al. (2007). The inflation targeting has been implemented in 1991 in Canada, therefore this model specification can help capturing the response of \( R_t \) to movements in \( \pi^c_t \) in the first third of our sample.

2.5 Exogenous processes

All the exogenous processes in the model introduced earlier follow

\[
\ln \Theta_t = (1 - \rho_\Theta) \Theta_{ss} + \rho_\Theta \Theta_{t-1} + \epsilon_\Theta^t,
\] (41)

where \( \Theta_t = \left\{ \epsilon_t^b, \epsilon_t^h, \epsilon_t^n, \epsilon_t^{\pi^c}, \epsilon_t^{R^m}, l_t, z_t^c, z_t^h, z_t^{ik} \right\} \) are the exogenous processes, \( \Theta_{ss} \) are their respective steady-state values, and \( \rho_\Theta \) their respective persistence parameters. The structural shocks in the model, \( \epsilon_\Theta^t = \left\{ \epsilon_t^b, \epsilon_t^h, \epsilon_t^n, \epsilon_t^{\pi^c}, \epsilon_t^{R^m}, \epsilon_t^l, \epsilon_t^c, \epsilon_t^h, \epsilon_t^{ik} \right\} \) along with the monetary policy innovation \( \epsilon_R^t \), are all zero-mean i.i.d. shocks with process-specific variance \( \sigma_{\epsilon_\Theta}^2 \), and are uncorrelated contemporaneously and at all leads and lags.
2.6 Market clearing

Consumption sector  The aggregations in the production and labour markets follow similar processes introduced in the New Keynesian literature. Integrating both sides of the intermediate goods production technology (18) yields

\[
\int_0^1 (y_{m,t}^c) \, dm = \int_0^1 z_t^c (k_{m,t}^c) \gamma^c \left( (n_{P,m,t}^{c,d})^\alpha^e (n_{I,m,t}^{c,d})^{1-\alpha^e} \right)^{1-\gamma^c} \, dm = z_t^c (u_t^{k^c} k_{t-1}^{k^c}) \gamma^c \left( (n_{P,t}^{c,d})^\alpha^c (n_{I,t}^{c,d})^{1-\alpha^c} \right)^{1-\gamma^c}.
\]

Substituting \(y_{m,t}^c\) in (42) and using demand function (20), we get

\[
\left[ \int_0^1 \left( \frac{P_{m,t}^c}{P_t^c} \right)^{-\theta^e} \, dm \right] y_t^c = s_t^y y_t^c = z_t^c (u_t^{k^c} k_{t-1}^{k^c}) \gamma^c \left( (n_{P,t}^{c,d})^\alpha^c (n_{I,t}^{c,d})^{1-\alpha^c} \right)^{1-\gamma^c},
\]

where \(s_t^y\) captures the inefficiencies associated with price dispersion arising from the price rigidity à la Calvo (1983). Schmitt-Grohe and Uribe (2007) show that these price dispersion indexes can be defined as

\[
s_t^y = (1 - \xi^c) (\bar{p}_t^c)^{-\theta^e} + \xi^c \left( \frac{(\pi_t^e)^{\psi^e}}{\pi_t^c} \right)^{-\theta^e} s_{t-1}^y.
\]

The market clearing condition for the consumption sector is therefore

\[
y_t^c = c_t + q_t^{k^c} k_{t}^{k^c} + q_t^{l^h} l_{t}^{l^h} + \epsilon_t^R m_t,
\]

where \(c_t = c_{P,t} + c_{I,t}\). Finally, the real profits are

\[
f_t^c = y_t^c - w_{P,t}^c n_{P,t}^{c,d} - w_{I,t}^c n_{I,t}^{c,d} - r_t^{k^c} u_t^{k^c} k_{t-1}^{k^c}.
\]
Housing sector  Given that total production, as expressed by (26), must satisfy the aggregate demand for the sector

\[ y^h_t = i^h_{P,t} + i^h_{I,t}, \]  

(47)

the real profits are

\[ f^h_t = q^h_t y^h_t - w^h_{P,t} n^h_{P,t} - w^h_{I,t} n^h_{I,t} - r^h_t k^h_{t-1} - r^I_{t-1}, \]  

(48)

and the total housing stock is \( h_t = h_{P,t} + h_{I,t}. \)

Labour input  The nominal wage rigidity induces a loss in the number of hours worked supplied due to nominal wage dispersions. Schmitt-Grohe and Uribe (2007) show that these price dispersions, for agents of type \( i \) in sector \( j \), can be expressed as

\[ s^j_{i,t} = \left( 1 - \xi^j \right) \left( \frac{\bar{w}^j_{i,t}}{w^j_{i,t}} \right)^{-\theta^j} + \xi^j \left( \frac{(\pi^c)^{i}}{\bar{\pi}^c_t} \right)^{-\theta^j} \left( \frac{w^j_{i,t-1}}{w^j_{i,t}} \right)^{-\theta^j} s^j_{i,t-1}, \]  

(49)

and the labour supply-demand relation is given by \( n^j_{i,t} = n^j_{i,t} s^j_{i,t} \).

Aggregate Economy  The real GDP is therefore given by

\[ y_t = y^c_t + q^h_t y^h_t. \]  

(50)

2.7 Competitive Equilibrium

An (imperfectly) competitive equilibrium is an allocation for:

- the lenders: \( \mathcal{C}_P = \{ c_{P,t}, h_{P,t}, n^j_{P,t}, b_{P,t}, i^j_{P,t}, k^j_{P,t}, u^j_{P,t}, d_t \}_{t=0,j \in \{c,h\}}^\infty \),
- the borrowers: \( \mathcal{C}_I = \{ c_{I,t}, h_{I,t}, n^j_{I,t}, b_{I,t}, m_t \}_{t=0,j \in \{c,h\}}^\infty \),
- the firms in consumption sector: \( \mathcal{F}^c = \{ y^c_{m,t}, K^c_{m,t}, n^{i,d}_{m,t}, F^c_{m,t} \}_{t=0,m \in [0,1],i \in \{P,I\}}^\infty \).
• the firms in housing sector: $F^h = \left\{ y^h_t, k^h_t, n^h_{i,t}, F^h_{t} \right\}_{t=0, i \in \{ P, I \}}^\infty$, and

• prices system: $P = \left\{ R_t, R^m_t, R^d_t, \pi^c_t, \pi^c_t, q^c_t, q^l_{i,t}, w^j_{i,t}, \tilde{w}^j_{i,t}, q^k_{j,t} \right\}_{t=0, i \in \{ P,I \}, j \in \{ c,h \}}^\infty$,

such that, given initial conditions on predetermined variables, the exogenous processes, and the prices system, the allocations $C_P$, $C_I$, $F^c$ and $F^h$ solve the households and firms problems, and all market clearing conditions in Section 2.6 are satisfied.

3 Empirical Strategy

In order to evaluate the performance of the model, we use a combination of calibrated and estimated parameters. Our choice to calibrate some of the parameters was mainly based on the lack of data for some of the model variables, particularly those describing the production functions and the wealth distribution. This section first describes our calibration approach, then presents the details regarding the estimation procedure, and concludes with a presentation of the data.

3.1 Calibration

The model is calibrated on a quarterly basis. Table 1 summarizes our calibration, while Table 2 displays the steady-states of the model and observed values of corresponding data. We calibrated this set of parameters because they are either difficult to estimate given the information contained in the model or because they are better identified using other information. For instance, some parameters are set to achieve target values for steady-states while others are set to commonly used values in the literature.

We set the steady-state annual inflation rate at 2 percent, this value being the target

\[ \text{For identification testing, we compute estimates of the Fisher information matrix. The Fisher matrix is a property of the model itself, and is independent of any data. It represents the maximum amount of information we can find in the data assuming the data are really generated by the model DGP. We compute two approaches: a time-domain one, and a frequency-domain one, and use a singular value decomposition to learn more about which parameters (or combinations of them) are identified the best or the worst. In our case, the intratemporal elasticity of substitution between sectoral labour supplies, the depreciation rates, the intratemporal elasticity of substitution across different varieties of intermediate goods and the wage-elasticities of demand has been revealed to be not or weakly identifiable.} \]
of the inflation-control policy implemented by the Bank of Canada. The steady-state of nominal and real interest rates reflect the lender’s degree of time preference, $\beta_P$, and the steady-state gross inflation rate. We use an annual real rate of return of 3.07 percent, which is the average over our sample. This implies that $\left(\frac{\pi^c_{ss}}{\beta_P}\right)^4$ is equal to 1.0307, which yields a $\beta_P$ of 0.9928. As for the calibration of the borrower’s time discount factor, we choose a value of 0.9847, which is the inflation rate divided by $R^m_t$ and is in range of other studies that estimated or calibrated this parameter (Krusell and Smith, 1998; Iacoviello, 2005; Iacoviello and Neri, 2010; Gelain et al., 2013) and which translates into a desire for borrowing. It is important to highlight that we are departing here from a common strategy used in previous studies estimating models of housing dynamics and which consists of assuming zero steady-state inflation. Given that, up to the first order, the steady-state represents the unconditional mean of the variables, our approach has the advantage of centering the model closer to the unconditional mean in the data.

The ratio of patient households relative to impatient households ($\alpha$) is 0.25, the former representing the top quartile of households in the model economy. Parameter $\alpha$ determines the labour share of income and, indirectly, the real estate wealth. In the model, patient households own all the physical capital wealth. By setting $\alpha$ at 0.25, the patient households own 75 percent of total wealth, which is broadly in line with financial and income data\textsuperscript{18}, and mortgage debt as a share of GDP is 0.92 (in the top of the distribution in our sample). It is important to highlight that we are departing here from the commonly used value in the literature, which is 0.79 (Iacoviello and Neri (2010); Lambertini et al. (2010, 2013)). By conducting identification tests, we found that it was not possible to identify this parameter in the absence of actual wealth data.

