Asymmetric shocks in a currency union: The role of central bank collateral policy

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Abstract

Currency unions limit the ability of the central bank to use interest rate policy to accommodate asymmetric shocks. I show that collateral policy can dampen asymmetric shocks in a currency area when the collateral portfolios of banks differ across countries. In the model, banks from 2 countries use collateral to borrow from the money market or the central bank. The distressed bank may be funding constrained due to a moral hazard problem. By relaxing its collateral policy, the central bank relaxes funding constraints in the distressed country without reducing its policy rate below the target level.

Keywords: Central banking, currency union, collateral policy, repo, monetary policy.

JEL Codes: E58, G01, G20.

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1 Introduction

The fragmentation of financial markets during the European sovereign debt crisis complicated the conduct of monetary policy in the euro area. From 2010 to 2013, countries that were negatively affected by the debt crisis experienced high interest rates, while countries that were relatively unaffected by (or benefited from) the shock experienced low interest rates. Data from the European Central Bank (ECB) suggest for instance that the funding cost of firms in Greece, Ireland, Italy, Portugal and Spain was 3% above the risk free rate versus 1.7% for firms in other Euro area countries from 2010 to 2013.¹ These differences contrast with the Taylor rate, which provides an indicator of the desired level of policy rates given economic conditions. According to this indicator, the central bank rate should have been 2.7% in low yield countries versus roughly 0% in high yield countries over the same period.²

According to the literature on optimum currency area, there is little that the central bank can do to respond to such asymmetric economic conditions. The reason is that its policy instrument, the interest rate, has to be the same for the whole currency area. If a country is hit by a negative demand shock while another is hit by a positive shock, the central bank must therefore choose between high interest rates to avoid overheating in the booming economy, or low interest rates to accommodate the shock in the distressed economy. The central bank however cannot pursue both policies since its policy rate must be the same for both countries (Mundell, 1961; McKinnon, 1963; Kenen, 1969).

In this paper, I build a microeconomic model of the bank lending channel of monetary policy to study an alternative instrument available to the central bank: collateral requirements. When lending to banks, central banks have collateral requirements that vary with the type of asset used. Can collateral policy help the central bank to respond to asymmetric shocks? The first contribution is to show how asymmetric shocks can lead to high rates in the distressed country and low rates in the booming one. I build a model where banks have asymmetric collateral portfolios due to a domestic bias. When economic shocks also affect the value of the collateral, there exists a “collateral crunch” regime where banks are constrained in their access to the money market. When the shock is asymmetric, the bank in the distressed country reduces the allocation of funds to the economy which increases the economy rate above the policy rate, whereas the economy rate in the stable country remains equal to the policy rate.

The second contribution is to derive the interest rate and collateral policy of the central bank consistent

¹Source: ECB Statistical datawarehouse, Lending spreads. Gilchrist and Mojon (2014), Horny et al. (2014) and Pianeselli and Zaghini (2014) show that the differences in funding cost of firms remain when controlling for firm and risk characteristics.
²As Nechio (2011), we compute the Taylor rate as 1+1.5*Inflation-1*Unemployment gap (group average). Low yield countries are AT, BE, FI, FR, DE, NL, SK, SI. High yield countries are GR, IE, IT, PT, ES.
with the objective of steering the real economy rate to a target level. If the distressed bank experiences a collateral crunch, I show that changes in the policy rate of the central bank have a heterogeneous effect across countries. By lowering its policy rate by one unit, the central bank reduces the interest rate in the booming country by one unit (away from the objective of the central bank) while the economy rate of the distressed country falls by less than a unit (so the positive effect is dampened). This implies that the interest rate policy of the central bank should remain closer to the target level for the whole currency area, while collateral policy may be used to relax the funding constraint of the distressed bank.

The theoretical model extends the work of Koulischer and Struyven (2014) to two banks A and B and two collateral types 1 and 2 and introduces a central bank deposit facility. This allows the interest rate instrument to be compared with the collateral policy instrument in the context of a currency area hit by asymmetric shocks. Macroeconomic shocks impact the investment opportunities available to banks and the value of their collateral. The central bank seeks to steer economy rates towards a target level. The game is in two stages: the central bank first sets its rate and collateral policy, then the banks choose how much and where to borrow.

As a benchmark, I first consider the case without moral hazard. I show that in this first best world the central bank can perfectly control the interest rate in the economy using only its deposit facility, where money market participants can place their funds at the policy rate. The central bank then simply sets its policy rate at the target level.

I then introduce moral hazard so that collateral becomes necessary to incentivize banks. As in Holmström and Tirole (2011), collateral requirements provide a penalty to banks in case of mismanagement since it is seized in case of default. I show that each bank is in one of two regimes depending on its stock of collateral. If a bank has sufficient collateral available, it can borrow the first-best amount and the regime is similar to the first best. If the available collateral falls below a threshold, the bank is constrained in its access to funding and enters the “collateral crunch” regime. The amounts lent fall and the economy rate increases relative to the policy rate of the central bank.

If the bank in the distressed economy is in a collateral crunch, the central bank faces a situation of high rates in the distressed country and low rates in the stable country. In this case the policy of the central bank consists in setting its policy rate close to the optimum of the booming country while reducing haircuts on the collateral owned mostly by the distressed bank. The rationale for a relatively high rate is that a reduction in the policy rate reduces the economy rate of country A by one unit while the reduction of the economy rate in the distressed country is less than one, so the positive effect of low rates in the
distressed country is dampened relative to the negative effect of low rates in the booming country.

The degree of asymmetry in the collateral portfolios of banks plays a key role in the policy of the central bank since both banks benefit from lower haircuts. If the portfolios are concentrated on specific assets (for example due to a home bias), the central bank can ensure that the increased exposure is done more exclusively at the benefit of the distressed bank, so that the cost of lowering the rate in the distressed country is reduced.

My model suggests that collateral policy can be useful in a situation of high rates in the distressed country and low rates in the stable one. However, it does not suggest that collateral policy can engineer a situation of high rates in the booming country and low rates in the depressed country. The reason is that, in using collateral policy, the central bank is competing with market participants. If it were to set collateral requirements above those of the market, banks would borrow from the market so collateral policy would have no effect on the final equilibrium. The best that collateral policy can do is to achieve a same economy rate across the currency area.

1.1 Related literature

There is a large literature on the role of unconventional central bank policy when financial intermediaries face balance sheet constraints (e.g. Gertler and Karadi (2011); Dedola et al. (2013)). The key novelty of this paper is to study the collateral policy of the central bank when banks are heterogeneous, which allows collateral policy in a currency union to be studied. Ashcraft et al. (2011), Bindseil (2013) and Koulisher and Struyven (2014) for example study the effect of collateral policy for symmetric banks. The mechanism proposed in this paper for the real effects of collateral policy builds on the work of Holmström and Tirole (2011) and Koulisher and Struyven (2014) and differs somewhat from that of Ashcraft et al. (2011). In their paper, securities are subject to exogenous margin requirements which may be directly changed by the central bank. In our model, haircut requirements are endogenous and the central bank may incur losses when lending to banks, so we include both the costs and benefits of collateral policy.

Our model includes both the interest rate and the collateral policy instrument. Focusing on the interest rate, Freixas et al. (2011) expand the model of Bhattacharya and Gale (1987) to study the optimal central bank policy when a crisis causes a disparity in the liquidity held among banks. One implication of their work is that the interest rate policy of the central bank should take into account developments in the financial sector (on this point see also Kashyap and Stein (2000); Freixas and Jorge (2008); Freixas et al. (2011); Jimenez et al. (2012)). The contribution of this paper is to show that when the disparity in the
access to liquidity differs across countries in a currency union, this can affect the optimal policy of the central bank in a currency union (on this point see also Kashyap and Stein (1997)).

The literature on optimum currency area (OCA) has focused on the use of fiscal policy to respond to asymmetric shocks (Mundell, 1961; McKinnon, 1963; Kenen, 1969; Farhi and Werning, 2012, 2013). Our paper shows that collateral policy may alleviate (but not solve) the “one instrument, two objectives” dilemma of the central bank. Our paper sheds light on the mechanism used by collateral policy to influence the real economy which took a central role in the European sovereign debt crisis (Acharya and Steffen, 2013; Dreschler et al., 2013).

