Decentralized School Finance and Metropolitan Suburbanization*

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Abstract

The residential patterns across the metropolitan areas have changed notably since 1950s; The population has suburbanized and the poor now live closer to the city center. The recent research by urban economists finds that the income elasticity of land demand is too low to explain the poor’s urbanization, which mainly comes from better access to public transportation in cities. We take the new findings seriously and develop a new hybrid Tiebout-Alonso model by explicitly introducing (i) a public transportation as an alternative mode of commute to automobile, and (ii) a housing production function that allows us to work with the income elasticity of land demand directly. We later extend the model in several other directions, including an extended model with a decentralized employment centers. Our model finds that the neighborhood amenities (i.e education and property taxes) have substantial effect on the residential patterns across metropolitan areas that cannot be ignored, and produces some testable predictions.

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1 Introduction

The suburbanization of population across the U.S. cities has been a long time phenomenon: In 1950, 57% of Metropolitan Statistical Area (MSA) residents lived in central cities; In the 2000s, about 33% of residents within MSAs live in the central cities. Interestingly, the suburbanization of population was not uniform; It was highly selective process by income and the residential patterns across the MSAs have substantially changed over time. Up until the 1970s, the rich moved to the suburban areas. Even though the rich returned to many city centers, displacing the poor later (reghentrification), mainly pre-1970 trend seems to shape today’s residential patterns in the MSAs. Consequently, the poor live closer to the city center than the rich\(^1\). While the suburbanization trend has proceeded farther and faster in the U.S., it was international in scope (Mills and Tan (1980)).

The suburbanization of population has received a great deal of attention from both scholars and policy makers around the world because the selection by income in the suburbanization process that led to the concentration of the poor in the central cities aggravated the fiscal and social problems of central cities: low quality public schools, high taxes, crime, congestion etc. One possible explanation for the suburbanization, offered by Leroy and Sonstelie (1983) and Gleaser et al. (2008), is based on an extension of the simple monocentric city model developed by Alonso (1964) and Muth (1969): The altered role of the automobile as a means of commuting to work. When automobile was first introduced, the poor could not afford it and relied heavily on public transportation. In the meantime, automobile gave the rich access to the cheaper suburban land. As a matter of fact, the poor resided in locations closer to the city center while the rich lived in suburban places. Over time, automobile has become more affordable for everyone mainly due to the decrease in the relative cost of

\(^1\)In 2000, 19.4% of the central city population within MSAs lived in poverty while 7.5% of suburban population was poor. See Margo (1992), Mieszkowski and Mills (1993), Mills and Lubuele (1997), and Gleaser et al. (2008).
commuting by automobile. As a result, the rich have begun to return to the city centers.

Although one major reason to study suburbanization is to understand the social and fiscal problems that come along with the concentration of poverty in central cities, the Leroy and Sonstelie (1983)’s explanation seems to ignore the neighborhood amenities and effects completely. On the other hand, there is a totally separate stream of literature, referred to as Tiebout Models, that focuses on neighborhood amenities to study those fiscal and social issues. In Tiebout model (1956), households voting with their feet shop for the community that best satisfies their preferences and form income stratified homogeneous communities. Income-stratification is motivated by the fact that competition between communities with varying public goods and taxes forces households to reveal their preferences; A household with high demand chooses the community with a higher local tax.

Communities are neither income homogeneous nor income stratified\(^2\). As a matter of fact, some communities in the 1970s and 1980s needed to use land use controls to exclude the poor from the rich communities\(^3\). Based on the evidence, some researcher have recently attempted to provide a unified theory of those two artificially separated streams of literature (e.g. Hanushek and Yilmaz (2007, 2011) and de Bartalome and Ross (2003)). Among other things, however, they ignored the altered role of public transportation and automobile as a means of commuting to work. Given the substantial empirical evidence that the urbanization of the poor mainly comes from better access to public transportation, we therefore compelled to take those attempts to unify those two artificially separated literatures one step further.

We develop a monocentric city model with two school districts. Households, differing in their incomes as well as the value they place on education, choose a school district (i.e. a quality of education and property tax package), a location in that school district, the


\(^3\)Hanushek and Yilmaz (2013) studies land use controls in a general equilibrium setup.
consumption of a composite commodity, housing size, leisure and more importantly, the mode of commute to maximize their utility. Commuting has both time and fixed cost components and households can commute to their workplaces by either an automobile or a bus. Also we introduce a housing production function in which firms uses land and capital as inputs. In addition to allowing for two different modes of commute – a public transport and a automobile–, the introduction of housing production function has important advantages for the spatial location of households across school the city: As opposed to the traditional urban location models, housing demand is not the same as land demand and household’s spatial location choice in a school district is determined by the income elasticity of demand for land, not by the income elasticity of demand for housing. Land is assumed to be owned by absentee landlords whose sole objective is to maximize their revenue from the land. As for education, school districts use local property taxes to finance schools. The property tax rate in each school district is assumed to be determined by majority voting. Besides, an education production function that uses peer effect and expenditure per pupil as inputs is introduced into the model.

The benchmark model and its implications are outlined in Sections 2 through 4. Next Section 5 extends the basic setup in several other dimensions: We eliminate neighborhood amenity differences (i.e. qualities of education and property taxes) by consolidating school districts, reduce the wage gap between the rich and the poor, and add a suburban employment center to capture decentralized employment pattern of U.S. cities. Finally, we conclude in Section 6.

2 Model

We begin with a simple model of a monocentric city, which is to be extended later in several other directions. On an xy plane, the CBD (Central Business District) is located at
the origin. The y-axis forms the boundary between the city’s two public school districts. Naturally, the school district on the west (east) is labeled west (east) school district. As in any other urban location model, the city has a radial transportation system.