Following Iacoviello and Neri (2010), the quarterly depreciation rates for housing, capital in the consumption sector and capital in the housing sector are set at 0.011,

\textsuperscript{18}See the Survey of Financial Security from Statistics Canada and the World Top Income Database available on Emmanuel Saez’s website.
0.025 and 0.03, respectively, implying annual depreciation rates of 4.06 percent, 10.38 percent and 12.55 percent, respectively. Likewise, the prices and wages markups $\theta^p$ and $\theta^w$ are set at 7.67, which yields steady-state mark-ups of 15 percent for intermediate goods producers and households.

The capital share of income in the consumption sector, $\gamma^c$, is set 0.25. In the housing sector, we set the capital and land share of income $\gamma^h$ and $\gamma^l$ at 0.10 and 0.35, respectively. These factor shares, along with a weight of housing service in the utility function $\epsilon_{ss}^h$ of 0.8, the intratemporal elasticities $\theta^p_T = \theta^w_T = 0.4$ and the depreciation rates, imply steady-state ratios of consumption, non-housing investment and housing investment to real GDP of approximately 73 percent, 15 percent and 10 percent, respectively. Moreover, these calibration choices imply ratios of business capital and housing wealth (together with $\alpha$) to annual GDP of around 1.5 and 2.2, respectively. Finally, along with the estimated parameters, the land share of income implies that the value of residential land is around 100 percent of GDP, a value close to the empirical data. The parameters $\eta_P$ and $\eta_I$ are set at 2.0 and 1.5, respectively, and the steady-state $\epsilon_{ss}^h$ is set so the steady-state labour supply is 0.37 for lenders and 0.45 for borrowers. The intratemporal elasticity of substitution between sectoral labour supplies are set at 10 for both households, yielding a share of total hours worked of 0.90 in the consumption sector.

Finally, the loan-to-value ratio is set at 0.85, which is the average value in Canada over the last 30 years, while $\epsilon_{ss}^{rm}$ is set at 0.066 to match the average quarterly spread between the risk-free rate and the 5-year mortgage rate over the last 30 years.

### 3.2 Bayesian Approach

The noncalibrated parameters, collected in vector $\Psi$, are estimated by using a Bayesian approach (see DeJong et al. (2000); Lubik and Schorfheide (2006); An and Schorfheide (2007)). Given the sample $X^T = [x_1, \ldots, x_T]$, we are interested in the joint posterior.
distribution of the parameters, given the empirical data,

\[ p(\Psi \mid X^T) = \frac{L(X^T \mid \Psi) p(\Psi)}{\int L(X^T \mid \Psi) p(\Psi) d\Psi}, \]

where \( L(X^T \mid \Psi) \) denotes the likelihood function, \( p(\Psi) \) is the prior distribution, and the denominator is known as the marginal distribution likelihood of the data.

In order to compute the likelihood for a given set of parameters, we solve a log-linear approximation of the equilibrium conditions in the neighborhood of the non-stochastic steady-state (Blanchard and Kahn, 1980). The local approximation method is first-order accurate to the extent that we limit the exogenous processes to be bounded in the neighborhood of the steady state, and the solution is obtained using QZ decomposition (Klein, 2000; Sims, 2002).\(^{19}\) The solution takes the form of a state-space model that is used to compute the likelihood function, and, given the linear solution and the assumption of normally-distributed shocks, the Kalman filter can be used to compute \( L(X^T \mid \Psi) \).

Given the likelihood function, we characterize the posterior distribution in two steps. First, we transform the data (described below) into a form suitable for computing the likelihood function, we use prior distributions for the noncalibrated parameters to incorporate additional information into the estimation, and we maximize the posterior using numerical methods. Finally, we use a metropolis posterior simulator to evaluate the behavior of the posterior distribution and to draw model parameters from the posterior distribution, using the mode obtained in the first step as a starting point.\(^{20}\)

**Prior Distributions** The advantage of using priors is to take our \textit{a priori} beliefs into account in estimating the parameters of the model. The choice of priors is described in the second, third and fourth columns of Tables 3 and 4 for the noncalibrated parameters, and Table 5 for the measurement errors. The priors’ distributions are guided

\(^{19}\)We use a modified version of the Klein (2000) algorithm available in the IRIS Toolbox (http://iristoolbox.codeplex.com/).

\(^{20}\)We use an Adaptive random-walk Metropolis posterior simulator with 500 000 draws with 100 000 burn-in draws and target acceptance ratio of 0.234.
by the constraints in these parameters and are either consistent with previous studies (Levin et al., 2006; Del Negro et al., 2007; Justiniano et al., 2010; Iacoviello and Neri, 2010; Schmitt-Grohe and Uribe, 2012) or fairly diffuse and relatively uninformative.

To reflect their strict positivity, we set a Gamma prior on the investment adjustment costs ($\phi^{k_c}$ and $\phi^{k_h}$) around 5 with a standard error of 2. We select a Beta prior for the Calvo price and wage parameters ($\xi^{p_c}, \xi^{w_c}$ and $\xi^{w_h}$) and the inflation indexation parameters ($\xi^{\pi_c}, \xi^{\pi_w}$ and $\xi^{\pi_h}$) because they belong to the interval $[0,1)$, and due to a lack of consensus on their values in the literature (Christiano et al., 2005; Smets and Wouters, 2007), we set the prior mean at 0.5, with a standard deviation of 0.22.

For all the persistence parameters governing the exogenous processes, we use a Beta prior with a mean equal to 0.80 and a standard deviation equal to 0.1. For all innovations’ and measurement errors’ standard deviations, we use an Inverse-gamma prior with a mean equal to 0.1 and a standard deviation equal to 0.2. These priors are quite disperse and were chosen to generate volatility in the endogenous variables that is broadly in line with the data. Their covariance matrix is assumed to be diagonal.

For the monetary policy specification, we base our priors on a standard Taylor rule with interest rate smoothing that responds gradually to inflation and output gap. We use a Beta prior for $\rho_r$ and $\rho_y$ with means of 0.85 and 0.125 and standard deviations of 0.1 and 0.025, respectively, and a Gamma prior with mean of 1.75 and a standard deviation of 0.25 for $\rho^{\pi_c}$. These priors are in line with previous Canadian studies (Christensen et al., 2009; Dorich et al., 2013).

Finally, we also implement model priors (Andrle and Benes, 2013). Model priors are priors about the model’s features and behavior as a system, such as the sacrifice ratio or the maximum duration of response of inflation to a particular shock, for instance. In our case, since we focus on housing-market related business cycles, we select correlation to be the most relevant model priors to implement. More specifically, we use the first to

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21 A Beta prior with a mean equal to 0.5 and a standard deviation equal to 0.22 yields the same estimation results on these parameters.
fourth-order cross-correlation between consumption, non-housing investment, housing investment, house price and mortgage debt, and we apply a Beta prior with the mean being the the sample first to fourth-order cross-correlation compute on data filtered using a Christiano-Fitzgerald filter (Christiano and Fitzgerald, 2003) with a frequency between 6 and 32 quarters and standard deviation set at 0.1.

**Data** To estimate the model, we use Canadian quarterly data for the period 1983Q2 to 2012Q2. The vector of observables used for the estimation includes fifteen variables: real consumption per capita, real residential investment per capita, real non-residential investment per capita, house price inflation, nominal short-term interest rate, nominal long-term interest rate, core CPI inflation rate, real mortgage debt per capita, capital price inflation, hours worked per capita, wage inflation and capacity utilization rate in the consumption sector, and hours worked per capita, wage inflation and capacity utilization rate in the housing sector. All series are expressed in annualized quarterly growth rate. Figure 1 plots the time series. A detailed description of the series used in the estimation is provided in Appendix A. In addition, we include i.i.d. measurement errors for hours worked, wages and capacity utilization rate.

It is relevant to notice that our set of observables includes more variables than most previous DSGE estimations for housing markets dynamic models. We consider series that are of general interest for policy analysis, such as consumption, investments, real wages, hours worked, inflation and interest rates, which are those usually used in the literature. Our data set also includes variables that may a priori help us identify several features of the model. For instance, disaggregated sectoral data, such as hours worked, wage and capacity utilization rates, will be useful in characterizing movements and correlation that are sectoral specific and could be hidden in aggregated data. Finally, mortgage debt contains information on the reallocation of debt between agents and the preference on consumption and housing services.

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22 We adjust for seasonality before any computation when necessary.
4 Empirical Results

In this section, we first describe the estimated posterior distribution, paying attention to the parameters describing the housing market dynamics. We then perform a posterior analysis to establish the extent to which the model can fit the data.

4.1 Posterior Distributions

The estimated posterior distributions of the noncalibrated parameters are summarized in Tables 3 and 4, while the measurement errors are presented in Table 5. In general terms, the information contained in the likelihood seems to significantly updates the assumed priors for all the parameters, given the marked differences in the statistics describing these two distributions.

The capital adjustment costs seem to differ across sectors. These results could imply that the model requires partial capital mobility across sectors in order to better approximate the data. Cumulated with the imperfect mobility in the labour market, this means that the real frictions caused by imperfect mobility play a significant role in the sub optimal allocation of resources relative to the perfect mobility scenario.

With autoregressive parameters being in general higher that 0.93, the estimated exogenous processes are in general pretty persistent, except for the land stock process and the technology process in the consumption sector, with parameters equal to 0.8 and 0.78, respectively. Housing demand is the most persistent process with an autoregressive parameter equal to 0.9963. In terms of volatility, among the estimated standard error of the exogenous processes, the investment-specific shock seems to be the most volatile, followed the preference shock. However, we will see in the next section that those shocks are not the one that drive the forecast error variance decomposition, mainly because of their persistence.