Section 2 describes the different elements of the model. Section 3 considers the “first-best” case without moral hazard and discusses how economic shocks affect the equilibrium. I then introduce moral hazard, focusing in Section 4 on the outcome for the bank and in section 5 on the policy of the central bank. Section 6 makes the link between the model and the Sovereign debt crisis and discusses policy implications.
2 Setup

Banks. Two cashless banks are located in two different countries (A and B) that share a single currency. Each bank \( i \in \{A, B\} \) has undertaken illiquid investments in the economy that need refinancing. For an investment \( q \), the bank earns \( R(q) \) with a probability \( p_i \) and zero with a probability \( 1 - p_i \). The return function \( R(\cdot) \) takes the Cobb-Douglas form \( R(q) = q^\alpha \) with \( \alpha < 1 \). For a total investment \( q_i \), the expected output is \( p_i R(q_i) \). The interest rate in the economy \( r_i \) is defined as the marginal return on investment

\[
r_i = p_i R'(q_i).
\]

This is the minimum return required for investment projects to be funded.

Banks can borrow from the money market or from the central bank (as illustrated in Figure 1) but there is moral hazard in the process: instead of channeling the funds to the real economy, banks can shirk and obtain a private benefit \( Hq \) that varies with the investment size \( q \). If a bank shirks, it defaults with probability 1.\(^3\) To prevent shirking, lenders can require collateral that is transferred in case of default. Since the bank defaults for sure when shirking, high collateral requirements reduce the payoff in case of default and therefore incentivize the bank not to shirk. There are two types of collateral in the economy, indexed 1 and 2. Bank \( i \) owns each collateral for a total worth of \( \theta_{i,1} \) and \( \theta_{i,2} \) respectively. The 2 collaterals are perfect substitutes and we denote bank \( i \)'s aggregate stock of collateral as \( \theta_i = \theta_{i,1} + \theta_{i,2} \). The stocks of assets are exogenous and may differ across banks (for instance if banks have a domestic bias).

Money market. The money market is composed of corporations with excess cash, money market funds or other financial institutions with excess liquidity wishing to lend their cash short term. There are no impediments to arbitrage and market participants may either fund the bank or place their cash at the central bank and earn the policy rate \( r_{cb} \) for sure.\(^4\) A contract in the money market is characterized by a (gross) interest rate \( r_i^p \) so that the bank pays \( r_i^p q_i^p \) in case of success when borrowing \( q_i^p \).\(^5\) The contract also specifies a collateral requirement or “haircut” \( h_{i,j}^p \) by bank \( i \) so that the bank must pledge \( h_{i,j}^p \) units of collateral \( j \) to borrow 1 unit. Since market participants and the borrowing banks are indifferent between either type of collateral, the no-arbitrage condition will cause haircuts for both assets to be the same, i.e.

\(^3\)An alternative interpretation is that the bank makes an unobservable choice of the probability of success \( p_B \) so that if it chooses \( p_B = 0 \) it obtains some private benefit which could result from lower monitoring costs or excessive perks (Holmström and Tirole, 2011).

\(^4\)In practice only banks have access to the deposit facility of the central bank. This is equivalent to our model if banks can place funds at the central bank on behalf of non-banks.

\(^5\)The gross interest rate is equal to 1 plus the net interest rate, \( r_i^p = 1 + \bar{r}_i^p \) where \( \bar{r}_i^p \) is the net interest rate. Similarly, I use the gross haircut \( h_i^p = 1 + \bar{h}_i^p \) so a net haircut of \( \bar{h}_i^p = 20\% \) is equivalent to a gross haircut of \( h_i^p = 1.2 \).
Let \( a_{i,1}^p \) and \( a_{i,2}^p \) be the amount of collateral pledged by the bank to the private money market and \( q_i^p \) the amount borrowed. A contract \((r_i^p, h_i^p)\) requires that:

\[
q_i^p = a_{i,1}^p + a_{i,2}^p.
\]

(Central bank) The central bank has two facilities (illustrated in Figure 1). First, the deposit facility allows money market participants to place their cash and earn the policy rate \( r_{cb} \) for each unit deposited. Second, the central bank offers loans to banks through open market operations. Loans are at the policy rate \( r_{cb} \) and are collateralized: for each unit borrowed the bank must pledge an amount \( h_{i}^{cb} \) of collateral, where \( h_{i}^{cb} \) and \( h_{i}^{cb} \) are the (gross) haircuts of the central bank. Let \( a_{i,1}^{cb} \) and \( a_{i,2}^{cb} \) be the amounts of collateral pledged by the bank to the central bank. To borrow an amount \( q_i^{cb} \), the collateral pledged by bank \( i \) must satisfy

\[
q_i^{cb} = \frac{a_{i,1}^{cb}}{h_{1}^{cb}} + \frac{a_{i,2}^{cb}}{h_{2}^{cb}}.
\]

At this point it is useful to compare and contrast the types of contracts offered by the central bank and money market participants. First, the money market can offer bank-specific contracts while the central bank must offer the same contract to all banks. This captures a key operational constraint faced by central

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\( h_{i,1}^p = h_{i,2}^p = h_i^p. \)

The reason for this is that for a given interest rate \( r_i^p \) the return of a loan with collateral 1 must be the same as for collateral 2, i.e.

\[
p_i r_i^p + (1 - p) h_{1,i}^p = p_i r_i^p + (1 - p) h_{2,i}^p,
\]

which implies that \( h_{1,i}^p = h_{2,i}^p = h_i^p \). Note that this does not necessarily imply that the two collateral have the same risk. Suppose that collateral 1 and 2 pay one with probability \( \theta_1 \) and \( \theta_2 \) respectively and such that \( \theta_1 < \theta_2 \). In this case to borrow one unit with full collateralization \( h_i^p = 1 \), the bank must pledge \( 1/\theta_1 \) units of collateral 1 and \( 1/\theta_2 \) units of collateral 2.
banks who must often lend to a very large number of counterparties amid tight time constraints. Second, the central bank can set asset-specific haircuts while competition between money market participants causes haircuts to be the same for both assets.

The central bank follows a policy rule that attempts to steer the real interest rate in each country to a target level \( r^* \). The distinction between the real economy rate \( r_i \) and a “target” or “neutral” interest rate \( r^* \) is meant to capture in a reduced form the influence of monetary policy on real activity (Svensson, 2011). The target rate \( r^* \) is the same for both countries which reflects the fact that in practice central banks have a single target rate for the whole currency area. The central bank also wishes to minimize its exposure to the banking sector which we measure as the loss given default \( LGD_{cb} \), i.e. the difference between the amount owed \( r_{cb}q_{cb} \) and the collateral seized in case of default:

\[
LGD_{cb} = \max \left\{ \sum_{i=A,B} \left( r_{cb}q_{cb} - \sum_{j=1}^{2} a_{cb}i,j \right), 0 \right\}.
\]

The indicator function \( 1_{\sum_{i=A,B} \left( r_{cb}q_{cb} - \sum_{j=1}^{2} a_{cb}i,j \right) \geq 0} \) ensures that a potential profit (negative LGD) has no effect on the loss function. The central bank minimizes the following loss function:

\[
L = \lambda \sum_{i=A,B} |r_i - r^*| + LGD_{cb}.
\] (3)

The parameter \( \lambda \) determines the relative importance to the central bank of minimizing rate deviations relative to minimizing losses. I assume that the central bank puts equal weight on each country.

The objective function followed by the central bank in (3) can be given several interpretations. One is that such a policy rule closely resembles practical monetary policy, such as the rule used by Taylor (1993) to describe the policy of the Federal Reserve. This work spurred a large literature on the use of monetary policy targeting the nominal rate (Benhabib et al., 2001). In such a context, the target rate in (3) would thus represent the real equivalent of the nominal target.

Another interpretation of the policy rule (3) is that it is welfare maximizing. Woodford (2003) for instance shows that when prices are sticky, deviations of output from its “natural rate” are proportional to the unexpected component of inflation. Under such a setting, the rate \( r^* \) in (3) can be interpreted as the rate that achieves a “potential” investment level \( q^*_i \) so that output is at its natural rate \( p_iR'(q^*_i) = r^* \).

An economy rate \( r_i \) above \( r^* \) could then be interpreted as expansionary whereas \( r_i < r^* \) would suggest

\(^{7}\text{One could also argue that if the central bank used bank specific contracts this may increase the risk of runs on the banks that are assessed as risky by the central bank.}\)
that output is close to or above potential, with an increased risk of inflation.

Yet another interpretation is that of Williamson (2012) who argues that in practice the actions of the central bank are constrained by those of the fiscal authority. He shows that if the fiscal authority fixes the real deficit forever, the optimal monetary policy may deviate from the Friedman rule. The policy rule (3) would then correspond to the rule that balances the costs and benefits of inflation.

