Parking and Urban Form

by

Jan K. Brueckner
Department of Economics
University of California, Irvine
3151 Social Science Plaza
Irvine, CA 92697
e-mail: jkbrueck@uci.edu

and

Sofia F. Franco
Nova School of Business and Economics
Campus de Campolide P-1099-032
Lisboa, Portugal
e-mail: sfranco@novasbe.pt

April 2015, Revised October 2015

Abstract

This paper analyzes the provision of residential parking in a monocentric city, with the ultimate goal of appraising the desirability and effects of regulations such as a minimum-parking requirement (MPR) per dwelling. The analysis considers three different regimes for provision of parking space: surface parking, underground parking, and structural parking, with the latter two regimes involving capital investment either in the form of an underground parking garage or an above-ground parking structure. Parking area is viewed as a dwelling attribute that, along with floor space, provides utility. In addition, road congestion in the neighborhood (which affects the commuting costs of local residents) depends on the average amount of off-street parking per dwelling, an externality that is ignored by profit-maximizing developers, making the equilibrium inefficient. The analysis explores the equilibrium spatial behavior of the two dwelling attributes as well as residential and parking structural density, and analysis of land rent shows which parking regimes are present in different parts of the city. Efficiency requires an increase in parking area per dwelling at each location, which can be achieved in a crude fashion by an MPR, whose effects are analyzed.

Parking and Urban Form

by

Jan K. Brueckner and Sofia F. Franco*

1. Introduction

Parking plays a central role in any effort to improve accessibility in urban areas. Parking supply (both residential and commercial) and parking policy affect the relative competitiveness of a central business district and influence where commercial as well as residential development occurs, affecting the cost and efficiency of daily commuting and influencing where people live and how they access work and shopping areas. In addition, congestion and parking supply are interrelated since looking for a parking space (known as "cruising" for parking) creates additional delays and impairs local circulation.

Most cities worldwide seek to avoid parking shortages by requiring commercial and residential developers to provide a certain amount of parking area, usually within the same premises, as a condition of zoning approval. These minimum parking requirements (MPRs) are usually set by city planners from standardized transportation planning manuals, which measure trip generation rates at peak periods with ample free parking and no public transit. The goal of MPRs is to ensure adequate parking at a low price so as to limit local congestion and stimulate local businesses (Shoup 1999).[1]

Even though off-street parking supply is a hotly debated element of parking policy, remarkably little analytical work exists studying the effects of parking supply on urban form. This omission is particularly puzzling, as a substantial proportion of urban land is devoted to parking (30% in multifamily-housing areas and 60% in commercial areas, according to Ferguson 2005). To help fill this gap in the literature, the present paper analyzes the provision of residential parking in a monocentric city, with the ultimate goal of appraising the desirability and effects of regulations such as residential minimum parking requirements. To maintain tractability, parking in the city center, which serves workers, is assumed to be freely available and is thus not considered in the model, which has a purely residential focus. In addition, the analysis assumes that alternate transportation modes are not available, so that all urban households own cars and thus value parking space. The consequences of relaxing these assumptions are briefly considered once the main results have been derived.

Most theoretical studies on parking examine parking in downtown areas, analyzing how parking pricing policy affects short-run commuter decisions regarding trip scheduling and frequency, transport mode choice, and parking location. Existing studies focus on the efficiency of second-best pricing of parking in the absence of congestion tolls (Arnott et al. 1991, Glazer and Niskanen 1992, Verhoef et al. 1995, Arnott and Rowse 1999, Anderson and de Palma 2004); the effects of curbside parking fees on cruising for parking in downtown areas (Arnott and Inci 2006);[2] the effects of underpricing of parking facilities on social welfare (Calthrop and Proost 2006); and the effects of parking and transit subsidies on the CBD's size (Voith 1998). Other analytical studies focus on the effect employer-paid parking on the level of optimal congestion charges (De Borger and Wuyts 2009) and on the optimal level of curbside parking capacity in downtown areas when both urban transport and curbside parking are underpriced (Arnott et al. 2013).[3] Yet, none of these previous studies has developed an urban spatial model that captures general-equilibrium interactions between residential parking and land-use, as is done is this paper.

Only two existing studies integrate parking into an urban spatial model. Anderson and De Palma (2007) analyze a city where the CBD is surrounded by a zone of parking lots, with the outlying residential area comprised of two zones. In the inner zone, residents walk all the way to work (crossing the parking lots). In the outer zone, residents drive to the parking area (crossing the zone where walkers live) and then walk from their parking spot to work. The paper characterizes the socially optimal configuration of the city and then shows that the optimum coincides with an equilibrium in which parking is provided by monopolistically competitive lot owners (each operating at a particular distance).

In a second paper, Franco (2015) develops a monocentric-city model with two transport modes, endogenous residential surface parking, and a form of bottleneck congestion at the CBD. Her goal is to explore the effects of changes in downtown parking supply on urban welfare, modal choice and urban spatial structure. Franco shows that CBD parking reforms such as supply limits that increase congestion costs in the short-run can be welfare improving if other commuting externalities such as air pollution can be reduced. The study also shows that parking limits may complement anti-sprawl policies by leading to a more-compact city in the long run.[4]

Our approach is complementary to the models of Franco (2015) and Anderson and De Palma (2007), although it differs by allowing the use of several alternative technologies for the provision of parking space. In the model, which borrows from Brueckner's (1983) model of urban yard space, housing developers use capital in standard fashion to produce floor space, which is divided into individual dwellings. Beyond floor space, another attribute is the parking area associated with the dwelling, which is also assumed to yield utility. Under the surface parking regime, parking area is provided via an outdoor parking lot, which requires a minimal capital investment, assumed to be zero (parking area is then analogous to the yard space in Brueckner (1983)). Under the structural parking regime, however, a parking structure built adjacent to the residential structure provides parking area. While capital cost is much higher than under surface parking, structural parking economizes on land through use of a multistory structure. Underground parking, by contrast, requires no additional land beyond that used for the residential building. Parking area is provided within an underground structure directly below the building, which involves higher capital cost than above-ground structural parking given the technical challenge of underground construction (see Cutter and Franco 2012 for evidence).

Under any of three parking regimes, the developer maximizes profit by choice of dwelling size, parking area per dwelling, residential structural density (capital per unit of residential land, an indicator of building height), and parking inputs. These inputs are land in the case of surface parking, land and capital in the case of structural parking, and capital alone in the case of underground parking (the land input is already available). Under the latter two regimes, parking structural density (capital per unit of parking land) is a choice variable of the developer, indicating the height of the above-ground parking structure or the depth of the underground structure. Under each regime, the maximization problem involves an important constraint that equates the development's total parking area to parking area per dwelling times the number of dwellings in the complex.

Focusing on urban form, the analysis first addresses the spatial behavior of the developer's choice variables, asking whether dwelling size and parking area per dwelling increase or decrease with distance to CBD while exploring the spatial behavior of both residential and (where relevant) parking structural density. Because of the complexity of the model, answers are available only through the assumption of specific functional forms for utility and production functions. Additional questions concern the relative locations within the city of the different parking regimes. Does the model imply (as intuition would suggest) that surface parking is found in the suburbs, with the non-surface parking regimes prevailing closer to the CBD?

The model is constructed to potentially include the phenomenon of parking-related congestion, which generates an additional neighborhood-level travel cost. With provision of off-street parking reducing this congestion, the extra travel cost falls as average parking area per dwelling in the neighborhood increases, a cost-side benefit that accompanies the utility gain from a dwelling's off-street parking area. Atomistic developers, however, are portrayed as ignoring the impact of their decisions on neighborhood-average parking area per dwelling, introducing an externality that makes the equilibrium inefficient.

The model solutions in the presence of parking-related congestion can be deduced using the solutions computed in its absence, and the analysis presents comparisons of the two sets of solutions. The discussion then turns to correction of the market failure from decentralized provision of parking. While intuition would suggest that a social planner would seek to raise parking area per dwelling above the equilibrium level, taking account of the congestion-related benefits overlooked by developers, the complexity of the model precludes a general demonstration of this conjecture. Instead, numerical analysis is used to confirm it, drawing on the numerical framework used earlier in the paper to illustrate the model's properties. The results indeed show that the parking area per dwelling is higher at the social optimum than in the equilibrium, while highlighting the other changes in urban form (dwelling size, structural densities) that accompany movement to the social optimum. The final step in the analysis is exploration of the effect on urban form of a minimum parking requirement, which specifies a spatially invariant minimum level of parking area per dwelling, failing to incorporate the spatial variation that the omniscient planner would dictate. With the exception of Cutter and Franco (2012), no other theoretical study examines the broad effects of parking requirements on urban form. In contrast to Cutter and Franco (2012), however, the current analysis focuses on residential MPRs instead of non-residential parking regulations.

The next section of the paper presents the general model, with subsections dealing with the three parking regimes. Section 3 presents model solutions in the absence of parking-related congestion, relying specific functional forms, and section 4 introduces this type of congestion and derives the effect on the equilibrium. Section 5 numerically characterizes the social optimum in the presence of parking-related congestion, comparing it to the equilibrium, while section 6 analyzes the effects of a minimum parking requirement, again numerically. Section 7 offers conclusions.

2. Model

2.1. Utility function and budget constraint

Dwellings in the model have two attributes: floor space, measured in square feet and denoted by q, and off-street parking area, again measured in square feet and denoted a.[5] Letting c denote consumption of a composite non-housing good, consumer utility is given by the common, well-behaved function $v(c, q, a)$. A crucial implicit assumption in this formulation is that every household in the city has a car and thus values parking space. Although the treatment of the decision to own a car is mostly beyond the scope of the paper, the discussion returns to this issue below, providing a sketch of how carless households could be added to the model. In addition, consumers in the model do not care which parking technology (surface, structural or underground) generates their parking area, valuing only a itself. This view matches the standard model's assumption that consumers have no preference over the type of building (high-rise or low-rise) that contains their dwelling floor space, which is also invoked here.

Off-street parking space generates utility by offering more convenience and safety than on-street parking. The consumer has a shorter and possibly safer walk to the vehicle, and off-street parking eliminates the search costs that may be incurred finding an on-street parking space. In addition, the vehicle is better protected from damage (and from the weather if covered) while parked in an off-street space. These benefits are assumed to be greater the larger amount of parking space associated with the dwelling, so that $v^a > 0$ (superscripts denote partial derivatives).

As usual in the monocentric model, all consumers work in the CBD and earn a common income w. Parking is freely available at the workplace.[6] Commuting cost from a residence x miles from the CBD is given by tx, where t is commuting cost per round trip mile. While the usual kind of road congestion is omitted from the model for simplicity, the consumer is assumed to experience parking-related congestion in the neighborhood of residence, which generates a cost of $\kappa - \tau\overline{a}$, where $\kappa, \tau > 0$ and $\overline{a}$ is the average amount of off-street parking in the neighborhood. This cost is due to the congestion caused when other residents cruise for parking while searching for an on-street space, which impedes movement in and out of the neighborhood as commuters access major roads heading to the CBD. An increase in the average amount of off-street parking in the neighborhood reduces this cost.[7] For simplicity, κ is absorbed into the income term (which becomes $y = w - \kappa$), so that the budget constraint of a consumer living at distance x is given by

$$c \; + \; R \;\; = \;\; y \; - \; tx \; + \; \tau\overline{a}, \tag{1}$$

where R is total rent for the dwelling (c is numeraire).

Along with the other variables in the model, $\overline{a}$ is implicitly a function of x. This average a value pertains to the neighborhood surrounding the residence, which can be viewed as a the adjacent portion of the circular ring at distance x that contains it. The size of this ring portion is not specified, but it is assumed to be large enough that individual developers view $\overline{a}$ as parametric and uninfluenced by their decisions, as explained further below. Note that this view of a ring as constituting a neighborhood is common in urban models.[8]

In equilibrium, the urban residents must reach a common utility level u, which is viewed as fixed for the purposes of the analysis (it thus relies on the "open-city" model). The fixed

u then determines the rent R that a consumer is willing to pay for a dwelling as a function of its location and attributes. This rent, denoted $R(q, a, x; \overline{a})$, is implicitly determined by eliminating c in the utility function using the budget constraint (1), and then setting the resulting expression equal to u:[9]

$$v(y - tx + \tau\overline{a} - R,\ q,\ a) \quad = \quad u. \tag{2}$$

Differentiating (2) shows that rent falls with x, rises with $\overline{a}$, and rises at a rate equal to the relevant marginal rate of substitution as q or a increases, so that

$$R^q \ = \ v^q/v^c \ > \ 0, \quad R^a \ = \ v^a/v^c \ > \ 0, \quad R^{\overline{a}} \ = \ \tau \ > 0, \quad R^x \ = \ -t \ < \ 0. \tag{3}$$

It is important to recognize that R embodies payments for both dwelling space and parking area, which are bundled by the developer, following actual real-world practice in residential markets. Thus, the consumer does not face separate prices for these dwelling attributes, as would be the case if dwelling floor space and parking area were provided by separate producers.

