On the Impact of Income per Capita on Health Outcomes: Is Africa Different?

Elizabeth Asiedu* University of Kansas
Neepa B. Gaekwad† University of Kansas
Malokele Nanivazo‡ University of Kansas
Mwanza Nkusu§ International Monetary Fund
Yi Jin¶ Monash University

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Abstract

This paper examines the link between income per capita, adult life expectancy and mortality rates for children. We construct an overlapping generations model and derive conditions under which the impact of income per capita on health outcomes is: (i) linear and positive; (ii) non-linear, positive and the marginal effect of income diminishes as income increases; and (iii) non-linear, positive and the marginal effect of income increases as income rises. We next estimate a dynamic panel model using panel data from 128 developing countries over the period 1994-2014. We find that: (i) Global factors (i.e., non country-specific factors) have a positive and significant impact on health outcomes, and this effect has increased over time; (ii) Countries in Sub-Saharan Africa (SSA) have a higher mortality rate and lower life expectancy than countries outside SSA; (iii) An increase in income per capita improves health outcomes and the effect is stronger at higher levels of income; and (iv) The effect of income per capita on health outcomes is different for SSA countries.

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*Corresponding author. Department of Economics, University of Kansas. Email: asiedu@ku.edu.
†University of Kansas, Lawrence, Email: neepag12@ku.edu
‡University of Kansas, Email: nanivazo@ku.edu
§International Monetary Fund, Email: mnkusu@imf.org
¶Monash University, Australia. Email: yi.jin@monash.edu.

The influence of economic conditions on mortality has been recognized at least since biblical times (Preston, 2007:484).

1 Introduction

To the extent that income per capita may be interpreted as the summary of the economic conditions in a country, the above quote suggests that per capita income has a causal effect on health outcomes. It is therefore not surprising that the large empirical literature on the determinants of health outcomes typically include GDP per capita as one of the explanatory variables that may have a significant effect on health outcomes. The overwhelming results from the literature is summarized by Pritchett and Summers (1996:863) who note that "wealthier nations are healthier nations" and "gains from rapid economic growth flow into health gains".[1] The idea that income may have a positive effect on health outcomes is plausible, for the simple reason that higher income permits households to spend more on the personal health of the family, which in turn improves the heath of the household. Under this scenario, the influence of a country's own level of income on the country's health outcome will depend on country-specific factors such as education, nutrition and factors that affect the delivery of health-related services. However, the health of a country's residents may also be influenced by exogenous factors. We elaborate on two such factors. The first pertains to the "global public good" aspect of health such as advances in medical technology and the diffusion of health technology.[2] Indeed, the importance of global factors is consistent with the findings of Preston (2007). Using data from 1900, 1930 and 1960, Preston (2007) finds that factors exogenous to a country's level of income accounted for about 84% of the increase in life expectancy during that period. He concludes that the importance of income per capita in explaining health outcomes has diminished over time. Clearly, globalization enhances technological diffusion. As a consequence, it is very likely that in this era of widespread globalization, the positive effect of per capita income on health outcomes observed in previous years may have diminished or completely disappeared in recent years. The second exogenous factor is "Geography". For example, countries in the tropics, in particular countries in Subsaharan Africa (SSA) are more prone to some specific diseases, such as malaria (Bloom and Sachs, 1998).[3] Indeed, the recent Ebola outbreak in West Africa lends credence to the effect

[1] A few exceptions include Anand and Ravalion (1993) who analyze the effect of income per capita on adult life expectancy and infant mortality across 22 countries. The authors find that the estimated coefficient of income per capita turns insignificant after controlling for incidence of poverty and public spending on health.

[2] A good example is the eradication of smallpox and measles in developing countries.

[3] Bloom and Sachs (1998) find that life expectancy at birth is lower for populations that reside in the tropics even after controlling for income levels.

of Geography on health outcomes (CDC, 2015).

This paper reassesses the relationship between income per capita and health outcomes. Specifically, the paper theoretically and empirically examines the link between GDP per capita, adult life expectancy and mortality rates for children. For the theoretical model, we construct an overlapping generations model where agents live for three periods: childhood, adulthood and old age. We assume that parents care about the quality of their children and that child mortality is a decreasing function of child quality. We derive the conditions under which: (i) income has a linear and a positive effect on child mortality rate and life expectancy; (ii) income has a non-linear and a positive effect on health outcomes, and the marginal effect of income diminishes as income increases; and (iii) income has a non-linear and a positive effect on health outcome, and the marginal effect of income increases as income rises.

The empirical analysis examines whether income per capita has a causal impact on health outcomes for children and adults. Specifically, we consider four measures of health, two for children and two for adults: under-1 year mortality rate, under-5 years mortality rates, and life expectancy for adult females and males. We answer four questions. All else equal, (i) How relevant are global factors (i.e., non-country specific factors) in explaining health outcomes? Has the effect of global factors on health outcomes increased over time? (ii) Do child mortality rates and adult life expectancy for Sub-Saharan Africa (SSA) vary significantly from the mortality rates and life expectancy of countries outside SSA; (iii) Does GDP per capita have a causal impact on health outcomes? (iv) Is the effect of GDP per capita on health outcomes for countries in SSA significantly different from the effect for non-SSA countries?