Finally, another interpretation is related to the bank-lending channel of monetary policy. Bernanke and Gertler (1995) and Kashyap and Stein (2000) have shown that bank lending conditions are instrumental in transmitting the monetary policy stance to the real economy. The central bank objective could thus be interpreted as the central bank targeting a specified amount of lending in the economy or a level of lending rates to firms and households.

This paper takes the objective of minimizing deviations $|r_i - r^*|$ as given and does not attempt to distinguish which interpretation is the most appropriate.

The second component of the policy rule (3) is the loss given default. This term is used to reflect the possibility that losses may endanger the independence of the central bank and its ultimate goal of price stability (Del Negro and Sims, 2014). The use of the realized losses instead of the expected losses $\left((1 - p_i) LGD^{cb}\right)$ captures in a reduced form the risk aversion of the central bank. Finally, our loss function implicitly assumes that the central bank is indifferent about potential redistributive effects of its interest rate policy, for example if one country is a net creditor while the other is a net debtor.

2.1 Funding choice and timing

The borrowing process has two stages (illustrated in figure 2). In the first stage, the central bank chooses its lending and borrowing policy and proposes a contract $(r^{cb}, h_1^{cb}, h_2^{cb})$. In the second stage, (a) the money market opens and offers contracts $(r_i^p, h_i^p)$ and (b) the bank chooses how much to borrow from each venue.

![Figure 2: Structure of the borrowing process](image-url)
in order to maximize its profit.

Money market participants are willing to finance the bank under two conditions. First, the contract in the money market \((r^p_i, h^p_i)\) must be offered by money market participants who require a return equal to the central bank’s policy rate \(r^{cb}\) (the participation constraint):

\[
p_i r^p_i + (1 - p_i) h^p_i = r^{cb}. \quad \text{(Participation)}
\]

Second, money market participants and the central bank only lend to bank \(i\) if it does not shirk, i.e. the chosen outcome across lending venues satisfies the incentive compatibility constraint of the bank:

\[
p_i R(q^p_i) - p_i \sum_s r^s_i q^s_i - (1 - p_i) \sum_{j,s} h^s_{i,j} q^s_i \geq H q_i - \sum_{j,s} h^s_{i,j} q^s_i. \quad \text{(IC)}
\]

where the left hand side term is the expected profit of the bank when it properly channels the funds to the real economy. The right hand side term is the payoff in case of shirking (where the bank loses the collateral pledged \(\sum_{j,s} h^s_{i,j} q^s_i\) and earns a private benefit \(\sum_s H q^s_i\)). With probability \(p_i\), the bank earns the return from its investments \(R\left(q^p_i + q^{cb}_i\right)\) and pays back \((r^p_i q^p_i + r^{cb} q^{cb}_i)\). With probability \((1 - p_i)\), the bank defaults and loses the collateral pledged \(\sum_{s=\{p,cb\}} \sum_{j=1}^{2} a^s_{i,j}\).

Given the contracts offered by the money market and the central bank ((Participation), (IC), (1) and (2)), the bank chooses the amounts to borrow from each venue \((q^p_i\) and \(q^{cb}_i)\) as well as how much collateral to pledge to each venue \(a^p_{i,1}, a^p_{i,2}, a^{cb}_{i,1}, a^{cb}_{i,2}\) to maximize its profit:

\[
\max_{q^p_i, q^{cb}_i, a^p_{i,1}, a^p_{i,2}, a^{cb}_{i,1}, a^{cb}_{i,2}} \quad p_i R\left(q^p_i + q^{cb}_i\right) - p_i \left(r^p_i q^p_i + r^{cb} q^{cb}_i\right) - (1 - p_i) \sum_{s=\{p,cb\}} \sum_{j=1}^{2} a^s_{i,j} \quad \text{(4)}
\]

subject to having enough collateral to meet the contract requirements (equations (1) and (2)). Finally, the bank cannot borrow or pledge negative amounts.

3 First best: no moral hazard

We begin with the model without moral hazard (thus ignoring the (IC)) and study the response of the central bank to economic shocks in this setting. To derive the equilibrium of the two stage game, we proceed by backward induction: in subsection 3.1 we first derive the optimal borrowing scheme of the bank and the contract offered by the money market \((r^p_i, h^p_i)\) for a given central bank policy \((r^{cb}, h^{cb}_1, h^{cb}_2)\)
and economic environment \((p_i, \theta_{i,1}, \theta_{i,2}, r^*)\). We then derive the optimal policy of the central bank in the first stage in case of symmetric and asymmetric shocks in subsection 3.2.

### 3.1 Money market and optimal bank borrowing (Stage 2)

The contract offered by the central bank is defined in equations (2). The contracts offered by the private market must satisfy the \((\text{Participation})\) constraint and the collateral requirements of equation (1). Since there is no moral hazard we may abstract from equation (IC).

Bank \(i \in \{A, B\}\) chooses the amounts to borrow \(q^p\) and \(q^{cb}\) in order to maximize its profit (4) subject to the collateral constraints (for \(j = 1, 2\)) and the contract offered by the private market and the central bank. For clarity, we drop the subscripts \(i\) in this subsection since we consider each bank separately.

In order to simplify the problem, we can break down central bank borrowing in two terms \(q^{cb} = q_1^{cb} + q_2^{cb}\) where \(q_j^{cb} = a_j / h_j^{cb}\) is the amount borrowed using collateral \(j\). We also define the cost of borrowing from the central bank using collateral \(j\) as

\[
r_j^{cb} = p r_j^{cb} + (1 - p) h_j^{cb}.
\]

By plugging \((\text{Participation})\) and (1) in the maximization problem we may rewrite the problem as

\[
\max_{q^p, q_1^{cb}, q_2^{cb}, a_1^p, a_2^p} p R \left( q^p + q^{cb} \right) - r^{cb} q^p - r_1^{cb} q_1^{cb} - r_2^{cb} q_2^{cb}
\]

subject to the collateral constraints (for \(j = 1, 2\))

\[
a_j^p + q_j^{cb} h_j^{cb} \leq \theta_j
\]

and the non-negativity constraints:

\[
q^p, q_1^{cb}, q_2^{cb}, a_1^p, a_2^p \geq 0
\]

Equation (5) shows that each funding source has a constant marginal cost, equal to \(r^{cb}\) for the private market and \(r_j^{cb}\) to borrow from the central bank using collateral \(j\). Money market participants are indifferent between a collateral transfer or an interest payment as long as they earn the policy rate in expectation (equation \((\text{Participation})\)) so the availability of collateral never acts as a constraint to private borrowing. This is not the case however for the central bank which requires collateral so that the bank can borrow
up to a maximum of \( \bar{q}^{cb}_j = \theta_j / h^{cb}_j \) using collateral \( j \).

Assume without loss of generality that \( h^{cb}_1 \leq h^{cb}_2 \) so that \( r^{cb}_1 \leq r^{cb}_2 \). The problem can be further simplified using the following marginal cost function:

\[
c(q) = \begin{cases} 
\min \{ r^{cb}_i, r^{cb}_f \} & \text{if } q \leq \bar{q}^{cb}_1 \\
\min \{ r^{cb}_i, \bar{q}^{cb}_2 \} & \text{if } \bar{q}^{cb}_1 < q \leq \bar{q}^{cb}_1 + \bar{q}^{cb}_2 \\
r^{cb} & \text{else}
\end{cases}
\]

(8)

The optimal aggregate investment level equalizes the marginal cost \( c(q) \) and the marginal return \( pR'(q) \).

**Proposition 1** (Bank strategy in stage 2 without moral hazard). The optimal aggregate investment level \( q_i \) solves \( \lim_{\varepsilon \to 0} c_i(q_i - \varepsilon) \leq p_i R'(q_i) \leq c_i(q_i + \varepsilon) \). The borrowing source is given by:

\[
(q^p, q^{cb}_1, q^{cb}_2) = \begin{cases} 
(q, 0, 0) & \text{if } r^{cb} < h^{cb}_1, h^{cb}_2 \\
(\max \{ q - \bar{q}^{cb}_1, 0 \}, \min \{ q, \bar{q}^{cb}_1 \}, 0) & \text{if } h^{cb}_1 < r^{cb} < h^{cb}_2 \\
(\max \{ q - \bar{q}^{cb}_1 - \bar{q}^{cb}_2, 0 \}, \min \{ q, \bar{q}^{cb}_1 \}, \min \{ q - \bar{q}^{cb}_1, \bar{q}^{cb}_2 - \bar{q}^{cb}_1, 0 \}) & \text{if } h^{cb}_2 < r^{cb}
\end{cases}
\]

The collateral pledged to the central bank is \( a^{cb}_j = q^{cb}_j h^{cb}_j \) and any \( a_j^p \in [0, \theta_j - a^{cb}_j] \) can be an optimal amount of collateral pledged to the private market.

