The Real Exchange Rate in the Long Run:
Balassa-Samuelson Effects Reconsidered

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Abstract

Historical data for over hundred years and 14 countries is used to estimate the long-run effect of productivity on the real exchange rate. We find large variations in the productivity effect across four distinct monetary regimes in the sample period. Although the traditional Balassa-Samuelson model is not consistent with these results, we suggest an explanation of the results in terms of contemporary variants of the model that incorporate the terms of trade mechanism. Specifically we argue that changes in trade costs over time may affect the impact of productivity on the real exchange rate over time. We undertake simulations of the modern versions of the Balassa-Samuelson model to show that plausible parameter shifts consistent with the behavior of trade costs can explain the cross-regime variation of the productivity effect.

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Key Words: Real exchange rates; Productivity; Balassa-Samuelson model; Terms of trade
1. Introduction

A basic hypothesis about the behavior of the real exchange rate is that the Purchasing Power Parity (PPP) holds in the long run and the real exchange rate converges to a constant value. Empirical testing of this hypothesis (based on post Bretton Woods period as well as longer spans of time) has produced mixed results.\(^1\) The evidence that the real exchange rates series are stationary is not conclusive, but even when stationarity is indicated the series exhibit a high degree of persistence. Estimates of the half-life of deviations from the mean value (typically from 3 to 5 years) suggest the PPP puzzle (Rogoff, 1996) that they are too long to be produced by monetary shocks (needed to account for exchange rate volatility) under plausible nominal rigidities.

One explanation of the highly persistent behavior of the real exchange rate is provided by the Balassa-Samuelson model that includes nontraded goods.\(^2\) In the standard version of this model, the long-run PPP holds only for traded goods, and the real exchange rate in the long run is a function of the relative productivity of traded to nontraded goods in the home and foreign countries. The time series properties of the real exchange rate in the model depend on the behavior of home and foreign productivity ratios (for traded relative to nontraded sectors). The Balassa-Samuelson effects also suggest an explanation of the PPP puzzle: estimates of the half-

\(^1\) For example, see Froot and Rogoff (1995), MacDonald (1995) and Taylor and Taylor (2004), for overviews of the extensive empirical research on PPP.

\(^2\) We use the conventional name for this model based on Balassa (1964) and Samuelson (1964). A number of studies also recognize the contribution of Harrod (1933) and call it the Harrod-Balassa-Samuelson model.
life would be biased if they are based on deviations from a constant mean value when the long-
run value of the real exchange rate is, in fact, time varying.\(^3\)

There is growing evidence for the post Bretton Woods period (for which sector-level productivity data are available) that the relative productivity of traded goods exerts a significant long-run effect on the real exchange rate.\(^4\) The magnitude of the effect, however, is typically smaller than the level predicted by the standard Balassa-Samuelson theory. A possible source of this result is that PPP does not hold for traded goods even in the long run.\(^5\) Departures from traded goods PPP can arise if, as suggested by recent trade and macroeconomic models, home and foreign goods are not perfect substitutes because of specialization in production or product differentiation. In this case, changes in traded goods productivity affect the real exchange rate not only through the relative price of nontraded goods (the conventional channel), but also via the relative price of home to foreign traded goods (the terms of trade). The terms of trade adjustment can diminish or even reverse the productivity effect operating through the conventional channel.\(^6\)

The empirical literature on the Balassa-Samuelson effects has not adequately explored how well the conventional model and its variants fit the data. To explore this issue, we use a large data set spanning over one hundred years and including 14 countries. As productivity data

\(^3\) See Taylor, Peel and Sarno (2001) and Imbs and others (2005) for alternative explanations of the PPP puzzle based, respectively, on nonlinearities and aggregation bias. Also see Macdonald and Ricci (2005) for an explanation highlighting the role of the distribution sector within the Balassa-Samuelson framework.

\(^4\) See, for example, Chinn and Johnston (1996), Choudhri and Khan (2005), Lee and Tang (2007), and Ricci, Milesi-Ferretti and Lee (2008).

\(^5\) There is considerable evidence of the failure of PPP for traded goods (e.g., Canzoneri, Cumby, and Diba, 1999, Engel, 1999). See MacDonald and Ricci (2005) for a theoretical discussion.

\(^6\) For an example of the reversal of the productivity effect within a DGE model (calibrated to UK-Euro area), see Benigno and Thoenissen (2003).
at the sectoral level are not available for earlier periods, our empirical analysis follows the usual practice of using the income per capita differential between the home and foreign countries as a proxy for the traded goods productivity differential, and identifying the Balassa-Samuelson effect with the coefficient of the income differential in the real exchange rate relation. The magnitude of the effect (so interpreted) depends on the behavior of the nontraded good productivity differential. We show that if, as typically assumed, the shares and productivity growth of nontraded goods do not differ between countries, then the Balassa-Samuelson effect in the conventional model equals the relative share of nontraded to traded goods. We also show that the effect would be smaller (but not change sign) in a variation of the model where the nontraded good productivity differential varies less than proportionately to changes in the traded good productivity differential.

We also examine the Balassa-Samuelson effects in two modern variants of the model, the first one based on specialization in production and the second on monopolistic competition. Both variants introduce the terms of trade channel, and the second variant also adds another channel operating via endogenous entry and exit of firms.\(^7\) We show that, as compared to the conventional model, the Balassa-Samuelson effects can be smaller or even of the opposite sign in the two variants, depending on the extent to which the terms of trade adjust in response to productivity changes.\(^8\) The two key determinants of the terms of trade response are the elasticity

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\(^7\) In the monopolistic competition model, we focus on the conventional case of homogeneous productivity (e.g., Krugman, 1980). Ghironi and Melitz (2005) suggest an alternative model of Balassa-Samuelson effects without nontraded good based on Melitz (2003) model of heterogeneous productivity. Bergin, Glick and Taylor (2006) develop a model where the share of nontraded goods (in an exogenous set of goods) is determined endogenously.

\(^8\) The terms of trade adjustment can cause a reversal of the Balassa-Samuelson effect even in a model with one factor of production. Other mechanisms such as labor market inefficiencies or biased technological change could also lead to a reversal of this effect in models with multiple factors: see, for example, Sheng and Xu (2011) and Gubler and Sax (2013).
of substitution between the home and foreign traded goods and the bias for home goods in the consumption of traded goods.\(^9\)

In our empirical analysis, we let the United States be the reference country and use Panel Dynamic Ordinary Least Squares (PDOLS) and Group Mean procedures to estimate the long-run Balassa-Samuelson effect. For the whole 1880-97 sample period, we find that the average long-run effect (across countries) is significantly positive, but is small and between 0.13 and 0.22 (depending upon which procedure is used and whether a trend is included or not). Assuming that the share of nontraded goods is at least 0.5, the conventional model predicts the effect to equal or exceed 1.0. It is difficult to explain such a large discrepancy between the estimated and predicted values. The discrepancy is smaller if the nontraded good productivity differential varies with the traded goods differential, but it is not reduced much under plausible assumptions. An explanation of the results is suggested by the modern versions that incorporate the terms of trade mechanism. We show that in these models, the size of the estimated effect can be accounted for by reasonable estimates of the home bias and the elasticity of substitution (within the range of values used in macroeconomic models).

The sample period involves major shifts in exchange-rate regimes and we distinguish four sub-periods representing different regimes: 1880-1913 (the classical gold standard), 1914-1945 (the wars and interwar), 1946-1971 (Bretton Woods), and 1972-1997 (managed floating). There were also important shifts in the structure of trade across these monetary regimes caused by changes in trade costs. To allow for differences in both the dynamics and the long-run effects among regimes, we also estimate the real exchange rate relation for each sub-period separately.

\(^9\) Choudhri and Schembri (2010) show that changes in the values of the substitution elasticity and the home bias (within the range suggested in the literature) can cause considerable variation in the Balassa-Samuelson effect.
We find that the average long-run productivity effect differs considerably across regimes: it is significantly positive but small in the 1880-1913 sub-period; becomes significantly negative and large in the 1914-1945 sub-period; is generally not significantly different from zero in the 1946-1971 sub-period; and is significant and positive for the 1972-1997 sub-period. The conventional model also does not provide a satisfactory explanation of these cross-regime differences. In the modern versions, however, we show that plausible changes in the home bias and the substitution elasticity caused by shifts in the behavior of trade costs could account for key differences in the Balassa-Samuelson effects across regimes.  

Although the Balassa-Samuelson effect tends to vary across regimes, the evidence suggests that it is present, and in the long-run the real exchange rate is not constant but conditioned on relative income levels. To explore the role of the Balassa-Samuelson effect in explaining the PPP puzzle, we examine both the conventional estimates of the half-life (based on the deviations from the mean) and the estimates controlling for the Balassa-Samuelson effect (based on the residuals in the real exchange rate regression). For the whole period, the (Nickel and time corrected) estimate of the half-life without the Balassa-Samuelson effect is between 3.4 and 5.3 years and is similar to conventional range of 3-5 years.  

Accounting for this effect lowers the estimated range to 1.9-2.6 years, which is much lower but still fairly long. The whole-period estimates, however, do not allow for a heterogeneous effect across regimes. Thus, we also estimate the half-life for each sub-period. The sub-period estimates (incorporating the

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10 We also explore an extended real exchange rate relation that includes trade balance (a proxy for net foreign assets) as an additional variable. The introduction of this variable makes little difference to the estimates of the Balassa-Samuelson effect (for the whole period as well as sub-periods).

11 The correction is based on the procedure suggested by Choi, Mark and Sul (2006) and is discussed in Section 4.1.

12 The estimates for the whole period as well as for the sub-periods discussed below are based on residuals in regressions with a time trend
Balassa-Samuelson effect) further reduce the half-life range to 0.4-0.7 years in the 1880-1913, 0.8-1.4 years in the 1914-1945, 0.3-0.6 years in the 1946-1971, and 0.7-1.2 years in the 1972-1997 sub-periods.  

These estimates suggest that the adjustment for a variable Balassa-Samuelson effect substantially lowers the persistence of the real exchange rate and helps resolve the PPP puzzle.

Our paper is related to Lothian and Taylor (2008) study, which uses data for three countries over a long span of time (nearly two centuries) to examine the importance of the Balassa-Samuelson effect. Their focus, however, is on testing the significance of this effect (assumed to be homogeneous over the entire period) within a framework that allows nonlinear reversion to the equilibrium real exchange rate and shifts in exchange rate volatility across regimes. The focus of our paper instead is on explaining the magnitudes and the signs of the Balassa-Samuelson effect under different regimes. Important contributions of our paper are to show that the estimates of the Balassa-Samuelson effect vary across regimes and to provide an explanation consistent with the modern versions of the Balassa-Samuelson model, which incorporate the terms of trade channel.

