Dynamic unstructured bargaining with private information and deadlines: theory and experiment

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Abstract

We study dynamic unstructured bargaining with deadlines and one-sided private information, via theory and experiment. We predict the incidence of bargaining failures ("strikes") and payoffs in each state by combining mechanism design and focal point approaches. Strikes are common in states with lower surpluses ("pies") and strike incidence is decreasing in the pie size. Subjects reach equal splits when strikes are efficient, while payoffs are unbalanced in states where strikes are inefficient, with additional surplus accruing to the informed player. We employ a machine learning approach to explore the information content of bargaining process data.

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1 Introduction

Bargaining is everywhere in economic activity: from price haggling in flea markets, to wage negotiations between unions and firms, to high-stakes diplomacy. Even in competitive, large-scale markets, sequences of market trades often result from individual buyer-seller partners bargaining over a range of mutually-agreeable contract terms, knowing their outside options from the market. Bargaining failures such as holdouts and strikes - due to disputes over what each side should get - are also common and reduce welfare.

Strikes are surprising because in almost every case, the bargain that was eventually struck after a costly strike could have been agreed to much earlier in the bargaining, which would have saved lost profits, legal bills and many other collateral costs. Then why do strikes happen? The standard approach in the game theory of private-information bargaining is that the willingness to endure a strike is the only way for one side to credibly convince the bargaining partner that their existing offer is inadequate. Thus, although strikes are ex-post inefficient, because of informational asymmetry strikes can be ex-ante efficient (in theory), and are sometimes unavoidable, even when both sides behave rationally given their information and beliefs (Kennan and Wilson, 1990).

Private information bargaining theories, and tests of these theories, have developed in two ways:

(1) The most popular way is bargaining theories based on highly structured settings, e.g. Stahl (1972) or Rubinstein (1982); for a review see Ausubel, Cramton and Deneckere (2002). “Structure” means that the rules of how bargaining proceeds are clearly specified in the theory. The rules typically define when bargaining must be completed (either a deadline or an infinite horizon), who can offer or counteroffer and at what time, when offers are accepted, whether communication is allowed (and in what form), and so on. Theoretical predictions of outcomes and payoffs depend sensitively on these structural features (see Cramton 1984, Gul and Sommerschein 1988, Fudenberg, Levine and Tirole 1985, Ausubel and Deneckere 1993, Grossman and Perry 1986, Rubinstein 1985). Following the burst of progress in game theory on structured private-information bargaining, a large experimental literature emerged testing these theories (Güth, Huck and Ockenfels 1996, Güth and Van Damme 1998, Mitzkewitz and Nagel 1993, Croson, Boles and Murnighan 2003, Kagel, Kim and Moser 1996, Kagel and Wolfe 2001, Kriss, Nagel and Weber 2013, Rapoport, Daniel and Seale 1998, Srivastava 2001, Ochs and Roth 1989). The clear assumptions about structure in the theory made experimental design and theory-testing straightforward.

(2) The less popular way of theorizing and experimentation is based on unstructured bargaining. Our paper returns to this less popular route, exploring unstructured bargaining
with one-sided private information in an experiment.

There are three good reasons to study unstructured bargaining.

First, most natural two-player bargaining is not highly structured. Conventional methods for conducting bargaining do emerge in natural settings, but these methods are rarely constrained, because there are no penalties for deviating from conventions. There may also be clear empirical regularities in unstructured bargaining—such as deadline effects (Roth, Murnighan and Schoumaker [1988])—that are evident in the data but not predicted by theory. Establishing these regularities can lead theorizing, rather than test theory.

Second, even when bargaining is unstructured, theory can still be applied to make clear interesting predictions. A natural intuition is that when bargaining methods are unstructured, no clear predictions can be made, as if the lack of structure in the bargaining protocol must imply a lack of structure (or precision) in predictions. This intuition is just not right. In the case we study, clear predictions about unstructured bargaining do emerge, thanks to the wonderful “revelation principle” (Myerson [1979, 1984]). This principle has the useful property of implying empirical predictions for all noncooperative equilibria, independently of the bargaining protocol, based purely on the information structure. For example, the application of the revelation principle in our setting leads to the prediction that strikes will become less common as the amount of surplus the players are bargaining over grows. This type of prediction is non-obvious and can be easily tested. Furthermore, if additional assumptions are made about equilibrium offers, and combined with the revelation principle, then exact numerical predictions about offers and strike rates can be made. That is, even if the bargaining protocol lacks structure, predictions can have plenty of restricted “structure” thanks to the beautiful game theory.

Third, unstructured bargaining creates a large amount of interesting data during the bargaining process. In unstructured bargaining, the behavioral freedom which is restricted in structured bargaining is unleashed—players can make offers at any time, can retract offers, can communicate, and so on. Of course, theories gain precision by ignoring these process data. However, if data about process variables are systematically associated with outcomes, these empirical regularities both challenge simple equilibrium theories and invite new theory development. For example, if players are in equilibrium then the probability of reaching a deal for all people in the same informational condition will be equal (adjusting for sampling error). However, if strike rates differs statistically across people - then players are not in an equilibrium, and process data can provide some clues about why some players bargain more successfully than others.

Process data are also useful because practical negotiation advice often consists of simple heuristics about how to bargain well. For example, negotiation researchers have long ago
postulated that initial offers might serve as bargaining anchors and that various psychological manipulations, such as perspective taking, could potentially bias bargaining outcomes \cite{Galsky2001, KristensenGarling1997, VanPouckeBuelens2002}. Advice like this can be easily tested by carefully controlled experimental designs that allows structure-free bargaining while keeping the process fully tractable, such as our paradigm.

2 Background

The experimental literature on bargaining is too vast to review here \cite[though see Chapter 4 of][]{Camerer2003, AusubelCrantonDeneckere2002, ThompsonWangGunia2010}. Therefore, we will only mention the studies most closely related to ours.

Before theoretical breakthroughs in understanding structured bargaining, most experiments used unstructured communication. The main focus of interest was process-free solution concepts such as the Nash bargaining solution \cite{Nash1950}, and important extensions \cite[e.g.][]{KalaiSmorodinsky1975}. We will refer to the amount of surplus available to share as the “pie”. Many bargains \cite{NydeggerOwen1974, RothMalouf1979} led to an equal split of the pie. Roth suggested that “bargainers sought to identify initial bargaining positions that had some special reasons for being credible... that served as focal points that then influenced the subsequent conduct of negotiation” \cite{Roth1985}. Under informational asymmetries, disagreements may thus rise due to coordination difficulties. Several papers by Roth and colleagues then explored what happens when players bargain over points which have different financial value to players \cite{RothMalouf1979, RothMaloufMurnighan1981, RothMurnighan1982, Roth1985}. In theory, there should be no disagreements in these games but a modest percentage of trials did result in disagreement (10-20%). Many of the disagreements could be traced to self-serving differences between which of two focal points should be adopted – whether to allocate points equally, or to allocate the money, resulting from points, equally. Focal points have remained an important theme in more recent work \cite{Schelling1960, Isoni2013, KristensenGarling1997, Janssen2001, BinmoreSamuelson2006, Roth1985, Janssen2006, Bardsley2010, HargreavesHeap2014}. Roth, Murnighan and Schonmaker \cite{RothMurnighanSchonmaker1988}, also drew attention to the fact that the large majority of agreements are made just before a (known) deadline, an observation called the “deadline effect”.

Several experiments have observed what happens in unstructured bargaining with two-sided private information \cite{Valley2002}. The typical finding is that in face-to-face and unstructured communication via message-passing, there are fewer disagreements than pre-
dicted by theory. However, when players bargaining can only make a single offer, disagreements are more common, and the key predictions of theory hold surprisingly well (Radner and Schotter, 1989; Rapoport, Erev and Zwick, 1995; Rapoport and Fuller, 1995; Daniel, Seale and Rapoport, 1998).

The closest precursor to our design is Forsythe, Kennan and Sopher (henceforth FKS), who studied unstructured bargaining with one-sided private information about the pie size (Forsythe, Kennan and Sopher, 1991; Kennan and Wilson, 1993). They used mechanism design to identify properties shared by all Bayesian equilibria of any bargaining game, using the revelation principle (Myerson, 1979, 1984). This approach gives a “strike condition” predicting when disagreements would be ex-ante efficient. FKS tested their theory by conducting several experimental treatments, with free-form communication. The results qualitatively match the theory quite closely.

Our theoretical framework generalizes the earlier work of FKS to a design with six equally-likely pies, with rapid bargaining (10 seconds per trial). Bargaining occurs only through visible offers and counter-offers. Furthermore, we address equilibrium selection by adding assumptions about the appeal of an equal pie split as a focal point.

Most related to our design from the literature studying structured bargaining is Mitzkowitz and Nagel (1993) (henceforth MN), who study ultimatum bargaining with incomplete information. In their “offer” game an informed proposer makes a take-it-or-leave-it offer to an uninformed responder. MN use the same distribution over pie sizes in ultimatum bargaining that we employ in unstructured bargaining. Contrasting our results with theirs allows comparing our unstructured design with an ultimatum bargaining setting with private information.

The reminder of this paper is organized as follows. We develop our bargaining theory in section 3. We describe our novel experimental design in section 4 and present its general results in section 5, where we specifically discuss how and to what extent the experimental data conform with our theoretical predictions. We examine whether bargaining process data can be associated with bargaining outcome variables in section 6. We discuss deviations from selfish-rational theory and compare our dynamic results to those obtained by previous studies of static bargaining in section 7 and conclude in section 8.