To answer these questions, we estimate a dynamic panel model and employ panel data from 128 developing countries over the period 1994-2014. We include ln (GDP per capita), $gdpc$, and $gdpc^2$ in our regressions to test the non-linear effect of income per capita on health outcomes. We also include a dummy variable, SSA, for countries in SSA, as well at the interaction term, $SSA \times gdpc$, to examine whether SSA is "different." We find that: (i) Global factors have a positive and significant impact on health outcomes and that the effect has increased over time, suggesting that country-specific factors, including GDP per capita, has become less relevant in explaining the variation in health outcomes across country and within country; (ii) Countries in SSA have a higher child mortality rate and lower adult life expectancy compared to countries outside SSA; (iii) An increase in GDP per capita significantly reduces mortality rates for children and increases adult life expectancy. However, the effect is non-linear: the negative effect on mortality and the positive effect on life expectancy becomes stronger at higher levels of income, pointing to a divergence in

health outcomes.[4] and (iv) The effect of GDP per capita on health outcomes is different for SSA countries. Specifically, the effect on mortality rate is less for SSA countries than non-SSA countries. In contrast, the effect on life expectancy is higher for SSA countries than for countries outside SSA. The results hold after controlling for health expenditure, primary school enrollment and HIV/AIDS prevalence.

The remainder of the paper is organized as follows. Section 2 examines the bivariate relationship between GDP per capita and health outcomes for our sample countries. Section 3 provides a brief literature review and it articulates our contribution to the literature. Section 4 presents the theoretical model, Section 5 describes the data and the variables employed in the empirical analysis and Section 6 discusses the estimation procedure. Section 7 presents the empirical results and Section 8 concludes.

2 Preliminary Analysis

The questions posed in the introduction are partly motivated by the data that we employ for the empirical analysis, which comprise of data from 128 developing countries over the period 1994-2014. Specifically, the data provides a glean of the relationship between GDP per capita and the four measures of health outcomes: under 1-year mortality rate, *infmort1*, under 5-years mortality rate, *infmort5*, adult female life expectancy, *lifeexpf* and adult male life expectancy, *lifeexpm*. The data generates four conjectures.

Conjecture 1: Global factors have a significant impact on health outcomes, and the effect has increased over time.

Evidence 1: Figures 1a-1d plot the four measures of health outcomes and income per capita in 1995 and 2014. The graphs show that for a given level *gdpc*, *infmort1* and *infmort5* in 2014 is lower than the mortality rate in 1995 (Figures 1a and 1b) and *lifeexpf* and *lifeexpm* in 2014 is higher than life expectancy in 1995 (Figures 1c and 1d). This suggests that non-income related factors are relevant in explaining the gains in health outcomes over time.

Conjecture 2: GDP per capita has a significant and positive effect on health outcomes.

Evidence 2: Table 1 shows the percentage change in GDP per capita and the measures of health outcomes by region and by income groups, from 1994 to 2014. The data show that for all the regions and all the income-groups, per capita income increased substantially from 1994-2014, and at the same time, there was a significant reduction in mortality rate and a substantial increase in life expectancy. For example from 1994-2014, real GDP per

[4]Clark (2011) also concludes that the effect of GDP per capita is nonlinear and it improves health outcomes, however, the marginal effect declines with income for life expectancy (convergence) but increases with infant mortality rates (divergence).

capita increased by about 65% in low-income countries, *infmort1* declined by 53%, *infmort5* decreased by 46% and *lifeexpf* and *lifeexpm* increased by 22% and 23%, respectively. This suggests that GDP per capita may have a causal and positive impact on health outcomes. Figures 2a-5b depict scatter plots of *gdpc* and health outcomes. The graphs show that a higher *gdpc* is associated with lower mortality rates and a higher life expectancy for all the countries (SSA and non-SSA countries). Thus, the data suggest that overall, *gdpc* may have a causal and positive effect on health outcomes.

Conjecture 3: The relationship between GDP per capita and health outcomes may be quadratic.

Evidence 3: Figures 2a-5b show the quadratic plot between *gdpc* and health outcomes for SSA and non-SSA countries. The graphs suggest that the relationship between *gdpc* and health outcomes may be quadratic.

Conjecture 4: The relationship between GDP per capita and health outcomes may be different for countries in SSA and non-SSA countries.

Evidence 4: Figures 2a-5b show that the curvature of the scatter plot for *gdpc* and health outcomes for SSA countries (Figures 2a, 3a, 4a and 5a) is different from that for non-SSA countries (Figures 2b, 3b, 4b and 5b). Specifically, the graph for *gdpc* and child mortality rates is concave for countries in SSA (Figures 2a and 3a), but convex for countries outside SSA (Figures 2b and 3b); and the graph for *gdpc* and life expectancy is convex for SSA countries (Figures 4a and 5a) but concave for non-SSA countries (Figures 4b and 5b).

A relevant question is whether the relationship between *gdpc* and health outcomes hypothesized above will hold after controlling for other relevant factors that may affect health outcomes.