Regarding parameters measuring nominal rigidities, the estimate of \( \theta_{pr} \) (0.55) implies that prices are reoptimized frequently, once every 2.25 quarters. However, given
the positive value of the indexation parameter ($\nu_c^p = 0.63$), prices change every period, although not in response to change in nominal costs. As for wages, we find that stickiness in the housing sector ($\theta_{wh} = 0.9731$) and in the consumption sector ($\theta_{wc} = 0.9693$) are almost equal. While being reoptimized infrequently, once every 33 quarters, wages are indexed every period to compensate on average almost 80 percent of the steady-state inflation ($\nu_{wh} = 0.7977$ and $\nu_{wc} = 0.7734$).

Finally, with a weight on inflation ($\rho_{\pi} = 2.46$) and a fairly small weight on output gap ($\rho_y = 0.27$), estimates of the parameters of the monetary policy rule are in line with previous evidence (Christensen et al., 2009; Dorich et al., 2013). In terms of the three monetary disturbances, the shock to interest rate spread seems to be the most volatile, but less persistent than the shock to inflation targeting. The monetary policy shock standard error is perfectly with previous studies with Canadian data (Christensen et al., 2009; Dorich et al., 2013).

### 4.2 Second Moments

Figures 4, 5 and 6 present first- and second-order autocorrelation for sets of selected model variables, as well as cross-autocorrelation along with data moments. These figures present moments from data filtered using a Christiano-Fitzgerald filter to isolate periodicity between 6 and 32 quarters, along with filtered theoretical (asymptotic) and simulated moments, both based on the model evaluated using the posterior mode. While the model overestimates the persistence (first- and second-order autocorrelation) of non-residential investment and wages in the housing sector and underestimates the persistence in hours worked in the consumption sector, it is able to replicate well all the other autocorrelations of the data, with the sample autocorrelation always being in the simulated distribution.

The model also matches both the sign and the level of the cross-correlation for most of the desired relationship being studied (figure 6). The theoretical and simulated cross-correlation of consumption with housing investment and house prices are both

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23Simulated moments are computed based on 1000 Monte Carlo simulations.
in line, in sign and level, with their data counterparts. However, the correlation between consumption and non-residential investment is estimated to be negative and mild, whereas it is positive and large in the data. This is due to the construct of the model, which assumes that consumption and non-residential investment are both produce in the same sector, making reallocation easy between them. Perhaps this simple assumption makes it difficult to match a more complex production structure in the real economy. The estimated correlation between consumption and mortgage debt is also of the good sign, but is twice the correlation seen in the data. Lastly, the model tends to imply a positive contemporaneous correlation between house prices and mortgage debt, while in the data we only observe a lagged relationship. Overall, the model seems to properly replicate the behavior of the variables in terms of autocorrelation and cross-correlation, but has some difficulties matching dynamics of consumption and investment in capital.

4.3 Variance Decomposition

After establishing the extent to which the estimated model can replicate the business cycle observations, we proceed with the variance decomposition. Table 6 presents the unconditional variance decomposition for both the observables (in growth rate) and the model variables representing the observables (in level) for all the shocks in the model, the last column being the sum of the contributions of all measurement errors. This section focuses on the model variables representing the observables (in level).

In terms of explaining the consumption fluctuations, the labour supplies shock and the inflation-targeting shock appear to be the most significant: over 60 percent of consumption volatility is due to the former, while the latter explains 16 percent. The labour supply shock affects directly the labour income of the agents, while the inflation-targeting shock cause, for a given increase in price level, variation in the reaction of the monetary policy across periods. In addition, both the housing demand shock and the investment specific shock explain over 8 percent of the consumption forecast error variance. The other drivers do not seem to play a significant role in
explaining consumption fluctuations. These results are in contrast with other studies that identified technology in the consumption sector shocks and monetary policy shocks among the main drivers of consumption volatility.

Together, the housing demand shock and the technology shock in housing sector account for almost 80 percent of the total house price volatility and 79 percent of the total housing investment volatility.

4.4 Shock Responses

In the last section, we analysed how well estimated model can replicate the business cycle observations. It is also important to understand what are the dynamics in the model implied by the shocks. In the section, we will focus on five of them that explained most the volatility in the model.

**Housing Demand**  Figure 9 plots impulse responses to the estimated housing demand shock. Overall, it raises on impact house prices and returns on housing investment. Since borrowers’ collateral is linked to house prices, they can increase their level of borrowing and consumption. Given their higher marginal propensity to consume, the effects on total consumption is positive and entirely driven by borrowers, their consumption increase being high enough to compensate the decrease in lenders’ consumption.

Housing demand shocks generate a co-movement between house prices, consumption, residential investment and hours worked (not shown) in both sectors of production observed in the data, especially during periods of housing booms. Shocks affect economic choices and, in particular, the housing and credit decisions of households. The occurrence of a positive housing demand shock prompts appreciations in housing prices and fuels current housing demand. Consequently, housing investment rises quickly on impact, with a peak increase of over 3 percent. House prices follow the same curve, with a peak increase of 2 percent on impact. Mortgage debt increases significantly, by more than 15 percent, reflecting the increase in housing investment but
also the increase in house price that affect the value of all the undepreciated housing stock. This increase in the collateral value boosts the consumption of borrowers and causes inflationary pressures, which has for effect of increasing interest rates. Moreover, due to limits to credit, borrowers increase their labour supply in order to raise funds for housing investments. However, coupled with a decrease in non-residential investment, wages rise. The increase in consumption and housing investment makes also GDP rise. Thus, housing demand shocks in this model generate pro-cyclicality among relevant variables.

**Housing Technology**  Figure 16 presents the model’s response to a one-standard deviation shock in housing technology. A positive technology shock in the housing sector leads to a rise in housing investment and a drop in housing prices due to the productivity gain. The negative net effect on the aggregate consumption masks two different realities. The positive shock in housing production technology induces an increase in real wage in both sectors, thus increasing labour income for both type of households. On the lenders side, this is translated in higher consumption and higher investments of all types (residential and non-residential in both sectors). On the borrowers side, this increase in labour income is not enough to compensate the decrease in house prices that translates into a decrease in the value of their collateral. This decrease in the value of the collateral, not compensated by the decrease in borrowing costs, has a negative impact on the level of consumption and mortgage debt. Finally, the persistence of the technology shock, cumulated with the persistence and the irreversibility of housing investment, maintains the real house prices lower than its steady-state value for a long period.

**Labour Supply**  Figure 10 presents the model’s response to a one-standard deviation shock in labour supply. This shock induces a greater disutility of hours worked to agents, causing an immediate decrease in hours worked in both sectors. This decrease leads to an increase in real wages in both sectors as the productivity increase slightly. Driving up the marginal cost of production, it is gradually transmitted to the inflation
in the consumption sector, which drives interest rates up via the monetary policy response. The decrease in labour income and the increase in borrowing costs leads to a decrease in housing investment from the borrowers and a real house price decline, thereby reducing collateral values.

**Monetary and Inflation Targeting** Figure 12 plots the effects of monetary policy shocks. The temporary shock leads to a rise in the nominal and real short-term interest rates, a fall in output, consumption and residential and non-residential investment. In line with the stylised facts on monetary policy shocks, real wages fall (not shown). The largest effect on consumption is about 1.5 times the one on non-residential investment. Overall, these effects are consistent with the evidence found in the literature.

Finally, figure 11 presents the model’s response following a one-standard deviation shock in inflation targeting. An increase in the inflation target means that, following an increase in inflation, the central bank will not increase the interest rate as much as in steady-state. The effects of a persistent change in the inflation objective are strikingly different from the monetary policy shock in one aspect. First, there is a liquidity effect, as nominal interest rates start increasing immediately as a result of the increased inflation expectations. Inflation picks up immediately, driven by an increase in consumption and housing investment. Interest rates continue to increase in response to higher inflation, this time at a lesser pace.

### 5 Housing Market Boom-Bust and Loan-to-Value Ratio

In this section, we first highlight key findings regarding the transmission mechanism of news shocks and describe the role of news shocks in housing market dynamics. We analyze by simulation the impact of news shocks on selected variables over the business cycle and show that news shocks can generate boom-bust housing market cycles. Then, we examine the effectiveness of implementing different countercyclical LTV ratios and compare it to the performance of a simple Taylor-type monetary policy rule augmented with house price inflation.
5.1 News Shocks and Housing Market Boom-Bust

It is well-known in the literature that rational-expectations DSGE models incorporating housing market and financial frictions can hardly generate boom-bust cycles. In general, macroeconomic models of housing markets mainly rely on fundamental developments in the economy to explain fluctuations in house prices and residential investment. However, survey evidence shows that house price dynamics are greatly related to macroeconomic expectations, especially to optimism about future house prices appreciations (Gomes and Mendicino, 2012). In our model, a natural way to incorporate expectations on housing prices is the exogenous process on housing demand.

Figure 18 presents, for key macroeconomic variables, the simulated impact of introducing anticipated shocks on housing demand within the model.\(^{24}\) During this exercise, we assume that economic agents have in period \(t\) an information set \(\Omega_t\) that goes beyond current and past realizations of \(\varepsilon_h\). The housing demand innovation \(\varepsilon_{t-i}^h\), \(\forall i\), is now composed of unanticipated and anticipated components:

\[
\varepsilon_{t-i}^h = \sum_{j=0}^\infty \varepsilon_{t-i-j}^h.
\]

In this formulation, agents observe the current and past values of the housing demand news shocks \(\varepsilon_{t-i}^h\). The notation \(\varepsilon_{t-i-j}^h\), \(\forall i, j\), means that the anticipated disturbances (or news shocks) learnt in \(t - i\) will affect the economy in \(j\) periods ahead (i.e. we learn in \(t - i\) a news that will happen in \(t - i + j\)). More specifically, the disturbance \(\varepsilon_{t}^{h,1}\) represents an innovation to \(\varepsilon_{t+1}^{h}\), which is announced in period \(t\) but materializes only in period \(t + 1\). Note that \(\varepsilon_{t}^{h,1}\) does not appear in the expression for \(\varepsilon_{t}^{h}\) given above. Rather, the above expression features \(\varepsilon_{t-1}^{h,1}\), the one-period-ahead announcement made in period \(t - 1\). Similarly, \(\varepsilon_{t}^{h,2}\), \(\varepsilon_{t}^{h,3}\) and \(\varepsilon_{t}^{h,4}\) would be observed in \(t\) and represent two-, three-, and four-period-ahead announcements of future changes in the housing demand.