**Households.** Our model has two types of households living in the monocentric city. Each household has one working member and workers differ by their skills and consequently, their earnings. Skilled workers are paid high wages, \( w_s \) and unskilled workers are paid a low wage, \( w_u \). Depending on the skill of the working member of a household, households are referred to as either skilled (\( S \)) or unskilled (\( U \)) households. For simplicity, it is assumed that wages at the CBD are exogenously given.

Each household \( i \in \{S,U\} \) has a child attending to the public school in her school district \( j \in \{w,e\} \). Also we assume households place a different valuation on education. Skilled households value education more (\( \alpha = \alpha_H \)) than unskilled households (\( \alpha = \alpha_L \)). Each household has a Cobb-Douglas utility function that depends on the quality of education her child receives, \( q \), the size of her dwelling, \( h \), the consumption of all other goods, \( z \), and the leisure time of the working member, \( l \). Thus, the utility function for a type \( i \in \{S,U\} \) household in school district \( j \in \{w,e\} \) at distance \( r \) is explicitly given by 

\[
U = q^\alpha_i h_j(r)^\eta_i z_j(r)^\gamma_i l_j(r)^\delta_i ,
\]

where \( z > 0, l \in [0, 24], \alpha_i \in \{\alpha_H, \alpha_L\}, \eta_i \in \{\eta_H, \eta_L\}, \gamma_i \in \{\gamma_H, \gamma_L\}, \) and \( \delta_i \in \{\delta_H, \delta_L\} \). Note that all the variables in the utility function vary with residential distance from the CBD, \( r \), or school district, \( j \). Also note that there are two different parameter values for each parameter in the utility function because we do have two different types of households: one for skilled (or high valuation) households and another one for unskilled (or low valuation) households. Without loss of generality, we choose to write down our constraints on a daily basis. As a result, leisure is supposed to be between 0 and 24 hours.

All workers commute to the CBD for work once a day. Commuters can choose between
two modes, denoted by $a$ and $b$. For example, $a$ may be automobile and $b$ may be bus. Users of mode $m \in \{a, b\}$ travels a mile in $t^m/2$ hours (i.e. time cost), and has a variable cost of $c^m/2$ per mile, and a fixed cost of $f^m$ per day. Thus, the total cost of using mode $m \in \{a, b\}$ at distance $r$ is $f^m + (c^m + w_i t^m) r$ for the worker of household $i \in \{S, U\}$ with wage $w_i$ in any school district. We assume mode $a$ to be faster and more expensive: $t_a < t_b$, $f_a > f_b$ and $c_a > c_b$. Naturally, the mode of commute choice depends on the utility each mode provides and the worker chooses the mode that provides the highest utility. It is easy to see that the utility maximizing worker will commute by bus if and only if $f^a + (c^a + w_i t^a) r > f^b + (c^b + w_i t^b) r$. Put it differently, the worker chooses to commute by bus (i.e. $m = b$) if and only if her commuting distance, $r$ is less than a cutoff distance, $r^*_i$ which is given by:

$$r^*_i = \frac{f^a - f^b}{c^a + w_i t^a - (c^b + w_i t^b)}$$

Under the assumption that commuting by car has a higher fixed cost (i.e. $f_a > f_b$), there is a positive cutoff distance, $r^*_i > 0$ for a worker with wage, $w_i$ as long as $c^a + w_i t^a < c^b + w_i t^b$. Otherwise, bus is adopted at any distance (i.e. $r^*_i = \infty$). Note that the formula for the cutoff distance, $r^*_i$ depends on wage, $w_i$. Since there are two different wage values, there are two different cutoff levels and, it can be shown that the cutoff distance gets smaller as wage increases. In other words, the cutoff distance is higher for unskilled workers (i.e. $r^*_S < r^*_U$) and the region on the xy plane over which unskilled workers choose to commute by bus is much larger than that of skilled workers.

Households have both time and money constraints. Total time available, 24 hours, must be split among work, commuting, and leisure. Skipping the simple details, the combined time and money constraint (i.e. the budget constraint) for the household can be written as:

$$z_j(r) + (1 + \tau_j) R^*_j(r) h_j(r) + w_i l_j(r) = Y_i(r; m) = 24w_i - (f^m + (c^m + w_i t^m) r)$$
where the mode of transportation is $m \in \{a, b\}$, the equilibrium house rent per unit of housing at distance $r$ and property tax rate in school district $j \in \{w, e\}$ are $R_j^*(r)$ and $\tau_j$, respectively. $24w_i - (f^m + (c^m + wt^m)r)$ is the the maximum amount of income the worker could earn, which is nothing but 24 hours of working less the commuting costs. It is worth emphasizing that in this formulation, commuting has a time cost component and it reduces the time endowment by $t^m r$ hours.

Clearly, a household’s problem at distance $r$ in school district $j \in \{w, e\}$ could be formulated as a utility maximization problem subject to her budget constraint in which she chooses the transportation mode, the size of dwelling, leisure and the consumption of all other goods:

$$V_i^j(r) = \max_{m, h, z, l} q_i^m h_i^j(r) \eta_i z_i^j(r) \gamma_i l_i^j(r)$$

subject to $z_i^j(r) + (1 + \tau_j) R_j^*(r) h_i^j(r) + w_i l_i^j(r) = Y_i(r; m)$

where $V_i^j(r)$ is the indirect utility function for household $i \in \{S, U\}$ at distance $r$ in school district $j \in \{w, e\}$. Also the household chooses the distance $r$ and school district $j$ that maximizes his utility function. Thus, the indirect utility function for the household is

$$V^i(.) = \max \{\max_{r} V_{j=w}^i(r), \max_{r} V_{j=e}^i(r)\}$$

**Housing.** In many urban location models, it is assumed that each household manages the construction of her house by herself. Thus, housing size becomes the same as lot size, but this is not true in general\(^4\). In our model, households derive utility from consuming housing space, and housing space is produced by firms in a competitive market through a constant returns to scale production function that uses capital, $k$ and land (lot size), $s$ as inputs:

$$h = Ak^b s^{1-b}$$

where $b \in (0, 1)$ is the share of capital in the production, and $A > 0$ is productivity

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\(^4\)As pointed out by Gleaser et al. (2008), the estimated income elasticity of demand for housing could be quite different from the estimated income elasticity of lot size. In case of larger income elasticity for housing, this would not drive the rich to live far from the city center.
parameter. Capital (expenditure) is in dollars and land is in square feet. The construction firm at location $d$ in school district $j \in \{w, e\}$ maximizes its profits, and its problem is given by

$$\max_{k, s} \Pi = R_j^*(r) A k_j(r)^b s_j(r)^{1-b} - k_j(r) - \Psi_j^*(r) s_j(r)$$

where $\Psi_j^*(r)$ is the equilibrium land rent per square foot.