Our paper also relates to Chong, Jorda and Taylor (2012), who use panel data including a large number of countries and covering the recent period to provide support for the existence of a long-run Balassa-Samuelson relation. Their main objective is to measure adjustment to long-run

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13 Even the sub-period estimates without the Balassa-Samuelson effect imply shorter half lives since they partially account for the time variation in the real exchange rate by allowing a different mean for each regime. These estimates, however, do not reduce half lives as much as the estimates with the Balassa-Samuelson effect.

14 Their estimate of the Balassa-Samuelson effect equals 0.125 for sterling-dollar and zero for the sterling-franc real exchange rates. Interestingly, the average value of their estimates (0.0625) is not much different than our estimate of the average effect for the whole period without a time trend (0.068).
equilibrium, which is purged of short-run frictions. Using a local projection approach, they find that the long-run adjustment has shorter half lives. Their results, however, do not explain the PPP puzzle related to short-run adjustment.\textsuperscript{15} Although explaining this puzzle is not the main concern of our paper, our empirical analysis does suggest that half lives of the short-run adjustment (after removing the long-run Balassa-Samuelson effect and allowing this effect to vary across periods) are not too long.

2. Data, Descriptive Evidence and Monetary Regimes

We have assembled annual data for 14 countries over the period 1880-1997 on real exchange rates relative to the U.S., real per capita GDP and the trade balance to GDP ratio. The countries covered, most of which are advanced countries are: Argentina, Australia, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, the UK and the US. For the data sources see the Data Appendix. Gaps in the annual data reflecting missing data during the World Wars for several countries were filled in by linear interpolation. The three series for each country ordered alphabetically are presented in figure 1 which shows both the actual data and the data filtered using a Hodrick-Prescott filter. The series presented are all relative to the U.S.

2.1 Monetary Regimes

Our data covers close to 12 decades. During that time span the world exhibited significant changes in the exchange rate regimes followed, as well as in the structure of the global economy. We demarcate the 12 decades into 4 distinct regimes: 1880-1913, the classical gold standard; 1914-1945, the wars and interwar; 1946-1971, Bretton Woods; and 1972-1997 managed floating.

\textsuperscript{15} In fact, they restate the puzzle as “How can one reconcile the high-frequency variability and flexibility of nominal exchange rates with the slow adjustment and persistence generated by frictions thought to have effects in the short term only?”
Our sample concludes just before the start of the formation of the euro zone. Including more up to date data would have resulted in five of the countries in our sample entering with identical exchange rates and monetary policy after the introduction of the Euro, thereby restricting our analysis.

*1880-1913*

This was the period of the classical gold standard in which most countries of the world had fixed gold parities (in our sample Italy adhered to gold for only part of the period as did Argentina, Spain never did). It was also a period characterized by relatively low tariffs and non-tariff barriers in most countries (with the principal exception of the US), open capital markets, relatively flexible wages and prices and limited government intervention in the economy.

In this period most countries had relatively stable real exchange rates reflecting the fixed parity and relatively stable price levels. It was also a period of sustained growth across the world although with the principal exception of Canada and Sweden, not growing as rapidly as the US (see figure 1). In this period, there were massive capital flows from the advanced countries of Western Europe to the emerging countries of the day: Argentina, Australia and Canada. This was reflected in persistent current account surpluses in the former countries and deficits in the latter (Bordo, Eichengreen and Kim 1998).

*1914-1945*

This turbulent period marked a significant change in the policy regime and in structure. Most countries abandoned the gold standard during and after World War I and allowed their exchange rates to float, or they imposed extensive exchange and capital controls (Obstfeld and Taylor
The gold standard was reinstated from 1925-1936 as a Gold-exchange standard. It was followed by a period of heavily managed floating.

The 1930s were characterized by large increases in tariff and non-tariff barriers and extensive capital controls. The 1930s also was a period with considerable nominal rigidity and in which government intervention in the economy increased dramatically to protect income and employment from the shocks of the Great depression. World War II continued the trends of the 1930s.

In this period the real exchange rate was highly unstable with no distinct trend. Per capita real growth, although high in the 1920s, declined dramatically in most countries during the 1930s until they cut the link with gold (Eichengreen 1992). There is little pattern of convergence with the US in the 1930s and substantial divergence in the war years (figure 1). The relationship between trade balances and the real exchange rate also appears to be weak in this period reflecting the disruption in trade and capital flows.

**1946-1971**

The Bretton Woods system established in 1944 required members to declare fixed parities in terms of the dollar. It also required current account convertibility. The postwar era was characterized by significant declines in tariff barriers, continued restrictions on capital mobility, considerable government intervention in the macro economy and remarkable macro stability (Bordo 1993). Real exchange rates were stable.

Economic growth in most countries was dramatic as they recovered from the war. The convergence that occurred in real per capita growth relative to the US is evident in figure 1. Also
evident in this period from figure 1 is a strong correlation between relative growth and the real exchange rate for a number of countries.

1972-1997

In this period, the Bretton Woods system collapsed and was replaced by generalized managed floating. For the first 10-15 years real exchange rate volatility was relatively high, with a positive trend in most countries. During this period trade protection continued to decline and trade mushroomed. Also restrictions on capital movements were gradually eliminated. After 1985 exchange rate volatility declined. The pattern of convergence in real per capita GDP with the US, and the positive correlation between relative per capita incomes and the real exchange rate, continued until the 1990s. Finally, in this period a negative correlation between the relative trade balance and the real exchange rate is apparent for a number of countries, especially Canada, Sweden and the UK.

In sum, as can be seen in figure 1, there is some evidence of the positive relationship between the real exchange rate and real per capita GDP relative to the U.S. suggested by the theory in section 3. There is also some evidence for the negative relationship between trade balances as a proportion of GDP relative to the U.S. for a number of countries. In section 4 of the paper we present econometric evidence that is consistent with the descriptive evidence presented here.

3. Theoretical Framework

This section develops a theoretical framework to compare different models of the long-run behavior of the real exchange rate and motivate the empirical relations estimated in the next
section. We first discuss a basic model based on the conventional Balassa-Samuelson theory. We then consider two variants of the basic model, which introduce specialization in production and monopolistic competition. We use a simple setup with two countries (home and foreign), one factor (labor), and two categories of goods (traded and nontraded goods). To facilitate comparisons between different models, we assume that there is a continuum of goods (or varieties) in each category. In the basic model, all traded goods can be produced in both countries. The first variant assumes that each country is specialized in the production of a subset of traded goods and thus home and foreign goods are not perfect substitutes. Productivity changes in this model lead to an adjustment in the terms of trade as well as the relative price of nontraded goods. The second variant assumes monopolistic competition (with each country producing different varieties of a differentiated traded good), and introduces an additional adjustment mechanism through endogenous entry and exit of firms.

3.1 Basic Model

We focus on the equations of the model for the home economy. Symmetric equations are assumed for the foreign economy with an asterisk used to denote foreign variables and parameters. The lifetime utility of a representative household is given by \( U_i = \sum_{s=t}^{\infty} \delta^{s-t} C_s \), where \( C_s \) is the household’s aggregate consumption index. As the paper is concerned only with the long-run effects, we focus on the steady-state conditions, and do not explicitly model the dynamics that would arise from nominal rigidities or international borrowing or lending (since the time subscripts are not needed, they are omitted). All prices in the model are defined as real prices in terms of \( C \) for the home relations and \( C^* \) for the foreign relations.

The aggregate consumption index is defined by the following Cob-Douglas function:
where $C_N$ and $C_T$ are consumption bundles for nontraded and traded goods, and $\gamma$ is the share of nontraded goods in expenditure. The indexes for the nontraded and traded consumption bundles are defined by the following CES aggregators:

$$
C_N = \left[ \int_{i \in \Omega_N} C_N(i)^{(\sigma-1)/\sigma} \, di \right]^{\sigma/(\sigma-1)},
$$

(0)

$$
C_T = \left[ \int_{j \in \Omega_T} C_T(j)^{(\sigma-1)/\sigma} \, dj \right]^{\sigma/(\sigma-1)},
$$

(0)

where $C_N(i)$ and $C_T(j)$ represent amounts consumed of individual nontraded and traded goods indexed by $i$ and $j$, respectively; $\Omega_N$ and $\Omega_T$ denote sets of nontraded and traded goods; and $\sigma$ is the elasticity of substitution between goods within each aggregate.

The production functions for nontraded and traded good are

$$
Y_N(i) = A_N L_N(i),
$$

(0)

$$
Y_T(j) = A_T L_T(j),
$$

(0)

where $Y_N(i)$ and $L_N(i)$ denote output and labor input for good $i$; $Y_T(j)$ and $L_T(j)$ are the corresponding variables for good $j$; and $A_N$ and $A_T$ represent labor productivities for nontraded and traded goods, which are assumed to be the same for all goods in each category.

Minimization of unit costs of $C_C, C_N$, and $C_T$ implies the following real prices of these indices:
where \( p_N(i) \) and \( p_T(j) \) are real prices of goods \( i \) and \( j \) in units of \( C \). Under perfect competition, these prices are set as

\[
p_N(i) = \frac{w}{A_N},
\]

\[
p_T(j) = \frac{w}{A_T},
\]

where \( w \) denotes the real wage rate. Assume that there are no trade costs. Under this assumption, the law of one price implies that

\[
q p_T(j) = p_T^*(j),
\]

where \( q \) is the real exchange rate defined as the real value of home currency (i.e., the relative price of \( C \) in terms of \( C^* \)).