\footnote{A comparable finding in sender-receiver games is that senders willingly share more private information than is selfishly rational; see Cai and Wang (2006); Wang, Spezio and Camerer (2010); Crawford (2003).}
3 Theory

In this section we develop a theory that provides testable predictions of disagreement rates and surplus division. Our model combines two methods to analyze bargaining: mechanism design and focal points. We extend the model of strikes developed in [Kennan (1986)] and Forsythe, Kennan and Sopher (1991) to an arbitrary finite number of states. This extension yields non-obvious predictions of the frequency of disagreement (the strike rate) in each state, using only the game structure, rationality and incentive-compatibility constraints, and assuming ex-ante efficiency. We then suggest a focal point approach to the problem of equilibrium selection. Combining these two approaches yields testable predictions about both strike rates and payoffs in each state.

3.1 Game and notation

Two players must agree on how to split a surplus (or “pie”), a random variable denoted by $\pi$. The informed player has private information about the actual size of the pie. The uninformed player knows that the informed player knows the pie size. States of the world are indexed by $k \in \{1, 2, \ldots, K\}$, and the pie size in state $k$ is $\pi_k$. Without loss of generality, we assume $\pi_k > \pi_j$ when $k > j$. The probability distribution of pie sizes $\Pr(\pi_k) = p_k$ is commonly known. The players have a finite amount of time $T$ to reach an agreement. They bargain over the payoff of the uninformed player, denoted by $w$, by continuously communicating their bids. Players cannot commit to a particular bargaining position. In case of agreement on an informed player’s payoff $w$, the informed player gets $y = \pi_k - w$. If no deal is made by time $T$, both players’ payoffs are zero.

3.2 The direct bargaining mechanism

By the revelation principle (Myerson, 1979, 1984), any Nash equilibrium payoffs of the bargaining game are equivalent to the payoffs from a direct mechanism in which the informed player truthfully reports the size of the pie, and each bargainer’s payoff and the probability of a strike is determined by that report (Forsythe, Kennan and Sopher, 1991). Following FKS, we assume that bargainers negotiate inscrutably over the set of direct mechanisms.

In the direct mechanism, the informed player announces the true size of the pie, $\pi_k$. The pie is then decreased by a known fraction, $1 - \gamma_k$, which can be interpreted as the strike probability in state $k$, leaving an expected pie size of $\gamma_k \pi_k$. The uninformed bargainer receives $x_k$, and the informed player gets the rest of the pie. To make predictions regarding observed behavior, we rely on the fact that the payoff $x_k$ in the direct mechanism is tantamount to the
expected payoff of the uninformed player in state \( k \) of the bargaining game: \( x_k = \gamma_k w_k \) such that \( w_k \) is the uninformed payoff conditional upon a deal in state \( k \). A mechanism therefore involves \( 2K \) parameters, \( \{\gamma_k, x_k\}_{k=1}^K \).

### 3.2.1 Individual rationality (IR)

Individual rationality requires that both players prefer to participate in the mechanism. Therefore, the IR requirement is that for all \( k \)

\[
\gamma_k \pi_k - x_k \geq 0 \tag{1}
\]

\[
x_k \geq 0 \tag{2}
\]

Note that the two equations impose boundaries on the uninformed player’s payoff; it is never greater than the expected remaining pie.

\[
\gamma_k \pi_k \geq x_k \geq 0 \tag{3}
\]

### 3.2.2 Incentive compatibility (IC)

A mechanism is IC if it is optimal for the informed player to tell the truth; i.e., her expected payoff is (weakly) maximized when she announces the true size of the pie. This requires

\[
\gamma_k \pi_k - x_k \geq \gamma_j \pi_k - x_j \quad \text{for all } k, \quad \text{for all } j \neq k. \tag{4}
\]

The IR and IC conditions together lead to the following result.

**Lemma 1.** If the bargaining mechanism satisfies IR and IC:

1. Disagreement rates are monotonically decreasing in the pie size \( k \).
2. The uninformed player’s payoffs are monotonically increasing in the pie size.
3. The uninformed player’s payoff is identical for all states in which the deal probability is 1.

Proof: See the Appendix, section A.1

### 3.2.3 Efficiency

In our setting a mechanism is efficient (more precisely, is “interim-incentive efficient” [Holmström and Myerson 1983]) if it is Pareto optimal for the set of \( K + 1 \) agents: the \( K \) informed players in each of the different states \( k \), and the uninformed player.
Lemma 2. The strike condition: For IC mechanisms, strikes in state $k$ are ex-ante efficient if

$$\frac{\pi_k}{\pi_{k+1}} < \frac{(1 - \sum_{j=1}^{k} p_j)}{(1 - \sum_{j=1}^{k-1} p_j)} = \frac{\Pr(\pi \geq \pi_{k+1})}{\Pr(\pi \geq \pi_k)}$$

(5)

Proof: See section A.2 of the Appendix.

The relations between pie size ratios and conditional probabilities of pie size in Eq. 5 are called “strike conditions”. By Lemma 1 (result 2) and Lemma 2, if there exists a cutoff state, $\pi_c$, in which $\gamma_c = 1$ (no strikes), then strikes are inefficient in all states $\pi_k$ such that $k \geq c$.

3.3 Equilibrium selection using focal points

In theory, the IR, IC, and efficiency constraints limit the scope of possible bargaining outcomes and predict when strikes are likely to occur. This is remarkable considering that the bargaining protocol is unstructured. However, these conditions do not precisely pin down the numerical strike rates $1 - \gamma_k$ and the equilibrium payoffs (conditional on a deal being reached) $w_k$ for each state. There are many such sets of parameter values that will satisfy the requirements for efficient mechanisms and which also satisfy IR, and IC and are equilibrium outcomes. To make a more precise prediction, we incorporate an equilibrium selection approach that relies on the extensive literature emphasizing the importance of focal points in bargaining games (Schelling, 1960; Isoni et al., 2013a, b, Kristensen and Gärling, 1997, Janssen, 2001, Binmore and Samuelson, 2006; Roth, 1985; Janssen, 2006; Bardsley et al., 2010). Absent other salient features of bargaining, the natural focal point is an equal split (i.e., $w_k = \pi_k/2$) (Roth and Malouf, 1979; Roth and Murnighan, 1982; Isoni et al., 2013a; Janssen, 2006). We therefore hypothesize that equilibrium payoff of the uninformed player, conditional on a deal, will equal half of the pie size, $(w_k = \pi_k/2)$ as long as an equal split satisfies the IC and PE conditions. It remains to check whether these offers, and the associated strike rates, form an equilibrium.

The IC constraints (Lemma 1, result 4) require that the uninformed player’s payoff must be the same for all pies for which the strike condition does not hold (i.e., where the strike rate is zero). If there is more than one such state (i.e., pie size) then the equilibrium offers in those states cannot all be equal splits of the pie. This implies that if there exists a cutoff state, $\pi_c$, such that $\gamma_c = 1$ (no strikes), then for all $\pi > \pi_c$, $\gamma_k = 1$ and the same equilibrium offer of $\pi_k/2$, will be the offer in the higher-pie-value states. Combining this result with equilibrium equal splits in lower-value pie states yields a clear prediction about the equilibrium uninformed
payoffs \( w_k^* \) in each state:

\[
  w_k^* = \begin{cases} 
    \frac{\pi_k}{2} & \forall \pi_k \leq \pi_c \\
    \frac{\pi_c}{2} & \forall \pi_k > \pi_c.
  \end{cases}
\]  

### 3.3.1 Strike rates in the focal equilibrium

In our experiment, \( \pi \) takes on values which are the integer dollar amounts between $1-6 with equal likelihood. It follows numerically that the strike condition (Eq. 2) holds for pies of size 1 and 2. When \( \pi = 3 \), the two sides of the inequality are equal so the strike rate is indeterminate. When \( \pi \geq 4 \) there should be no strikes.

As noted just above, the IC constraints (Lemma 1, condition 4) require that the uninformed player’s payoff must be the same for all pies for which strikes are inefficient (i.e. the strike condition does not hold). Combined with the focal principle of equal splitting, this implies that an equal split of \( \pi = 4 \) can be an equilibrium, but then the same offer, 2, will be the equilibrium offer for the larger pie sizes 5 and 6.

The inequalities constraining when the strike condition applies (equation 5) and the constraints provided by focal offers enable us to pin down the exact numerical strike rates for all pie sizes. We set \( \gamma_4 = \gamma_5 = \gamma_6 = 1 \), as required by the strike condition when pie sizes are uniformly distributed over \( \{1, 2, 3, 4, 5, 6\} \). Noting again that the uninformed player’s payoff in each state \( x_k \) in the direct bargaining mechanism is equal to the payoff in case of a deal times the strike rate, we fix \( x_k = \gamma_k(0.5\pi_k) \) for all \( k < 4 \), and \( x_k = \gamma_k2 = 2 \) for all \( k \geq 4 \). Consequently, we can use the IC condition (Eq. 4) to make explicit predictions of the strike rates in the efficient equilibrium:

\[
  \gamma_j \leq \frac{0.5\pi_k}{\pi_k - 0.5\pi_j}\gamma_k \quad \forall \ k \leq 4, \ j \neq k.
\]  

Solving this set of inequalities numerically (see Section A.3 of the Appendix) and picking the highest possible values of \( \gamma_k \) (for maximal efficiency) yields the prediction of \( \gamma_3 = 0.8 \), \( \gamma_2 = 0.6 \) and \( \gamma_1 = 0.4 \).

### 4 Experiment

In this section, we present a novel experimental paradigm of dynamic bargaining, that allows both parties to communicate offers whenever they please, while keeping their behavior tractable.
4.1 Design

Our experimental design is a continuous-time bargaining game with one-sided private information. At the start of each session, participants were randomly divided into two equally-sized type groups, informed and uninformed. The types were fixed for the session’s 120 bargaining periods. Each period had the following steps:

1. Each player was randomly matched with a partner from the other group in a stranger protocol (to prevent sequential effects such as reputation building).