Land Market. Land is owned by absentee landlords. The objective of landlords is to maximize land rent revenues they collect. In addition to skilled and unskilled households, farmers also can put the land into use. Farmers are willing to pay a fixed bid of $\bar{R}_a$ per unit of housing. Put it differently, landlords hold an auction at each location in which land goes to the highest bidder as long as it exceeds $\bar{R}_a$. Otherwise, it is undeveloped and left for agricultural use.

The bidding process has substantial implications for the equilibrium. In equilibrium, it must be the case that identical households obtain identical utility levels. To see that, suppose we have two, say, skilled households: Household A and Household B. If Household A achieves a higher utility than Household B, then Household B can outbid Household A by replicating Household A’s choices and offering a slightly higher house (hence, land) rent. By doing so, Household B could be better off as well. Therefore, identical households with different utilities cannot be the case in equilibrium. Put it differently, in equilibrium, all locational and school district advantages must be capitalized into prices (i.e. house and land rents) so that identical households obtain identical utilities.

We can make our theoretical and computational work much easier by exploiting the fact that identical households obtain identical utilities in equilibrium. Given the quality of education, $q_j$ and property tax, $\tau_j$ in school district $j \in \{w, e\}$ as well as the utility level, $\bar{u}_i$ for household $i \in \{S, U\}$, we can define her house bid-rent function at location $r$ as
\[ R^i_j(r) = R(r; q_j, \tau_j, \bar{u}_i) = \max_{m,h,z,l} \left\{ \frac{Y_i(r) - z - w_i l}{(1 + \tau_j) h} \mid U(q_j, h^i_j(r), z^i_j(r), l^i_j(r)) = \bar{u}_i \right\} \]

Given the Cobb-Douglas form of the utility function, following the footsteps of Hanushek and Yilmaz (2007) yields the following functional forms for the house bid-rent and size\(^5\):

\[ R^i_j(r) = k^{1/\eta_i} q_i^{\alpha_i/\eta_i} Y_i^*(r) \frac{\eta_i + \gamma_i + \delta_i}{\eta_i} \bar{u}_i^{-1/\eta_i} \]

\[ h^i_j(r) = \eta_i \frac{\eta_i}{(\eta_i + \gamma_i + \delta_i)(1 + \tau_j) R^i_j(r)} Y_i^*(r) \]

where \( Y_i^*(r) \) is the household’s income net of commuting costs: \( Y_i^*(r) = 24 w_i - (f^b + (c^b + wt^b)r) \) if \( r < r^*_i \), \( 24 w_i - (f^a + (c^a + wt^a)r) \) otherwise. Basically, both the house bid-rent and size functions have two pieces that intersect at \( r^*_i \). In the first piece where \( r < r^*_i \), the mode of commute is bus because it is cheaper. In the second piece, the cheaper mode is commuting by car and thus, it is adopted. It is easy to see the house bid-rent is decreasing in distance as long as \( Y_i^*(r) \) is decreasing in distance. Moreover, it is convex as long as \( \frac{\eta_i + \gamma_i + \delta_i}{\eta_i} > 1 \). We need to find the land bid-rent (i.e. land rent to be offered to the landlord) at any location \( r \) in school district \( j \in \{w, e\} \) to talk about spatial equilibrium. Given the quality of education, \( q_j \) and property tax, \( \tau_j \) in school district \( j \in \{w, e\} \) as well as the utility level, \( \bar{u}_i \) and house bid-rent, \( R^i_j(r) \) for household \( i \in \{S, U\} \), it is simple to show that the profit maximizing house construction firms offer the following land bid-rent to the landlord:

\[ \Psi^i_j(r) = \Psi(r; q_j, \tau_j, \bar{u}_i) = A^{\frac{1}{1-\pi}} b^{\frac{b}{1-\pi}} R^i_j(r)^{\frac{b}{1-\pi}} \]

Obviously, the main determinant of the land bid-rent is the house bid-rent. The land bid-rent function is nothing but a convex transformation of the house bid-rent function. It is because as the land rent increases, construction firms substitute in capital for land and due to bidding, the additional revenue generated goes to the landlord. Moreover, if a household offers a higher house bid-rent at any location, then she also offers a higher land bid-rent at

\(^5\)where \( k_i = \frac{\eta_i^{\gamma_i} \gamma_i^{\delta_i} d_i^{\eta_i}}{(\eta_i + \gamma_i + \delta_i)^{\eta_i} \gamma_i^{\delta_i}} \) is a constant.
the same location\(^6\). The amount of lot size construction firms use to produce a house for household \(i\) at location \(r\) in school district \(j\) is given by

\[
s^i_j(r) = s(r; q_j, \tau_j, \bar{u}_i) = \frac{(1-b)n_i}{(\eta_i + \gamma_i + \delta_i)(1+\tau_j)\bar{\Psi}_j(r)}
\]

Now, we are ready to formally define the bidding process. The household receives three bids for the land at location \(r\) in school district \(j\) and the land is offered to the use of highest bid. Formally,

\[
\Psi^*_j(r) = \max \{\Psi^S_j(r), \Psi^U_j(r), \bar{R}_a\}
\]