3 Brief Literature Review

Most of the empirical papers on the determinants of health outcomes include income per capita as one of the explanatory variables. Table B1 in the Appendix B shows a summary of 19 papers with publication dates from 1990-2011, that have included per capita income as an explanatory variable. There are eight notable points. First, none of the papers include a theoretical model. Second, Figures 2a-5b suggest that the relationship between *gdpc* and health outcomes may be quadratic. Yet, only one paper, Clark (2011) includes the square of *gdpc* as an explanatory variable. The remaining papers assume that the relationship between income per capita and health outcomes is linear. Third, only five papers include time dummy variables. This is problematic because the data suggest that time-specific effects

may be relevant in explaining health outcomes. Fourth, the data employed in the papers are old. Specifically, the latest data are from 2007 (Biggs et. al., 2010). As pointed out earlier, it is possible that the effect of income per capita on health outcomes has changed over time. There is therefore a need for an analysis that employs more recent data. Fifth, only three studies (Klasen, 2006; Scanlan, 2010; Clark, 2011) include HIV/AIDS as an explanatory variable. Controlling for HIV prevalence is important because as noted in (UNDP, 2005: 10), "HIV/AIDS is a global epidemic and the disease has inflicted the single greatest reversal in human development in modern history." Sixth, the data suggest that SSA is "different". Yet only 4 papers include a dummy variable for SSA (Cornia and Mwabu, 1997; Ranis et.al., 2000; Klasen, 2006 and Clark, 2011). Here, it is important to distinguish between the "intercept" effect and the "slope" effect. Note that including a dummy variable for SSA permits one to test the "intercept effect", i.e., whether health outcomes in SSA countries is significantly different from health outcomes in non-SSA countries. None of the studies include an interaction term between SSA and *gdpc* to examine "slope effect" i.e., whether the effect of *gdpc* on health outcomes is different for countries in SSA. Seventh, similar to many macroeconomic variables, health outcomes are likely to be persistent, i.e., current values of health outcomes are likely to be correlated with previous values of health outcomes. If that is the case, then lagged health outcome should be included as an explanatory variable in the regressions. Yet, only one study includes lagged health as an explanatory variable (Neumayer, 2004). In addition, the analysis is based on Ordinary Least Squares (OLS) estimations. This is problematic because OLS estimates are biased and inconsistent when a lagged dependent variable is included as an explanatory variable in the estimations. The last point pertains to the estimation procedures employed in the papers. Specifically, the studies employ OLS, two-stage least squares (2SLS) and random effects estimations. There is also the issue of reverse causality between health outcomes and *gdpc*. For example higher adult life expectancy may lead to an increase in per capita income (Bloom et. al., 2004 and Cervellati and Sunde, 2011), and lower mortality rates for children may have a positive effect on income per capita (Bhargava et. al., 2001; Lorentzen et. al., 2008). It is important to note that reverse causality and the inclusion of a lagged dependent variable introduces endogeneity and this problem cannot be addressed by OLS, 2SLS or random effects estimations.

This paper makes several contributions to the literature. First, unlike previous studies, the empirical estimations is based on a model with micro foundations. Furthermore, we employ the dynamic panel the system General Method of Moments estimator proposed by Blundell and Bond (1998) to addresses the endogeneity and dynamic issues not considered in previous studies. We also utilize more recent data (i.e., 1994 to 2014), include time dummy variables and we control for HIV prevalence. In addition, we include *gdpc* and the square

of *gdpc* as explanatory variables to allow the effect of *gdpc* on health outcomes to vary by income. Finally, we include a dummy variable for SSA and interact the SSA dummy variable with *gdpc* to examine whether health outcomes in SSA countries differ from outcomes in non-SSA countries with regard to the "intercept" and "slope". This is important because several studies have concluded that SSA is unique in that the effect of some socioeconomic and institutional factors are different for SSA countries and non-SSA countries (e.g., Asiedu, 2002; Dalgaard et al., 2004).[5] Indeed, examining the "Africa" effect on health outcomes is one of the innovations of the paper. To the best of our knowledge this is the first paper to examine this issue for health outcomes. The paper also contributes to the discussion on whether the increase in growth observed during the past two decades in many developing countries, in particular, countries in SSA, has led to a significant reduction in poverty levels (Sala-i-Martin and Pinkovsky, 2010; Thorbecke, 2013).[6] This paper focuses on one aspect of poverty reduction, i.e., an increase in adult life expectancy and a decline in mortality rate for children. The paper is timely and has important policy implications. Specifically, the target for Goal 4 of the United Nations Millennium Development Goal (MDG) is to reduce under-five mortality rate by two thirds by 2015 compared to 1990, the benchmark year for the MDGs.[7] However, as noted in (UNICEF, 2014:6) this goal is unlikely to be met: "Progress is insufficient to meet MDG 4. If current trends continue in all countries, the target will only be reached globally by 2026, 11 years behind schedule." Thus, by providing a rigorous analysis of the determinants of health outcomes in developing countries, this paper will assist policy makers and the international community to craft effective policies to improve health outcomes in poor countries. Finally, the paper has important policy implications for countries in SSA. There are at least two reasons. First, SSA has the highest under-five mortality rate. In 2013, about 50% of global under-five deaths occurred in sub-Saharan Africa, although the region accounts for only 13% of global population. Furthermore, there are 12 countries with an under 5-years mortality rate of at least 100 deaths per 1000 live births in year 2013, and all the countries are in SSA (UNICEF, 2014).[8] The second reason

[5]For example, Asiedu (2002) shows that the determinants of FDI to SSA is different from the factors that drive FDI to other regions, and the analysis of Dalgaard et al., (2004) suggests that aid may be less effective in SSA countries.

[6]Thorbecke (2012) documents that poverty in Africa remains high despite gains in growth. In contrast, Sala-i-Martin and Pinkovsky (2010) find a substantial reduction in poverty.

[7]The United Nations (UN) adopted eight Millennium Development Goals (MDGs) in September 2000. The eight MDGs are (i) eradicate extreme poverty and hunger; (ii) achieve universal primary education; (iii) promote gender equality and empower women; (iv) reduce child mortality; (v) improve maternal health; (vi) combat HIV/AIDS, malaria and other diseases; (vii) ensure environmental sustainability; and (viii) develop a global partnership for development.