\(^{24}\)Based on the forward expansion available in IRIS Toolbox.
In this set-up, $\varepsilon_{t}^{h, 0}$ can be viewed as the usual contemporaneous (i.e. unanticipated) disturbances to $\varepsilon_{t}^{h}$.

Three simulations are presented in the figure 18:

**Case 1: Unanticipated shocks**  The baseline scenario is a series of four unanticipated shocks (i.e. $\varepsilon_{t}^{h, 0}$) on housing demand. We assume an increasing value of $\varepsilon_{t}^{h, 0}$ from $t + 4$ to $t + 7$:

- $\varepsilon_{t+4}^{h, 0} = 0.5\sigma_{e_{h}}$
- $\varepsilon_{t+5}^{h, 0} = 1.0\sigma_{e_{h}}$
- $\varepsilon_{t+6}^{h, 0} = 1.5\sigma_{e_{h}}$
- $\varepsilon_{t+7}^{h, 0} = 2.0\sigma_{e_{h}}$

**Case 2: Anticipated shocks with revision**  It consists of a series of four anticipated shocks learnt in $t$ but unrealized (i.e. revised by an equivalent negative unanticipated shock in the period it was supposed to happen):

- $\varepsilon_{t}^{h, 0} = 0.5\sigma_{e_{h}}$, but revised with $\varepsilon_{t+4}^{h, 0} = -0.5\sigma_{e_{h}}$
- $\varepsilon_{t}^{h, 5} = 1.0\sigma_{e_{h}}$, but revised with $\varepsilon_{t+5}^{h, 0} = -1.0\sigma_{e_{h}}$
- $\varepsilon_{t}^{h, 6} = 1.5\sigma_{e_{h}}$, but revised with $\varepsilon_{t+6}^{h, 0} = -1.5\sigma_{e_{h}}$
- $\varepsilon_{t}^{h, 7} = 2.0\sigma_{e_{h}}$, but revised with $\varepsilon_{t+7}^{h, 0} = -2.0\sigma_{e_{h}}$

**Case 3: Anticipated shocks with revision but no monetary policy reaction**  This case is the same as Case 2, but we exogenise the interest rate so it always stays at its steady-state value.

As expected, the unanticipated shocks plot on figure 18 start to have an impact in $t + 4$, when the shock happen. Therefore, the impacts under this scenario are the same as the impulse response following a housing demand shock described in section 4. We observe a positive co-movement between housing investment, house price and consumption, but also a monetary policy reaction following the slight increase in
inflation and the deviation of GDP from its steady-state value. Finally, housing demand increases following the four positive shocks from $t + 4$ to $t + 7$, and stays nearly flat afterward given that the persistence parameter is close to 1.

The story is different for the anticipated shock scenario. News shocks generate co-movement between house prices, consumption and residential investment, but also hours worked (not shown) in both sectors of production observed in the data, especially during periods of housing booms. News shocks affect economic choices and, in particular, the housing and credit decisions of households differently than unanticipated shocks. Expectations about the occurrence of positive housing demand shocks immediately generate beliefs of future appreciations in housing prices and fuel current housing demand. All the agents learn about the positive news shocks at the same moment, in $t + 1$, in contrast with the unanticipated shock case, where the information was slowly diffused over four periods. Consequently, housing investment rise quickly on impact, with a peak increase of near 11 percent in $t + 4$. House prices follow the same curve, with a peak increase of 9 percent in $t + 4$. Mortgage debt increases significantly, by more than 60 percent, reflecting the increase in housing investment, but also the increase in house prices that affect the value of all the undepreciated housing stock. This increase in the collateral value boosts the consumption of borrowers and fuels inflation, inducing a rise in interest rates. Overall, as news spread, the value of housing collateral increases and the rise in house prices is, thus, coupled with an expansion in household’s credit and consumption. Moreover, due to limits to credit, borrowers increase their labour supply in order to raise internal funds for housing investments. For the decrease in non-residential investment to be coupled with an increase in hours, wages rise. The increase in consumption and housing investment also causes GDP to rise. Thus, news shocks in this model generate pro-cyclicality among relevant variables. However, in $t + 4$, agents learn about the housing demand and revise their views on the current state of the economy: positive housing demand shock has not occur. Housing investment and house prices start to decline on impact, followed by mortgage debt. The collateral value then starts to drop and agents have
to revise their consumption level. The same mechanism occurs every time when the positive housing demand news shock does not materialize. In $t+8$, housing investment declines by 12 percent and house prices by 9 percent. Moreover, from peak to through, consumption level decline by near 25 percent, and the real GDP decline by close to 10 percent, generating a recession. During that period, housing demand never moves. All of this resulted only from unrealized expectations.

Finally, the last case uses the same path of news and unanticipated shocks, but exogenises the interest rate and makes it non-reactive to changes in the economic state. The resulting dynamic is the same as in for anticipated shocks with monetary policy reaction, but with stronger macroeconomic variables reactions due to the non-reaction of interest rates to inflation and GDP increases.

Overall, the three case scenarios suggests that news shocks could play an important role in boom-bust housing market cycles as they can generate co-movement between consumption, housing investment and house prices, similar to what is observed in the data, especially during periods of housing booms. News shocks contribute to the boom- and bust-phases in house prices.

5.2 Countercyclical LTV

We now study the effectiveness of implementing a countercyclical LTV ratio to reduce or eliminate the amplitude of the boom-bust cycle describes above (i.e. Case 2, anticipated shocks). First, we consider two countercyclical LTV ratios. In both cases, we assume that the monetary policy authority continues to follow the estimated Taylor-type rule and we allow the LTV ratio to vary around its long-run setting of 85 percent. The first rule considered is based on the deviation of house prices from their steady-state

$$\omega_t = \omega_{ss} \left( \frac{q^h_t}{q^h} \right)^{-\phi}$$
while the second rule is based on the deviation of the debt-to-GDP ratio from its steady-state
\[ \omega_t = \omega_{ss} \left( \frac{M_t/y_t}{M/y} \right)^{-\phi_\omega}, \]
with \( \phi_\omega \) being the countercyclical parameter.\(^{25}\) Finally, we compare the results of these regulatory LTV policies with the performance of a Taylor-type monetary policy rule augmented with house prices inflation
\[ \frac{R_t}{R_{ss}} = \left( \frac{R_{t-1}}{R_{ss}} \right)^{\rho_r} \left( \frac{\pi_t}{\pi_{ec}} \right)^{(1-\rho_r)\rho_{\pi c}} \left( \frac{Y_t}{Y_{ss}} \right)^{(1-\rho_r)\rho_y} \left( \frac{\pi_h}{\pi_h} \right)^{(1-\rho_r)\rho_{\pi h}} \exp \left( \varepsilon_t^R \right). \]

Figures 19, 20 and 21 show the simulation results. We implemented three cases of macroprudential policy. For both rules the countercyclical LTV, we implemented the rules with \( \phi_\omega = \{0.0, 0.5, 1.0\} \), the case with the countercyclical parameter equal to 0 replicating Case 2 (not countercyclical LTV) to facilitate comparisons. In the case of monetary policy, three value of the parameters are considered, namely \( \rho_{\pi h} = \{0.0, 1.0, 2.0\} \), the case with the house price inflation parameter equal to 0 replicating Case 2 (not countercyclical LTV) to facilitate comparisons. When policy is based on house prices, for both parameters values the countercyclical LTV ratio does not reduce the surge in housing investment and house prices. Expectations about the occurrence of positive housing demand shocks still immediately generate beliefs of future appreciations in housing prices and fuel current housing demand. However, the transmission mechanism creating a spillover effect on consumption via loosening of the collateral constraint is greatly reduced when \( \phi_\omega = 0.5 \) and eliminated when \( \phi_\omega = 1.0 \). Therefore, we still experience a house price correction of near 10 percent and a housing investment decrease of 12 percent when agents realize that the expectations do not materialize, but this does not lead to a recession. When policy is based on the debt-to-GDP ratio, as in the previous rule studied, house prices and housing investment are not affected by LTV, because the housing demand news

\(^{25}\)This choice of tested policies is based on the literature but remains arbitrary. Many other rules for countercyclical LTV could be tested, but a full comparative study based on welfare criterion would be necessary. This paper can serve as a starting point to welfare studies.
shocks dominates, but the transmission mechanism is greatly reduced. However, when \( \phi^\omega = 1.0 \), the wealth effect vis the collateral constraint is still materialize and we still observe a decline in consumption.

Finally, we consider a modified Taylor-type rule augmented with house price inflation. As expected, the effects of including the house price inflation in the monetary policy rule are more diffuse and affect all the macroeconomic variables. It helps reducing the house prices and housing investment impacts of news shocks on housing demand; however, it is less effective than a sectoral policy like the LTV ratio to target uniquely the spillover of the wealth effect on consumption via the loosening of the collateral constraint.

6 Conclusion

The objective of this chapter was twofold. We wanted to study if there is any statistical evidence suggesting an important collateral link between the housing market and the rest of the economy, and, if such a link exists, to determine whether it is mostly demand- or supply-driven. We also followed the news shocks literature and looked if a housing-market boom-bust cycle can arise endogenously following unrealized expectations of a rise in housing demand. To this end, we constructed a New Keynesian model in which a fraction of households borrow against the value of their houses. We estimated the model with Canadian data using Bayesian methods and assessed the model’s ability to capture key features of consumption and house price data. Finally, we performed an analysis to determine how well the introduction of a countercyclical loan-to-value (LTV) ratio can reduce household indebtedness and housing price fluctuations compared to a monetary policy rule augmented with house price inflation.