For the rest of the paper, it is quite useful to introduce an equilibrium type function that keeps track of land use at location \(r\) in school district \(j\):

\[
\theta^*_j(r) = \arg\max \{\Psi^S_j(r), \Psi^U_j(r), \bar{R}_a\}
\]

where, for example, \(\theta = S\) is the case if the highest land bid-rent at location \(r\) in school district \(j\) belongs to a skilled household. At this point, we can push the theory to understand the factors behind the spatial ordering of households (i.e. the forces that pull or push households towards the CBD) in school districts. In line with urban economics literature, the spatial ordering of households is determined by the steepness of (land) bid-rent functions. Households with a steeper bid-rent curve locate closer to the CBD. Formally, the land bid-rent for household \(i_1\) in school district \(j\) is steeper than the bid-rent for household \(i_2\) in the same school district if and only if the following condition\(^7\) holds at each \(r^*\) such that \(\Psi^i_j(r^*) = \Psi^{i_2}_j(r^*)\):

\[
\frac{\partial \Psi^i_j(r^*)}{\partial r} \frac{\partial \Psi^{i_2}_j(r^*)}{\partial r} = \frac{(\eta_{i_1} + \gamma_{i_1} + \delta_{i_1})\eta_{i_2}^* Y^*_i(r^*) (c^{m_{i_1}} + w_{i_1} c^{m_{i_1}})}{n_{i_1}(n_{i_2} + \gamma_{i_2} + \delta_{i_2}) Y^*_i(r^*) (c^{m_{i_2}} + w_{i_2} c^{m_{i_2}})} > 1
\]

\(^6\)Actually, houses are produced by house construction firms and they offer land bid-rents to landlords. To keep tractability, we describe the model as if the land bid-rent is offered by households.

\(^7\)See Hanushek and Yilmaz (2007) for details.
Household $i_1$ has a steeper bid-rent curve (and hence attracted more to the CBD) if either her income or her budget share of land expenditures is lower. Moreover, the marginal commuting cost, $(c^{m_i} + w_iT^{m_i})$, has an impact on the steepness of bid-rent curves and the higher it is, the higher the pull force towards the CBD is. As discussed above, there are cut off distances for households below which everybody commutes by bus and the cutoff distance is lower for skilled worker households. Thus, it make sense to check the relative steepness of bid-rent functions over three regions: (i) $r^* \leq r^*_S$ where both skilled and unskilled workers commute by bus, (ii) $r^*_S < r^* \leq r^*_U$ where skilled (unskilled) workers commute by car (bus) and (iii) $r^* > r^*_U$ where both skilled and unskilled workers commute by car. The rest of the details are verified numerically.

The incorporation of different modes of commuting is a crucial point in our model. The empirical literature finds that the income elasticity for land demand is far less than one\footnote{Gleaser et al. (2008) estimates the income elasticity can be as high as 0.5, but it seems more likely to be around 0.25.}. Since households can be either skilled or unskilled households in our model, it must be the case that the budget share of land for skilled households is lower than that for unskilled households. Thus, if both skilled and unskilled households had access only to the same commuting mode of transportation, skilled households would have a steeper land bid-rent and be more attracted to the CBD. As pointed out by Gleaser el al. (2008), the income elasticity of demand for land is too low for urban poverty to come from skilled households’ desire to live where land is cheap (the traditional explanation of urban poverty). Our model, as in LeRoy and Sonstelie (1983), assumes that households can use a different mode of transportation: bus or automobile. Clearly, the relative steepness of bid-rents is determined by not only the income elasticity for land demand but also the marginal cost of commuting (hence, the mode of commute as well). As discussed above, only on $r^*_S < r^* \leq r^*_U$ households use different means of commuting. Over that region, skilled households commute by car.
while unskilled households make use of bus. Assuming that the marginal cost of commuting were higher for unskilled households over there (i.e \((c^b + w_ut^b) \gg (c^e + w_st^e)\)), skilled households are more likely to have a flatter land bid-rent and attracted more towards to the suburban areas. Thus, the spatial ordering of households from the CBD towards suburban areas in any school district should be as follows: Skilled households with a bus commuter worker, unskilled households with a bus commuter worker, skilled households with a car commuter worker, and unskilled households with a car commuter worker. Needless to say, in equilibrium, some household types can be absent in any school district.

Our city is closed in the sense that the the population of skilled households, \(\tilde{N}^S\) and unskilled households, \(\tilde{N}^U\) are exogenously given. Explicitly, the population constraints for skilled and unskilled households are given by

\[
\int_{\theta^*_w(r)=i} \frac{\pi}{s_w^i(r)} dr + \int_{\theta^*_e(r)=i} \frac{\pi}{s_e^i(r)} dr = \tilde{N}^i, \forall i \in \{S,U\}
\]

In equilibrium, it must be the case that semi-rings around the CBD in any school district are occupied by identical households due to radial symmetry. The amount of land available for housing between the distance \(r\) and \(r + dr\) (i.e. the size of the semi-ring or land density at \(r\)), is \(\pi r dr\) in any school district. Each household’s land consumption at \(r\) in school district \(j\) is given by \(s_j^i(r)\) and hence, the population of type \(i\) households at the semi-ring is \(\frac{\pi dr}{s_j^i(r)}\).

As expected, type \(i\) households live either in the west school district or east school district. The former integral calculates the number of type \(i\) households in the west school district, \(N_w^i\). The latter integral finds the population of type \(i\) households in the east school district, \(N_e^i\). Needless to say, the population constraints above are written under the assumption that the land market clears in both east and west school districts.

