Competition for Status Creates Superstars
An Experiment on Public Good Provision and Network Formation

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\textbf{ABSTRACT:} We investigate a mechanism that facilitates the provision of public goods in a network formation game. We show how competition for status encourages a core player to realize efficiency gains for the entire group. In a laboratory experiment we systematically examine the effects of group size and status rents. The experimental results provide very clear support for a competition for status dynamic that predicts when, and if so which, repeated game equilibrium is reached. Two control treatments allow us to reject the possibility that the supergame effects we observe are driven by social preferences.

\textbf{Keywords:} Network formation, public goods, competition, status. 

\textbf{JEL codes:} C91, D85, H41

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\textsuperscript{*}This paper supersedes our working paper “Superstars Need Social Benefits: An Experiment on Network Formation” (van Leeuwen et al., 2013).
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1 Introduction

The provision of public goods often benefits from the exemplary performance of a small subset of players. People volunteering to help out at amateur sports teams, for example, usually show extraordinary dedication and spend a substantial part of their free time working at the club instead of being with their families. Academics spend much more time organizing workshops than can reasonably be expected in a one-shot game and editors dedicate a lot of their time to their journals without proper contingent reimbursement. A very small set of people is usually responsible for developing open source software (OSS; Lerner and Tirole 2002; Crowston et al. 2006) and a limited number of people make most contributions to Wikipedia (Voss 2005; Ortega et al. 2008).

The ease with which examples of efficient public good provision by a small subset of a group come to mind contrasts sharply with observed behavior in laboratory experiments. In applications where the efficient outcome can only be supported as an equilibrium of the repeated game, coordination on this efficient outcome is rarely observed in the laboratory. In fact, such experimental supergame results are by and large limited to games with two players, and even there efficient play tends to be fragile (see for instance the evidence reviewed in Huck et al. 2004). An additional behavioral mechanism is usually needed to support the emergence of the efficient outcome. Examples of such mechanisms include the possibility to punish defectors in public good games (Fehr and Gächter 2000) and the possibility to exclude badly behaving members from consuming the public good (Cinbyabuguma et al. 2005). These mechanisms cannot explain the efficient provision of a public good by only a few members as in the examples above, however.

In this paper, we explore the effectiveness of a novel behavioral mechanism that allows players to realize efficiency gains in repeated games. This is based on the intuition that in many examples of successful public good provision, competition for ‘status’ plays an essential role. Status may yield an internalized psychological reward; for example, contributors may be driven by the prestige or warm glow that their exemplary behavior generates (Lakhani and Wolf 2005; von Krogh and von Hippel 2006; Fershtman and Gandal 2011). Alternatively, status may yield expectations of material returns, e.g., contributors like OSS-developers may recognize that their conspicuous contributions can serve as a stepping stone to a better job in the future (Lerner and Tirole 2002; 2005); or may lead to payments by a third party, e.g. through advertisements (Roberts et al. 2006). In this paper, we will refer to all such benefits (psychological and material) as ‘status rents’. Status rents will encourage
players to compete in terms of ‘good’ behavior. The most important contribution of our paper is that we show how – if the rents are high enough – competition for status encourages a single player to realize efficiency gains for the entire group. We identify the circumstances under which inefficient provision, efficient provision and even overprovision of the public good are to be expected.

The model introduced by Galeotti and Goyal (2010) – GG hereafter – provides a fruitful theoretical structure for our analysis. In their network formation game, each player simultaneously chooses links to other players and their own investment to the public good. Players consume some public good, for instance OSS code, which they can do either by investing personally (writing code) or by making links to others who invest in the public good (using someone else’s publicly available code).

In GG’s baseline model there are no status rents. We introduce these in their model by awarding players a monetary payoff for each incoming link. GG’s main result is that in every strict equilibrium of the game, the number of players who invest in the public good is limited. These players – ‘the influencers’ – form the core of the network. Other (periphery) players link to the core, without contributing themselves. Together, the players form a core-periphery network. If the core consists of only one player, we say that a star has formed. Important for our purpose is that GG’s main result is unaffected by the introduction of status rents.¹

In the one-shot game that GG study, efficient cooperation is not supported in equilibrium. In the finitely repeated game that we are interested in, a plethora of equilibria can be supported in equilibrium, including one with efficient provision of the public good almost until the end. To shed light on which of these repeated game equilibria is to be expected, we introduce a simple behavioral model in which selfish players compete for attractive network positions. In this dynamic level-k model, we assume that players are forward looking with limited foresight. The model predicts whether a repeated game equilibrium will be reached, and if so, which one.

In situations without status rents, the periphery position is more attractive than the core position. In these circumstances, players prefer others to do the painful job of providing the public good, and the behavioral model predicts that the star networks are unstable and inefficiency will result. With status rents, the core position can become more attractive than a

¹ GG’s model explains findings reported in early work by Lazarsfeld et al. (1944) and Katz and Lazarsfeld (1955). These suggest that individuals’ roles in the network are distributed in a specific way, where a limited number of individuals influence the majority. This has been observed in applications as diverse as fashion, opinions and voting. These observations imply that information is typically acquired and shared in networks with a core-periphery structure, where a small core acquires information and a large periphery free rides.
periphery position. As a consequence, players will compete to be in the core. We show how
the person willing to invest most in the public good attracts all links and becomes the core
player in a star network. Competition for status forces the core player to invest up to the level
where payoffs across network positions are roughly equalized, i.e. to the point where
periphery players no longer have an incentive to challenge the core player by investing more.
This process yields a repeated game equilibrium with an investment in the public good that
follows from the parameters of the game.

In the laboratory, we investigate how competition for status affects public good
provision in an environment where players decide both on their contributions and on their
network connections. In particular, we consider two network characteristics that according to
the behavioral model will systematically affect the extent to which the public good is
provided and the structure and stability of the network. The first is the extent of status rents
that a player derives from incoming links; these rents are absent, of medium value or of high
value. The second characteristic is group size, which is either small (4 players) or large (8
players). In a full 3x2 design, we systematically vary status rents and group size in such a
way that the (stage-game) equilibrium predictions of the GG model remain unaffected.

In contrast, our behavioral model predicts that behavior will vary systematically
across our two treatment dimensions. Only with sufficiently large status rents, we predict
convergence to a stable core-periphery network. The particular equilibrium selected depends
on the two characteristics. Essentially, provision of the public good benefits from an increase
in status rents per link as well as from an increase in group size. 2 Finally, we add two control
treatments to the design, in which the network structure is exogenously imposed and based on
actual networks formed in the endogenous counterparts. This allows us to isolate the
competition-for-status explanation from other possible explanations of contributions by the
core (e.g., certain kinds of other-regarding preferences).

In our experiments, we implement the game in a straightforward manner. Subjects
participate in only one of the eight (3x2+2) treatments. They are informed that they remain in
the same group for 75 periods and they are informed of the relevant parameters (most
importantly, group size, status rents, linking costs, the costs of investing in the public good
and the benefits that they derive from having access to the public good). They know that they
have access to their own public good investments and to the investments of the players to

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2 In the working paper that this paper supersedes (van Leeuwen et al. 2013), we considered four treatments that
differed in many respects from the treatments reported here. The behavioral model that we propose in this paper
was inspired by studying the individual data of our working paper. Importantly, all tests of the model are out of
sample. None of the data collected for the previous paper are used here.
whom they have created links. In each period, subjects simultaneously make their links and investment decisions (except in the control treatments, where they only make investment decisions).

Our experimental results for the treatments with endogenous network formation provide clear evidence that participants compete for status rents. The existence of status rents is necessary for stable star networks to form and the extent of such rents and group size both boost the provision of the public good. Without status rents, even in the final 25 periods star networks are only observed in 10% of the cases. This means that in almost all cases, public good provision is decentralized and subjects access on average less of the public good than the stage-game Nash amount. As a result, outcomes are highly inefficient and average experimental earnings are even below what could be expected if there was no scope for networks to form. At the other extreme (with high status rents), in the final 25 periods subjects in the core of a star contribute close to the efficient amount (on average 97% of the efficient amount) when groups are small and they vastly overcontribute when group are large (on average 173% of the efficient amount). In these cases a star network is formed in 53% and 86% of the cases, respectively. Note that in our network game group size has a positive influence on contributions to the group. This is in sharp contrast to previous experiments on supergame effects. Finally, in the treatments with high status rents, but also in the treatment with medium status rents and large groups, we observe that groups mostly converge to ‘superstars’, in which the core player invests in more units of the public good than is expected in the stage game equilibrium.

Our behavioral model predicts the comparative statics that we observe. Groups tend to converge to the repeated game equilibria selected by our model. Further support comes from the process by which this occurs. In the first half of these treatments, in most groups multiple subjects compete by investing heavily in the public good. They then converge to a superstar in the second half of the experiment. We believe that our paper is the first to generate convergence to stable supergame effects in experimental network games. Finally, our results confirm a central prediction of the GG model, that is, that the maximum number of players who invest and form the core is independent of group size.

The results for the endogenous network formation treatments are consistent with the hypothesis that players compete for status. There are, however, other possible explanations for the results. Bloch and Jackson (2007) argue that an exchange of transfers can lead to efficiency gains in repeated network games. Alternatively, core players may feel that it is their duty to reciprocate by investing more in the public good if they receive high rents from
incoming links. Or, altruism or inequity aversion may motivate them to share some of the windfall gains that high status rents bring. Notice that all these alternative mechanisms are fundamentally different from a mechanism that is based on selfish players competing for status. In particular, our status dynamics predict that endogenous network formation is crucial for supergame effects to emerge. In this mechanism, supergame effects are not expected when players participate in an exogenously determined network. In contrast, if one of the other mechanisms drives the results, the emergence of supergame effects should not depend on the way the network is formed. This is the reason why we include two control treatments with exogenous networks.

The results for these controls provide strong support for our conjecture that supergame effects are primarily driven by competition for status. We observe many more superstars when networks are formed endogenously than when they are exogenously imposed. In comparison, the positive role of social motivations is negligible. With exogenous networks, only a handful of pro-socially motivated core players contribute more than would be expected on the basis of selfishness. We conclude that superstars need status rents; these trigger competition between the players, which has a substantial impact on the provision level of the public good, and on the shape and stability of the network.

The remainder of this paper is structured as follows. We start with a brief discussion of previous studies in Section 2. We present the theoretical framework in Section 3. Section 4 describes the experimental design and procedures and in Section 5 we provide equilibrium and efficiency predictions for the game with the parameters of the experiment. The results of the experiment are described in Section 6 and Section 7 concludes.

2 State of the Art

There is a relatively large theoretical literature on network formation and the provision of public goods in networks, either with endogenously formed networks or exogenously given networks.3 Most relevant for our study is GG (Galeotti and Goyal 2010), who extend the network public goods game of Bramoullé and Kranton (2007) by adding endogenous network formation using the protocol designed by Bala and Goyal (2000). As mentioned above, we employ the GG framework in our experiment.

Closest to the current study are two other papers that use the GG-framework in

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3 For an overview of the theoretical literature on network formation, see for example Goyal (2007) or Jackson (2008). Other theoretical papers that study public good provision on endogenously formed networks include Cho (2010) and Cabrales et al. (2011). Galeotti et al. (2010), Boncinelli and Pin (2012) and Bramoullé et al. (2014) study public good provision on exogenous networks.
laboratory experiments. However, both papers focus on other treatment variables than we do. The first is Rong and Houser (2012), who use the best-shot version of the GG-model, i.e. players face a binary choice whether or not to invest. Between treatments, they vary the strategy set of the players and consider the effects of sequential decision making. They find that a restricted strategy set yields more equilibrium (star) networks, while sequential decision making does not lead to more equilibrium behavior. The second is Goyal et al. (2013), who study the effects of varying the costs of linking and introducing individual heterogeneity. They find that increasing link costs leads to fewer links being made and lower aggregate earnings. Their results, as well as those of the baseline treatment in Rong and Houser (2012), line up well with the results in our treatments without status rents. In all cases, (equilibrium) core-periphery networks are rarely observed and social welfare is low due to the ineffective network structures.

In our setup, players decide both on their network connections and their investments in a (local) public good where investments are strategic substitutes. These two elements have also been studied in isolation. In experiments purely concerned with network formation (i.e. players only decide on their links) a typical result is that groups rarely converge to equilibrium (star) networks when equilibrium payoffs between different positions are asymmetric (Falk and Kosfeld 2012). Introducing heterogeneity in values can reduce payoff asymmetries; as a result star networks form more often (Goeree et al. 2009). Other experimental studies consider public good games with strategic substitutes, but on fixed networks (Rosenkranz and Weitzel 2012; Charness et al. 2014).4

To the best of our knowledge, we are the first to study endogenous network formation in combination with public goods investment and status rents. Our introduction and analysis of status rents also sheds light on results observed in previous field and laboratory studies. In a natural field experiment, Zhang and Zhu (2011) investigate contributions to Chinese Wikipedia. They interpret the repeated blockings of Chinese Wikipedia in mainland China as an exogenous variation in group size and observe that contributions increase when groups are larger. Restivo and van de Rijt (2012) provide an example of how status rents may be operationalized in the field. They show that informal rewards (‘barnstars’) encourage

4 Other papers that experimentally study public goods on exogenous networks include Fatas et al. (2010), Carpenter et al. (2012) and Leibbrandt et al. (2014). Eckel et al. (2010) study public good provision on fixed star networks and assign the core positions in some treatments to either ‘high-status’ or ‘low-status’ individuals. Status is determined by performance on a trivia quiz. High-status core players are more often mimicked by the periphery than low-status core players. Several other experimental papers that investigate games on an endogenous network (e.g. Ule, 2005, Corbae and Duffy, 2008, Knigge and Buskens, 2010, Berninghaus et al., 2012, Wang et al., 2012) use games that differ substantially from ours.
contributors on Wikipedia to increase their contributions. In laboratory experiments, providing rankings based on pro-social behavior can positively affect giving in dictator games (Duffy and Kornienko 2010). Finally, the positive effect that intergroup competition has on cooperation (e.g. Bornstein et al. 1990; Schram and Sonnemans 1996; Nalbantian and Schotter 2007; Reuben and Tyran 2010) may also be attributable to intragroup status.

Aside from status rents, one could interpret the benefits from an incoming link as a transfer between players. Transfers (or side payments) can be an effective way to sustain otherwise unstable networks (Jackson and Wolinsky 1996; Bloch and Jackson 2007). However, our focus is on the competition for links that arise when there are status rents. This turns out to be important. Our two control treatments with exogenous networks show that transfers per se are insufficient to generate the supergame effects that we observe with endogenous networks.