2. In each game, an integer pie size, $\pi \in \{1, 2, 3, 4, 5, 6\}$, was drawn from a commonly known discrete uniform distribution:
   $$\Pr(\pi_k) = \frac{1}{6} \forall \pi \in \{1, 2, 3, 4, 5, 6\}.$$

3. The informed player was told the true value of $\pi$ for that period.

4. Each pair bargained over the uninformed player’s payoff, denoted by $w$. Players communicated their monetary offers, in multiples of $0.2$, using mouse clicks on a graphical interface that was designed for this purpose by z-tree software [Fischbacher 2007] (see Fig. 1). The offer values were between $0$ and $6$.

5. During the first two seconds of bargaining, both players fixed their initial offers, without seeing the offers of their partner (see Fig. 1a).

6. Once the initial offers were set, players bargained continuously for 10 seconds using mouse clicks (see Fig. 1b).

7. When players’ positions matched each other, visual feedback was given to both of them in the form of a vertical stripe connecting their offer lines (see Fig. 1c). If none of the players changed their position for the next 1.5 seconds following the offer-match feedback, a deal was made. Thus, in order to make a deal, the latest time in which players’ bids could match was $t = 8.5$ seconds.

8. If no deal had been made within 10 seconds of bargaining, both players’ payoffs from that period were $0$.

9. After the period had ended, both players were told their payoffs and the actual pie size (see Fig. 1d).

A video demonstration of the task is available on [https://www.youtube.com/watch?v=y7pKh1EJsvMk](https://www.youtube.com/watch?v=y7pKh1EJsvMk).
Figure 1: Bargaining interface. (a) Initial offer screen: in the first two seconds of bargaining, players set their initial position, oblivious to the initial demand of their partner. The pie size at the top left corner appears only for the informed type. (b) Players communicate their offers using mouse click on the interface. (c) When demands match, feedback in the form of a green vertical stripe appears on the screen. If no changes are made in the following 1.5 seconds, a deal is made. (d) Following the game, both players are notified regarding their payoffs and the pie size.
4.2 Methods

We conducted eight experimental sessions, five at the Caltech SSEL and three at the UCLA CASSEL labs. There were a total of N=110 subjects (mean age: 21.3 SD: 2.4; 47 females). The number of subjects varied slightly across sessions due to show-up differences (see Appendix B.1 for details). In the beginning of each session, subjects were randomly assigned to isolated computer workstations and were handed printed versions of the instructions (see Appendix C). The instructions were also read aloud by the experimenter (who was the same person in all sessions). All of the participants completed a short quiz to check their understanding of the task. Subjects played 15 practice rounds in order to become familiar with the game and the interactive interface before the actual play of 120 periods. Participants’ payoffs were based on their profits in randomly chosen 15% of the periods, plus a show-up fee of $5. Each session lasted approximately 90 minutes.

5 Experimental results

5.1 Main findings

Our data consists of each subject’s bargaining positions and the outcomes of 120 periods. We first note that strike rates and offer amounts were not significantly different in the two subject groups (Caltech vs. UCLA). Strike rates do appear to decline somewhat with experience, but we report results across all periods and include controls for period number (see Appendix B for details). Therefore, in further analyses we will pool all these data together across subject groups and periods.

We observed the following empirical regularities:

**Result 1.** Disagreement rates are monotonically decreasing with the pie size.

The mean deal rates for pie are summarized in Table 1 and Fig. 2a. While the probability of disagreement decreased with the pie size, the mean amount of surplus lost due to strikes (Table 1) was positively correlated with the pie, as relatively small amounts of money are lost when strikes occur in small pie games.

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3 There is a negative correlation \( r = -0.48 \) between session size and overall deal rate, which is largely due to smaller high-deal rate sessions being conducted at Caltech (controlling for location reduces the correlation to \( -0.09 \)). The difference between (regression-predicted) average deal rates in the smallest and largest session sizes is also not large in magnitude, dropping from 65% to 58%.

4 A small fraction (less than 2.5 percent) of the games were excluded from analysis, due to a software bug in the first sessions conducted.
Figure 2: Deal rates and mean payoffs across pie sizes

(a) Deal rates by pie size

(b) Mean payoffs by pie size and subject type, periods ending in a deal
### Table 1: Average payoffs* and deal rates by pie size**

<table>
<thead>
<tr>
<th>Pie size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informed payoff (theory)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>2.0</td>
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<td>Informed payoff (data)</td>
<td>0.37</td>
<td>0.95</td>
<td>1.56</td>
<td>2.23</td>
<td>3.07</td>
<td>3.87</td>
<td>2.01</td>
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<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uninformed payoff (theory)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Uninformed payoff (data)</td>
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<td>1.05</td>
<td>1.44</td>
<td>1.77</td>
<td>1.93</td>
<td>2.13</td>
<td>1.49</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td></td>
<td></td>
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<tr>
<td>Deal rate (theory)</td>
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<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
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<td>Deal rate (data)</td>
<td>0.42</td>
<td>0.48</td>
<td>0.54</td>
<td>0.69</td>
<td>0.73</td>
<td>0.81</td>
<td>0.61</td>
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<td>(0.06)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.02)</td>
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<td></td>
</tr>
<tr>
<td>Surplus Loss (Theory)</td>
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<td>0.8</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.33</td>
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<tr>
<td>Surplus Loss (data)</td>
<td>0.58</td>
<td>1.04</td>
<td>1.39</td>
<td>1.25</td>
<td>1.36</td>
<td>1.16</td>
<td>1.13</td>
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<tr>
<td>(0.06)</td>
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<td>(0.10)</td>
<td>(0.10)</td>
<td>(0.10)</td>
<td>(0.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information value*** (theory)</td>
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<td>0.0</td>
<td>0.0</td>
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<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Information value*** (data)</td>
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<td>0.83</td>
<td>1.39</td>
<td>0.40</td>
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<td>(0.03)</td>
<td>(0.03)</td>
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<td>(0.04)</td>
<td>(0.07)</td>
<td>(0.10)</td>
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</tbody>
</table>

* Averages are calculated for deal games only.
** Means and standard errors are calculated by treating each session’s mean as a single observation.
*** Information value = the mean difference between the informed and uninformed payoffs.

**Result 2.** When the pie is small or medium ($\pi \leq 4$), players split it equally (the mode of the distribution of uninformed players’ payoffs is roughly half of the pie); In large pie games ($\pi > 4$) the share of the informed player increases and the mode of the uninformed players’ payoff distributions is $2$.

The distributions of uninformed players’ payoffs are plotted in Fig. 3 and the mean payoffs (conditional upon a deal being reached) are in Fig. 2b. We note that the distributions for large pie games ($\pi > 4$) also have smaller, yet clearly observable local maxima at a half of the pie.

**Result 3.** The informed players’ offers were monotonically increasing with time; the uninformed players’ demands were monotonically decreasing with time.

Result 3 is illustrated by the plots of mean bargaining positions shown in Fig. 4.

**Result 4.** Most of the deals were made just before the deadline.

More than half of the deals were made in the last two seconds of bargaining. Fig. 5 shows the cumulative distribution function (CDF) of deals over time for all pies, which sharply increased as the deadline approached in all states. Generally, deals were reached sooner when the pie was larger.
Figure 3: Uninformed players’ payoff distributions by pie size (deal periods only). The red lines locate the half of the pie in each distribution; the dashed black lines indicate the focal equilibrium payoff.
Figure 4: Mean bargaining position for pie (all periods)

Figure 5: Cumulative distribution of deal times by pie size. Median deal times are marked by an asterisk
5.2 Theory testing

We now turn to testing the predictions derived from the bargaining theory. We note that all bargaining positions lacked the players’ ability to commit, with the following exceptions: (a) positions at the deadline (i.e., 8.5 seconds into the bargaining process); (b) positions at the time a deal is made, 1.5 seconds after the positions’ initial match had occurred. We define these bargaining positions as Last Feasible Offers (LFOs) for the informed players and Last Feasible Demands (LFDs) for the uninformed players.

5.2.1 Use of focal points

In our experimental interface, players could communicate offers by multiples of $0.2; thus, offering exactly the half of a pie was feasible only for even pies. Accordingly, the empirical distributions of both players’ bargaining positions had sharp maxima (local and global) at bargaining positions matching exactly the half of even pie sizes, $1, $2 and $3, and smoother, yet clearly visible maxima at the offer values that are closest to the half of odd pie sizes, i.e., $0.5, $1.5 and $2.5. These modes are evident in the distribution of bargaining positions at all times, including the initial offers (Fig.6) and “final” offers (LFOs and LFDs), shown in Fig.7.

No other local maxima were observed in any of the informed player’s offer distributions (across time and pie-size), providing support for the use of half of the pie as bargaining focal points. For uninformed bargainers we also found local maxima in integer values of $3 – $6, though they were significantly smaller in size, and none of them existed in the LFD and LFO distributions.

5.2.2 The strike condition

Empirical disagreement rates fall smoothly with increasing pie size, and therefore do not fall as sharply as the theory predicts (Table 1, Fig. 2a). Strikes are common even at the largest pie size of $6, with a relative frequency of 0.19. It is important to note that in some interesting models, strikes can occur even with complete information (e.g. Haller and Holden, 1990). If the forces operating in such models also apply in our private-information settings, the strike rates could be larger than those predicted by the mechanism design approach.