The housing developer maximizes profit taking the $R(q, a, x; \overline{a})$ function into account. Reliance on this function, rather than a price p per square foot of housing, distinguishes the current developer's problem from that in the standard exposition of the urban model. In that exposition, consumers choose q conditional on a price p per square foot of housing. On the production side, housing floor space in the standard model is produced by combining land and capital, with the capital completely covering the lot (leaving no open space). Housing output per unit of land is given by $h(S)$, where S equals capital per unit of land (structural density) and $h' > 0,\ h'' < 0$. This is the intensive version of a constant-returns production function whose arguments are capital and land. The developer's profit per unit of land is then written $ph(S) - iS - r$, where i is the price per unit of capital and r is rent per unit of land. Note that $ph(S)$ is revenue per unit of land, equal to price per square foot times square feet per unit of land, and that the developer chooses S. If the $R(\cdot)$ function were used instead of p, with a and $\overline{a}$ constrained to be zero to match the standard model, profit would be written

$R(q,x)h(S)/q - iS - r$, where revenue is now rent per dwelling (R) times dwellings per unit of land, $h(S)/q$, equal to square feet per unit of land divided by square feet per dwelling. In addition, the developer would now choose both S and q. It is easy to see that these two versions of the standard model are equivalent.

With multiple housing attributes (q and a), the second of the above approaches, which relies on the $R(\cdot)$ function and assigns attribute choices to the developer, is more natural and straightforward. The previous profit function, however, must be adjusted to include the provision of parking. The subsequent sections carry out the required adjustments for the cases of surface parking, structural parking, and underground parking, while characterizing the developer's optimal choices.[10]

2.2. Surface parking

To provide housing with surface parking, the developer must acquire the land on which the building sits, denoted "residential" land, as well as additional land for parking. Let ℓ denote residential land and $\widetilde{\ell}$ denote the parking land area. Then, assuming that surface parking entails no capital cost, the developer's profit is given by

$$\left[\frac{h(S)}{q}R(q,a,x;\overline{a}) - iS - r\right]\ell \;-\; r\widetilde{\ell}, \tag{4}$$

where the bracketed term gives profit per unit of land of residential land. The amount of land $\widetilde{\ell}$ required for parking equals the number of dwellings in the building times parking area a per dwelling, or

$$\widetilde{\ell} \;=\; \ell\frac{h(S)}{q}a. \tag{5}$$

Note the RHS equals residential land times dwellings per unit of residential land (total dwellings) times parking area per dwelling. Substituting for $\widetilde{\ell}$ in (4) using (5) and factoring out ℓ, profit per unit of residential land with surface parking equals

$$\pi_{su} \;=\; \frac{h(S)}{q}R(q,a,x;\overline{a}) \;-\; iS \;-\; r\left[1+\frac{h(S)}{q}a\right], \tag{6}$$

where the su subscript denotes surface parking.

The developer chooses S, q and a to maximize (6). In doing so, the developer behaves perfectly competitively with respect to land rent r, which is viewed as parametric, and takes the level of the rent function R as given.[11] Consistent with this competitive behavior, the developer also views $\overline{a}$, the average amount of off-street parking in the immediate neighborhood, as parametric. This average is the joint result of decisions by many developers, and any particular developer views it as uninfluenced by his decisions.[12] Ultimately, however, the a choices of developers end up determining $\overline{a}$, as seen below. The first-order conditions for the maximization problem are

$$S: \qquad (R - ra)\frac{h'}{q} \; - \; i \;\; = \;\; 0 \tag{7}$$

$$q: \qquad R^q\frac{h}{q} \; - \; (R - ra)\frac{h}{q^2} \;\; = \;\; 0 \tag{8}$$

$$a: \qquad R^a \; - \; r \;\; = \;\; 0. \tag{9}$$

These equations have the usual marginal-revenue-equals-marginal-cost interpretations. Eq. (9), which says that the increase in rent per dwelling from more parking area should equal the marginal land cost r, is especially transparent. In (7), a higher S leads to an increase in dwellings per unit of residential land (h'/q), but the resulting revenue R per dwelling is partly offset by the parking-land cost of ra per dwelling. The resulting marginal revenue is then equated to the cost i of capital. In (8), a higher q reduces dwellings per unit of residential land, with the net revenue loss captured by the second term, but rent per dwelling also rises (first term), with q being optimal when these effects just balance.

While land rent r is viewed as parametric by the developer, it is ultimately endogenous, necessitating an additional equilibrium condition beyond (7)–(9). This condition is the zero-profit requirement, which is written $\pi_{su} = 0$. Along with (7)–(9), this condition determines equilibrium values of S, q, a and r as functions of the parameters of the problem, the most important of which are x and $\overline{a}$. A final equilibrium requirement is that $\overline{a}$, the average amount of off-street parking in the neighborhood, is consistent with the decisions of developers. Since developers in a neighborhood all share a common value of x, the consistency requirement can be written as $\overline{a} = a$ (the average a is the value chosen by developers at the given x). This equilibrium condition shows that the choice of a generates an externality, with the developer's individual choice of a affecting parking-related congestion in the neighborhood and thus the profits of other developers. As a result, equilibrium in the model will be inefficient, as discussed in detail in section 4.

Of major interest is the spatial behavior of dwelling and building characteristics as well as the behavior of land rent. In principle, the derivatives of these variables with respect to distance x can be analyzed by total differentiation of (7)–(9) and the zero-profit condition. Although no general results are available when $\overline{a}$ enters the R function, some general conclusions can be stated when $\tau = 0$, implying the absence of parking-related congestion (and thus the absence of $\overline{a}$ from $R(\cdot)$). In fact, these conclusions follow directly from the results of Brueckner's (1983) analysis of a model of urban yard space. To understand this commonality, note that as depicted, surface parking is formally identical to the yard space associated with a dwelling, which consumes land in identical fashion. Therefore, when $\tau = 0$, the surface-parking model just involves a relabeling of Brueckner's yard-space variable. Appending su subscripts to the variables, Brueckner's Theorem 1 can be restated as

Proposition 1. *Suppose that $\tau = 0$ holds in the surface-parking model. Then, $\partial S_{su}/\partial x$, $\partial r_{su}/\partial x < 0$. If q and a are complements or independent goods, then $\partial q_{su} \partial x$, $\partial a_{su}/\partial x > 0$. If q and a are substitutes, then at least one of the inequalities $\partial q_{su} \partial x > 0$, $\partial a_{su}/\partial x > 0$ must hold.*

Complementarity or independence of q and a obtains when R's cross partial derivative satisfies $R^{qa} \geq 0$, while q and a are substitutes when $R^{qa} < 0$ (the sign of R^{qa} is opposite to that of the pure substitution term in the Slutsky equation). Brueckner (1983) also proves that the second-order conditions for the developer's problem are satisfied with well-behaved utility and production functions.

Proposition 1 shows that, with surface parking, structural density S and land rent r behave as in the standard urban model, decreasing with distance x to the CBD. If q and a are complements or independent goods, then q increases with x, as in the standard model, while parking area shows the same spatial pattern. In the substitutes case, however, either q or a

(but not both) could be decreasing in x. Although yard space and dwelling floor space would appear to be substitutes, the intuition is less clear for parking area and floor space. However, in the case of Cobb-Douglas preferences, considered in section 3, q and a are substitutes.

2.3. Structural parking

Instead of relying on surface parking, the developer could economize on land by building an above-ground parking structure connected to the residential building. The technology for providing structural parking mirrors that for floor space, with parking area per unit of land given by $f_{st}(\widetilde{S})$, where $\widetilde{S}$ is parking structural density (capital per unit of parking land) and $f'_{st} > 0$, $f''_{st} < 0$ (the st subscript denotes structural parking). Generalizing (4), profit with structural parking is given by

$$\left[\frac{h(S)}{q}R(q,a,x;\overline{a}) - iS - r\right]\ell \;-\; (r + i\widetilde{S})\widetilde{\ell}, \tag{10}$$

where the term $i\widetilde{S}\widetilde{\ell}$ gives capital cost for the parking structure. The constraint relating $\widetilde{\ell}$ and ℓ is now written

$$\frac{h(S)}{q}\ell a \;=\; f_{st}(\widetilde{S})\widetilde{\ell} \tag{11}$$

where the LHS is the total required parking area when area per dwelling is a, while the RHS is total area in the parking structure. Using (11) to substitute for $\widetilde{\ell}$ in (10), and then factoring out residential land ℓ, profit per unit of residential land is

$$\pi_{st} \;=\; \left[\frac{h(S)}{q}R(q,a,x;\overline{a}) - iS - r\right] \;-\; (r + i\widetilde{S})\frac{h(S)}{q}\frac{a}{f_{st}(\widetilde{S})}. \tag{12}$$

The developer's first-order condition are

$$S: \qquad \left[R - (r + i\widetilde{S})\frac{a}{f_{st}}\right]\frac{h'}{q} \;-\; i \;=\; 0 \tag{13}$$

$$q: \qquad R^q\frac{h}{q} \;-\; \left[R - (r + i\widetilde{S})\frac{a}{f_{st}}\right]\frac{h}{q^2} \;=\; 0 \tag{14}$$

$$a: \qquad R^a \;-\; (r + i\widetilde{S})\frac{1}{f_{st}} \;=\; 0 \tag{15}$$

$$\widetilde{S}: \qquad (r + i\widetilde{S})f'_{st} \;-\; if_{st} \;=\; 0. \tag{16}$$

To interpret these conditions, observe that, instead of equaling $R - ra$ as in (7)–(9), revenue per dwelling net of parking cost with structural parking is $R - (r + i\widetilde{S})a/f_{st}$. To understand the second term in this expression, note that $r + i\widetilde{S}$ is land plus capital cost per unit of parking land, while f_{st}/a is the number of dwellings served per unit of parking land (parking area per unit of land divided by parking area per dwelling). Therefore the second term above, which can be written $(r + i\widetilde{S})/(f_{st}/a)$, has units of $(cost/parking\ land)/(dwellings/parking\ land)$, or parking cost per dwelling. Conditions (13)–(15) are the same as (7)–(9) after substitution of this new parking-cost-per-dwelling expression in place of ra (in (15), replacement is by the expression's a derivative). The previous interpretations again apply. Note that condition (16), which pertains to the new choice variable $\widetilde{S}$, is the condition for minimizing parking cost per dwelling holding a constant, which requires minimizing $(r + i\widetilde{S})/f_{st}(\widetilde{S})$. As before, a zero-profit condition must hold, which is now written $\pi_{st} = 0$, and the additional equilibrium condition $\overline{a} = a$ must be imposed.

Although it can be shown that the second derivatives of the profit function (12) are negative, as required by the second-order conditions, the determinant requirements on Hessian matrix cannot verified, so that satisfaction of these conditions must be assumed. In addition, because of the greater complexity of the structural-parking model, no general results on the spatial behavior of the choice variables are available. Such conclusions can be derived, however, when specific functional forms are imposed, as seen in section 3.

2.4. Underground parking

With underground parking, capital is invested to construct a parking garage underneath the building rather than adjacent to it.[13] As a result, no additional land beyond the residential land ℓ is used. Underground parking area per unit of parking land is given by $f_{ug}(\widetilde{S})$, where $\widetilde{S}$ is again parking structural density (capital per unit of parking land), and since underground parking is more costly to build, the inequality $f_{ug}(\widetilde{S}) < f_{st}(\widetilde{S})$ holds along with $f'_{ug} > 0$, $f''_{ug} < 0$.

