A Theory of Subprime Mortgage Lending, with Applications to the Rise and Fall of the Subprime Conduit Market*

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Abstract

We present a general equilibrium model of a subprime economy characterized by limited recourse mortgages, asymmetric borrower credit quality information, and mortgage lenders that either own or sell the loans they originate. Portfolio lenders have access to soft information, but cannot securitize mortgages and are capacity constrained. Conduit lenders originate mortgages based on hard information only, but have access to the liquidity from the securitized investment market. The trade-off between borrower adverse selection and secondary market liquidity determines the equilibrium size of the portfolio and conduit loan markets in our model. House prices, mortgage rates, loan amounts, and consumers’ tenure choice are also endogenous in our model. We show how changes in the available credit scoring technology and secondary market liquidity can trigger a change in the equilibrium regime. Our theory rationalizes the rise and fall of the subprime mortgage market.

Key words: subprime lending; credit scoring technology; portfolio lenders; conduit lenders; general equilibrium; endogenous mortgage market segmentation.

JEL Classification numbers: R21, R3, R52, D4, D5, D53.

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1 Introduction

One of the developments preceding the Great Recession was the segmentation of the subprime mortgage market (traditional portfolio lenders versus remote conduit lenders) and the huge expansion of subprime lending and mortgage securitization and subsequent collapse. We build a theory based on informational and liquidity differences that can explain the observed market segmentation and the rise and fall of the subprime mortgage market.

In particular, we show that a crude or even non-existent hard credit scoring technology was enough to explain why traditional relationship lenders, whose business model was to “originate-to-own”, were the only ones that operated in the subprime mortgage market before mid-1990s. These lenders - also referred to as “portfolio lenders” - had cheap access to soft credit risk information, and this allowed them to screen between subprime borrowers of different default risk type. However, these traditional portfolio lenders were capacity constrained, and this left many potential high quality subprime borrowers without a mortgage - these leftovers preferred to rent than borrowing at a prohibitive high mortgage rate from other potential lenders who only relied on poor hard credit information. For this context we identify an equilibrium regime where only portfolio lenders are active, and a small number of high quality consumers are able to buy a house with a subprime mortgage.

We then show that the new and better hard credit scoring technologies that became available in the early to mid 1990s, such as FICO scores and consumer’s credit history, was sufficient to trigger the emergence of the subprime conduit mortgage. With a better hard credit information, conduit lenders, who only relied on hard information and whose business model was primarily (but not exclusively) “originate-to-distribute”, were able to attract low risk consumers (good type) by offering them a better mortgage rate than before, but still at worse terms than portfolio lenders. We identify the parameter thresholds for this equilibrium regime where both portfolio lenders and conduit lenders actively lent to different pools of borrowers at different mortgage rates.

Afterwards, in the early 2000s, the conduit lender’s “originate-to-distribute” business model became predominant: all higher quality borrowers preferred to migrate to the subprime conduit mortgage market leaving traditional portfolio lenders with a small market share of leftovers. In our model, this new equilibrium regime is generated by stronger investor’s appetite for subprime mortgage-backed securities (MBS) together with a wider confidence in the existing hard credit scoring technology. These changes were enough to make the subprime conduit loan market the first option for good type consumers; the fundamental proportions of higher quality borrowers that attempted to borrow from conduit lenders improved accordingly, and this in turn decreased the conduit lenders’ default expectations among their pools of borrowers. In this equilibrium regime, there was a a lot of credit in the subprime economy because conduit lenders could accommodate any “number” of borrowers as long as the hard credit scoring technology identified them as good borrowers. This boom of subprime credit is accompanied in our model by a jump in house prices and subprime home ownership rates. Moreover, because investors were effectively pricing conduit mortgage rates, the conduit mortgage rate fell below what traditional portfolio lenders charged for their low default risk mortgages.

Finally, we rationalize the fall of the conduit mortgage market in 2006 by appealing to three possibilities: (1) a negative shock to the subprime borrowers’ ability to repay their
mortgage debt (e.g., higher unemployment), which decreased the proportion of good type subprime consumers in the economy and made the pool of borrowers with conduit loans riskier; (2) the confidence on the existing hard credit scoring technology was shaken as lending credit standards worsened - we rationalize a subsequent reversal of lending standards in a setting with costly soft information acquisition and an increasing mortgage securitization rate; and (3) a significant reduction in liquidity from secondary market investors. In this environment we can show that conduit mortgages became an expensive and less attractive option for good type consumers compared to renting - the subsidy paid by the higher quality borrowers to support a pooling loan rate became so high that discouraged home ownership. When high credit quality consumers run away from the conduit loan market, conduit lenders’ pool of borrowers is only composed of risky bad type consumers and the market collapses. Importantly, the drop in available subprime credit makes house prices plummet.

Our theory of subprime mortgage lending relies on a general equilibrium model that incorporates the following important elements that are characteristic of a subprime economy: (1) limited recourse mortgages, (2) asymmetric borrower credit quality information, and (3) two funding sources for consumers, the portfolio mortgage market and the conduit mortgage market. We allow consumers to choose between portfolio loans and conduit loans, and show that in equilibrium the consumers’ market choices are consistent with lenders’ beliefs on proportions of low risk consumers. Thus, the subprime mortgage market segmentation is endogenous in our model. In addition, the loan amounts, the mortgage rates, the house prices, and the household’s tenure choice (owning versus renting) are all endogenous determined in equilibrium. We also show that a competitive equilibrium with endogenous segmented mortgage markets exists for our economy.

Explicitely recognizing the possibility of two different funding sources for consumers is important to understand the change between equilibrium regimes. To simplify our analysis, we assume that portfolio lenders’ access to soft information allows them to perfectly screen between borrower types; however, we require that they keep all the mortgages originated in their portfolio. Conduit lenders distribute a fraction of their mortgages originated to the investors, thus have access to the liquidity from the secondary mortgage market; however, conduit lenders lack soft information and their credit scoring technology is imperfect. These differences imply that mortgage rates are different between the two types of lenders. In particular, while portfolio lenders incorporate soft information into the determination of a (borrower specific) risk-based subprime loan rate, conduit lenders recognize that their borrower-lending clientele is lower credit quality on average. Thus, the conduit mortgage rate contains an adverse selection component, captured by the lack of soft information, but also a liquidity component coming from the conduit lender’s access to the securitized investment market. These two components move the conduit loan rate in opposite directions. On the one hand, securitization allows customization (conduit loans are priced using the investors’ time discount rate), which lowers the cost of capital in the conduit loan market. On the other hand, adverse selection in the primary mortgage increases the cost of capital in the conduit loan market. This trade-off between secondary market liquidity and adverse selection in the primary mortgage market is the key driver of the rise and fall of the subprime lending market in our model.
Relationship with the literature: The subprime crisis that started in 2007 and its aftermath has been coined as the Great Recession. Much of its discussion has focused on the problems around the secondary mortgage market, see e.g. Gennaioli, Shleifer, and Vishny (2012) and Gorton and Ordonez (2014). This paper focuses instead on the changes that occurred in the supply side of the primary mortgage market (underwriting standards and growth of mortgage securitization), as pointed out by Mian and Sufi (2009), to rationalize the emergence of the subprime conduit market in mid-1990s, its subsequent dominance over the traditional relationship lending model in the early 2000s, and its posterior collapse in early 2007.

The literature on collateralized lending with asymmetric information is vast and has captured attention in recent years in light of the subprime mortgage lending and financial crisis. In brief, and at a high level, this paper contributes to the literature that studies how both information frictions and mortgage securitization possibilities affect subprime mortgage originations, securitization, and house prices. See Jaffe and Russell (1976), Stiglitz and Weiss (1981) and Akerlof (1986) for classic papers on the effects of information frictions on screening, sorting and borrower default. For recent work that focuses on how different lenders’ information sets affect mortgage loan outcomes, borrowers’ default, and market unraveling, see, e.g., Karlan and Zinman (2009), Adams et al. (2009), Edelberg (2004), Rajan, Seru and Vig (2010), and Einav et al. (2013). See Miller (2014) for a related analysis of the importance of information provision to subprime lender screening. More generally, see Stein (2002) for a description of how private information includes soft information, and how difficult is to communicate soft information to other agents at a distance.1

Our equilibrium analysis of the subprime mortgage market also contributes to the recent empirical literature that attempts to identify the pricing determinants of differences between portfolio loans and conduit loans, and also differences among different types of conduit loans themselves (see Keys, Mukherjee, Seru, and Vig (2010) and Krainer and Laderman (2014))2. Agarwal, Amromin, Ben-David, Chomsisengphet and Evano (2011) recognized the lack of a theoretical model. To this extent, our paper provides a framework that enables to decompose the conduit mortgage spread into a credit information component, a foreclosure recovery rate component, and a component that captures the access to liquidity in the securitized investment market. We then show how these different pricing components can drive the rise and fall of the subprime conduit mortgage market.

Our paper is also related to the literature of shadow banking and subprime lending. As in Gennaioli, Shleifer, and Vishny (2012), our model can also illustrate that investors’ wealth drives up securitization, but in addition our model is able to generate the result that adverse selection in the loan origination market can be the only reason why the conduit loan market shuts down, even when there is investors’ appetite for mortgage-backed securities. This provides a different angle to the role of adverse selection on the rise and fall of subprime mortgage lending, which so far has focused on adverse selection in the

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1See also Inderst (2008) for a model that suggests a strong complementarity between competition and the adoption of hard-information lending techniques.
2See also Adelino, Gerardi and Willen (2013), Agarwal, Chang, and Yavas (2012), Agarwal, Amromin, Ben-David, Chomsisengphet and Evano (2011), Ambrose, Lacour-Little, and Sanders (2005), Bubb and Kaufman (2014), and Piskorski, Seru and Vig (2010)).
secondary mortgage market - see e.g., Gorton and Ordonez (2014) leading paper. Our paper also departs from Mayer, Piskorski, and Tchistyi (2013), Makarov and Plantin (2013), and Piskorski and Tchistyi (2011) by distinguishing between shadow bank and formal bank funding models, and relating their change in market share to different equilibrium subprime mortgage configuration regimes that result from changes in the credit scoring technology, securitization, foreclosure costs, or lenders’ capacity constraints. Importantly, house prices in our model as well as market segmentation are endogenous.

Our model captures the ebbs and flows of shadow bank activity, often peaking just prior to a downturn. The peak corresponds with poor access to soft information acquisition by conduit lenders and high liquidity flowing from security investors to conduit lenders (which is their largest if not exclusive source of funds). This is consistent with Purnanandam’s (2010) evidence that lack of screening incentives coupled with leverage-induced risk-taking behavior significantly contributed to the current subprime mortgage crisis. Our equilibrium mechanism links subprime mortgage lending standards to the run-up and eventually collapse in home-prices, and thus fills a gap in the literature that studies mortgage leverage and the foreclosure crisis (Corbae and Quintin (2015)). This is different from the previous macroeconomic general equilibrium models where high house prices relative to housing fundamentals can only reflect expectations of future owner housing returns (Favilukis, Ludwigson and Van Nieuwerburg (2015)). Our model also differs from Ordonez’s (2014) theory that crisis appear when mortgage-backed security investors neglect systemic risks by focusing instead on the information problems that are specific to the primary conduit loan origination market.

Our interpretation of the credit scoring technology is similar to Chatterjee, Corbae, and Rios-Rull (2011) and Guler (2014) in that the technology dictates the fraction of borrowers of a given type. However, in their models they do not distinguish between hard information and soft information, nor between portfolio lender versus conduit lender, and also assume the same technology for all lenders. Also, our work is unique in considering limited recourse mortgages, which are specific to subprime mortgages. Our result that an improvement in hard credit scoring technology leads to increases in the quantity of lending and also more lending to relatively opaque risky borrowers is similar to the effects of the small business credit scoring on commercial bank lending, as empirically documented by Berger, Frame and Miller (2005).