We postulated that inefficient disagreements were the results of false revelations made by the informed players. To test our hypothesis we estimated three logistic regression models with the dependent measure \(deal = 1\) (i.e., \(strike = 0\)), that included subject-level dummy

\footnote{Our results are robust to the definition of LFO and LFD; specifically, we successfully replicated all of the analyses while setting the time of the LFO and LFD to 8 and 9 seconds into the bargaining process.}
Figure 6: Initial bargaining position distributions

(a) Informed player’s initial offers for pie (USD). The dashed red lines locate the half of the pie in each distribution.

(b) Uninformed player’s initial demands (SUD)
variables (for both informed and uninformed players) and controls for both period and location (Caltech or UCLA). We estimated a model that includes the pie size alone (Model A), the LFO alone (B) and both pie size and LFO (C). Our analysis (Table 2) reveals that Model B which includes the informed player’s LFO fits the data better than Model A which includes the pie size, as implied by a lower Akaike Information Criterion (AIC) score. Furthermore, when including both the pie size and the informed LFO in the model, the marginal effect of the latter was almost 6 times greater. Interestingly, the effect of period on the likelihood of reaching a deal was highly significant in model (A), but was eliminated by including the LFO, suggesting that all learning effects are mediated by the LFO.

Using our regression model, we estimated the empirical deal rates as a function of the informed player’s LFOs (see Table 3, second row) and found that the fitted likelihood of a strike in high stake games was much smaller (0.15 and 0.09) when the LFO was half of the pie ($2.5 and $3). The disagreement rate was 0.24 when the LFO was $2, higher than the efficient strike condition prediction (of no strikes) under the focal equilibrium offer. In smaller pie-sizes ($1, 2, 3), disagreement rates were also greater than predicted while qualitatively conforming with our equilibrium theory.

5.2.3 Incentive compatibility

In accordance with Lemma 1 (parts 1 and 3), both the deal rates and the uninformed players’ mean payoffs were monotonically increasing with the pie size (Fig.2, Table 1). The mode of the uninformed payoff distribution (Fig.3) was identical ($2) in all the pies where the strike condition did not hold, in accordance with the fourth condition. In Table 3 we calculated the informed player’s expected payoff in state $k$, $E[y_k]$ as a function of her announcement, by

$$E[y_k] = \hat{\gamma}_k (\pi_k - \hat{w}_k),$$

where $\pi_k$ is the realization of the pie, $\hat{w}_k$ is the informed player’s LFO and $\hat{\gamma}_k$ is its corresponding empirical deal rate (estimated by fitting model B, see Table 2). In accordance with the IC and PE predictions, the expected informed player’s payoff (Table 3) was maximal if she offered a half of the pie when the strike condition held and $2 when it did not hold - in three of the six states. The exceptions were the pies of $2, 3 and $4, where true telling was second to a riskier option of reporting a lower state; the differences were small and could

\[6\] The regression effects are robust to inclusion/exclusion of these controls.

\[7\] The regression results are robust to the inclusion of quadratic terms for the pie size and period (which effects are statistically insignificant) and to variation in the definition of LFO, by setting its time to $t = 8$ and $t = 9$. 

19
Table 2: Logistic regression, dependent variable: Deal = 1

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<th></th>
<th>A</th>
<th>B</th>
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<td>Pie size</td>
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<td>0.04***</td>
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<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td></td>
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<tr>
<td>Period</td>
<td>0.0007***</td>
<td>0.0002</td>
<td>0.0003</td>
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<td></td>
<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
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<tr>
<td>Informed LFO</td>
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<td>0.31***</td>
<td>0.23***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
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<td>0.01</td>
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<tr>
<td></td>
<td>(0.10)</td>
<td>(0.01)</td>
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<td>yes</td>
<td>yes</td>
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<tr>
<td>U sub. dummies</td>
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<td>yes</td>
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<td>7110.6</td>
<td>7066.8</td>
</tr>
<tr>
<td>No. observations</td>
<td>6432</td>
<td>6432</td>
<td>6432</td>
</tr>
</tbody>
</table>

Notes: Coefficient are reported as marginal effects.

*p < 0.1  **p < 0.05  ***p < 0.01
potentially be reconciled by attributing risk-averse preferences to our subjects.

Table 3: Expected informed player’s payoffs as a function of LFO and pie size, USD.

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<tr>
<th>LFO, $x_k$</th>
<th>0.5</th>
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<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical deal rate, $\gamma_k$</td>
<td>0.36</td>
<td>0.50</td>
<td>0.64</td>
<td>0.76</td>
<td>0.85</td>
<td>0.91</td>
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<td></td>
<td>(0.020)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.007)</td>
</tr>
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<td>$1$ pie</td>
<td>0.18</td>
<td>0.00</td>
<td>-0.32</td>
<td>-0.76</td>
<td>-1.27</td>
<td>-1.82</td>
</tr>
<tr>
<td>$2$ pie</td>
<td>0.54</td>
<td>0.50</td>
<td>0.32</td>
<td>0.00</td>
<td>-0.42</td>
<td>-0.91</td>
</tr>
<tr>
<td>$3$ pie</td>
<td>0.90</td>
<td>1.00</td>
<td>0.96</td>
<td>0.76</td>
<td>0.42</td>
<td>0.00</td>
</tr>
<tr>
<td>$4$ pie</td>
<td>1.26</td>
<td>1.50</td>
<td>1.60</td>
<td>1.52</td>
<td>1.27</td>
<td>0.92</td>
</tr>
<tr>
<td>$5$ pie</td>
<td>1.62</td>
<td>2.00</td>
<td>2.24</td>
<td>2.28</td>
<td>2.12</td>
<td>1.82</td>
</tr>
<tr>
<td>$6$ pie</td>
<td>1.98</td>
<td>2.50</td>
<td>2.88</td>
<td>3.05</td>
<td>2.97</td>
<td>2.73</td>
</tr>
<tr>
<td>Uninformed exp. payoff</td>
<td>0.18</td>
<td>0.50</td>
<td>0.96</td>
<td>1.52</td>
<td>2.12</td>
<td>2.73</td>
</tr>
</tbody>
</table>

Notes: Maximal values are **bold**, even splits are *italic.*

5.3 Dynamics and the deadline effect

As players cannot commit to their positions (and at the absence of temporal discounting), strike threats are not especially credible early in the game. Thus, it is not surprising that on average, informed players prefer to start by making low offers (in hope that some of them would be accepted), and that uninformed players “respond” to this behavior by waiting for better offers (Fig. 4). As a consequence most deals were made in the two seconds before the deadline (see Fig. 5).

To better understand the temporal dynamics of state revelation, we turn to the distribution of initial bargaining positions (Fig. 6). For all pies, the mode of the distribution of initial offers of the informed players was no greater than $1. Nevertheless, all of the distributions had a smaller local maxima at the true value, indicating players at times revealed the pie size immediately. Even the $6$ state also had a mode at $3$, implying unnecessary (from a self-regarding rational point of view) information sharing already at the initial offer stage. We further discuss this phenomena in Section 7.2. As the game progressed, more informed players revealed the true state by increasing their offers (see Fig. 4, left panel), such that as the deadline approached, all the LFOs distributions peaked at the values predicted by equilibrium (Fig. 7).

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8A risk averse informed player would increase her LFO and forgo some of her expected payoff in order to increase the probability of reaching a deal.
Figure 7: Final offer distributions.

(a) Informed player’s last feasible offers (LFOs) for pie, USD. The red lines mark the half of the pie in each distribution.

(b) Uninformed player’s last feasible demands (LFDs), USD.
6 Using process data

Our unstructured paradigm records bargaining process data that could be associated with outcome variables. This process data may be used to predict disagreements before the deadline has arrived. For example, suppose that at the 5 second mark, neither player has changed her offer for more than 3 seconds. This mutual stubbornness might be associated with an eventual strike. Our approach is to consider a large number of such candidate observable features in search of a small set that is predictive, using cross-validation (Stone, 1974) to control for overfitting. This machine learning approach has been used in many, many applications in computer science and neuroscience, and a few in economics (Einav and Levin, 2014; Varian, 2014; Belloni et al., 2012; Krajbich et al., 2009; Smith et al., 2014; Mullainathan, 2014).

One possibility is that there is little predictive information in such features, after controlling for overfitting. Indeed, if players know what the predictive features are, they should alter their behavior in order to avoid costly disagreements, erasing the features’ predictive power. Another possibility is that there are numerous small influences on disagreement that the players simply do not notice and which may be picked up by our modeling.

We chose 26 behavioral features recorded during bargaining (such as the current difference between the offers and the time since the last position change, see Fig. 10 for the full list), and randomly split the entire set of trials into ten groups. For each of the 10 holdout groups, we trained a model to classify trials into disagreements or deals, using the remaining 90% of the data, by estimating a logistic regression with a Least Absolute Shrinkage and Selection Operator (LASSO) penalty (Tibshirani, 1996; Smith et al., 2014; Varian, 2014; Belloni et al., 2012). By applying these trained models, we then conducted out-of-sample classification of the binary bargaining outcomes for each of the 10 holdout samples.

Obviously, the pie size is a strong predictor of disagreements. The challenge for our machine learning approach is whether process features have predictive power similar to the pie size when studied alone, and whether these features add predictive power when used together with the pie size. To investigate the predictive power of process data, we estimated

9A LASSO-penalized logistic regression maximizes the standard logistic regression log-likelihood function minus a penalty term equal to the the sum of their absolute values of the regression coefficients (their $L_1$ norm) to overcome potential overfitting of the training data. The procedure includes a pre-processing stage of standardizing the dependent variables to have mean 0 and standard deviation 1.