Profit per unit of residential land is now given by

$$\pi_{ug} \quad = \quad \frac{h(S)}{q} R(q, a, x; \overline{a}) - i(S + \widetilde{S}) - r, \tag{17}$$

where the parking land cost from the surface case disappears but the new underground capital cost $i\widetilde{S}$ appears. The relevant parking-availability constraint now relates $\widetilde{S}$ to other variables. The constraint is written

$$\frac{h(S)}{q}\ell a \quad = \quad f_{ug}(\widetilde{S})\ell, \tag{18}$$

where the RHS is total available parking. Canceling ℓ, and inverting the f_{ug} function to solve for $\widetilde{S}$ yields

$$\widetilde{S} \quad = \quad f_{ug}^{-1}(h(S)a/q) \quad \equiv \quad \widetilde{S}(S,q,a), \tag{19}$$

where $\widetilde{S}^S, \widetilde{S}^a > 0$ and $\widetilde{S}^q < 0$. Substituting (19) in (17), the first-order conditions are

$$S: \qquad R\frac{h'}{q} \;-\; i(1+\widetilde{S}^S) \quad = \quad 0 \tag{20}$$

$$q: \qquad R^q\frac{h}{q} \;-\; R\frac{h}{q^2} \;-\; i\widetilde{S}^q \quad = \quad 0 \tag{21}$$

$$a: \qquad R^a\frac{h}{q} \;-\; i\widetilde{S}^a \quad = \quad 0. \tag{22}$$

From (20), a higher S increases revenue per unit of residential land (first term), but in addition to the direct capital cost (i), the resulting increase in dwellings per unit of residential land raises the amount of required underground parking capital, leading to an extra cost of $i\widetilde{S}^S$. In (21), the first two terms give the direct effects of an increase in q, while the last term captures the reduction of parking capital allowed by a larger q, which lowers dwellings per unit of residential land. In (22), an increase in a raises revenue per dwelling (first term), but the increase in required total parking space necessitates more parking capital, leading to an extra cost of $i\widetilde{S}^a$. The zero-profit condition $\pi_{ug} = 0$ must hold, and the additional equilibrium condition $\overline{a} = a$ must again be imposed.

As in the case of structural parking, it can be shown that the second derivatives of the profit function (17) are negative, but satisfaction of the second-order conditions related to the Hessian determinant is not guaranteed and must be assumed. In addition, general results on the choice variables are again not available. However, the next section of the paper dispells this ambiguity and that for the structural-parking case by imposing specific functional forms.

2.5. Spatial behavior of land rent

Although no general results are available on the spatial behavior of the choice variables under the structural and underground parking regimes, it can be shown that land rent is decreasing with distance x to the CBD, as under the surface parking regime. Totally differentiating the zero-profit conditions for two regimes with respect to x yields

$$\frac{\partial \pi_j}{\partial R} R^x \;+\; \frac{\partial \pi_j}{\partial r}\frac{\partial r}{\partial x} \;=\; 0, \quad j = st,\ ug, \tag{23}$$

where the x-derivatives of the choice variables vanish due to the envelope theorem. Rearranging yields

$$\frac{\partial r}{\partial x} \;=\; -\frac{\partial \pi_j/\partial R}{\partial \pi_j/\partial r} R^x \;<\; 0, \quad j = st,\ ug, \tag{24}$$

where the inequality follows because $R^x < 0$, $\partial \pi_j/\partial R > 0$, and $\partial \pi_j/\partial r < 0$ (the last two inequalities can be seen from inspection of (12) and (17)). Summarizing yields

Proposition 2. *Land rent is a decreasing function of distance to the CBD under the structural and underground parking regimes.*

3. Solutions with Specific Functional Forms

3.1. The solutions

This section imposes specific functional forms for preferences and the floor space and parking production functions, which allows solutions for all the variables in the model to be computed.[14] Preferences and the production functions are assumed to take a Cobb-Douglas form, making the h and f functions power functions with exponents less than one. Formally, the functional form assumptions are

$$v(c,q,a) \;=\; c^{1-\alpha} q^{\alpha} a^{\gamma}, \quad h(S) \;=\; S^{\beta}, \quad f_{st}(\widetilde{S}) \;=\; \mu_{st}\widetilde{S}^{\theta}, \quad f_{ug}(\widetilde{S}) \;=\; \mu_{ug}\widetilde{S}^{\theta}, \tag{25}$$

where the parameters are all positive satisfy $\alpha,\ \beta,\ \theta < 1$ and $\mu_{ug} < \mu_{st}$. Note that the exponents of the f_{st} and f_{ug} functions are the same, while $\mu_{ug} < \mu_{st}$ implies that a given amount of capital per acre yields less parking area underground than in a structure, reflecting the higher cost of underground parking. From (2), the form of preferences implies

$$R(q, a, x; \overline{a}) \;=\; y \;-\; tx \;+\; \tau\overline{a} \;-\; u^{\frac{1}{1-\alpha}} q^{-\frac{\alpha}{1-\alpha}} a^{-\frac{\gamma}{1-\alpha}}. \tag{26}$$

Recall that, with Cobb-Douglas preferences, q and a are substitutes ($R^{qa} < 0$).

In the case without parking-related congestion, where $\tau = 0$, closed-form solutions for all the variables can be derived (the case where $\tau > 0$ is considered below). While the derivations are lengthy, the steps for the underground (surface) case are shown in the appendix (online appendix), and the structural derivations are available on request. The solutions for residential structural density, S, are as follows:

$$S_{su} \;=\; A_{su}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma}} \tag{27}$$

$$S_{st} \;=\; A_{st}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{28}$$

$$S_{ug} \;=\; A_{ug}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{29}$$

$$\frac{\partial S_{su}}{\partial x},\; \frac{\partial S_{st}}{\partial x},\; \frac{\partial S_{ug}}{\partial x} \;<\; 0. \tag{30}$$

The A's in (27)–(29) are complicated constants, and the conclusions in (30) follow because all the exponents are positive. Thus, residential structural density under all three parking regimes follows the pattern in the standard urban model, decreasing with distance x to the CBD (recall from Proposition 1 that (27) holds generally). Note that the exponents in the S_{st} and S_{ug} solutions are identical, a pattern that holds in the solutions for all the remaining variables. This pattern reflects the interaction of the Cobb-Douglas assumptions with the general structure of the model, and it presumably would not be repeated if the model could be solved under different functional form assumptions.

Letting B's denote constants, the solutions for dwelling size q are given by

$$q_{su} \;=\; B_{su}(y - tx)^{\frac{\beta\gamma-(1-\beta)(1-\alpha)}{\alpha(1-\beta)+\gamma}} \tag{31}$$

$$q_{st} = B_{st}(y-tx)^{-\frac{(1-\beta)(1-\alpha)+(\theta-\beta)\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{32}$$

$$q_{ug} = B_{ug}(y-tx)^{-\frac{(1-\beta)(1-\alpha)+(\theta-\beta)\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{33}$$

$$\frac{\partial q_{su}}{\partial x},\ \frac{\partial q_{st}}{\partial x},\ \frac{\partial q_{ug}}{\partial x}\ >(<)\ 0. \tag{34}$$

Thus, the spatial behavior of q is ambiguous under all three parking regimes. Note that, for surface parking, this result is consistent with Proposition 1 (see below for the behavior of a). In addition, observe that if γ, the utility exponent on a, is sufficiently small (so that parking is relatively unimportant), then the q exponent in (31) will be negative and q will increase with x, as in the standard model. Note also that the exponents in (32) and (33) are the same, implying that q behaves identically over space in the structural and underground-parking regimes. In addition, observe that if $\theta \geq \beta$, so that the common capital exponent in structural and underground parking-area production is at least as large as the residential floor-space production exponent, then q_{st} and q_{ug} both increase with x, as in the standard model.

Letting C's denote constants, the solutions for parking area a per dwelling are given by

$$a_{su} = C_{su}(y-tx)^{-\frac{\alpha\beta+(1-\alpha)}{\alpha(1-\beta)+\gamma}} \tag{35}$$

$$a_{st} = C_{st}(y-tx)^{-\frac{(1-\theta)-\alpha(1-\beta)}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{36}$$

$$a_{ug} = C_{ug}(y-tx)^{-\frac{(1-\theta)-\alpha(1-\beta)}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{37}$$

$$\frac{\partial a_{su}}{\partial x}\ >\ 0, \qquad \frac{\partial a_{st}}{\partial x},\ \frac{\partial a_{ug}}{\partial x}\ >(<)\ 0. \tag{38}$$

Since a_{su} is increasing in x by (35) while the behavior of q_{su} is ambiguous by (31), the results are consistent with Proposition 1's requirement that at least one of q and a increases with x in the surface-parking case. As in the case of q, the exponents in (36) and (37) are the same, indicating the same spatial behavior of a_{st} and a_{ug}. Note that if $\theta \leq \beta$, the exponents are both negative, so that a_{st} and a_{ug} both increase with x.

Letting D's denote constants, the solutions for parking structural density, $\widetilde{S}$, are

$$\widetilde{S}_{st} = D_{st}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{39}$$

$$\widetilde{S}_{ug} = D_{ug}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{40}$$

$$\frac{\partial \widetilde{S}_{st}}{\partial x}, \quad \frac{\partial \widetilde{S}_{ug}}{\partial x} < 0. \tag{41}$$

Comparing (39)–(40) and (28)–(29), parking structural density and residential structural density have common exponents across the structural and underground regimes, with all densities decreasing in x.

Finally, letting E's denote constants, the solutions for r are given by

$$r_{su} = E_{su}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma}} \tag{42}$$

$$r_{st} = E_{st}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{43}$$

$$r_{ug} = E_{ug}(y - tx)^{\frac{1+\gamma}{\alpha(1-\beta)+\gamma(1-\theta)}} \tag{44}$$

$$\frac{\partial r_{su}}{\partial x}, \quad \frac{\partial r_{st}}{\partial x}, \quad \frac{\partial r_{ug}}{\partial x} < 0. \tag{45}$$

As seen above, the inequalities in (45) hold generally.

Summing up, the solutions show that structural densities, both residential and parking, decline with distance x, matching the general behavior of land rents. However, the spatial behaviors of dwelling size and parking area per dwelling are more complex. In the case of surface parking, a increases with x while q could either increase or decrease, although the former case is ensured if parking is relatively unimportant, with γ small. The spatial patterns of q and a, which are the same under the structural and underground parking regimes, are both ambiguous. However, when $\theta > \beta$, q increases with x while a could increase or decrease. When $\theta < \beta$, a increases with x while q could increase or decrease. But in the intermediate case where $\theta = \beta$, both a and q increase with x. This case, where production exponents are equal for floor space and parking area, thus fully mimics the standard urban model, with both dimensions of dwelling "size" increasing with distance, while structural densities and land rent fall with x.

3.2. The relative locations of the different parking regimes

Different parking regimes coexist in real-world cities, with surface parking present in some areas and structural or underground parking present in others. The model makes sharp predictions about the relative locations of the three regimes, making use of the land-rent solutions in (42)–(44). As usual, a particular parking regime will be present in a given area if developers using that regime bid more for land than developers using the other regimes. Therefore, the relative locations of the parking regimes can be deduced by considering the heights and the slopes of the the land-rent curves generated by the r solutions.

Since r_{st} and r_{ug} have common $y - tx$ exponents, the two corresponding land rent curves differ only in the multiplicative constants E_{st} and E_{ug}, which are highly complex expressions that are both naturally increasing in the respective productivity parameters μ_{st} and μ_{ug} (see (25)). Therefore, one of the two regimes will dominate the other throughout the city, generating higher land rent. To assess the direction of the dominance, note that the advantage of underground parking relative to structural parking is that it avoids the need for additional land beyond the residential land area. But underground parking has two disadvantages, the first being lower capital productivity and thus higher capital costs, as reflected in the shift factor $\mu_{ug} < \mu_{st}$ in the production function. The second and subtler disadvantage is the requirement that the amount of parking land equal the amount of residential land, a constraint that is not present in the structural case, where residential land and parking land can be adjusted independently.