The equilibrium conditions for nontraded goods and the labor market are

\[
C_N(i) = Y_N(i),
\]

\[
L = \int_{\omega_N} L_N(i) \, di + \int_{\omega_T} L_T(j) \, dj,
\]

where \( L \) is the fixed supply of labor. Following the standard approach, we assume that there exists a debt-dependent transaction cost or a risk premium that induces a unique steady state with the ratio of real net foreign assets to \( C \) equal to a certain (possibly zero) value. Letting \( b \)

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\(^{16}\) Trade costs are introduced in the variants of this model. In the present model, however, absence of trade costs is needed to have both diversified production (all goods in set \( \Omega_T \) are produced in both countries) and trade in goods.
denote the corresponding ratio of the real trade balance to $C$ (i.e., the net foreign asset ratio times the real interest rate), we express the equilibrium condition for the current account balance as

$$\int_{j \in \Omega} p_T(j)[C_T(j) - Y_T(j)] dj = bC . \quad (0)$$

To derive a relation determining the real exchange rate, we use a first-order log-linear approximation around the initial steady state. Let a hat over a variable denote the log deviation from its initial steady state value. Using (6)-(9), their foreign counterparts and (10), we can derive a general form of the conventional Balassa-Samuelson relation for the real exchange rate as

$$\hat{q} = \gamma(\hat{A}_T - \hat{A}_N) - \gamma^*(\hat{A}_T^* - \hat{A}_N^*) . \quad (0)$$

Empirical analysis often uses a simple form of this relation, where the real exchange depends on the home-foreign differential in traded goods productivity or income per capita. This form can be derived by making the following two assumptions:

**Assumption 1.** $\gamma = \gamma^*$.

**Assumption 2.** $\hat{A}_N = \hat{A}_N^*$.

The first assumption abstracts from international differences in tastes (i.e., the share parameters in the aggregate consumption index). The second assumption accords with the view often associated with the Balassa-Samuelson theory that nontraded goods generally represent services, which are produced by similar technology across countries. Under assumptions 1 and 2, relation (14) simplifies to: $\hat{q} = \gamma(\hat{A}_T - \hat{A}_T^*)$. Thus in the simple version, the home-foreign differential in
the traded goods productivity is the only determinant of the real exchange rate and the coefficient of the productivity differential equals the share of nontraded goods.

The simple version also implies a relation that links the real exchange rate to the differential between real income per capita in the home and foreign country. Let \( y \) denote real income per capita and assume that it is proportional to real income per worker. Also let \( b = 0 \) in the initial steady state so that real income per worker simply equals the real wage (since labor is the only factor and there are zero profits). Set \( y = w \), and use (6)-(9) to get

\[
\hat{y} = \gamma \hat{A}_N + (1 - \gamma) \hat{A}_F.
\]  

(0)

Next, Use (15) and its foreign counterpart to substitute for \( \hat{A}_F \) and \( \hat{A}_F^* \) in (14) and get:

\[
\hat{q} = \frac{\gamma}{1 - \gamma} (\hat{y} - \hat{y}^*) - \frac{\gamma}{1 - \gamma} (\hat{A}_N - \hat{A}_N^*) - \frac{\gamma - \gamma^*}{(1 - \gamma)(1 - \gamma^*)} (\hat{y}^* + \hat{A}_N^*).
\]  

Under assumptions 1 and 2, this relation reduces to

\[
\hat{q} = \frac{\gamma}{1 - \gamma} (\hat{y} - \hat{y}^*).
\]  

(0)

Note that the coefficient of income differential (the Balassa-Samuelson effect) is larger than the share of the nontraded goods. The magnitude of this coefficient, however, is sensitive to the assumption about the productivity differential for nontraded goods. For example, consider a modified version, which replaces assumption 2 by the following assumption:

**Assumption 3.** \( \hat{A}_N - \hat{A}_N^* = \lambda (\hat{A}_F - \hat{A}_F^*), \ 0 < \lambda < 1 \).
Assumption 3 allows the nontraded good productivity differential to vary, but for simplicity, assumes a proportional relation between the productivity differentials for nontraded and traded goods. To capture the view that international productivity differences are less important for nontraded than traded goods, the elasticity of the nontraded good productivity differential with respect to the traded good differential \( \lambda \) is assumed to be less than one. Using assumptions 1 and 3 along with (14), (15) and its foreign counterpart, we modify (16) as

\[
\hat{q} = \frac{\gamma (1 - \lambda)}{1 - \gamma + \lambda \gamma} (\hat{y} - \hat{y}_*) .
\]

The alternative assumption 3 still allows the real exchange rate to be expressed as a function of the income differential, but this differential’s coefficient is smaller than under assumption 2.

### 3.2 Specialization in Production

This section examines how the results change if the basic model is modified to let the home and foreign countries be specialized in the production of traded goods. To motivate this variation, assume that each country’s productivity differs between two groups of traded goods. Partition the traded good set into two subsets, \( \Omega_H \) and \( \Omega_F \), and modify the production function (5) for traded goods as

\[
Y_t(j) = \begin{cases} 
A_H L_t(j) & \text{if } j \in \Omega_H \\
A_F L_t(j) & \text{if } j \in \Omega_F
\end{cases}.
\]

Let \( A_H / A_H^* > A_F / A_F^* \), so that the home country has a comparative advantage for goods in the subset, \( \Omega_H \). We assume that this advantage will lead to equilibrium with the home country producing goods in this subset and the foreign country producing goods in the other subset (goods in the two subsets are referred to as home and foreign traded goods).
Also, modify the consumption index (3) for traded goods as

\[ C_T = \left[ (\chi_H C_H)^{\eta-1} + (\chi_F C_F)^{\eta-1} \right]^{\eta/(\eta-1)}, \]

\[ C_H = \left[ \int_{j \in \Omega_H} C_H(j)^{(\eta-1)/\sigma} dj \right]^{\sigma/(\sigma-1)}, \quad C_F = \left[ \int_{j' \in \Omega_F} C_F(j')^{(\eta-1)/\sigma} dj' \right]^{\sigma/(\sigma-1)}, \]

where \( \eta \) is the elasticity of substitution between home and foreign goods; \( \chi_H \) and \( \chi_F \) are preference parameters for these goods; and \( C_H(j) \) and \( C_F(j') \) represent domestic consumption of home (exported) good \( j \) and foreign (imported) good \( j' \). The above specification allows for home bias in preferences as well as for asymmetric elasticities of substitution between and within home and foreign bundles.

We also introduce trade costs in this model and assume that they take the form of iceberg costs such that \( \tau(>1) \) units of a product need to be exported to deliver 1 unit in the importing country. The price indexes for \( C_T, C_H \) and \( C_F \) are given by

\[ p_T = \left[ \left( \frac{p_H}{\chi_H} \right)^{1-\eta} + \left( \frac{\tau p_F}{\chi_F} \right)^{1-\eta} \right]^{\eta/(1-\eta)}, \]

\[ p_H = \left[ \int_{j \in \Omega_H} p_H(j)^{1-\sigma} dj \right]^{\eta/(1-\eta)}, \quad \tau p_F = \left[ \int_{j' \in \Omega_F} \{ \tau p_F(j') \}^{1-\sigma} dj' \right]^{\eta/(1-\sigma)}, \]

where \( p_H(j) \) is the home real price (in terms of \( C \)) of home good \( j \), \( p_F(j') \) is the foreign real price (also in terms of \( C \)) of foreign good \( j' \), and \( \tau p_F(j') \) the home real prices of the foreign good. In view of the production function (18),

\[ p_H(j) = w/A_H. \]
The real prices of individual goods in terms of \( C \) are connected by the real exchange rate to those in terms of \( C^* \) (denoted by an asterisk) as

\[
q_{p_H}(j) = p^*_H(j), \quad q_{p_F}(j^*) = p^*_F(j^*).
\]

The labor market condition (12) still holds, but note that \( L_T(j) = 0 \) for \( j \in \Omega_F \). The condition for the current account balance can now be written as

\[
p_F C_F - p_H C^*_H = bC. \tag{0}
\]

Let \( z = \frac{p_H}{(\tau p_F)} \) denote the terms of trade for the home country. Normalize \( p_T = p_H = p_F = 1 \) in steady state, use (6), (8), (20), (21), the corresponding foreign equations, and (22) to modify the relation for the real exchange rate as

\[
\hat{q} = \gamma(\hat{A}_H - \hat{A}_N) - \gamma^*(\hat{A}^*_F - \hat{A}^*_N) + [\gamma(1 - \theta) + \gamma^* \theta^* + \theta - \theta^*] \hat{z}, \tag{0}
\]

where \( \theta = (\chi_H)^{\theta^{-1}} = 1 - (\chi_F / \tau)^{\theta^{-1}} \) is the steady-state share of home goods in the home traded good basket while \( \theta^* = (\chi_H^*)^{\theta^{-1}} = 1 - (\chi_F^*)^{\theta^{-1}} \) is the share of home goods in the foreign basket.

Note that \( \theta \) increases and \( \theta^* \) decreases as trade cost index, \( \tau \), increases. There is a home bias for the national traded good if \( \theta > \theta^* \). In the presence of trade costs, home bias arises even if preferences for the national good are the same in the two countries (\( \chi_H = \chi_H^* \)). The home bias, moreover, increases in trade costs. The real exchange rate relation (24) now includes a terms of trade effect [represented by the third term in (24)], thorough which productivity differentials can potentially offset the standard Balassa-Samuelson effects [captured by the first two terms in (24)], as discussed below. Real income per worker also depends on the terms of trade. Letting \( y = w \), and using (6), (8), (20), (21), we can express...
\[
\hat{y} = \gamma \hat{A}_w + (1 - \gamma) \hat{A}_h + (1 - \gamma)(1 - \theta) \hat{z} \tag{0}
\]

Letting \( \beta = \theta - \theta' \) denote home bias (with a value between zero and one) and using assumption 1, the solution for the terms of trade is derived in the Appendix A as

\[
\hat{z} = \frac{-(\hat{A}_h - \hat{A}_r^*)}{\eta(1 + \beta) - \beta}. \tag{0}
\]

Note that if \( \eta > \beta / (1 + \beta) \), then \( \eta(1 + \beta) - \beta > 0 \), and we have the typical result that an improvement in home productivity (relative to foreign productivity) will worsen the terms of trade.\(^{17}\)

To relate the real exchange rate to the income differential, we first substitute the value of \( \hat{z} \) from (26) into (24), (25) and its corresponding foreign equation, use assumption 2 (as in the basic model), and obtain

\[
\hat{q} = \gamma \left( \frac{\eta(1 + \beta) - 1 - \beta / \gamma}{\eta(1 + \beta) - \beta} \right) (\hat{A}_h - \hat{A}_r^*), \tag{0}
\]

\[
\hat{y} - \hat{y}^* = (1 - \gamma) \left( \frac{\eta(1 + \beta) - 1}{\eta(1 + \beta) - \beta} \right) (\hat{A}_h - \hat{A}_r^*), \tag{0}
\]

We then use (27) and (28) to get

\[
\hat{q} = \frac{\gamma}{(1 - \gamma)} \Phi(\hat{y} - \hat{y}^*), \tag{0}
\]