10We use cross-validation to determine the weight placed on the penalty term in the LASSO regression. In our setting cross-validation involves partitioning the training data into $k$ subsets, holding out one of the subsets, and calculating coefficient values (models) over a range of penalty weights. For each penalty weight, the model’s out-of-sample predictive performance is calculated on the hold-out sample. The process is then repeated by holding out each of the other $k - 1$ subsets, and the final penalty weight is chosen as the value of the penalty that results in the best out-of-sample predictive performance over all $k$ hold-out samples.
three strike prediction models at seven different points in the bargaining process, separated by 1 second intervals (e.g. 1, 2, ..., 7 seconds after bargaining started). One model relies only on the pie size, the second uses only process features, and the third combines both pie size and process features.\textsuperscript{11}

We evaluate our results using “Receiver Operating Characteristic” (ROC) curves (Hanley and McNeil\textsuperscript{[1982]} Bradley\textsuperscript{[1997]}). ROC is a standard tool in signal detection theory, used for quantifying the performance of a binary classifier under different trade-offs between type I and type II errors. A familiar example is a household smoke alarm: The alarm can be tuned to be very sensitive, indicating a fire when frying chicken creates too much smoke. Or it can be tuned to be insensitive, ignoring both the smoke from fried chicken and from a genuine fire caused by Grandpa’s half-lit cigar, after he fell asleep and knocked it onto his copy of the Daily Prophet. Devices can be tuned to create different percentages of false alarms and missed true alarms; the resulting ROC curve reflects the Pareto frontier of these two types of detection errors.

The use of an ROC curve reflects the fact that one can always create more true positives—in our example, predicting more strikes— but doing so comes at the cost of then predicting more false positives (predicted strikes that don’t happen). When using these methods, one would often like to know the tradeoff between correctly detecting true positives more accurately and also reducing the probability of false positives. A curve mapping all pairs of true and false positive levels therefore allows choosing an optimal policy for every given relative cost of the two types of errors.

To calculate the ROC, we subjected the out-of-sample predicted deal probabilities (calculated by applying the estimated logistic LASSO regression weights to the out-of-sample process data) to different decision thresholds; i.e., for a decision threshold \( TH \), all predicted values less than \( TH \) were classified as ”strike” where predicted values greater than or equal to \( TH \) were classified as ”deal”.\textsuperscript{12} Every point on the ROC, therefore, represents a decision threshold, such that its coordinates represent the empirical false positive and true positive rates, calculated using the threshold.

For a random classifier, the true positive and false positive rates are identical (the 45-degree line in Fig. 8). A good classifier increases the true positive rate (moving up on the y-axis) and also decreases the false positive rate (moving left on the x-axis). The difference between the ROC and the 45-degree line, in the upper-left direction, also known as the ”area under the curve” (AUC)(Bradley\textsuperscript{[1997]}) is an index of how well the classifier does.\textsuperscript{13}

\[\text{11} \text{We included only trials that were still in progress (when a deal has not yet been achieved), and excluded trials in which the offer and demand were equal at the relevant time stamp.}\]

\[\text{12} \text{We used decision threshold between 0 and 1 on a grid with a resolution of 0.01}\]

\[\text{13} \text{The AUC is closely related to the Mann-Whitney-Wilcoxon } U\text{-statistic (Hanley and McNeil} \textsuperscript{[1982]})\]
Our ROC analysis shows that process data does better than random for every time stamp (for illustration, see Fig.8). Furthermore, the mean out of sample prediction error of the classifier using solely process features is as low as a classifier using the pie size for time instances greater than 5 seconds into the bargaining process. Combining pie size and process features improves accuracy further: a classifier using both pie size and process data outperforms the classifier using the pie size alone as early as 2 seconds into the bargaining process (Fig. 8, 9).

To further investigate which behavioral process features predict disagreements, we used a “post-LASSO” procedure ([Belloni and Chernozhukov 2009]; Belloni et al. 2012). Our analysis reveals a rich set of behavioral features that reliably predict disagreements throughout the bargaining process (Fig. 10). For example, an absence of movement of the uninformed player (i.e., stubbornness) is associate with an eventual disagreements already 4 seconds into the bargaining process. Increased activity on the informed player side (i.e., many position changes) is a precursor of an upcoming deal, as early as one second into the bargaining process (see Fig. 10 for the complete list of features and their predictive power).

The statistical value of process measures is important for studying bargaining in naturalistic settings. The accuracy of features alone for predicting strikes (even without pie size) suggests that it could be possible to use this type of analysis to do statistical mediation. That is, an important, often overlooked body of theory in mechanism design shows that if the designer has an independent measure of private information (which the informed player cannot manipulate or hide), efficiency can be enhanced by conditioning mechanism outcomes on this independent measure.

Intuitively, suppose in our setting the pie size is $6. For the IC constraint to bind, the mechanism must impose strikes when a lower pie size is (untruthfully) reported, to prevent an informed player from misreporting that the pie is worth less than $6. But what if there were another indicator measure of pie size - which is sufficiently accurate and not manipulable? Then the mechanism could combine this indicator with the reported pie size, penalizing the informed player if her report and the indicator disagree.

A proof of principle that such a mechanism can work was offered by [Krajbich et al. 2009]. They used neural measures of private value for a public good in a threshold public goods game. In their domain, it was shown that the mechanism satisfies the voluntary participation (IR) constraint provided the mechanism is sufficiently accurate and agents are not too risk-averse.

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14The “post-LASSO” procedure consisted of three steps. First, we optimized the LASSO tuning parameter \( \lambda \) using 10-fold cross validation on the entire data set. Second, we conducted model selection by fitting a logistic LASSO regression using the optimized tuning parameter to the data. Finally, we fitted an ordinary logistic regression to the data, using the features with non-zero LASSO coefficients from the second stage.
In future work, process measures could be used as indicators of likely strikes, or as indicators of pie sizes, to create behaviorally-enhanced mechanisms which avoid disagreements. Such a process-informed mechanism can, in principle, reduce strikes and improve efficiency, while also satisfying voluntary participation constraints so that bargainers will agree to use them.

Figure 8: Strike prediction using bargaining process data, Receiver Operating Characteristic (ROC)

7 Further analyses

7.1 Inefficient disagreements

Contrary to the predictions of our theoretical model, bargaining failures occur relatively often at higher pie sizes. We conjecture that these unpredictably high strike rates are not due to risk-aversion, although deriving closed-form results under risk-aversion is beyond the scope of this paper. Intuitively, risk-aversion compresses the utilities from good and great bargaining outcomes closer together, which implicitly makes strike outcomes (= 0 payoff) much worse. Through the IR constraint, more concave utility should therefore motivate more information revelation which lowers strikes. As we have discussed in section 5.2.3, these incidents could be partly explained by informed players’ offering less than a half of the
Figure 9: Out of sample strike prediction mean square error across time

Figure 10: Bargaining process features used for outcome prediction (deal=1) and their significance levels (post LASSO t-values, trimmed at $|t| = 4$).

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<th>3</th>
<th>4</th>
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<td>Uninformed number of changes</td>
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<td>Informed weighted average</td>
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<td>Uninformed weighted average</td>
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\(\pi_c\) (which is $4 in our game). However, disagreements still occurred even when a large pie had been reported (by offering $2 or more).

We investigated the uninformed players’ LFDs in high stake disagreement games to shed light on these bargaining failures. A possible source of disagreements when the strike condition does not hold lies in our experimental design: our task leaves room for mis-coordination at the end of the game, if both players simultaneously change their position at the very final moment \((t = 8.5s)\). Looking at the LFD distribution, we observe that such incidents occur only in a small fraction (10.8\%) of inefficient disagreement games. In the vast majority of these games, the uninformed players demanded more than $2, the half of the cutoff pie. The LFD CDF (Fig. 11) shows that 73.3\% of these disagreements occurred because demands of less than $3 but more than $2 (with a mode peaking at the focal point of $3 and local modes at $2.4 and $2.6); an additional 13\% occurred due to demands of more than $3. Given that disagreements in games where the informed players offer $2 or more are much less frequent in an ultimatum version of our game (see Mitzkewitz and Nagel (1993) Fig.3 p.178) it is unlikely that such disagreements could be attributed to preferences over payoff distributions per se, but rather relate to dynamics of the bargaining procedure-in line with the predictive power of process data for bargaining outcomes (section 6).

Figure 11: Uninformed LFDs during inefficient disagreements (when LFO is greater than $2), empirical cumulative distribution

7.2 Information sharing in high-stake games

Our efficient focal equilibrium analysis predicted that informed players would reveal the true pie size (by offering half of it) only when the strike condition does not hold \((\pi \leq 4)\). The modes of the distributions of the informed player’s LFO do match this prediction. However,
a fraction of the offers were greater than half of the pie even when the strike condition did not hold, resulting in local maxima of the LFO distribution at 2.5 and 3 for \( \pi = 5, 6 \). Such high offers are also apparent in the ultimatum version of our game (see Mitzkewitz and Nagel (1993) Table 1a p.176). Why do informed players reveal so much information in these high-value states?

One possible account is strategic: given that a fraction of the uninformed players does not play the efficient equilibrium (i.e. demand more than a half of the cutoff pie), informed players should make higher offers that increase their probability of reaching a deal at the expense of their share of the pie (see Table 3). Risk aversion may increase this incentive to reach a deal. A second possible explanation is that players are motivated by additional factors besides their own payoffs. The literature on other-regarding preferences suggests that such motivations might include inequality aversion (Fehr and Schmidt, 1999), social preferences and reciprocity (Rabin, 1993; Charness and Rabin, 2002; Dufwenberg and Kirchsteiger, 2004), social image (Andreoni and Bernheim, 2009) or lying aversion (Gneezy, Rockenbach and Serra-Garcia, 2013; Gibson, Tanner and Wagner, 2013).