Because of these offsetting factors, the relationship between the complex E_{st} and E_{ug} constants is ambiguous, even when $\mu_{ug} = \mu_{st}$ holds, eliminating the capital cost disadvantage (in this case, the second disadvantage is still present).[15] But since E_{ug} is increasing in μ_{ug}, two cases are possible: (i) $E_{st} > E_{ug}$ holds when $\mu_{ug} = \mu_{st}$, implying that $E_{st} > E_{ug}$ also holds for any $\mu_{ug} < \mu_{st}$; (ii) $E_{ug} > E_{st}$ holds when $\mu_{ug} = \mu_{st}$, implying that there exists some μ^*_{ug} such that $E_{ug} > (<)\ E_{st}$ holds when $\mu_{ug} > (<)\ \mu^*_{ug}$. These conclusions yield

Proposition 3. *When parking related congestion is absent, one of the non-surface parking regimes dominates the other (generating higher land rent) everywhere in the city. Structural parking either dominates underground parking for all possible values of the underground production-function shift factor μ_{ug}, or there exists a critical μ_{ug} value, denoted μ_{ug}^*, such that underground parking dominates (is dominated by) structural parking as $\mu_{ug} > (<)\ \mu_{ug}^*$.*

Thus, the model implies that only one of the non-surface parking regimes will be observed in the city. The next question is where this regime will be located relative to the area with surface parking. The answer is immediate from inspection of the land-rent solutions. Since its larger denominator means that the r_{su} exponent in (42) is smaller than the common exponents in the r_{st} and r_{ug} solutions, it follows that the r_{su} curve will be flatter than either of the other curves at a point of intersection. Therefore, the r_{su} curve will be higher (lower) than the other regime's curve at locations outside (inside) the x value where they intersect, implying

Proposition 4. *If parking-related congestion is absent and multiple parking regimes are present in the city, then the surface parking will be observed in the city's suburban area, with one of the other parking regimes (structural or underground) observed in the central part of the city.*

This conclusion makes intuitive sense given that both non-surface parking regimes conserve on land, making their use natural in the central part of the city where land is expensive.

Note, however, that the proposition is contingent on the presence of multiple parking regimes in the city. It is possible in principle for a city to have a single regime, an outcome that depends in part on the level of agricultural land rent r_A, which determines the distance to the edge of the city. For example, if r_A is greater than the rent level at the intersection of relevant non-surface and surface rent curves, then the city's edge will lie inside the intersection point and only the non-surface regime will be observed in the city (recall that urban rent is independent of r_A when the city is open).[16]

Tables 1, 2 and 3 show model solutions for $r,\ a,\ q,\ S$, and $\widetilde{S}$ under the following parameter values: $u = 100,\ y = 100,\ t = 0.3, \alpha = 0.4,\ \delta = 0.1,\ \beta = 0.8,\ \theta = 0.8,\ \mu_{st} = 0.08,\ \mu_{ug} = 0.06$, and $i = 0.05$. Note that the exponents of the floor space and parking production functions are assumed to be the same and equal to 0.8. Applying these parameter values to the solutions from section 3.1, Table 1 shows land rents under the three regimes for both $\tau = 0$ and for a positive τ value of 0.2, a case discussed in section 4 below. The columns for $\tau = 0$ show that the underground regime generates higher land rent than the structural regime, dominating it at all distances in a reflection of Proposition 3. Following Proposition 4, the underground (surface) parking regime generates the highest land rent inside (outside) an x value of 35, with the two regimes thus being present in the shaded distance ranges in the $\tau = 0$ columns of Table 1. Figure 1 illustrates these outcomes.[17]

Tables 2 and 3 show solutions for the remaining variables in the three regimes, with the solutions with $\tau = 0.2$ discussed in the next section. The shading in the columns again shows the distance ranges over which the solutions are relevant, with shading in a particular cell of the table indicating that land rent for the given parking regime exceeds rent for the other two regimes at that location. Thus, a shaded cell contains the value of a particular variable (q, for example) that is observed at the location. This property is further illustrated in Figures 2 and 3, where the dotted curve shows the observed values of a and q, which track the curves showing the separate solutions for the underground and surface regimes, with tracking occurring over the distance range where land rent for that regime is maximal.

Under the given parameter values, a (first part of Table 2) and dwelling size q (second part of Table 3) both increase with distance under all regimes. Note that parking area per dwelling exhibits a discontinous upward jump in the transition from underground to surface parking, while dwelling size exhibits a discontinuous downward jump (these jumps are also seen in Figures 2 and 3). As is true in general, both residential and parking structural density decrease with distance under the given parameter values, and like parking area, residential structural density exhibits a discontinous upward jump in the transition between the underground and surface regimes. The patterns seen in Tables 2 and 3 are highly robust to changes in the parameter values.

It should be noted that, given the open-city assumption, the distance $\overline{x}$ to the edge of the city can be fixed arbitrarily by specifying the value of agricultural land rent r_a. The value of $\overline{x}$ is therefore not of interest, although it is assumed to lie beyond the distance range shown in the tables.

3.3. Carless households

As noted above, the model assumes that every household has a car, with no alternate transportation modes available. These assumptions are not accurate for some large cities, where carless households who rely on public transit appear to be common in central neighborhoods.[18] This section of the paper explores the implications of adding such carless households to the model.

With no car owned, a household would not value parking space, making the a exponent γ equal to zero. The absence of a car would also change the commuting-cost parameter t. While cars are costly to operate, they are usually faster than public transit, and if this effect dominates, t would be higher for a carless household relying on public transit than for a car-owning household. In the open-city context, carless and car-owning households would be constrained to reach the same utility level.

To find the location of carless households in the city, the slope of their associated land-rent curve would be compared to the land-rent slopes in (42)-(44). The carless land-rent curve is found by setting $\gamma = 0$ in any of these solutions (yielding $1/\beta(1-\gamma)$) while raising t (the multiplicative constant also changes). The common r_{st} and r_{ug} exponents are easily seen to be decreasing in γ, which makes the r exponent greater for carless households. This fact, along with the high carless value of t, means that the carless land-rent curve is steeper than the r_{st} and r_{ug} curves. As a result, if the city contains any carless households, they will occupy the land closest to the center, surrounded by a zone of car owners.

Even though this model does not have an explicit car-ownership choice, such a choice is implicit. For example, if the carless bid-rent curve lies everywhere below the curves with car-ownership, then the city has no carless area. But this rent relationship in effect says that carlessness is an inferior choice, not being able to generate enough land rent at a fixed utility level to outbid car owners for land anywhere in the city. If, on the other hand, the carless rent curve dominates in the city center, then carlessness is a superior choice in that area, generating higher land rent than car ownership. Therefore, while consumers are indifferent between carlessness and car ownership by imposition of a common utility level, the relative attractness of these options at different locations in the city is registered in the land market.

4. Solutions With Parking-Related Congestion

Consider now the case where $\tau > 0$, so that parking-related congestion is present. Since $\overline{a}$ is taken as parametric by developers, the previous a formulas still apply but with $y - tx$ replaced by $y - tx + \tau\overline{a}$. Imposing the equilibrium requirement that $\overline{a} = a$, these formulas then take the form

$$a \quad = \quad C(y - tx + \tau a)^{\phi}, \tag{46}$$

where C and the exponent ϕ vary across regimes. Rather than giving a closed-form solution, (46) instead gives an implicit solution for a. However, the solution remains unique, a consequence of the fact that $\phi < 1$ holds in each regime, implying that the RHS of (46) is a concave function of a. With the RHS greater than the LHS when $a = 0$ and the LHS linear in a, concavity implies that a single a value exists where the two expressions are equal.

Recall that when $\phi < 0$, $\partial a/\partial x > 0$ holds when $\tau = 0$. This same conclusion can be demonstrated when $\tau > 0$, as follows. Rearranging (46) yields $a^{\frac{1}{\phi}}C^{-\frac{1}{\phi}} - \tau a = y - tx$, and differentiating with respect to x then gives

$$\left[\frac{a^{\frac{1}{\phi}-1}k}{\phi} - \tau\right]\frac{\partial a}{\partial x} \quad = \quad -t \quad < \quad 0, \tag{47}$$

where $k = C^{-\frac{1}{\phi}} > 0$. With $\phi < 0$, the bracketed term in (47) is negative, implying that $\partial a/\partial x > 0$ must hold for the LHS to be negative. By contrast, the sign of this derivative is ambiguous when $\phi > 0$.

In addition, it can be shown that, despite $\partial a/\partial x > 0$, $y - tx + \tau a$ is decreasing in x when $\phi < 0$. Solving for $\partial a/\partial x$ using (47), the x derivative of $y - tx + \tau a$ is

$$\begin{aligned} -t \;+\; \tau\frac{\partial a}{\partial x} \quad &= \quad -t \;-\; \tau\frac{ta}{(a^{\frac{1}{\phi}}k/\phi) - \tau a} \\ &= \quad -\frac{ta^{\frac{1}{\phi}}k/\phi}{(a^{\frac{1}{\phi}}k/\phi) - \tau a} \quad < \quad 0. \end{aligned} \tag{48}$$

Recall that the previous solutions for S, q, $\widetilde{S}$, and r all involved $y-tx$ raised to an exponent. When $\tau > 0$, the solutions for these variables are given by the same expressions with $y - tx$ replaced by $y - tx + \tau a$, with a given by the solution to (46). Since $y - tx$ is decreasing in x, the sign of the x derivative of the previous solutions depended on the sign of the exponent, being negative (positive) when the exponent was positive (negative). Since $y - tx + \tau a$ is also decreasing in x when ϕ is negative given (48), all the previous conclusions about the signs of the x derivatives of S, q, $\widetilde{S}$, and r are unaffected, as long as ϕ for the particular regime is negative. Summarizing yields

Proposition 5. *If parking space per dwelling under a particular regime is increasing in x in the absence of parking-related congestion, then the same conclusion holds in the presence of such congestion. In addition, under this condition, the spatial behavior of S, q, $\widetilde{S}$ and r is the same in the presence of parking-related congestion as in its absence, with the distance derivatives having the same signs.*

To state the implications of the proposition most simply, suppose that parameter values are such that all the ambiguities in the solutions with $\tau = 0$ are absent: $\theta = \beta$ and γ is small. Then $\partial a/\partial x$ is positive under each regime when $\tau = 0$, while q increases with x and r along with the structural densities decrease with x. Then, Proposition 5 says that *these vary same patterns emerge when $\tau > 0$*. Appropriate qualifications to this statement would be made when ambiguities are present in the solutions with $\tau = 0$.[19]

While the previous discussion has focused on slopes, the levels of the solutions also change when $\tau > 0$. Since $y - tx + \tau a$ is then larger than $y - tx$, it follows that all solutions with a positive exponent (which are decreasing in x) are larger at a given x when $\tau > 0$ than when $\tau = 0$, while all solutions with a negative exponent (which are increasing in x) are smaller when $\tau = 0$. For example, the S and r solutions are all larger when $\tau > 0$ than when $\tau = 0$.

Tables 1, 2 and 3 show the solutions when τ takes the positive value of 0.2. Under the land rent patterns for $\tau = 0.2$, shown in Table 1, underground parking continues to dominate the structural regime at all locations. In addition, the underground parking area extends farther from the CBD than when $\tau = 0$, with the shaded areas in the table's $\tau = 0.2$ columns showing that surface parking now first appears at $x = 40$ instead of $x = 35$. While the direction of this change could not be predicted, the changes in the other variables, as seen in the second of the two columns for each regime in Tables 2 and 3, follow Proposition 5 and the subsequent discussion. In particular, given that a is increasing with distance under both the underground and surface regimes when $\tau = 0$, this pattern is preserved when $\tau = 0.2$. In addition, as predicted by Proposition 5, the spatial patterns of q (increasing in x), S, and $\widetilde{S}$ (both decreasing in x) are preserved as well, as is the pattern of land rent (decreasing in x) from Table 1. Moreover, as predicted by the discussion following Proposition 5, the levels of the variables that are increasing in x (a and q) are lower at each location when $\tau = 0.2$, while the levels of variables that are decreasing in x (S, $\widetilde{S}$) are higher.

5. Inefficiency of Equilibria with Parking-Related Congestion

5.1. General considerations

Because developers view themselves as unable to affect the average level of off-street parking in their neighborhoods, they ignore the collective beneficial effects of their a choices on parking-related congestion. This observation suggests that the equilibrium level of off-street parking when $\tau > 0$ will be too low.