Recent papers in the literature of shadow banking and subprime lending are Ashcraft and Schuermann (2008), Bernake (2008), European Central Bank (2008), Keys, Mukherjee, Seru and Vig (2010), Geanakoplos (2010a, 2010b), Mishkin (2008), Purnanandam (2011), Quintin and Corbae (2015), and Keys et al (2013); see also Calem, Covas, and Wu (2013) and Fuster and Vickery (forthcoming) for evidence of a collapse of the private label RMBS market during the financial crisis.

As Ashcraft, Adrian, Boesky and Pozsar (2012) point out, at the eve of the financial crisis, the volume of credit intermediated by the shadow banking system was close to $20 trillion, or nearly twice as large as the volume of credit intermediated by the traditional banking system at roughly $11 trillion.

Other relevant papers that study foreclosure dynamics while taking exogenous house prices are Guler (2014) and Cambell and Cocco (2014).

Guler (2014) considers non-recourse contracts, whereas Arslan, Guler and Taskin (2015), Chatterjee, Corbae, Nakajima and Rios-Rull (2007), and Chatterjee, Corbae, and Rios-Rull (2008), and Chatterjee, Corbae, and Rios-Rull (2011) consider unsecured consumer loans (see Chatterjee and Eyigungor (2012) for a departure from these models where long-maturity debt is issued against collateral which value may fluctuate over time).
In our baseline model we focus on adverse selection problems in the primary mortgage market. We show that even without any adverse selection in the secondary securities market, we are able to generate boom and bust episodes similar to the recent subprime crisis. This is different from previous works and Frankel and Jin (2015) that considered adverse selection in the secondary mortgage market as the main reason of the expansion and collapse of lending (see also Gorton and Ordonez (2014)).

Finally, our model is able to relate the activity in the financial market with the urban economy. Land use regulations prevent subprime borrowers with small loans from buying houses with lot size above a minimum threshold. This lower bound on house size rules out a mortgage market for high risk (bad type) consumers, and forces subprime consumers without a mortgage to go to the rental market. This result illustrates how housing regulations prevent the least well-endowed subprime consumers who cannot afford from purchasing a house with a minimum lot size. Thus, the structural details underlying mortgage contract design and market organization consequently feed back to affect the rent versus own decision in our model.

**Paper structure:** The rest of this paper is as follows. In Section 2 we present the baseline model. Section 3 gives the equilibrium definition, states the equilibrium existence result, and discusses the pricing implications on mortgage rates. Section 4 identifies the different equilibrium regimes that our model can generate and discusses several potential factors that can trigger a change in the equilibrium regime. In Section 5 we discuss the rise and fall of the subprime mortgage market under the lenses of our model. Section 6 concludes.

## 2 The model

Our baseline model consists of a two-periods (periods 1 and 2) deterministic economy with asymmetric information in the primary conduit mortgage market and the following types of agents: subprime households ($h$), portfolio lenders ($pl$), conduit lenders ($cl$), and security investors ($i$). We will also refer to a portfolio lender and a conduit lender as PL and CL, respectively. Subprime households are also called subprime consumers. By abuse of notation, we will write $l$ to denote a lender independently of his type (PL or CL). We focus on the market of subprime mortgages, and study the equilibrium pricing and sorting of consumers between PL and CL. At the same time, CL sell a fraction of their originated mortgages to the investors.

We find convenient to denote an agent type by $a = h, pl, cl, i$, with respective sets $A(H), A(PL), A(CL)$, and $A(I)$. We denote the whole set of all agents in the economy by $\mathbf{A} = A(H) \cup A(PL) \cup A(CL) \cup A(I)$. The non-atomic measure space of agents in this economy is given by $(\mathbf{A}, \mathcal{A}, \lambda)$, where $\mathcal{A}$ is a $\sigma$-algebra of subsets of the set of agents $\mathbf{A}$, and $\lambda$ is the associated Lebesgue measure. For simplicity, the measures of portfolio lenders, conduit lenders and investors are all set to be equal to 1, i.e., $\lambda(A(PL)) = \lambda(A(CL)) = \lambda(A(I)) = 1$. 


2.1 Main assumptions

The general equilibrium model we are about to describe has the following main assumptions:

**Two types of subprime households:** In our economy all subprime households fall below some subsistence poverty line and have a subsistence income in period 1 equal to $\omega^{SR} > 0$ units of the numeraire good. We think of $\omega^{SR}$ as a government subsidy that is fungible and can be used to either rent a house or to fund a down payment on an owner-occupied house should the borrower qualify for a sub-prime mortgage. When the subprime consumer uses $\omega^{SR}$ to rent a house, this is equivalent as getting access to one of the housing affordable units provided by local governments. In the second period some of the subprime consumers experience a positive income shock (e.g., get a better job) $\omega^+ > \omega^{SR}$, while the rest of the pool remains at their current (poverty) income level $\omega^{SR}$. Label the consumers that experience an increase in their second period endowment as a G-type (or good type) and those who don’t as a B-type (or bad type). Consumers know their type in period 1, but G-type consumers are unable to verifiably convey their unrealized increase in income level to outside parties. This is an important aspect of our model with subprime consumers - as discussed below, the lenders’ credit scoring technology that screens borrower types is coarse in absence of soft information, and, in general, considerably worse that the credit scoring technology in the prime lending market. The measures of types G and B households in the economy are $\lambda_G \equiv \lambda(A(G))$ and $\lambda_B \equiv \lambda(A(B))$, respectively. In the Appendix ?? we provide further details that characterize subprime consumers, subprime housing markets and subprime mortgage markets as compared to their prime counterparts.

**Limited recourse mortgages:** Recourse mortgages are specific to the subprime market in the US and Europe, except few special cases such as purchase money mortgages in California and 1-4 family residences in North Dakota. As it happens in real life, most of these recourse mortgages are subject to some limited liability. This is especially true for subprime borrowers that have few resources (wealth). We will consider recourse mortgage contracts that are subject to some ungarnishable minimum subsistence consumption ($\omega^{SR}$) by the borrower (“limited recourse”). This limited liability nature of the contract is similar to a mortgage exemption that protects the subprime borrower from consuming less than a subsistence rent (see Davila (2015) for an analysis of bankruptcy exemptions from a welfare point of view). Under this contract a “good type” consumer (with no default risk) can credibly commit to pay back the loan even if the loan repayment is higher than the house value, but a “bad type” consumer (with default risk) cannot. Hence, the recourse nature of the contract introduces a potential for adverse selection in the primary subprime mortgage market. Also, this type of contract implies that bad type borrowers, who by assumption are

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7 The literature on general collateral equilibrium is vast. See Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014) for leading models, and Fostel and Geanakoplos (2014) for a review of the theory of leverage developed in collateral equilibrium models with incomplete markets. See also Geanakoplos (2010) for a more applied view of the role of these models in the understanding of the recent credit crisis.

8 Lenders cannot take everything and leave a consumer homeless when he defaults and becomes bankrupt. In fact, bankruptcy is designed to shield consumers from too much recourse on mortgage loans. See [law...]

9 See Poblete-Cazenave and Torres-Martinez’s (2013) for a recent descriptive analysis of a model with limited liability mortgage loans.
only endowed with a subsistence rent at time of mortgage repayment, end up defaulting and giving their housing asset to the lenders. Hence, the limited recourse mortgage is effectively a non-recourse mortgage for the bad type borrowers. In the Appendix ?? we elaborate on the details of limited recourse mortgage contracts, their implications for adverse selection, and also explain the differences if we were to consider non-recourse mortgages instead where adverse selection would be absent in our baseline model.

Two funding sources for consumers: Portfolio lenders (PL) originate mortgages to be held in the entity’s asset portfolio (“originate-for-ownership”). In contrast, conduit lenders (CL) are transactional, specializing only in originating mortgages for sale to a third party (“originate-for-distribution”). This access to secondary mortgage markets can possibly reduce the cost of capital when secondary subprime mortgage markets are liquid and competitive. Another difference is that PL and CL have different credit scoring technologies. CL generally work out of a small office with computers, with no established presence in a community. In the baseline model below we assume that CL have access to hard credit information (e.g., credit history and FICO scores), which is always accurate, but it does not necessarily lead to a perfect assessment of consumer type. PL have soft information as a supplement to the available hard credit information, and by assumption this is enough to fully reveal the borrower’s type (PL know their borrowers and their communities and borrowers maintain checking and other personal accounts with them).\(^\text{10}\) As such CL are not capable of resolving asymmetric information over and above what is available with hard information and their credit scoring technology. The lack of soft information by CL introduces asymmetric information in the primary CL mortgage market. Later in the paper (see Section 6) we will allow lenders to choose their optimal amount of soft information and show that the assumed differences in soft information acquisition between lender types do not speak against optimality. Also, in our discussion of the rise and fall of subprime mortgage lending, we will be able to accommodate some possible change in the PL’s business model by changing parameters \(\lambda(A(PL))\) and \(\lambda(A(CL))\).

Capacity constrained portfolio lenders: Another assumption is that portfolio lenders cannot lend to more than \(v(PL)\) consumers. In particular, we assume \(\lambda_C > v(PL)\) (portfolio lenders can only lend to some but not all good type consumers). This assumption is motivated by additional constraints faced by portfolio lenders, such as the time constraint to originate loans that require face-to-face contact between borrower and lender (one important source of soft information). In addition, other considerations may also apply here, such as the inability of portfolio lenders in the short run to raise equity to finance new mortgages. The assumption of capacity constrained PL then implies that when portfolio loans are the first choice among consumers, the rest of good type consumers who did not get a portfolio loan have no other option but to go to the conduit loan market in order to get a mortgage. Also, bad type consumers, who are identified as such by the portfolio lender’s additional soft credit information, only can get a mortgage if misrepresenting their

\(^{10}\)Soft information may include listening to and analyzing the borrower’s explanation for past difficulties in making credit payments and determining whether the hard numbers for the borrower or property make sense given what a loan agent can perceive about them. For a discussion of how securitization discourages lenders from engaging in “soft” mortgage underwriting, see “Comments to the Federal Deposit Insurance Corporation” by the National Association of Consumer Advocates on February 22, 2010.
type in the conduit mortgage market. Table 1 summarizes the main distinctions between traditional portfolio lenders and conduit lenders, which can be seen as representative of the subprime mortgage market in the 1980s and early 1990s.\footnote{The pre-1990s US depository model was of thousands of small portfolio lenders that generally operated over very narrow geographic areas. Thrifts were a particular type of depository that were designed to make residential mortgage loans—the subject of this paper. The banking crisis in the 1980s, coupled with the relaxation of many banking laws involving geographic- and product-market expansion, led to fewer and larger depositories. This consolidation and expansion was further propelled by the IT revolution.}

<table>
<thead>
<tr>
<th>Soft information</th>
<th>Originate-to-distribute</th>
<th>Capacity constrained</th>
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<td>(traditional) PL</td>
<td>YES</td>
<td>NO</td>
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<td>CL</td>
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Table 1

**Adverse selection in the primary conduit mortgage market:** Lenders cannot perfectly screen the type of borrowers using hard information only; only additional soft information can identify the type of subprime borrower.\footnote{Chatterjee, Corbae, and Rios-Rull (2011) allow consumers to borrow multiple times to study the role of reputation acquisition where the individual’s type score is updated every period according to some exogenous rule. These are characteristics of prime borrowers who build some credit reputation over time by borrowing in multiple occasions. In our paper we study subprime consumers whose access to credit is rather limited and in general can borrow only once. Thus, there is no reputation acquisition in our model, nor a need to update the individual’s type score.}

In our baseline model investors rely on the same credit scoring technology than those lenders without soft information, thus leaving aside the possibility of adverse selection in the secondary market of mortgage backed securities. Later, in Section 6, we examine the implications of dropping this assumption.

**Inelastic owner-occupied housing supply:** The owner-occupied housing consumption space is $[0, \bar{H}]$ where $\bar{H}$ denotes the aggregate amount of owner-occupied housing in the economy.\footnote{Below, in Section 4, we will study the impact of introducing a minimum housing consumption $H_{\text{min}} > 0$ resulting from local land use regulation in the form of minimum quality standards for owner occupied houses.} For simplicity, we take the aggregate supply of owner-occupied housing in the first period and the aggregate demand of owner-occupied housing in the second period as exogenously given and equal to $\bar{H} = 1$.