[8]The countries are Angola (167), Sierra Leone (161), Chad (148), Somalia (146), Central African Republic (139), Guinea-Bissau (124), Mali (123), Democratic Republic of the Congo (119), Nigeria (117), Niger (104), Guinea (101) and Cote d'Ivoire (100). The numbers in parenthesis refer to the number of deaths per 1000

is that SSA has made the least progress in reducing under-five mortality rates. Under-five mortality rates in SSA declined by 48% from 1990 to 2015. This compares with a reduction of 56% for South Asia, 56% for Middle East & North Africa, 67% for East Asia and 67% for Latin America (UNICEF, 2014).

4 A Model of Endogenous Mortality and Life Expectancy

We consider a simple overlapping generations model of an endowment economy where agents live for three periods: childhood, adulthood, and old age. Only a fraction $\rho \in (0,1)$ of children born survive to adulthood, and a fraction $p \in (0,1)$ of adults survive to old age. All decisions are made at the beginning of adulthood. Adults of generation t care about adult consumption c_t, old-age consumption d_{t+1}, number of survived children n_t, and the quality of children q_t. Following Becker (1960), child quality is modeled as child expenditure.[9] Thus, the more parents spend on children's health, the higher probability that the children will survive to adulthood (Strulik, 2004).[10] The utility function is assumed to be

$$\ln c_t + \alpha \ln (n_t - n_0) + \gamma \ln q_t + p \ln d_{t+1}. \tag{1}$$

where $n_0 > 0$ indicates that parents want to have at least n_0 survivors and α and γ are parents' altruism factors.[11] The old-age consumption is weighted with the survival probability to old age p, which is assumed to be constant for simplicity.[12] We assume that the child survival rate $\rho(q_t)$ is an increasing function of q_t.

4.1 Adults' Optimization Problem

Let y be the income endowment. An adult chooses a consumption profile c_t and d_{t+1}, savings for old age s_t, via an actuarially fair annuity market, number of survived children n_t, and

live births.

[9] In most studies of child quality, researchers focus on education expenditures which improve children's human capital and hence their income earning ability in adulthood. Since we are interested in how increases in income affect child mortality and survival rate, we abstract from education and endogenous income determination, and therefore we link child quality to just health expenditures.

[10] See Strulik (2004) for a growth model where heallth and fertility are endogenously determined. Here, child quality has two components: child health and schooling.

[11] Later we will see that the presence of n_0 in the current setup ensures child quality to be related to income. Otherwise q will be solely dependent on fundamental parameters and independent of income level.

[12] One can allow agents to also invest in their own health, in which case the adulthood survival rate, p, will depend on adulthood health expenditure. Here, income level will affect life expectancy through both the endogenous, ρ, and the endogenous p. We abstract from this because incorporating endogenous p makes it difficult to obtain tractable analytical results.

child quality q_t.[13] Then the budget constraint in adulthood is given by:

$$c_t + s_t + \frac{n_t}{\rho(q_t)} q_t = y, \tag{2}$$

The expenditure on children is evenly distributed among all children born $n_t/\rho(q_t)$. Here, the fertility rate is simply $n_t/\rho(q_t)$. For a fixed amount of expenditures on children, there exists a trade-off between child quantity $n_t/\rho(q_t)$ and child quality q_t. Let R be the constant interest rate. Then if an adult survives to old age, she would receive the annuity income from savings, Rs_t/p and the old-age budget constraint is simply

$$d_{t+1} = Rs_t/p. \tag{3}$$

The agent's optimization problem is to maximize (1) subject to (2) and (3).

Assuming interior solutions, the first order conditions yield

$$s_t \quad : \quad \frac{p}{s_t} = \frac{1}{c_t}, \tag{4}$$

$$n_t \quad : \quad \frac{\alpha}{n_t - n_0} = \frac{1}{c_t} \frac{q_t}{\rho(q_t)}, \tag{5}$$

$$q_t \quad : \quad \frac{\gamma}{q_t} = \frac{n_t}{c_t} \frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t}. \tag{6}$$

implying that for each of the choice variables, optimality is achieved when marginal benefit equals marginal cost. Note that if $n_0 = 0$, (5) and (6) are able to pin down child quality q_t, which will be independent of income y. With $n_0 > 0$, equations (4)-(6) yield the following:

$$\frac{s_t}{c_t} = p, \tag{7}$$

$$\frac{1}{c_t} = \frac{1}{n_0}\left[\frac{n_t}{c_t} - \alpha \frac{\rho(q_t)}{q_t}\right], \tag{8}$$

$$\frac{n_t}{c_t} = \frac{\gamma}{q_t}\left[\frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t}\right]^{-1}. \tag{9}$$

Substituting the above three equations into the adulthood budget constraint (2), we obtain an equation that determines q_t :

$$1 + p + \frac{y}{n_0}\alpha \frac{\rho(q_t)}{q_t} = \left[\frac{y}{n_0} - \frac{q}{\rho(q_t)}\right] \frac{\gamma}{q_t}\left[\frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t}\right]^{-1}. \tag{10}$$

[13] The convenience of an annuity market keeps the model simple by avoiding the need to explicitly model accidental bequests.