**Proof.** We first show that \( c(q) \) is the funding cost function with lowest funding cost for an investment level \( q \). The general funding cost function is

\[
r^{cb} q^p + r^{cb}_1 q^{cb}_1 + r^{cb}_2 q^{cb}_2.
\]

The funding scheme uses the cheapest funding source, subject to the collateral constraints. If \( r^{cb} < h^{cb}_1 \), the best option is to borrow only from the private market. Collateral availability cannot constrain the investment level since money market participants are indifferent between any contract that satisfies (Participation) (they are willing to fund the bank without collateral if the interest rate is sufficiently high).

If \( r^{cb}_1 < r^{cb} < r^{cb}_2 \), the bank first borrows from the central bank at cost \( r^{cb}_1 \) using collateral 1 until it has exhausted all its collateral \( (q < \bar{q}^{cb}_1) \). It then borrows from the private market at cost \( r^{cb} \). If \( r^{cb}_1 < r^{cb}_2 < r^{cb} \), the bank borrows using collateral 1 until \( \bar{q}^{cb}_1 \), then collateral 2 until \( \bar{q}^{cb}_1 + \bar{q}^{cb}_2 \) and then borrows from the private market. This shows that \( c(q) \) is the minimum funding cost function.
Consider now the optimal aggregate investment level. The function $c(q)$ provides the marginal funding cost for a given investment level $q$. The condition $\lim_{\epsilon \to 0} c(q - \epsilon) \leq pR'(q) \leq c(q + \epsilon)$ is a first-order condition suited to the non-linearity of the cost function $c(q)$.

Figure 3 illustrates proposition 1 for the case where $h_1^{cb} < h_2^{cb} < r^{cb}$. The optimum marginal cost function faced by the bank is a step function, with discrete jumps at the thresholds $\bar{q}_1^{cb}$ and $\bar{q}_1^{cb} + \bar{q}_2^{cb}$. The marginal revenue $pR'(q)$ is a standard downward sloping and convex curve and the optimum investment level is at the intersection of the marginal revenue and the marginal cost. The value of the marginal revenue at this point is the interest rate in the economy. The quantities borrowed are to the left of the intercept - in the figure the bank pledges all its collateral 1 and 2 to the central bank and also borrows from the private market.

The choice of collateral by banks could be interpreted as following a “Gresham’s law of collateral”: banks use primarily the collateral that is overvalued by the central bank and keep the undervalued collateral. There is a continuum of contracts in the private market since private market participants are indifferent between collateral or interest payments.

**Proposition 2** (Money Market Contract). Any contract $(r^p, h^p)$ that satisfies $r^{cb} = pr^p + (1 - p) h^p$ and for which the bank has sufficient collateral $\left(\theta_1 + \theta_2 - q_1^{cb} h_1^{cb} - q_2^{cb} h_2^{cb}\right) / q^p \geq h^p$ can be an equilibrium, where $q^p$, $q_1^{cb}$ and $q_2^{cb}$ are defined in proposition 1.

Proposition 2 implies that contracts with less collateral command higher interest rates, which is what we observe in practice where interest rate on secured loans are lower than the unsecured rates. The premium...
between covered and uncovered loans is increasing in the probability of default \((1 - p)\), as observed in practice.

### 3.2 Optimal central bank policy (Stage 1)

We now derive the optimal policy of the central bank in stage 1 given the behavior in stage 2 of money market participants and the bank that we derived above. Economic shocks affect the probabilities of success of projects in each country, the value of the collateral held by banks and the target rate \(r^*\). In particular we consider the case where countries are hit by an asymmetric shock that lowers the probability of success in country \(B\) relative to \(A\), \(p_A > p_B\) and increases the value of collateral 1 and reduces that of collateral 2.

If the central bank chooses not to lend to banks and sets \(h_{j}^{cb} \geq r^{cb}\), each bank \(i\) borrows a quantity \(q_i\) in the private market so that the economy rate is equal to the policy rate of the central bank.

\[
r^{cb} = p_i R'(q_i) = r_i.
\]  

(9)

Since there is no spread between the policy rate and the target rate, there is a perfect pass-through of the policy rate of the central bank and the central bank can reach its target \(r_i = r^*\) by setting its policy rate equal to the target rate. Notice in particular that the economy rate is independent of the probability of success of real economy projects \(p_i\); while riskier entrepreneurs will pay a higher interest rate without collateral, the risk-adjusted cost of funding will be the same for firms in the booming or distressed country.

Since banks only borrow from the central bank when it has a risk exposure \((h_{j}^{cb} < r^{cb})\), it is never in the interest of the central bank to lend against collateral to banks. The central bank adjusts its rate policy to the change in the target rate and is indifferent to changes in the probability of success \(p_i\) or collateral values \(\theta_{i,j}\).

**Proposition 3** (Central Bank Policy Without Moral Hazard). *Without moral hazard the central bank can perfectly gear the interest rate in the economy and sets its policy rate \(r^{cb} = r^*\). All lending goes through the money market \((h_{j}^{cb} > r^{cb} \forall j)\).*

*Proof.* If the central bank sets \(h_{j}^{cb} > r^{cb} \forall j\), the bank only borrows from the private market by proposition 1 and the economy rate is equal to the policy rate. The target rates are equal so that by setting the policy rate at \(r^{cb} = r^*\) the central bank’s loss function is equal to zero. 

\[\square\]
In the absence of moral hazard there is a perfect transmission of monetary policy so the interest rate in the economy is equal to the policy rate of the central bank. Since the economy rate (i.e. the return required by banks to its borrowers) does not depend on local economic conditions such as the probability of success $p_i$, the central bank therefore sets its policy rate at its target in proposition 3. This rate is uniformly transmitted across the currency area. The central bank does not lend to banks since they only borrow from the central bank if the required return is lower than that of the market, which increases the exposure of the central bank and increases its loss function.

4 Bank borrowing with moral hazard (Stage 2)

We now introduce moral hazard so collateral becomes necessary to incentivize the bank. We start in this section with the second stage and derive the money market contract and the optimal borrowing of each bank for a given central bank policy $(r^{cb}, q^{cb})$. We derive in the next section the policy of the central bank in stage 1. As in subsection 3.1, we drop the subscript $i$ since we consider each bank separately.

The main difference with moral hazard is that money market participants and the central bank are no longer indifferent between contracts $(r^p, h^p)$ that solve

$$pr^p + (1 - p) h^p = r^{cb}.$$ 

(10)

The reason is that not all contracts induce compliance by the bank. To ensure that the bank effectively invests the funds in the project, the incentive compatibility constraint (IC) must be satisfied:

$$\begin{align*}
pr(q) - p \sum_{s=p,cb} r^s q^s - (1 - p) \sum_{j,s} a^s_j &\geq Hq - \sum_{j,s} a^s_j.
\end{align*}$$

(11)

Notice in particular that unless the net returns in case of success are especially high (so that the incentive compatibility constraint is automatically satisfied), collateral will help meet the IC constraint. This means that the available quantity of collateral will possibly act as a constraint to lending.

The problem faced by the bank is identical to the case without moral hazard but now includes the IC (11), since the money market will not offer contracts that violate the IC. The general problem is:

$$\max_{q^p, q^{cb}, a^1, a^2, a^1_{cb}, a^2_{cb}} pR \left(q^p + q^{cb}\right) - p \left(r^p q^p + r^{cb} q^{cb}\right) - (1 - p) \sum_{s \in \{p, cb\}} \sum_{j=1}^2 a^s_j$$

15
such that

\[
\begin{align*}
    a_1^p + a_1^{cb} & \leq \theta_1 \\
    a_2^p + a_2^{cb} & \leq \theta_2 \\
    q^{cb} & = \frac{a_1^{cb}}{h_{1}^{cb}} + \frac{a_2^{cb}}{h_{2}^{cb}} \\
    q^p & = \frac{a_1^p + a_2^p}{h^p}
\end{align*}
\]

(12) \hspace{1cm} (13) \hspace{1cm} (14) \hspace{1cm} (15)

\[
pR \left( q^p + q^{cb} \right) - p \left( r^{cb} q^p + r^{cb} q^{cb} \right) - \left( 1 - p \right) \sum_{s=\{p,cb\}} \sum_{j=1}^{2} a_j^s \geq Hq - \sum_{j,s} a_j^s
\]

(16) \hspace{1cm} (17)

The solution to this problem can be derived in 3 steps. First, solve the problem when the new constraint is slack. Second, solve the problem when it binds and third, determine in which parameter space the constraint binds.

