Optimal Incentive Contract with Costly and Flexible Monitoring

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Motivation

Choice of monitoring technology has significant impact on employee productivity.

Standard agency models take the monitoring technology as exogenously given.

Need strong assumptions to justify

1. Simple and intuitive contracts;
2. Heterogeneity in managerial practices.
A principal-agent model with flexible and costly monitoring:

- **Flexibility**: specify the qualitative and quantitative natures of the monitoring technology;
- **Cost**: increasing in the entropy of the agent’s compensation.

Endogenize the choice of monitoring technology as part of the contract design problem.

Use factors that affect the monitoring cost to explain
  - Simple and intuitive contracts;
  - Heterogeneity in human resource practices.
Agenda

1. Baseline model
2. Extensions
3. Conclusion
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A risk-neutral principal and a risk-averse agent.

Agent payoff $u(w) - c(a)$:

- Consumption $w \geq 0$, $u(0) = 0$, $u' > 0$, $u'' < 0$;
- Effort $a \in \{0, 1\}$, $c(1) = c > c(0) = 0$.

Each effort level $a$ generates a probability space $(\Omega, \Sigma, P_a)$.

Principal’s goal: elicit high effort from the agent.
Incentive Contract

A pair of monitoring technology $\mathcal{P}$ and wage scheme $w(\cdot)$:

1. $\mathcal{P}$: a partition of $\Omega$ whose elements belong to $\Sigma$;
2. $w : \mathcal{P} \rightarrow \mathbb{R}_+$.

Timeline:

- Parties commit to $\langle \mathcal{P}, w(\cdot) \rangle$;
- The agent privately exerts $a \in \{0, 1\}$;
- Nature draws $\omega \in \Omega$ according to $P_a$;
- $A(\omega) \in \mathcal{P}$ is publicly realized;
- The principal pays the promised wage $w(A(\omega))$. 
The contract defines a signal $X$ and a random wage $W$.

For each effort level $a$ and $A \in \mathcal{P}$:
- $X$ takes value $A$ with prob. $P_a(\omega \in A)$;
- $W$ equals $w(A)$ with prob. $P_a(\omega \in A)$. 
Monitoring Cost and Total Cost

Monitoring cost for each given \( a \):

\[ \mu \cdot H_a(W) \]

1. \( H_a(W) \): entropy of the random wage.
2. \( \mu > 0 \): cost and benefit of monitoring the agent.

Total cost for each given \( a \):

\[
\underbrace{E_a[W]}_{\text{incentive cost}} + \underbrace{\mu \cdot H_a(W)}_{\text{monitoring cost}}
\]
For each $A \in \Sigma$, define

$$z(A) = 1 - \frac{dP_0}{dP_1}(A)$$

A contract is incentive compatible for the agent if

$$\int_{A \in \mathcal{P}} u(w(A))z(A)\,dP_1 \geq c$$
The optimal incentive contract $\langle P^*, w^*(\cdot) \rangle$ solves

$$\min_{\langle P, w(\cdot) \rangle} \mathbb{E}_1[W] + \mu \cdot H_1(W)$$

s.t. (IC) and (LL)
Standard agency models take $\mathcal{P}$ as exogenously given and solve for

$$\min_{w: \mathcal{P} \to \mathbb{R}^+_+} \mathbb{E}_1[W], \text{ s.t. (IC) and (LL)}$$

Denote the solution by $w^*(\cdot; \mathcal{P})$.

**Lemma 1.**

*For any given $\mathcal{P}$, there exists $\lambda > 0$ such that for each $A \in \mathcal{P}$, $u'(w^*(A; \mathcal{P})) = \frac{1}{\lambda z(A)}$ if and only if $w^*(A; \mathcal{P}) > 0$.***
Increasing Wage Scheme and MLRP

**Definition 1.**

Suppose $\mathcal{P}$ is totally ordered under $\preceq$. Then the distributions of the signal induced by $\mathcal{P}$ satisfy the monotone likelihood ratio property if any $A, A' \in \mathcal{P}$ such that $A \preceq A'$, we have $z(A) < z(A')$.

**Lemma 2.**

Suppose $\mathcal{P}$ is totally ordered under $\preceq$. Then $w^*(\cdot; \mathcal{P})$ is increasing if and only if the distributions of the signal induced by $\mathcal{P}$ satisfy MLRP.
Why May MLRP Fail?

For an arbitrary monitoring technology,

1. $\mathcal{P}$ may not be totally ordered, e.g., multi-source feedback;
2. Even if $\mathcal{P}$ is totally ordered, MLRP is still a strong property.
Theorem 1.