Finally, there is some related work that supports the approach of our behavioral model. Previous theoretical work has studied network formation with forward-looking players (Dutta et al. 2005; Herings et al. 2009). In our behavioral model, we assume that players are forward-looking with limited foresight. This assumption is supported by experimental work on network formation (Callander and Plott 2005; Berninghaus et al. 2012; Kirchsteiger et al. 2013; Caldara and McBride 2014).

3 Theory

3.1 Stage game and static analysis
We study the one-way flow variant of the static game in GG and extend the model to allow players to enjoy status rents for each incoming link. Wherever possible, we follow the notation in GG.

Denote the set of players by \( N = \{1, 2, \ldots, n\} \). Every player \( i \in N \) simultaneously decides on her (public good) investment level \( x_i \) and the links \( g_i \) that she forms. Investments are a non-negative integer, i.e. players select their investments from the set \( X = \{0, 1, 2, \ldots, x_{\text{max}}\} \). The vector \( g_i = (g_{i,1}, g_{i,2}, \ldots, g_{i,n}) \) specifies the links that \( i \) forms, where \( g_{i,j} = 1 \) if \( i \) forms a link to \( j \) and \( g_{i,j} = 0 \) if not. Hence, a strategy for player \( i \) consists of the combination of her public good investment and links and we denote this by \( s_i = (g_i, x_i) \), and i’s strategy space is denoted by \( S_i \). The linking decisions of all players jointly define the (directed) network architecture \( g = (g_1, g_2, \ldots, g_n) \) and \( x = (x_1, x_2, \ldots, x_n) \) denotes the vector of investments. A strategy profile is then denoted by \( s = (x, g) \). The set of
all possible strategy profiles is denoted by $S$.

Forming a link to another player $j$ allows $i$ to access $j$’s investment. Let $N_i(g) = \{j \in N : g_{i,j} = 1\}$ denote the set of players that $i$ links to and $\eta_i(g) = |N_i(g)|$ the number of links that $i$ forms. Likewise, we denote the number of links that are formed to $i$ by $\omega_i(g) = |j \in N : g_{j,i} = 1|$. The total investment that $i$ accesses is then given by $y_i = x_i + \sum_{j \in N_i(g)} x_j$. The benefits $f(y_i)$ of accessing units are increasing and concave in $y_i$. Note that the investments of $i$ and of the players she has linked to are perfect substitutes: $i$ values her own investment the same as any investments by any $j \in N_i(g)$.

Investing in units of the good comes at a constant marginal cost of $c > 0$ per unit and making a link comes at a cost $k > 0$. Players receive status rents $b \geq 0$ from each incoming link. We take $k > b$, which ensures that making links has a net cost to society. This results in the following payoff function:

$$\Pi_i(s) = f(y_i) - cx_i - k\eta_i(g) + b\omega_i(g).$$

If we assume self-regarding preferences, a strategy profile $s^N$ is a strict Nash equilibrium if for every player $i \in N$ it holds that

$$\Pi_i(s^N_i, s^{N \setminus i}) > \Pi_i(s_i, s^{N \setminus i}) \forall s_i \in S_i,$$

where $\Pi_i(s^N_i, s^{N \setminus i})$ is the stage-game payoff of player $i$ given that she chooses $s^N_i$ and the other players choose $s^{N \setminus i}$.

In any strict Nash equilibrium, a core-periphery network is formed where the players in the core invest in the public good and players in the periphery do not invest. This is the main result in GG. The proof of this and subsequent results is relegated to Appendix A.\textsuperscript{5,6} In a core-periphery network, any player forms links to all the core players but not to any of the periphery players. In equilibrium, the players in the core jointly invest in $\bar{y}$ units, defined as the optimal public good investment if players would act in isolation. The maximal number of players that can be sustained in the core (and invest) is independent of group size and status rents. A special case is the Nash star. In this outcome, a single player forms the core and invests in $\bar{y}$ units. When we refer to ‘stars’, we always mean periphery-sponsored stars.

\textsuperscript{5} This, and the following results carry over from GG. As we study a slightly modified game with discrete investments and one-way flow, we provide the proofs in Appendix A for completeness.

\textsuperscript{6} We only consider the (arguably most interesting) case where $cy > k$. If the reverse holds, the unique Nash equilibrium is the empty network (i.e. no links are formed) where all players invest in $\bar{y}$ units.
If $c \gamma > k$, the Nash star is always a strict Nash equilibrium. The intuition is the following. The marginal benefits of the public good exceed the costs of investing up to $\gamma$ units of the good. This implies that every player wants to access at least $\gamma$ units of the good. Suppose there exists some player $i$ that invests in $\gamma$ units. When forming a link is strictly less costly than investing in $\gamma$ units, i.e. $c \gamma > k$, the best response of any other player than $i$ would be to link to $i$ and not invest, hence a star forms where the core invests and all others free-ride and link to the core. Finally, for $i$, given that no other player is investing, it is optimal to invest in $\gamma$ units. There are $n$ such equilibria: one for each player being in the core.

Note that the rents from incoming links play no role in this intuition. This is why the stage-game equilibria are independent of the level of status rents. More formally, given a collection of strategies $s_{-i}$ of all players $j \neq i$, player $i$ will prefer strategy $s_i$ over $s'_i$ if

$$f(y_i) - f(y'_i) - c_i(x_i - x'_i) - k(\eta_i(g) - \eta_i(g')) + b(\omega_i(g) - \omega_i(g')) \geq 0.$$  

As the strategies of all other players are fixed, it must be that $\omega_i(g) = \omega_i(g')$ and the final term on the left hand side of (1) cancels. Then, $i$’s decision is independent of the status rents $b$ and the set of Nash equilibria must be independent of $b$. Moreover, as we show in Appendix A, the number of players in the core is independent of group size.

We define social welfare $W$ resulting from a strategy profile $s$ as the sum of all individual payoffs, i.e. $W(s) = \sum_{i \in N} \Pi_i(s)$. A strategy profile $s$ is called efficient if

$$W(s) \geq W(s') \forall s' \in S.$$  

Based on this definition, the efficient outcome is a star in which the core player invests in (weakly) more units than in the Nash star, while the periphery players do not invest. This is the case because all players – either in the periphery or the core – benefit from additional investments by the core. The efficient investment by the core is denoted by $\bar{\gamma} \geq \gamma$ (see Appendix A). Note that any investment by the core above $\gamma$ units will lead to welfare losses. In our further analysis, we refer to superstars. We call an outcome a superstar if it is a star where the core invests in strictly more units than in the Nash star. Note that efficient outcomes are superstars if $\gamma > \gamma$.

\footnote{As we will describe in section 5, the Nash star is the only type of stage-game Nash equilibrium in our experiments.}
3.2 Subgame perfect equilibria

The finitely repeated game that we study hosts a plethora of equilibria. Appendix D provides more details. In Proposition D.1 we show that efficient superstars can be sustained in a subgame perfect repeated game equilibrium. Both with and without status rents, efficient superstars can be sustained by rotating the core position. Such rotation requires tremendous coordination, however, and is unlikely to be observed in laboratory play (e.g. Goeree et al. 2009; Falk and Kosfeld 2012). Considering only subgame perfect equilibria with fixed positions, Proposition B.2 shows that efficient superstars can be sustained in equilibrium only when there are status rents.

Although status rents affect the set of repeated-game equilibria, subgame perfection in itself provides little guidance which equilibrium, if any, to expect. As argued before, we hypothesize that players will compete for attractive network positions. We capture this hypothesis in a simple behavioral model, which we use to derive predictions for the experimental treatments.

3.3 Competition for status

Our competition for status dynamic is motivated by the idea that players jockey for a position in the periphery or the core, whichever they consider most attractive. As noted before, status rents do not affect the set of stage-game equilibria, but they do affect the payoffs of players in the core. In any strict Nash equilibrium, the core players earn less than the periphery players in the absence of status rents. Introducing status rents increases the payoffs of core players, without affecting those in the periphery. The effect on core payoffs depends on group size. Hence, status rents and group size jointly determine the relative payoffs between players in the periphery and the core.

Players can use their investment decision to obtain a more attractive position. Note that the linking decisions are less useful to jockey for a position than the investment decision. The reason is that other players decide with their links whether a player becomes part of the periphery or the core. In contrast, the investment decision is entirely at the disposal of the individual herself. To capture this difference, we assume that for the investment decision, when players form expectations about how play will unfold in the future, they look forward with limited foresight. With their linking decisions, players simply choose myopic best responses to the current investment profile. To generate predictions of our competition for status dynamic, we have to make further, more specific, assumptions. In doing so, we assign a lot of weight to simplicity.
In the model, each period consists of two phases. In the first, one randomly assigned player reconsiders her investment in the public good. No other player changes her investment.\(^8\) This determines the investments \(x_t\).\(^9\) In the second phase of period \(t\) every player \(i \in N\) revises her linking decision \(g_{i,t}\) by playing a myopic best response to \(x_t\). When reconsidering their investment, we assume that players are forward looking with limited foresight, taking future responses of the other players into account. If a player is selected to revise her investment choice, she chooses the level that maximizes her expected payoff in the current period \(t\) and the immediate next period \(t + 1\), anticipating how others will respond in the near future and assuming that her investment decision will be the same in both \(t\) and \(t + 1\). When imagining how play will unfold for each possible investment level, the player assumes that the other players will occasionally revise their investment decisions with limited foresight.

The way we model forward-looking behavior can be interpreted as a ‘dynamic level-\(k\) model’. We assume that a player who updates her investment considers the next two periods, while she expects that others will look only one period ahead. This is akin to ‘level-\(k\)’ models, where a player with level \(k\) plays a best response to other players who reason \(k - 1\) steps ahead. Our assumption that \(k = 2\) lines up well with experimental evidence on beauty contest games (Nagel 1995; Duffy & Nagel 1997).

If some player \(i\) updates her investment \(x_{i,t}\) at time \(t\), she expects that others will play myopic best response with their links in response to her investment \(x_{i,t}\) and the investments of all other players (which did not change from period \(t - 1\)). Denote this vector of investments at time \(t\) by \(x_t = (x_{1,t}, x_{2,t}, \ldots, x_{n,t})\) where \(x_{j,t} = x_{j,t-1}, \forall j \neq i\). That is, \(i\) expects a myopic best response where she, and all other players, choose \(g_{\ell,t}\) such that:

\[
\Pi_{\ell,t}(g_{\ell,t} | x_t) \geq \Pi_{\ell,t}(g'_{\ell,t} | x_t), \forall g'_{\ell,t} \neq g_{\ell,t}, \forall \ell \in N.\(^{10}\)
\]

Furthermore, \(i\) expects that at \(t + 1\) one other player \(j\) will consider updating her investment \(x_{j,t+1}\) and that \(j\) will do so taking the (myopic best response) linking decisions at \(t + 1\) of all

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\(^8\) In the experiment, all subjects decide simultaneously on their investments and their links. We depart from this in the behavioral model for tractability.

\(^9\) To enable description and analysis of the dynamics, we add subscript \(t\) to all relevant variables from here onward.

\(^{10}\) Note that a player’s myopic best response for her links is independent of the links of other players in our game with one-way flow. For some \(x_t\), there exist multiple best replies in the linking decisions, i.e. when two players invest in exactly the same amount. In this case, we assume that players will not change their links, i.e. for player \(i\) to attract the links that player \(j\) has at \(t - 1\) she should invest strictly more than \(j\).
players into account. This implies that \( i \) expects that investments at \( t + 1 \) will be \( x_{i,t+1} = x_{i,t} \) for herself, \( x_{\ell,t+1} = x_{\ell,t-1} \) \( \forall \ell \neq i, j \) and she expects \( j \) to choose \( x_{j,t+1} \) such that this maximizes \( j \)'s payoffs at \( t + 1 \) after all players have played myopic best response with their links.

A player \( i \) who updates her investment evaluates the future payoffs by taking a convex combination over the expected payoffs at \( t \) (the present) and \( t + 1 \) (the future):\(^{11}\)

\[
\mathbb{E} \Pi_i = (1 - \alpha) \mathbb{E} \Pi_{i,t} + \alpha \mathbb{E} \Pi_{i,t+1}.
\]

This means that when \( \alpha = 0 \), \( i \) puts all weight on the immediate expected payoff while \( \alpha = 1 \) puts all weight on the future expected payoff. A player may well put relatively more weight on the future than on the present \((\alpha \geq \frac{1}{2})\). This would reflect the belief that the outcome in \( t + 1 \) will persist. A strategy profile is stable if for all players \( i \in N \):

\[
\mathbb{E} \Pi_i(x_{i,t} = x_{i,t-1}, x_{t-1}) > \mathbb{E} \Pi_i(x_{i,t} = x_{i}', x_{t-1}), \forall x_i' \neq x_{i,t-1},
\]

holds. As the link decisions follow mechanically from the model, we suppress the dependence of the expected payoffs on the network architecture.

Here, we discuss the conditions under which star networks are stable under the behavioral model (the proofs are given in Appendix B).\(^{12}\) A star network is stable if neither the core-player \( i \) nor any periphery player \( j \) changes her investments when given a chance to reconsider her choice.

Consider first the core-player \( i \). The myopic best response of \( i \) is to invest \( x_i = \mathcal{P} \), which yields a Nash star. However, if the sum of status rents is sufficiently low, the core position earns less than the periphery and a forward-looking core player may try to move to the periphery by choosing \( x_i < \mathcal{P} \). On the other hand, if the sum of status rents is sufficiently large, the core position earns more than the periphery. In this case, a forward-looking core player will anticipate that her position may be challenged by one of the periphery players. As a result, \( i \) considers choosing \( x_i > \mathcal{P} \) to avoid being challenged. Player \( i \) expects to be challenged by periphery player \( j \) if:

\[
\mathbb{E} \Pi_j(x_{j,t+1} = x_{i,t} + 1) > \mathbb{E} \Pi_j(x_{j,t+1} = 0).
\]

---

\(^{11}\) To simplify notation, we suppress indicating in the LHS that the evaluation is taking place in period \( t \).

\(^{12}\) In Appendix B we also show that stable networks are either stars or have inefficiently low investment.
With $x_{j,t+1} = x_{i,t} + 1$, all players $\ell \neq i,j$ will link to $j$ at $t + 1$ and $\mathbb{E}l_j(x_{j,t+1} = x_{i,t} + 1) = f(x_{i,t} + 1) - (x_{i,t} + 1)c + (n - 2)b$. If $j$ does not invest, she will remain in the periphery of the star with $i$ in the core and $\mathbb{E}l_j(x_{j,t+1} = 0) = f(x_{i,t}) - k$. Using this, $i$ expects that no periphery player will challenge when:

$$f(x_{i,t}) - k > f(x_{i,t} + 1) - (x_{i,t} + 1)c + (n - 2)b.$$  

Rewriting gives:

$$\text{(2)} \quad (n - 2)b < (x_{i,t} + 1)c - k - [f(x_{i,t} + 1) - f(x_{i,t})].$$

If (2) holds, core player $i$ expects not to be challenged. Note that the RHS of (2) is strictly increasing in $x_i$ for $x_i \geq \mathcal{y}$, because by construction $c > f(x_{i,t} + 1) - f(x_{i,t})$ for investments larger than or equal to the Nash quantity $\mathcal{y}$. Denote the lowest value of $x_i$ for which (2) holds by $y^*$.\textsuperscript{13} This is the lowest investment level at which the core does not expect to be challenged. It follows from the LHS of (2) that $y^*$ is increasing in both $n$ and $b$, provided that $b$ is strictly positive and $n > 2$. Payoffs for the core and each of the individual periphery players are roughly equalized at $y^*$.