4.2 Child Survival Rate

We now specify the child survival rate. Let $\rho(q_t) = \theta + (1-\theta)\rho_0 (q_t)^{\mu}$, where $\theta \in [0,1]$, $\mu \in (0,1)$, and $\rho \in (0, \bar{\rho})$. The parameter θ captures the component of the survival rate that is not directly controlled by parents. In order to derive analytical results, we set $\theta = 0$. The parameter μ captures the effectiveness of child quality in increasing the child survival rate. Later we will see that the magnitude of μ crucially affects the nonlinearity of our result. The log utility ensures a positive child quality hence the survival rate can never be zero. On the other hand, the survival rate being less than 1 requires ρ_0 be small enough. The upper bound $\bar{\rho}$ will be determined later. With this particular functional form of the survival rate, (10) yields the optimal child quality, denoted by q^*,

$$q^* = \left[\frac{\rho_0}{n_0} \frac{\gamma - \alpha(1-\mu)}{\gamma + (1+p)(1-\mu)} y \right]^{\frac{1}{1-\mu}} \tag{11}$$

Assumption 1: $\gamma > \alpha(1-\mu)$.

Under Assumption 1, q^* is clearly positive. With the optimal q^*, the restriction that the survival rate is less than 1, $\rho(q^*) < 1$, yields the value for $\bar{\rho}$:

$$\bar{\rho} = \left[\frac{n_0}{y} \frac{\gamma + (1+p)(1-\mu)}{\gamma - \alpha(1-\mu)} \right]^{\mu}. \tag{12}$$

The remaining variables can also be determined:

$$\begin{aligned} c^* &= \frac{1-\mu}{\gamma + (1+p)(1-\mu)} y, \\ s^* &= pc^*, \\ d^* &= Rc^*, \\ n^* &= \frac{\gamma}{\gamma - \alpha(1-\mu)} n_0 > n_0. \end{aligned}$$

Child mortality rate is simply

$$1 - \rho(q^*) = 1 - \rho_0 (q^*)^{\mu}.$$

Based on the age structure of the current model, the life expectancy at birth (i.e., age 0), denoted by e_0, is given by (see Appendix A for the details of the derivation of e_0):

$$e_0 = \frac{1}{2} + \rho(q^*)(1+p).$$

4.3 Effect of an Increase in Income on Child mortality rate and Life expectancy

Totally differentiating (11) with respect to y yields

$$\frac{dq^*}{dy} = \frac{\rho_0}{n_0} \frac{\gamma - \alpha(1-\mu)}{\gamma + (1+p)(1-\mu)} \frac{1}{1-\mu} (q^*)^{\mu} > 0,$$

and

$$\frac{d^2q^*}{dy^2} = \frac{\rho_0}{n_0} \frac{\gamma - \alpha(1-\mu)}{\gamma + (1+p)(1-\mu)} \frac{\mu}{1-\mu} (q^*)^{\mu-1} \frac{dq^*}{dy} > 0.$$

Therefore

$$\begin{aligned} \frac{d[1-\rho(q^*)]}{dy} &= -\rho_0 \mu (q^*)^{\mu-1} \frac{dq^*}{dy} < 0. \\ \frac{de_0}{dy} &= (1+p)\rho_0 \mu (q^*)^{\mu-1} \frac{dq^*}{dy} > 0. \end{aligned}$$

When income increase, parents invest more in child quality and this reduces child mortality rate and increases life expectancy.[14] The second order derivatives are given by:

$$\begin{aligned} \frac{d^2[1-\rho(q^*)]}{dy^2} &= \frac{\rho_0}{y^2} (q^*)^{\mu} \frac{\mu(1-2\mu)}{(1-\mu)^2}, \\ \frac{d^2e_0}{dy^2} &= \frac{(1+p)\rho_0}{y^2} (q^*)^{\mu} \frac{\mu(2\mu-1)}{(1-\mu)^2}. \end{aligned}$$

Note that the sign of the second order derivatives depends on whether $\mu >= \frac{1}{2}$.

We summarize the above results in Proposition 1. Note that the nonlinearity result in the empirical evidence (see Section 7 for details) is consistent with case (*iii*) below.

Proposition 1 *Under Assumption 1 and $\rho_0 \in (0, \bar{\rho})$ with $\bar{\rho}$ given by (12), an increase in income reduces the child mortality rate ($d(1-\rho)/dy < 0$) and it raises life expectancy at birth ($de_0/dy > 0$). Furthermore,*

(i) when $\mu = \frac{1}{2}$, $d^2[1-\rho(q^*)]/dy^2 = 0$ and $d^2e_0/dy^2 = 0$, implying that income has a linear effect on child mortality rate and life expectancy;

(ii) when $\mu \in \left(0, \frac{1}{2}\right)$, $d^2[1-\rho(q^*)]/dy^2 > 0$ and $d^2e_0/dy^2 < 0$, implying that income has a nonlinear effect and the marginal effect of income diminishes as income increases;

(iii) when $\mu \in \left(\frac{1}{2}, 1\right)$, $d^2[1-\rho(q^*)]/dy^2 < 0$ and $d^2e_0/dy^2 > 0$, implying that income has a nonlinear effect and the marginal effect of income gets stronger as income rises.

[14] Note that although the survival number of children is fixed, the fertility rate will be affected by the level of income.

5 The Data and the Variables

The empirical analysis employs data from 128 developing countries (45 SSA countries and 83 non-SSA countries) over the period 1994-2014. As it is standard in the literature, the data are averaged over three years. The list of countries included in the regressions and a detailed description of the variables is reported in Table B2 in Appendix B.