To demonstrate this conclusion analytically, the perspective of a social planner would be taken. Since consumer utility is fixed by the open-city assumption, the planner's goal would be to maximize total land rent in the city, which accrues to absentee landowners as income. This maximization problem, in turn, requires maximizing land rent at each location through proper choice of the variables S, q, a, and $\widetilde{S}$ (under the non-surface regimes) at that location, a goal that serves to maximize total rent on both the intensive and extensive margins. In other words, while rent maximization raises landowner incomes above the equilibrium level at previously urbanized locations, higher rents will also lead to a spatial expansion of the city, raising rental incomes at previously undeveloped locations. With land rent again determined by a zero-profit condition for developers, the planner can achieve the highest r by maximizing the developer's profit *while taking a's effect on parking-related congestion into account.*

Under the surface parking regime, for example, the planner would maximize

$$\pi_{su} \;=\; \frac{h(S)}{q}R(q,a,x,a) \;-\; iS \;-\; r\left[1+\frac{h(S)}{q}a\right], \tag{49}$$

which is identical to the previous profit expression (6) except that the $\overline{a}$ argument of R is replaced by a. Instead of (9), the first-order condition for choice of a would be

$$R^a \;+\; R^{\overline{a}} \;-\; r \;=\; 0. \tag{50}$$

With $R^{\overline{a}} = \tau$ and $R^{aa} < 0$, (50) would lead to a higher a value than solution where the $\overline{a}$ effect is ignored, other things equal. But magnitude of R^a depends on the value of q, which in turn is linked to the value of S. Indeed, since all the choice variables are interdependent, the comparison between the planning solution and the equilibrium cannot be made by simply focusing on the first-order condition for a. This difficulty suggests that the entire planning solution should be computed using the previous functional-form assumptions and then compared to the equilibrium. But this path is infeasible given that a appears in the linear part of R while also being raised to an exponent, which prevents derivation of analytical solutions for any of the variables. As a result, the comparison between the planning solution and the equilibrium is carried out numerically, with the results shown in Table 4.

5.2. Numerical results

In the upper panel of Table 4, the first column again shows equilibrium land rent when $\tau = 0.2$ (now extended to three decimal places for comparison purposes). The numbers represent the upper envelope of the rents in Table 1, with the shaded values showing rent for the underground regime and the unshaded values corresponding the surface regime. The second column shows the optimal land rent, where the term "optimal" refers to the planning solution. Since the parking congestion effect is taken into account, optimal land rent is slightly higher than the equilibrium rent at each location. The shaded range remains unchanged, indicating that the switch from the underground regime to the surface regime again occurs at $x = 40$ (structural parking is again dominated).[20] The third column of the upper panel is discussed below.

The second half of Table 4's upper panel shows parking area per dwelling, with the first column showing equilibrium values and the second showing optimal values. Comparing the columns, the conjecture that the optimal a is higher than the equilibrium value is borne out. The optimal parking area per dwelling is higher at each location under both the underground and surface regimes, reflecting the planner's recognition of the congestion-reducing effect of a higher a. Note that the shaded range represents a values for the underground regime, while the unshaded range represents a values for the surface regime.

The lower panel of Table 4 shows the differences between the optimal and equilibrium values of dwelling size and residential structural density, which are impossible to predict a priori. The first two columns of the panel show that the optimal dwelling size is smaller than the equilibrium size at each location. Thus, the shift to larger parking areas in moving to the planning solution is accompanied by a decline in dwelling sizes, so that the two dwelling attributes change in opposite directions. In addition, while Table 4 shows that the optimal residential structural density is lower than the equilibrium density at the shaded locations, where underground parking is provided, the reverse is true under surface parking, with residential structural density increasing in moving from the equilibrium to the planning solution.[21] Although, with the decrease in dwelling sizes, a decline in S under both regimes might have been expected given the need for less floor space, the solutions show that this conjecture is incorrect.[22] But the change in parking structural density for the underground regime follows this intuition: with a rising, $\widetilde{S}$ increases in moving from the equilibrium to the planning solution, a result that is not shown in Table 4. These conclusions are summarized as follows:

Patterns in the numerical solution. *In the numerical solution, the optimal parking area per dwelling is higher than the equilibrium area throughout the city, while the optimal dwelling size is lower than the equilibrium size. The optimal residential structural density is higher (lower) than the equilibrium density under the surface (underground) regime, while the optimal underground parking structural density is higher than the equilibrium density.*

6. A Minimum Parking Requirement

While the planning solution requires raising a above the equilibrium value at each location, cities often take a cruder approach by imposing a minimum parking requirement (MPR). An MPR consists of a spatially invariant value of a, denoted $\widehat{a}$, along with the constraint $a \geq \widehat{a}$. It is useful to investigate the effect of an MPR, using the numerical approach while again setting $\tau = 0.2$.[23]

When a mild MPR with $\widehat{a} = 3.8$ is imposed, the MPR is binding out to $x = 15$, and while a rises over this range, there are no effects outside it. A more-stringent MPR with $\widehat{a} = 7$ has broader effects, as seen in the MPR columns of Table 4. As can be seen in the second part of Table 4's upper panel, the equilibrium violates the MPR over almost the entire range of distance values, with the MPR nonbinding only at $x = 60$, where surface parking is provided. Therefore, when the MPR is imposed, a rises at all distances closer to the CBD, with the increase being substantial close to the CBD but smaller farther out.

Since a marginal increase in a raises land rent starting at the equilibrium, the initial movement toward the MPR value raises land rent at all locations. If $\widehat{a}$ is below the optimal a, then land rent continues to rise until $\widehat{a}$ is reached. But if the movement toward $\widehat{a}$ causes a to rise above the optimal a value, then land rent starts to fall from the maximum achieved at $\widehat{a}$. In this case, whether rent under the MPR is higher or lower than the equilibrium value depends on how far $\widehat{a}$ lies above the optimal a value. If the difference is small, land rent under the MPR will be higher, and otherwise it will be lower.

These conclusions can be seen in Table 4. Since $\widehat{a}$ is far above the optimal values for underground parking, the MPR reduces land rent everwhere under that regime relative to the equilibrium. But since $\widehat{a}$ is smaller than the optimal a for surface parking at $x = 55$, land rent rises at this location, staying the same at $x = 60$ since the MPR is not binding there. Moreover, since $\widehat{a}$ is only slightly larger than the optimal a values at $x = 45$ and $x = 50$, the MPR raises land rent at these locations as well. But at all distances closer to the CBD, $\widehat{a}$ is sufficiently far above the optimal a that the MPR reduces land rent under the surface regime at these locations, a result that can be seen at $x = 40$ (where the surface regime prevails before and after the imposition of the MPR). Finally, since the MPR imposes a bigger constraint for the underground regime, land rent falls by more for this regime than for the surface regime in the vicinty of boundary between the regimes, leading to an inward shift in the boundary from $x = 40$ to $x = 30$, with the surface parking area expanding (as seen in the change in the shaded areas).

The lower panel of Table 4 shows that the MPR reduces dwelling sizes relative to the equilibrium under both regimes, an effect that follows the dwelling-size reduction in moving to the planning solution. Apparently, any increase in a prompts a reduction in q, regardless of whether or not the higher a overshoots the optimal value. The effects of the MPR on residential structural density, however, do not always follow the changes that occur in moving to the social optimum. The pattern is the same for underground parking, with the MPR greatly reducing S under that regime over the entire range of distances in the city. But the MPR also leads to a lower S under the surface regime (rather than the higher one reached in moving to the planning solution) out to a distance of $x = 40$. This effect can be seen at $x = 40$, where surface parking exists both in the equilibrium and under the MPR. S rises, however, at $x = 45$, 50, and 55, as in the shift to the planning solution (staying constant at $x = 60$). Apparently, an increase in a only raises S under the surface regime when the increase leaves a close to the optimal value. When the increase far overshoots the optimal value, as happens when $x \leq 40$, the higher a leads to a reduction rather than an increase in residential structural density. Figures 4 and 5 illustrate the results from Table 4 for two of the variables: parking area per dwelling and dwelling size.

While the MPR is binding almost everywhere in this numerical example, Cutter and Franco (2012) provide empirical results designed to test whether MPRs in the Los Angeles area represent binding constraints on developers, using two approaches relying on a commercial-property database. The first compares a building's available parking area to the area mandated by the MPR, and the evidence showing that the two areas tend to be close suggests that the MPR constraint is typically binding. The second approach estimates the value of additional parking using an hedonic price model, and then compares this value to the cost of providing additional parking. The results show that value is less than cost, suggesting that MPRs lead developers to provide more parking than they would voluntarily, making the constraints binding.

7. Conclusion

This paper has analyzed provision of parking in a monocentric city, focusing in a realistic fashion on three different parking regimes reflecting different technologies for producing parking space. The analysis derives the equilibrium spatial behavior of parking area and floor space per dwelling, residential structural density, and parking strutural density (when relevant) under the three regimes, while deducing (through an analysis of land rent) the relative locations of the regimes in the city. Since failure by developers to account for the effect of their decisions on neighborhood parking congestion makes the equilibrium inefficient, a social planner would seek to raise parking area per dwelling throughout the city, with attendant effects on the other decision variables. A minimum parking requirement offers a crude way of addressing this inefficiency, and its effects are derived.

The novelty of the paper is its consideration of parking as an element of urban form, achieved by adding parking area per dwelling and any associated capital investment to the list of land-use variables chosen by housing developers. This innovation builds on Brueckner's (1983) incorporation of yard space in the standard urban model. However, because the paper has a purely residential focus, embodying the usual and unrealistic restriction of the city's business area to a point in space, it abstracts from a host of questions related to work-related parking. These questions include the optimal provision of employee parking space, which in turn has broad implications for transport mode choice and road congestion. A related question concerns decentralized provision of employee parking, particularly the effects of different charging schemes (worker-paid parking vs. provision of free space by employers). By extending the model to include land-use in the CBD, and by adding mode choice as well as road congestion, such questions could be addressed. See Franco (2015) for an initial effort in this direction.

Appendix

A.1. Derivation of underground-parking solutions when $\tau = 0$

Under the assumed functional forms, the condition (18) and the $\widetilde{S}$ function are written

$$\frac{aS^{\beta}}{q} \quad = \quad \mu\widetilde{S}^{\theta}; \quad \widetilde{S} \quad = \quad \delta\left(\frac{a}{q}\right)^{\frac{1}{\theta}} S^{\frac{\beta}{\theta}}, \tag{$a1$}$$

where $\delta = \mu^{-1/\theta}$. Profit in (17) then becomes

$$R\frac{S^{\beta}}{q} \; - \; i\left[S \; + \; \delta\left(\frac{a}{q}\right)^{\frac{1}{\theta}} S^{\frac{\beta}{\theta}}\right] \; - \; r. \tag{$a2$}$$

From (20), the first-order condition for S is

$$R\frac{\beta S^{\beta-1}}{q} \; - \; i\left[1 \; + \; \frac{\delta\beta}{\theta}\left(\frac{a}{q}\right)^{\frac{1}{\theta}} S^{\frac{\beta}{\theta}-1}\right] \quad = \quad 0, \tag{$a3$}$$

and the condition (22) for a, after some rearrangement, can be written as

$$i\delta\left(\frac{a}{q}\right)^{\frac{1}{\theta}} S^{\frac{\beta}{\theta}-1} \quad = \quad \frac{\theta\gamma}{1-\alpha}\nu q^{-\frac{\alpha}{1-\alpha}} a^{-\frac{\gamma}{1-\alpha}} \frac{S^{\beta-1}}{q}, \tag{$a4$}$$

where $\nu = u^{\frac{1}{1-\alpha}}$. Substituting ($a4$) for the last term in ($a3$), gathering terms, and substituting for R using (24) while imposing $\tau = 0$, ($a3$) reduces to

$$\frac{\beta S^{\beta-1}}{q}\left[y - tx - \frac{1+\gamma-\alpha}{1-\alpha}\nu q^{-\frac{\alpha}{1-\alpha}} a^{-\frac{\gamma}{1-\alpha}}\right] \quad = \quad i. \tag{$a5$}$$

From (21), the first-order condition for q is, after rearrangement

$$-R\frac{S^{\beta}}{q} \; + \; \frac{i\delta}{\theta}\left(\frac{a}{q}\right)^{\frac{1}{\theta}} S^{\frac{\beta}{\theta}} \; + \; \frac{\alpha}{1-\alpha}\nu q^{-\frac{\alpha}{1-\alpha}} a^{-\frac{\gamma}{1-\alpha}} \frac{S^{\beta}}{q} \quad = \quad 0. \tag{$a6$}$$

Combining ($a4$) and ($a6$), eliminating R using (24), and simplifying yields

$$\nu q^{-\frac{\alpha}{1-\alpha}} a^{-\frac{\gamma}{1-\alpha}} \quad = \quad \frac{1-\alpha}{1+\gamma}(y - tx). \tag{$a7$}$$

Substituting ($a7$) in ($a5$) and substituting for R, $a(5)$ becomes

$$\frac{\beta S^{\beta-1}}{q}\frac{\alpha}{1+\gamma}(y - tx) \quad = \quad i, \tag{$a8$}$$

and solving for S yields

$$S \quad = \quad \left(\frac{\alpha\beta}{iq(1+\gamma)}\right)^{\frac{1}{1-\beta}} (y - tx)^{\frac{1}{1-\beta}}. \tag{$a9$}$$

Substituting ($a9$), ($a4$) can be written (after substantial simplification) as

$$a \quad = \quad F(y - tx)^{\frac{\theta-\beta}{1-\beta}} q^{\frac{1-\theta}{1-\beta}}, \tag{$a10$}$$

where F is a constant.