\(^{17}\) If \( \eta < \beta / (1 + \beta) \), then we obtain the case highlighted in Corsetti, Dedola and Leduc (2008), where higher home productivity, in fact, improves the terms of trade and thus enhances the Balassa-Samuelson effect.
where $\Phi = 1 - \frac{\beta / \gamma}{\eta(1 + \beta) - 1}$. As (29) indicates, the Balassa-Samuelson effect in the specialization model is different than in the conventional model [relation (16)]. In the (typically-assumed) case of $\eta > \beta / (1 + \beta)$, $\Phi$ is less than one and decreases as $\beta$ increases or $\eta$ decreases. In this case, moreover, $\Phi$ is negative if $\eta \gamma / (1 + \gamma) < \beta / (1 + \beta)$. Thus, incorporating the terms of trade effect not only decreases the Balassa-Samuelson effect [compare (29) with (16)], but also reverses the sign of the effect if the substitution elasticity is sufficiently low or home bias is sufficiently high.\(^{18}\)

### 3.3 Monopolistic Competition

Finally, we briefly discuss how monopolistic competition affects the results. Assume that the nontraded and traded products now represent varieties of differentiated goods produced under monopolistic competition. Let $\Omega_N$ now denote the set of domestic nontraded varieties, and $\Omega_H$ and $\Omega_F$ the sets of home and foreign traded varieties. The production function for a nontraded variety (indexed by $i$) is still given by (4), but the production functions for home and foreign traded varieties (indexed by $j$ and $j^*$) are now revised as

$$Y_H(j) = A_H L_H(j), \quad Y_F(j^*) = A_F L_F(j^*),$$

where $Y_H(j), L_H(j)$ and $A_H$ denote output, labor input and labor productivity for a home traded variety and $Y_F(j^*), L_F(j^*)$ and $A_F$ the corresponding variables for a foreign traded variety. The consumption and price indices for traded goods are the same as under specialized production, as given in (19) and (20).

\(^{18}\) The productivity coefficient is similarly influenced by the terms of trade effect.
Under monopolistic competition, the optimal prices for nontraded varieties, and for traded varieties in the home and foreign markets are

\[ p_N(i) = \frac{\sigma^w}{(\sigma - 1)A^N}, \quad p_H(j) = \frac{\sigma^w}{(\sigma - 1)A^H}, \quad qp_H^*(j) = \frac{\sigma^w}{(\sigma - 1)A^H}. \]  

(0)

The number of firms is determined endogenously in each sector by the condition that free entry and exit leads to zero profits. Assume that fixed amounts of labor equal to \( \phi_N / A_N \) and \( \phi_H / A_H \) are required to start the production of a variety of the nontraded and traded goods, respectively.

Under the zero-profit condition, the price of each variety also equals its average cost, and thus

\[ p_N(i) = (w / A_N)(\phi_N / Y_N(i) + 1), \quad p_H(j) = (w / A_H)(\phi_H / Y_H(j) + 1). \]

(0)

Given symmetric prices in each set, the aggregate prices for the sets of nontraded and traded varieties according to (6) and (20) are

\[ p_N = (n_N)^{1/(1-\sigma)} p_N(i), \quad p_H = (n_H)^{1/(1-\sigma)} p_H(j), \quad \tau p_F = (n_F)^{1/(1-\sigma)} \tau p_F(j^*). \]

(0)

where \( n_N, n_H \) and \( n_F \) are the numbers (mass) of varieties in the three sets. The trade-balance condition (23) still holds, but the condition for the labor market is modified as

\[ L = n_N[\phi_N / A_N + L_N(i)] + n_H[\phi_H / A_H + L_H(j)]. \]

(0)

Make use of (6), (20), (31), (33) and their foreign counterparts, let \( y = w \), and derive the relations for the real exchange rate and real income per worker for the monopolistic-competition case as

\[ \hat{q} = \gamma(\hat{A}_H - \hat{A}_N) - \gamma(\hat{A}_F - \hat{A}_N) + [\gamma(1-\theta) + \gamma^*\theta^* + \theta - \theta^*]\hat{z}  
+ [\gamma(\hat{n}_H - \hat{n}_N) - \gamma(\hat{n}_F - \hat{n}_N)] / (\sigma - 1), \]

(0)

\[ \hat{y} = \gamma\hat{A}_N + (1-\gamma)\hat{A}_H + (1-\gamma)(1-\theta)\hat{z} + [\gamma\hat{n}_N + (1-\gamma)\hat{n}_H] / (\sigma - 1). \]

(0)
Both of these relations now include an additional effect [represented by the fourth term in each relation] operating through changes in the number of firms. Note that if the number of firms is fixed in the monopolistic competition model (so that the adjustment occurs only on the intensive margin), the relations for the real exchange rate and income per capita are no different from the perfect competition model with specialization.

As shown in Appendix A, the solution of the model (with assumption 1) yields

\[ \hat{\epsilon} = \frac{-\sigma(\hat{A}_H - \hat{A}_F^*)}{(\sigma - 1)[\eta + \beta(\eta - 1)]}, \]  

(0)

\[ \hat{n}_H - \hat{n}_N = \hat{A}_H - \hat{A}_N, \quad \gamma \hat{n}_N + (1 - \gamma)\hat{n}_H = \frac{\sigma - 1}{\sigma}(\hat{y} - (1 - \gamma)(1 - \theta)\hat{\epsilon}). \]  

(0)

Next, use (35)-(38), foreign counterparts of (36) and (38), and (16) to obtain

\[ \hat{q} = \frac{\sigma \gamma}{\sigma - 1} \left( \frac{\eta(1 + \beta) - 1 - \beta / \gamma}{\eta(1 + \beta) - \beta} \right)(\hat{A}_H - \hat{A}_F^*), \]  

(0)

\[ \hat{y} - \hat{y}^* = \frac{\sigma(1 - \gamma)}{\sigma - 1} \left( \frac{\eta(1 + \beta) - 1}{\eta(1 + \beta) - \beta} \right)(\hat{A}_H - \hat{A}_F^*). \]  

(0)

In comparison with the specialization model [see (27) and (28)], the effect of the traded-goods productivity differential on both the real exchange rate and the income differential is magnified in the present model. Remarkably, however, the degree of magnification is the same for both variables. The relation between \( \hat{q} \) and \( \hat{y} - \hat{y}^* \) implied by (39) and (40) is the same as (29). Thus, although endogenous entry/exit (adjustment on the extensive margin) in the monopolistic competition model increases the productivity coefficient, it does not alter the coefficient of the income differential, which remains the same as that in the specialization model.
3.4 Extensions and Empirical Implementation

Our empirical model is based on the long-run theoretical relation that links the real exchange rate to the income differential. In empirically implementing the two-country model to a multi-country world, we let the home country represent an individual country and the foreign country stand for the rest of the world. Using subscripts \( c \) and \( w \) to denote variables for a country and the rest of the world, and assuming that the coefficients (except the constant term) are homogeneous across countries, we specify our empirical relations as

\[
\ln q_{c,t} = \alpha_{0c} + \alpha_1 (\ln y_{c,t} - \ln y_{w,t}) + e_{c,t},
\]

where \( e_{c,t} \) is the error term. In the simple version of the conventional Balassa-Samuelson model (relation 16 based on assumptions 1 and 2), \( \alpha_1 \) equals \( \gamma / (1 - \gamma) \). The value of this coefficient would be smaller in the modified version, which allows the nontraded good productivity differential to vary (relation 17 based on assumptions 1 and 3). In this case, \( \alpha_1 \) equals

\[
(1 - \lambda) \gamma / (1 - \gamma + \lambda \gamma) \quad \text{with} \quad \lambda < 1.
\]

In the modern version of the Balassa-Samuelson model which incorporates specialized production or monopolistic competition (relation 29), \( \alpha_1 \) equals \( \Phi \gamma / (1 - \gamma) \) where \( \Phi = 1 - (\beta / \gamma) / (\eta(1 + \beta) - \beta) \) is less than one, and can even be negative under sufficiently low substitution elasticity or high home bias.

Estimation of (41) requires data for all economies to measure the real exchange rate of a country with respect to all other countries (\( q_c \)), and the income variable for the rest of the world (\( y_w \)). As our historical data set includes data for a subset of countries, we use a bilateral version of (41). Choosing the United States as the reference country and letting subscript \( u \) denote US variables, and defining the country \( c \)'s bilateral real exchange rate with US as \( q_{cu} = q_c / q_u \) (so
that it represents the real value of the currency of country $c$ in terms of US dollar, we subtract relation (41) for US from that for country $c$, and derive

$$
\ln q_{c,u,t} = \alpha'_0 + \alpha_1 (\ln y_{c,t} - \ln y_{u,t}) + e'_{c,t} \quad (0)
$$

where $\alpha'_0 = \alpha_{0c} - \alpha_{0u}$ and $e'_{c,t} = e_{c,t} - e_{u,t}$.

We estimate this relation in the next section using panel data. If assumption 1 does not hold, $\alpha_1$ would differ across countries and we allow for this possibility. In this case, world variables could have an additional effect that is omitted in (42).\(^{19}\) This omission would introduce cross-sectional dependence between relations for different non-US countries and we introduce time effects in our panel estimates to account for this possibility. We also estimate the model with and without a time trend. The trend term could allow for the possible effect of changes in parameters such as trade costs and the share of nontraded goods, which are treated as constants in our model for simplicity. This term could also capture the effect of deterministic trends in the productivity differential for nontraded goods if there are departures from assumptions 2 or 3.\(^{20}\)

Finally, we also consider the possibility that the net foreign assets ratio does not converge to a unique value in steady state. In this case, trade balance (relative to income) would not be constant in the long run, and this variable would also be included in the real exchange rate

\(^{19}\) Note that without assumption 1 (but with assumption 2), (14), (15) and its counterpart imply that $\hat{q} = \frac{\gamma}{1-\gamma}(\hat{y} - \hat{y}^*) - \frac{\gamma - \gamma^*}{(1-\gamma)(1-\gamma^*)}(\hat{y}^* + \hat{A}_n)$. The second term for country $c$ and US would now be different and introduce additional world variables in (42).