We show in Appendix B that the level of status rents and group size determine whether star networks are stable, and if so, which investment by the core can be expected. When the sum of status rents is sufficiently high, i.e. when the core earns more than the periphery in the Nash star, $y^* \geq \mathcal{y}$ holds and the star where the core invests in $y^*$ units is the only stable outcome. Any investment by the core lower than $y^*$ will make her lose her attractive position. As she anticipates this, she will keep her investment at $y^*$. If status rents are low and the core earns less than the periphery in the Nash star, Nash stars and all superstars are unstable. This is because any forward-looking core player will lower her investment, expecting that some other player will invest the Nash amount $\mathcal{y}$ in the next period if she does so.

Some of the simplifying assumptions we make are not crucial for the results. For instance, when a superstar has emerged and the core player is considering her investment, the model assumes she commits to the same investment for two periods. A core player might instead consider to first deviate to $\mathcal{y}$ and to invest 0 in the subsequent period, anticipating that

\textsuperscript{13} In Appendix B we show that when $y^* < \mathcal{y}$ all Nash stars and superstars are unstable if players are sufficiently forward-looking.
she will be able to link to another player who takes over the core position. Such an extension will not change the predicted outcome. What is necessary for the stability predictions is that players converge to a profile in which the core position is (slightly) more attractive than a periphery position. This is true under most parameter configurations, among which the parameters in our treatments with status rents. As long as the periphery position is less attractive than the core, the latter will not want to give up her position for a short-term gain if she assigns sufficient value to the future.

A superstar would only become unstable if the core player could harvest short-term gains by free riding, knowing that she could subsequently immediately regain the core position with certainty. This involves the strong assumption that other players refrain from competing when the attractive core position has become available. Whether or not such a destabilizing force will materialize is ultimately an empirical matter to which we will pay special attention in the analysis of the experimental results.

Our behavioral model also generates predictions for the treatments with exogenous networks. Here, selfish players have no incentive to overinvest to obtain a better position. Hence, the dynamics simply predict that the core will invest in the stage-game Nash amount $\hat{y}$ and that periphery players will not invest.

4 Experimental design and procedures

In the experiment, subjects play the stage-game described in Section 3 repeatedly for 75
periods. Across treatments, we systematically vary two parameters: group size \( n \) and the level of status rents \( b \). Table 1 summarizes this design: we have groups of either 4 or 8 subjects, who play the experimental game either with no \((b = 0)\), medium \((b = 22)\) or high \((b = 66)\) status rents. In addition, we ran two treatments with high status rents where the links are exogenously imposed.

In the treatments with endogenous network formation we implement a partners design: i.e. subjects are randomly assigned to a group and play the experimental game with fixed partners.\(^{14}\) These partners are identified by letters ranging from A to D or A to H, depending on the group size and the letters refer to the same subject throughout the experiment. The number of periods is announced in the experimental instructions (see Appendix E). In every period, all subjects simultaneously decide on whom to link to and how much to invest. On their decision screen, subjects can review all previous decisions in a history box. Once everyone in the session has made a decision, subjects are informed of the resulting outcome and their own payoffs. Examples of key screenshots are provided in Appendix F.

In the treatments with exogenous linking, everything is the same as in the treatments with endogenous linking except that we impose the linking decisions observed in the endogenous linking treatments. This means that subjects are informed of the links they will form in the current period and only decide on their investment in the public good. In the instructions, subjects were informed that subjects could in no way affect the links by their

\(^{14}\) This corresponds to many cases in the world outside of the laboratory. For example, on many OSS projects, the key contributors remain active over several years (Robles et al., 2005, Crowston et al., 2012).
decisions. Note that subjects do pay for outgoing links and receive rents for incoming links. As with endogenous linking, subjects have access to the history box. Hence, in the treatments with exogenous networks, subjects face exactly the same link structures as subjects in the corresponding endogenous network treatments.

In all treatments, earnings are denoted in ‘points’. In addition to a starting capital of 2000 points, subjects earn points in every period. Total point earnings are exchanged at the end of the experiment at a rate of 0.10 euro for every 30 points. Table 2 gives the benefits function $f(y_i)$ (in points), as well as the costs of linking, $k$, the costs of investment $c$ and the status rents $b$. As specified in Section 3, the function $f(y_i)$ is increasing and concave in $y_i$, and $k > b$.

Sessions were run between May and July 2014 in the CREED laboratory of the University of Amsterdam and lasted about two hours. For each treatment with $n = 4$, we had 8 groups in total while for each treatment with $n = 8$ we had 6 groups. In total, 320 subjects participated in the experiment, each in only one session. We conducted 15 sessions where, depending on show-up, the number of subjects per session varied between 12 and 32, but in most sessions 24 subjects participated. We randomized treatments within a session: in each session with endogenous network formation at least two different treatments were conducted. Each subject participated in one treatment only. Subjects were recruited from the local CREED database, which consists mostly of undergraduate students from various fields. Of the subjects in our experiments, 49% are female and 61% were studying at the Amsterdam School of Economics or the Amsterdam Business School. Cash earnings were between 5.10 euro and 125.10 euro, with a mean of 30.63 euro.

The experiment was computerized using PHP/MySQL and was conducted in English. Upon entering the laboratory, subjects were randomly allocated to a separate cubicle. Communication was prohibited throughout the session. Before starting the network experiment, we elicited risk preferences using a procedure similar to the one of Gneezy and Potters (1997). In this procedure, each subject decided how much to invest of a capital of 600 points. The amount invested was lost or multiplied by 2.5 where each possibility occurred with probability 0.5. The result of the investment was then added to the amount not invested. Subjects were only informed of the outcome of this part at the very end of the experiment.

After this, subjects read the instructions of the network game at their own pace, on-screen. While reading the instructions, a printed summary was handed out. To ensure that all
subjects understood the instructions, they were required to answer several test questions. The experiment did not continue before everyone had answered all questions correctly.\textsuperscript{15}

We ended each session with a short questionnaire after which we privately informed subjects of the outcome of the risk elicitation task and their aggregate earnings in the experiment. Subjects were privately paid in cash for all periods of the network game and the risk-elicitation task.

5 Predictions for the experiment

In all experimental treatments with endogenous linking, the Nash equilibria of the stage game are the same. As argued in subsection 3.1, the set of Nash equilibria is independent of our treatment variables; status rents and group size. Figure 1 illustrates these equilibria for the parameters of the experiment. In the figure, circles represent the players and the numbers inside these circles display their investment. A link is represented by an arrow, which points away from the player who makes it. Hence, we see a Nash star, where the core player invests in $\mathcal{y} = 2$ units and the other players form links to the core and do not invest. Furthermore, the efficient outcome is also the same across treatments: in all cases it is a superstar where the core invests in $\mathcal{y} = 4$ units. As noted before, status rents and group size do not affect the set of stage-game equilibria, but they do affect the payoffs of players in the core.\textsuperscript{16} According to our behavioral model, these payoff differences determine whether stars are stable, and therefore affect which investment by the core player we can expect.

Table 3 summarizes the predictions of the behavioral model. To start, note that without status rents, we do not expect star networks to be stable. This is because the core

\textsuperscript{15} The experimental instructions and test questions can be found in Appendix E.

\textsuperscript{16} In Appendix C, we provide a table with the payoffs in different star networks for our parameters.
earns less than the periphery and any forward-looking core player will lower her investment to zero, expecting that some other player will invest the Nash amount 2 in the next period. This changes when status rents are introduced. In n4b22, we expect stable Nash stars to form. With higher status rents or larger groups, we expect competition for the core position. More specifically, in treatments n8b22 and n4b66 we expect that competition leads to the formation of efficient superstars where the core invests in four units. In n8b66, we expect that competition for the core position will be so intense that it encourages severe overinvestment by the core. Here, we expect the emergence of star networks where the core invests in eight units. Note that stable superstars require that $i$ puts sufficient weight on her future payoff (e.g. because she expects it to extend beyond the next period), so that she resists the temptation to increase her payoff in the current period (by lowering her investment) in order to sustain the core position in the future. The final row of Table 3 gives the lower bounds for this weight.

When there are status rents, the stable stars predicted by our behavioral model can each be supported as part of a subgame perfect equilibrium. This can be seen by comparing the stable outcome $y^*$ to the subgame perfect equilibria reported in Appendix D. In this way, our model provides an equilibrium selection for the supergame. Moreover, the equilibrium selected varies across treatments. Note also that the model predicts that no equilibrium will be reached in the supergame when there are no status rents. Together, this means that our set of treatments provides a powerful test bed for the theory.

17 For our experiment, we deliberately chose the values of $n$ and $b$ such that we keep $(n - 2)b$ - and thus $y^*$-constant between treatments n8b22 and n4b66.
6 Results

We have organized the presentation of the experimental results as follows. In Section 6.1, we start with an overview of the outcomes that are observed in our treatments with endogenous network formation. We complement this overview with a discussion of cross-treatment differences in the provision of the public good. Then we provide an overview of the efficiency levels that follow from the networks formed, together with the public good provision. In Section 6.2, we study the behavioral dynamics in the experiment and compare them to our theoretical predictions. We deal with the question of which treatments trigger a competition for status, and we present an analysis of the frequency and stability of the outcomes that we observe. Finally, in Section 6.3 we present the results of our exogenous network treatments, which allow us to shed light on the motives underlying our results. Unless stated otherwise, all tests reported are Mann-Whitney tests (henceforth, MW). Throughout, we use two-sided tests using average statistics per group as units of observation.

6.1 Overview: star networks and public good provision

Figure 2 plots the relative frequency of stars over time. At the start of the experiment, we hardly observe any stars in any treatment. Starting around period 10, a clear distinction emerges between the two treatments without status rents and those with. With status rents, the frequency of stars steadily increases over time. In the last 25 periods of these treatments, this frequency rises to 76%. In treatment n8b66 stars are even observed in 88% of the last 10

![Figure 2: Development of star networks](image_url)

Notes: Lines show the relative frequencies of periphery-sponsored stars by treatment and period. Lines are smoothed by taking the moving average over periods $t - 3$ to $t + 3$ for every period $t$. 
periods. In stark contrast, there is no clear trend in the occurrence of stars in the treatments without status rents. There, such networks remain rare throughout the experiment.

Table 4 makes these results more precise and tests whether the observed differences are significant. The table confirms the picture emerging from Figure 2. Stars form substantially and systematically more often in the treatments with status rents than in the treatments without. Within these classes of treatments, differences are much smaller and mostly insignificant.

Status rents and group size also have profound effects on the provision of the public good. To compare investment choices while holding network composition constant, we focus on the investment of core players in periods where stars were formed. The results are presented in Table 5. This table shows that, conditional on a star being formed, public good provision is inefficiently low (that is, below four units) in the treatments without rents and the treatment with medium rents and small group size. In treatments n4b66 and n8b22 public good provision is close to the efficient level of four units. In treatment n8b66 the core player vastly overinvests with an average contribution level that is close to double the efficient amount. With status rents, any increase in group size or status rents leads to higher investment by core players. The sizable differences between treatments are all significant, except for the comparisons between n4b0 and n8b0 and between n8b22 and n4b66. By and large, the average and median investment levels accord very well with the predictions of the behavioral model.

### Table 4: Frequency of star networks

<table>
<thead>
<tr>
<th>Status rents</th>
<th>Group size</th>
<th>Relative frequency of periphery-sponsored stars</th>
<th>P-values pairwise MW tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>All periods</td>
<td>Final 25 periods</td>
</tr>
<tr>
<td>b = 0</td>
<td>n = 4</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>n = 8</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>b = 22</td>
<td>n = 4</td>
<td>0.58</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>n = 8</td>
<td>0.50</td>
<td>0.73</td>
</tr>
<tr>
<td>b = 66</td>
<td>n = 4</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>n = 8</td>
<td>0.58</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Notes: the left panels provide the relative frequencies of periphery-sponsored stars in all periods and in the final 25 periods. The right panels provide the results of MW tests for the differences in occurrence between treatments using the observations in all periods. In Appendix C we provide a table with the p-values for differences in the final 25 periods.
Investments in the public good are one of the factors that affect efficiency in this environment. The other is the links made to access the public good. We now consider both factors simultaneously by looking at treatment differences in observed efficiency. Table 6 shows the relative frequency of efficient star networks (where the core invests in 4 units), mean earnings and mean earnings net of status rents per treatment.

Efficient star networks are almost exclusively observed in n8b22 and n4b66. As noted before (cf. Table 4), stars rarely form at all without status rents. In n4b22, stars are formed but investments by the core are typically lower than the social optimum (cf. Table 5). In n8b66, we also frequently observe stars but here the core vastly overcontributes. As for earnings, as expected, these increase as status rents rise. We correct for this effect of adding money to the system by deducting the status rents from the earnings. This yields a clear difference between the treatments with and without status rents. The treatments without status rents perform particularly badly in terms of (net) earnings. Here subjects do not benefit from interacting with others and actually do worse than if they had completely refrained from making links and simply investing in two units themselves.\(^{18}\) This mirrors previous experimental results reported in the literature on endogenous network formation without status rents (e.g., Falk and Kosfeld, 2012). Net earnings are much higher when there are status rents. In pairwise comparisons, either of the treatments without status rents reaches

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**Table 5: Investment by Core Players in Star Networks**

<table>
<thead>
<tr>
<th>Status rents</th>
<th>Group size</th>
<th>Predicted Mean (s.e.)</th>
<th>Median</th>
<th>P-values pairwise MW tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>n = 4 n = 8 n = 4 n = 8 n = 4 n = 8</td>
</tr>
<tr>
<td>(b = 0)</td>
<td>n = 4</td>
<td>1.83 (0.11)</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>n = 8</td>
<td>1.82 (0.09)</td>
<td>2</td>
<td>0.92 -</td>
</tr>
<tr>
<td>(b = 22)</td>
<td>n = 4</td>
<td>2.22 (0.06)</td>
<td>2</td>
<td>0.01 0.02 -</td>
</tr>
<tr>
<td></td>
<td>n = 8</td>
<td>3.49 (0.33)</td>
<td>4</td>
<td>0.01 0.03 0.03 -</td>
</tr>
<tr>
<td>(b = 66)</td>
<td>n = 4</td>
<td>3.61 (0.18)</td>
<td>3.5</td>
<td>0.00 0.02 0.00 0.75 -</td>
</tr>
<tr>
<td></td>
<td>n = 8</td>
<td>7.07 (0.45)</td>
<td>7.5</td>
<td>0.00 0.02 0.00 0.00 0.00 -</td>
</tr>
</tbody>
</table>

Notes: The left panel lists the predicted, mean and median investment in the public good by core players, conditional on a periphery-sponsored star being formed. The predicted investment is equal to \(y^*\), except in the treatments without status rents, where no stable star is predicted. Standard errors of the mean are presented in parentheses, based on mean investments per group. The median is obtained by taking the median within each group first and then the median of these numbers per treatment. The right panel presents \(p\)-values for tests whether mean core investments differ between treatments, conditional on a periphery-sponsored star having formed.