**Overlapping generations economy:** Our model can also be conceived as an overlapping generations (OLG) economy, where in each period there are new lenders and investors (alternatively, we could assume instead that lenders and investors cannot share risk across time among different generations of households).\footnote{Extending the OLG model to a setting with infinitely lived agents and more than one good is subtle because the presence of such agents may preclude equilibrium existence due to the possibility of Ponzi schemes (see Seghir (2006)).} In that case, our baseline two periods economy becomes similar to an OLG economy where households in the second period choose to sell their houses to the new generation of younger households (the stock of owner-occupied housing changes hands from old households to young households).
2.2 Subprime households

Consumption can take two forms: owner-occupied housing ($H_\tau$) and rental housing ($R_\tau$), where $\tau = 1, 2$ denotes the corresponding time period. In period 1 a household can buy a house of size $H_1$ at price $p_1$ per house size unit or rent a house of size $R_1$ at per unit price 1 (numeraire good). House purchases are long term contracts, once signed the house can be “consumed” in both periods (if the consumer buys a house in period 1, the same house enters in period 2 budget constraint as an asset endowment evaluated at market price $p_2$). On the other hand, buying good $R$ can be seen as a one-period contract: it only allows consumption of this good during one period only. The rental housing market for subprime consumers usually requires of a government voucher. We assume that these rental housing units (e.g., shelter) are provided by the government in exchange of a voucher. To streamline our analysis, we assume that the voucher is fungible and can be used to fund a down payment on an owner-occupied house should the borrower qualify for a sub-prime mortgage. This is similar to a situation where the government is subsidizing the home equity part of a mortgage. Once the second period starts, households expect to die at the end of the period. Thus, we refer to households in period 2 as old households, and households in period 1 as young households. When households are old, they can also choose to consume owner-occupied housing $H_2$ and the numeraire good $R_2$. Household $h$’s preferences are represented by utility function: $u^h(R_1, H_1, R_2, H_2)$ that is continuous, concave and monotonic.

In period 1 (impatient) households can increase their consumption by borrowing from either a portfolio lender (PL) or a conduit lender (CL). Both types of lenders originate mortgages in a competitive environment, although they differ in the terms of their contracts (see below). The matching between consumers and lenders is endogenous in our model and will be addressed later. Now, we describe the optimization problem of a consumer whose choice and access to a primary mortgage market $l = PL; CL; \emptyset$ has already been determined (we write $l = \emptyset$ if the consumer is not able to borrow from a PL or a CL). By assumption, a consumer can only access one type of primary mortgage market $l$. Denote the consumer’s loan amount in the subprime mortgage market $l$ by $q^l \psi^l \geq 0$, where $q^l$ and $\psi^l \geq 0$ denote the $l$-type mortgage discount price and loan repayment due at the beginning of the second period (when $l = \emptyset$, we write $\psi^\emptyset = 0$). For simplicity, we normalize the loan interest rate to 0. Equilibrium existence requires an upper bound $B > 0$ on $\psi^l$, but this bound can be chosen arbitrarily (in our characterization of equilibrium below we will choose this bound such that this short sale constraint is non-binding):

$$\psi^l \leq B$$

(1)

The period 1 budget constraint of a consumer with access to primary mortgage market $l$ is:

$$p_1 H_1 + R_1 \leq q^l \psi^l + \omega^{SR}$$

(2)

The consumer’s mortgage down payment is endogenous in this model; for example, if $R_1 = 0$, then the downpayment is equal to $\omega^{SR}/p_1 H_1$.

Sub-prime loans are subject to a limited recourse mortgage contract that stipulates that a borrower is allowed to consume his subsistence income $\omega^{SR}$ if default occurs. Accordingly,
we write the second period budget constraint as follows:

\[ p_2 H_2 + R_2 \leq \max \{ \omega^S_R, \omega^*_2 + p_2 H_1 - \psi^d \} \]

where \( \omega^*_2 \) denotes the period 2 endowment of a consumer of type \( t = G, B \) and is such that \( \omega^G_2 = \omega^*_2 \) and \( \omega^B_2 = \omega^{SR}_2 \). The term \( p_2 H_1 \) in the right hand side of the inequality captures the value of the house purchased in the previous period and is interpreted as a sale at market price \( p_2 \) per house size unit. The consumer can then use the proceeds of this sale for consumption after repaying his mortgage.\(^{15}\) The maximum operator in \( (3) \) allows the household to strategically default and consume at least the minimum subsistence income \( \omega^{SR}_2 \).\(^{16}\) There is no default if \( p_2, H_1, \) and \( \psi^d \) are such that \( \omega^{SR}_2 \leq \omega^*_2 + p_2 H_1 - \psi^d \). Loan payment is (partially) enforced by the nature of the limited recourse loan in our model.

Denote the household \( h \)'s consumption bundle by \( x^h = (H^1_h, R^1_h, H^2_h, R^2_h) \). The pair \( (x^h, \psi^h) \) is feasible if it satisfies constraints \( (2), (1) \) and \( (3) \). Finally, the households’ optimization problem is as follows: each household maximizes his utility function subject to constraints \( (2), (1) \) and \( (3) \).

**Remark:** To demonstrate the robustness of our model we have included in the Appendix ?? the technical details of extending the baseline deterministic economy with one state of nature in the second period to an stochastic economy with uncertainty in the consumer’s second period endowment realization. In the first state the consumer’s endowment, irrespective of his type, is \( \omega^* \), whereas in the second state his endowment is \( \omega^{SR} \) and thus defaults on the loan payment. Subprime consumers differ in their probabilities attached to each of these two states. We show that the baseline model is in fact a particular case of the extended model and that the predictions of the model do not change in qualitative terms.

### 2.3 Lenders

We require that to receive a mortgage, the consumer must be identified as a G-type, i.e., \text{rating=G}.\(^{17}\) Denote the probabilities that a lender \( l \) gives a good rating to a G-type type borrower and a B-type type borrower by \( CST^G_l \equiv \Pr_l(\text{rating=G}|G) \) and \( CST^B_l \equiv \Pr_l(\text{rating=G}|B) \), respectively. By assumption, conduit lenders only rely on hard information and thus \( CST^G_{CL} < 1 \). Portfolio lenders have access to soft information on top of the hard credit information, and thus, by assumption, always assign a good signal to G-type consumers, i.e., \( CST^G_{PL} = 1 \). Therefore, given the PL’ capacity constraint, PL end

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\(^{15}\)A consumer with an owner-occupied house at the beginning of period 2 decides whether to sell it at market price, or to consume it. The latter is equivalent to the joint transactions of selling the house the consumer owns at the beginning of period 2 and then buying immediately after a house with same size.

\(^{16}\)Strategic default is simply an optimality condition in which the borrower, subject to the relevant recourse requirements, decides whether mortgage loan payoff to retain ownership of the house or default with house forfeiture generates greater utility. For a discussion of the default option, see Deng, Quigley, and Van Order (2000). See Davila (2015) for an exhaustive analysis of exemptions in recourse mortgages.

\(^{17}\)As explained in Section 3, we can rule out a market for B-type consumers by appealing to land use regulations in the form of a minimum house size, or to common practice where lender don’t want to lend to a consumer that is known to default. Also, observe that the adverse selection problem in the mortgage market would not disappear if we allow for a market of “bad ratings”, since B-type consumers would still prefer to misrepresent their type and borrow a large loan amount as G-type consumers do.
up lending to a mass $v(PL)$ of G-type consumers. The measure of consumers that receive a loan from CL is equal to

$$\mu^{CL}(\text{rating}=G) = CST^l_G \cdot \mu^G_G + CST^l_B \cdot \mu^B_B$$

where $\mu^G_G$ and $\mu^B_B$ denote the measure of G-type and B-type consumers that attempt to borrow from CL ($\mu^G_G$ and $\mu^B_B$ are endogenous in the model, and so is $\mu^{CL}(\text{rating}=G)$).

We can use Bayes’ rule and write the expected probability of lending to a G-type consumer, given that the lender $l$’s hard credit scoring technology assigns him a good rating, as follows:

$$\Pr^l(G|\text{rating}=G) = \frac{CST^l_G \cdot \hat{\pi}^l_G}{CST^l_G \cdot \hat{\pi}^l_G + CST^l_B \cdot \hat{\pi}^l_B}$$

(4)

where $\hat{\pi}^l_G$ denotes the fundamental proportion of G-type consumers among all consumers that attempt to borrow from conduit lenders.\(^{18}\) For example, if a CL’s pool of borrower contains 60 B-type consumers and 40 G-type consumers, then $\hat{\pi}^l_G = 0.4$. For simplicity, we assume that $\Pr^l(\text{rating}=G|G) = \Pr^l(\text{rating}=B|B)$ and $\Pr^l(\text{rating}=G|B) = \Pr^l(\text{rating}=B|G)$, which imply $CST^l_G = 1 - CST^l_B$.

To simplify notation, we write the lender $l$’s belief on the proportion of G-type consumers in its pool of borrowers as

$$\pi^l \equiv \Pr^l(G|\text{rating}=G).$$

Then, by assumption, we can write $\pi^{PL} = 1$ and $\pi^{CL} < 1$.

Lenders are subject to an “originate-to-distribute” type constraint, which says that a lender $l$ cannot distribute more than a fraction $d^l$ of its mortgages originated; in particular,

$$z^l \leq d^l \varphi^l$$

(5)

where $\varphi^l \geq 0$ denotes the total amount of mortgages originated by lender $l$, $z^l \geq 0$ is the amount of mortgages the lender sells to the investors, and $d^l$ is the fraction of mortgages that are originated for distribution.\(^{19}\) $\varphi^l$ and $z^l$ are choice variables, and $d^l$ is a parameter that takes value 0 if the lender is a portfolio lender ($l = PL$), and $d^l \in (0, 1]$ if the lender is a conduit lender ($l = CL$). In general, $d^{CL}$ is typically close to 1 for CL. A distribution rate smaller than 1 can be the result of a regulation or a self-imposed constraint due to reputation concerns (not modelled here). We then say that a pair ($\varphi^l, z^l$) is feasible if it satisfies (5).

Given the nature of the limited recourse mortgage contract, when there is borrower default, the lender garnishes all borrower’s income above the subsistence rent $\omega^{SR}$. This

\(^{18}\)In the Appendix we show that $\Pr^l(G|\text{rating}=G)$ can be expressed in a linear way as follows: $\Pr^l(G|\text{rating}=G) = 1 - \varepsilon \hat{\pi}(B)$, where $\varepsilon$ denotes the amount of asymmetric information between the lender and the borrower and $\hat{\pi}^{CL}(B) = 1 - \hat{\pi}^{CL}(G)$.

\(^{19}\)In our model, homogeneous loans are pooled and securitized into one asset (see Aksoy and Basso (2014) for a model with tranching). We also ignore agency issues regarding securitization and its implications on distressed loans (see Cordell, Dynan, Lehner, Liang, and Mauskopf (2009), Piskorski, Seru, and Vig (2010), Agarwal, Anrormin, Ben-David, Chomsisengphet, and Evano (2011), Ghent (2011), and Adelino, Gerardi, and Willen (2013) for a discussion of the role of securitization on residential mortgages).
includes repossessing the house and reselling it if the borrower happened to buy one in the first period. However, the foreclosure process is costly for the lender: foreclosure cost and other indirect costs associated with foreclosure delays result in a loss \( (1 - \delta)p_2H_1 \) to the lender, where \( \delta \in [0, 1] \) denotes the foreclosure recovery rate.