Substituting ($a10$) in ($a7$) and solving for q yields the solution (33). Substituting (31) into ($a10$) then yields the a solution (37), and substituting (31) into ($a9$) yields the S solution (29). Substituting the S, q and a solutions into ($a1$) yields the S_{ug} solution (40).

Land rent is given by

$$r \quad = \quad R\frac{S^\beta}{q} \; - \; iS \; - \; i\delta\left(\frac{a}{q}\right)^{\frac{1}{\theta}} S^{\frac{\beta}{\theta}} \tag{$a11$}$$

Combining (24) and ($a7$) yields

$$R \quad = \quad \frac{\alpha+\gamma}{1+\gamma}(y - tx), \tag{$a12$}$$

so that ($a8$) can be rewritten as

$$\frac{\beta S^\beta}{q}\frac{\alpha}{\alpha+\gamma}R \quad = \quad iS. \tag{$a13$}$$

Using $(a12)$ and $(a13)$, the first two terms in $(a11)$ equal

$$\left(1-\frac{\alpha\beta}{\alpha+\gamma}\right)R\frac{S^{\beta}}{q} \quad = \quad \frac{\alpha(1-\beta)+\gamma}{1+\gamma}\frac{S^{\beta}}{q}(y-tx), \tag{a14}$$

using $(a12)$. Then, using $(a4)$ and $(a7)$, the last term in $(a11)$ is

$$\frac{\theta\gamma}{1-\alpha}\nu q^{-\frac{\alpha}{1-\alpha}}a^{-\frac{\gamma}{1-\alpha}}\frac{S^{\beta}}{q} \quad = \quad \frac{\theta\gamma}{1-\alpha}\frac{1-\alpha}{1+\gamma}(y-tx)\frac{S^{\beta}}{q}. \tag{a15}$$

Substituting $(a14)$ and $(a15)$ into $(a11)$ then yields

$$r \quad = \quad \frac{\alpha(1-\beta)+\gamma(1-\theta)}{1+\gamma}\frac{S^{\beta}}{q}(y-tx) \tag{a16}$$

Finally, substituting the S and q solutions into $(a16)$ yields the r solution (44).

Table 1: Land rent when τ = 0 and τ = 0.2

	Land Rent					
x	**Underground**		**Surface**		**Structural**	
	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2
0	5.43	5.87	3.23	3.34	4.39	4.75
5	4.60	4.98	2.94	3.05	3.72	4.03
10	3.88	4.22	2.68	2.79	3.14	3.41
15	3.27	3.57	2.44	2.54	2.65	2.88
20	2.75	3.00	2.21	2.32	2.22	2.43
25	2.30	2.53	2.00	2.11	1.86	2.04
`30	1.92	2.12	1.81	1.92	1.56	1.71
35	1.60	1.77	1.64	1.75	1.30	1.43
40	1.33	1.47	1.48	1.59	1.08	1.19
45	1.10	1.23	1.33	1.44	0.89	0.99
50	0.91	1.01	1.20	1.30	0.73	0.82
55	0.75	0.84	1.07	1.18	0.60	0.68
60	0.61	0.69	0.96	1.07	0.49	0.56

Note: This table shows the solutions, when τ=0 and τ =0.2, for land rent in the three parking regimes, with shading showing the distance ranges over which the solutions are relevant (part of the equilibrium land-rent profile).

Table 2: Parking area and parking structural density when τ = 0 and τ = 0.2

	Parking Area per Dwelling						*Parking Structural Density*			
x	**Underground**		**Surface**		**Structural**		**Underground**		**Structural**	
	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2
0	3.57	3.54	2.82	2.74	3.60	3.57	86.88	93.90	351.17	379.78
5	3.64	3.61	3.04	2.95	3.67	3.64	73.57	79.72	297.38	322.45
10	3.71	3.67	3.29	3.18	3.74	3.70	62.15	67.52	251.19	273.12
15	3.78	3.74	3.56	3.44	3.81	3.77	52.36	57.05	211.62	230.75
20	3.85	3.81	3.87	3.71	3.88	3.84	43.99	48.08	177.80	194.46
25	3.92	3.88	4.20	4.01	3.96	3.92	36.85	40.41	148.96	163.44
30	4.00	3.96	4.56	4.35	4.04	3.99	30.79	33.87	124.44	136.99
35	4.08	4.04	4.97	4.71	4.12	4.07	25.64	28.31	103.65	114.51
40	4.17	4.12	5.41	5.10	4.20	4.15	21.29	23.59	86.07	95.43
45	4.25	4.20	5.91	5.54	4.29	4.24	17.62	19.60	71.24	79.30
50	4.34	4.29	6.46	6.02	4.38	4.33	14.54	16.24	58.77	65.69
55	4.44	4.38	7.08	6.54	4.47	4.42	11.95	13.41	48.31	54.24
60	4.53	4.48	7.77	7.11	4.57	4.51	9.79	11.03	39.57	44.64

Note: This table shows the solutions, when τ=0 and τ =0.2, for parking area per dwelling and parking structural density in the three parking regimes, with shading showing the distance ranges over which the solutions are relevant (part of the equilibrium profiles).

Table 3: Residential structural density and dwelling size when τ = 0 and τ = 0.2

	Residential Structural Density						*Dwelling Size*					
x	**Underground**		**Surface**		**Structural**		**Underground**		**Surface**		**Structural**	
	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2	τ=0	τ=0.2
0	347.53	375.60	258.16	266.93	351.17	379.78	180.55	179.02	191.61	191.37	180.17	178.64
5	294.30	318.90	235.38	244.14	297.38	322.45	183.85	182.25	192.25	192.00	183.47	181.86
10	248.59	270.10	214.31	223.06	251.19	273.12	187.27	185.58	192.91	192.63	186.88	185.18
15	209.42	228.20	194.84	203.57	211.62	230.75	190.80	189.02	193.58	193.27	190.41	188.62
20	175.95	192.30	176.88	185.59	177.80	194.46	194.46	192.59	194.26	193.92	194.06	192.17
25	147.42	161.63	160.32	169.01	148.96	163.44	198.25	196.27	194.95	194.58	197.84	195.85
30	123.15	135.47	145.07	153.75	124.44	136.00	202.18	200.09	195.66	195.25	201.76	199.66
35	102.58	113.23	131.06	139.72	103.65	114.51	206.25	204.04	196.39	195.93	205.82	203.60
40	85.17	94.37	118.20	126.83	86.07	95.44	210.48	208.14	197.13	196.62	210.04	207.69
45	70.50	78.41	106.41	115.02	71.24	79.30	214.87	212.39	197.88	197.32	214.42	211.92
50	58.16	64.95	95.62	104.20	58.77	65.69	219.43	216.80	198.65	198.03	218.97	216.32
55	47.81	53.63	85.76	94.31	48.31	54.24	224.16	221.37	199.44	198.75	223.70	220.89
60	39.17	44.14	76.77	85.28	39.58	44.64	229.09	226.13	200.25	199.48	228.62	225.62

Note: This table shows the solutions, when τ=0 and τ =0.2, for residential structural density and dwelling size in the three parking regimes, with shading showing the distance ranges over which the solutions are relevant (part of the equilibrium profiles).

Table 4: Comparison of equilibrium, optimum and MPR when τ = 0.2

	Land Rent			*Parking Area per Dwelling*		
x	**Equilibrium**	**Optimal**	**MPR: a ≥ 7**	**Equilibrium**	**Optimal**	**MPR: a ≥ 7**
0	5.869	5.882	4.440	3.54	3.76	7
5	4.983	4.995	3.840	3.61	3.84	7
10	4.220	4.231	3.312	3.67	3.91	7
15	3.566	3.576	2.849	3.74	4.00	7
20	3.005	3.014	2.443	3.81	4.08	7
25	2.525	2.534	2.089	3.88	4.17	7
30	2.117	2.124	1.856	3.96	4.26	7
35	1.769	1.776	1.714	4.04	4.36	7
40	1.585	1.592	1.575	5.10	5.75	7
45	1.438	1.445	1.441	5.54	6.32	7
50	1.303	1.310	1.310	6.02	6.96	7
55	1.179	1.188	1.185	6.54	7.69	7
60	1.066	1.076	1.066	7.11	8.51	7.11

	Dwelling size			*Residential Structural Density*		
x	**Equilibrium**	**Optimal**	**MPR: a ≥ 7**	**Equilibrium**	**Optimal**	**MPR: a ≥ 7**
0	179.02	178.27	178.00	375.60	370.34	223.42
5	182.25	181.45	180.96	318.90	314.30	194.71
10	185.58	184.74	184.03	270.10	266.09	169.24
15	189.02	188.13	187.21	228.20	224.71	146.69
20	192.59	191.65	190.52	192.30	189.27	126.78
25	196.27	195.28	193.95	161.63	159.00	109.25
30	200.09	199.03	186.94	135.47	133.20	148.48
35	204.04	202.92	188.74	113.23	111.27	137.11
40	196.62	194.21	190.66	126.83	127.35	126.03
45	197.32	194.63	192.71	115.02	115.59	115.26
50	198.03	195.03	194.92	104.20	104.84	104.84
55	198.75	195.40	197.29	94.31	95.02	94.78
60	199.48	195.72	199.48	85.28	86.06	85.28

Note: The equilibrium values represent the unregulated market equilibria when congestion is not internalized; the optimal values represent the social optimum, and the values with MPR: a≥7 represent market equilibria under a MPR equal to 7. The shaded values show each variable value (e.g. rent, parking area per dwelling, dwelling size or residential structural density) for the underground regime and the unshaded values show the values of the variable under the surface regime.