Schools. School district \(j \in \{w,e\}\) provides a public education, \(q_j\) that is financed through a property tax, \(\tau_j\) on residential property\(^9\). The local government in each school

\(^9\)Over the years, we see a growing involvement by the states to reduce the spending disparities across
district balances its budget and the expenditure per pupil in school district $j$ is given by

$$e_j = \tau_j \frac{\int_{R^*_j(r) > R_a} R^*_j(r) \pi dr}{N_j}, \quad \forall j \in \{w, e\}$$

where $N_j = N_j^S + N_j^U$ is the population in school district $j$. Note that the integration is over the locations where $R^*_j(r) > R_a$ because only residential property is taxed to finance schools.

Unfortunately, characterizing an education production function proved difficult due to the fact that it is not quite clear (i) how expenditures on education affect the quality of education$^{10}$ and (ii) what the effect of peers on achievement is$^{11}$. Nevertheless, we assume the following education production function in school district $j$:

$$q_j = c_0 \exp\left(-c_1 \frac{N^U_j}{N_j} \right) e^{c_2}$$

where $c_0$, $c_1$, and $c_2 < 1$ are positive constants. The expenditures has a positive impact on the quality; However, since $c_2 < 1$, the expenditures increase the quality at a diminishing rate. Also, skilled households generate a positive peer effect and hence, a higher quality of education while the peer effect of unskilled households is a negative one.

In each school district, the property tax is determined by majority voting. For the sake of simplicity, we assume the voters are myopic$^{12}$. Besides, they perceive the expenditure per pupil on education in their school district as a measure of quality$^{13}$, $\tilde{q}_j$: The higher the school districts. Both the method and the degree of school finance equalization differ by state. We believe that the main findings of the paper is invariant to the alternative specification of school finance policy. See Hanushek and Yilmaz (2007, 2013) for a study of some prominent school finance equalization policies in the U.S.

$^{10}$Hanushek (1996) finds that the education production literature has failed to find a consistent relationship between spending and quality.

$^{11}$The literature seems to fail to find a clear result on the effect of peers on achievement. See, for instance, Nechyba (2006) and Sacerdote (2011) for survey of the literature.


$^{13}$For a voting behavior in which households have a knowledge of the education production function, see Hanushek and Yilmaz (2013).
expenditure per pupil is, the better the quality of education their school district provides for them. As such, a type \(i\) household at distance \(r\) in school district \(j\) has a preferred property tax rate given by

\[
\max_{\tau_j} V(.) = \frac{k_i}{R_j(r)^{\eta_i}(1+\tau_j)^{\eta_i} \tilde{q}_j^{\alpha_i} Y^*_i(r)^{\gamma_i+\delta_i}} \quad \text{subject to} \quad \tilde{q}_j = e_j
\]

\[
e_j = \tau_j \int_{R^*_j(r)>R_a} \frac{R^*_j(r)\pi dr}{N_j}
\]

As long as \(\eta_i > \alpha_i > 0\), the household has a single peaked preference in \(\tau_j\) and her unique preferred tax rate is given by \(\tilde{\tau}_i = \frac{\alpha_i \eta_i}{\alpha_i - \alpha_i}\). In our formulation, the preferred tax rate has nothing to do with the household’s income; it is a function of her taste parameters for education, \(\alpha_i\) and house size, \(\eta_i\). Since we have two different household types, we do end up with two possible preferred tax rates such that \(\tilde{\tau}_S > \tilde{\tau}_U\).

Here is the timeline of events: Utility maximizing households with the expectation that the last period’s education and property tax packages in school districts would prevail in the current period make their school district, residential and household choice decisions. Once they move in, they are stuck. A majority voting takes place to determine the property tax rates (and qualities of education) in school districts for the current period. Next period they update their expectations and the sequence of events starts over again.

**Definition:** An equilibrium is a set of utility levels, \(\bar{u}^*_S\) and \(\bar{u}^*_U\), market house rent functions, \(R^*_w(r)\) and \(R^*_e(r)\), quality of education and property tax pairs \((q_j, \tau_j)\) \(j \in \{w, e\}\), and equilibrium type functions \(\theta^*_j(r)\) \(j \in \{w, e\}\) which show the equilibrium occupant of the location at distance \(r\) in school district \(j\) such that

- Given market house rents, qualities of education and property tax rates in school districts, households chooses composite good, leisure, house size, distance to the CBD, the mode of commute, and a school district to maximize their utilities.

- Housing is produced by competitive firms, which use land and capital as inputs.
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<td>$b$</td>
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Table 1: Calibration Parameters

- Land is owned by absentee landlords. They receive bids from housing construction firms and farmers. Land goes to the highest bidder.

- Identical households get identical treatment. In other words, the same type households attain the same utility level in equilibrium.

- Our city is closed. The land markets clear and thus, population constraints hold.

- The property taxes are determined by majority voting. Each school district runs a balanced budget and provides a public education.

3 Calibration

Our model is calibrated to replicate an average U.S. city around 2010. Fortunately, some parameter values can directly be taken from several resources:

i. In 2009, the median annual incomes of households with a high school graduate householder and with a college graduate householder are $39,647 and $82,722, respectively.

Assuming working members of households provide a labor of 40 hours per week\textsuperscript{14},

\textsuperscript{14}The average weekly working hours for full time workers is 38.2 in 2010. From now on, the statistical facts come from the Statistical Abstract of the United States, 2012 unless otherwise is indicated.
hourly wage rates for unskilled and skilled workers are calibrated to be $w_u \approx 19.06$ and $w_s \approx 39.77$, respectively.