Lenders are risk neutral with time discount factor \( \theta^l > \theta^h \) and belief \( \pi^l \) on the fraction of G-type borrowers in the pool. In particular, we consider the following linear separable profit function for a lender \( l \):

\[
\Phi^l(\varphi^l, z^l) = (\omega^l - q^l\varphi^l + \tau z^l) + \theta^l(1 - d^l)(\pi^l\varphi^l + (1 - \pi^l)\delta p_2H_1^G),
\]

where \( \tau \) denotes the sale price of a mortgage in the secondary market. The lender’s first period endowment is positive, \( \omega^l_1 > 0 \) (for simplicity, we assumed \( \omega^l_2 = 0 \)). The lenders optimization problem is as follows. Each lender \( l \) chooses \( \varphi^l \) and \( z^l \) to maximize \( \Phi^l(\varphi^l, z^l) \) subject to the originate-to-distribute constraint (5). Risk-neutrality implies that the lender’s first order condition determines the competitive mortgage price \( q^l, l = PL, CL \) (see Section 3).

Only a fraction \( 1 - d^l \) of mortgages affect the lender’s profit function in the second period as he distributes a fraction \( d^l \) of the mortgage payment proceeds to investors. Notice that the interaction between the originate-to-distribute constraint (5) and the profit function (6) determines the two possible loan origination models. On the one hand, CL can distribute a fraction \( d^{CL} > 0 \) of the originated mortgages, but lack soft information (so \( \pi^{CL} < 1 \)). PL, on the other hand, have soft information (\( \pi^{PL} = 1 \)) but don’t sell their mortgages to the investors (\( d^{PL} = 0 \)).

Finally, denote the lender \( l \)’s consumption bundle by \( x^l = (\omega^l_1 - q^l\varphi^l + \tau z^l, (1 - d^l)(\pi^l\varphi^l + (1 - \pi^l)\delta p_2H_1^G)) \in \mathbb{R}_+^2 \). The lender \( l \)’s choice set is denoted by \( C^l \subseteq \mathbb{R}_+^2 \) and is composed of all pairs \((\varphi^l, z^l)\) that are feasible.

### 2.4 Investors

Investors assign a smaller weight to period 1 consumption than lenders do, and therefore we write \( \theta^l < \theta^i \). We assume that both conduit lenders and investors only rely on hard credit information \( CST^{CL} = CST^i \) and their beliefs are such that \( \pi^{CL} = \pi^i < 1 \). This assumption is convenient as it allows us to focus on the adverse selection problem in the primary mortgage market, leaving aside potential information problems that may arise between conduit lenders and secondary mortgage investors (later, in Section 6 we will discuss the implications of dropping this assumption).

The investor \( i \)’s optimization problem consists of choosing \( z^i \) to maximize the following profit function:

\[
L^i(z^i) = \omega^i_1 - \tau z^i + \theta^i(\pi^i z^i + (1 - \pi^i)d^i\delta p_2H_1^G)
\]

where \( \omega^i_1 > 0 \) denotes the investor’s endowment in period 1 (for simplicity, we assume \( \omega^i_2 = 0 \)). The term \( \pi^i z^i + (1 - \pi^i)d^i\delta p_2H_1^G \) captures the investor’s second period revenue.

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from buying mortgages in the first period. The first term, \( \pi^i z^i \), corresponds to the payment from the fraction \( \pi^i \) of G-type borrowers. The second term, \((1 - \pi^i) d^l \delta p_2 H^G_i \), corresponds to the income from lending to a fraction \((1 - \pi^i)\) of B-type borrowers. The term \( d^l \) stands for the percentage of mortgages that lenders sell and hence investors are entitled to that revenue. Because B-type consumers are not able to honor the loan payment corresponding to a G-type loan contract, the investor only receives the depreciated value of the foreclosed house, \( \delta p_2 H^G_i \), from these defaulted mortgages. Finally, denote the investor \( i \)'s consumption bundle is denoted by \( x^i = (\omega^i_1 - \tau z^i, \pi^i z^i + (1 - \pi^i) d^l \delta p_2 H^G_i) \in \mathbb{R}^+_2 \).

3 Equilibrium and Mortgage Pricing

In this section we propose an equilibrium notion of a competitive economy with endogenous segmented markets, assert that an equilibrium exists, and examine its pricing implications.

3.1 Equilibrium definition and the existence result

A consumer can choose among three possibilities: (1) borrow in the PL market, (2) borrow in the CL market, or (3) not borrow. We denote these possibilities by \( m^{PL}, m^{CL}, \) and \( m^\emptyset \), respectively. Thus, the set of consumer’s market “membership” possibilities is \( M = \{m^{PL}, m^{CL}, m^\emptyset\} \). A “membership” is consumer-type \((t(h) = G, B)\) and market-type \((l = PL, CL, \emptyset)\) specific, and is denoted by \( m^l_{t(h)} \equiv (t(h), l) \). A list is a function \( \iota : M \rightarrow \{0, 1\} \), where \( \iota(t(h), l) \) denotes the number of memberships of type \((t(h), l)\). We assume that a consumer \( h \) can only belong to one market in \( M \) (i.e., \( \sum_{l=PL,CL,\emptyset} \iota(t(h), l) = 1 \)). We write \( Lists = \{\iota : \iota \text{ is a list}\} \), define the consumer’s market membership choice function by \( \mu : A \rightarrow Lists \), and denote the aggregate of type \((t(h), l)\)-memberships by \( \hat{\mu}(t(h), l) = \int_{A(t(h))} \mu^h(t(h), l) d\mu \). We find convenient to rewrite the consumer \( h \)'s utility function as a function of his consumption and market type, e.g., \( u^h(x^h, \mu^h(m)) \). We assume that the utility mapping \((h, x, \mu) \rightarrow u^h(x, \mu)\) is a jointly measurable function of all of its arguments. The consumer \( h \)'s choice set \( X^h \subset \mathbb{R}^+_3 \times Lists \) consists of the feasible set of elements \((x^h, \psi^h, \mu^h)\) that this consumer can choose. The consumption set correspondence \( h \rightarrow X^h \) is a measurable correspondence.

Below we formally define our notion of equilibrium, which is similar to the standard concept of a competitive equilibrium in general equilibrium with the additional condition that lenders’ beliefs \( \pi \equiv (\pi^{PL}, \pi^{CL}) \) must be consistent with the distribution of consumers into markets given by the membership function \( \mu \). In particular, we say that the aggregate membership vector \( \hat{\mu} \in \mathbb{R}^M \) is consistent with \( \pi \) if there is a continuous function \( f = \hat{\mu} \circ \pi \) such that \( \hat{\pi}^l = f(\hat{\pi}^l; CST^l, \theta^l, \theta^h, d^l) \), where \( \pi \) is given by function (4). In addition, the the membership function \( \mu \) must be such that the PL’s capacity constraint holds, i.e.,

\[
\int_{A(G)} \mu^h(G, PL) d\mu \leq v(PL)
\]  

(8)

**Definition 1:** Given the triplet \((CST^{PL}, CST^{CL}, CST^h)\), an equilibrium for this economy consists of a vector of memberships \( \mu \), prices \((p_1, p_2, q^{PL}, q^{CL}, \tau)\) and allocations \((x^h,\)
\(\psi^h)_{h \in A(G) \cup A(B)}, (x^d, \psi^l, z^l)_{l \in A(I), l = PL, CL}, (x^m, z^m)_{i \in A(I)}\) such that:

1. Each consumer \(h\) chooses \((x^h, \psi^h, \mu^h)\) in \(X^h\) that maximizes \(u^h(x^h, \mu^h(m))\).

2. Each lender \(l\) chooses \((\psi^l, z^l)\) in \(C^l\) that maximizes \(\Phi^l(\psi^l, z^l)\).

3. Each investor \(i\) chooses \(z^i \in \mathbb{R}_+\) that maximizes \(\Lambda^i(\psi^i, z^i)\).

1.4) \(\hat{\mu}\) is consistent with \(\pi\).

1.5) Market clearing:

\[
\begin{align}
MC.1 \quad & \int_{A(H)} \psi^h_{PL} \mu^h(t(h), PL) dh = \int_{A(PL)} \varphi^PL dx \forall \tau \\
MC.2 \quad & \int_{A(H)} \psi^h_{CL} \mu^h(t(h), CL) dh = \int_{A(CL)} \varphi^CL dk \\
MC.3 \quad & \int_{A(CL)} z^l dl = \int_{A(I)} z^l di \\
MC.4 \quad & \sum_{l \in \{PL, CL\}} \int_{A(H)} P^h_l \mu^h(t(h), l) dh + \int_{A(CL) \cup A(PL)} \int_{A(I)} x^l_i dl + \int_{A(I)} x^l_i di = \int_{A} \omega^L da, \\
MC.5 \quad & \sum_{l \in \{PL, CL\}} \int_{A(H)} H^h_l \mu^h(t(h), l) dh = \sum_{l \in \{PL, CL\}} \int_{A(H)} H^h_l \mu^h(t(h), l) dh = \tilde{H}
\end{align}
\]

**Theorem 1 (Existence):** There exists an equilibrium.

We leave the details of the existence proof for the Appendix ??.

**Remarks about the notion of equilibrium:**

1. We focus on the concept of pooling equilibrium in the conduit loan market (the market where there is adverse selection). A separating equilibrium can exist because CL would be willing to lend to B-types if the loan amount is small. This is because, even if B-types always default in the second period (since their income in the second period equals their subsistence rent), the CL would get some income by foreclosing the house, being this income a fraction \(\delta\) (foreclosure cost) of the house value \(p_2H_1\). This is profitable for the CL when the loan amount is small, because by foreclosing the house the conduit lender is able to seize the equity that the borrower put to buy the house \((\omega^{SR})\). Formally, the CL gets \(\delta p_2H_1\) in \(t = 2\) and pays \(q^B \varphi^B\) in \(t = 1\). So, one possibility to rule out a conduit loan market for B-types is to prevent CL from writing this type of contracts that always involve foreclosure. This is common sense, as nobody wants to knowingly lend to bad types where the probability of default is high. So we can rule out this type of mortgage contracts as a matter of common practice. Another possibility to rule out a conduit loan market for B-type borrowers is to assume that there is a land use regulation in the form of a minimum house size \((H_{CL}^{min})\). In the Appendix ?? we show that there is a threshold \(H_{CL}^{min}\) that prevents B-type consumers with a small conduit loan to buy a house with a size larger than

\[
H_{CL}^{min} = \frac{\tilde{H}(\omega^{SR} + \tilde{L})}{2\omega^{SR} + \tilde{L} + \tilde{G}}
\]

where \(\tilde{L} = \theta^i \delta \omega^{SR} / (1 - \theta^i \delta)\) is the maximum loan amount that a conduit lender would give to a B-type consumer being compatible with non-negative profits for the lender, and \(L^G = \tilde{\theta}(\omega^{SR} + \tilde{L}) / (1 - \tilde{\theta})\) is the loan amount that a G-type consumer would obtain from a conduit lender when mortgage markets are segmented (using the CL’s first order condition and G-type consumer’s first period budget constraint). Land use regulations have applied in the U.S. and other countries for decades, and it is well known that consumption standards such as minimum lot sizes can exclude low-income groups if they are set too high - see
e.g. Malpezzi and Green (1996) and Duraton, Henderson and Strange (2015). These two reasons together (common practice in mortgage lending and land use regulations) make it clear that separation will not happen, and therefore we can ignore it without compromising the robustness of the model.

2. In our model default risk is the result of the CL’s inability to perfectly screen between borrower types, and thus it can be attributed to the endogenous behavior of consumers with whom they are matched in equilibrium. We treat this risk as idiosyncratic in the sense we assume that the matching of lenders with consumers is independent and uniform, and that the law of large numbers applies.\footnote{See also NAHB Research Center (2007) and the Wharton Housing Regulation Index for measures of housing regulation.}

3. Our notion of equilibrium assumes that lenders and investors form beliefs about the composition of the lenders’ pool of borrowers. These beliefs are common, degenerate and governed by the lender’s credit scoring technology. Lenders and investors take their beliefs as given and optimize without taking into account the consumers’ choice of mortgage market.\footnote{See Zame (2007) and Duffie and Sun (2007, 2012).} Equilibrium condition (1.4) requires that lenders’ beliefs are consistent with the distribution of consumers into mortgage markets.