We find statistical evidence suggesting an important collateral link between the housing market and the rest of the economy, and this link is mainly driven by demand factors. We also find that the introduction of news shocks can generate a housing market
boom-bust cycle, the bust following unrealized expectations on housing demand. Our estimated model explains several features of the data. At the cyclical frequencies, it matches the observation that both housing prices and housing investments are strongly procyclical with consumption (and then with GDP), volatile and sensitive to interest rates.

Our study also suggests that higher loan-to-value ratios can amplify housing-market boom-bust cycles by encouraging speculative housing investments by credit-constrained borrowers, but the amplification effect is mainly concentrated via the collateral constraints. However, our study suggests that, in line with preceding dynamic equilibrium models with credit-constrained borrowers considered by Iacoviello (2005) and Kiyotaki et al. (2010), the loan-to-value ratio does not significantly alter aggregate house-price dynamics, but it is a viable tool to mitigate the spillover effects via the collateral constraint.

As pointed out in Iacoviello and Neri (2010), a good part of the fluctuations in housing prices and housing investment observed in the data are viewed by the model as the outcome of the exogenous shift in housing demand. This shock potentially includes unmodeled features of the model. The housing investment is mainly made at the household level, while our data are per capita. With the constant decrease in the number of persons per household observed since the beginning the 70s, this dynamics is probably capture within the housing demand shock. Also, using perturbation methods, it is hard to model exogenous change in policy, as the one we observed in regulatory LTV ratios over the last 15 years. Changes in LTV requirements could potentially have been captured in the housing demand shock. These elements are interesting questions for further research.
References


IMF (2012). Chapter 3 dealing with household debt.


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Table 1: Calibrated Parameters

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<th>Parameter</th>
<th>Value</th>
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## Table 2: Steady-State Ratios

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Table 4: PRIOR AND POSTERIOR DISTRIBUTIONS OF EXOGENOUS PROCESSES

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### Table 6: Unconditional Forecast Error Variance Decomposition

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<th>$\varepsilon^{R^m}$</th>
<th>$\varepsilon^l$</th>
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Figure 1: DATA, FROM 1983Q2 TO 2012Q2
Figure 2: Estimated (Smoothed) Shocks, from 1983Q2 to 2012Q2
Figure 3: Histograms of Estimated (Smoothed) Shocks, from 1983Q2 to 2012Q2.
Figure 4: Theoretical, Simulated and Sample 1st Order Autocorrelation of Selected Model Variables
Figure 5: Theoretical, Simulated and Sample 2nd Order Autocorrelation of Selected Model Variables
Figure 6: Theoretical, Simulated and Sample Cross-Correlation for Selected Model Variables

- Consumption
- Non-Residential Investment
- Residential Investment
- Mortgage Debt
- House Price
Figure 7: Forecast Error Variance Decomposition for Selected Model Variables
Figure 8: Shock Response Function - Preference Shock

- GDP Deviation

- Consumption

- Housing Investment

- Capital Investment

- Mortgage Debt

- Housing Price

- Short-Term Interest Rate

- Inflation

- Quarterly Deviation

- Quarterly Quarters
Figure 9: Shock Response Function - Housing Demand Shock
Figure 10: SHOCK RESPONSE FUNCTION - LABOUR SUPPLY SHOCK
Figure 11: Shock Response Function - Inflation Targeting Shock
Figure 12: SHOCK RESPONSE FUNCTION - INTEREST RATE SHOCK

**GDP**

**Consumption**

**Capital Investment**

**Housing Investment**

**House Price**

**Mortgage Debt**

**Short-Term Interest Rate**

**Inflation**
Figure 13: SHOCK RESPONSE FUNCTION - MORTGAGE PREMIUM SHOCK

- GDP Deviation
- Consumption
- Housing Investment
- Housing Prices
- Mortgage Debt
- Capital Investment
- Inflation
- Short-Term Interest Rate
Figure 14: SHOCK RESPONSE FUNCTION - LAND STOCK SHOCK

- GDP Deviation
- Consumption
- Housing Investment
- Capital Investment
- Mortgage Debt
- House Price
- Housing Investment
- Short-Term Interest Rate
- Inflation
- Mortgage Debt

Quarters: 5 10 15 20 25 30 35 40
Figure 15: Shock Response Function - TFP (Consumption Sector) Shock

- GDP Deviation
- Consumption
- Housing Investment
- Capital Investment
- Mortgage Debt
- Housing Price
- Inflation
- Short-Term Interest Rate

Quarters

GDP Deviation
Consumption
Housing Investment
Capital Investment
Mortgage Debt
Housing Price
Inflation
Short-Term Interest Rate

Quarters
Figure 16: Shock Response Function - TFP (Housing Sector) Shock

- Capital Investment
- Consumption
- Housing Investment
- Mortgage Debt
- GDP
- House Price
- Inflation
- Short-Term Interest Rate

Quarters
Figure 17: Shock Response Function - Investment Shock

- Capital Investment
- Consumption
- House Price
- Inflation
- GDP
- Housing Investment
- Short-Term Interest Rate

Quarters
5 10 15 20 25 30 35 40
-0.14 -0.12 -0.1 -0.08 -0.06 -0.04 -0.02 0
Inflation
Quarters
5 10 15 20 25 30 35 40
-0.6 -0.4 -0.2 0 0.2 0.4 0.6
Mortgage Debt
Quarters
5 10 15 20 25 30 35 40
-2 -1.5 -1 -0.5 0 0.5 1 2 3 4 5
House Price
Quarters
5 10 15 20 25 30 35 40
-0.2 0 0.2 0.4 0.6
Housing Investment
Quarters
5 10 15 20 25 30 35 40
-0.08 -0.06 -0.04 -0.02 0
Short-Term Interest Rate
Quarters
5 10 15 20 25 30 35 40
-0.5 0 0.5 1 1.5 2
GDP
Quarters
5 10 15 20 25 30 35 40
-0.6 -0.4 -0.2 0 0.2 0.4 0.6
Consumption
Quarters
5 10 15 20 25 30 35 40
-1 -0.5 0 0.5 1 1.5 2
GDP
Figure 18: Anticipated and Unanticipated Housing Demand Shock
Figure 19: COUNTERCYCLICAL LTV POLICY BASED ON HOUSE PRICE

- GDP
- Consumption
- House Price
- Inflation
- Housing Investment
- Short-Term Interest Rate

Countercyclical parameter equal to 0
Countercyclical parameter equal to 0.5
Countercyclical parameter equal to 1.0
Figure 20: Countercyclical LTV Policy based on Debt-to-GDP Ratio

- GDP
- Consumption
- House Price
- Housing Demand
- Capital Investment
- Mortgage Debt
- Housing Investment
- Short-Term Interest Rate
- Inflation
Figure 21: Monetary Policy Responsive to House Price Inflation

- House price inflation parameter equal to 0
- House price inflation parameter equal to 1
- House price inflation parameter equal to 2

Variables:
- Capital Investment
- Consumption
- House Price
- Inflation
- GDP
- Mortgage Debt
- Housing Demand
- Housing Investment
- Housing Price
- Short-Term Interest Rate
- Capital Investment
- Consumption
- House Price
- Inflation
- GDP
- Mortgage Debt
- Housing Demand
- Housing Investment
- Housing Price
- Short-Term Interest Rate

Quarters:
- 

Y-axes:
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10

GDP Percent vs. Quarters

Consumption Percent vs. Quarters

Inflation Percent vs. Quarters

House Price Percent vs. Quarters

Mortgage Debt Percent vs. Quarters

Housing Demand Percent vs. Quarters

Housing Investment Percent vs. Quarters

Short-Term Interest Rate Percent vs. Quarters
Appendices

A Definitions and Data Sources

Consumption ($c_t$)

1983Q2 to 2012Q2 data

Real (chained 2007 dollars) household consumption expenditure on non-durable goods, semi-durable goods and services per capita (number of persons of working age, 15 years and over), seasonally adjusted at annual rates. We compute the annualised quarterly growth rate and remove the mean.

Source: Statistics Canada (Cansim Tables 282-0001 and 380-0064), Internal Calculations

Core CPI inflation rate ($\pi_c^t$)

1983Q2 to 2012Q2 data

All-items CPI excluding eight of the most volatile components and the core CPI. We splice both series, compute the annualized quarterly growth rate and remove the Bank of Canada’s inflation target of 2 percent.

Source: Statistics Canada (Table 326-0020), Internal Calculations

Residential investment ($y_h^t$)

1983Q2 to 2012Q2 data

Real (chained 2007 dollars) business gross fixed capital formation in residential structures per capita (number of persons of working age, 15 years and over), seasonally adjusted at annual rates. We compute the annualised quarterly growth rate and remove the mean.

Source: Statistics Canada (Cansim Tables 282-0001 and 380-0064), Internal Calculations

House price inflation rate ($\pi_h^t$)

1983Q2 to 2012Q2 data
Nominal house prices. We compute the annualised quarterly growth rate and remove the Bank of Canada’s inflation target of 2 percent.

*Source:* *Multiple Listing Service (MLS)*

**Non-residential investment** \((i_t^k)\)

*1983Q2 to 2012Q2 data*

Real (chained 2007 dollars) business gross fixed capital formation in non-residential structures, machinery and equipment per capita (number of persons of working age, 15 years and over), seasonally adjusted at annual rates. We compute the annualised quarterly growth rate and remove the mean.