\(^{20}\) A deterministic trend could also be present in the traded goods productivity differential. However, although the income differential and the real exchange rate relations would have a deterministic trend in this case, the relation between these variables would not include the trend term.
relation. To allow for this possibility, we also estimated the bilateral version of the empirical model with trade balance differential between country c and the US as an additional variable.\(^{21}\)

4. Empirical Analysis

4.1 Stationarity tests and real exchange rate persistence

In Table 1 we present both univariate and panel unit root tests for the full sample period. The univariate tests are calculated using the method of Elliot, Rotenberg and Stock (1996) with the optimal lag order calculated using a sequential t-test proposed by Ng and Perron (2000). Three different tests are reported for the panel unit root tests and these are the Adj-t\(^{*}\) test of Levin, Lin and Chu (2002), the W-t-bar test of Im, Pesaran and Shin (2003) and the Z[t-bar] test of Pesaran (2003). As can be seen, the evidence with respect to the stationarity properties of the univariate series is rather mixed for all three variables (although the main focus of our analysis is the real exchange rate /productivity relationship we also include the relative trade balance term here since that features in our robustness tests). For example, in 14 out of 42 cases, non-stationarity of the variables can be rejected in the no time trend regression and on 13 occasions out of 42 when a time trend is included. In the univariate case the majority of rejections of non-stationarity arise for the real exchange rate series.

In terms of the panel unit root results, presented in the bottom half of Table 1, we always reject non-stationarity at the 5% or 1% level for the real exchange rate and the trade balance term. Productivity differentials (measured by per capita income differentials) prove to be stationary at both the 1% and 5% levels on the basis of all three tests when a time trend is included.

\(^{21}\) Letting \(b_c\) denote the balance of trade of a country, the global exchange rate relation is modified as

\[
\ln q_{c,t} = \alpha_{0c} + \alpha_t (\ln y_{c,t} - \ln y_{w,t}) + \alpha_2 b_{c,t} + e_{c,t},
\]

which implies the following bilateral relation:

\[
\ln q_{cu,t} = \alpha_{0c} + \alpha_t (\ln y_{c,t} - \ln y_{u,t}) + \alpha_2 (b_{c,t} - b_{u,t}) + e'_{c,t}.
\]
included. The panel unit root tests for our four different sub samples are presented in Table 2 and it is interesting to note that the rejection of nonstationarity is not as clear cut in the cases of the real exchange rate and trade balance differential as it was for the full sample period. In terms of the real exchange rate, there is more evidence in favor of stationarity for the two post world war II sub samples.

In table 3 we present the half lives calculated for the real exchange rates using the method of Choi, Mark and Sul (2006). The full period results fall within the conventional 3 – 5 year range. As would be expected, the two fixed rate periods – the Gold Standard and Bretton Woods – produce the fastest adjustment speeds, with Bretton Woods delivering the tightest range of 0.5 to 0.8. The two floating rate periods – inter war and post Bretton Woods - produce slower, although similar, adjustment speeds.

4.2 Panel Estimates

The long-run effect of productivity differential on the real exchange rate can be estimated by Group Mean (GM) and Panel DOLS procedures. These procedures provide unbiased estimates of the coefficients in the long run relation if the variables in the relation are nonstationary or mixed stationary and non-stationary and cointegrated. As discussed above, the evidence for nonstationarity of the real exchange rate and productivity differential based on sub samples is mixed. If these variables are assumed to be nonstationary, the evidence on half lives of the residuals (discussed below) suggests that they are cointegrated. Thus, we use GM and Panel DOLS to estimate the long-run productivity effect, but as a robustness exercise, we also explore alternative estimates based on an error-correction specification, which are appropriate if
all variables are nonstationary and cointegrated and allow the short run dynamics to be influenced by the deviation from equilibrium.

In tables 4 and 5 we present the individual, Group Mean and panel DOLS estimates for our base line productivity specification. Since we found evidence of cross-sectional dependence in our panel estimates (discussed further below), we only report estimates where cross-sectional dependence is accounted for by time dummies and demeaning the series. The individual coefficients on the relative productivity term, reported in Table 4 reveal a preponderance of positive coefficients in the range of 0.03 to 1.67 with weak statistical significance. The pooled results, however, show a much clearer picture. The results in the GM row are for the simple group mean and these give a statistically significant coefficient of 0.19 in the specification without a trend and 0.22 in the specification with a time trend. The PDOLS estimates are similar and significant both with and without a common trend, although the result of the latter test is significant only at 10% when an individual trend is used.

The results for the various sub sample periods are reported in Table 5 and show considerable variation across regimes. In the Classical Gold standard period, 1880-1913, the coefficient values on the relative productivity term are all positive, significant and of a similar order of magnitude with and without a trend (irrespective of the trend being common or individual). The values, in the range of 0.13 to 0.25 are clearly smaller than that expected in the traditional Balassa-Samuelson narrative but (as explained below) are consistent with the modern versions presented in Section 3. Moving into the period 1914-1945, we note that the sign flips from being positive to significantly negative in all cases and the magnitude of the coefficient

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22 As discussed in Section 3.4, departures from assumption 1 (symmetric shares) could lead to cross-sectional dependence.
using the Panel DOLS estimators is approximately double the size of the coefficient in the case of the GM estimator.

In the Bretton Woods period the results differ both with respect to whether a trend is included or not and also depending on the estimator used. For example the GM results produce an insignificantly negative outcome in the no trend case and a significantly negative outcome in the trend case. With the Panel DOLS estimator the results all produce a small but insignificantly positive coefficient on the relative productivity term. In the post Bretton Woods period the results show a clear positive and statistically significant coefficient with the coefficient varying from 0.20 to 1.18 depending on the specification. An interesting aspect of the post Bretton Woods results is that the coefficient on the relative income term falls from 1.18 to 0.58 when an individual trend is used rather than a common trend. These results do not qualitatively change when, as a robustness exercise, we re-estimate the Balassa-Samuelson effects after introducing the trade balance as an additional regressor (see Appendix B).

Again, changes in the sign and the magnitude of the coefficient across sub samples are puzzling. An explanation of these results in terms of the modern version of the Balassa-Samuelson theory is suggested in the next section.

4.3 Half Lives of Residuals

The half-life estimates discussed above do not account for the influence of the productivity differential. To control for this effect, Table 6 shows estimates of half-lives of residuals obtained from relations that contain the productivity variable. We consider two specifications based on panel DOLS: the first excludes the first difference terms (no fd’s) and the second includes these terms (fd’s). The first specification controls only for the long-run effect of
the productivity differential while the second specification also controls for the short-run effects of the differential captured by the first difference terms. For each specification, half-life estimates are presented for relations with and without a time trend. As already noted above, even without the productivity effect half-lives tend to be shorter for sub samples than for the whole sample. Controlling for this effect, sub sample half-lives are significantly reduced, especially if short term effects are also incorporated. The half-lives are further reduced when a time trend is included, thereby perhaps suggesting that our productivity term may not capture the totality of productivity trends. The statistical significance of these results indicates clear evidence of cointegration in the panel DOLS estimates. It is interesting to note that it is possible to provide a resolution for the PPP puzzle in the context of a linear model by controlling for productivity effects and allowing a time trend. For example, the half-life range of the real exchange rate in Table 2 (without controls) is from 1.9 to 5.7 years for the post Bretton Woods period. In the presence of a time trend, the half-life range in Table 6 reduces sharply to 0.8-1.4 years after controlling for the long-run productivity effect and 0.7-1.2 years after also controlling for the short-run effect. The interwar period also has a long half-life range, and controls also significantly shorten this range.

4.4 Error Correction Specification

Given the mixed evidence from tables 1 and 2 on the (non)stationarity of some of our variables, as a robustness check, we also report estimates of the real exchange rate - productivity relationship using Mean Group (Pesaran and Smith, 1995) and Pooled Mean Group (Pesaran, Shin and Smith, 2004) estimators which consider the case when all variables are I(1) and cointegrated. Then, it is possible to adopt an Error Correction Specification of the form:
\[ \Delta q_{it} = \phi_i \left( q_{i,t-1} - \alpha_{it} - \alpha_{iy} (y-y^*)_{it} \right) + \xi_{it} \Delta (y-y^*)_{it} + \nu_{it}, \]

where \( \phi \) should be significantly negative if the variables show a return towards the long-run equilibrium. These estimators are indicated in case of nonstationary heterogeneous panels, especially if the number of groups or time periods is large. In order to control for potential cross-sectional dependence, the series are first demeaned. The two estimators differ in the way they deal with panel heterogeneity: while the Mean Group estimates are simple unweighted averages of the N individual regressions, the pooled mean group (PMG) estimator constrains the long-run elasticity to be equal across countries. Pooling is consistent and efficient if restrictions apply, but inconsistent if the true model is heterogeneous. Hausman tests are performed under the null that the difference in coefficients is not systematic. The results in table 7 seem to suggest that pooling is always consistent and efficient except in one case for the post Bretton Woods period, when a trend is included. Overall the PMG results seem to confirm, in terms of sign, size and significance the results obtained using the PDOLS and GM estimators, especially if individual trends are included. In table 7, we also use the basic formula to extract half-lives based on the error correction parameters. These show sizes similar to those of the unbiased half lives reported in table 6, confirming our overall baseline results.

5. Explaining the Results

In this section, we examine if the Balassa-Samuelson theory is capable of explaining the results discussed above. We consider both the conventional and the modern versions of the theory.
5.1 The Conventional Version

The estimates of the income coefficient in the PDOLS and GM regressions are difficult to reconcile with the conventional Balassa-Samuelson model. The share of nontraded goods ($\gamma$) is typically assumed to be greater than 0.5. For $\gamma \geq 0.5$, the productivity effect in (16) predicted by the conventional model (simplified by assumptions 1 and 2) would equal or exceed 1.0. Our estimates of the coefficient of the productivity differential for the full sample range from 0.13 to 0.22 and are much below the predicted value.

Small values of the productivity coefficient could be potentially explained by the modification of the model that allows productivity differential for nontraded goods to vary (as in relation (17) based on assumption 3 instead of assumption 2). However, $\lambda$ (the elasticity of the nontraded traded good productivity differential with respect to the traded good differential) would have to be fairly large to account for the low estimates of the productivity coefficient. For example, to explain the full-sample range for the productivity coefficient, we would need values, of $\lambda$ between 0.64 and 0.77 for $\gamma = 0.5$, and between 0.70 and 0.81 for $\gamma = 0.6$. Such high values of $\lambda$ seem implausible as they imply that international differences in nontraded goods productivity growth tend to be substantial relative to the productivity growth differences for traded goods.

Estimates of the coefficients for the subsamples show considerable variation across periods and are even harder to explain by the conventional Balassa-Samuelson model. Estimated values are positive but small (below 0.25) in the gold standard period; are negative (between -0.17 and -0.42) in the interwar period; have an ambiguous sign (range from -0.33 to 0.12) in the Bretton-Woods period; and are positive, and on average, larger (within a wide range from 0.2 to
1.18) in the post Bretton-Woods period. The conventional model or its modification do not suggest an explanation of why the productivity coefficient would be negative in some periods and why it would differ so much from one period to another.