4. Given the PL’s capacity constraint and the CL’s imperfect credit scoring technology, consumers of the same type may end up with different loan amounts, and thus different realized housing consumption and ex-post utility (e.g., there will be an equilibrium configuration where some G-type consumers are lucky and obtain a portfolio loan, some G-type consumers obtain a conduit loan, and the remaining G-type consumers cannot borrow and must rent). Our approach to equilibrium existence is consistent with this interpretation.

5. The continuum of consumers allows us to deal with three types of non-convexities in the equilibrium existence proof: the non-convexity associated with the maximum operator in the consumer’s second period budget constraint, the non-convexity associated to the consumer’s choice set \((H_1 \in \{0\} \cup [H_{\min}, \bar{H}])\), and the non-convexity associated with the consumer’s choice of loan market \((m_{PL}, m_{CL}, m^0)\).

Next, we derive asset pricing conditions that any equilibrium in this economy must satisfy using the lender and investor’s optimality conditions.

### 3.2 Mortgage Discount Prices

Using the lender and investor’s first order conditions we obtain the following conduit loan discount price:

\[
q^{CL} = \frac{\bar{\pi} \bar{\theta}}{1 - \delta (1 - \bar{\pi}) \theta}
\]

where \(\bar{\pi} \equiv \pi^{CL} = \pi^l, \bar{\theta} \equiv d^l \theta^l + (1 - d^l) \theta^t\), and \(d^l\) is the lender \(l\)’s mortgage distribution rate. Since \(\theta^i > \theta^t\), a higher distribution rate implies a higher \(q^{CL}\). A belief \(\bar{\pi}\) smaller than 1 captures the negative effect of adverse selection on the mortgage discount price. The term \(1 - \delta (1 - \bar{\pi}) \bar{\theta}\) in (10) is the “default loss” that the conduit lender incurs when its pool of borrowers contains an expected fraction \(1 - \bar{\pi}\) of B-type borrowers: the higher is the...
default loss, the lower is the discount price that the conduit lender offers to its borrowers. We can rewrite expression (10) in a more intuitive way as follows:

\[ q_{CL} = \frac{\text{hard info predictive power} \times d^l \text{-weighted discount factor}}{\text{default loss}} \]

The inability of CL to fully resolve information asymmetries with their the hard information-based screening technology (\( \pi < 1 \)) implies that some borrowers in their pool are of bad type. Since bad type borrowers (endogenously) fail to comply with mortgage payment contract terms and conditions, with the net post-foreclosure sales proceeds less than the promised payment (as \( \delta \in (0,1) \)), the conduit lender incurs in a “default loss”. As a result, based on observables and expectations at the time of mortgage loan origination, the lender finds it optimal to tack on a pooling rate premium to the base loan rate to account for adverse selection risk. However, the loan rate may move indirectly with the credit risk of its borrowers if the lender’s access to liquidity in the secondary market is sufficiently high (i.e., high \( \theta \)). Roughly speaking, securitization allows customization, which lowers the cost of capital (\( 1/q_{CL} \)) in a conduit loan market where lemons are present.

Similarly, the discount price that investors pay for the subprime mortgages is

\[ \tau = \frac{\bar{\pi}\theta^l}{1 - \delta(1 - \bar{\pi})\theta} \quad (11) \]

Finally, the PL, who by assumption has \( d^{PL} = 0 \) and \( \pi^{PL} = 1 \), finds optimal to set the mortgage price equal to its discount factor \( \theta^l \), i.e.,

\[ q^* = \theta^l \quad (12) \]

Since \( \pi^{PL} = 1 \) implies no default, \( q^* \) can be thought as a risk free discount price that does not incorporate liquidity gains from distribution of originated loans to investors. We see that, when the fraction of lemons in the CL’s pool of borrowers converges to zero - that is, when its hard credit scoring technology is such that \( 1 - \bar{\pi} \rightarrow 0 \) - , \( q_{CL} \) converges to \( q^* \) if it is not possible to distribute mortgages to investors (\( d^{CL} = 0 \)) or if there are no investors that buy subprime mortgage securities (e.g., if \( \omega_i^1 = 0 \)).

Pricing conditions (10) and (12) can be compared as follows:

\[ q_{CL} < q^* \text{ if } \pi^{CL} < \pi_2 \equiv \frac{\theta^l(1 - \delta\bar{\theta})}{\bar{\theta}(1 - \delta\theta^l)}. \]

Threshold \( \pi_2 \) defined in the expression above will appear again in the next section when we characterize the different equilibrium regimes. Interesting, as the distribution rate \( d^{CL} \) increases, threshold \( \pi_2 \) decreases, and hence more information is needed to sustain an environment where the conduit mortgage rate is below the risk free rate.

By excess premium (EP) we mean the difference between the rate of return of conduit loans and the risk free rate of portfolio loans, i.e.,

\[ EP \equiv (1/q^{CL}) - (1/q^*) \quad (13) \]
Proposition 1: The excess premium increases with default losses and decreases with the predictive power of hard credit information, a higher distribution rate of mortgages to investors, and a higher risk free rate.

Remark: Our pricing results have some analogies with Sato’s (2015) analysis of transparent versus opaque assets. Sato shows that transparent firms own transparent assets and opaque firms own opaque assets in equilibrium. This is analogous to us showing PL hold only higher quality loans and CL own a mix. The reasons for such holdings are different in the two models, however. In our model, CLs are intermediaries that originate and sell opaque subprime MBS. Sato also shows that opaque assets trade at a premium to transparent assets. This is primarily due to agency distortions in the opaque firm. For us a premium in opaque asset prices comes through the investors’ demand for subprime MBS. For a similar result in the commercial mortgage market, see An, Deng and Gabriel (2011), who find that conduit loans enjoyed a 34 basis points pricing advantage over portfolio loans in the CMBS market.

4 Equilibrium regimes

In Section 3 we showed that an equilibrium for this economy exists under quite mild conditions. In order to streamline our analysis, we focus on a more analytically tractable setting where owner-occupied housing ($H$) and rental housing ($R$) are perfect substitutes and consider the following linear separable utility function:

$$u^h(R_1, H_1, R_2, H_2) = R_1 + \eta H_1 + \theta^h(R_2 + H_2),$$

where $\theta^h < 1$ denotes the consumer’s discount factor and $\eta > 1$ denotes a preference parameter that captures that, all else equal, in the first period young households prefer to consume owner-occupied housing over rental housing (this can be possibly due to a better access to schools, for example; see Corbae and Quintin (2015) for a model with also an “ownership premium” in preferences, and Hochguertel and van Soest (2001) for empirical evidence). When households are old, the utility from consumption of owner-occupied housing $H_2$ and the utility from consumption of rental housing $R_2$ are the same. Also, to get simple closed form solutions, we assume $\omega_2^+ = 1$, $\omega^{SR} = 1/2$, $v(PL) = 1$, and $\lambda_G = 1.5$.

4.1 House prices and land use regulation

This subsection discusses the effect of the owner-occupied housing price on consumers’ housing choices, and then examines the role of land use regulations on the exclusion of subprime consumers from mortgage markets. First, recall that the aggregate demand for owner-occupied housing in the first period and the aggregate supply of owner-occupied housing in the second period are inelastic, both equal to $H = 1$. A constant stock of owner-occupied housing is convenient to get simple closed form equilibrium solutions because the market clearing house prices are such that $p_1 = p_2 = p$.\footnote{The owner-occupied market clearing equations in periods 1 and 2 and the households’ optimal choice $H_2^h = 0$ (shown in the Appendix) imply that $p_1 = p_2 = p$.} Defaults occur in our model...
due to the imperfect hard credit scoring technology used by CL, and not due to house price movements.\textsuperscript{25} Secondly, in equilibrium $p > 1$, which implies that old households with a mortgage will sell their house in the second period and move to rental housing, as the benefits to owning go away as the younger household transitions to older age.\textsuperscript{26} In the first period, however, young consumers with a mortgage will find it optimal to buy a house, provided that the credit scoring technology parameter $\pi^{CL}$ exceeds a certain threshold, as argued below. Thirdly, PLs can in general lend to G-type consumers or to B-type consumers. Similarly to our discussion on the effect of $H^{\min}_{CL}$ on a conduit mortgage market specific for B-type consumers, we can also find a threshold $H^{\min}_{PL}$ that rules out a portfolio mortgage market specific for B-type consumers. In particular, we find that G-type consumers crowd out B-type consumers from the portfolio mortgage market if there is a local policy that requires a minimum house (lot) size equal to

$$H^{\min}_{PL} \equiv \omega^{SR}/p(1 - \delta^t)$$

(14)

This housing policy implies that subprime consumers with a small portfolio loan (or no loan) have no other option but to rent in the first period, because when $p > 1$ these consumers can only afford buying a house of size $\omega^{SR}/p$, which is certainly below $H^{\min}_{PL}$.

Finally, define $H^{\min} \equiv \max\{H^{\min}_{PL}, H^{\min}_{CL}\}$, which is the minimum house size that rules out both a portfolio mortgage market and a conduit mortgage market specific to B-type consumers. $H^{\min}$ then captures how local land regulations, in the form of a minimum lot size, affects the bottom of the housing market by excluding subprime borrowers of bad type from the mortgage market.

### 4.2 Mortgage market collapses

This section identifies three thresholds, $\pi_0$, $\pi_1$ and $\pi_2$, for the CL’s belief $\pi^{CL}$ on the proportion of good type borrowers in its pool. These thresholds determine different subprime mortgage market configurations, and all can be expressed as a function of the parameters of our economy, including key parameters such as $\theta^t$, $\theta^f$ and $d^f$. Also notice that the CL’s belief $\pi^{CL}$ is an increasing function of the CL’s credit scoring technology $Pr^t(\text{rating}=G|G)$ and the fundamental proportion $\bar{\pi}^{CL}_G$ of G-type consumer that attempts to borrow from CL (see expression (4)).

1. In presence of land regulation constraints, the conduit market can collapse if belief $\pi^{CL}$ sufficiently deteriorates. In particular, there is a threshold $\pi_0$ that solves the following equation:

$$H^G_{CL}(\pi_0) = H^{\min}$$

(15)

such that when $\pi^{CL} < \pi_0$ conduit loans are so small that borrowers with these loans cannot afford to buy a house with size above $H^{\min}$.

\textsuperscript{25}For a model where default is triggered by a fall in house prices, see Chatterjee and Eyigungor (2015) and Arslan, Guler and Taskin (2015) where mortgages are non-recourse.

\textsuperscript{26}Also, as households get older, their needs may change and may prefer independent living, assistance living, or even nursing care than living by their own in a big owner-occupied house. See Hochguertel and van Soest (2001) for evidence that young households buy a house to accommodate the new family members and possibly to get access to better schools, but when they are old and the family size decreases, these households sell their houses and move to smaller rental houses.
2. There is a conduit mortgage market as long as G-type consumers prefer to borrow from conduit lenders than renting in the first period. When $\pi_{CL}$ decreases below a given threshold $\pi_1$, the implicit conduit mortgage rate is so high that G-type consumers prefer to rent in both periods ($R_1 = \omega_{SR}$ and $R_2 = \omega_2^+$) than borrowing from CL and buying a house in the first period. Threshold $\pi_1$, at which indifference between buying a house with a conduit loan and renting in both periods occurs, solves the following equation: \[ \eta H_{1,CL}^{G} + \theta^h \omega_{SR} = \omega_{SR} + \theta^h \omega^+ \] (16)

When $\pi_{CL} < \pi_1$, conduit loans are so small that G-type consumers prefer to rent in both periods.