*Source:* *Statistics Canada (Cansim Tables 282-0001 and 380-0064), Internal Calculations*

**Capital price inflation rate** \((\pi_t^k)\)

*1983Q2 to 2012Q2 data*

Nominal implicit price index of business gross fixed capital formation in non-residential structures, machinery and equipment. We compute the annualised quarterly growth rate and remove the Bank of Canada’s inflation target of 2 percent.

*Source:* *Statistics Canada (Cansim Table 380-0066), Internal Calculations*

**Mortgage debt** \((b_{i,t})\)

*1983Q2 to 2012Q2 data*

Real (core CPI) residential mortgage credit per capita (number of persons of working age, 15 years and over), seasonally adjusted. We compute the annualised quarterly growth rate and remove the mean.

*Source:* *Statistics Canada (Cansim Tables 282-0001 and 176-0069), Internal Calculations*

**Nominal short-term interest rate** \((R_t)\)

*1983Q2 to 2012Q2 data*

Treasury bills rate, 3-months. We remove a linear trend.

*Source:* *Statistics Canada (Cansim Table 176-0043), Internal Calculations*
Nominal long-term interest rate ($R^m_t$)

1983Q2 to 2012Q2 data

Average residential mortgage lending rate, 5 years. We remove a linear trend.

Source: Statistics Canada (Cansim Table 176-0043), Internal Calculations

Hours worked in consumption sector ($n^c_t$)

1983Q2 to 2012Q2 data

Hours worked in the consumption sector per capita (number of persons of working age, 15 years and over). The full computation methodology for this series is available upon request. We compute the annualised quarterly growth rate and remove the mean.

Source: Statistics Canada (Cansim Tables 281-0001, 281-0002, 281-0023, 281-0026 and 282-0001), Internal Calculations

Wage inflation rate in consumption sector ($\pi^w^c_t$)

1983Q2 to 2012Q2 data

Nominal wages in the consumption sector. The full computation methodology for this series is available upon request. We compute the annualised quarterly growth rate and remove the Bank of Canada’s inflation target of 2 percent.

Source: Statistics Canada (Cansim Tables 281-0004 and 281-0031), Internal Calculations

Capacity utilization rate in consumption sector ($u^k^c_t$)

1983Q2 to 2012Q2 data

Capacity utilization rate in the consumption sector. The full computation methodology for this series is available upon request. We compute the annualised quarterly growth rate and remove the mean.

Source: Statistics Canada (Cansim Tables 028-0001, 028-0002 and 031-0003), Internal Calculations

Hours worked in housing sector ($n^h_t$)

1983Q2 to 2012Q2 data

Hours worked in the housing sector per capita (number of persons of working age, 15 years and over).
years and over). The full computation methodology for this series is available upon request. We compute the annualised quarterly growth rate and remove the mean.

Source: Statistics Canada (Cansim Table 281-0001, 281-0002, 281-0023, 281-0026 and 282-0001), Internal Calculations

Wage inflation rate in housing sector \( (\pi_t^{wh}) \)

1983Q2 to 2012Q2 data

Nominal wages in the housing sector. The full computation methodology for this series is available upon request. We compute the annualised quarterly growth rate and remove the Bank of Canada’s inflation target of 2 percent.

Source: Statistics Canada (Cansim Tables 281-0004 and 281-0031), Internal Calculations

Capacity utilization rate in housing sector \( (u_t^{kh}) \)

1983Q2 to 2012Q2 data

Capacity utilization rate in the housing sector. The full computation methodology for this series is available upon request. We compute the annualised quarterly growth rate and remove the mean.

Source: Statistics Canada (Cansim Tables 028-0001, 028-0002 and 031-0003), Internal Calculations
B Optimization

Lagrangian for lenders optimization problem  The Lagrangian for the lenders optimization problem takes the following form:

\[ L = E_0 \sum_{t=0}^{\infty} \beta_P^t e^h U (c_{P,t}, h_{P,t}, n_{P,t}^c, n_{P,t}^h) + \lambda_{P,t}^c \times \]

\[
\left\{ \begin{array}{l}
n_{P,t}^c \int_0^1 w_{P,t,t}^c \left( \frac{w_{P,t}^c}{w_{P,t}} \right)^{-\theta^p} \frac{d \ell}{\ell} + n_{P,t}^h \int_0^1 w_{P,t,t}^h \left( \frac{w_{P,t}^h}{w_{P,t}} \right)^{-\theta^p} \frac{d \ell}{\ell} \\
+ r_{t}^c u_{i}^k k_{t-1}^c + r_{t}^h u_{i}^k k_{t-1}^h + (q_{t}^c + r_{t}^c) l_{t-1} + \frac{R_{t-1}^b P_{t-1}^h}{\sigma_t^h} + f_t^c + f_t^h + f_{t}^l \\
+ \frac{1}{\phi_m} \sum_{s=1}^{\phi_m} \frac{d_{t-s}}{\prod_{v=s}^{\phi_m} \pi_{t+v}} + \sum_{s=1}^{\phi_m} (R_{t-s}^d - 1) \left( \frac{\phi_{m-s}^m}{\phi_m} \right) \frac{d_{t-s}}{\prod_{v=s}^{\phi_m} \pi_{t+v}} \\
- c_{P,t} - q_{t}^c k_{t}^c - q_{t}^h k_{t}^h - q_{t}^l l_t - b_{P,t} - d_t \\
- q_{t}^c \left[ h_{P,t}^c - (1 - \delta^c) h_{P,t-1} \right] \\
- q_{t}^c \left[ k_{t}^c - (1 - \delta^c) k_{t-1}^c - z_i^c i_{t}^c \left[ 1 - \frac{\phi^c}{2} \left( \frac{i_{t}^c}{i_{t-1}^c} - 1 \right) \right] \right] \\
- q_{t}^h \left[ k_{t}^h - (1 - \delta^c) k_{t-1}^h - z_i^h i_{t}^h \left[ 1 - \frac{\phi^h}{2} \left( \frac{i_{t}^h}{i_{t-1}^h} - 1 \right) \right] \right] \\
+ \frac{w_{P,t}^c}{\lambda_{P,t}^c} \left[ n_{P,t}^c - n_{P,t}^c \int_0^1 \left( \frac{w_{P,t}^c}{w_{P,t}} \right)^{-\theta^c} \frac{d \ell}{\ell} \right] + \frac{w_{P,t}^h}{\lambda_{P,t}^h} \left[ n_{P,t}^h - n_{P,t}^h \int_0^1 \left( \frac{w_{P,t}^h}{w_{P,t}} \right)^{-\theta^h} \frac{d \ell}{\ell} \right] \end{array} \right\} 
\]

The first-order necessary conditions for \( c_{P,t}, h_{P,t}, n_{P,t}^c, n_{P,t}^h, k_{t}^c, k_{t}^h, l_t, u_{t}^c, u_{t}^h, i_{t}^c, i_{t}^h, \) \( b_{P,t} \) and \( d_t \) are, respectively,

\[
\lambda_{P,t}^c = \frac{(1 - \epsilon_t^h)^{\frac{1}{\phi_p} c_{P,t}}}{(1 - \epsilon_t^h)^{\frac{1}{\phi_p} c_{P,t} + \frac{1}{\phi_p} b_{P,t}}}, \quad (51) 
\]

\[
\lambda_{P,t}^c q_{t}^h - \beta_P \left( 1 - \delta^h \right) E_t \left[ \lambda_{P,t+1}^c q_{t+1}^h \right] = \frac{(\epsilon_t^h)^{\frac{1}{\phi_p} h_{P,t}}}{(1 - \epsilon_t^h)^{\frac{1}{\phi_p} c_{P,t} + \frac{1}{\phi_p} b_{P,t}}}, \quad (52) 
\]

\[
e_t^h \left( n_{P,t}^c \right)^{\frac{\phi_{p+1}}{\phi_p}} + \left( h_{P,t} \right)^{\frac{\phi_{p+1}}{\phi_p}} \right)^{\frac{\phi_{p+1}}{\phi_p}} \left( n_{P,t}^c \right)^{\frac{1}{\phi_p}} = \frac{\lambda_{P,t}^c w_{P,t}^c}{\lambda_{P,t}^c}, \quad (53) 
\]
\[
\epsilon_t^n \left( n_{P,t}^c \frac{\delta^p + 1}{\delta^p_p} + n_{P,t}^h \frac{\delta^p + 1}{\delta^p_p} \right) \frac{\delta^p + 1}{\delta^p_p} = \frac{\lambda_{P,t} u^h_{P,t}}{\lambda_{P,t}^n}, \tag{54}
\]

\[
\lambda^{c}_{P,t} q^{k_e}_{t} = \beta_P E_t \left[ \lambda^{c}_{P,t+1} (u_{t+1}^{k_e} r_{t+1}^{k_e} + q_{t+1}^{k_e} (1 - \delta_{t+1}^{k_e})) \right] \tag{55}
\]

\[
\lambda^{c}_{P,t} q^{k_h}_{t} = \beta_P E_t \left[ \lambda^{c}_{P,t+1} (u_{t+1}^{k_h} r_{t+1}^{k_h} + q_{t+1}^{k_h} (1 - \delta_{t+1}^{k_h})) \right], \tag{56}
\]

\[
\lambda^{c}_{P,t} q^{l}_{t} = \beta_P E_t \left[ \lambda^{c}_{P,t+1} (q_{t+1}^{l} + r_{t+1}^{l}) \right], \tag{57}
\]

\[
r^{k_e}_{t} = q^{k_e}_{t} \left[ \delta^{k_e}_{1} + \delta^{k_e}_{2} (u^{k_e}_{t} - 1) \right], \tag{58}
\]

\[
r^{k_h}_{t} = q^{k_h}_{t} \left[ \delta^{k_h}_{1} + \delta^{k_h}_{2} (u^{k_h}_{t} - 1) \right], \tag{59}
\]