ii. Without loss of generality, we normalize the sum of Cobb-Douglas utility function parameters (i.e. $\eta_i + \gamma_i + \delta_i = 1 \forall i \in \{U, S\}$) to have the nice interpretation that the parameters correspond to the budget shares. Each worker is assumed to work for 40 hours per week, which implies the budget share of leisure, $\delta_i$ as $\frac{\delta_i}{\eta_i + \gamma_i + \delta_i} = 1 - \frac{40w_i}{24 \times 7 \times w_i} \approx 0.76$. A household with an annual income of $39,647 spends about 21.8% of her earnings on housing (shelter only) in 2009 (i.e. $\frac{\eta_U}{\eta_U + \gamma_U + \delta_U} = (1 - 0.77) \times 0.22 \approx 0.05$). Thus, the budget shares of housing expenditures and composite goods for an unskilled household become $\eta_U = 0.051$ and $\gamma_U = 0.182$, respectively. Recalling that an unskilled household’s preferred tax rate is given by $\tilde{\tau}_U = \frac{\alpha_U}{\eta_U - \alpha_U}$, we set $\alpha_U$ to be $0.015$ so that her preferred property tax rate is about 0.8%$^{15}$.

iii. The fixed cost and variable material cost of commuting by car are about $f_a \approx $16.75 per day and $c_a \approx 35.5$ cents per round trip mile, respectively$^{16}$. We assume a single fare is charged for a bus ride regardless of distance, so that the variable material cost of commuting by bus, $c_b$ is zero. The Public Transportation Fare Database (American Public Transportation Association, 2010) reports the adult single-trip bus fares for major metropolitan areas in the U.S., which ranges from $7 in Woodbridge, VA to $0.50 in Los Angeles. As such, we set the round trip fixed cost of commuting to be $f_b = 2 \times 3.5 = 7$. The time cost of car transportation, $t_a$ is assumed to be 0.067 hours per round trip mile, while the corresponding number for the bus, $t_b$ is set to be 0.182 hours per round trip mile$^{17}$.

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$^{15}$In 2009, the median property tax rates range from 1.89% in New Jersey to 0.18% in Louisiana. Also note that the property tax rates we provide are out of value of houses. Source: American Community Survey, 2009.

$^{16}$Source: Bureau of Transportation Statistics; United States Department of Transportation, 2011.

$^{17}$The average commuting speeds for car commuting and bus commuting are 28.87 and 11.42 miles per
iv. The population of the city in our model is set to be 1,500,000 households. For persons
25 years and older, 39% has an associate’s degree or more in 2010. Thus, 40% of the
population is assumed to be skilled workers while the remaining 60% is assumed to be
unskilled households.

So far, we have calibrated the parameters we can get direct estimates from the data.
For the remaining parameters, we take an indirect approach. We set calibration targets for
some endogenous model parameters, and simultaneously calibrate remaining parameters to
achieve those calibration targets. Below is the list of parameters simultaneously calibrated
along with the endogenous calibration targets we aim to achieve:

v. The parameters of the education production function are set to be \( c_0 = 7, c_1 = 0.0285, \)
\( c_2 = 0.284 \) so that the expenditures on education and peer effect patterns produce
the quality of education calibration targets along with the degree of interaction target
between peer effects and expenditures on education.

vi. To generate a population density of approximately 3,215 households per square mile\(^1^8\),
the agricultural rent, \( \bar{R}_a \) is set to be $15,600 per square mile per day.

vii. The budget share of housing for a skilled household is set to be \( \eta_S = 0.033 \) to yield an
income elasticity of lot size demand 0.35\(^1^9\). Due to the normalization of utility function
parameters, the budget share of composite good becomes \( \gamma_S = 0.20 \). Once again, we set
\( \alpha_S \) to be \( \alpha_S = 0.017 \) so that a skilled household’s preferred property tax rate is about
2.1%.

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\(^1^8\)The median population per square mile of cities with 1,000,000 or more but 2,000,000 or less population
is 3,544 per square mile in 2005. Source: County and City Data Book, 2007.

\(^1^9\)Gleaser et al. (2008) finds the elasticity of land (lot size) demand with respect to income ranges from
0.25 to 0.5.
viii. As for the house production function parameters, \( A \) and \( b \) set to be 0.009 and 0.37, respectively to have a lot size-house size ratio of about 0.25 at the fringe and about 0.6 around the CBD.

4 Benchmark

\(<\text{Figures 1 and 2 about here}>\>

In Figures 1 through 4, we describe the benchmark equilibrium. In both school districts, rents go down as we move away from the CBD. What we see is nothing but the capitalization of accessibility; the closer locations have higher rents. Workers choose to commute by bus if the distance to their workplace is less than the cutoff distance, \( d^* \): Approximately, \( d^*_U = 5.3 \) miles for unskilled workers and \( d^*_S = 2.3 \) miles skilled workers, respectively. The cutoff distance for skilled workers is smaller because bus is a slower mode of commute and has a higher time cost for skilled workers (i.e. \( (t^b - t^a)w_S >> (t^b - t^a)w_U \)). As discussed above, the spatial ordering of households (i.e. steepness of land bid-rents) are determined mainly by the push-pull (towards CBD) effects of the following three forces: the budget share of housing, household’s income, and marginal commuting costs. Skilled households are attracted to the suburban areas; They have higher incomes and housing is a normal good. Unskilled households are pulled towards the suburban areas; They spend a higher percentage of their income on housing (i.e. \( \eta_U > \eta_S \)). In benchmark equilibrium, the marginal commuting costs pull the skilled towards the CBD if they commute by the same means of transportation, but pushes the skilled away from the CBD if the skilled commutes by automobile and the unskilled commutes by bus. The degree and net effect of those forces are determined by our calibration parameters. Overall, at locations where both types commute by either automobile \((d > 5.3)\) or bus \((d < 2.3)\), the skilled household’s willingness to live in big houses where the land is cheap is not that strong. Those households have steeper bid-rent curves than
unskilled households. At locations where skilled households commute by automobile and unskilled households commute by car ($2.3 < d \leq 5.3$), the marginal commuting costs change from a pull force to a push force and change the balance of power. Unskilled households now have a much higher marginal commuting costs and locate closer to the CBD. To sum up, the spatial ordering from the CBD to the suburban areas in any school district is given as skilled household with a bus commuter, unskilled household with a bus commuter, skilled household with a automobile commuter, and unskilled household with a automobile commuter. School districts are clearly heterogeneous and all four types are present in either school district. As shown in figure 2, the relationship between house size and distance is no longer a monotone increasing relationship even though there is an increasing house size trend as households move away from the CBD. We believe the pattern we have is more consistent with the empirical evidence.