---

\(^{18}\) In this case, a player earns 42 points.
significantly lower net earnings than any of the treatments with status rents (all $p<0.05$). Net earnings are the highest in treatment n8b22. This is also the treatment where we observe efficient 4-stars the most frequently. Net earnings are higher in this treatment than in all other treatments ($p<0.10$ for all pairwise comparisons).

Next, we turn to the convergence of our data across periods. Figure 3 displays for each treatment the proportion of groups that converge to a stable outcome. If a group converges, it is almost always to a star network. Without status rents groups almost never converge to any stable outcome, independent of group size. When there are status rents, group size and status rents have a beneficial effect on the provision of the public good. With medium status rents and small groups, groups usually converge to a star network in which the core player consistently invests the stage game Nash amount of two units. When group size is doubled in treatment n8b22, all groups converge to a stable network. The most frequently observed end-point of the dynamics is a superstar in which the core player invests the efficient amount of four units. Similarly, with high status rents, investments are higher in large groups than in small groups. In fact, in this case all large groups converge to a star

<table>
<thead>
<tr>
<th>Status rents</th>
<th>Group size</th>
<th>Efficient stars</th>
<th>Mean earnings</th>
<th>Mean net earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 0$</td>
<td>$n = 4$</td>
<td>0.00</td>
<td>33.4 (3.0)</td>
<td>33.4 (3.0)</td>
</tr>
<tr>
<td></td>
<td>$n = 8$</td>
<td>0.00</td>
<td>38.9 (1.9)</td>
<td>38.9 (1.9)</td>
</tr>
<tr>
<td>$b = 22$</td>
<td>$n = 4$</td>
<td>0.01</td>
<td>75.9 (2.0)</td>
<td>60.9 (2.0)</td>
</tr>
<tr>
<td></td>
<td>$n = 8$</td>
<td>0.36</td>
<td>99.9 (4.2)</td>
<td>81.6 (3.9)</td>
</tr>
<tr>
<td>$b = 66$</td>
<td>$n = 4$</td>
<td>0.29</td>
<td>106.1 (6.4)</td>
<td>59.7 (6.6)</td>
</tr>
<tr>
<td></td>
<td>$n = 8$</td>
<td>0.00</td>
<td>124.6 (7.4)</td>
<td>67.9 (6.5)</td>
</tr>
</tbody>
</table>

Notes: The first column gives the relative frequency of efficient outcomes. The efficient outcome is a periphery-sponsored star where the core invests in four units and no periphery player invests. Mean earnings are denoted per subject in points per period. For mean net earnings we subtract the status rents. Standard errors of the means are computed using each group as an individual observation. The panels on the right give $p$-values for tests whether mean net earnings differ between treatments.
network in which the core player overinvests in the public good. In this treatment, the large
majority of groups converge to superstar networks in which the core invests in seven or eight
units, which is on average almost double the efficient investment of four units.

These results agree well with the predictions of the behavioral model. In accordance with this
model, stable Nash stars (where the core invests in two units) should only be observed in
treatment n4b22 and the occurrence of large overinvestments should be limited to treatment
n8b66 (cf. Table 3). Of particular interest is the comparison of treatments n4b66 and n8b22.

These treatments allow us to investigate whether the earnings for the player in the core
relative to the players in the periphery is essential for the results. Only when the core earns
more, does our behavioral model predict that there will be competition for this position.

According to the model, a periphery player has the same incentive to challenge the core in
either of these two treatments; the model assumes that with a slightly higher investment than
the core player a challenger will attract \( n - 2 \) links. In either treatment this yields a total
benefit of 132 points (2x66=132; 6x22=132). In agreement with the model, the data of these
two treatments are quite similar. We observe slightly more superstars in n8b22, but the
difference is not significant \( (p=0.60) \). This result allows us to better understand the group size
effect noted above. It does not matter for our subjects if a rise in potential status rents is
created by an increase in potential linkers or by an increase in rents per linker. In agreement
with the behavioral model, they compete for the core position to the same extent in both
cases.

---

**FIGURE 3: PROPORTION OF GROUPS CONVERGING AND END-POINT OF THE DYNAMICS**

*Notes: A group converges to a network if all players repeat decisions at least 5 times. Most groups converge
to ‘\( x \)-star’ outcomes, periphery-sponsored stars in which the core player invests in \( x \) units of the public good.
Most groups converge only once: only 4 of the 42 groups converged to two or more different networks. In
these cases, we include the last stable network.*
Behavioral dynamics in the experiment

The key element of our behavioral model is that players compete for the core position if status rents make it more attractive than a periphery position. Without such rents, players prefer that others fulfill the costly job of providing the public good to the group. Indeed, our data show that subjects (only) compete for the core position in the treatments where we expect them to do so. Here, we focus on the competition itself.

Figure 4 shows how the distribution of competitors for the core position develops over time in our treatments with endogenous network formation. Without status rents, there are basically no subjects who consistently invest in more than two units of the good. In treatment n4b22 the core position in a Nash star is slightly more attractive than a periphery position, but not enough to support a superstar where the core consistently invests in three units or more. In agreement with this observation, we see few investments above two units and very rarely observe that multiple players invest at the same time. Clearer competition is observed in treatments n4b66 and n8b22 where the efficient superstar is predicted to emerge. In the first 25 periods, we frequently observe multiple players who invest in more than two units and compete for the core position (in 54% and 21% of the observations in n8b22 and n4b66 respectively). In the final 25 periods, the dust settles and typically only one overinvesting player remains. Very fierce competition is observed in our treatment n8b66. In the first 25 periods, we observe up to five players who simultaneously invest in more than two units. In all cases at least one player overinvests and in the majority of observations
multiple players compete for the core position. The competition diminishes towards the end, and in the final 25 periods a single surviving player manages to deter the competition.

More details on how subjects compete are offered in Figure 5, which shows the top three investment levels per group over time. In the treatments without status rents, there are usually two or three subjects investing two units. This finding illustrates that in these treatments subjects are not able to coordinate on a fixed star network. The picture is different in treatment n4b22, where after some time on average only one subject consistently invests two units and a stable star network is formed. In treatments n4b66 and n8b22 we observe higher investments by the top contributor than in n4b22: the subject in the core learns that an efficient level of four units is needed to prevent being challenged by periphery players. Once it has become clear who is the superstar in a group, the investments by the other group members gradually fade out.\(^{22}\)

The most interesting dynamics are observed in treatment n8b66. At the start of the experiment, the mean investments for the second highest contributor are almost as sizable as for the top contributor: on average they invest in 4.00 units in the first 25 periods. Only after a while does this runner up start to give up. To further investigate what is going on in this treatment, Figure 6 provides a more detailed view. Here, we show the top-3 investments

\(^{22}\) In a regression, we find a negative correlation between risk aversion and being in the core of a star in treatments where we expect competition (i.e. when networks are formed endogenously and \(b > 0\)). We find no significant correlation between risk aversion and core positions in the other treatments. Moreover, we do not find any relation between gender and network positions. Similar results are obtained when we use public good investments as the dependent variable. More details are available upon request.
across periods in each of the six groups in n8b66. In all groups, we see that in early periods at least two players compete for the core position by investing in very high amounts. At some point all but one player give in and a superstar forms with a core player who (over)invests in 7 or 8 units. At these levels, payoffs are to a large extent equalized and periphery players stop challenging the core player.

To investigate the stability of the decisions made, Table 7 shows the frequency of various stars and how often they were repeated, after having been formed. The outcomes predicted by our behavioral model are listed in bold. In agreement with this model, star networks occur only sporadically in the treatments without status rents, and if they occur, they tend to be unstable. In n4b22, the predicted 2-star is the most frequently observed and also most stable outcome. In treatments n4b66 and n8b22, the efficient 4-star is predicted to occur, and indeed it is most often observed in both treatments. In these treatments the efficient star is very stable. If it is formed, it remains unchanged in 90% of the cases. Finally in n8b66 a star network is predicted in which the superstar invests in 8 units. Here, in the experiment most often superstars are observed where the core invests in 7 or 8 units, and these outcomes are again remarkably stable. In 88% of the cases that such a superstar is formed, it is exactly repeated in the next period.

Finally, we investigate what happens if a core player in a $y^*$-star deviates by lowering her investment. As already noted, $y^*$-stars are very stable: core players rarely lower their investments: of the 319 $y^*$-stars that are formed up to period 65, only 12 core players deviate

23 A similar figure of the other treatments is available upon request.
by lowering their investment.\textsuperscript{24} This strongly suggest that subjects anticipate that lowering their investment is costly, as they could be challenged and lose the attractive core position. Given the small number of core players who deviate, it is hard to make strong claims about the resulting payoffs, but still we illustrate what subjects might expect in Figure 7. Here we plot the normalized earnings of core players in $-\text{stars}$ and we split them by whether they lower their investment (at period $t = 0$) or not. As expected, the core players who lower their investment earn a higher immediate payoff (at period $t = 0$), but on average pay the price in subsequent periods. These losses outweigh the short-term gains: the aggregated (normalized) losses in periods $t = 1$ to $t = 10$ are roughly 7 times the size of the normalized gains at $t = 0$.

\textbf{6.3 Motives underlying investments to the public good}

The evidence presented so far is consistent with the explanation based on competition for status. Once the core position becomes more attractive than the periphery position because of the status rents that it generates, subjects start competing for it. To prevent being challenged, star players are forced to invest to such an extent that the payoffs across positions are

\textsuperscript{24} We focus on treatments where $y^* > \mathcal{G}$, to ensure that core-players can deviate by lowering their investment.
approximately equalized. However, we cannot exclude that the results are driven by social motives. For instance, subjects may be inequity averse, and therefore choose higher investments to compensate others for the gains that status rents bring. Inequity aversion may also explain why subjects prefer to avoid the core position if it is relatively unattractive in the absence of status rents. Another possibility is that core players feel that they have to return the favor if they receive status rents, or that they are simply driven by altruism and prefer to give something to others when they become richer. To distinguish between the competition for status dynamics and the possibility that the subjects are driven by social motives, we included the two treatments in which we impose the networks that were naturally formed in the corresponding treatments with endogenous network formation. If the competition for status dynamics drive the results, we should not observe higher investments in these control treatments than the individually rational number of two units. If on the other hand, the results are driven by social motives, we should not observe any difference with the corresponding endogenous network treatments.

Table 8 displays the mean, median and predicted public good investments by core players in star networks in the exogenous treatments, together with the benchmarks provided by the endogenous treatments. The results provide convincing support for the competition for status dynamics. The results are particularly strong for the case where group size is small and status rents are large; here, the mean and the median are exactly at two units as predicted. Even more so, there are almost no cases where the core player invests in more than two units of the public good. But also in the case with large group size and large status rents, the results are striking: average investment is only a fraction of the level that is achieved when networks

**Figure 7: Payoffs from Lowering Investments as a Core Player**

*Notes: Mean payoffs of core players in $y^*$-stars. Earnings are normalized by normalizing the payoff in $y^*$-star that is formed at $t = -1$ to 0. The figure is based on $y^*$-stars where $y^* > y$, to ensure that core-players can deviate by lowering their investment.*
are endogenously formed. The differences between the investments in the endogenous and exogenous network treatments are significant at the 5% level.

Figure 8 shows the distribution of investment choices by core players for each of the treatments with \( b = 66 \), both for endogenous and exogenous networks. It is clear that with exogenous networks, core players invest substantially less than with endogenous networks. In both n4b66EXO and n8b66EXO, in periphery-sponsored stars the modal core investment in the stage-game Nash amount is two units, while we hardly ever observe such low investments with endogenous networks. An alternative explanation would be that endogenous network formation allows selection of socially motivated types to the core position. Our data provide little support for this selection effect of competition. If it were to hold, one would expect to see these supergame effects in at least 1 out of \( n \) exogenous groups. Indeed, in n8b66EXO, we observe some cases where core players invest in more than two units, but these only account for a small number of cases. Moreover, even in these groups investments remain well below the level of 7 or 8 units that we frequently observe in the endogenous network treatments. Hence, such traces of social motives are negligible compared to the competition effect of endogenizing network formation.

### 7 Conclusion

We have investigated the effectiveness of a novel behavioral mechanism to generate supergame effects in an environment where players decide both on the network structure and on their contributions to a local public good. For this, we introduced status rents for incoming links to the endogenous network formation game of Galeotti and Goyal (2010). Our conjecture was that players would jockey to obtain attractive network positions. In the presence of status rents, this implies that players would compete for status and the winner of
the competition for status would be forced to take the interests of the other group members into account. To structure the interaction, we developed a simple dynamic behavioral model that predicts when a subgame perfect equilibrium outcome is to be expected, and which equilibrium is selected. We subsequently tested these predictions in a laboratory experiment, where we varied group size and status rents across treatments. These are two factors that should systematically affect the equilibrium that is selected if people compete for status. Thereby, these factors determine the extent to which the public good is provided and the shape and stability of the outcome.

We find that the presence of status rents is crucial for the results. With status rents, subjects start by jockeying for the core position. Once the dust has settled and subjects have implicitly agreed on who will be in the core of the network, a stable outcome arises in which the core player consistently contributes an amount that roughly equalizes payoffs across positions. For specific environments, this entails contributing more than the stage-game Nash quantity and in some even more than what is efficient. In this way, the core player in a superstar prevents being challenged by the others, and periphery players maintain their links, which the superstar-core needs for her exemplary behavior. Across treatments, we observe that these supergame effects – i.e., the adjustment of contributions aimed at maintaining the network (core) position – increase with the size of the status rents and with group size. In stark contrast, inefficiency reigns and groups rarely converge to equilibria when there are no status rents. These experimental results correspond closely to the predictions of the

**Notes:** The figure shows the distributions of investment of core players in treatments with $b = 66$, conditional on a periphery-sponsored star having formed. All period/group observations are included.
competition for status dynamic and the selection of a subgame perfect equilibrium (if any) that it entails.