The unstructured offer dynamics allow us to separate the strategic and distributional preference explanations. Given that the majority of offers over $2 are accepted, and as such offers yield greater expected payoffs than more generous offers (see Table 3), we hypothesize that strategic information sharing should not occur in the initial offers that can be updated further in the game. On the contrary, preference-driven information sharing should occur right at the start of the game.

Out of 1038 total trials with pies of 5-6, 350 trials had offers greater than $2 before the deadline. Of those 350 trials, 156 (44.5%) had offers greater than $2 already in the initial offer stage and 121 (34.6%) were made in the last two seconds before the deadline, as reflected by a steep increase in the cumulative offers distribution as the deadline approaches (see Fig. 12). These dynamics suggest that both preference-based factors (leading to initial offers that are overly generous) and strategic or process-driven factors (leading to an increase in last-second generous offers) contribute to unnecessary information sharing.

8 Conclusion

Much of the recent literature on bargaining has studied structured bargaining. We reiterate here our motivations for studying unstructured bargaining in dynamic and uncertain environments. First, much real-world bargaining is unstructured and involves private information. Next, theoretical methods are available to make predictions regarding behavior in such games. Finally, unstructured bargaining generates process data which may stimulate
In this paper we study dynamic unstructured bargaining in a game with one-sided private information. We combine mechanism design theory with an equilibrium selection model based upon focal points which reflect an equal split of the surplus available in each state of the world. Our approach is agnostic regarding the driving force behind equal splits. A large theoretical literature attempts to address the question of why equal splits are focal; equal splits might result, for example, from inequality aversion, concerns about fairness, or social norms. Another explanation might be lying aversion, and our experimental design, which incorporates feedback after each round of bargaining, may encourage truthful revelation. However, our design also involves random, anonymous re-matching of bargaining partners after each game, which might be expected to act in the opposite direction.

We acknowledge that our laboratory bargaining institution abstracts from a number of features of real-world bargaining: much real-world bargaining occurs face-to-face and with less anonymity than in our design. Real-world bargaining may involve repetition and reputation. In addition, real-world bargaining often involves two-sided, rather than one-sided, private information. Furthermore, our laboratory design is context-free, and contextual cues may influence behavior in real-world bargaining. Yet, our design is straightforward to implement in the laboratory and for subjects to understand, and it allows for players to employ broad range of dynamic bargaining strategies.

Our theoretical model predicts that the rate of bargaining failures will be monotonically decreasing in the pie size and that the distribution of surplus will favor the informed player when the pie size crosses a threshold. We find support for both of these hypotheses in our
data. However, we also observe an interesting departure from the theoretical benchmark, that bargaining failures arise even at the highest pie levels and even after many rounds of play.

We use process data to investigate how bargaining dynamics affects outcomes and whether it might help to explain these deviations from our theoretical model. Our machine learning approach shows that process data is incrementally informative for predicting strikes when the pie size is included in the model. This result suggests that bargaining failures may result from process “mistakes” that could have been avoided if players had behaved differently. Process data may be used to avert strikes and other inefficient disagreements by offering ‘course corrections’ in the bargaining process. We argue that further understanding of these process-driven disagreements will require more study of unstructured bargaining.

References


Roth, Alvin E. 1985. “Toward a focal point theory of bargaining.” *Game-theoretic models of bargaining*, 259–268. 4 8


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A Mathematical Appendix

A.1 Proof of Lemma 1

As noted in the main text (Equation 4) incentive compatibility for the informed player requires

$$\gamma_k \pi_k - x_k \geq \gamma_j \pi_k - x_j \text{ for all } j \neq k.$$ 

We first prove by induction that $$\gamma_k$$ is decreasing in $$k$$ (Lemma 1.1), and then rely on Lemma 1.1 for the proofs of Lemmas 1.2 and 1.3.

Proof. First, incentive compatibility for pie sizes $$\pi_1$$ and $$\pi_2$$ requires

$$\gamma_1 \pi_1 - x_1 \geq \gamma_2 \pi_1 - x_2$$

$$\gamma_2 \pi_2 - x_2 \geq \gamma_1 \pi_2 - x_1$$
which implies that 
\[(\gamma_2 - \gamma_1)\pi_2 \geq x_2 - x_1 \geq (\gamma_2 - \gamma_1)\pi_1\]
and therefore 
\[(\gamma_2 - \gamma_1)(\pi_2 - \pi_1) \geq 0\]
implies that \(\gamma_2 \geq \gamma_1\).

Next, consider \(\pi_k\) and \(\pi_{k+1}\). Incentive compatibility requires
\[
\gamma_k\pi_k - x_k \geq \gamma_{k+1}\pi_{k+1} - x_{k+1}
\]
\[
\gamma_{k+1}\pi_{k+1} - x_{k+1} \geq \gamma_k\pi_k - x_k
\]
These two equations imply that
\[
(\gamma_{k+1} - \gamma_k)\pi_{k+1} \geq x_{k+1} - x_k \geq (\gamma_{k+1} - \gamma_k)\pi_k
\]
and therefore
\[
(\gamma_{k+1} - \gamma_k)(\pi_{k+1} - \pi_k) \geq 0
\]
By definition, \(\pi_{k+1} \geq \pi_k\), so then \(\gamma_{k+1} \geq \gamma_k\), and therefore the strike rate \(1 - \gamma_k\) is monotonically decreasing in the pie size.

The remaining results follow directly from Equations 9 and Lemma 1.1. By Lemma 1.1, 
\[(\gamma_{k+1} - \gamma_k)\pi_k \geq 0, \text{ so by Equation 9 } x_{k+1} - x_k \geq 0, \text{ and therefore the uninformed player’s}
\]
payoffs are monotonically increasing in the pie size (Lemma 1.2).

Furthermore, replacing \(\gamma_k = \gamma_{k+1} = 1\) in the righthand inequality of Equation 9, it immediately follows that \(x_k = x_{k+1}\) (Lemma 1.3).

\[\square\]

A.2 Proof of Lemma 2

A mechanism is efficient if it is Pareto optimal for the set of \(K + 1\) agents: the informed player in for each pie size, and the uninformed player.

We first show that strikes in the “best” pie size \(\pi_K\) are never efficient for the class of direct mechanisms that we consider. That is, if the mechanism \(\mu = \{\gamma_k, x_k\}_{k=1}^K\) is efficient, then it must be the case that \(\gamma_K = 1\).

If \(\mu\) is an efficient mechanism, then the incentive compatibility conditions must hold and so by Lemma 1 \(\gamma_K \geq \gamma_k\) for all \(k \leq K\). If \(\gamma_K = 1 - \delta < 1\), we can define a new mechanism \(\mu^*\) with \(\gamma^*_K = 1\), \(\gamma^*_k = \gamma_k + \frac{\delta}{K-1}\), for all \(k < K\), and \(x^*_k = x_k\), for all \(k\). The mechanism \(\mu^*\) does not affect the uninformed player’s expected payoff, but it increases the informed player’s payoff by \(\delta \pi_K\) in state \(K\) and by \(\frac{\delta}{K-1}\pi_k\) in states \(1, \ldots, K - 1\), so the original mechanism
cannot be efficient.

Next, if $\gamma_k, k < K$, can be increased without violating the IC constraint, the uninformed bargainer is unaffected as is the informed bargainer in states $j \neq k$, while player $I_k$, the informed bargainer in state $k$, is made better off. Therefore, following Eq. 9 efficiency requires that the following condition holds:

$$x_{k+1} - x_k = (\gamma_{k+1} - \gamma_k)\pi_{k+1}. \quad (11)$$

**The Strike Condition**

Preliminaries: consider a generic mechanism is $\mu = ((\gamma_k)_{k=1}^K, (x_k)_{k=1}^K)$. An alternative mechanism is $\mu^* = ((\gamma_k + \delta_k)_{k=1}^K, (x_k + d_k)_{k=1}^K)$. If $\mu^*$ satisfies IC, then

$$(x_{k+1} + d_{k+1}) - (x_k + d_k) = ((\gamma_{k+1} + \delta_{k+1} - (\gamma_k + \delta_k))\pi_{k+1}$$

If both $\mu$ and $\mu^*$ satisfy IC, we have, by subtracting IC from IC$^*$,

$$d_{k+1} - d_k = (\delta_{k+1} - \delta_k)\pi_{k+1} \quad (12)$$

We now extend the strike condition from FKSs 2-state model to a model with K states. Consider a mechanism $\mu = ((\gamma_k)_{k=1}^K, (x_k)_{k=1}^K)$, and suppose that strikes are inefficient in state 2, ..., $K$, so that $\gamma_2 = \cdots = \gamma_K = 1$.

**Example 1: Efficient strikes in State 1 only**

If strikes are not efficient in any state, including state 1, there is another mechanism $\mu^* = ((\gamma_k + \delta_k, 1, \ldots, 1), (x_k + d_k)_{k=1}^K)$ that dominates $\mu$. Let $\Delta V_k, \Delta U$ denote the difference in payoffs between $\mu^*$ and $\mu$ for the informed players in state $k$ and the uninformed player, respectively. Then

$$\Delta V_1 = \delta_1\pi_1 - d_1$$
$$\Delta V_k = -d_2 = \delta_2\pi_2 - d_1, \forall k > 1$$
$$\Delta U = \sum_{k=1}^K p_k d_k$$
$$= d_1 - (1 - p_1)\delta_1\pi_2$$

Where the second and third lines follow from the fact that $d_K = d_{K-1} = \cdots = d_2 = d_1 - \delta_1\pi_2$ by Equation 12 and the fact that all $\delta_k = 1$ if $k \geq 2$. If all $K+1$ changes are
nonnegative and one is positive, then \( \delta_1 \) is not zero, and

\[
\begin{align*}
\delta_1 \pi_1 & \geq d_1 \geq (1 - p_1) \delta_1 \pi_1 \\
\delta_1 \pi_2 & \geq d_1 \geq (1 - p_1) \delta_1 \pi_2
\end{align*}
\]

corresponding to the equations at the bottom of page 256 of FKS. The first set of conditions imply that \( \delta_1 \) is nonnegative, and the second set of conditions imply that \( \pi_1 > (1 - p_1) \pi_2 \). In this case strikes in state 1 cannot be efficient, since values of \( \delta_1 \) and \( d_1 \) can be chosen such that \( \mu^* \) dominates \( \mu \). If the ‘strike condition’ \( (1 - p_1) \pi_2 > \pi_1 \) holds, then mechanisms exist where striking in state 1 is efficient.