For the dependent variable, we consider 4 measures of health outcomes: under 1-year mortality rate, under 5-years mortality rate, female life expectancy and male life expectancy. Note that the variable of interest is GDP per capita. Following the literature, we use GDP per capita based on purchasing power parity (PPP). The data are in constant 2011 international dollars. The control variables are total health expenditure, primary school enrollment and HIV prevalence rate. These variables have been used in previous studies.[15] All the data are from the 2015 World Development Indicators, published by the World Bank. The summary statistics of the variables are reported in Table 2.

6 Estimation Procedure

We estimate a linear dynamic panel-data (DPD) model to capture the effect of lagged health outcomes on current health outcomes. DPD models contain unobserved panel-level effects that are correlated with the lagged dependent variable, and this renders standard estimators inconsistent. The Arellano and Bond (1991) difference GMM estimator provides consistent estimates for such models. This estimator differences the data first and then uses lagged values of the endogenous variables as instruments. However, as pointed out by Arellano and Bover (1995), lagged levels are often poor instruments for first differences. Blundell and Bond (1998) proposed a more efficient estimator, the system GMM estimator, which mitigates the weak instruments problem by using additional moment conditions.[16] We therefore use the more efficient and less biased system GMM estimator for our regressions.

We now point out some potential caveats of the system GMM estimator and discuss how these problems are addressed. The first issue relates to the validity of the instruments.

[15] We considered other explanatory variables that have been used in previous studies, including measures of income inequality, gender inequality, access to safe water, degree of urbanization, institutional quality, etc, however, the variables did not display a consistent relationship after controlling for total health expenditure, primary school enrollment and HIV prevalence.

[16] The system GMM uses more instruments than the difference GMM, and therefore one might expect the system estimator be more biased than the difference estimator. However, Hayakawa (2007) shows that the bias is smaller for the system than the difference GMM. Specifically, the bias of the system GMM estimator is smaller because it is a weighted sum of the biases of the difference and the level estimator, and that these biases move in opposite directions.

Second, the procedure assumes that there is no second order autocorrelation in the idiosyncratic errors. Another pertinent issue is that the test for autocorrelation and the test for the validity of the instruments lose power when the number of instruments, i, is large relative to the cross section sample size (in our case, the number of countries), n. Specifically, when the instrument ratio, r, defined as $r = \frac{n}{i}$, is less than 1, the assumptions underlying the two procedure are likely to be violated (Roodman, 2007). Furthermore, a low r raises the susceptibility of the estimates to a Type 1 error—i.e., producing significant results even though there is no underlying association between the variables involved. The easiest solution to this problem is to restrict the number of lags of the dependent variable used for instrumentation to the point where $r \geq 1$ (Roodman, 2007).

To address these potential problems, we test for autocorrelation and the validity of instruments for each regression. Specifically, for each regression, we report the p-values for the test for second order autocorrelation as well as the Hansen-J test for overidentifying restrictions. We report the results for 20 regressions, and the p-values indicate that the assumption of no second order autocorrelation is satisfied in each of the regressions. Furthermore, the instruments are valid in 14 out of the 20 regressions. The 6 regressions where the instrument requirement is violated pertain to the regressions that do not control for HIV prevalence. Our preferred specification is the one that includes all the explanatory variables. Thus, the two assumptions are satisfied in our "preferred" specifications. Furthermore, in all the 20 regressions, $r \geq 1$, and therefore we do not restrict the number of lags of the dependent variable used for instrumentation. Finally, including time dummy variables in the regressions has two advantages: it reflects the global factors that affect health outcomes and it also increases the likelihood that the assumption of no correlation across individuals in the idiosyncratic disturbances will be satisfied (Roodman, 2007). We therefore include time dummy variables in all our regressions.

We end the section by providing some details about our estimation strategy. First, we use the two-step GMM estimator, which is asymptotically efficient and robust to all kinds of heteroskedasticity. Second, the independent variables are treated as strictly exogenous in all the regressions. In addition, our regressions utilize only internal instruments—we do not include additional (external) instruments.[17]

[17]Note that the system estimator uses the first difference of all the exogenous variables as standard instruments, and the lags of the endogenous variables to generate the GMM-type instruments described in Arellano and Bond (1991). Furthermore, the system estimations include lagged differences of the endogenous variables as instruments for the level equation. See Asiedu and Lien (2011) for a detailed discussion.

7 Estimation Results

We estimate a variant of the equation:

$$\begin{aligned} health_{it} &= \rho health_{it-1} + \alpha gdpc_{it} + \beta gdpc_{it}^2 + \varphi SSA \\ &\quad + \delta gdpc_{it} \times SSA + \Sigma_{j=1}^{J} \gamma_j Z_{jit} + \Sigma_{t=1}^{T} \lambda_t P_t + \theta_i + \varepsilon_{it} \end{aligned} \tag{1}$$

where i refers to countries, t to time, *health* is a measure of health outcome, *SSA* takes on value one if the country is located in SSA and zero otherwise, $SSA \times gdpc$ is the interaction term, Z is a vector of control variables, P is a vector of dummy variable that takes on value one in period t, and θ_i is the country-specific effect.[18] All the variables are measured in logs.

Each regression includes a dummy variable for SSA, time dummy variables and a lagged dependent variable. The other control variables are included incrementally. Specifically, Column (1) controls for income per capita, Column (2) includes health expenditure and primary school enrollment and Column (3) controls for HIV prevalence. The results reported in Column (3) is our preferred specification, where all the control variables are included.