With the help of our two control treatments in which we imposed exogenous networks, we excluded other motivations that might explain our results. In agreement with our conjecture that the results are driven by competition for status among selfish players, the observed supergame effects largely disappear when the possibility to compete for the core position is excluded by design. Instead, if our core players in superstars had been driven by other motivations, like exchange of favors, altruism or inequity aversion, results should have been independent of how the positions in the network are assigned.

We therefore conclude that free competition for a favorable position is the key ingredient for the emergence of superstars in our environment. Beyond the specific setting of our experiment, this may help explain why so often a small minority in a group contributes so heavily to a public good. There is no need to assume that pure altruism or other pro-social motivations drive this behavior. Even purely self-interested individuals may do so if they care enough about the status rents that are closely linked to being at the core of the group in this way. In short, volunteers at sports teams, workshop organizers and OSS developers all contribute much more to their group than can be expected in a one-shot environment because the supergame they are involved in allows them to collect status rents from the central position that they have.

References


**Appendix A: Stage-game equilibria and efficient outcomes**

This appendix is greatly indebted to the analysis in Galeotti and Goyal (2010). In agreement with the theoretical literature on network formation, we will restrict our attention to pure-strategy equilibria.

*Some additional notation*
Most of the notation is introduced in section 3 of the main text, but for the proofs we will use the following additional definitions. The marginal benefit of accessing the $m^{th}$ unit of the good is given by $MB(m) = f(m) - f(m - 1)$.

A network is a core-periphery network if there are two sets of players $\mathcal{N}_C$ and $\mathcal{N}_P$ for which it holds that $\mathcal{N}_i(\mathbf{g}) = \mathcal{N}_C \setminus \{i\}, \forall i \in \mathcal{N}_C$ and $\mathcal{N}_j(\mathbf{g}) = \mathcal{N}_C, \forall j \in \mathcal{N}_P$. In such a network, the players in $\mathcal{N}_C$ form the core and $\mathcal{N}_P$ form the periphery. All core players form links to the other core players, but not to the periphery players and any periphery player links to all the core players but to none of the other periphery players. A core-periphery network with a single player in the core is called a star network.

**Nash equilibria of the stage game**

We start by stating our variant of Lemma 1 in Galeotti and Goyal (2010):

**Lemma A.1** In any Nash equilibrium $\mathbf{s}^N$, all players $i \in N$ access at least $\mathcal{Y}$ units of the good, $y_i^N \geq \mathcal{Y}$, and all players who acquire units personally will access exactly $\mathcal{Y}$ units of the good, i.e. if $x_i^N > 0$ then $y_i^N = \mathcal{Y}$.

**Proof.** Suppose that a player $i \in N$ accesses fewer than $\mathcal{Y}$ units of the good, i.e. $y_i < \mathcal{Y}$. If this is the case $i$ can strictly increase her payoff by investing as the marginal benefits strictly exceed the marginal costs for $y \leq \mathcal{Y}$, i.e. $MB(m) > c, \forall m \leq \mathcal{Y}$. If a player $i$ invests in units personally, i.e. $x_i > 0$, and she accesses more than $\mathcal{Y}$ units of the good, $y_i > \mathcal{Y}$, she can strictly increase her payoff by lowering $x_i$ as $MB(m) < c, \forall m > \mathcal{Y}$.

Next, we can state our version of Galeotti and Goyal’s Proposition 2.

**Proposition A.1** In any strict Nash equilibrium $\mathbf{s}^N$, (i) a core-periphery network is formed where (ii) the core players all invest, (iii) the periphery players do not invest, (iv) aggregate investment equals $\mathcal{Y}$ units and (v) the maximum number of players in the core is independent of $n$ and $b$ and is given by the largest integer smaller than $\frac{c\mathcal{Y}}{k}$.

**Proof.** We start by showing that in every strict Nash equilibrium $\mathbf{s}^N$, $\sum_{i \in N} x_i^N = \mathcal{Y}$ should hold. It is easy to see that by Lemma A.1, it must be that aggregate investment in $\mathbf{s}^N$ equals at least $\mathcal{Y}$ units, i.e. $\sum_{i \in N} x_i^N \geq \mathcal{Y}$. We define the set of all players with incoming links by
\( \mathcal{N}_1 = \{ i : \omega_i > 0 \} \) and all players without incoming links by \( \mathcal{N}_0 = \{ j : \omega_j = 0 \} \). Note that in a strict equilibrium, any player \( i \in \mathcal{N}_1 \) should invest in strictly more units than any player \( j \in \mathcal{N}_0 \). If not, players that link to \( i \) could (weakly) improve by moving their links to \( j \). Also, \( cx^N_i > k \) should hold for all \( i \in \mathcal{N}_1 \), as otherwise no player would link to \( i \) (but rather invest personally), hence \( x^N_i > \frac{k}{c} \), \( \forall \ i \in \mathcal{N}_1 \). Note that \( \sum_{i \in \mathcal{N}_1} x^N_i < \mathcal{Y} \) cannot hold, as then for every \( i \in \mathcal{N}_1 \), \( y^N_i < \mathcal{Y} \) follows and this contradicts Lemma A.1. We will now show that \( \sum_{i \in \mathcal{N}_1} x^N_i > \mathcal{Y} \) cannot hold in a strict equilibrium. Assume that it holds. This implies that any \( i \in \mathcal{N}_1 \) does not link to all other players in \( \mathcal{N}_1 \) as otherwise \( y^N_i > \mathcal{Y} \) follows which contradicts Lemma A.1. Hence, for every \( i \in \mathcal{N}_1 \), there exists some \( \ell \neq i \in \mathcal{N}_1 \), for whom \( g_{i, \ell} = 0 \). Without loss of generality we can order the players in \( \mathcal{N}_1 \) such that \( x^N_1 \geq x^N_2 \geq \cdots \geq x^N_m > \frac{k}{c} \) holds. As we are considering strict equilibria, this means that no \( i \in \mathcal{N}_1 \) will link to the last player \( m \) in the sequence, as otherwise \( i \) could (weakly) improve by linking to someone higher in the sequence. Aggregate investment by all players \( i \in \mathcal{N}_1 \setminus \{ m \} \) should still be at least \( \mathcal{Y} \) units, as by Lemma 1 \( y^N_i = \mathcal{Y}, \forall \ i \in \mathcal{N}_1 \) should hold. This means that there exists some subset \( \mathcal{N}_1' \subseteq \mathcal{N}_1 \setminus \{ m \} \), whose investments add up exactly to \( \mathcal{Y} \): \( \sum_{i \in \mathcal{N}_1'} x^N_i = \mathcal{Y} \). Take the smallest possible subset \( \mathcal{N}_1' \) for which \( \sum_{i \in \mathcal{N}_1'} x^N_i = \mathcal{Y} \) holds. Then, as \( x^N_i > \frac{k}{c}, \forall \ i \in \mathcal{N}_1 \), player \( m \) could strictly improve by not investing and linking to all \( i \in \mathcal{N}_1' \). This contradicts \( m \) being in \( \mathcal{N}_1 \) in an equilibrium. Hence, it cannot be that \( \sum_{i \in \mathcal{N}_1'} x^N_i > \mathcal{Y} \) and it follows that \( \sum_{i \in \mathcal{N}_1} x^N_i = \mathcal{Y} \).

If \( \sum_{i \in \mathcal{N}_1} x^N_i = \mathcal{Y} \), this implies that any \( j \in \mathcal{N}_0 \) will have \( x^N_j = 0 \), as they can access \( \mathcal{Y} \) units (at lower costs than investing personally) by linking to all \( i \in \mathcal{N}_1 \). Thus aggregate investment equals \( \mathcal{Y} \) and \( S^N \) must be a core-periphery network where all players \( i \in \mathcal{N}_1 \) form the core and all \( j \in \mathcal{N}_0 \) form the periphery. Recall, that \( x^N_i > \frac{k}{c}, \forall \ i \in \mathcal{N}_1 \) holds. As \( \sum_{i \in \mathcal{N}} x^N_i = \mathcal{Y} \), this provides an upper bound for the number of players that invest personally, and hence the size of the core: \( |\mathcal{N}_1| < \frac{c\mathcal{Y}}{k} \).

**Efficient outcomes**

In any efficient outcome, it must be that aggregate investment is at least \( \mathcal{Y} \) units, i.e. \( \sum_{i \in \mathcal{N}} x_i \geq \mathcal{Y} \). If not, all players will access fewer than \( \mathcal{Y} \) units and they can strictly increase their payoff (and hence the sum of payoffs) by investing in additional units as \( MB(m) > c, \forall \ m \leq \mathcal{Y} \). As \( c\mathcal{Y} > k \), the aggregate costs of accessing at least \( \mathcal{Y} \) units of the good are
minimized by forming a periphery-sponsored star where only the core invests, i.e. \( x_i \geq y, x_j = 0, g_{ji} = 1, g_{l,j} = 0, g_{l,i} = 0, \forall j \neq i, l \neq i,j \). The efficient level of investment \( x_i = y \) by the core is such that the sum of all marginal benefits just exceeds the marginal costs of investing. That is, \( y \) is set such that it satisfies \( nMB(y) \geq c \) and \( nMB(y + 1) < c \).

**Appendix B: Dynamic stability**

Here we discuss the dynamic properties of our behavioral model. For the analysis, we make two simplifying assumptions.

**Assumption 1:** The benefit function is sufficiently concave: \( k > f(x + x') - f(x) \) if \( x \geq y \) and \( x \geq x' \); \( f(0) = 0 \) and \( f(1) > k \).

Assumption 1 lists a requirement on the concavity of \( f(\cdot) \) that ensures that no player will link to multiple players if there exists some player that invested in at least \( y \). In addition, it says that there is no public good without investments and that the benefits associated with access to one unit exceed the costs of linking.

**Assumption 2:** \( k > f(y) - f\left(\frac{1}{x} y\right)\).

Assumption 2 provides a condition on the linking costs that prevents networks with multiple hubs being formed in equilibrium. We note that the parameterization in our experiment fulfills Assumptions 1 and 2.

**Proposition B.1:** The behavioral model generates the following predictions:

(i) When status rents are sufficiently high, i.e. when the player in the core of the Nash star earns more than a player in the periphery, there exists a lower bound on \( x \) above which the only stable outcomes are periphery-sponsored stars where the core player invests \( y^* \) units and no periphery player invests.

(ii) When status rents are low or absent, i.e. when in the Nash star the core player earns less than a player in the periphery, there exists a lower bound on \( x \) above which all Nash stars and superstars are unstable.

**Proof.** We first derive the decisions of the core players in periphery-sponsored stars. We do this for two separate cases. In case I, we consider the case where status rents are relatively high, i.e. when \( y^* > y \) and, in case II, we look at the decisions of core players when the
reverse holds. Then, we derive the conditions under which periphery players in a periphery-sponsored star leave their investments unchanged.

**Core player case I: \( y^* > \hat{y} \)**

First, we show that if no other player \( j \neq i \) invests at \( t - 1 \), then if core player \( i \) is given the chance to revise her choice, she will choose \( x_{i,t} = y^* \). This rules out all other periphery-sponsored stars (where the periphery does not invest) than the \( y^* \)-star as stable outcomes.

Consider the situation where core player \( i \) is given the chance to change her investment. At \( t - 1 \), none of the other players invested, i.e. \( x_{j,t-1} = 0 \ \forall \ j \neq i \). For expositional reasons, we suppress that all expected payoffs below are conditional on the investments of others at \( t \). For the \( y^* \)-star to be stable, \( \mathbb{E} \Pi_i(x_{i,t} = y^*) > \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') \ \forall \ x_{i,t}' \neq y^* \) must hold. This implies that

\[
\text{(B.1)} \quad \mathbb{E} \Pi_i(x_{i,t} = y^*) > (1 - \alpha) \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') + \alpha \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') \ \forall \ x_{i,t}' \neq y^*,
\]

should hold. If \( i \) invests in \( y^* \) units, she expects to be in the core of the \( y^* \)-star at \( t \) and \( t + 1 \). Her expected payoff is then \( \mathbb{E} \Pi_i(x_{i,t} = y^*) = f(y^*) - cy^* + (n - 1)b \), for which we simply write \( \mathbb{E} \Pi_i(x_{i,t} = y^*) = \pi^*_C \). Below, we will show that any other investment \( x_{i,t}' \neq y^* \) will lead to lower expected payoffs at \( t + 1 \), i.e. \( \pi^*_C > \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}'), \ \forall \ x_{i,t}' \neq y^* \). Then, expected payoffs can only be higher if there exists a profitable deviation in the current period, i.e. when \( \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') > \pi^*_C \). In this case, (B.1) holds when:

\[
\text{(B.2)} \quad \alpha > \frac{\mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') - \pi^*_C}{\mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') - \mathbb{E} \Pi_{i,t+1}(x_{i,t} = x_{i,t}')}, \ \forall \ x_{i,t}' < y^*, \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') > \pi^*_C.
\]

Note that in this case, \( \mathbb{E} \Pi_i(x_{i,t} = x_{i,t}') > \pi^*_C > \mathbb{E} \Pi_{i,t+1}(x_{i,t} = x_{i,t}') \) and there exists a lower bound \( \bar{\alpha} \in (0,1) \) for which (B.2) holds. Hence, if \( i \) is sufficiently forward looking \( (\alpha > \bar{\alpha}) \), she will choose \( x_{i,t} = y^* \).