### 5.2 Modern Versions

We next examine whether the modern version of the Balassa-Samuelson model based on specialization in production or monopolistic competition can explain the range of estimates of the productivity effect. For both versions, the productivity effect is given by (29) and depends not only on $\gamma$, but also on $\eta$ (the elasticity of substitution between home and foreign tradables) and $\beta$ (the home bias). Shifts in these parameters could potentially account for the cross-period variation in the productivity coefficient. An important reason for such shifts is the behavior of trade costs, which changed dramatically across the four sub-periods.\(^{23}\) Trade costs fell prior to World War I mainly as a result of reduction in transport costs and improvements in information technology. They rose sharply in the interwar period largely due to the escalation of tariff and nontariff barriers caused by the Great Depression. Trade costs started to fall again after World War II. The decrease in trade costs accelerated in the post Bretton Woods period because of significant reductions in trade barriers resulting from regional and international trade agreements and in transportation costs due to technological improvements.

$$\beta = \theta - \theta'$$ (the difference between the shares of home goods in the home and foreign bundles of traded goods), changes in trade costs would shift $\beta$ via their effect on $\theta$ and $\theta'$.

\(^{23}\) Jacks, Meissner and Novy (2009) assign an important role to trade costs in explaining the twin booms in international trade before World War I and after World War II as well as the internation trade bust in the interwar period. They also discuss the behavior of trade costs from 1870 to 2000 based on their estimates derived from bilateral trade data using the gravity model of international trade.
An increase in trade costs would raise the prices of imports and exports, and for $\eta > 1$, would increase $\theta$ and decrease $\theta^*$ and hence increase $\beta$.\(^{24}\) Thus we would expect $\beta$ to increase in the interwar period, decrease in the Bretton Woods period, and decrease further in the period after Bretton Woods. Higher trade costs, especially larger nontariff barriers, could also reduce the range of foreign goods available in the home market as well as the range of home good available abroad. Such changes could make imported and exported goods less substitutable with local goods, and decrease $\eta$. Thus $\eta$ could also have changed across the four regimes in a direction opposite to that of $\beta$.

It is interesting to explore whether realistic values of $\gamma$, $\beta$ and $\eta$ for the four regimes could explain the magnitude and the signs of the productivity coefficient in each regime. There is much interest in estimating $\eta$ (often referred to as the “Armington elasticity”), which plays an important role in a wide variety of macroeconomic and international trade models. Macroeconomic models typically calibrate or estimate its value at the aggregate level to be between 0.5 and 2.0.\(^{25}\) Studies using disaggregated international trade data suggest much larger estimates of the elasticity, but these estimates are based on a specification (different from ours) which assumes that the elasticity between a pair of varieties is the same regardless of where they are produced.\(^{26}\) However, in an alternative specification that allows the Armington elasticity to differ from the elasticity between foreign varieties, Feenstra, Obstfeld, and Russ (2012) estimate

\(^{24}\) Trade costs could also affect the share of nontraded good if there are departures from the typical assumption that the elasticity of substitution between traded and nontraded goods equals one (i.e., aggregate consumption index is a Cobb-Douglas function of traded and nontraded bundles of goods).

\(^{25}\) Estimation of macroeconomic models typically yields an estimate of the elasticity close to the lower half of this range (e.g., see Bergin, 2004, Lubik and Schorfheide, 2005).

\(^{26}\) See, for example, Imbs and Mejean (2011) for estimates of the elasticity, allowing it to be either heterogeneous or homogeneous across sectors.
the Armington elasticity to be closer to unity and within the range assumed in the macroeconomic models. In the simple case of symmetric import shares in tradable bundles \((1 - \theta = \theta^*)\), the home bias can be expressed as \(\beta = 2\theta - 1\). Estimates of \(\gamma\) and \(1 - \theta\) can be derived from sectoral production and trade data.\(^{27}\) For OECD countries, such data are available since 1970, and estimates based on this data suggest values of \(\gamma\) above 0.6 and of \(\theta\) below 0.80 (implying \(\beta\) below 0.6).\(^{28}\)

The role of \(\eta\) and \(\beta\) in determining the productivity coefficient in the model with specialization or monopolistic competition is illustrated in Figure 2. For \(\gamma = 0.65\), the figure shows the relation between the productivity coefficient and \(\eta\) for two values of \(\beta\), a low value of 0.4 and a high value of 0.8. We let \(\eta\) vary between 1.0 and 2.0. The productivity coefficient is an increasing function of \(\eta\), and a higher \(\beta\) shifts the function down (for \(\eta > 1\)).

Interestingly, the ranges for \(\eta\) and \(\beta\) shown in the figure are capable of accounting for not only large negative values of the productivity coefficient estimated for the interwar period, but also large positive values generally estimated for the post Bretton Woods period. For \(\beta\) between 0.4 and 0.8, values of \(\eta\) in the 1.2-1.4 range could generate low positive values of the productivity coefficient consistent with the gold standard period (as well as the whole period). As can be seen from the figure, however, an increase in \(\beta\) alone would not be explain the sharp drop in the estimate of the productivity coefficient for the interwar period. A small decrease in \(\eta\) would

\(^{27}\) The estimates of \(\gamma\) are sensitive to how traded goods are classified. A narrow measure identifies traded goods with Manufacturing, Agriculture and Mining sectors. A broader measure would also include portions of Elecricity and Gas, Transportation and Communication, and Financial Services sectors that have significant trade.

\(^{28}\) See, for example, Choudhri and Marasco (2013), who estimate average values of \(1 - \gamma\) and \(1 - \theta\) for a set of OECD countries to equal 0.36 and 0.22, respectively.
also be needed to account for this result. A combination of an increase in \( \eta \) and a decrease in \( \beta \) could bring about the estimated changes in the productivity coefficient between the Bretton Woods and the interwar periods. Further changes in this direction could produce the results for the post Bretton Woods period.

The modern versions also provide an explanation of the PPP puzzle. The long-run real exchange rate depends on the productivity differential and possibly on a time trend.\(^{29}\) The productivity and time trend coefficients, moreover, could vary across regimes. Thus the model is consistent with the evidence that half-lives drop sharply as controls are introduced for productivity and a time trend and cross-regime variation in coefficients is allowed for.

5. Conclusions

There has been a revival of interest recently in the Balassa-Samuelson hypothesis regarding the effect of productivity on the real exchange rate. The empirical testing of this hypothesis, however, has not produced clear-cut results. On the one hand, there is considerable evidence that productivity is a determinant of the real exchange rate. The size and the sign of the productivity effect, on the other hand, appears to be sensitive to the data sets and varies from one study to another.

In this paper, we use historical data for over hundred years and 14 countries to examine the Balassa-Samuelson productivity effect. Our sample period can be divided into 4 distinct monetary regimes and we find large variations in the productivity effect across regimes. The

\(^{29}\) As noted in Section 3.4, a time trend could capture the effects of departures from assumptions 2 or 3, or intra-regime trends in trade costs.
The conventional Balassa-Samuelson theory does not provide an explanation of the cross-regime variation of the productivity effect or even the size of the effect for most regimes.

We consider two modern variants of the Balassa-Samuelson model (based on specialization or monopolistic competition) to examine whether these variants are capable of explaining our results. These variants include a new terms of trade channel which can offset the traditional Balassa-Samuelson channel (operating via the relative price of nontraded goods) and significantly modify the productivity effect. The strength of the terms of trade effect depends on the home bias in consumption of traded goods and the elasticity of substitution between home and foreign bundles of these goods. We argue that variations in trade costs caused shifts in these parameters across regimes. We undertake simulations of the modern variants of the Balassa-Samuelson model to show that plausible parameter shifts consistent with the cross-regime behavior of trade costs can explain the size of the productivity effect in each regime as well as the variation in this effect across regimes. The Balassa-Samuelson theory modified to account for the terms of trade effect thus has the potential to explain the observed variation in the productivity effect over a long period.
Appendix A

Determination of the Terms of Trade and the Number of Firms

Specialization in production

Let a hat over a variable denote the log deviation of the variable from its initial steady state value (denoted by a bar over the variable). In the model with specialization in production, the number of firms is fixed. We derive the relation determining the terms of trade as follows. From (1), we have \( \hat{C}_r = \hat{C} - \hat{p}_T \). Then noting that \( \hat{p}_T = -\gamma(\hat{p}_N - \hat{p}_T) \) from (6); \( \hat{p}_T = \hat{p}_H - (1-\theta)\hat{z} \) from (20) under the normalization that \( \hat{p}_H = \hat{p}_F = 1 \); and \( \hat{p}_N - \hat{p}_H = \hat{A}_H - \hat{A}_N \) from (8) and (21); we obtain

\[
\hat{C}_r = \hat{C} + \gamma(\hat{A}_H - \hat{A}_N) + \gamma(1-\theta)\hat{z}. \quad (A1)
\]

Since (19) implies that \( \hat{C}_F = \hat{C}_r - \eta(\hat{p}_F - \hat{p}_T) \) and (20) implies that \( \hat{p}_T = \hat{p}_F - \theta\hat{z} \), we can use these expressions and (A1) to express

\[
\hat{C}_F = \hat{C} + \gamma(\hat{A}_H - \hat{A}_N) + (\gamma(1-\theta) + \eta\theta)\hat{z}. \quad (A2)
\]

The foreign counterpart of this relation is

\[
\hat{C}_H^* = \hat{C} + \gamma(\hat{A}_F^* - \hat{A}_N^*) - [\gamma^*\theta^* + \eta(1-\theta^*)]\hat{z}. \quad (A3)
\]

Since \( \hat{w} = \hat{A}_N + \hat{p}_N \) from (8), we can use (6), (20) and (21) to obtain

\[
\hat{w} = \gamma \hat{A}_N + (1-\gamma)\hat{A}_H + (1-\gamma)(1-\theta)\hat{z}. \quad (A4)
\]

The corresponding relation for the foreign economy is

\[
\hat{w}^* = \gamma^* \hat{A}_N^* + (1-\gamma^*)\hat{A}_F^* - (1-\gamma^*)\theta^*\hat{z}. \quad (A5)
\]

Use (23) with \( b = 0 \) in the initial steady state to express
\( \hat{z} = \hat{C}_F - \hat{C}^*_H. \)  
(A6)

Since \( C = wL, \ C^* = w^*L^* \) and \( \hat{L} = \hat{L} = 0, \ \hat{C} = \hat{w} \) and \( \hat{C}^* = \hat{w}^* \), use (A4) and (A5) to substitute for \( \hat{C} \) and \( \hat{C}^* \) in (A2) and (A3), then use the resulting expressions to substitute for \( \hat{C}_F \) and \( \hat{C}^*_H \) in (A6) and let \( \gamma^* = \gamma \) under Assumption 1 to obtain

\[
\hat{z} = \hat{A}_H - \hat{A}^*_F + [1 + \eta + \eta(\theta - \theta^*) - (\theta - \theta^*)]\hat{z}.
\]
(A7)

Relation (26) can be readily derived from (A7).