\[
\lambda^{c}_{P,t} \left( 1 - q^{k_e}_{t} z_{t} \right) \left( 1 - \frac{\phi^{k_e}_{t}}{2} \left( \frac{i_{t+1}^{k_e}}{i_{t-1}^{k_e}} - 1 \right)^2 \phi^{k_e}_{t} \left( \frac{i_{t+1}^{k_e}}{i_{t-1}^{k_e}} - 1 \right) \right) = \beta_P E_t \left[ \lambda^{c}_{P,t+1} q^{k_e}_{t+1} z_{t+1} \phi^{k_e}_{t} \left( \frac{i_{t+1}^{k_e}}{i_{t}^{k_e}} - 1 \right)^2 \left( \frac{i_{t+1}^{k_e}}{i_{t}^{k_e}} \right)^2 \right] \tag{60}
\]

\[
\lambda^{c}_{P,t} \left( 1 - q^{k_h}_{t} z_{t} \right) \left( 1 - \frac{\phi^{k_h}_{t}}{2} \left( \frac{i_{t+1}^{k_h}}{i_{t-1}^{k_h}} - 1 \right)^2 \phi^{k_h}_{t} \left( \frac{i_{t+1}^{k_h}}{i_{t-1}^{k_h}} - 1 \right) \right) = \beta_P E_t \left[ \lambda^{c}_{P,t+1} q^{k_h}_{t+1} z_{t+1} \phi^{k_h}_{t} \left( \frac{i_{t+1}^{k_h}}{i_{t}^{k_h}} - 1 \right)^2 \left( \frac{i_{t+1}^{k_h}}{i_{t}^{k_h}} \right)^2 \right], \tag{61}
\]

\[
\lambda^{c}_{P,t} = \beta_P R_t E_t \left[ \frac{\lambda^{c}_{P,t+1}}{\sigma^m_{t+1}} \right], \tag{62}
\]

and

\[
\lambda^{c}_{P,t} = \sum_{s=1}^{\phi^m} \beta_P E_t \left[ \lambda^{c}_{P,t+s} \left( \frac{1}{\phi^m} + (R^d_t - 1) \left( \frac{\phi^m - s + 1}{\phi^m} \right) \right) \right], \tag{63}
\]

where $\lambda^{c}_{P,t}$ is the Lagrange multiplier on budget constraint \((6)\), $\frac{\lambda^{c}_{P,t} u^j_{P,t}}{\lambda^{c}_{P,t}}$ and $\lambda^{c}_{P,t} q^{j}_{t}$ are the Lagrange multipliers on labour supply constraints \((5)\) and the law of motion of capital \((7)\), respectively. Equation \((51)\) describes the marginal utility of current consumption of non-durable goods. Equation \((52)\) requires that households equate
the marginal utility of current consumption goods to the marginal utility increase of housing stock services, the latter being composed of two parts: (i) the direct utility gain of an additional unit of housing, and (ii) the expected utility stemming from the consumption of the resale value of housing purchased in previous periods. Equations (53) and (54) link real wages in both sectors to households’ marginal rate of substitution between consumption goods and leisure. In equilibrium, real wages in the consumption and housing sectors are equal. Equations (55) and (56) requires that households equate their marginal utility of current consumption goods to the marginal utility increase of an additional unit of capital, which includes two parts: (i) the rental rate of capital, and (ii) the expected utility stemming from the consumption of the resale value of undepreciated capital purchased in previous periods. Equations (58) and (59) link the variable capacity utilization rate to the rental rate of capital. Equations (60) and (61) require that households equate the investment cost, in terms of the foregone marginal utility of consumption goods, to the expected value of the rebate in adjustment cost in the following period. Equation (62) is the typical Euler condition that equates the cost of sacrificing one unit of consumption goods to the benefit of investing in the bond market.\footnote{Since lenders own all firms and financial intermediaries, it also determines the pricing kernel of the economy.} Finally, equation 63 equates the cost of sacrificing one unit of consumption goods to the benefit of making deposits generating a flow of revenues for $\phi^m$ periods.
Lagrangian for borrowers optimization problem

The Lagrangian for the borrowers optimization problem takes the following form:

\[
L = E_0 \sum_{t=0}^{\infty} \beta_t \epsilon_t h_t \left( c_{I,t}, h_{I,t}, n_{I,t}^c, n_{I,t}^h \right) + \lambda_{I,t}^c \times \\
\left\{ \begin{array}{l}
\int_0^{\infty} \frac{1}{w_{I,t}} \left( \frac{w_{f,t}^c}{w_{f,t}^c} \right)^{-\theta_{n}^c} dl + \int_0^{\infty} \frac{1}{w_{I,t}} \left( \frac{w_{f,t}^h}{w_{f,t}^h} \right)^{-\theta_{n}^h} dl \\
+ \frac{v_{f,t}^h}{n_{f,t}^c} \left[ n_{f,t}^c - n_{f,t}^c \frac{1}{0} \left( \frac{w_{f,t}^c}{w_{f,t}^c} \right)^{-\theta_{n}^c} dl \right] + \frac{v_{f,t}^h}{n_{f,t}^h} \left[ n_{f,t}^h - n_{f,t}^h \frac{1}{0} \left( \frac{w_{f,t}^h}{w_{f,t}^h} \right)^{-\theta_{n}^h} dl \right] \\
+ \lambda_{I,t}^c \left[ M_t + \omega h_{I,t} q_{I,t}^h \right]
\end{array} \right\}
\]

The first-order necessary conditions for \( c_{I,t}, h_{I,t}, n_{I,t}^c, n_{I,t}^h, b_{I,t} \) and \( m_t \) are, respectively,

\[
\lambda_{I,t}^c = \frac{(1 - \epsilon_t^h) \frac{1}{\gamma_t} c_{I,t}^c}{(1 - \epsilon_t^h) \gamma_t c_{I,t}^c + (\epsilon_t^h) \gamma_t h_{I,t}^c}, \\
\lambda_{I,t}^c q_{I,t}^h - \beta_t (1 - \delta_t) E_t \left[ \lambda_{I,t+1}^c q_{I,t+1}^h \right] = \\
\lambda_{I,t}^c q_{I,t}^h - \beta_t (1 - \delta_t) E_t \left[ \lambda_{I,t+1}^c q_{I,t+1}^h \right] = \\
\frac{(\epsilon_t^h) \frac{1}{\gamma_t} h_{I,t}^c}{(1 - \epsilon_t^h) \gamma_t c_{I,t}^c + (\epsilon_t^h) \gamma_t h_{I,t}^c} + \lambda_{I,t}^c \lambda_{I,t}^h \omega q_{I,t}^h,
\]

\[
\epsilon_t^c \epsilon_t^h \left( n_{I,t}^{c} \frac{\theta_{q}^{c+1}}{\theta_{q}^{c}} \right) + \left( n_{I,t}^{h} \frac{\theta_{q}^{c+1}}{\theta_{q}^{h}} \right) \frac{\theta_{q}^{c+1}}{\theta_{q}^{h+1}} + \left( n_{I,t}^{c} \frac{\theta_{q}^{c+1}}{\theta_{q}^{c}} \right) \frac{\theta_{q}^{c+1}}{\theta_{q}^{h+1}} = \frac{\lambda_{I,t}^c q_{I,t}^c}{\lambda_{I,t}^h q_{I,t}^h},
\]

\[
\epsilon_t^c \epsilon_t^h \left( n_{I,t}^{c} \frac{\theta_{q}^{c+1}}{\theta_{q}^{c}} \right) + \left( n_{I,t}^{h} \frac{\theta_{q}^{c+1}}{\theta_{q}^{h}} \right) \frac{\theta_{q}^{c+1}}{\theta_{q}^{h+1}} + \left( n_{I,t}^{c} \frac{\theta_{q}^{c+1}}{\theta_{q}^{c}} \right) \frac{\theta_{q}^{c+1}}{\theta_{q}^{h+1}} = \frac{\lambda_{I,t}^c q_{I,t}^c}{\lambda_{I,t}^h q_{I,t}^h},
\]

\[
\lambda_{I,t}^c = \beta_t R_t E_t \left[ \frac{\lambda_{I,t+1}^c}{\gamma_{I,t+1}^c} \right],
\]

79
and

$$\lambda^c_i (1 - \lambda^b_i) = -\sum_{s=1}^{\phi^m_s} \beta^s \lambda^c_i \lambda^b_i \left[ \frac{\lambda^c_i \Pi_{s=1}^{m-1} (1 - \phi_{c+1}^{m-s+1})}{\Pi_{s=1}^{m} (1 - \phi_{c+1}^{m-s+1})} + \lambda^b_i \left( \frac{\phi_{c+1}^{m-s}}{\phi_{c+1}^{m}} \right) \right],$$

(69)

where $\lambda^c_i$ is the Lagrange multiplier on budget constraint (10), $\lambda^b_i$ and $\lambda^b_i \lambda^b_i$ are the Lagrange multipliers on labour supply constraint (5) and the borrowing constraint (12), respectively. Equations (64), (66), (67) and (68) have the same interpretation as for the lenders. Finally, equations (65) and (69) depend on the same two components as the lenders' equations, but also on the marginal utility of relaxing the borrowing constraint.