Now we will show that \( \pi^*_C > \mathbb{E} \Pi_{i,t+1}(x_{i,t} = x_{i,t}'), \ \forall \ x_{i,t}' \neq y^* \) is indeed true. When \( i \) invests in more than \( y^* \) units, she expects not to be challenged at \( t + 1 \), but she will earn a strictly lower payoff than in the \( y^* \)-superstar in both periods.\(^{25}\) Hence, profitable deviations can only follow from lower investments than \( y^* \). If \( x_{i,t} < y^* \), \( i \) expects to be challenged at

\(^{25}\) Remember that for values above \( \hat{y} \), the marginal benefits of investing are strictly lower than the marginal costs.
\[ t + 1, \text{i.e. there will be some player} j \text{ who she expects will invest} x_{j,t+1} = \max\{y, x_{i,t} + 1\} \]
and no player will link to \( i \) at \( t + 1 \). At \( t + 1 \), \( i \) could either link to \( j \) or not. (i) If she does not link to \( j \) (or any other player), her expected payoff at \( t + 1 \) will be \( \mathbb{E}I_{i,t+1}(x_{i,t} \in [0, y^*]) = f(x_{i,t}) - cx_{i,t} \). Note that this payoff is maximal for \( x_{i,t} = y \), but that even in this case it is
strictly smaller than the payoff of a periphery player in a Nash star: \( \pi^N = f(y) - k \). The
payoff of a periphery player in the \( y^* \)-star (\( \pi^P = f(y^*) - k \)) is strictly larger than \( \pi^N \)
as \( y^* > y \). When \( \pi^C > \pi^P \), this implies \( \pi^C > \pi^P > \pi^N > \mathbb{E}I_{i,t+1}(x_{i,t} \in [0, y^*]) \); hence,
investing \( x_{i,t} < y^* \) leads to lower expected payoffs at \( t + 1 \) than investing \( x_{i,t} = y^* \). \( \pi^C > \pi^P \)
is true in most cases, among which all of our experimental treatments.\(^{26} \) (ii) If \( i \) does link to \( j \),
her expected payoff at \( t + 1 \) is given by: \( \mathbb{E}I_{i,t+1}(x_{i,t} \in [0, y^*]) = f(x_{i,t} + x_{j,t+1}) - cx_{i,t} - k \).
Given that \( x_{j,t+1} = \max\{y, x_{i,t} + 1\} \), this expected payoff must be strictly smaller than \( \pi^N \),
and therefore it must also be strictly smaller than \( \pi^P \). Again, if \( \pi^C > \pi^P \) this implies that any
investment \( x_{i,t} < y^* \) will lead to lower expected payoffs at \( t + 1 \). All in all, \( \mathbb{E}I_{i,t+1}(x_{i,t} = y^*) > \mathbb{E}I_{i,t+1}(x_{i,t} = x_{i,t}^*) \forall x_{i,t}^* \neq y^* \).

**Core player case II: \( y^* < y \)**

We will show that in this case superstars will not form and Nash stars are stable for all values of \( \alpha \) when the core in the Nash star earns more than the periphery in the Nash star. This is the
case when \( (n - 1)b > c\bar{y} - k \). When the reverse holds, there exists a lower bound on \( \alpha \)
above which Nash stars must be unstable.

If \( i \) invests in the stage game Nash amount or more, i.e. \( x_{i,t} \geq \bar{y} \), she expects not to be
challenged as \( y^* \leq \bar{y} \). Note that in this case \( \mathbb{E}I_i(x_{i,t} = \bar{y}) > \mathbb{E}I_i(x_{i,t} > \bar{y}) \), which implies
that superstars will not be formed when \( y^* \leq \bar{y} \). If \( i \) chooses \( x_{i,t} = \bar{y} \), a Nash star is expected
to result at both \( t \) and \( t + 1 \) and \( i \) will be in the core and her expected payoffs are \( \pi^N \).

For a Nash star to be stable, \( \mathbb{E}I_i(x_{i,t} = \bar{y}) > \mathbb{E}I_i(x_{i,t} = x_{i,t}^*) \forall x_{i,t}^* \neq \bar{y} \) should hold.
We just established that \( i \) will not invest more than \( \bar{y} \), but she may want to invest less
than \( \bar{y} \) in order to induce others to invest. First note that the expected payoffs at \( t \) are
maximized by investing \( x_{i,t} = \bar{y} \). If \( i \) invests less, she earns a strictly lower payoff at \( t \) as no
other invests, and she will access fewer than \( \bar{y} \) units. Hence, any profitable deviation should

\(^{26} \) If \( \pi^C < \pi^P \), the core could potentially earn a higher payoff by investing less than \( y^* \). If this is the case, the
behavioral model predicts that competition would push investments up to \( y^* \), but that the \( y^* \)-star will be
unstable.
arise from higher expected payoffs at \( t + 1 \), i.e. a Nash star is stable when:

\[
\alpha < \frac{\pi_C^N - \mathbb{E} \Pi_{t+1}(x_{i,t} = x'_{i,t})}{\mathbb{E} \Pi_{t+1}(x_{i,t} = x_{i,t}) - \mathbb{E} \Pi_{t+1}(x_{i,t} = x'_{i,t})}, \forall x'_{i,t} < \mathcal{y}.
\]

Note that both the denominator and the numerator are strictly positive, as \( \pi_C^N > \mathbb{E} \Pi_{t+1}(x_{i,t} = x'_{i,t}) \) and the only relevant cases are those for which \( \mathbb{E} \Pi_{t+1}(x_{i,t} = x'_{i,t}) > \mathbb{E} \Pi_{t+1}(x_{i,t} = x_{i,t}) \) holds. Consider that \( i \) invests \( x_{i,t} < \mathcal{y} \). At \( t + 1 \), if some other player \( j \) updates, there are four possible outcomes: (1) \( j \) invests \( x_{j,t+1} = \mathcal{y} - x_{i,t} \) and links to \( i \), and \( i \) links to \( j \); (2) \( j \) invests \( x_{j,t+1} = \mathcal{y} - x_{i,t} \) and links to \( i \) (and \( i \) does not link to \( j \)); (3) \( j \) invests \( x_{j,t+1} = \mathcal{y} \) and \( j \) does not link to \( i \) and \( i \) does not link to \( j \); or (4) \( j \) invests \( x_{j,t+1} = \mathcal{y} \) and \( j \) does not link to \( i \) but \( i \) does link to \( j \). The expected payoffs for \( i \) in outcomes (1), (2) and (3) are strictly lower than \( \pi_C^N \), which means that the RHS in (B.3) is strictly larger than one. Hence, the only possible restriction on \( \alpha \) comes from the situations described in outcome (4). In these situations, \( \mathbb{E} \Pi_{t+1}(x_{i,t} \in [0, \mathcal{y}]) = f(\mathcal{y} + x_{i,j}) - cx_{i,t} - k \), which is maximized when \( x_{i,t} = 0 \) and in this case \( \mathbb{E} \Pi_{i,t+1}(x_{i,t} = 0) = \pi_P^N \). When \( i \) does not invest, she expects \( \mathbb{E} \Pi_{i,t}(x_{i,t} = 0) = 0 \) and (B.3) yields:

\[
\alpha < \frac{\pi_C^N}{\pi_P^N}.
\]

When status rents are absent or low, the core earns less than the periphery in a Nash star, i.e. \( \pi_C^N < \pi_P^N \). This is the case when \( (n - 1)b < c\mathcal{y} - k \). It follows from (B.4) that Nash stars are unstable if players are sufficiently forward looking.

The core also earns less than the periphery in a Nash star (and Nash stars are unstable for sufficiently forward-looking players) when \( \mathcal{y}^* < \mathcal{y} \). Suppose this is not the case and \( \mathcal{y}^* < \mathcal{y} \) and \( \pi_C^N \geq \pi_P^N \) both hold. \( \pi_C^N \geq \pi_P^N \) implies that \( f(\mathcal{y}) - c\mathcal{y} + (n - 1)b \geq f(\mathcal{y}) - k \), or:

\[
b \geq \frac{c\mathcal{y} - k}{n - 1}.
\]

Recall that the core player expects that she will not be challenged when she invests \( \mathcal{y}^* < \mathcal{y} \). A periphery player \( j \) will not challenge the core \( i \) when investing \( x_j = \mathcal{y} - \mathcal{y}^* \) and linking to \( i \) leads to a strictly higher payoff than investing \( x_j = \mathcal{y} \). This means, that \( f(\mathcal{y}) - (\mathcal{y} - \mathcal{y}^*)c - k > f(\mathcal{y}) - c\mathcal{y} + (n - 1)b \) should hold. Rewriting gives:
However, (B.5) and (B.6) cannot simultaneously hold as the RHS in (B.5) is strictly larger than the RHS in (B.6) for $y^* < \mathcal{Y}$. Hence, $y^* < \mathcal{Y}$ implies $\pi_C^N < \pi_P^N$ and in this case all Nash stars and superstars are unstable if players are sufficiently forward looking.

If status rents are relatively high, the core earns more than the periphery in a Nash star, i.e. $\pi_C^N > \pi_P^N$ (i.e. when $(n - 1)b > c\mathcal{Y} - k$). Hence, if $y^* = \mathcal{Y}$ and $\pi_C^N > \pi_P^N$, the core in a Nash star will not change her investment and Nash stars may be stable.

**Periphery players (cases I and II)**

We have now derived the investment level of core players. That is, when $\pi_C^N > \pi_P^N$ the core player will invest in $x_{i,t} = y^*$ units. For the corresponding Nash star or superstar to be stable, a periphery player $j \neq i$ should not want to challenge the core player $i$. Consider a periphery player $j$. If $j$ sticks to investing $x_{j,t} = 0$, she expects to be in the periphery of a $x_{i,t}$-star both at $t$ and $t + 1$, unless the core player is allowed to change her investment at $t + 1$. In this case she expects a Nash star to form at $t + 1$.\(^{27}\) This implies that she expects to earn:

$$
\mathbb{E}P_j(x_{j,t} = 0) = (1 - \alpha)\pi_P^* + \alpha \left( \frac{1}{n-1} \pi_P^N + \frac{n-2}{n-1} \pi_P^* \right) = \pi_P^* - \frac{\alpha}{n-1} (\pi_P^* - \pi_P^N),
$$

where $\pi_P^*$ is the payoff of $j$ being in the periphery of the $y^*$-star, i.e. $\pi_P^* = f(y^*) - k \geq \pi_P^N$.

First note that investing $x_{j,t} \in (0, y^*]$ will lead to lower expected payoffs than not investing. In this case, $x_{j,t} \leq y^* \leq x_{i,t}$ and (by Assumption 1) no-one will link to $j$ at $t$ or $t + 1$. Hence, $j$ will be better off by not investing and linking to $i$ at $t$ and $t + 1$. Then, the only possible profitable deviation is when $j$ challenges $i$ by investing in more units. If $x_{j,t} > x_{i,t}$, $j$ expects that all others than $i$ will link to her at $t$ and $t + 1$, and if $i$ is given the chance to update at $t + 1$ she will lower her investment and also link to $j$. This implies that she expects to earn:

$$
\mathbb{E}P_j(x_{j,t} > y^*) = (1 - \alpha)\left( f(x_{j,t}) - cx_{j,t} + (n - 2)b \right) + \alpha \left( \frac{1}{n-1} \left( f(x_{j,t}) - cx_{j,t} + (n - 2)b \right) + c \right).
$$

Note that the payoffs in (B.7) are the largest when $j$ invests in the smallest possible amount above $i$, i.e. when $x_{j,t} = x_{i,t} + 1$. Then, if $P_j(x_{j,1} = 0) > \mathbb{E}P_j(x_{j,1} = y^* + 1)$ the $y^*$-star is

\(^{27}\) Of course, the $x_{i,t}$-star can also be a Nash star. This is the case in n4b22.
stable. This is the case when:

\[(B.8) \quad (n - 2)b < (y^* + 1)c - k - (f(y^* + 1) - f(y^*)) - \frac{a}{n - 1}(f(y^*) - f(\hat{y}) + b)\]

Note the similarity between the inequalities in (2) and (B.8). Inequality (2) was used to derive the value of \(y^*\), i.e. the investment level at which a core player does not expect to be challenged. As the RHS in (B.8) is smaller than the RHS in (2), the core might actually be challenged in some cases. Note however that differences are very small and decreasing in \(n\).\(^{28}\) Thus, with sufficiently large status rents and forward-looking players, periphery-sponsored stars where the core invests in \(y^*\) units are stable.

**Uniqueness**

Now we will show that the \(y^*\)-stars are the only stable outcomes when \(\pi^N_C > \pi^N_P\). First note that if a pair of players \(i, j\) exists for which \(x_{j,t-1} \geq x_{i,t-1} > 0\) and \(x_{j,t-1} \geq \hat{y}\) holds, the outcome is unstable as \(i\) will earn a higher expected payoff by not investing. If \(i\) continues to invest \(x_{i,t} = x_{i,t-1}\), she expects (by Assumption 1) not to attract any links at \(t\) and \(t + 1\) as \(x_{j,t-1} \geq x_{i,t-1}\) and \(x_{j,t-1} \geq \hat{y}\). Then, at \(t + 1\) she could either link to \(j\) or not. If she does so, she expects to earn \(f(x_{j,t-1} + x_{i,t-1}) - cx_{i,t-1} - k\), while if she does not she expects to earn \(f(x_{i,t-1}) - cx_{i,t-1}\). Note that both are strictly smaller than \(\pi^N_P\), the payoff that would result if at \(t + 1\) she does not invest. This implies the following lemma:

**Lemma B.1:** If there is some player \(j\) who invests in the stage-game amount \(\hat{y}\) or more, i.e. \(x_{j,t-1} \geq \hat{y}\), any outcome where some other player \(i\) has positive investment is unstable.

Which gives us the following useful corollary:

**Corollary B.1:** All stable outcomes are either:

(i) periphery-sponsored stars where the core invests in a positive amount and no periphery player invests,

(ii) inefficient outcomes where all players have lower investments than \(\hat{y}\)

If \(\pi^N_C > \pi^N_P\), (and, therefore, \(y^* \geq \hat{y}\)) the \(y^*\)-star is the only stable outcome when all

\(^{28}\) If this does happen, a cycle between \(y^*\)-stars and \((y^* + 1)\)-stars is predicted by the model.
outcomes described in situation (ii) of Corollary B.1 are unstable. In situation (ii) of Corollary C.1, \( x_{i,t-1} < \hat{y} \forall i \in N \). There exists some value of investment \( x_{\text{low}} \), such that \( k > f(y^* + x_{\text{low}}) - f(x_{\text{low}}) \). That is, players who choose an investment of at least \( x_{\text{low}} \) will not form a link to any player \( i \) who invests \( x_i \leq y^* \). Denote the number of players who are below this threshold and who may choose to form a link by \( n_0 = |\{j \in N : x_j \leq x_{\text{low}}\}| \). If \( n_0 \geq (n - 1) \), the strategic situation boils down to the choice of a core player in a periphery-sponsored star, which we described above. When \( i \) updates, she will choose \( x_{i,t} = y^* \geq x_{i,t-1} \) (provided she is sufficiently forward-looking) and the outcome must be unstable. At the other extreme, when \( n_0 = 0 \), player \( i \) expects that any other player \( j \) who revises her investment at \( t + 1 \) will choose \( x_{j,t+1} = y \), unless \( i \) increases her investment. If she remains at \( x_{i,t} = x_{i,t-1} \in (0, x_{\text{low}}] \), she expects to earn \( f(x_{i,t-1}) - cx_{i,t} \) at \( t + 1 \), while if she does not invest, she expects to earn \( \pi^N_p \) which is strictly larger. Hence, not investing will lead to a higher expected payoff for \( i \) than not changing her investment (provided she is sufficiently forward-looking) and the outcome must be unstable. Extending this argument shows that for any value of \( n_0 \), a player who updates will have a profitable deviation by either not investing in \( \hat{y} \) or above to attract links. Hence, any outcome in (ii) of Corollary B.1. is unstable when \( \pi^N_C > \pi^N_P \) and only the \( y^* \)-star is stable. Recall that all Nash stars and superstars are unstable when the reverse holds.