Question 1. How relevant are global factors in explaining health outcomes? Has the effect of these factors on health outcomes increased over time?

The regressions include six time dummy variables, 1997-99, 2000-02, 2003-05, 2006-08, 2009-11, 2012-14, and the reference period is 1994-96. The parameter of interest is the estimated coefficient of the time dummy variable, $\widehat{\lambda_t}$. Note that $\widehat{\lambda_t}$ is significant at the 1% level in all the regressions (Tables 3, 4, 5, 6, 7 and 9). In addition, $\widehat{\lambda_t}$ is negative for the mortality regressions (Tables 3, 4 and 8) and positive for the life expectancy regressions (Tables 5, 6 and 9), and for each specification, the magnitude of $\widehat{\lambda_t}$ increases as the time periods increase. These results suggest that global factors have a positive and significant impact on health outcomes and that the effect has increased over time. More importantly, the results imply that country-specific factors, including GDP per capita, has become less relevant in explaining the variation in health outcomes across country and within country.

Question 2. Do child mortality rates and adult life expectancy for countries in SSA vary significantly from the mortality rates and life expectancy of countries outside SSA?

The estimated coefficient of the SSA dummy variable, $\widehat{\varphi}$, is significant at the 1% level in all the regressions. Furthermore, $\widehat{\varphi}$ is positive for the mortality regressions and negative for the life expectancy regressions. This suggests that overall, mortality rate is higher and life

[18] We estimate a parsimonous model for two reasons. The Blundell and Bond (1998) procedure mitigates the potential problem of ommitted variable bias. In addition including more variables increases the number of instruments, i, and therefore raises the instrument count, $r = \frac{n}{i}$.

expectancy is lower in SSA countries than non-SSA countries. For example, all else equal, the under 1-year mortality rate is about 0.459 percentage points higher for SSA countries than non-SSA countries (Column 3 of Table 3), and the under 5-years mortality rate is about 0.505 percentage points higher for SSA countries (Column 3 of Table 4). There is a gap for life expectancy, however, the gap reduces significantly when one controls for HIV/AIDS. Specifically, the gap shrinks from 0.1698 to 0.0674 percentage points for females (Columns 2 and 3 of Table 5), and it declines from 0.1642 to 0.0707 for males (Columns 2 and 3 of Table 6). This result clearly underscores the importance of controlling for HIV/AIDS. In addition, it reflects the adverse effect of the HIV epidemic on adult life expectancy.

Question 3. Does GDP per capita have a causal impact on health outcomes?

To answer this question, we estimate equation (1) without the interaction term, $SSA \times gdpc$. Note that $\partial health/\partial gdpc = \alpha + 2\beta \times gdpc$, and $\widehat{\alpha}$ and $\widehat{\beta}$ are significant at the 1% level in all the regressions (Tables 3, 4, 5, and 6). In order to keep the paper focused and also to conserve on space, the discussion will focus on the results for under 1-year mortality rate and life expectancy for females. The results for under 5-years mortality rate and male life expectancy are qualitatively similar and are available upon request.

The estimated marginal effect of *gdpc* for *infmort1* (Column 3 of Table 3) and *lifeexpf* (Column 3 of Table 5) are given by (2) and (3), respectively:

$$\frac{\partial health}{\partial gdpc} = 0.1439 - 2 \times 1.5956 \times gdpc \tag{2}$$

$$\frac{\partial health}{\partial gdpc} = -0.1543 + 2 \times 1.0938 \times gdpc \tag{3}$$

The mean of *gdpc* for the sample is 8.32 (see Table 2). Thus, our results imply that all else equal, a 1% in GDP per capita in the average country will reduce *infmort1* by about 26.4% in the short-run and by about 69.4% in the long-run.[19] A similar increase in GDP per capita is expected to raise *lifeexpf* by 18% and 55% in the short-run and long-run, respectively. To further elucidate the discussion, we compute the average of GDP per capita, $\overline{gdpc}$, over the sample period, 1994-2014, for each country and we evaluate the estimated value of $\partial health/\partial gdpc$ at reasonable values of $\overline{gdpc}$. Specifically, we evaluate $\widehat{\alpha} + 2\widehat{\beta} \times \overline{gdpc}$ at the $10^{th}, 25^{th}, 50^{th}, 75^{th}$ and the 90^{th} percentile of $\overline{gdpc}$ for the *infmort1* and *lifeexpf* regressions. The $10^{th}, 25^{th}, 50^{th}, 75^{th}$ and the 90^{th} percentile correspond to the value of $\overline{gdpc}$ for Sierra Leone, Haiti, Bolivia, Ecuador and Romania, respectively.

[19] This follows from the fact that the short-run effect of a Δ change in *gdpc* on *health* is given by $\partial health/\partial gdpc = (\widehat{\alpha} + 2\widehat{\beta} \times gdpc) \times \Delta$ and the long-run effect is $(\widehat{\alpha} + 2\widehat{\beta} \times gdpc) \times \Delta/(1 - \widehat{\rho})$, where $\widehat{\rho}$ is the estimated coefficient of $health_{it-1}$. Here, $\Delta = 1$, $gdpc = 8.32$, $\widehat{\alpha} = 0.1439$, $\widehat{\beta} = -1.5956$ and $\widehat{\rho} = 0.6835$.