**Lagrangian for wages optimization problem (for $i \in \{P, I\}$ and $j \in \{c, h\}$)**

The Lagrangian for wages optimization problem takes the following form:

$$L = E_t \sum_{s=0}^{\infty} \left( \beta^s \lambda^c_i \lambda^b_i \right) \frac{\tilde{w}^j_{i,t,s}}{w^j_{i,t,s}} \prod_{k=1}^{s} \left( \frac{1}{\pi_{k+1}^c} \right) \left[ \tilde{w}^j_{i,t,s} \prod_{k=1}^{s} \left( \frac{1}{\pi_{k+1}^c} \right) - \frac{w^j_{i,t,s}}{\lambda^b_i \lambda^b_i} \right].$$

The households' first-order necessary condition with respect to the optimally set wage rate in the current period in the production sector $j$, $\tilde{w}^j_{i,t}$, is

$$E_t \sum_{s=0}^{\infty} \left( \beta^s \lambda^c_i \lambda^b_i \right) \frac{\tilde{w}^j_{i,t,s}}{w^j_{i,t,s}} \prod_{k=1}^{s} \left( \frac{1}{\pi_{k+1}^c} \right) \left[ \frac{\theta^j_{n_j} - 1}{\theta^j_{n_j} \tilde{w}^j_{i,t,s}} \prod_{k=1}^{s} \left( \frac{1}{\pi_{k+1}^c} \right) - \frac{w^j_{i,t}}{\lambda^b_i \lambda^b_i} \right] = 0. \quad (70)$$
Using (53), (54), (66) and (67) to eliminate $\lambda_{i,t}^{n_j}$ yields

$$
E_t \sum_{s=0}^{\infty} \left( \beta_t \xi^{w_j} \right)^s \lambda_{i,t+s}^c \eta_{i,t+s}^{j,d} \left( \frac{\bar{w}_{i,t}^j}{w_{i,t+s}^j} \right)^{-\theta_{n_j}} \prod_{k=1}^{s} \left( \frac{\left( \pi_c^{c} \right)^{i,w_j}}{\pi_c^{c+k}} \right)^{-\theta_{n_j}} \left[ \frac{\theta_{n_j} - 1}{\theta_{n_j}} \right] \frac{1}{\lambda_{i,t+s}^c} e_t^{b} n_{i,t+s}^{n_j} \left( \frac{n_{i,t+s}^c}{\theta_{n_j}+1} + \left( n_{i,t+s}^h \frac{\theta_{n_j}+1}{\theta_{n_j}} \right) \frac{n_{i,t+s}^j}{\theta_{n_j}+1} \right) \frac{1}{\theta_{n_j}} \left( \frac{\bar{w}_{i,t}^j}{\bar{w}_{i,t+1}^j} \right)^{-\theta_{n_j}} \eta_{i,t}^{j,d} + \right. 
$$

This equation states that, in labour market in which the wage rate is re-optimized in period $t$, the real wage is set to equate the expected future average marginal revenue to the average marginal cost of supplying labour. In this equation, $\frac{\theta_{n_j}}{\theta_{n_j}-1}$ represents the markup of wages over the marginal cost of labour that would prevail in the absence of wage stickiness and trend inflation. To write the wage-setting equation in recursive form, we need to define intermediate variables $f_{i,t}^{1,j}$ and $f_{i,t}^{2,j}$. This yields

$$
f_{i,t}^{1,j} = \frac{\theta_{n_j} - 1}{\theta_{n_j}} \left( \frac{\bar{w}_{i,t}^j}{w_{i,t}^j} \right)^{1-\theta_{n_j}} \left( \frac{1}{w_{i,t}^j} \right)^{-\theta_{n_j}} \lambda_{i,t}^c \eta_{i,t}^{j,d} + \beta_t \xi^{w_j} E_t \left[ \left( \frac{\left( \pi_c^{c} \right)^{i,w_j}}{\pi_c^{c+1}} \right) \left( \frac{\bar{w}_{i,t}^j}{w_{i,t+1}^j} \right)^{1-\theta_{n_j}} \frac{1}{\theta_{n_j}} \left( \frac{\bar{w}_{i,t}^j}{\bar{w}_{i,t+1}^j} \right)^{-\theta_{n_j}} \eta_{i,t}^{j,d} \right], 
$$

$$
f_{i,t}^{2,j} = \beta_t \xi^{w_j} E_t \left[ \left( \frac{\left( \pi_c^{c} \right)^{i,w_j}}{\pi_c^{c+1}} \right) \left( \frac{\bar{w}_{i,t}^j}{w_{i,t+1}^j} \right)^{-\theta_{n_j}} \frac{1}{\theta_{n_j}} \left( \frac{\bar{w}_{i,t}^j}{\bar{w}_{i,t+1}^j} \right)^{-\theta_{n_j}} \eta_{i,t}^{j,d} \right], 
$$

$$
f_{i,t}^{1,j} = f_{i,t}^{2,j}. 
$$

(72)

\(^{27}\)Which is necessary for the representation of the model in state-space form.
Lagrangian for prices optimization problem

The Lagrangian for wages optimization problem takes the following form:

\[
L = E_t \sum_{s=0}^{\infty} \left( \beta P^c \right)^s \frac{\lambda_{P,t}^c}{\lambda_{P,t}} \frac{P_t^c}{P_{t+s}^c} \left\{ \begin{array}{c}
\tilde{P}_t \left( \frac{\tilde{P}_c}{P_{t+s}^c} \right)^{-\theta^c} y_{t+s} - r_{t+s}^k k_{m,t+s}^c \\
-\omega_{P,t+s}^c n_{P,m,t+s}^c + \omega_{i,t+s}^c n_{i,m,t+s}^c \\
\gamma^c \left( \left( \frac{\pi_{c}^t}{\pi_{c}^{t+k}} \right)^{1-\alpha} \left( \frac{n_{i,m,t+s}^d}{n_{P,m,t+s}^d} \right)^{\alpha} \right)^{-1-\gamma^c} \\
+ MC_{m,t+s} \times
\left( k_{m,t+s}^c \right)^{\gamma^c} \left( \left( \frac{\pi_{c}^t}{\pi_{c}^{t+k}} \right)^{1-\alpha} \left( \frac{n_{i,m,t+s}^d}{n_{P,m,t+s}^d} \right)^{\alpha} \right)^{-1-\gamma^c}
\end{array} \right\}
\]

To maximize the expected present value of their real dividends, the producers of intermediate goods in the consumption sector must meet the following first-order necessary condition with respect to \( \tilde{P}_t^c \)

\[
E_t \sum_{s=0}^{\infty} \left( \beta P^c \right)^s \frac{\lambda_{P,t+s}^c}{\lambda_{P,t}} \frac{P_t^c}{P_{t+s}^c} \left( \frac{\tilde{P}_t}{P_{t+s}^c} \right)^{-\theta^c} \frac{\left( \frac{\pi_{c}^t}{\pi_{c}^{t+k}} \right)^{-\theta^c}}{\prod_{k=1}^{s} \left( \frac{\pi_{c}^t}{\pi_{c}^{t+k}} \right)^{-\theta^c}} y_{t+s}^c
\left[ \frac{\theta^c - 1}{\theta^c} \left( \frac{\tilde{P}_t}{P_{t+s}^c} \right) \prod_{k=1}^{s} \left( \frac{\pi_{c}^t}{\pi_{c}^{t+k}} \right)^{-\theta^c} - MC_{m,t+s} \right] = 0,
\]

with \( \beta P^c \frac{\lambda_{P,t+s}^c}{\lambda_{P,t}} \frac{P_t^c}{P_{t+s}^c} \) being the Lagrange multiplier on demand function (20), and \( MC_{m,t+s} \) is the firm’s nominal marginal cost. Since firms are assumed to act in the best interest of their owners (i.e. the lenders), the Lagrange multiplier is the marginal rate of substitution for consumption goods over time (i.e. Equation (62)). According to this expression, optimizing firms set nominal prices so that average future expected marginal revenues equate average future expected marginal costs.

The expression above does not have a direct recursive formulation, making the computation difficult. However, writing the price-setting equation in recursive form eases
this process. To do so, we need to define intermediate variables $x_t^{c,1}$ and $x_t^{c,2}$:

$$x_t^{1} = \frac{\theta^c - 1}{\theta^c} p_t^{1-\theta^c} y_t^c + \beta_P \xi^P E_t \left[ \frac{\lambda^c_{P,t+1}}{\lambda^c_{P,t}} \left( \frac{\pi^c_t}{\pi^c_{t+1}} \right)^{1-\theta^c} \left( \frac{\tilde{p}_t^c}{\tilde{p}_{t+1}^c} \right)^{1-\theta^c} x_{t+1}^1 \right],$$

$$x_t^{2} = p_t^{1-\theta^c} y_t^c m_{c,m,t} + \beta_P \xi^P E_t \left[ \frac{\lambda^c_{P,t+1}}{\lambda^c_{P,t}} \left( \frac{\pi^c_t}{\pi^c_{t+1}} \right)^{-\theta^c} \left( \frac{\tilde{p}_t^c}{\tilde{p}_{t+1}^c} \right)^{-\theta^c} x_{t+1}^2 \right],$$

$$x_t^1 = x_t^2. \tag{73}$$
C  Macroprudential measures to deal with housing and mortgage market booms

Loan-to-Value Ratio

• In March 2004, the CMHC *Flex Down* program broadened the eligible sources of funds for the minimum down payment (set at 5 percent).

• In March 2006, the CMHC allowed 0 percent down payment and extend the maximum amortization period to 30 years.

• In April 2007, the LTV limit for insured loans increased to 80 percent from 75 percent and the maximum amortization period is extended to 40 years.

• In October 2008, the maximum LTV for insured loans was reduced from 100 percent to 95 percent and the maximum amortization period for new government backed insured mortgages was lowered from 40 to 35 years.

• In April 2010, the maximum LTV for refinanced mortgages was lowered from 95 percent to 90 percent and the minimum down payment on properties not occupied by owner was raised from 5 percent to 20 percent.

• In March 2011, the maximum LTV for refinanced mortgages was lowered from 90 percent to 85 percent and the maximum amortization for new government backed insured mortgages was lowered from 35 to 30 years.

• In June and July 2012, the maximum LTV on HELOCs was lowered from 80 percent to 65 percent, the maximum LTV for refinanced mortgages was lowered from 85 percent to 80 percent and the maximum amortization for new government backed insured mortgages was lowered from 30 to 25 years.