The West School District provides a better education. Be it commuting by automobile or by bus, most skilled households choose to reside in the west school district; Explicitly, 79.5% of skilled (high valuation) households resides in the west while 89.4% of unskilled (low valuation) households resides in the east. 61.8% of all households has a residence in the east school district because most households –60% of the population– are assumed to be unskilled worker households. The findings related to commuting behavior might provide useful insights. 26.4% of the population commutes by bus, of which 66.3% lives in the East School District. About 8.2% of skilled workers and 38.5% of unskilled workers commute to their workplace by bus. Not surprisingly, most unskilled (skilled) households with a bus commuter reside in the east (west): Approximately 74% (87%) of unskilled (skilled) households with a bus commuter. We can make similar arguments about the car commuters; Most unskilled (skilled) households with a car commuter reside in the east (west)
As expected, the equilibrium property tax rate is the preferred property tax rate of a skilled (high valuation) household in the west while the property tax preference of unskilled (low valuation) households prevails in the east. More explicitly, $\tau_w = 2.1\%$ and $\tau_e = 0.8\%$. The West School District also has relatively much better peers and thus, it is more efficient. In the west, the better education is clearly capitalized into housing prices; Gross rents are higher in the West School District. Needless to say, we can see the capitalization of education into housing prices by the jump as we cross the school district boundary from the east to the west. The West School District with a much higher property tax rate and housing prices raises more than as twice expenditure per pupil on education as the East School District does. Combining a higher expenditure with better peers, it remains the best school district in the city.

5 Extensions

To learn more about the implications of our model, we now turn to three additional simulations that extend the benchmark model.

<< Figures 3 and 4 about here >>

5.1 A Unified School District

Our model provides a unified treatment of natural evolution and fiscal problem theories of suburbanization. The two theories have a number of interactions and interrelations, and as a result, it is difficult to distinguish between them empirically. However, our calibrated model presents a unique opportunity just to do that. In the first extension, we assume East and West School Districts are unified as a single school district operated by the city\textsuperscript{20}. Basically, our model just becomes a pure natural evolution theory model.

\textsuperscript{20}For a more detailed analysis of school district consolidation on welfare and equality in educational opportunity, see Hanushek and Yilmaz (2007, 2013).
As seen in Figures 3 and 4, the differences in the fiscal capacity of school districts have been completely eliminated; The property taxes are equalized at $\tau_w = \tau_e = 0.8\%$ and both school districts spend the same amount of money on each pupil. In equilibrium, we end up with two identical communities. The compositions of population in both school districts are the same and consequently, the peer effects are equalized as well.

Comparing the new equilibrium to the benchmark, there are substantial differences in the distribution of households in both school districts. Even though the mode of commute distribution of households in the city are similar – about 35\% of unskilled workers and 8\% of skilled workers commute by bus –, the population compositions of school districts are totally different in both equilibria: In contradiction with an even split of all household types in the new equilibrium, almost all skilled (unskilled) households – be it with a bus or (an automobile) commuter – reside in the west (east) school district in the benchmark equilibrium. Our model clearly predicts that the return of the skilled to the locations around the CBD (regentrification) mainly takes place in the school district with a better fiscal capacity and thus, with a better education. Moreover, the relocation of the unskilled (of course with a automobile commuter) to suburban areas mainly occurs in the school district with a lower fiscal capacity and quality of education. In conclusion, the fiscal problem theory seems to make important contribution to explain suburbanization and regentrification phenomena that evidently, cannot be ignored.

5.2 A Lower Wage Gap

One key parameter in our model is the wage gap between skilled and unskilled workers: Not only, wages determine the housing (and land) demand but they also determine the marginal cost of commuting through the time cost component of commuting. Also, there is a large variation in wages (and hence, the wage gap) across the U.S. cities. In this extension,
we alter the benchmark model by increasing the wage of unskilled workers by 15%. A summary of new equilibrium is shown in Figures 3 and 4.

Similar to the benchmark, the West School District with a better fiscal capacity—a higher property tax and expenditure per pupil—provides a better education. While qualities of education in the west are really close to each other in both benchmark and new equilibria, the east provides a better education than the benchmark. This is mainly due to the fact that unskilled households are richer in the extended model and as a result, the rents along with property taxes collected (and thus, expenditure per pupil) in the east are higher than those values of the benchmark.

The size (i.e. both the area and population) of each school district in the extended model resembles to the size of the same school district in the benchmark. The main difference from the benchmark seems to be the proportion of households with a bus commuter. It goes down from the benchmark ratio of 26.4% to 15.1% in the new equilibrium. More explicitly, the ratio of unskilled households with a bus commuter almost halves while the ratio of skilled workers with a bus commuter nearly doubles. Since the quality of education difference between school districts in the extended model is smaller than the benchmark, we see a less degree of segregation and more even distribution of households than the benchmark. In other words, the number of skilled (unskilled) households increases in the east (west). To sum up, the lower the wage gap between the skilled and unskilled is, we expect to see a more even regentrification of the skilled (with a bus commuter) and a better access to suburban places by the unskilled (with a automobile commuter).

5.3 A Decentralized City

While a monocentric city setup is theoretically elegant and analytically tractable, its applicability to modern American cities is quite limited; The decentralization of unemploy-
ment has been a long time phenomenon in the U.S. and most jobs are located outside the CBD, in suburban areas\textsuperscript{21}. In line with the empirical evidence, we extend the benchmark model by introducing a suburban employment ring at 6 miles off the CBD in this section. The wages at the suburban employment center are determined by the wages at the CBD discounted by the wage gradient of 1\%\textsuperscript{22}. Based on the fact that public transportation systems rely on high densities, we assume that only jobs at the CBD is accessible through a public transportation and suburbs do not. In other words, the only mode of commute to the suburban employment center is an automobile. The outcomes of the new equilibrium are described in Figures 3 through 6.