Figure 1: Land rent when τ = 0

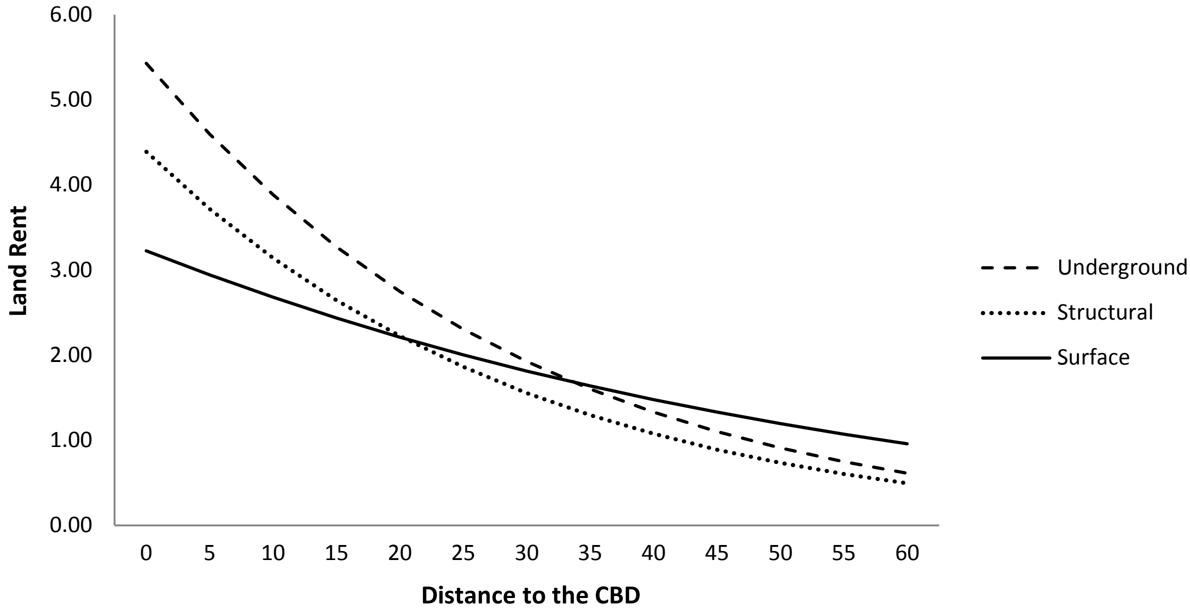


Fig. 2: Parking area when τ = 0

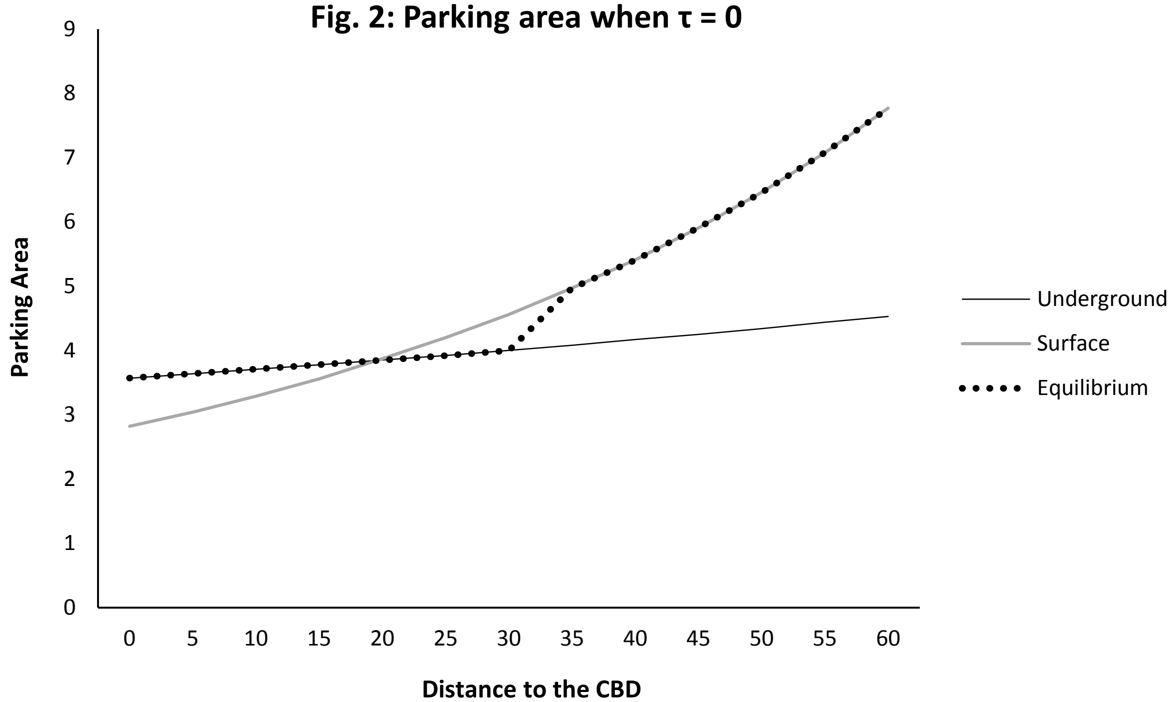


Fig. 3: Dwelling size when τ = 0

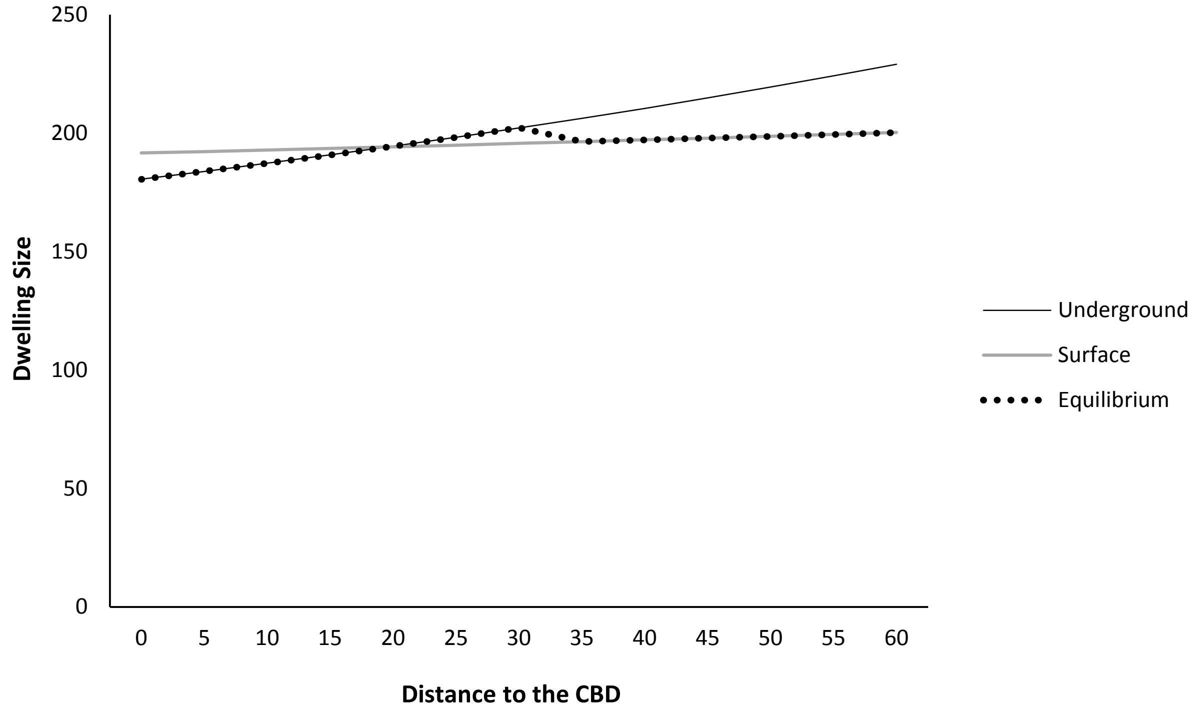


Fig. 4: Parking area when τ = 0.2

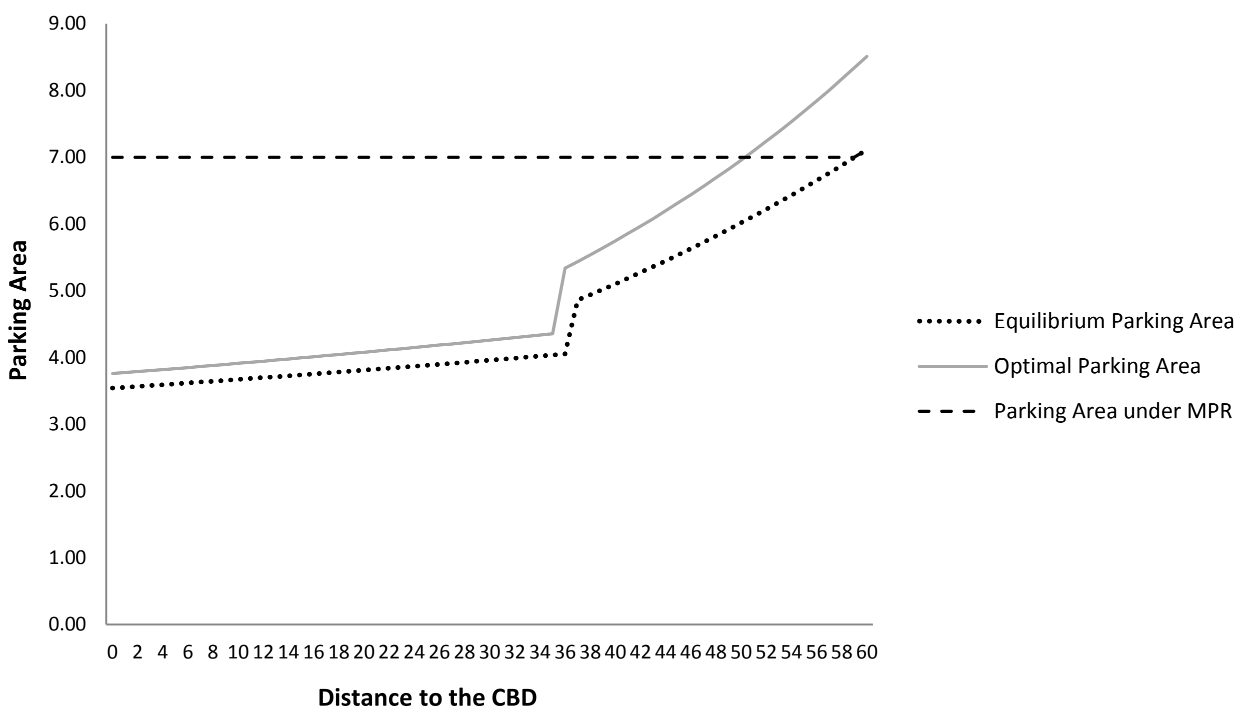
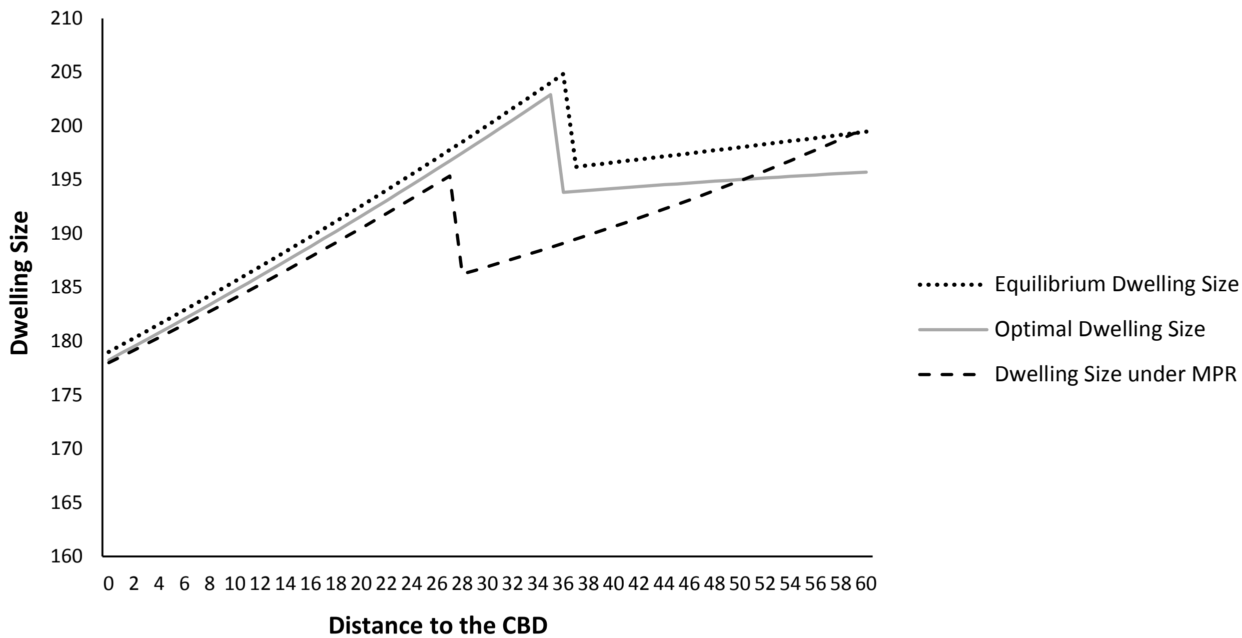


Fig. 5: Dwelling size when τ = 0.2



References

Anderson, S., de Palma, A. (2004) The economics of pricing parking, *Journal of Urban Economics*, 55: 1-20.

Anderson, S., de Palma, A. (2007) Parking in the city, *Papers in Regional Science*, 86: 621-632.

Arnott, R.J., de Palma, A., Lindsey, R. (1991) A temporal and spatial equilibrium analysis of commuter parking, *Journal of Public Economics*, 45: 301-335.

Arnott, R.J., Rowse, J. (1999) Modeling parking, *Journal of Urban Economics*, 45: 97-124.

Arnott, R.J., Inci, E. (2006) An integrated model of downtown parking and traffic congestion, *Journal of Urban Economics*, 60: 418-442.

Arnott, R.J., Inci, E., Rowse, J. (2015) Downtown curbside parking capacity, *Journal of Urban Economics*, 86: 83-97.

Brueckner, J.K. (1983) The economics of urban yard space: An 'implicit-market' model for housing attributes, *Journal of Urban Economics*, 13: 216-234.

Brueckner, J.K., Thisse, J.-F., Zenou, Y. (1999) 'Why is central Paris rich and downtown Detroit poor? An amenity-based theory, *European Economic Review*, 43: 91-107.

Calthrop, E., Proost, S. (2006) Regulating on-street parking, *Regional Science and Urban Economics*, 36: 29-48.

Cutter, W.B., Franco, S.F. (2012) Do parking requirements significantly increase the area dedicated to parking? A test of the effect of parking requirements values in Los Angeles County, *Transportation Research Part A*, 46: 901-925.