We derived the solution of the first case when the IC (equation (17)) is slack in the previous section (proposition 1). We denote for future reference the total aggregate borrowing in the first-best as \( q^{FB} \). If the IC binds in optimum, then the collateral constraints (12) and (13) also have to bind since by increasing the collateral pledged, the bank relaxes the IC which allows to increase borrowing and move closer to the optimal borrowing \( q^{FB} \).

In the second case, the collateral constraints and the IC bind. Plugging equations (12), (13) and (17) and rearranging the constraints, the maximization problem of the bank can be written as:

\[
\max_{q^p,q_1^{cb},q_2^{cb}} H \left( q^p + q_1^{cb} + q_2^{cb} \right) - \theta
\]

subject to the non-negativity constraints (7) and

\[
pR \left( q \right) - \left( H + r^{cb} \right) q + \theta + \left( 1 - p \right) \sum_{j=1}^{2} \left( r^{cb} - h_j^{cb} \right) q_j^{cb} = 0.
\]

(18) \hspace{1cm} (19)

In other words, the bank wishes to maximize the total amount borrowed \( q \), which includes borrowings from the private market \( q^p \) and the central bank using collateral 1 and 2, (with \( q_j^{cb} = \frac{ao_j^{cb}}{h_j^{cb}} \)). As we see in equation (19), the amount borrowed from the central bank will depend on the sign of \( r^{cb} - h_j^{cb} \). If the haircut on collateral \( j \) is below the policy rate, \( h_j^{cb} < r^{cb} \), the bank will pledge all collateral \( j \) to the central
bank and borrow the maximum amount possible from the central bank, \( q_j^{cb} = \theta_j^{cb}/h_j^{cb} \). If \( h_j^{cb} > r_j^{cb} \), the bank does not pledge collateral \( j \) to the central bank.

The last term in (19) can be interpreted as the risk exposure taken by the central bank since it is the difference between the amount owed and the value of collateral seized in case of bank default. Defining \( LGD_i^{cb} \) as the loss incurred by the central bank in case of a default of bank \( i \),

\[
LGD_i^{cb} = \sum_{j=1}^{2} \left[ r_j^{cb} - h_j^{cb} \right] \frac{\theta_{i,j}}{h_j^{cb}} 1_{r_j^{cb} > h_j^{cb}},
\]

we may rewrite equation (19) as:

\[
pR(q) - \left( H + r_j^{cb} \right) q + (1 - p) LGD_i^{cb} + \theta = 0.
\]

The solution for \( q \) of this equation provides the total investment level of the bank. When the bank lacks collateral, this can constrain its investment capacity. The exposure taken by the central bank relaxes the constraint as would an increase in the available collateral.

We have now derived the investment level in the 2 regimes under moral hazard: first-best and collateral crunch. The final step is to derive the parametric regions of each regime. Let \( q^{FB} \) be the aggregate investment level of the first-best case of proposition 1. The collateral crunch occurs if the IC is violated when the bank pledges all its collateral and borrows \( q^{FB} \). We call this parametric region the “condition 1” (which we refer to in short as C1).

**Condition 1.** \( C_1 \) and \( C_1^* \): The parametric region where the first-best investment level satisfies the IC is such that

\[
pR(q^{FB}) - \left( H + r_j^{cb} \right) q^{FB} + (1 - p) LGD_i^{cb} + \theta \geq 0
\]

where \( q^{FB} \) is the first-best investment of proposition 1. We label the case where \( r_j^{cb} = r^* \) as \( C_1^* \).

If \( C_1 \) is violated, the IC is violated at the first-best investment level \( q_i^{FB} \). The bank then enters the collateral crunch regime where it is constrained by the availability of collateral. The investment level falls below the unconstrained, first-best level.

**Proposition 4** (Stage 2: Bank borrowing with moral hazard). *Given an incentive compatible loan offer from the central bank \((r_j^{cb}, h_1^{cb}, h_2^{cb})\) and the private market \((r_p, h_p)\), the optimal strategy \((q^p, q^{cb}, a_1^p, a_2^p, a_1^{cb}, a_2^{cb})\) of the bank in stage 2 is*
If C1 holds, the strategy is the same as in the first best case of proposition 1.

Else, the equilibrium aggregate investment level is below that of the first best ($q < q^{FB}$) and solves

$$pR(q) - \left(H + r^{cb}\right)q + (1 - p)LGD^{cb} + \theta = 0$$

where $LGD_i$ is defined in (20). The central bank borrowing is $q_j^{cb} = \frac{\theta j H_j^{cb}}{h_j^{cb} < r^{cb}}$ and the private market borrowing is $q^p = q - q_1^{cb} - q_2^{cb}$. The collateral pledged is $a_j^{cb} = \theta j 1_{h_j^{cb} < r^{cb}}$ and $a_j^p = \theta j - a_j^{cb}$.

Proof. If C1 holds, the first-best contract of proposition 1 is incentive compatible and therefore also maximizes the profit of the bank under the (non-binding) IC constraint. Else, the bank solves (18) subject to (19). Since the objective is linear and increasing in total borrowing $q$, the bank chooses its borrowing at the central bank $q_j^{cb}$ in order to maximize its total aggregate borrowing. It is clear from equation (19) that borrowing from the central bank with collateral $j$ increases the total borrowing capacity of the bank only if $h_j^{cb} < r^{cb}$, so the bank only borrows from the central bank with collateral $j$ if $h_j^{cb} < r^{cb}$ and in this case pledges all its collateral $j$ to the central bank.

The contract in the private market is similar to the first best case with the addition of the incentive constraint.

Proposition 5 (Money market contract). If C1 holds and $q^p > 0$ in proposition 4, any contract $(r^p, h^p)$ that satisfies $r^{cb} = pr^p + (1 - p) h^p$, for which the bank has sufficient collateral $(\theta 1_{h_1^{cb} > r^{cb}} + \theta 2_{h_2^{cb} > r^{cb}}) \leq q^p h^p$) and that is incentive compatible, i.e. satisfies

$$pR(q) - \left(H + r^{cb}\right)q + (1 - p)LGD^{cb} + h^p q^p + \theta 1_{h_1^{cb} > r^{cb}} + \theta 2_{h_2^{cb} > r^{cb}} \geq 0$$

is part of an equilibrium.

Comparative statics. We now study how shocks to the economic environment affect the outcome for a given central bank policy $(r^{cb}, h_1^{cb}, h_2^{cb})$. We consider economic shocks $\epsilon$ that affect the parameters $p$, $\theta$ and $r^*$ so that high values of $\epsilon$ are associated with high $p$, $\theta$ and $r^*$ and relax the IC constraint, i.e.

$$\left| \frac{\partial \left(pR(q^{FB}) - (H + r^{cb}) q^{FB}\right)}{\partial \epsilon} \right| \leq \left| \frac{\partial \theta}{\partial \epsilon} \right|$$

where $q^{FB}$ is the first-best investment level of proposition 1.
Equation (23) ensures that incentive problems worsen in a downturn, when investment opportunities are less attractive and collateral values are low. We let \( E_i \) be the value of the shock \( \epsilon \) for which C1 holds if \( \epsilon > E_i \) and C1 is violated if \( \epsilon < E_i \).

The economic shock can be interpreted as a common factor that influences the level of the economic parameters. We can show that, when the central bank does not lend to banks, the collateral crunch regime creates a spread between the economy rate and the policy rate.

**Proposition 6** (Interest rates and economic shocks). For a given central bank policy \((r^{cb}, h^1, h^2)\) such that \( h^cb > r^{cb} \),

If the collateral constraint is slack (C1 holds), the interest rate in the economy \( i \) is equal to the policy rate of the central bank.

If it binds (C1 does not hold), the interest rate in economy \( i \) is above the policy rate of the central bank.