**Example 2: Efficient Strikes in States 1 and 2 but not 3.**

Consider mechanisms \( \mu, \mu^* \) that meet the IC constraints. Suppose that \( \gamma_1 < \gamma_2 < 1 \), but that \( \gamma_k = 1 \) for all \( k \geq 3 \).

We make use of Equation 12 repeatedly. If \( \mu^* \) dominates \( \mu \), then

\[
\begin{align*}
\Delta V_1 &= \delta_1 \pi_1 - d_1 \geq 0 \\
\Delta V_2 &= \delta_2 \pi_2 - d_2 \geq 0 \\
\Delta V_k &= -d_3 = \delta_2 \pi_3 - (\delta_2 - \delta_1) \pi_2 - d_1 \geq 0, \forall k > 2 \\
\Delta U &= \sum_{k=1}^{K} p_k d_k \\
&= p_1 d_1 + p_2 (d_1 + (\delta_2 - \delta_1) \pi_2) + (1 - p_1 - p_2) (d_1 + (\delta_2 - \delta_1) \pi_2 - \delta_2 \pi_3) \\
&= d_1 + (1 - p_1)(\delta_2 - \delta_1) \pi_2 - (1 - p_1 - p_2) \delta_2 \pi_3 \geq 0
\end{align*}
\]

The condition for \( \Delta U \) implies

\[
d_1 + (1 - p_1)(\delta_2 - \delta_1) \pi_2 \geq (1 - p_1 - p_2) \delta_2 \pi_3
\]

Multiplying the conditions for \( \Delta V_1 \) and \( \Delta V_2 \) by \( p_1 \) and \( 1 - p_1 \) respectively, and adding up the two conditions, gives

\[
\begin{align*}
p_1 \delta_1 \pi_1 + (1 - p_1) \delta_2 \pi_2 & \geq p_1 d_1 + (1 - p_1) d_2 \\
p_1 \delta_1 \pi_1 + (1 - p_1) \delta_2 \pi_2 & \geq d_1 + (1 - p_1)(\delta_2 - \delta_1) \pi_2
\end{align*}
\]
Therefore we have

\[ p_1 \delta_1 \pi_1 + (1 - p_1) \delta_2 \pi_2 \geq d_1 + (1 - p_1)(\delta_2 - \delta_1)\pi_2 \geq (1 - p_1 - p_2)\delta_2 \pi_3 \]

If we assume \( \delta_1 = 0 \) and \( \delta_2 \geq 0 \) then this implies that strikes in state 2 are inefficient if

\[ \frac{\pi_2}{\pi_3} \geq \frac{1 - p_1 - p_2}{1 - p_1} \]

And therefore strikes are efficient in state 2 if

\[ \frac{\pi_2}{\pi_3} < \frac{1 - p_1 - p_2}{1 - p_1} \]

This is the extension of the “strike condition” from FKS.

**Example 3: Efficient strikes in state k but not state k+1**

We next extend the example to the situation where strikes are efficient in state \( k \) but not in state \( k + 1 \). Extending the above condition, we expect to find that strikes are inefficient in state \( k \) (but not necessarily state \( k - 1 \)) if

\[ \frac{\pi_k}{\pi_{k+1}} \geq \frac{\Pr(\pi \geq \pi_{k+1})}{\Pr(\pi \geq \pi_k)} \tag{13} \]

and hence if

\[ \frac{\pi_k}{\pi_{k+1}} < \frac{\Pr(\pi \geq \pi_{k+1})}{\Pr(\pi \geq \pi_k)} \tag{14} \]

then strikes are efficient in \( k \).

To derive the strike condition, we let \( \mu = ((\gamma_j)_{j=1}^K, (x_j)_{j=1}^K) \) and \( \mu^* = ((\gamma_j + \delta_j)_{j=1}^K, (x_j + d_j)_{j=1}^K) \). Assume that strikes are not efficient in states \( k + 1, \ldots, K \), so that \( \gamma_j = 1 \) if \( j > k \),
and note that this implies that $d_{k+1} = \ldots = d_K$. If $\mu^*$ dominates $\mu$, then

$$
\Delta V_1 = \delta_1 \pi_1 - d_1 \geq 0 \\
\vdots \\
\Delta V_j = \delta_j \pi_j - d_j \geq 0, \; j < k \\
\Delta V_k = \delta_k \pi_k - d_k \geq 0 \\
\Delta V_j = \pi_{k+1} \delta_k - d_k \geq 0, \; j > k \\
\Delta U = \sum_{j=1}^{K} p_j d_j \\
= \sum_{j=1}^{k} p_j d_j + (1 - \sum_{j=1}^{k} p_j) d_{k+1} \geq 0
$$

Multiplying the conditions for players $I_1, \ldots, l_k$ by $p_j$ and summing them up gives

$$
\sum_{j=1}^{k} p_j \pi_j \delta_j \geq \sum_{j=1}^{k} p_j d_j
$$

Multiplying the equation for player $k$ by $(1 - \sum_{j=1}^{k-1} p_j)$ gives

$$
(1 - \sum_{j=1}^{k-1} p_j) \delta_k \pi_k \geq (1 - \sum_{j=1}^{k-1} p_j) d_k
$$

Adding up these two conditions gives:

$$
\sum_{j=1}^{k} p_j \pi_j \delta_j + (1 - \sum_{j=1}^{k-1} p_j) \delta_k \pi_k \geq \sum_{j=1}^{k} p_j d_j + (1 - \sum_{j=1}^{k-1} p_j) d_k \quad (15)
$$
Next the condition for $\Delta U$ gives:

$$\begin{align*}
\sum_{j=1}^{k} p_j d_j + (1 - \sum_{j=1}^{k} p_j) d_{k+1} &\geq 0 \\
\sum_{j=1}^{k} p_j d_j + (1 - \sum_{j=1}^{k} p_j) (d_k - \delta_k \pi_{k+1}) &\geq 0 \\
\sum_{j=1}^{k-1} p_j d_j + p_k d_k + (1 - \sum_{j=1}^{k} p_j) d_k - (1 - \sum_{j=1}^{k} p_j) \delta_k \pi_{k+1} &\geq 0 \\
\sum_{j=1}^{k-1} p_j d_j + (1 - \sum_{j=1}^{k-1} p_j) d_k - (1 - \sum_{j=1}^{k} p_j) \delta_k \pi_{k+1} &\geq 0 \\
\sum_{j=1}^{k-1} p_j d_j + (1 - \sum_{j=1}^{k-1} p_j) d_k &\geq (1 - \sum_{j=1}^{k} p_j) \delta_k \pi_{k+1} \quad (16)
\end{align*}$$

Combining Equations (15) and (16) gives

$$\begin{align*}
\sum_{j=1}^{k-1} p_j \pi_j \delta_j + (1 - \sum_{j=1}^{k-1} p_j) \delta_k \pi_k &\geq \sum_{j=1}^{k-1} p_j d_j + (1 - \sum_{j=1}^{k-1} p_j) (d_k) \geq (1 - \sum_{j=1}^{k} p_j) \delta_k \pi_{k+1} \quad (17)
\end{align*}$$

And this implies that

$$\begin{align*}
\sum_{j=1}^{k-1} p_j \pi_j \delta_j + (1 - \sum_{j=1}^{k-1} p_j) \delta_k \pi_k &\geq (1 - \sum_{j=1}^{k} p_j) \delta_k \pi_{k+1} \quad (18)
\end{align*}$$

If we assume the strike rate is optimal for all states $j < k$, then $\delta_j$ equals 0 for all $j \leq k - 1$, implying that

$$\begin{align*}
(1 - \sum_{j=1}^{k-1} p_j) \delta_k \pi_k &\geq (1 - \sum_{j=1}^{k} p_j) \delta_k \pi_{k+1} \quad (19)
\end{align*}$$

If $\delta_k > 0$, then strikes are inefficient in state $k$ if

$$\frac{\pi_k}{\pi_{k+1}} \geq \frac{(1 - \sum_{j=1}^{k} p_j)}{(1 - \sum_{j=1}^{k-1} p_j)}$$

implying that strikes are efficient in state $k$ if

$$\frac{\pi_k}{\pi_{k+1}} < \frac{(1 - \sum_{j=1}^{k} p_j)}{(1 - \sum_{j=1}^{k-1} p_j)}$$
or alternatively
\[
\frac{\pi_k}{\pi_{k+1}} < \frac{\Pr(\pi \geq \pi_{k+1})}{\Pr(\pi \geq \pi_k)}
\] (20)
which matches our conjecture in Equation 14.