**Monopolistic Competition**

In the model with monopolistic competition, the number of firms is determined endogenously and we also derive the relation determining the number of firms. As (33) implies that

\[
\hat{p}_N = \hat{p}_N(i) - \hat{n}_N / (\sigma - 1) \quad \text{and} \quad \hat{p}_H = \hat{p}_H(j) - \hat{n}_H / (\sigma - 1),
\]
and (31) implies that

\[
\hat{A}_N + \hat{p}_N(i) = \hat{A}_H + \hat{p}_H(j),
\]
we use these relations to obtain

\[
\hat{p}_N - \hat{p}_H = \hat{A}_H - \hat{A}_N + (\hat{n}_H - \hat{n}_N) / (\sigma - 1).
\]
(A8)

Since (1), (6) and (20) imply that \( \hat{C}_N = \hat{C} - (1 - \gamma)(\hat{p}_N - \hat{p}_H + (1 - \theta)\hat{z}) \) and

\[ \hat{C}_T = \hat{C} + \gamma(\hat{p}_N - \hat{p}_H + (1 - \theta)\hat{z}), \]
we can use (A8) to substitute for \( \hat{p}_N - \hat{p}_H \) in these expressions to express

\[
\hat{C}_N = \hat{C} - (1 - \gamma)(\hat{A}_H - \hat{A}_N) - (1 - \gamma)(1 - \theta)\hat{z} - (1 - \gamma)(\hat{n}_H - \hat{n}_N) / (\sigma - 1), \quad \text{(A9)}
\]

\[
\hat{C}_T = \hat{C} + \gamma(\hat{A}_H - \hat{A}_N) + \gamma(1 - \theta)\hat{z} + \gamma(\hat{n}_H - \hat{n}_N) / (\sigma - 1). \quad \text{(A10)}
\]
Next, using (19) and (20) to express \( \hat{C}_t = \hat{C}_T - \eta \theta \hat{z} \) and \( \hat{C}_h = \hat{C}_T - \eta \gamma (1 - \theta) \hat{z} \), and using (A10) to substitute for \( \hat{C}_t \) in these expression, we get

\[
\hat{C}_h = \hat{C} + \gamma (\hat{A}_h - \hat{A}_N) + [\gamma (1 - \theta) - \eta (1 - \theta)] \hat{z} + \gamma (\hat{n}_h - \hat{n}_N) / (\sigma - 1), \quad (A11)
\]

\[
\hat{C}_f = \hat{C} + \gamma (\hat{A}_h - \hat{A}_N) + (\gamma (1 - \theta) + \eta \theta) \hat{z} + \gamma (\hat{n}_h - \hat{n}_N) / (\sigma - 1). \quad (A12)
\]

From (31) and (33), we have \( \hat{w} = \hat{A}_N + \hat{p}_N + \hat{n}_N / (\sigma - 1) \). Then using (6) and (20) to express \( \hat{p}_N = (1 - \gamma)(\hat{p}_N - \hat{p}_H + (1 - \theta) \hat{z}) \) and using (A8), we obtain

\[
\hat{w} = (1 - \gamma)(1 - \theta) \hat{z} + \gamma \hat{A}_N + (1 - \gamma) \hat{A}_h + [(1 - \gamma) \hat{n}_h + \gamma \hat{n}_N] / (\sigma - 1). \quad (A13)
\]

Using (19), (11), (23) with \( b = 0 \), and noting that \( \theta = \frac{\hat{C}_h}{\hat{C}_h + \hat{C}_f} \), and \( 1 - \theta = \frac{\hat{C}_f}{\hat{C}_h + \hat{C}_f} \)
under our normalization, we derive \( \hat{C}_N = \frac{\sigma}{\sigma - 1} \hat{n}_N + \hat{Y}_N(i) \), and

\[
\theta \hat{C}_h + (1 - \theta) \hat{C}_h^* = \frac{\sigma}{\sigma - 1} \hat{n}_h + \theta \hat{C}_h(j) + (1 - \theta) \hat{C}_h^*(j) = \frac{\sigma}{\sigma - 1} \hat{n}_h + \hat{Y}_h(j). \quad (A14)
\]

Since (30) and (32) imply that \( Y_N(i) = (\sigma - 1) \phi_N \) and \( Y_H(j) = (\sigma - 1) \phi_H \), \( \hat{Y}_N(i) = \hat{Y}_H(j) = 0 \) in the above expressions, we have [note that \( \theta \hat{C}_h + (1 - \theta) \hat{C}_h^* = \theta \hat{C}_h + (1 - \theta)(\hat{C}_f - \hat{z}) \)]

\[
\hat{n}_N = \frac{\sigma - 1}{\sigma} \hat{C}_N, \quad (A14)
\]

\[
\hat{n}_h = \frac{\sigma - 1}{\sigma} (\theta \hat{C}_h + (1 - \theta)(\hat{C}_f - \hat{z})). \quad (A15)
\]

Now use (A9), (A11) and (A12) to substitute for \( \hat{C}_N, \hat{C}_h \) and \( \hat{C}_f \) in (A14) and (A15), and utilize the resulting expressions to derive
\[
\hat{n}_H - \hat{n}_N = \hat{A}_H - \hat{A}_N, \quad (A16)
\]

\[
[(1 - \gamma)\hat{n}_H + \gamma \hat{n}_N)] / (\sigma - 1) = [\hat{C} - (1 - \gamma)(1 - \theta)\hat{z}] / \sigma, \quad (A17)
\]

which, given \(\hat{C} = \hat{y}\), yield (38).

Using (A13) and (A17) and letting \(\hat{C} = \hat{w}\), we obtain

\[
\hat{C} = \frac{\sigma}{\sigma - 1}[\gamma \hat{A}_N + (1 - \gamma)\hat{A}_H] + (1 - \gamma)(1 - \theta)\hat{z}. \quad (A18)
\]

Next, using (A16) and (A18) to substitute for \(\hat{n}_H - \hat{n}_N\) and \(\hat{C}\) in (A12), we restate

\[
\hat{C}_F = \frac{\sigma}{\sigma - 1}\hat{A}_H + [(1 - \theta) + \eta\theta]\hat{z}. \quad (A19)
\]

The corresponding foreign relation can be similarly derived as

\[
\hat{C}_H^* = \frac{\sigma}{\sigma - 1}\hat{A}_H^* + [\theta^* + \eta(1 - \theta^*)]\hat{z}. \quad (A20)
\]

Finally, using (A6), (A19) and (A20), we obtain the solution for \(\hat{z}\) given by (37).
## Appendix B

<Insert table B1 here >

<Insert table B2 here>

### Table B1 – Univariate and Panel DOLS (1880-1997)

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<th>Country</th>
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<th>b-b*</th>
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**Notes:** DOLS equations are specified as: $q_{it} = \alpha_0 + \mu_i + \delta_i + \alpha_t(y - y^*)_{it} + \sum_{k=p}^p \delta_{ik} \Delta(y - y^*)_{it-k} + \epsilon_{it}$. $GM = N^{-1} \sum_{i=1}^{N} \hat{\alpha}_{it}$ and \( \tau = N^{-1/2} \sum_{i=1}^{N} I_{d_{it}} \). To account for cross-sectional dependence, time dummies are included in the GM equations. GM significance is based on heteroskedasticity and autocorrelation consistent standard errors for the underlying univariate regressions. Panel DOLS significance is based on Driscoll and Kraay standard errors, which are heteroskedasticity consistent and robust to general forms of cross-sectional (spatial) and temporal dependence (see Hoeckle, 2007).
### Table B2 – Panel Estimates (sub-periods)

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**Notes:** DOLS equations are specified as: \( q_{it} = \alpha_0 + \mu_i + \delta_i + \alpha_t (y_{it} - y_{it}^*) + \sum_{k=-p}^{p} \delta_k \Delta (y_{it} - y_{it}^*)_{t-k} + \epsilon_{it} \).

\[
GM = N^{-1} \sum_{i=1}^{N} \hat{\alpha}_{it} .
\]

To account for cross-sectional dependence, time dummies are included in the PDOLS regressions and series are cross-sectionally demeaned before applying the GM estimator. Italics denote rejection at 10%, Bold denote rejection at 5%, Italics and Bold denote rejection at 1%. GM significance is based on heteroskedasticity and autocorrelation consistent standard errors for the underlying univariate regressions. Panel DOLS significance is based on Driscoll and Kraay standard errors, which are heteroskedasticity consistent and robust to general forms of cross-sectional (spatial) and temporal dependence (see Hoeckele, 2007).
Data Appendix


Data for 15 Countries – Argentina, Australia, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden, UK and USA.

For a full list of the data references, see Michael Bordo’s web page.