**Proof.** Since \( h^cb > r^{cb} \), the bank does not borrow from the central bank. In the first-best, the investment level solves \( r^{cb} = pR' (q) = r \) so the economy rate is equal to the policy rate. In the collateral crunch regime (C1 does not hold), the investment level is below the first-best level (see proof of Lemma (4)) so the economy rate is above the policy rate since \( R'(q) \) is decreasing in \( q \).

Figure 4 illustrates this result for the case where \( R(q) = \sqrt{q}, \theta = 0.015 + 0.01 \cdot \epsilon, p = 0.4 + \epsilon \cdot 0.1, H = 1.5, r^{cb} = 1.08 \) and \( \epsilon \) goes from -1 to 1. In the first best, the investment level \( q_i \) falls and the minimum haircut increases as the shock \( \epsilon \) falls. More importantly, the economy rate is not influenced by the shock and remains equal to the central bank’s policy rate. If the symmetric shock is too negative and below the threshold \( E_i \) (so that condition 1 is not satisfied any more), the banks enter the “collateral crunch” regime. In this case, the lack of available collateral leads money participants to restrict the amount lent to banks below the first-best level in order to ensure that they remain sufficiently incentivized. This causes the interest rate in the economy \( r = pR'(q^p) \) to increase relative to the central bank’s policy rate.

To sum up, the introduction of moral hazard creates a new regime for the bank. If it lacks collateral, the bank will be constrained in its access to funding. The economy rate then rises above the policy rate, and investment falls below the first-best level.

5 Central bank policy (Stage 1)

We now consider the policy of the central bank in the presence of moral hazard. We consider the case where bank A is located in a booming country and bank B is in a distressed country, for instance because
the two countries have been hit by an asymmetric shock. The probability of success of projects in economy $A$ is higher than in economy $B$, $p_A > p_B$. The value of the collateral owned by banks is also asymmetric. Collateral 1, owned mostly by bank $A$, has a high value whereas the value of collateral 2 is low so that $\theta_{1,A} > \theta_{1,B}$, $\theta_{2,A} < \theta_{2,B}$ and $\theta_{1,A} + \theta_{2,A} > \theta_{1,B} + \theta_{2,B}$. We will focus here on the case where the collateral owned by bank $A$ is such that bank $A$ is always in the first-best regime.

We solved in section 4 the bank’s borrowing scheme in the second stage. In this section, we derive the policy of the central bank in stage 1. For a given central bank policy, banks are in a collateral crunch or the first-best depending on whether C1 is satisfied or not in their case. so that bank $A$ is in the first-best regime for common values of the policy rate. In stage 1, the central bank solves

$$
\min_{r^{cb}, K^{cb_1}, K^{cb_2}} L = \lambda (|r_A - r^*| + |r_B - r^*|) + LGD^{cb}.
$$

The central bank faces 2 possible situations. If both banks are in the first best regime, the problem is similar to the case without moral hazard of section 3. There is a perfect transmission of the policy rate of the central bank in both countries, and the central bank sets its rate at the target level so $r^{cb} = r^*$.

If bank $B$ is in a collateral crunch, the economy rate in country $B$ is higher than the policy rate whereas the economy rate in country $A$ is equal to the central bank rate. When $r^{cb}$ is between $r_A$ and $r_B$, the loss
function can be rewritten as

$$\min_{r^{cb}, h^{cb}} L = \lambda (r_B - r_A) + LGD^{cb}$$

(24)

so the spread between the economy rates strictly increases the loss function.

To derive the optimal interest rate and haircut policy of the central bank in this case, we rely on an intermediate result that explains how the spread changes with the policy rate of the central bank. We will show that the spread is a decreasing function of the policy rate because of an imperfect pass-through of the policy rate in the distressed country. When the central bank reduces its policy rate by one unit, the economy rate in the collateral crunch country falls by less than a unit, so the spread increases.

**Lemma 1 (Imperfect pass-through).** In the collateral crunch regime (i) there is an imperfect pass-through of the policy rate so that a unit change in the policy rate of the central bank changes the economy rate by less than a unit,

$$\frac{\partial r_i}{\partial r^{cb}} = \frac{\partial (p_i R(q))}{\partial r^{cb}} < 1,$$

where $q$ solves $p_i R(q) - r^{cb}q - Hq + \theta_i = 0$. (ii) The spread between the policy rate and the economy rate is decreasing in the policy rate of the central bank;

**Proof.** See appendix A. □

To understand Lemma 1, suppose that the bank is in the collateral crunch regime and that the central bank increases its policy rate. This has 2 opposing effects. First, it reduces the investment level required for the bank to be in the first-best regime which brings the bank closer to the first-best. Second, it increases the funding cost of the bank which reduces the profit of the bank, tightens the incentive constraint and reduces the amount that the bank can borrow. This brings the bank further away from the first-best. We show in appendix A that when $R(q) = q^a$ the first effect is larger than the second one because while the gain from shirking is linear in $q$, the output function is concave so if $q$ is sufficiently small, the output per unit of investment will be greater than the per unit gains from shirking $H$. An increase in the policy rate therefore moves bank $B$ closer to the first-best by reducing the first-best investment level.

Given this intermediate result, consider now the policy of the central bank and assume that its policy rate is at the target level $r^*$ and bank $B$ is in the collateral crunch. In country $A$, the economy rate is equal to the target rate. In country $B$, the economy rate is above target. A first observation is that the central bank never increases its policy rate above $r^*$ because this increases the economy rate both in $A$ and $B$ away from the target level.
If the currency union only consisted of bank $B$, the central bank would only have to lower its policy rate below the target until the economy rate in country $B$ is on target. A lower rate indeed relaxes the incentive compatibility constraint since it reduces the cost of funding of banks. The central bank would therefore take the spread into account when setting its policy rate, however that would not affect its ability to reach the target.

In a heterogeneous currency union, the central bank faces a dilemma: to reach the target in country $A$, it would need to set the policy rate on target $r^*$ whereas for country $B$ only lower rates would allow to reach the target. Lemma 1 shows however that the central bank does not reduce its policy rate below $r^*$ because this increases the spread. A lower policy rate therefore strictly increases the Loss function in (24), so the central bank keeps its policy rate at $r^*$.

Collateral policy can however be used to alleviate the funding constraints on bank $B$ and increase the investment level in that country. By setting $h_{cb}^B < r_{cb}^B$, the central bank increases its exposure to banks. This also increases the expected profit of the distressed bank which relaxes its incentive constraint and allows it to borrow more from the money market. However, while below market haircuts can have positive welfare consequences by lowering rates in country $B$, they also create a transfer to bank $A$, which is already in the first best so these transfers only increase the loss function. This implies that the central bank will reduce its haircut on the asset that is owned mostly by the distressed bank (asset 2 in our case). The portfolio asymmetry thus plays a key role in the response of the central bank. If it is large, the central bank can target its exposure to the distressed bank which lowers the total amount of exposure required to lower rates in country $B$.

**Proposition 7.** If $C1^*$ holds, bank $B$ is in the first-best regime. As in proposition 3, the central bank sets $r_{cb}^B = r^*$ and does not lend to banks ($h_{cb}^B > r_{cb}^B \forall j = 1, 2$).

Else, bank $B$ is in the collateral crunch regime. The central bank sets $r_{cb}^B = r^*$, $h_{cb}^B > r_{cb}^B$ and relaxes collateral policy on collateral 2 by setting

$$h_{cb}^2 = \sqrt{r^* \frac{\theta_{A,2} + \theta_{B,2}}{\lambda \xi_{r,h,B}}}$$

(25)

where $\xi_{r,h,B} = \partial r_B / \partial h_{cb}^B$, $r_B = p_B R'(q_B)$ and $q_B$ solves $p_B R(q_B) - r_{cb}^B q_B - H q_B + \theta_B - (r_{cb}^B - h_{cb}^B) \frac{\theta_{B,2}}{h_{cb}^B} = 0$.

**Proof.** See appendix B.

Without moral hazard (proposition 3), the central bank never relaxes its collateral policy since it can
reach its target in both countries even if the probabilities of success $p_i$ or the available collateral $\theta_i$ differ. When we introduce moral hazard, the central bank may face a situation where the lack of collateral of banks in the country hit negatively may create a credit crunch in that country and increase the economy rate above the policy rate. In this case, the central bank may want to relax its collateral policy to increase lending in the distressed economy.