A.3 Calculating strike rates using focal points

The strike condition implies that disagreement is inefficient when the pie size is 4, 5 or 6, so we first fix \( \gamma_4 = \gamma_5 = \gamma_6 = 1 \). Based on the strike condition and the equal split principle, payoffs conditional on a deal are \( x_6 = x_5 = x_4 = 2, x_3 = 1.5, x_2 = 1 \) and \( x_1 = 0.5 \). As no disagreement should occur for \( \pi \in \{4, 5, 6\} \) and as the predicted equilibrium conditional payoff is \( x_4 = \$2 \) for these pies, it follows that the informed player’s payoff for \( \pi \in \{5, 6\} \) is always greater than for \( \pi = 4 \). Therefore, we set \( x_k = 0.5\gamma_k \pi_k \) and then solve the IC inequalities for \( \pi \leq \$4 \):

\[
\gamma_j \leq \frac{0.5\pi_k}{\pi_k - 0.5\pi_j} \gamma_k \text{ for all } k \leq 4, j \neq k.
\] (21)

Solving the inequalities for \( k = 4 \) and \( j = 3, 2, 1 \) yields

\[
\gamma_3 \leq \frac{2}{2.5}
\] (22)

\[
\gamma_2 \leq \frac{2}{3}
\] (23)

\[
\gamma_1 \leq \frac{2}{3.5}.
\] (24)

Solving the inequalities for \( k = 3 \) and \( j = 4, 2, 1 \) yields

\[
\gamma_3 > \frac{2}{3}
\] (25)

\[
\gamma_2 < \frac{1.5}{2} \gamma_3
\] (26)

\[
\gamma_1 < \frac{1.5}{2.5} \gamma_3.
\] (27)

Solving the inequalities for \( k = 2 \) and \( j = 4, 3, 1 \) yields

\[
\gamma_2 > 0
\] (28)

\[
\gamma_2 > 0.5\gamma_3
\] (29)
Table 4: Session information, I-Informed, U-Uninformed

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<td>32</td>
<td>23</td>
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<td>24</td>
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\[ \gamma_2 > 1.5\gamma_1. \] (30)

Finally, for \( k = 1 \) it is always optimal to report the truth if \( \gamma_1 > 0 \), as offers exceeding 1 would generate a non-positive payoffs.

Maximal efficiency requires the largest possible values of \( \gamma_1, \gamma_2, \gamma_3 \) which are compatible with the IC inequalities. The only upper constraint on \( \gamma_3 \) is equation (22): thus, we set \( \gamma_3 = \frac{2}{25} = 0.8 \). The lowest upper constraint on \( \gamma_2 \) is equation (26); accordingly we set \( \gamma_2 = \frac{1.5}{2} \gamma_3 = 0.6 \). The value of \( \gamma_1 \) is constrained by equation (26), to be less than \( .8 \times \frac{1.5}{25} = .48 \) and is constrained by equation (30) to be \( \gamma_1 < \gamma_2/1.5 = 0.4 \). Therefore, the maximal value is \( \gamma_1 = .4 \).

B Pooling data

B.1 Caltech SSEL vs. UCLA CASSEL

Summary information of all of the experimental sessions (location, number of subjects and gender by role) is recapitulated in Table 4. For comparing the sessions taking places at Caltech vs. UCLA, we first calculated the mean deal rates and payoffs (in case of a deal) for each subject and pie size, and contrasted the group averages (see Table 5). Qualitatively, deal rates and payoff were monotonically increasing with the pie for both groups. The most significant difference observed between the groups was a 9 percent increase of deal rates in the largest pie ($6) at Caltech sessions. We used a 2-sided t-test to compare Caltech and UCLA subjects; while for some of the pies we found statistically significant differences at the 0.05 level, none of the differences survived correction for multiple hypothesis (\( p_{max} = 0.096 \) using the Bonferroni correction, for deal rates at $6 pie).
Table 5: Average payoffs (case of deal) and deal rates by pie size, Caltech vs. UCLA

<table>
<thead>
<tr>
<th>Pie size</th>
<th>Venue</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deal rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Caltech</td>
<td>0.43</td>
<td>0.50</td>
<td>0.56</td>
<td>0.71</td>
<td>0.75</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>UCLA</td>
<td>0.34</td>
<td>0.42</td>
<td>0.51</td>
<td>0.63</td>
<td>0.71</td>
<td>0.76</td>
</tr>
<tr>
<td>p-value*</td>
<td></td>
<td>0.14</td>
<td>0.29</td>
<td>0.42</td>
<td>0.11</td>
<td>0.36</td>
<td>0.03</td>
</tr>
<tr>
<td>Payoff, informed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Caltech</td>
<td>0.39</td>
<td>0.98</td>
<td>1.60</td>
<td>2.23</td>
<td>3.02</td>
<td>3.83</td>
</tr>
<tr>
<td></td>
<td>UCLA</td>
<td>0.36</td>
<td>0.95</td>
<td>1.55</td>
<td>2.31</td>
<td>3.19</td>
<td>4.06</td>
</tr>
<tr>
<td>p-value*</td>
<td></td>
<td>0.67</td>
<td>0.61</td>
<td>0.56</td>
<td>0.40</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Payoff, uninformed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Caltech</td>
<td>0.61</td>
<td>1.05</td>
<td>1.45</td>
<td>1.82</td>
<td>2.01</td>
<td>2.19</td>
</tr>
<tr>
<td></td>
<td>UCLA</td>
<td>0.66</td>
<td>1.12</td>
<td>1.50</td>
<td>1.75</td>
<td>1.85</td>
<td>2.01</td>
</tr>
<tr>
<td>p-value*</td>
<td></td>
<td>0.44</td>
<td>0.21</td>
<td>0.40</td>
<td>0.37</td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>

*Two-sided t-tests, uncorrected for multiple comparisons.

B.2 First vs. second half of the trials

To compare the first and second halves of bargaining periods, we calculated the mean deal rates and payoffs (in case of a deal) for each subject at any given pie size, and contrasted the averages of the first and second halves of the periods (see Table 6). Qualitatively, deal rates and payoff were monotonically increasing with the pie for both groups. The largest difference observed was 8 percent increase of efficiency (deal rates) in the second half compared to the first one, when the pie was $6. We further used a 2-sided t-test to compare the two halves. While for some of the pies we found statistically significant differences at the 0.05 level (in particular, deal rates were higher and informed players’ payoffs in case of a deal were lower at the Caltech pool), none of the differences survived correction for multiple hypothesis ($p_{max} = 0.24$ using Bonferroni correction).

C Instructions

This is an experiment about bargaining. You will play 120 rounds of a bargaining game.

In the game, one participant (the informed player) is told the total amount of money (pie size) in each round. This amount will be $1, 2, 3, 4, 5, or 6, chosen randomly in each trial. The amount will appear on the top left corner of the screen.

The other player is not informed of the pie size.
Table 6: Average payoffs (case of deal) and deal rates by pie size, first vs. second half of the trials

<table>
<thead>
<tr>
<th>Pie size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deal rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First 60</td>
<td>0.38</td>
<td>0.47</td>
<td>0.49</td>
<td>0.63</td>
<td>0.72</td>
<td>0.76</td>
</tr>
<tr>
<td>Last 60</td>
<td>0.39</td>
<td>0.45</td>
<td>0.58</td>
<td>0.70</td>
<td>0.73</td>
<td>0.84</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.97</td>
<td>0.61</td>
<td>0.07</td>
<td>0.09</td>
<td>0.68</td>
<td>0.02</td>
</tr>
<tr>
<td>Payoff, informed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First 60</td>
<td>0.43</td>
<td>1.02</td>
<td>1.63</td>
<td>2.32</td>
<td>3.17</td>
<td>4.03</td>
</tr>
<tr>
<td>Last 60</td>
<td>0.31</td>
<td>0.91</td>
<td>1.52</td>
<td>2.23</td>
<td>3.05</td>
<td>3.89</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.08</td>
<td>0.03</td>
<td>0.08</td>
<td>0.15</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>Payoff, uninformed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First 60</td>
<td>0.60</td>
<td>1.04</td>
<td>1.41</td>
<td>1.74</td>
<td>1.88</td>
<td>2.00</td>
</tr>
<tr>
<td>Last 60</td>
<td>0.68</td>
<td>1.13</td>
<td>1.53</td>
<td>1.82</td>
<td>1.99</td>
<td>2.17</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.14</td>
<td>0.07</td>
<td>0.04</td>
<td>0.15</td>
<td>0.10</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*Two-sided t-tests, uncorrected for multiple comparisons.

During each round, participants bargain over the uninformed player’s payoff.

The roles are randomly selected and fixed for the duration of the experiment. Before each round, informed and uninformed players are randomly matched.

Participants negotiate by clicking on a scale from $0 to 6 (see figure 1). Amounts on the scale represent the uninformed player’s payoff.

During the first 2 seconds, participants select their initial offers. Note that the initial location of the cursors is random. In the following 10 seconds, the participants bargain, using the mouse to select payoffs for the uninformed player. Clicking the mouse on a different part of the scale moves the cursor.

A deal occurs when the cursors are in the same place for 1.5 seconds. When both cursors are in the same place on the scale, a green rectangle will appear (see figure 2).

If a deal is made, the informed player’s payoff is equal to the pie size minus the negotiated uninformed player’s payoff. If the agreement exceeds total amount of money, the payoff will be negative.
If no deal has been made after 10 seconds of bargaining, both participants get $0.

Following each trial, the uninformed player will be shown of the pie size.

The game has total 120 trials.

Before the experiment begins there will be 15 training trials, to allow you to practice.

At the end of the game, you will receive payment based on randomly selected 10% of your trials.

You will receive a $5 participation fee in addition to whatever you earn from playing the game.

Quiz

Total amount is $3. Cursors were matched in $1. How much money does the informed participant get? How much does the uninformed participant get?

Total amount is $2. Cursors were matched in $4.1. How much money does the informed participant get? How much does the uninformed participant get?

One second before the end of the trial, both participants have agreed on payoff of $2 and the green rectangle appears. What is going to happen when the trial ends?

Both participants have agreed on payoff of $2 and the green rectangle appears. After one second, the uninformed player changed his offer to $2.5. What is going to happen?