For any $\mu > 0$,

(i) $\mathcal{P}^* = \{A_1, A_2, \cdots, A_n\}$ for some $n \in \mathbb{N}$;

(ii) $z(A_1) < z(A_2) < \cdots < z(A_n)$;

(iii) $w^*(A_1) = 0 < w^*(A_2) < \cdots < w^*(A_n)$. 
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2. Extensions
   - Multi-task
   - Multi-agent
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A risk-neutral principal and a risk-averse agent.

The agent can exert $a_i \in \{0, 1\}$ in each of two tasks $i = 1, 2$.

Each effort profile $\bar{a} \in \{0, 1\}^2$ generates $(\Omega, \Sigma, P_{\bar{a}})$.

Principal’s goal: elicit high effort in both tasks.
Detect Deviation

For each $A \in \Sigma$ and each $\vec{a} \in \{10, 01, 00\}$, define

$$z_{\vec{a}}(A) = 1 - \frac{dP_{\vec{a}}(A)}{dP_{11}(A)}$$

A contract is incentive compatible for the agent if for each $\vec{a} \in \{10, 01, 00\}$,

$$\int_{A \in \mathcal{P}} u(w(A)) z_{\vec{a}}(A) dP_{11} \geq c(11) - c(\vec{a})$$
Theorem 2.

For each $\mu > 0$,

(i) $P^* = \{A_1, \ldots, A_n\}$;

(ii) $w^*(A_1) = 0 < w^*(A_2) < \cdots < w^*(A_n)$;

(iii) There exist $\lambda_{\bar{a}}, \bar{a} \in \{10, 01, 00\}$, such that for all $k = 2, \ldots, n$,

$$u'(w^*(A_k)) = \frac{1}{\sum_{\bar{a}} \lambda_{\bar{a}} z_{\bar{a}}(A_k)}$$
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Multiple Agents

A risk-neutral principal and two risk-averse agents \( i = 1, 2 \).

Each agent \( i \) exerts \( a_i \in \{0, 1\} \).

Each \( a_i \) independently generates \((\Omega, \Sigma, P_{a_i})\), where
- \( \Omega = \{0, 1\} \), \( \Sigma = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\} \);
- \( P_1(1) = p \in (0, 1) \) and \( 1 - \frac{dP_0(1)}{dP_1(1)} = z \in (0, 1) \).

Each \( \vec{a} = (a_1, a_2) \) generates \((\Omega \times \Omega, \Sigma \otimes \Sigma, P_{a_1} \times P_{a_2})\).
Principal’s goal: elicit high effort from both agents.

A monitoring technology $\mathcal{P}$ and a wage scheme $\tilde{w}(\cdot)$:

1. $\mathcal{P}$: a partition of $\Omega \times \Omega$ whose elements belong to $\Sigma \otimes \Sigma$;
2. $\tilde{w} : \mathcal{P} \rightarrow \mathbb{R}_+^2$. 
Individual Reward

\[ w_1 = 0 \quad w_1 > 0 \]
\[ w_2 > 0 \quad w_2 = 0 \]

\[ w_1 = 0 \quad w_1 > 0 \]
\[ w_2 = 0 \quad w_2 = 0 \]

Figure: $\Gamma_4$
Tournament

\[ w_1 = 0 \]
\[ w_2 = \max \]

\[ w_1 = \text{median} \]
\[ w_2 = \text{median} \]

\[ w_1 = \max \]
\[ w_2 = 0 \]

Figure: $\Gamma_{3b}$
Group Compensation

Figure: $\Gamma_{2a}$

\[ w_1 > 0 \]
\[ w_2 > 0 \]

\[ w_1 = 0 \]
\[ w_2 = 0 \]
Group Compensation

Figure: $\Gamma_{2b}$

- $w_1 > 0$
- $w_2 > 0$
- $w_1 = 0$
- $w_2 = 0$
Optimal Multi-Agent Contract

Figure: Individual reward vs. group compensation
Result

1. Difference in $\mu$ yields various kinds of incentive schemes.
2. Lack of individual performance appraisal when $\mu$ is big.

Explain variation in managerial practices by factors that affect $\mu$:

- **Cost**: information technology, labor market regulation, tacit knowledge transfer;
- **Benefit**: human capital share, product market competition.
Conclusion

A principal-agent model with costly and flexible monitoring.

Endogenize the choice of monitoring technology.

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