The interest rate policy of the central bank is determined by the imperfect pass through result of Lemma 1. The haircut policy reflects the trade-off faced by the central bank when using this instrument: it avoids an excessive increase in the economy rate of $B$ on the one hand but it increases the exposure of the central bank to the banking system. This is reflected in equation (25) by the numerator, where higher amounts of available collateral by banks directly increase the exposure of the central bank by unit of haircut reduction, while the denominator reflects the more positive effects of haircut policy which is to relax the credit constraints in the distressed economy and lower the associated economy rates. Similarly, if the weight placed on hitting the target economy rate $\lambda$ increases, the haircut of the central bank is lower. The central bank haircut in (25) also increases if the target rate $r^*$ increases. The reason for this is that the exposure of the central bank is determined in relative terms, with respect to the private market required rate of return which is $r^*$. If the target rate $r^*$ increases, this increases the credit exposure and thus the haircut.

In particular, while increased exposure to the distressed bank allows to improve lending conditions in that country, the exposure towards bank $A$ only increases the loss function. This implies that, all else equal, an increase in the collateral $2$ owned by bank $A$ increases the haircut of the central bank. The reason is that it increases the cost of using collateral policy since, for a unit of exposure to bank $B$, the central bank also increases its exposure to bank $A$. The portfolio asymmetry will therefore play a key role in determining to what extent the central bank can use collateral policy. Ex-post, once the shock has struck, the central bank prefers a high degree of asymmetry since this reduces the cost of using collateral policy. However ex-ante the central bank prefers a low degree of asymmetry since this helps reduce the negative impact of the shock on bank $B$ (if collateral values are correlated to the local economic conditions).

6 Discussion

_Collateral availability during the Sovereign debt crisis._ The model suggests that the availability of collateral is one potential mechanism driving the “high-rate / low rate” equilibrium during the Sovereign debt crisis.
In an empirical exploration of the collateral policy of the ECB from 2009 to 2011, Cassola and Koulischer (2015) use a specificity of the ECB’s operating framework to construct a proxy for collateral availability. The proxy is based on the fact that the Eurosystem uses a “pooling system” for collateral management, so banks pledge any amount of collateral they desire and obtain a credit line that may or may not use. In practice, banks tend to pledge more collateral than required for their borrowing need. However, one can expect that when banks are short on collateral they will save on the use of the extra collateral and pledge only the amount required for their borrowing need.

Figure 5 (from Cassola and Koulischer (2015)) plots the ratio of the total amount of collateral pledged (after haircut) to the amount borrowed against the interest rate spread of non-financial corporations (NFC). As predicted by the model, Cassola and Koulischer (2015) find a non-linear correlation pattern between the economy rate and the availability of collateral. When banks have plenty of collateral, the economy rate does not seem to correlate with collateral availability. However if collateral is scarce, we observe an almost vertical relationship between the economy rate and the availability of collateral.

The figure also illustrates the trade-off faced by the central bank when using collateral policy. It is when banks are short on collateral that a relaxation of requirements (i.e. moving the vertical red bar to the left) is likely to have the largest effect on economic conditions in the distressed country. But this comes at the cost of increasing the exposure of the central bank to the riskiest banks.

Collateral valuation and central bank exposure. In the model, the key mechanism of collateral policy is that there is an “overvaluation” of the collateral by the central bank relative to the market. This creates an exposure of the central bank to the banking system. In practice, the type of exposure will depend on how one interprets the value premium assigned by the central bank. If the true value is the market value, then the central bank may be exposed to credit risk and a loss in case of default. However the value of the collateral could also be owner-specific (as in e.g. Parlatore (2012)). If market prices include a large liquidity discount, which can be plausible in a severe turmoil, the central bank could have an advantage with respect to market participants since it is a long-term market participant. Our model is thus compatible with the Bagehot (1873) principle that the central bank may take on liquidity risk but not credit risk, however it also highlights that distinguishing the two may be difficult in case of turmoil (Goodhart, 1999).

The exposure of the central bank also makes clear that collateral policy is not a “free” policy instrument like the policy rate. This contrasts with the work of Ashcraft et al. (2011) or Acharya et al. (2012) where the central bank is not explicitly included in the model and simply influences the economic fundamentals.
Figure 5: Interest rate on loans to non-financial corporations (NFC) and overcollateralization rate by country and quarter. The NFC rate is a proxy for the real economy rate in the theoretical model. The overcollateralization rate is the ratio of the amount of assets pledged to the amount borrowed and can be interpreted as a proxy for the availability of collateral. We exclude cases where the overcollateralization rate is higher than 5 and countries with less than €4 billion of assets pledged. Source: Cassola and Koulischer (2015).
In our model, the central bank directly competes with the private market and must make a more attractive offer (i.e. take on risk) if it wants to influence the market outcome. In particular if the value of the collateral was endogenous to the collateral policy of the central bank, the conclusions would be reinforced: lower haircuts on an asset increases its attractiveness and incentivizes banks to hold this asset. This increases the value of the collateral so if it is owned mostly by the distressed bank, this further reduces rates in the distressed economy.

**Collateral and macro-prudential policy.** The fact that the central bank competes with the money market also suggests that collateral policy is not ideally suited to “lean against the wind”. The reason is that if the central bank sets its haircuts above market levels, banks will simply choose to borrow from the market so the haircut does not influence the final outcome. Of course the model abstracts from potential effects on asset prices which is the main mechanism behind the proposed used of collateral policy as a macro-prudential tool (Buiter and Sibert, 2005; Brunnermeier, 2010). Still, it seems reasonable to think that it is in case of severe turmoil that collateral policy takes the center stage, as was the case during the Sovereign debt crisis. This “one-sidedness” of the collateral policy instrument to alleviate downturns also raises questions of moral hazard and of the link to other macro-prudential tools. These are discussed in detail by Koulischer and Struyven (2014).

**Collateral policy versus asset purchases.** An alternative policy could be for the central bank to purchase the collateral directly. This would provide cash for the bank to invest, however, in general the bank will still need to borrow from the market. The reason is that banks generally borrow both covered and uncovered so that in effect one unit of collateral is used to borrow more than one unit (and $h^P < 1$ in our model). In that case the final equilibrium remains unchanged: the bank uses the cash received as collateral and remains constrained by the lack of skin in the game. The central bank could however choose to purchase the assets above their market value. This could be justified for example if the assets are illiquid and the central bank attaches a lower liquidity premium than the market. In that case the value differential will relax the incentive constraint and increase lending, just as in the model. A recapitalization or a direct transfer would similarly help to relax the incentive problem.

In the model the real effect of this policy would depend on how it influences the budget constraint. If the bank purchases the collateral at its market values, this exchanges the collateral against currency. This however does not change the problem faced by the bank, since it uses collateral to solve the moral hazard problem.

**Optimum Currency Area.** The literature on optimum currency area emphasizes that the central bank
faced a “one tool / two objectives” dilemma when responding to an asymmetric shock in a currency union. I show in this paper that collateral policy can be used to overcome this dilemma in a specific case: when the distressed country experiences a collateral crunch. In this situation, where rates are high in the distressed country (where they should be low) and low where they should be high, collateral policy can help by reducing the interest rate in the distressed economy. However, the best that the central bank can do with collateral policy is to bring back interest rates to a same level in both countries. It cannot engineer high rates in the booming country and low rates in the distressed country. In that sense our model is most suited to the Sovereign debt crisis period, but not to the preceding years (2002-2007) when interest rate spreads were similar for all countries whereas they should have been high in booming countries.

Dynamic moral hazard and links to fiscal policy. As discussed above, the model captures several features of collateral policy that may be relevant in practice. However there are at least two aspects that are not captured by the model. The first one is the fact that using collateral policy in case of turmoil could lower banks’ incentives to hold collateral. This could lead to increased intervention by the central bank in the long run. As explained above, one option to minimize the negative incentives created by collateral policy could be to combine its use with macro-prudential tools such as the Liquidity Coverage Ratio. Another aspect that the model does not capture is the link between collateral policy and fiscal policy. The eligibility criteria may be an important factor for the decision of banks to invest in an asset. Collateral requirements could then be viewed as a tool to ensure fiscal discipline by governments (Buiter and Sibert, 2005). In practice it is likely that both the mechanism described in this paper and the interaction with fiscal discipline are active. This paper focuses on the former and leaves the latter for future research.

7 Conclusion

I build a model of the transmission of monetary policy where banks can borrow from the central bank or the money market using collateral. I show that in the presence of moral hazard a lack of collateral can constrain banks’ access to the interbank market which increases the funding cost of firms and households in the real economy.