Real GDP: Argentina (GDP) – 1884-1913, Gerardo della Paollera: (1988), 1914-1988,


References


Figure 1. Real Exchange Rates relative to the US, Real per Capita GDP relative to the US, Trade Balance to GDP relative to the US, 1880-1997, 14 Countries
Figure 2. Model Predictions of the Productivity Coefficient

Beta = 0.4

Beta = 0.8

Eta
Table 1. Univariate and Panel Unit Root Tests (1880-1997)

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<td>$\mu$ $\tau$ $\mu$ $\tau$ $\mu$ $\tau$</td>
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<td>-3.49 -3.52 -1.40 -1.64 -1.81 -1.94</td>
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</tr>
<tr>
<td>Japan</td>
<td>-1.54 -1.57 -0.38 -1.72 -0.97 -1.48</td>
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<tr>
<td>Netherlands</td>
<td>-2.31 -2.79 -2.07 -2.17 -0.32 -1.86</td>
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<tr>
<td>Norway</td>
<td>-2.22 -3.67 -1.31 -2.58 -1.11 -1.71</td>
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<tr>
<td>Spain</td>
<td>-1.66 -1.74 -0.83 -1.08 -2.28 -2.70</td>
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<tr>
<td>Sweden</td>
<td>-2.22 -4.05 -0.93 -3.64 -0.20 -2.24</td>
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</tr>
<tr>
<td>UK</td>
<td>-1.69 -1.88 -0.80 -1.66 -0.69 -3.89</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Univariate Unit Root Tests are based on Elliot, Rothenberg and Stock (1996) with optimal lag order (not reported) is calculated using the Modified AIC criterion proposed by Ng and Perron (2000). Levin, Lin, Chu and Im, Pesaran and Shin tests are based on demeaned data to account for cross-sectional dependence. Pesaran’s test is robust to cross-sectional dependence. Italics denote rejection at 10%, Bold denote rejection at 5%, Italics and Bold denote rejection at 1%.
### Table 2. Panel Unit Root Tests (sub-periods)

<table>
<thead>
<tr>
<th></th>
<th>1880-1913</th>
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<th></th>
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<tbody>
<tr>
<td></td>
<td>trend</td>
<td>y-y*</td>
<td>b-b*</td>
<td>trend</td>
<td>y-y*</td>
</tr>
<tr>
<td>Levin Lin Chu Adj- t*</td>
<td>-1.56</td>
<td>-1.37</td>
<td>0.26</td>
<td>-0.97</td>
<td>-4.07</td>
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<tr>
<td>Pesaran Z[t-bar]</td>
<td>-2.84</td>
<td>-1.24</td>
<td>0.49</td>
<td>-1.36</td>
<td>-0.69</td>
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</tbody>
</table>

Notes: Italics denote rejection at 10%, Bold denote rejection at 5%, Italics and Bold denote rejection at 1%. Levin, Lin, Chu and Im, Pesaran and Shin tests are based on demeaned data to account for cross-sectional dependence. Pesaran’s test is robust to cross-sectional dependence.

### Table 3. Real Exchange Rate Half Lives

<table>
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</thead>
<tbody>
<tr>
<td>Uncorrected</td>
<td>6.9</td>
<td>3.1</td>
<td>3.8</td>
<td>1.7</td>
<td>2.8</td>
</tr>
<tr>
<td>Nickell</td>
<td>6.6</td>
<td>3.1</td>
<td>4.7</td>
<td>1.1</td>
<td>5.6</td>
</tr>
<tr>
<td>Time</td>
<td>3.6</td>
<td>1.5</td>
<td>1.9</td>
<td>0.5</td>
<td>1.9</td>
</tr>
<tr>
<td>Nickell and Time Bias corrected</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>$H_{0.025}$</td>
<td>3.4</td>
<td>1.4</td>
<td>1.9</td>
<td>0.5</td>
<td>1.9</td>
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<tr>
<td>$H_{0.5}$</td>
<td>4.1</td>
<td>1.8</td>
<td>2.6</td>
<td>0.6</td>
<td>2.9</td>
</tr>
<tr>
<td>$H_{0.975}$</td>
<td>5.3</td>
<td>2.3</td>
<td>4.2</td>
<td>0.8</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Notes: Following Choi, Mark and Sul (2006), $H_{0.025}$, $H_{0.5}$, and $H_{0.975}$ are the 2.5, 50, and 97.5 percentiles of the half-life distribution.
Table 4. Individual and Panel DOLS (1880-1997)

<table>
<thead>
<tr>
<th>Country</th>
<th>$y-y^*$</th>
<th>$t$-stat</th>
<th>$y-y^*$</th>
<th>$t$-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1.67</td>
<td>4.70</td>
<td>-1.00</td>
<td>-1.87</td>
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<tr>
<td>Australia</td>
<td>0.27</td>
<td>1.11</td>
<td>0.08</td>
<td>0.35</td>
</tr>
<tr>
<td>Canada</td>
<td>-0.27</td>
<td>-3.33</td>
<td>0.06</td>
<td>0.31</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.63</td>
<td>1.25</td>
<td>0.53</td>
<td>1.57</td>
</tr>
<tr>
<td>Finland</td>
<td>0.12</td>
<td>1.22</td>
<td>-0.19</td>
<td>-1.58</td>
</tr>
<tr>
<td>France</td>
<td>0.10</td>
<td>1.03</td>
<td>0.18</td>
<td>1.31</td>
</tr>
<tr>
<td>Germany</td>
<td>0.66</td>
<td>2.58</td>
<td>0.73</td>
<td>2.73</td>
</tr>
<tr>
<td>Italy</td>
<td>0.03</td>
<td>0.19</td>
<td>-0.06</td>
<td>-0.54</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.11</td>
<td>-0.80</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.23</td>
<td>1.25</td>
<td>0.34</td>
<td>2.57</td>
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<tr>
<td>Norway</td>
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<td>1.28</td>
<td>0.32</td>
<td>1.01</td>
</tr>
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<td>Spain</td>
<td>0.19</td>
<td>1.14</td>
<td>0.39</td>
<td>2.39</td>
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<tr>
<td>Sweden</td>
<td>0.44</td>
<td>2.80</td>
<td>-0.30</td>
<td>-1.48</td>
</tr>
<tr>
<td>UK</td>
<td>0.22</td>
<td>1.70</td>
<td>0.34</td>
<td>1.29</td>
</tr>
<tr>
<td>GM</td>
<td>0.19</td>
<td>5.87</td>
<td>0.22</td>
<td>3.86</td>
</tr>
<tr>
<td>Panel DOLS</td>
<td>0.13</td>
<td>2.30</td>
<td>0.17</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Notes: The DOLS equations are specified as:

$$q_{it} = \alpha_0 + \mu_i + \delta_t + \alpha_1(y - y^*)_{it} + \sum_{k=-p}^{p} \delta_{ik} \Delta(y - y^*)_{i,t-k} + \epsilon_{it}.$$  The Group Mean estimates are 

$$GM = N^{-1} \sum_{i=1}^{N} \hat{\alpha}_{ti}$$  and  

$$\tau = N^{-1/2} \sum_{i=1}^{N} t_{\hat{\delta}_{it}}.$$  To account for cross-sectional dependence, time dummies are included in the PDOLS regressions and series are cross-sectionally demeaned before applying the GM estimator. Univariate DOLS and GM significance is based on heteroskedasticity and autocorrelation consistent standard errors. Panel DOLS significance is based on the Driscoll and Kray (1998) standard errors, which are heteroskedasticity consistent and robust to general forms of cross-sectional (spatial) and temporal dependence (see Hoeckle, 2007).
### Table 5. Panel DOLS (Sub-samples)

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>trend</td>
<td>trend</td>
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</tr>
<tr>
<td>y-y*</td>
<td>τ</td>
<td>y-y*</td>
<td>τ</td>
<td>y-y*</td>
<td>τ</td>
<td>y-y*</td>
<td>τ</td>
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<tr>
<td>GM</td>
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<td>8.27</td>
<td>0.25</td>
<td>7.90</td>
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<td>-0.17</td>
<td>-4.83</td>
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<tr>
<td>Panel DOLS</td>
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<td>2.72</td>
<td>0.17</td>
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<td>-0.42</td>
<td>-4.33</td>
<td>-0.31</td>
<td>-2.92</td>
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</tbody>
</table>

Notes: DOLS equations are specified as: 

\[ q_{it} = \alpha_0 + \mu_i + \delta_i + \alpha_t \left( y - y^* \right)_t + \sum_{k=-p}^{p} \delta_{tk} \Delta \left( y - y^* \right)_{t-k} + \varepsilon_{it} \] 

GM are group mean estimates, where \( GM = N^{-1} \sum_{i=1}^{N} \hat{\alpha}_i \) and \( \tau = N^{-1/2} \sum_{i=1}^{N} t_{\delta_{it}} \). To account for cross-sectional dependence, time dummies are included in the PDOLS regressions and series are cross-sectionally demeaned in the GM estimator. GM significance is based on heteroskedasticity and autocorrelation consistent standard errors for the underlying univariate regressions. Panel DOLS significance is based on the Driscoll and Kraay (1998) standard errors, which are heteroskedasticity consistent and robust to general forms of cross-sectional (spatial) and temporal dependence (see Hoeckle, 2007).
Table 6. Panel residuals half-lives.

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<td>y-y*</td>
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<td><strong>-3.52</strong></td>
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<tr>
<td>HL_{0.5}</td>
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<td>5.0</td>
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<td>1.6</td>
</tr>
<tr>
<td>HL_{0.975}</td>
<td>5.2</td>
<td>6.4</td>
<td>3.3</td>
<td>1.3</td>
<td>2.2</td>
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<table>
<thead>
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<tr>
<td>y-y*</td>
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<td>0.17</td>
<td>-0.003</td>
<td>0.17</td>
<td><strong>-0.25</strong></td>
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<tr>
<td>LLC Adj t*</td>
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<td><strong>-1.59</strong></td>
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<tr>
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<td>1.9</td>
<td>0.8</td>
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<td>1.0</td>
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<tr>
<td>HL_{0.5}</td>
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<td>2.2</td>
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<td>1.3</td>
</tr>
<tr>
<td>HL_{0.975}</td>
<td>2.5</td>
<td>2.6</td>
<td>1.4</td>
<td>0.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Notes: Italics denote rejection at 10%, Bold denote rejection at 5%, Italics and Bold denote rejection at 1%. PDOLS significance is based on the Driscoll and Kraay (1998) standard errors. Time effects are included in the regressions. Following Choi, Mark and Sul (2006), H_{0.025}, H_{0.5}, and H_{0.975} are the 2.5, 50, and 97.5 percentiles of the half-life distribution.
Table 7. Error Correction Specification

<table>
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<td>MG</td>
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<td>0.9</td>
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<tr>
<td>Trend</td>
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<tr>
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<td>HL</td>
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<td>1.6</td>
<td>1.3</td>
<td>0.7</td>
</tr>
<tr>
<td>Hausman Test</td>
<td>$\chi^2(1)$</td>
<td>0.63</td>
<td>1.08</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Trend</td>
<td>$\chi^2(2)$</td>
<td>3.72</td>
<td>2.20</td>
<td>3.46</td>
<td>1.58</td>
</tr>
</tbody>
</table>

Notes: $\phi$ and $\gamma$ are the adjustment and long run parameters. Half Life (HL) estimates are based on the parameter $\phi$. Italics denote rejection at 10%, Bold denote rejection at 5%, Italics and Bold denote rejection at 1%. Series are demeaned to account for cross-sectional dependence. The Mean Group (MG) estimates are simple unweighted averages of the N individual regressions. The pooled mean group (PMG) estimator constrains the long-run elasticity to be equal across countries. Pooling is consistent and efficient if restrictions are true, but inconsistent if the true model is heterogeneous. MG is consistent in any case. The Hausman test is performed under the null that the difference in coefficients is not systematic. Pooling seems always consistent and efficient except for the post Bretton Woods period, when a trend is included.