De Borger, B., Wuyts, B. (2009) Commuting, transport tax reform and the labour market: Employer-paid parking and the relative efficiency of revenue recycling instruments, *Urban Studies*, 46: 213-233.

Feitelson, E., Rotem, O. (2004) The case for taxing surface parking, *Transportation Research Part D*, 9: 319-333.

Ferguson, B.K. (2005) *Porous pavements.* Boca Raton, Florida: Taylor & Francis.

Franco, S.F. (2015) Downtown parking supply, work-trip mode choice and urban spatial structure, Working paper, Nova School of Business and Economics.

Fujita, M. (1989) *Urban Economic Theory.* Cambridge: Cambridge University Press.

Glazer A., Niskanen, E. (1992) Parking fees and congestion, *Regional Science and Urban Economics*, 22: 123-132.

Hasker, K., Inci, E. (2014) Free parking for all in shopping malls, *International Economic Review*, 55: 1281-1304.

Inci, E. (2015) A review of the economics of parking, *Economics of Transportation*, 4: 50-63.

Litman, T. (2011) Why and how to reduce the amount of land paved for roads and parking facilities, *Environmental Practice*, 13: 38-46.

Mukhija V., Shoup, D. (2006) Quantity versus quality in off-street parking requirements, *Journal of the American Planning Association*, 72: 296-308.

Schaller Consulting. (2007). *The Skewed Economic Incentives to Drive in Manhattan.* New York: Transportation Alternatives.

Shoup, D. (1997). Evaluating the effects of cashing out employer-paid parking: Eight case studies. *Transport Policy*, 4: 201-216.

Shoup, D.C. (1999) The trouble with minimum parking requirements, *Transportation Research Part A*, 33: 549-574.

Shoup, D. (2002) *The Trouble with Minimum Parking Requirements.* Victoria, BC: Victoria Transport Policy Institute.

Shoup, D.C. (2005) *The High Cost of Free Parking.* Chicago: Planners Press.

Shoup, D.C., Pickrell, D.H. (1978) Problems with parking requirements in zoning ordinances, *Traffic Quarterly*, 32: 545-561.

Van Ommeren, J.N., Wentink, D., Dekkers, J. (2011) The real price of parking policy, *Journal of Urban Economics*, 70: 25-31.

Van Ommeren, J.N, de Groote, J., Mingardo, G. (2014) Residential parking permits and parking supply, *Regional Science and Urban Economics*, 45: 33-44.

Van Ommeren, J.N., Wentink, D., Rietveld, P. (2012) Empirical evidence on cruising for parking. *Transportation Research A*, 46: 123-130.

Verhoef, E.T., Nijkamp, P., Rietveld, P. (1995) The economics of regulatory parking policies: The im(possibilities) of parking policies in traffic congestion, *Transportation Research Part*

A, 29: 141-156.

Voith, R. (1998) Parking, transit, and employment in a central business district, *Journal of Urban Economics*, 44: 43-58.

Willson, R.W. (1995) Suburban parking requirements: A tacit policy for automobile use and sprawl, *Journal of the American Planning Association*, 61: 29-42.

Willson, R., Shoup, D. (1990). The effects of employer-paid parking in downtown Los Angeles: A study of office workers and their employers. Working paper, Graduate School of Architecture and Urban Planning, UCLA.

Willson, R., Shoup, D. (1990). Parking subsidies and travel choices: Assessing the evidence. *Transportation*, 17: 141-157.

Footnotes

[*]We thanks Kristian Behrens and several referees for helpful comments. Franco is a permanent researcher at the UECE-Research Unit on Complexity and Economics of ISEG/ULisboa. She acknowledges the support of a grant from the Multicampus Research Program and Initiative (MPRI) of the University of California (award number 142934). However, the views expressed in this paper are solely those of the authors.

[1]While minimum parking requirements alleviate some urban parking problems, it is increasingly recognized that they can create others. Critics of these regulations allege that MPRs force developers to use more land per square foot of livable area than the market would dictate while raising the cost and reducing the profitability of development in areas with high land values (Willson 1995, Shoup 2002, 2005). As a result, MPRs may lead to an oversupply of parking (Cutter and Franco 2012) while shifting the location of new development and making infill projects and historic building retrofits less attractive and feasible (Shoup and Pickrell 1978). In addition, by creating large, unsightly parking lots impervious to precipitation, MPRs may compromise urban design (Mukhija and Shoup 2006) and contribute to environmental problems (Feitelson and Rotem 2004, Litman 2011). Despite these criticisms of MPRs, Hasker and Inci (2014) provide a rationale for mild minimum parking requirements in shopping malls. The logic is that the social planner cares about the loss in utility of customers who would like to purchase a good but cannot for lack of a parking space, while the developer does not.

[2]Van Ommeren, Wentink and Rietveld (2012) provide evidence on the extent of cruising for parking in Dutch downtown areas, finding it not to be substantial (a conclusion that may not generalize beyond the Dutch case).

[3]See Inci (2015) for a survey of this literature.

[4]Hasker and Inci's (2014) model of shopping-mall parking does not have a spatial component, but as in the present analysis, provision of parking contributes to the profit of the developer, in this case the shopping-mall owner. The model has no parking quantity choice, with the analysis instead focusing on the pricing of both mall parking and the retail goods on sale. To entice shoppers to the mall, some of whom may not find the good they seek (thus leaving with zero shopping surplus), the optimal price of parking is zero, matching real-world practice.

[5]An alternative measure of parking area would be the number of parking spaces associated with the dwelling, but this discrete measure is replaced with a continuous square-footage measure for analytical convenience.

[6]Shoup (1997, 2005) estimates that 95% of commuters receive free parking at their workplace, with over 50% of commuters in the central business districts of large cities like Los Angeles, New York, and London receiving parking paid for by their employer (Wilson and Shoup 1990, Schaller Consulting 2007). On the other hand, downtown visitors and shoppers may face significant parking congestion costs in cruising for a parking space. For simplicity, and since the focus is on residential parking, the analysis abstracts from such costs in workplaces and shopping areas.

[7]To maintain simplicity, this formulation does not include on-street parking search costs in the budget constraint. If a consumer's search costs were instead captured in this fashion, the costs would decrease with a, since more off-street parking means less need to search for on-street parking. This possibility is suppressed, with the benefits of off-street parking captured solely in the utility function. Correspondingly, the $\overline{a}$ term in the budget constraint is unrelated to parking search costs, capturing only the increase in CBD commuting cost caused by congestion due to cruising for parking in the neighborhood. Therefore, off-street residential parking provides a local direct private benefit to households (convenience, safety), which is captured in the utility function via a, and also a local public benefit through its local aggregate level (less local congestion from cruising for a vacant curb parking spaces), which is captured in the budget constraint. Note that commuters living at greater distances do not experience this neighborhood congestion (which is on local streets rather than major arteries) as they pass by neighborhood on their way to the CBD.

[8]For example, Fujita (1989, Ch. 7) portrays consumers as caring about neighborhood population density, and consumers in Brueckner, Thisse and Zenou (1999) care about the neighborhood's amenities, with both given by values for the residential ring. Note that a model where neighborhoods have radial width, corresponding to a discrete range of x values, would be analytically cumbersome.

[9]The semicolon preceding $\overline{a}$ emphasizes its special status, being neither a direct choice variable nor an immutable locational characteristic like x.

[10]Under one of the parking regimes analyzed below (underground parking), the model can be presented in a way that allows the consumer to choose q, as in the standard model. With q not appearing anywhere in the profit function aside from in the revenue term Rh/q, R/q can be replaced by p (which now depends on a), and the consumer can be allowed to choose q. But with a appearing both in p and elsewhere in the profit function, it must be chosen by the developer. Under the other two regimes, however, both q and a appear outside the revenue expression, so that both variables must be chosen by the developer. A different approach to consumer choice is sketched in footnotes 6 and 10 of Brueckner (1983), where the consumer is portrayed as taking the role of the developer, purchasing inputs of land and capital to generate floor and yard space. This approach is shown to be equilavent to the one in which the developer makes all the choices, but it lacks realism.

[11]The competitive developers in the standard urban model view the price p per square foot of floor space as parametric, and in the same fashion, the current developers view the level of the R function as parametric. While the current open-city assumption, by fixing u, pins down the level of R in any case, u and hence R's level would be endogenous in a closed-city version of the model, but that level would be nevertheless be viewed as parametric by the competitive developers.

[12]It would be possible in principle to analyze strategic behavior on the part of developers, with each developer recognizing the effect of his a choice on $\overline{a}$, taking the decisions of other developers as parametric. However, as in the case of the planner's problem discussed in section 5 below, the model would then not be solvable in closed form.

[13]Underground parking could be viewed as including the case where a single-story parking garage is built under the first residential floor of the building but above ground.

[14]The spatial behavior of the variables under these solutions seems mostly natural, but is hard to judge the generality of this behavior. Given the algebraic difficulty of generating the solutions, it appears likely that solutions would be infeasible under more general utility and production functions such as the CES form, requiring a retreat to purely numerical methods whose outcome is hard to predict.

[15]In expression for these constants, the various parameters of the model are raised to exponents that themselves are functions of the model parameters. The noncomparability of the magnitudes of E_{st} and E_{ug} follows because the exponent on any particular parameter, say θ, differs between E_{st} and E_{ug} expressions, with the relation between the two exponents ambiguous and dependent on the magnitudes of the parameters involved in the exponent expressions. As result, the overall magnitudes of the constants cannot be compared. The expression for the E_{st} constant is shown in the online appendix.

[16]In principle, another possibility is that one regime (surface or one of the non-surface regimes) dominates at all possible x values irrespective of the value of r_A. To understand this possibility, observe that the rent for each regime equals zero at an x value of y/t. If the rent curve for one of two regimes rises faster than the other curve as x falls toward zero from y/t, that regime will dominate the other one at all x values. Whether such an outcome arises depends on the relationship between the slopes and multiplicative constants for the two regimes. Given the complexity of the constants, no general statement can be made.

[17]Calculations show that the second-order conditions involving the Hessian determinant hold under these solutions at each location. In addition, while the assumptions $\mu_{st} = 0.08$ and $\mu_{ug} = 0.06$ lead to dominance of the underground regime over the structural regime, this dominance continues to hold when $\mu_{ug} = 0.06$ as long as $\mu_{st} < 0.099$. Conversely, when $\mu_{st} = 0.08$, underground dominance continues to hold as along as $\mu_{ug} > 0.0485$.

[18]A 2014 study of the Transportation Research Institute at University of Michigan found that in 2012, 9.22% of US households were carless. The cities with the largest share of carless households were New York at 56%, Chicago at 28%, and Detroit at 26%, with San Jose having the lowest share, at 5.8%. Although the share of carless households varies greatly across cities and regions, with suburban jurisdictions having greater car ownership than more densely-populated cities, Census estimates indicate average ownership of 1.8 vehicles per US household in 2013.

[19]In addition to this preservation of spatial behaviors when $\phi < 0$, it is easily seen that the relative rate of change in the variables of the model as x increases is smaller in the presence of parking-related congestion. Focusing on a, for example, differentation of (46) yields

$$\frac{1}{a}\frac{\partial a}{\partial x} \quad = \quad \phi\, \frac{-t + \tau \frac{\partial a}{\partial x}}{y - tx + \tau a}. \tag{$f1$}$$

Compared to the case where $\tau = 0$, the negative numerator of the ratio in $(f1)$ is closer to zero when $\tau > 0$ while the denominator is larger. As a result, the relative rate of change of a is smaller when $\tau > 0$ than when $\tau = 0$. Differentiation of the solutions for the other variables after replacing $y - tx$ by $y - tx + \tau a$ yields analogous conclusions.

[20]Finer spatial resolution, of course, would show a change in the switch point.

[21]These patterns prevails within each regime over the entire range of distances in the city, with S falling (rising) at all locations for the underground (surface) regime.

[22]Possible intuition for the result in the surface case is that, since more parking land must be provided as a rises (without the benefit of offsetting adjustment in $\widetilde{S}_{ug}$), it is optimal to economize on overall land use by raising S, even though q is falling.

[23]Van Ommeren, de Groote and Mingardo (2014) explore the welfare losses from another kind of policy intervention: the use of residential parking permits (a variant of local parking caps) in Dutch shopping districts within (historic) city centers. More generally, Van Ommeren, Wentink and Dekkers (2011) estimate the willingness to pay for parking permits in purely residential areas, which reduce residents' need to cruise for parking (such permits are not very common in the US).