**Lemma 1:** *The conduit mortgage market collapses when $\pi_{CL} < \max\{\pi_0, \pi_1\}$.***

3. Consumers may prefer to borrow from CL if the conduit loan is larger than the portfolio loan. Formally, there is a threshold $\pi_2$ at which the G-type consumer is indifferent between a conduit loan and a portfolio loan. This threshold solves the following expression: \[ \eta H_{1,CL}^{G} + \theta^h \omega_{SR} = H_{1,PL}^{G} + \theta^h \omega_{SR} \] (17)

Observe that when $\pi_{CL} > \pi_2$, consumers prefer conduit loans even when conduit lenders risk-price the presence of lemons and their subsequent default into the mortgage discount price. In this case, the CL’s fundamental proportions of G-type consumers, $\hat{\pi}_{CL}^G$, improves as now conduit loans are the first best option for G-type consumers. Also interestingly, when the mortgage distribution rate $d_{CL}$ increases, threshold $\pi_2$ decreases and the conduit mortgage market expands.

**Lemma 2:** *The portfolio mortgage market becomes the first choice for G-type consumers when $\pi_{CL} > \pi_2$.***

Below we summarize the different possible market configurations in terms of the CL’s belief $\pi_{CL}$ and indicate the size of the portfolio and conduit mortgage markets for each of these configuration. For simplicity, we assumed that CLs are not capacity constrained,$^{29}$ so whenever a G-type is not able to borrow from a PL, he can always try to borrow from a CL. However, not all G-type consumers that attempt to borrow from a CL end up with a loan. This is because the conduit lender’s credit scoring technology identifies a G-type consumers as a bad consumer with positive probability.

**Proposition 2** (Mortgage market configurations):

27In the left hand side term of equation (16) both portfolio loan and conduit loan markets are active and the market clearing house price is computed accordingly.

28The left hand side term in equation (17) represents the G-type consumer’s utility from buying a house in the first period with a conduit loan and then renting (in a setting where only the conduit loan market is active). The right hand side term in equation (17) represents the G-type consumer’s utility from buying a house in the first period with a portfolio loan and then renting (in a setting where both portfolio loans and conduit loans markets are active).

29Alternatively, $v(CL) > \text{Pr(rating=G|G)} \lambda_G + \text{Pr(rating=G|B)} \lambda_B$. 

20
• If $\pi_{CL} < \max\{\pi_0, \pi_1\}$, the conduit mortgage market collapses and only a mass $v(PL)$ of G-type consumers can borrow to buy a house. The rest of consumers, with mass $\lambda_G - v(PL) + \lambda_B$, rent in both periods.

• If $\pi_{CL} > \pi_2$, G-type consumers prefer the conduit mortgage market. Consumers that receive a good rating are able to borrow at the conduit loan rate and buy a house. Those G-type consumers without a conduit loan will borrow from their second best option, the portfolio loan market. The rest of consumers will rent in both periods.\(^{30}\)

• When $\pi_{CL} \in [\max\{\pi_0, \pi_1\}, \pi_2]$, portfolio lenders lend to a mass $v(PL)$ of G-type consumers. Consumers that receive a bad rating in the conduit loan market have no option but to rent in both periods.\(^{31}\)

The proof follows immediately from our previous analysis and is thus omitted. Next, we explain the effect of changes of key parameters on $\pi_0, \pi_1$ and $\pi_2$. First, when the predictive power of the hard credit scoring technology worsens, $\pi_{CL}$ decreases, and there is more asymmetric information between borrowers and CLs, and all else equal, the conduit market is closer to its collapse (or enters in the collapse region). Second, when the consumer’s discount factor $\theta^i$ increases and the owner-occupied preference parameter $\eta$ decreases, consumers find renting in the first period relatively more attractive than borrowing-to-own, and thus the conduit loan market shrinks as $\pi_1$ increases. Third, when the investor’s discount factor $\theta^g$ and/or the distribution rate $d_{CL}$ increase, all else equal, the conduit loan market expands (as threshold values $\pi_0, \pi_1$ and $\pi_2$ decrease). This is because conduit mortgages become more attractive due to the higher investors’ willingness to pay for mortgages. Fourth, a higher foreclosure cost expands the region where both portfolio and conduit loan markets are active, as a lower $\delta$ decreases the value of thresholds $\pi_0, \pi_1$ and increases the value of $\pi_2$.

Next, we illustrate how the excess premium (EP) and the PL and CL loan amounts ($q^{PL}g^{PL}$ and $q^{CL}g^{CL}$, respectively) change when we vary the CL’s belief $\pi_{CL}$. We represent these functions in Figures 1 and 2, respectively. For that, we assume that $d_{CL} = 0.8$, $\theta^i = 0.4$, $\theta^g = 0.7$, $\theta^d = 0.9$, $\eta = 4$, $\delta = 0.5$, $\lambda_G = 1.5$, $\lambda_B = 1$ and $v(PL) = 1$. The $\pi$-thresholds for these parameters are $\pi_0 = 0.15$, $\pi_1 = 0.63$ and $\pi_2 = 0.71.\(^{32}\)$ In Figure 1 we observe two lines. The line $d_{CL} = 0$ in Figure 1 computes $EP$ when CL cannot distribute mortgages to investors. In this case, the G-type consumers always prefer portfolio loans over conduit loans and the conduit mortgage rate is always above $1/q^*$ (so $EP > 0$). When belief $\pi_{CL}$, the $EP$ decreases. The second line in Figure 1 illustrates $EP$ when $d_{CL} = 0.8$. It changes from positive to negative at $\pi_{CL} = \pi_2 \approx 0.71$. At this point the CL’s gains from

\(^{30}\) In particular, a mass $Pr(\text{rating}=G|G)\lambda_G + Pr(\text{rating}=G|B)\lambda_B$ of consumers receive a good rating and get a conduit loan; a mass $\min\{1 - Pr(\text{rating}=G|G)\lambda_G, 1\}$ of G-type consumers without a conduit loan borrow from portfolio lenders; and a mass $(1 - Pr(\text{rating}=G|G))\lambda_G + (1 - Pr(\text{rating}=G|B))\lambda_B - \min\{1 - Pr(\text{rating}=G|G)\lambda_G, 1\}$, of consumers rent in both periods.

\(^{31}\) In particular, a mass $Pr(\text{rating}=G|G)(\lambda_G - v(r)) + Pr(\text{rating}=G|B)\lambda_B$ of consumers get a conduit loan; and a mass $(1 - Pr(\text{rating}=G|G))(\lambda_G - v(r)) + (1 - Pr(\text{rating}=G|B))\lambda_B$ of consumers receive a bad rating from conduit lenders’ credit scoring technology and rent in both periods.

\(^{32}\) Observe that threshold $\pi_2$ that solves equation (17) exactly coincides with the threshold that solves equation $q^* = q^{CL}$ (or equivalently, $EP = 0$) and also equation $q^{PL}g^{PL} = q^{CL}g^{CL}$.
intermediation exactly offset its loss from bad type (defaulted) loans, and the EP coincides with the risk-free rate \(1/q^*\). When \(\pi_{CL} > 0.71\), the CL’s mortgage rate is smaller than the PL’s rate, and G-types consumers prefer conduit loans to portfolio loans in equilibrium. In Figure 2 we can see that it is exactly at \(\pi_{CL} = \pi_2 \equiv 0.71\) when the conduit loan amount coincides with the portfolio loan amount, and when \(\pi_{CL} > 0.71\), CLs offer a higher loan amount than PLs.

4.3 CL’s CST and regime change

In this section we want to examine the role that the CL’s credit scoring technology \(CST_{CL}^{CL}\) in triggering regime changes. We have in mind two well known facts: (1) in the early 1990s the hard credit scoring technology improved for the subprime market (e.g., computer software allowed lenders to have access to better borrower’s credit histories and FICO scores); and (2) lending standards deteriorated as subprime mortgage securitization increased during period 2001-2005 (Keys, Mukherjee, Seru, and Vig’s (2010)).\(^{33}\) We can think of two stages:

- **Stage 1:** The economy inherits a state where the portfolio loan market is the preferred choice for G-type consumers. With an initial low \(\pi_{CL}^{CL}\), we can find a \(CST_{CL}^{CL}\) high enough that triggers a change of regime where \(\pi_{CL} \geq \pi_2\) and \(\tilde{\pi}_{CL}^{CL}\) is high.

- **Stage 2:** The economy inherits a state where the conduit loan market is the preferred choice for G-type consumers (high \(\tilde{\pi}_{CL}^{CL}\)), but lending standards deteriorate \((CST_{CL}^{CL}\) decreases).

Stage (1) is characteristic of a “boom”, as the CL’s amount of mortgage lending house size and owner-occupied house price increase when \(CST_{CL}^{CL}\) is high enough to bring \(\pi_{CL} \geq \pi_2\).

\(^{33}\) There is some debate about this result. See for instance Bubb and Kaufman (2014) who study the effect of the moral hazard of securitization on lenders creening, and conclude that securitization did not lead to lax screening.
Figures 3 and 4 capture the evolution towards this change of regime. There we can observe three equilibrium regimes: Regime 1 with \((\hat{\pi}_G^{CL}, CST_G^{CL})\) such that \(\pi^{CL} < \pi_1\), Regime 2 with \((\hat{\pi}_G^{CL}, CST_G^{CL})\) such that \(\pi^{CL} \in (\pi_1, \pi_2)\), and Regime 3 with \((\hat{\pi}_G^{CL}, CST_G^{CL})\) such that \(\pi^{CL} \geq \pi_2\).

Stage (2) is characteristic of an economy where lending standards are relaxed and thus there is an inevitably high number of subsequent defaults \((\lambda_B (1 - CST_G^{CL})\) is high because \(CST_G^{CL}\) is low). Later, in Section 4.5 we will rationalize the transition from Stage 1 to Stage 2.

We now illustrate the process towards Stage 1 where \(CST_G^{CL}\) improves similar to a positive shock to the technology available to lenders. We will see how a higher \(CST_G^{CL}\) can trigger a discontinuity in the equilibrium values of mortgage lending\(^{34}\) (Figure 3), house price (Figure 4), house size \(H_{PL}^{CL}\) for borrowers with portfolio loans and house size \(H_{CL}^{CL}\) for borrowers with conduit loans (Figure 5), and rental market size (Figure 6). See the Appendix ?? for the corresponding closed form solutions. We can identify three regimes:

- **Regime 1 (low \(CST_G^{CL}\), low \(\hat{\pi}_G^{CL}\), \(\pi^{CL} < \pi_1\)):** There are only PLs in the subprime mortgage market, whose loan amount is independent of the CL’s CST, and therefore the house price is low and constant, and the size of the owner-occupied house is large precisely because house price is low. This regime is characteristic of a dormant conduit loan market.

- **Region 2 (moderate \(CST_G^{CL}\), low \(\hat{\pi}_G^{CL}\), \(\pi^{CL} \in (\pi_1, \pi_2)\)):** The conduit mortgage market emerges because CLs offer loan amounts that are sufficiently attractive to G-type households that did not get a loan from PLs than renting. In this regime there are new consumers with a mortgage when compared to Regime 1. The higher credit supply increases the demand for housing and in turn increases the house price. Also, because housing supply is inelastic, more credit coming from the conduit loan market decreases the equilibrium house size that consumers with portfolio loans can buy. On the other hand, consumers with conduit loans can buy a larger house size as \(\pi^{CL}\) keeps increasing. This regime is characteristic of an emergent conduit mortgage market that coexists with the portfolio mortgage market (still the preferred option for G-type consumers).

- **Regime 3 (high \(CST_G^{CL}\), high \(\hat{\pi}_G^{CL}\), \(\pi^{CL} \geq \pi_2\)):** This regime starts when \(CST_G^{CL}\) is high enough such that \(\pi^{CL}\) attains value \(\pi_2 = 0.71\) with low \(\hat{\pi}_G^{CL}\). At that point G-type consumers migrate to the conduit loan market and the CL’s fundamental proportion of G-type consumers, \(\hat{\pi}_G^{CL}\), jumps from 0.33 (low) to 0.75 (high).\(^{35}\) Transition from Regime 2 to Regime 3 captures the “boom” of the subprime mortgage market, driven by an expansion of CLs’ mortgage credit, high house prices, larger house sizes for borrowers with conduit loans, and a small size of the rental market (as more consumers get access to a mortgage to buy a house). This regime is characteristic of a dominant conduit mortgage market and a small or negligible portfolio mortgage market.