**Predictions for the experiment**

For all treatments with \( b > 0 \), we have \( \pi^N_C > \pi^N_P \) and the behavioral model predicts that the \( y^* \)-star will form. This means, that we expect a 2-star in treatment n4b22, a 4-star in treatments n4b66 and n8b22 and an 8-star in treatment n8b66. In the treatments without rents (n4b0 and n8b0), we expect that all periphery-sponsored stars will be unstable as \( \pi^N_C < \pi^N_P \) in these treatments. We also compute the required lower bound on \( \alpha \) for each treatment. For n4b0, n8b0 and n4b22 \( y^* = \hat{y} = 2 \). For n4b22, \( \pi^N_C > \pi^N_P \) and the 2-star (the Nash star) is stable for any \( \alpha \). For n4b0 and n4b22, we obtain the following expected payoffs:

\[
\begin{align*}
\mathbb{E}\Pi_i(x_{i,t} = 0) &= \alpha(f(2) - k) = 82\alpha, \\
\mathbb{E}\Pi_i(x_{i,t} = 1) &= (1 - \alpha)(f(1) - c + (n - 1)b) + \alpha(f(3) - c - k) = 37 + 15\alpha, \\
\mathbb{E}\Pi_i(x_{i,t} = 2) &= f(2) - 2c + (n - 1)b = 42.
\end{align*}
\]
Nash stars are unstable when (B.3) does not hold, i.e. when one of the following two conditions is satisfied:

\[
\alpha > \frac{\pi^N - \mathbb{E}L_i(x_{i,t}=0)}{\mathbb{E}L_{i+1}(x_{i,t}=0)-\mathbb{E}L_i(x_{i,t}=0)} = \frac{\pi^N}{\pi^B} \geq \frac{f(2)-2c+(n-1)b}{f(2)-k} = \frac{42}{82},
\]

\[
\alpha > \frac{\pi^N - \mathbb{E}L_i(x_{i,t}=1)}{\mathbb{E}L_{i+1}(x_{i,t}=1)-\mathbb{E}L_i(x_{i,t}=1)} = \frac{(f(2)-2c+(n-1)b)-(f(1)-c+(n-1)b)}{f(3)-c-k-(f(1)-c+(n-1)b)} = \frac{f(2)-f(1)+c}{f(3)-f(1)-k} = \frac{1}{3}.
\]

Hence, Nash stars are unstable when \( \alpha > \frac{1}{3} \) and all periphery-sponsored stars are unstable when \( \mathbb{E}L_i(x_{i,t} = 0) > \mathbb{E}L_i(x_{i,t} = 1) \), which is the case when \( \alpha > \frac{37}{67} \).

For treatments \( n4b66, n8b22 \) and \( n8b66, y^* > \mathcal{g} \) and (B.2) should hold for the \( y^* \) to be stable. Below, we give the expected payoffs for our game for investing in \( y^* \) or below.

\[
\mathbb{E}L_i(x_{i,t} = 0) = \alpha(f(2) - k),
\]

\[
\mathbb{E}L_i(x_{i,t} = 1) = (1 - \alpha)(f(1) - c + (n - 1)b) + \alpha(f(3) - c - k).
\]

\[
\mathbb{E}L_i(x_{i,t} = 2) = (1 - \alpha)(f(2) - 2c + (n - 1)b) + \alpha(f(2) - 2c),
\]

\[
\mathbb{E}L_i(x_{i,t} \in (2, y^*) ) = (1 - \alpha)(f(x_{i,t}) - cx_{i,t} + (n - 1)b) + \alpha(f(x_{i,t}) - cx_{i,t}),
\]

\[
\mathbb{E}L_i(x_{i,t} = y^* ) = f(y^*) - cy^* + (n - 1)b.
\]

It is easily computed that for any level of \( \alpha \) both \( \mathbb{E}L_i(x_{i,t} = y^*) > \mathbb{E}L_i(x_{i,t} = 0) \) and \( \mathbb{E}L_i(x_{i,t} = 2) > \mathbb{E}L_i(x_{i,t} \in (2, y^*) ) \) hold. Hence, the only relevant bounds on \( \alpha \) come from \( \mathbb{E}L_i(x_{i,t} = y^*) > \mathbb{E}L_i(x_{i,t} = 1) \) and \( \mathbb{E}L_i(x_{i,t} = y^*) > \mathbb{E}L_i(x_{i,t} = 2) \). Computing these bounds yields:

\[
\alpha > \max \left\{ \frac{c(y^*-1)-(f(y^*)-f(1))}{k+(n-1)b-(f(3)-f(1))}, \frac{c(y^*-2)-(f(y^*)-f(2))}{(n-1)b} \right\}.
\]

For \( n8b22 \) this gives \( \alpha > \max\{0.44, 0.43\} = 0.44 \), for \( n4b66 \) \( \alpha > \max\{0.33, 0.33\} = 0.33 \) and for \( n8b66 \) \( \alpha > \max\{0.61, 0.60\} = 0.61 \).
Appendix C: Additional tables

Table C.1: Payoffs and welfare in different star networks

<table>
<thead>
<tr>
<th>Core investment</th>
<th>( \pi_p )</th>
<th>( \pi_c )</th>
<th>W</th>
<th>( \pi_p )</th>
<th>( \pi_c )</th>
<th>W</th>
<th>( \pi_p )</th>
<th>( \pi_c )</th>
<th>W</th>
<th>( \pi_p )</th>
<th>( \pi_c )</th>
<th>W</th>
<th>( \pi_p )</th>
<th>( \pi_c )</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_c = 1 )</td>
<td>22</td>
<td>37</td>
<td>103</td>
<td>37</td>
<td>191</td>
<td>103</td>
<td>169</td>
<td>191</td>
<td>345</td>
<td>235</td>
<td>301</td>
<td>499</td>
<td>653</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_c = 2 )</td>
<td>82</td>
<td>42</td>
<td>288</td>
<td>42</td>
<td>616</td>
<td>108</td>
<td>354</td>
<td>196</td>
<td>770</td>
<td>240</td>
<td>486</td>
<td>504</td>
<td>1078</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_c = 3 )</td>
<td>107</td>
<td>12</td>
<td>333</td>
<td>12</td>
<td>761</td>
<td>78</td>
<td>399</td>
<td>166</td>
<td>915</td>
<td>210</td>
<td>531</td>
<td>474</td>
<td>1223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_c = 4 )</td>
<td>126</td>
<td>-24</td>
<td>354</td>
<td>-24</td>
<td>858</td>
<td>42</td>
<td>420</td>
<td>130</td>
<td>1012</td>
<td>174</td>
<td>552</td>
<td>438</td>
<td>1320</td>
<td></td>
<td></td>
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<tr>
<td>( x_c = 5 )</td>
<td>129</td>
<td>-76</td>
<td>311</td>
<td>-76</td>
<td>827</td>
<td>-10</td>
<td>377</td>
<td>78</td>
<td>981</td>
<td>122</td>
<td>509</td>
<td>386</td>
<td>1289</td>
<td></td>
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<tr>
<td>( x_c = 6 )</td>
<td>132</td>
<td>-128</td>
<td>268</td>
<td>-128</td>
<td>796</td>
<td>-62</td>
<td>334</td>
<td>26</td>
<td>950</td>
<td>70</td>
<td>466</td>
<td>334</td>
<td>1258</td>
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<tr>
<td>( x_c = 7 )</td>
<td>133</td>
<td>-182</td>
<td>217</td>
<td>-182</td>
<td>749</td>
<td>-116</td>
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<td>-28</td>
<td>903</td>
<td>16</td>
<td>415</td>
<td>280</td>
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<tr>
<td>( x_c = 8 )</td>
<td>134</td>
<td>-236</td>
<td>166</td>
<td>-236</td>
<td>702</td>
<td>-170</td>
<td>232</td>
<td>-82</td>
<td>856</td>
<td>-38</td>
<td>364</td>
<td>226</td>
<td>1164</td>
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<tr>
<td>( x_c = 9 )</td>
<td>135</td>
<td>-290</td>
<td>115</td>
<td>-290</td>
<td>655</td>
<td>-224</td>
<td>181</td>
<td>-136</td>
<td>809</td>
<td>-92</td>
<td>313</td>
<td>172</td>
<td>1117</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_c = 10 )</td>
<td>136</td>
<td>-344</td>
<td>64</td>
<td>-344</td>
<td>608</td>
<td>-278</td>
<td>130</td>
<td>-190</td>
<td>762</td>
<td>-146</td>
<td>262</td>
<td>118</td>
<td>1070</td>
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<td></td>
</tr>
</tbody>
</table>

Notes: Payoffs for core (\( \pi_c \)) and periphery (\( \pi_p \)) players in different star networks where the periphery players do not invest. W denotes the welfare level, which is defined as the sum of payoffs.

Table C.2: MW tests for differences in the relative frequencies in periphery-sponsored stars in the final 25 periods.

<table>
<thead>
<tr>
<th>p-values</th>
<th>( b = 0 )</th>
<th>( b = 22 )</th>
<th>( b = 66 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>final 25 periods</td>
<td>( n = 4 )</td>
<td>( n = 8 )</td>
<td>( n = 4 )</td>
</tr>
<tr>
<td>( b = 0 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( n = 8 )</td>
<td>0.49</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( b = 22 )</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td>0.02</td>
<td>0.02</td>
<td>0.29</td>
</tr>
<tr>
<td>( n = 8 )</td>
<td>0.04</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>( b = 66 )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Appendices for online publication

Appendix D: Subgame perfect equilibria of the repeated game

The finitely repeated game has a plethora of repeated game equilibria, including those where a stage-game equilibrium is played in each of the $T$ periods. Here, we are interested in equilibria in which players earn higher payoffs than when they repeatedly play a stage-game equilibrium. We focus on equilibria in which superstars are consistently formed, which includes the efficient superstar. In addition, we restrict our attention to strict subgame perfect equilibria. In the GG model, all efficient outcomes are superstars. We denote the efficient investment level by the core player in the superstar by $\gamma$.

One way to support efficient equilibria is by the use of punishment strategies. Like Benoit and Krishna (1985) we consider the use of ‘optimal punishments’. The optimal punishment strategy yields the worst possible payoff for the punished player $i$ that is feasible in a subgame perfect equilibrium. The average payoff for the punished player $i$ from the optimal punishment in $K$ periods of punishment is denoted by $w_i(K)/K$, where $w_i(K)$ is the total payoff of the punished player in these $K$ periods. Benoit and Krishna show that this payoff is bounded by:

$$v_i \leq w_i(K)/K \leq w_i(1),$$

where $v_i$ is $i$’s minmax payoff and $w_i(1)$ her worst possible payoff in a stage-game equilibrium.

One way to support (efficient and inefficient) superstars is by rotating the core position.

**Proposition D.1:** In all of our experimental games with endogenous network formation, efficient superstars with a rotating core position can be supported as part of a subgame perfect equilibrium until period $T - Q$, where $1 \leq Q < T$.

**Proof.** Suppose that all players observe a perfectly correlated signal $\theta_t$ at the beginning of each period $t$. This signal is an independent draw from the set $\{1, \ldots, n\}$. Each integer is drawn with equal probability. Consider the strategy profile $s^N$, where each player’s strategy $s^N_t$ adheres to:
“In each period, the player who is assigned the core position is determined by the draw of \( \theta_t \), i.e. if \( \theta_t = j \), player \( j \) will be in the core position in period \( t \). In a period, the core player does not form any links, and all other players only form a link to this core player. In the first \( T-Q \) periods the core player invests \( y^S > \gamma \) and in the final \( Q \) periods the core player invests the stage-game Nash level \( \gamma \). If some player \( i \) deviates in period \( t \), play switches to the punishment regime and the deviating player will be punished in periods \( t + 1 \) to \( T \) by the optimal punishment strategy.”

The strategy profile \( s^N \) constitutes a subgame perfect equilibrium if it satisfies the one-stage-deviation principle. That is, the strategy profile \( s^N \) is a subgame perfect equilibrium if and only if:

\[
\pi_i(s^N_i, s^N_{-i}) > \pi_i(s_i^*, s^N_{-i}) \quad \forall \ i \in N, \forall \ s_i^*,
\]

where \( s_i^* \) indicates a deviant strategy of player \( i \), which differs from the equilibrium strategy \( s^N \) only in period \( t \) and conforms to \( s^N \) thereafter (see for instance, Theorem 4.1 in Fudenberg and Tirole 1991, p.109, for a proof).

Let \( \pi_c^S \) and \( \pi_p^S \) denote the stage-game payoff of being, respectively, in the core or the periphery of the superstar where the core invests in \( y^S \). Likewise, we write \( \pi_c^N \) and \( \pi_p^N \) for the payoff of being, respectively, in the core or the periphery of the Nash star where the core invests in \( \gamma \).

In \( s^N \), the final \( Q \) periods consist of a sequence of stage-game equilibria, hence, a profitable one-stage-deviation can only exist in the first \( T - Q \) periods. First consider deviations by the core in these periods. The optimal deviation by the core player in a superstar is to invest in the Nash level \( \gamma \), which implies that a Nash star would result. Hence, by deviating in period \( t \leq T - Q \), the sum of payoffs in periods \( t \) to \( T \) of core player \( i \) will be:

\[
\pi_i(s_i^*, s^N_{-i}) = \pi_c^N + (T - t) \frac{w_i(T-t)}{T-t}.
\]

The expected payoff from following \( s_i^N \) is \( \bar{\pi}_i^S = \frac{n-1}{n} \pi_c^S + \frac{1}{n} \pi_p^S \) in each of the first \( T - Q \) periods and \( \bar{\pi}_i^N = \frac{n-1}{n} \pi_c^N + \frac{1}{n} \pi_p^N \) in each of the final \( Q \) periods. In every period \( t \leq T - Q \), the core player in the superstar will not deviate if:
As \( \bar{\pi}^S > \bar{\pi}^N > w_i(1) \geq \frac{w_i(T-t)}{T-t} \), it suffices to consider only period \( t = T - Q \). Using this, we can rewrite (D.1) to obtain:

\[
Q > \frac{\pi^N_C - \pi^S_C}{\pi^N_C - w_i(Q)/Q},
\]

which gives a condition for the minimal length of the ‘Nash phase’ that is needed to avoid deviation by the core.\(^29\) As \( \bar{\pi}^N > w_i(1) \geq \frac{w_i(T-t)}{T-t} \) and \( \pi^N_C > \pi^S_C \), the right hand side of (D.2) is always positive.