In the case of two banks and two assets, I show how asymmetric shocks that have a different effect on collateral values can lead to a situation with high rates in the distressed country and low rates in the booming country. The collateral crunch also affects the transmission of monetary policy so that a unit reduction in the policy rate translates into less than a unit reduction in the economy rate. This reduces
the effectiveness of the policy rate instrument and suggests that higher policy rates that are closer to the optimum of the stable country are more desirable.

Collateral policy can nevertheless be used to improve lending conditions in the distressed country. By reducing the haircut on assets owned mostly by the distressed bank, the central bank can relax the incentive problem and lower the interest rate in that economy. The asymmetry of banks’ collateral portfolios plays a key role in the effectiveness of the haircut instrument. If banks have significant differences in their collateral portfolios, the central bank can better target its policy to economic conditions across regions. From an ex-ante point of view, however, portfolio asymmetries may worsen the extent of the shock by increasing the correlation between banks’ collateral stocks and economic conditions.

The paper suggests several fruitful avenues for future research. One is to make asset prices endogenous to the haircut policy of the central bank which would allow to distinguish the role played by asset price changes versus increased risk of the central bank that is studied in the paper. A second one is to benchmark the collateral policy instrument to other instruments that could be used to respond to asymmetric shocks.
References


NeChio, F. (2011): “Monetary policy when one size does not fit all,”


Proof. Let us begin with point (i). In the first-best case, investment is given by

$$p_i R'(q) = r^{cb}$$

(26)

In the collateral crunch, investment solves

$$p_i R(q) - r^{cb} q - H q + \theta_i = 0,$$

which may be rewritten as

$$p_i R(q) + \theta_i = r^{cb} + H.$$  

(27)

Let $F(q) = p_i R'(q)$ and $G(q) = \frac{p_i R(q) + \theta_i}{q}$ denote the left hand side of equations (26) and (27) respectively. Since $R(q) > 0$ and $R''(q) < 0$, the average output is always higher than the marginal output so $F(q) < G(q)$. Moreover since $R(q) = q^\alpha$ we have $F'(q) > G'(q)$, i.e. the (negative) slope of $G(q)$ is larger than that of $F(q)$ for a given $q$.

To see this point, note that

$$\frac{\partial [G(q) - F(q)]}{\partial q} = \frac{p_i R'(q) q - (p_i R(q) + \theta_i)}{q^2} - p_i R''(q)$$

Since $R(q) = q^\alpha$:

$$\frac{p_i \alpha q^{\alpha-1} q - (p_i q^\alpha + \theta_i)}{q^2} - p_i \alpha (\alpha - 1) q^{\alpha-2}$$

This may be rewritten as:

$$-p_i q^\alpha (\alpha - 1)^2 - \frac{\theta_i}{q^2} < 0$$

which is always negative. Hence $F'(q) > G'(q)$.

Consider now a decrease of the policy rate from $r_0^{cb}$ to $r_1^{cb}$ in the collateral crunch regime. Let $q_0$ and $q_1$ be the investment levels before and after the rate change. We want to show that the economy rate decreases less than the change in policy rate, i.e. $F(q_0) - F(q_1) < r_0^{cb} - r_1^{cb}$ (since $F(q)$ is the economy rate for a given $q$). The investment level in the collateral crunch solves (27), which implies that

$$(G(q_0) - G(q_1)) = r_0^{cb} - r_1^{cb}.$$  

Since $F(q)$ decreases more slowly than $G(q)$, $F'(q) > G'(q)$, we have that

A Proof of Lemma 1

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$$(G(q_0) - G(q_1)) = r_0^{cb} - r_1^{cb}.$$  

Since $F(q)$ decreases more slowly than $G(q)$, $F'(q) > G'(q)$, we have that
\( G(q_0) - G(q_1) > F(q_0) - F(q_1) \) which proves that the change in economy rate in the collateral crunch is lower than the change in policy rate, which implies a pass-through of interest rate lower than one.

Finally, the imperfect pass-through implies that the spread is decreasing in \( r^{cb} \). To see this, suppose that the policy rate is at the limit case where \( r^{cb} = r_i^{cb} \). If the central bank lowers its policy rate, the bank enters the collateral crunch and the economy rate falls by less than the policy rate. The larger the fall, the larger the spread between the economy rate and the policy rate.

B Proof of proposition 7

Proof. We showed in the text that it is always optimal for the central bank to set its policy rate at \( r^{cb} = r^* \) if \( r_B^{cb} > r^* \). We now derive the optimal collateral policy when bank \( B \) in that case. First, it is important to see that the central bank will always relax its policy on collateral because this asset is the one owned mostly by bank \( B \). This allows the central bank, for a unit of risk taken, to maximize the effect on the economy rate in country \( B \). The central bank therefore solves:

\[
\min \ L = \lambda (r_A - r^* + r_B - r^*) + LGD^{cb}.
\]

Taking out the constant terms and since \( r^{cb} = r^* \), we may rewrite this as\(^8\)

\[
\min \ L = \lambda r_B + LGD^{cb}.
\]

where \( r_B = p_B R'(q_B) \), \( q_B \) solves

\[
p_B R(q_B) - r^{cb} q_B - H q_B + \theta_B + LGD_B^{cb} = 0, \tag{28}
\]

where

\[
LGD_i^{cb} = \left( r^{cb} - h_2^{cb} \right) \frac{\theta_i.2}{h_2^{cb}}
\]

and \( LGD^{cb} = \sum_{i=A,B} LGD_i^{cb} \) (as in proposition 4, both banks borrow fully against collateral 2 since it has a lower funding cost). The objective function can be written as

\[
\min \ L = \lambda r_B + \left( r^{cb} - h_2^{cb} \right) \frac{\theta_{A.2} + \theta_{B.2}}{h_2^{cb}}
\]

\(^8\)Since \( \lambda < \frac{1}{3} \), it is never optimal for the central bank to set \( h_2^{cb} \) such that it influences the economy rate in country \( A \) (\( \partial r_A / \partial h_2^{cb} > 0 \)).
where \( r_B = p_B R'(q_B) \) and \( q_B \) solves (28). The FOC yields:

\[
\lambda \xi_{r,h_2^b,B} - r^c \theta A,2 + \theta B,2 \left( \frac{h_2^b}{(h_2^b)^2} \right) = 0
\]

where \( \xi_{r,h,B} = \partial r_B / \partial h_2^b \). This can be written as:

\[
h_2^b = \sqrt{r_A^2 \theta A,2 + \theta B,2 \lambda \xi_{r,h,B}}
\]

This is indeed a minimum since \( L \) is a convex function of \( h_2^b \). The second-order derivative of \( L \) is:

\[
\lambda \xi'_{r,h,B} + 2r^c \theta A,2 + \theta B,2 \left( \frac{h_2^b}{(h_2^b)^2} \right)^3
\]

The second term is positive. Regarding the first term, we have

\[
\xi_{r,h,B} = \frac{\partial pR'(q_B)}{\partial h_2^b}
\]

i.e.

\[
\xi_{r,h,B} = pR''(q_B) q'_B
\]

where \( q'_B = \partial q_B / \partial h_2^b \). The second order derivative is:

\[
\partial \xi_{r,h,B} / \partial h_2^b = pR''(q_B) (q'_B)^2 + pR''(q) q''_B
\]

The first term is positive since \( R'' > 0 \). The second term is also positive since \( q''_B \) is negative. To see this, take the implicit differentiation of the investment level function

\[
p_B R(q_B) - r^c q_B - H q_B + \theta B - \left( r^c - h_2^b \right) \frac{\theta B,2}{h_2^b} = 0
\]

with respect to \( h_2^b \). This yields:

\[
p_B R'(q_B) q'_B - r^c q'_B - H q'_B + r^c \frac{\theta B,2}{(h_2^b)^2} = 0.
\]

Derive again:

\[
p_B R''(q_B) (q'_B)^2 + p_B R'(q_B) q''_B - r^c q''_B - H q''_B - 2r^c \frac{\theta B,2}{(h_2^b)^3} = 0.
\]
Rearrange:

\[ q_B'' = \frac{p_B R''(q_B) (q'_B)^2 - 2 r^{cb} \frac{\theta_R}{(h^p)} }{-p_B R'(q_B) + r^{cb} + H}. \]

This is always negative since \( R''(\cdot) < 0 \) and \( p_B R'(q_B) < +r^{cb} + H \) in the collateral crunch (else the bank could increase its borrowing and generate a return higher than the cost of funding \( r^{cb} \) and the private benefit \( H \)).