\(^{34}\)The total amount of PL lending and CL lending is given by expressions \(q^{PL}\psi^{PL}(\min(\lambda_G - CST_G^{CL}\mu_G^{CL}, 1))\) and \(q^{CL}\psi^{CL}\mu_G^{CL}(\text{rating}=G)\), respectively.

\(^{35}\)i.e., increases from \((\lambda_G - v(PL))/(\lambda_G - v(PL) + \lambda_B) = 0.33\) to \((\lambda_G)/(\lambda_G + \lambda_B) = 0.75.\)
The thresholds for the $\text{CST}_{G}^{CL}$ in Figures 3-6 are computed using the following expression derived from expression (4):

$$CST_{G}^{CL} = \frac{1 - \hat{\pi}_{G}^{CL}}{\hat{\pi}_{G}^{CL}} \frac{1}{1 + \frac{1}{\hat{\pi}_{G}^{CL}}} - 2 \quad (18)$$

Replacing $\pi^{CL}$ for thresholds $\pi_{0} = 0.15$, $\pi_{1} = 0.63$ and $\pi_{2} = 0.71$, we obtain thresholds $CST_{G,0}^{CL} = 0.27$, $CST_{G,1}^{CL} = 0.77$ and $CST_{G,2}^{CL} = 0.45$. Observe that $CST_{G,1}^{CL} > CST_{G,2}^{CL}$. This is because $CST_{G,1}^{CL}$ and $CST_{G,2}^{CL}$ are computed with $\hat{\pi}_{G}^{CL} = 0.33$ (low) and $\hat{\pi}_{G}^{CL} = 0.75$ (high), respectively (using (18)).

Roughly speaking, to maintain a given level of $\pi^{CL}$, a high $\hat{\pi}_{G}^{CL}$ tolerates a lower $CST_{G}^{CL}$ than a low $\hat{\pi}_{G}^{CL}$. In Figures 3 and 4 we can clearly see that the different stages and regimes identified above. In Stage 1, the portfolio mortgage market is the preferred choice for $G$-type consumers, so $\hat{\pi}_{G}^{CL} = 0.33$ (low). There can be two regimes in this first stage: Regime 1 with a low CL’s CST ($CST_{G}^{CL} < 0.27$), and Regime 2 with moderate CL’s CST ($CST_{G}^{CL} \in [0.27, 0.77]$). The transition to Regime 3, with $CST_{G}^{CL} \geq 0.77$, triggers a change of $\hat{\pi}_{G}^{CL}$ from 0.33 (low) to 0.75 (high). Once $\hat{\pi}_{G}^{CL} = 0.75$ (high), Stage 2 can be established where CLs reduce $CST_{G}^{CL}$ within the interval $CST_{G}^{CL} \in [0.45, 1]$, without changing G-type consumers’ preference for the conduit loan market as their first choice. We will examine this possibility in section 4.5.

Figures 5 and 6 deserve some additional explanations. In Figure 5 we see that when the economy enters Regime 3, the house size of borrowers with conduit loans is larger than for borrowers with portfolio loans. This is consistent with the idea that the portfolio mortgage market is not the consumers’ first option in Regime 3. We also see that the equilibrium house size of consumers with portfolio loans plummets when the conduit loan size enters in Region 3, as the expansion of the conduit loan market injects more credit in the economy and house price jumps. Also notice that there is a discontinuity in the equilibrium house size purchased with conduit loans when $\hat{\pi}_{G}^{CL}$ and $CST_{G}^{CL}$ are such that $\pi^{CL} = \pi_{2}$ even when the jump in the conduit loan amount is partially offset by the jump in the equilibrium house price at that point.

Figure 6 show that the size of the rental market is largest in Regime 1 (only the portfolio mortgage market exists). When Regime 2 starts ($\pi^{CL}$ attains $\pi_{1}$) and the rental market shrinks as new consumers get (conduit) mortgages. Then, we see how the rental market shrinks again at when Regime 3 starts ($\pi^{CL}$ attains $\pi_{2}$), as the conduit mortgage market absorbs a substantial larger fraction of G-type and B-type consumers, while the portfolio mortgage market also absorbs those G-type consumers without a conduit loan. In Regime 3 a mass $(1 - CST_{G}^{CL})\lambda_{B}$ of B-type consumers are able to get a conduit loan. However, as $\pi^{CL}$ gets closer to 1, CLs better differenciate between G-type and B-type consumers and reject more and more B-type consumers, and as a results the size of the rental market converges to the “number” of B-type consumers in the economy ($\lambda_{B} = 1$). See the Appendix ?? for a more detailed explanation of Figure 6 together with the specific equilibrium expressions of the size of the rental market.
4.4 Liquidity from securities market fuels the “boom”

Here we focus on changes of the investor’s time discount factor \( \theta^i \) while keeping the lender’s time discount factor \( \theta^l \) constant. Previous simulation exercises assumed \( \theta^l = 0.9 \) and \( \theta^i = 0.7 \). If we increase \( \theta^i \) to 0.95, the CL’s mortgage discount price and loan amount increase because a fraction \( \delta^{CL} \) of the mortgages originated by the CL is now priced at a higher price \( \theta^i = 0.95 \) (see the pricing equation (10) and the equilibrium loan amount expression in the Appendix ??). As mentioned above, when the investor’s discount factor \( \theta^i \) increases, all else equal, the conduit loan market expands and threshold values \( \pi_0, \pi_1 \) and \( \pi_2 \) decrease. Same reasoning applies when the distribution rate \( \delta^{CL} \) increases, all else
equal. We refer to a higher \( \theta^i \) and/or higher \( d^{CL} \) as more liquidity from the securities market.

**Claim:** More liquidity from the securities market reduces threshold \( \pi_2 \), and thus a lower \( CST_G^{CL} \) is necessary to transition from Regime 2 to Regime 3.

In terms of our numerical example above, when \( \theta^i \) goes from 0.9 to 0.95, \( \pi_2 \) falls from 0.71 to 0.66, and \( CST_G^{CL} \) falls from 0.45 to 0.39.

### 4.5 Endogenous transition from Stage 1 to Stage 2

In our previous analysis we identified Stage 1 as an episode where an improvement in \( CST_G^{CL} \) brings \( \hat{\pi}_G^{CL} \) from 0.33 (portfolio loan market preferred) to 0.75 (conduit loan market preferred). On the other hand, Stage 2 was characterized as an episode with high \( \hat{\pi}_G^{CL} \) but where \( CST_G^{CL} \) worsens. How can we rationalize an endogenous transition from Stage 1 to Stage 2? To answer this question we focus on the role of the mortgage securitization rate variable \( d^{CL} \), which is known to have increased during the booming expansion period of the subprime conduit mortgage market. Let start with Regime 3, where \( (CST_G^{CL}, \hat{\pi}_G^{CL}) \) are such that \( \pi^{CL} \geq \pi_2 \). Assume that the distribution rate \( d^{CL} \) is still moderate (e.g., as it happened in the period 1998-2001). Then, let us increase \( d^{CL} \) (as it occurred during period 2002-2006). We want to show that when \( d^{CL} \) increases, CLs acquire less soft information to screen between borrower types. For this, let us consider the functional form \( CST_G^{CL} = h + \sqrt{s} \), where the first and second components correspond to the hard and soft information components, respectively. Hard information is freely available and used by all lenders, whereas soft information can be acquired at cost \( s \). Also, let us modify the profit function \( \Phi^{CL}(\varphi^{CL}, z^{CL}) \) as follows:

\[
\Phi^{CL}(\varphi^{CL}, z^{CL}) = (\omega_1^{CL} - s - q^{CL}\varphi^{CL} + \tau z^{CL}) + \theta^i(1-d^{CL})(\pi^{CL}(s) \varphi^{CL} + (1 - \pi^{CL})\delta p_2 H_1^G),
\]

where \( s \) denotes the cost of acquiring soft information in the first period, and \( \pi^i(s) \) is the CL’s belief on the proportion of G-type consumers in its pool of mortgages. From expression (4) we know that \( \partial \pi^i / \partial CST_G^{CL} > 0 \), and from functional form \( CST_G^{CL} = h + \sqrt{s} \) we know that \( CST_G^{CL} \) is a continuous, increasing and concave function of \( s \). We will first assume that \( \hat{\pi}_G^{CL} \) is high and show that

**Proposition:** CLs find optimal to decrease \( CST_G^{CL} \) when the mortgage securitization rate \( d^{CL} \) increases.

Because we allow lenders to acquire soft information, this exercise can be seen as an extension of our previous setting to one with endogenous soft information acquisition. Taking the partial derivative with respect to \( s \), with (5) binding, and writing \( D^i = \varphi^i - \delta p_2 H_1^G \) to denote the lender’s default loss, we get:

\[
[s] : 1 = \theta^i(1 - d^{CL}) \frac{\partial \pi^{CL}}{\partial CST_G^{CL}} \frac{\partial CST_G^{CL}}{\partial s} D^i \tag{FOC[s]}
\]

In the FOC[s], we have that the marginal cost (MC) to acquire soft information is constant and equal to 1, while the marginal benefit (MB) decreasing with slope \(-1/(4s^{3/2})\).
The intersection between MC and MB pins down the optimal amount of soft information acquired by conduit lenders.\textsuperscript{36} Because $\partial CST_{G}^{CL}/\partial s = 0.5s^{-1/2}$, FOC$[s]$ implies that the CL finds optimal to acquire less soft information the higher is its mortgage distribution rate $d^{l}$, ceteris paribus.\textsuperscript{37} This proves our proposition. The result also rationalizes our simplifying assumption in the baseline model that PL with $d^{PL} = 0$ have access to soft information, whereas CLs with $d^{CL} > 0$ don’t.

**Example:** Let $CST_{G}^{CL} = h + \sqrt{s}$ with $h = 0.6$. Take the same parameters used in previous simulations with the exception that $d^{CL}$ can now take two possible values: $d^{CL} = 0.5$ (moderate securitization) and $d^{CL} = 0.8$ (high securitization). In Figure 7 we plot two MB curves corresponding to each of these securitization rates. Table 1 represents the optimal values of soft information acquisition ($s$), the value of $CST_{G}^{CL}$, the CL’s belief $\hat{\pi}_{G}^{CL}$, and the consistency between $\hat{\pi}_{G}^{CL}$ and $\hat{\pi}_{G}^{CL}$ (when $\pi^{CL} \geq \pi_{2}$, $\hat{\pi}_{G}^{CL}$ should equal 0.75; and when $\pi^{CL} < \pi_{2}$, $\hat{\pi}_{G}^{CL}$ should equal 0.33). As expected, when $d^{CL}$ increases, the amount of soft information $s$ acquired by a CL decreases. For instance, going from a moderate $d^{CL} = 0.5$ to a high $d^{CL} = 0.8$, decreases $s$, and this change in turn decreases $CST_{G}^{CL}$ from 0.82 to 0.68. Then, using expression (4) together with these values for $CST_{G}^{CL}$ and $\hat{\pi}_{G}^{CL} = 0.75$ (CL is the best option for G-type consumers), we see that $\pi^{CL}$ decreases from 0.93 to 0.83. Notice that when $d^{CL}$ increases, $\pi_{2}$ decreases, but still we have $\pi^{CL} \geq \pi_{2}$ for both cases. Thus, fundamental proportion $\hat{\pi}_{G}^{CL} = 0.75$ is consistent with region $\pi^{CL} \geq \pi_{2}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Figure 7}
\end{figure}

\textsuperscript{36}In equilibrium default losses can in turn be expressed as a function of the parameters of our economy as follows: $D(\omega^{2}_{1}, \delta, d^{CL}, \pi^{CL}, \theta^{l}, \theta^{i}) = \omega^{2}_{1}(1-\delta) - \frac{\delta \omega^{2}_{2}}{2}$. 