If \( b = 0 \), we have \( v_i = w_i(1) = \pi^N_C \). Using this in (D.2), we can compute for our game that \( Q \geq 3 \) if \( n = 4, b = 0 \) and \( Q \geq 2 \) if \( n = 8, b = 0 \).\(^30\) Similarly, in our games with status rents, the worst stage-game equilibrium payoff is the periphery position in the Nash star. Using this as the average punishment payoff \( w_i(Q)/Q \) in case of deviation, we find that \( Q \geq 11 \) if \( n = 4, b = 22 \), \( Q \geq 5 \) if \( n = 8, b = 22 \), \( Q \geq 2 \) if \( n = 4, b = 66 \) and \( Q \geq 2 \) if \( n = 8, b = 66 \) are needed to sustain the efficient superstar with \( y^S = y^N = 4 \). This shows that no core player will deviate in the first \( T - Q \) periods for the above bounds on \( Q \).

Now consider deviations by a periphery player \( j \). Note that within a period, the periphery player best responds by linking to the core in the superstar. If a periphery player deviates, she will be punished in all \( T - t \) remaining periods. The expected future payoffs in case of deviation are thus \( w_i(T-t) \). As \( \pi^S > \pi^N > w_i(1) \geq \frac{w_i(T-t)}{T-t} \), the expected future payoff of adhering to the strategy \( s^N_j \) is strictly larger than the future payoffs of deviation. Hence, no periphery player will deviate, which shows that the strategy profile \( s^N \) constitutes a subgame perfect equilibrium. Q.E.D.

**Superstars with a fixed core player**

Given that in practice rotation schemes are rarely implemented, we now focus on equilibria where a fixed periphery-sponsored star is formed in all periods. We divide the game in two phases: a ‘superstar phase’ where a superstar with a fixed core is played in the first \( T - Q \) periods, and an ‘end phase’ which consist of the final \( Q \) periods.

\(^29\) Proposition D.1 holds whenever (D.2) is fulfilled. We have restricted the proposition to our experimental games in order to keep the notation as simple as possible.

\(^30\) More specifically, if \( n = 4, b = 0 \) we have \( Q > \frac{11}{5} \) and if \( n = 8, b = 0 \) we have \( Q > \frac{66}{35} \).
Proposition D.2: Consider the set of equilibria where (1) on the equilibrium path a periphery-sponsored star with a fixed core player is formed and (2) in a punishment phase the same network is formed in each of the remaining periods; then status rents are necessary for the formation of superstars in the repeated game equilibrium.

Proof. In this type of equilibrium, the efficient superstar is played in periods 1 to $T - Q$ and a Nash star is played in the final $Q$ periods. In every period, the same player $i$ fills the core position. Note that we rule out rotations on the equilibrium path as well as in a possible punishment phase. Now, consider the following strategy profile $s^N$:

“In each period, the same player $i$ is assigned the core position. This player does not form any link, and all other players only form a link to this core player. In the first $T - Q$ periods the core player invests the superstar level $y^S > \mathcal{Y}$ and in the final $Q$ periods the core player invests the stage-game Nash level $\bar{y}$. If some player $j$ deviates in period $t$, play switches to the punishment regime and the deviating player will be punished in periods $t + 1$ to $T$ by the optimal punishment strategy.”

First, consider deviations by the core player. As before, the optimal deviation by the core in the superstar phase is to lower her investment to $\bar{y}$, which results in a Nash star. Hence, the core will not deviate in the first $T - Q$ periods if:

\begin{equation}
(T - Q - t + 1)\pi_\mathcal{C}^S + Q\pi_\mathcal{C}^N > \pi_\mathcal{C}^N + (T - t)w_i(1)
\end{equation}

and will have a profitable deviation if the reverse holds. If $\pi_\mathcal{C}^S \leq w_i(1)$ it is sufficient to consider only period $t = 1$, while if $\pi_\mathcal{C}^S > w_i(1)$ it is sufficient to consider only the final period of the superstar phase, i.e. $t = T - Q$.

First consider the game without status rents. As before, $w_i(1) = \pi_\mathcal{C}^N$. As $\pi_\mathcal{C}^N > \pi_\mathcal{C}^S$ for any $y^S > \mathcal{Y}$, we consider period $t = 1$. Using this in (D.3) reduces the condition to $\pi_\mathcal{C}^S > \pi_\mathcal{C}^N$ which is not true. Thus, the reverse sign holds in (D.3) and a profitable deviation exists for the core player. Hence, the strategy profile $s^*$ does not constitute a subgame perfect equilibrium in the absence of status rents. Q.E.D.
Now, we will show that superstars with a fixed core can indeed be sustained in our treatments with status rents. In our treatments with status rents \( w(1) = \pi_p^N \). In treatment n4b22, \( \pi_p^N > \pi_c^S \) for any \( y^S > y \), therefore we consider period \( t = 1 \). In this case, the minimal length of the Nash phase becomes:

\[
Q > \frac{\pi_c^S - w_i(1) + \pi_i(1) - \pi_c^S}{\pi_c^S - \pi_c^N}.
\]

Computing this for \( y^S = y \), gives \( Q \geq 46 \), which implies that an efficient superstar with a fixed core can be supported but only from period 1 to 29. In the other treatments with status rents (i.e. n8b22, n4b66 and n8b66) \( \pi_c^S > \pi_p^N \), which implies that it is sufficient to consider the last period of the superstar phase. Using this in eq. (D.3) yields:

\[
Q > \frac{\pi_p^N - \pi_c^S}{\pi_c^S - w_i(1)},
\]

which provides a condition for the minimal length of the Nash phase. As \( w(1) = \pi_p^N \), and the payoff in the periphery of the Nash star is strictly smaller than \( \pi_c^S \) when \( y^S = y = 4 \), condition (D.4) implies that \( Q \geq 1 \) in n8b22, n4b66 and n8b66. In the final \( Q \) periods, the Nash star will be played, which is a stage-game Nash network. Hence, the core player has no profitable one-stage-deviation. Note that also superstars with higher core-investment than 4 units could be supported in this manner. Superstars can be supported in this way until the penultimate period as long as \( \pi_c^S > \pi_p^N \). Table C.1. in Appendix C lists the payoffs in different star networks. In n8b22, superstars where \( y^S \leq 4 \) can be supported until the penultimate period, in n4b66 this is the case for \( y^S \leq 5 \) and in n8b66 for \( y^S \leq 10 \).

Finally, consider the periphery players in the game with status rents. Again, within a period, the periphery players best respond by linking to the core in the superstar. If a periphery player deviates, she will be punished in all \( T - t \) remaining periods. The average future payoffs in case of deviation are thus \( w_i(1) \). As \( \pi_p^S > \pi_p^N \geq w_i(1) \), the future payoffs of adhering to the strategy are strictly larger than the future payoffs of deviation. Hence, no periphery player will deviate, which shows that the strategy profile \( s^N \) constitutes a subgame perfect equilibrium in the treatments with status rents, and that efficient superstars can be sustained until period \( T - 1 \).
Appendix E: Experimental instructions and test questions

All text in red italics is treatment specific. Treatment specific text is denoted by: <n4> and <n8> for \( n = 4 \) and \( n = 8 \) respectively, <ENDO> and <EXO> for endogenous and exogenous networks respectively and <b0>, <b22> and <b66> for the respective level of status rents. All public good investments and links in examples and test questions are independently and randomly generated for each subject.

Welcome!

Welcome to this experiment on decision-making. During the experiment, you are not allowed to communicate with other participants. If you have any questions, please raise your hand. One of the experimenters will come to you to answer your question.

During the experiment you can earn points. These points are worth money. How many points (and hence how much money) you earn depends on your own decisions, the decisions of others and chance. Your decisions are anonymous. They will not be linked to your name.

Every 30 points are equivalent to 0.10 euro.

At the end of the experiment the points that you earned will be converted to euros and the amount will be paid to you privately, in cash.

Today's experiment consists of two parts. You will spend most time on the second part. The second part will be explained after you have finished the first part. Your decision in the first part has no influence on the proceedings of the second part and your decisions in the second part do not affect the proceedings of the first part.

Instructions first part

You are now given 600 points. You must decide how many points you want to invest in a lottery. The points that you do not invest will be added your total earnings at the end of the experiments and paid out to you in cash.

The lottery: You have a chance of 50% of losing the amount you invest and a 50% chance of winning two and a half times the amount you invested.
Whether you win or lose in the lottery is determined by chance. For this, you choose whether you want to play heads or tails. If the outcome of the lottery is the same as your choice, you win. The chance of heads or tails is equal: both occur with 50% probability. The outcome of the lottery will be announced to you at the end of the experiment.

In summary, your earnings in the lottery are determined as follows. If you decide to invest $X$ points in the lottery and you win the lottery, you earn the number of points that you did not invest in the lottery plus two and a half times the number of points that you did invest in the lottery. Thus, your earnings will then be:

$$600 - X + 2.5 X.$$

If you lose then you will only earn the points that you did not invest. Your earnings will then be:

$$600 - X.$$

If you have any questions, please raise you hand and one of the experimenters will come to you to answer your question. If everything is clear, click below to make you decision for the lottery.

Instructions second part

Please read the following instructions carefully. After reading the instructions we will ask you several questions to test whether you understand the experiment. The experiment will
continue after you answered all questions correctly. While reading the instructions, you can browse back and forth between pages by using the menu on the top of your screen.

Your total earnings consist of the points you earn in the first part of the experiment (the lottery) and the sum of all points that you earn in the second part of the experiment. At the beginning of the second part you will receive a starting capital of 2000 points. This will also be added to your earnings.

As before, every 30 points are equivalent to 0.10 euro

The second part of the experiment consists of 75 rounds. You have now been randomly placed in a group of 4 participants. The composition of this group will not change during the experiment. In this group you will be randomly assigned a role. This role will be indicated by a letter: "A", "B", "C" or "D". The letters "A", "B", "C", "D", "E", "F", "G" or "H". The letters "A", "B", "C" and "D" will thus refer to the same participant throughout the entire experiment.

Everybody in your group has received the exact same instructions. However, it may be that people that are not in your group will participate in a different experiment.

Costs and benefits

Every round you can earn points by having ‘access’ to units of a good. The number of points that you earn depends on the number of units that you have access to. This is shown in the following table:

<table>
<thead>
<tr>
<th>Units</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>10+i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefits</td>
<td>0</td>
<td>92</td>
<td>152</td>
<td>177</td>
<td>196</td>
<td>199</td>
<td>202</td>
<td>203</td>
<td>204</td>
<td>205</td>
<td>206</td>
<td>206+i</td>
</tr>
</tbody>
</table>

The table shows for instance that you earn 152 points if you have access to 2 units and that you earn 204 points if you have access to 8 units of the good.

There are two ways to access units of the good.

1. You buy units of the good yourself.
2. You make have a ‘link’ to another participant. In this case you have access to the units that the other participant has bought.

In addition, you will earn points if other participants make a link to you. For each link that another participant makes to you, you will receive 22 points. 66 points.

Buying units and making having links is costly.

The cost of making having a link is 70 points for each role. Every round, you can maximally make have one link to each of the other roles. This means that you cannot make have more than 3 links.

You yourself will not decide on your links, like others will not decide on their links. When you decide on how many units you want to buy, you will be informed about the links that you will have in the current round. The number of units that you buy does not affect links in the current or future periods. Similarly, the links that others buy neither affect links in the current of future rounds. The participants in this experiment do not have any influence on how the links evolve.

The cost of buying units is 55 points per unit. Every round, you can maximally buy 10 units of the good.

In summary:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per unit</td>
<td>55</td>
</tr>
<tr>
<td>Cost per link</td>
<td>70</td>
</tr>
<tr>
<td>Benefits per link to your role</td>
<td>22</td>
</tr>
</tbody>
</table>
Example

The table and the figure above show a possible outcome of a round. The table and figure merely serve as an example, the content does not give any information on what to expect in the experiment. The numbers chosen for this example have been chosen randomly and are different for each participant.

The decisions of your role are displayed in orange and the decisions of the other roles are displayed in blue. In the example, your role is A. In the figure, roles are indicated by the letters A, B, C and D. A, B, C, D, E, F, G and H. The number of units a participant bought is indicated by the colored circles. The larger the acquisition of a participant is, the darker is the circle at the corresponding role. In the example, the participant in role B bought 3 unit(s) and the participant in role D bought 10 units. The blue circle at role D is thus darker than the blue circle at role B. The acquisitions of all participants are also listed in the table.

In the figure, links are indicated by arrows. The arrow points away from the one who made the link. In this case, A made has a link to B, B made has a link to A, C made has links to A and D and the participant in role D roles D, E, F, G and H made no links. These decisions are also listed in the table.

In the example your role is A. In the example above, your earnings would be calculated as follows:
You bought 3 units
You made have 1 link(s)
Access to 6 units
2 link(s) made to you
2 link(s) to you

Earnings this round

Cost/benefits

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>You bought 3 units</td>
<td>-165</td>
</tr>
<tr>
<td>You made have 1 link(s)</td>
<td>-70</td>
</tr>
<tr>
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<td>202</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>2 link(s) to you</td>
<td></td>
</tr>
</tbody>
</table>

Practice questions I

Your group of four participants:

- Is the same in every round
- Changes from round to round

Which statement is correct:

- Your role is the same in every round
- Your role is determined randomly every round

How many points do you earn if you access 3 units of the good?

_____ points

How many points does it cost to make have a link to another participant?

_____ points

How many points do you earn for each link that is made to you?

_____ points

How many points do you earn for each link to you?

----- points
Practice questions II (identical setup for Practice questions III)

The table and the figure above show a possible outcome of a round. The table and figure merely serve as an example, the content has been generated randomly and gives no information on what to expect in the experiment.

What are your total costs for <ENDO> making <EXO> having links in the example above? _____ points

In the example above you bought 4 unit(s) of the good. How many points does this acquisition cost? _____ points

How many units of the good do you access in total in the example above? _____ points

What are your benefits of accessing units of the good in the example above? _____ points

<ENDO> How many points do you earn for the links that are made to your role? <EXO> How many points do you earn for the links to your role? _____ points

What would be your earnings in the example above? _____ points
**End of instructions**

You have reached the end of the instructions. You can still go back by using the menu above. If you are ready, click on 'continue' below. If you need help, please raise your hand.

**Hand-out printed summary**

**Summary**

Your total earnings consist of the points you earn in the first part of the experiment (the lottery) and the sum of all points that you earn in the second part of the experiment. At the beginning of the second part you will receive a starting capital of 2000 points. This will also be added to your earnings.

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<tr>
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</table>
Appendix F: Screen shots