<< Figures 5 and 6 about here >>

As shown in Figure 5, the relationship between distance and rents are no longer a monotone relationship. House rents attain a local maximum around the suburban employment ring in any school district. The pattern is not surprising because we have two employment centers and two different commuting patterns; some households commute to their workplace at the CBD while some other households commute to their job at the suburban employment ring. The hill about the suburban employment center in each school district belongs to households with a suburban employment center commuter. Similarly, the big hill around the CBD belongs to the households with a CBD commuter. We see a similar pattern for the house size. While it is not monotone relationship, households have a tendency to occupy bigger houses as they move away from the employment center; Skilled households reside in houses with a size above the trend and unskilled households reside in houses with a size

\textsuperscript{21}75.9 percent of metropolitan area employment is more than three miles from the Central Business District in 2000. However, monocentric city model is still applicable for a diminishing number of American cities. See Anas, Arnott and Small (1998), Mieszkowski and Smith (1991), Giuliano and Small (1991), Glaeser and Kahn (2001, 2004) for a further discussion.

\textsuperscript{22}Ihlandfeldt (1992) finds that wages decline by approximately one percent per mile of distance from the CBD.
below the trend.

Schooling outcomes—fiscal capacities of schools (i.e. property tax rates, expenditures per pupil), peer effects, education qualities—in the extended model look very similar to those in the benchmark. Moreover, the sizes—both area and populations—of school districts are very similar to the benchmark. However, most jobs are located at the suburban employment center and only about 31.8% of workers has a job at the CBD. As a result, the introduction of suburban employment center has major implications for the spatial distribution of households and their working member’s commuting patterns. We do not see any unskilled household with an automobile commuter to her job at the CBD: 35% of unskilled households commutes to their workplace at the CBD by bus; The rest commutes by an automobile to the suburban employment center and resides in the east school district. Note that the only unskilled households in the west school we have—are the ones with a bus commuter to her job at the CBD. As for skilled workers, about 73% has a job at the suburban employment center. For skilled households with a workplace at the CBD, 22.8% commutes to the CBD by bus. At any workplace and mode of commute, most skilled (unskilled) households reside in the west (east). The predictions of the decentralized employment center extension are (i) regentrification occurs mainly in the school district with a better education; (ii) all unskilled households with an automobile commuter have a job at the suburban employment center and all unskilled workers with a job at the CBD commutes by bus; (iii) all unskilled households with an automobile commuter reside in the school district with a lower quality of education and property tax.

6 Conclusions

It is our judgment that local public goods and taxes are also important determinants of residential patterns across MSAs. Recently some scholars have tried to merge Tiebout
Models with Urban Location Models. In the light of some recent developments on urban location models, our model extends this attempt in two major directions: The introduction of (i) public transportation as an alternative means of commute; (ii) the housing production function that makes land demand and housing demand different. The urban economics literature finds that the primary reason for the concentration of the poor around the CBD is the existence of public transportation system. It is because the automobile is relatively more expensive for some of the poor and they mainly choose to reside around the CBD. Moreover, the literature highlights the fact that the common assumption in urban location models — land demand is the same as housing demand — is wrong and the income-land demand (not housing demand) relationship determines the spatial residential patterns across the MSAs. Besides, we can use the recent reliable estimates of the income elasticity of land demand. The land demand elasticity is much smaller than the housing demand elasticity, suggesting that the income elasticity of land is too small to explain the poor’s urbanization. The existence of local public goods and taxes adds an interesting dimension to our model, that cannot be ignored: The urbanization of the poor usually takes place in the school district with lower taxes and worse education; The poor with an automobile commuter to her job at the suburban place live in the school district with lower taxes and lower quality of education; The regentrification takes a much faster pace in the school district with a better education; The public transportation reliance is higher in the worse school district.

As for future research, we hope we can get to test the empirical predictions of our model in the data. The —population density gradient based— previous attempts (e.g. Mills and Price (1984) or Mills (1986)) found that the set of measures of central city problems — crime, educational attainment, and taxes— adds nothing to the understanding of population suburbanization. The problem with this indirect approach was that, among other things, small errors translate large absolute quantities. Given the rich dataset available nowadays,
we hope we can finally settle this important issue.
References


Mills, Edwin S.1986. "Metropolitan Central City Population and Employment Growth During the


Figure 1: Monthly Market Gross Rent (cents per square foot)

\(q_e = 10, \ e_e = \$6.78 \text{ per day}\)

\(q_w = 15, \ e_w = \$14.95 \text{ per day}\)

Figure 2: House Size (square feet)
Figure 3: Quality of Educations and Property Tax Rates
(property tax rates over the bars)

- Bench: East 0.8% West 2.1%
- Cons.: East 0.8% West 0.8%
- Lower prem.: East 0.8% West 2.1%
- Decentr.: East 0.8% West 2.1%

Figure 4: Distribution of Households Across School Districts
(in millions)

- Unskilled: Auto
- Unskilled: Bus
- Unskilled: SUB
- Skilled: Auto
- Skilled: Bus
- Skilled: SUB
Figure 5: Monthly Market Gross Rent (cents per square foot)

\[ q_e = 9.8, \quad e_w = \$6.56 \text{ per day} \]

\[ q_w = 14.9, \quad e_w = \$14.6 \text{ per day} \]

![Chart showing market gross rent for different distances from the CBD for unskilled and skilled workers using auto and bus transportation.](chart5)

Figure 6: House Size (square feet)

![Chart showing house size for different distances from the CBD for unskilled and skilled workers using auto and bus transportation.](chart6)