\textsuperscript{37}Also, from the FOC$[s]$ we find that the optimal amount of soft information is lower the lower is its discount factor $\theta^{l}$, the lower is the default loss $D(\varphi^{l})$, and the weaker is the effect of $s$ on $CST_{G}^{CL}$ (here given by the square root function).
4.6 A decadent labor market can trigger the “bust”

Now consider parameters $\lambda_G$ and $\lambda_B$, which stand for the measure of G-type and B-type consumers in the economy, respectively. A shock to the labor market that increases unemployment among subprime consumers can be seen in terms of our model as a decrease in $\lambda_G$ and increase in $\lambda_B$. In this case, B-type consumers are thought as those consumers that remain with the subsistence rent $\omega^S_R$ and fail to get a job that gives them the extra income $\omega^+ - \omega^S_R$. A negative shock to the labor market does not change equilibrium threshold $\pi_2$, which can be written as

$$\pi_2 = \frac{\theta^i(1 - \delta(d^{CL}\theta^i + (1 - d^{CL})\theta^i))}{(d^{CL}\theta^i + (1 - d^{CL})\theta^i)((1 - \theta^i) + \theta^i(1 - \delta))}.$$ 

However, a negative shock to the labor market decreases $\pi^{CL}$ because it depends on $\tilde{\pi}^{CL}_G$ (see expression (4)), which decreases when $\lambda_G$ goes down and $\lambda_B$ goes up. It is possible to decrease $\tilde{\pi}^{CL}_G$ enough to bring $\pi^{CL}$ below threshold $\pi_2$, in which case the conduit mortgage market shrinks or even collapses.

Claim: For a given $CST^{CL}_G$, a shock to $(\lambda_G, \lambda_B)$ is sufficient to bring the economy from Region 3 to Regions 2 or 1.

In addition, a lower liquidity from the securities market (i.e., lower $\theta^i$) also contributes bringing down the economy to Regions 2 or 1 (this argument is the opposite as our previous claim).

5 Discussion

5.1 A short narrative of the boom and bust

Here we provide a short narrative for each of the equilibrium regimes identified in the previous section.

- Stage 1, Regime 1: Only portfolio lenders

Consider the period pre-middle 1990s in which subprime loan credit scoring technology was crude and there did not exist powerful summary statistics on consumer credit quality (FICO score). This meant that it was very difficult for subprime loan originators to reliably distinguish between good and bad credit borrowers based on hard information (low $CST^{CL}_G$, low $\tilde{\pi}^{CL}_G$, and $\pi^{CL} < \pi_1$). If transaction-based lending were to occur based on hard information only, the high likelihood to confusing good and bad types in underwriting decisions would increase loan rates substantially due to adverse selection concerns, thus
potentially pricing all borrowers out of the conduit mortgage market. But relationship (portfolio) lenders, such as local depository financial institutions, were capable to soliciting soft information to improve their underwriting decision outcomes. Potentially based on regulatory requirement (e.g., CRA), localized relationship lenders were the only available source of subprime loans, but were subject to capacity constraints that resulted in the rationing of credit (to good types) in subprime neighborhoods. Also, in addition to adverse selection concerns as related to loan pricing with transaction-based lending, in this period there was also little demand for subprime loans packaged as securities (e.g., low \( \theta^i \) could be such that the gain from intermediation, \( \theta^i - \theta^l \), was low or even null). There were not strong regulatory or tax reasons to invest in pooled-tranched securities backed by mortgage or other types of loans. Capital flows into bond markets were “normal” and were not distorted by factors such as foreign capital flows looking for dollar-denominated low-risk investments. This implies that a private-label subprime MBS market was non-existent, since the high cost of loan sales was not offset by any other benefits that might be associated with subprime loan securitization.

- **Stage 1, Regime 2: Conduit lenders enter into the subprime mortgage market**

Now consider the evolved period from the middle 1990s to early 2000s in which credit information was then available to improve credit scoring decisions (FICO is introduced and provides accurate assessments of borrower credit quality), and where credit scoring models \( (CST^{CL}_G) \) themselves improved. This created a foundation where it is now possible to more credibly distribute subprime loans into a secondary market. Concurrent with this is the introduction of capital reserve regulation (Basel II) that increased the attractiveness of owning low credit risk (AAA-rated) securities. There has also been shocks (the Asian and Russian financial crises) that have shifted foreign capital flows towards dollar-denominated U.S. Treasuries and close substitutes. This shift in demand has decreased yields of riskless and near riskless bonds, causing fixed-income investors to move further out the credit risk curve in search for higher yields. The search for higher yields and favorable capital treatment causes demand for AAA-rated securities to skyrocket. But these securities were not in sufficient supply to meet all of the demand. The subprime mortgage market represented a vast untapped market, where the pooling of such loans could then be converted (in part, but large part) into AAA-rated securities in large quantities to help satisfy the demand. Improved credit scoring technology along with a high demand for manufactured AAA-rated securities sets the stage for the rise of the subprime mortgage market. A reduction in the pooling rate on subprime loans due to better (perceived if not actual) sorting of good and bad types made it feasible for low-cost transaction-based lenders (brokers and other conduit lenders) to set up shop to apply automated underwriting based on hard information only.

- **Stage 1, Regime 3: Conduit lenders dominate the subprime lending market**

By the early to middle 2000s, demand for AAA-rated securities had intensified (\( \uparrow \theta^i \)). With this intensified demand and increasing confidence in the basic conduit loan business model \( (CST^{CL}_G) \), conduit loans rates declined to the point where the pooled conduit loan
rate fell below the portfolio loan rate, and the traditional portfolio loan market shrink (relative to the total size of the subprime mortgage market) as good subprime borrower types migrated to the conduit loan market to take advantage of the low rates ($\pi_{CL}^{G}$ increases). There was a housing market boom.

- **Stage 2: The “bust” of the subprime conduit mortgage market**

Finally, starting in 2006, with the start of a sustained increase in unemployment ($\downarrow \lambda_{G}$, $\uparrow \lambda_{B}$) and also concerns about the performance of subprime mortgages due to lenders’ potential lax screening (high $\pi_{CL}^{G}$ but moderate $CST_{CL}^{G}$, as in Stage 2 above), confidence in the credit scoring based conduit loan business model was shaken. Also, demand for credit-risky MBS fell (e.g., investors became more impatient, i.e., $\downarrow \theta$), and consequently investors increased the required pooling loan rate. All these events brought the conduit loan market into the collapse region ($\pi_{CL} < \pi_1$). Subprime home ownership rates stalled and the housing boom ended (badly).

- **The conduit mortgage market reemerges**

Lastly, as a post-script, imagine it is 2018 and the U.S. economy is now “normalized”. The “broken” securitized lending business model is declared to be “fixed” as improved scoring variables are introduced and mechanisms are put into place to improve the quality of credit model assessments ($\uparrow CST_{CL}^{G}$). Also, the percentage of good types in the subprime population increases due to an improved job outlook and increasing wages at the low end of the labor market ($\uparrow \lambda_{G}$). Demand for highly rated securities has persisted ($\uparrow \theta$), and once again a conduit subprime mortgage market emerges to provide financing for the lower end of the housing market.

5.2 Adverse selection in the secondary mortgage market

In this paper we have focused on information problems that occur in the primary mortgage market, and let aside potential adverse selection problems in the secondary mortgage market. In this setting we have shown how changes in the hard credit scoring technology, liquidity from the securities market and labor market conditions may trigger a change to a different equilibrium regime, in turn affecting house prices, mortgage rates, loan amounts and housing affordability.

Information problems in secondary mortgage markets have been now widely studied, and can be seen as complementary to our analysis of boom and bust episodes in the subprime mortgage market. For instance, Fishman and Parker (2015) consider a setting where investors may acquire more information than intermediaries (CLs in terms of our model, as CLs only rely on hard information). In their model valuation by sophisticated investors creates an adverse selection problem. This is because investors who do valuation fund only good assets, leaving bad ones to approach unsophisticated investors. This worsens the pool of assets purchased by unsophisticated investors who do not do valuation, in turn lowering the price, in turn making valuation even more profitable. In that setting, a move from an equilibrium with valuation to an equilibrium without valuation has the features of a credit crunch: lower prices, lower levels of investment, and profitable valuation. Some of
these features also appear in Gorton and Ordonez (2014) theory of short-term collateralized debt. In their setting, when the economy relies on informationally insensitive debt, firms with low quality collateral can borrow, generating a credit boom and an increase in output. A crisis occurs when a (possibly small) shock causes agents to suddenly have incentives to produce information, leading to a decline in output.

Another strand of the literature has focused instead on the strategic considerations that lenders have when securitizing their mortgages for distribution to security investors. For instance, Frankel and Jin (2015) show that under securitization ignorance is bliss: a remote bank can compete successfully for applicants with strong observables because investors will not suspect the remote lender of choosing only bad loans to sell.

Our baseline model rules out the possibility of adverse selection in the secondary mortgage markets because (1) CLs and investors rely on the same (hard) information to screen between borrower types (i.e., \( \pi^i = \pi^{CL} \)), and (2) PLs, who have access to soft information in the baseline model, are not allowed to distribute mortgages to investors. This set of assumptions eliminates the possibility of adverse selection in the secondary mortgage markets. Adverse selection in secondary markets may arise if investors who only rely on hard information buy mortgage-backed securities from lenders that have superior (soft) information. In the Appendix ?? we explore this possibility and its implications on the equilibrium regime, mortgage spreads and realized defaults. There we consider sophisticated portfolio lenders that are able to securitize mortgages and distribute them to the investors. We consider two situations: one where sophisticated PLs only distribute mortgages that the hard credit scoring technology assigns a good rating; and another where sophisticated PLs behave strategically and sell bad mortgages to naive investors whose CST identifies as good mortgages. In the first case, we show that sophisticated PLs become the first choice for G-type consumers as they can distribute mortgages as CLs do, but also have better information than CLs. In the second case, we show how investors are selected against by informed mortgage originators and, as a result, investor’s default expectations are lower than their realized default. In future work we are planning to construct a more elaborated model along these lines.

6 Conclusions

We have provided a general equilibrium model of a subprime economy with endogenous market segmentation, endogenous tenure choice, endogenous house prices, and endogenous mortgage rates and loan amounts. The distinction between the two different sources of funding for consumers (portfolio vs. conduit lenders) was important to capture the trade-off between access to soft information and access to the liquidity from the secondary securities market, as well as to illustrate the migration from one market to another and their respective market sizes. Another important element of our theory was the limited recourse nature of the subprime mortgages, which gives rise to borrower’s adverse selection problems in the primary mortgage conduit market. Our analysis provided a new way to characterize the emergence, and posterior boom and bust of the subprime conduit mortgage market, by focusing on the improvements of the hard credit scoring technology associated to that specific market, as well as changes in liquidity from the securities market and labor market.
There are several other interesting theoretical extensions that we leave for future work. First, we think that it would be interesting to examine whether pre-payment penalties and mortgage refinancing have any role in implementing a Pareto superior equilibrium when adverse selection is present. Secondly, it also seems interesting to allow for a house price bubble and study its implications on the rise and fall of the subprime mortgage market. This possibility, although it has already been widely studied in the literature, could bring new insights on how the bubble relates to innovations in the credit scoring technology and subsequent beliefs. Finally, our model can be enriched by incorporating agency issues regarding securitization and examining its implications on distressed loans.

Our model and results provide new insights for empirical work. For instance, one would like to compare the severity of the adverse selection problem in the subprime mortgage market between non-recourse US states and limited recourse US states following our discussion in the Appendix. For that, Ghent and Kudlyak’s (2011) table 1 serves as an excellent summary of the different state recourse laws. Also, it would be interesting to examine how the severity of the adverse selection problem changed during the different securitization regimes, or when differences along time in foreclosure costs, banks’ lending capacities, or the credit scoring technology are observed. Last, but not least, it would be interesting to see the economic and statistical significance of the components that we identify in the mortgage credit spread.

References