The results reported in Table 7 show that $\widehat{\alpha} + 2\widehat{\beta} \times \overline{gdpc}$ is negative for *infmort1* and positive for *lifeexpf*. In addition the magnitude of $\widehat{\alpha} + 2\widehat{\beta} \times \overline{gdpc}$ increases with $\overline{gdpc}$. For example, a 1% increase in GDP per capita reduces *infmort1* by about 24.12% in the 25^{th} percentile country and by 30% in the 90^{th} percentile country. A similar increase in $\overline{gdpc}$ raises *lifeexpf* by about 16.5% in the 25^{th} percentile country and by 20.6% in the 90^{th} percentile country. These results imply that income per capita significantly reduces mortality rates and increases life expectancy. However, the effect is non-linear: the positive effect is stronger at higher levels of income, pointing to a divergence in health outcomes. This result contrasts with Clark (2011),who also concludes that the effect of GDP per capita is nonlinear and it improves health outcomes, however, the marginal effect declines with income for life expectancy (convergence) but increases with infant mortality rates (divergence).

Question 4. Does the effect of GDP per capita on health outcomes for countries in SSA differ significantly from the effect for non-SSA countries?

Here we estimate (1) and the parameter of interest is the estimated coefficient of $SSA \times gdpc$, $\widehat{\delta}$. The regressions for mortality and life expectancy are reported in Tables 8 and 9, respectively. Note that $\widehat{\delta}$ is significant at the 1% level in all the regressions, suggesting that the effect of income per capita on health outcomes is significantly different for SSA countries. In addition, $\widehat{\delta}$ is positive for both the mortality and life expectancy regressions. This implies that the positive effect of GDP per capita on mortality rate is less for SSA countries, however, the effect on life expectancy is higher for SSA countries than for countries outside SSA. For example, all else equal, a 1% increase in income per capita reduces under 1-year mortality rate by about 0.1128 percentage points more in non-SSA countries than SSA countries (Column 2 of Table 8). In contrast a similar change raises female life expectancy by 0.0256 percentage points more in SSA countries than non-SSA countries (Column 2 of Table 9).

We now turn our attention to the other explanatory variables. Health care expenditure and school enrollment have a positive impact on health outcomes and HIV/AIDS prevalence has a negative effect. In addition, the estimated coefficient of lagged health, $\widehat{\rho}$, is positive and significant in all the regressions, suggesting that health outcomes are persistent. Indeed this underscores the importance of including lagged health as an explanatory variable in the regressions.

8 Conclusion

This paper has theoretically and empirically examined the link between GDP per capita, adult life expectancy and mortality rates for children. We find that: (i) Global factors (i.e.,

non country-specific factors) have a positive and significant impact on health outcomes, and this effect has increased over time; (ii) Countries in SSA have a higher mortality rate and lower life expectancy than countries outside SSA; (iii) An increase in GDP per capita improves health outcomes and the effect is stronger at higher levels of income; and (iv) The effect of GDP per capita on health outcomes is different for SSA countries.

There are four main policy implications to be drawn from the findings of the paper. First, as rising per capita GDP has been associated with better health outcomes for children as well as adults, policies that translate into increasing per capita income would be associated with inclusive health benefits. Second, the significance of the impact of other control variables (health expenditures, school enrollment and HIV prevalence) on health outcomes suggests that policy interventions in other areas are important for healthier populations irrespective of the level of income per capita. Third, the result that SSA is "different" suggest that the region would benefit from specific interventions beyond the ones that could be envisaged for other developing areas. The paper's findings suggest that controlling for all relevant factors, populations in the SSA region face location-related health challenges that translate into poorer health outcomes. The geographic location in malaria-prevalent or drought-prone areas makes populations vulnerable to malnutrition and poor health from tropical diseases and other diseases non-existent in non-SSA regions. Finally, there is a need for more investments in health related research, foreign aid in health (private and public) and efforts should be made to diffuse the results globally—in particular to countries in SSA since they have worse health outcomes. Indeed, several papers (Mishra and Newhouse, 2009; Ndikumana and Pickbourn, 2013) have found that aid in health has an impact on health outcomes in recipient countries.

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9 Appendix A

To derive the formula for life expectancy in the model economy, we define the following variables. Let x be the age category. Agents live for three periods in the model, thus x takes the value of 0, 1, and 2. Let N_x be the number of individuals in the population who survive to the beginning of age category x. Define $l_x = N_x/N_0$ be the proportion of individuals who survive to the beginning of age category x and $L_x = (l_x + l_{x+1})/2$ be the proportion of individuals surviving to the midpoint of age category x assuming survivorship is linear within age categories. Next, let $T_x = T_{x-1} - L_{x-1}$ with $T_0 = \Sigma_x L_x$ be the total number of age categories left to be lived by all individuals who survive to the beginning of age category x. Finally, the life expectancy, $e_x = T_x/l_x$, is defined as the mean number of age categories remaining until death for individuals surviving to the beginning of age category x.

Based on the model specification, we present calculations in the following table. We focus on a cohort of age-0 children with the size of their parents being normalized to 1.

	x	N_x	l_x	L_x	T_x	e_x
Child	0	$\frac{n^*}{\rho(q^*)}$	1	$\frac{1}{2}\left[1+\rho\left(q^*\right)\right]$	$\frac{1}{2}+\rho\left(q^*\right)(1+p)$	$\frac{1}{2}+\rho\left(q^*\right)(1+p)$
Adult	1	n^*	$\rho\left(q^*\right)$	$\frac{1}{2}\rho\left(q^*\right)(1+p)$	$\rho\left(q^*\right)\left(\frac{1}{2}+p\right)$	$\left(\frac{1}{2}+p\right)$
Old	2	pn^*	$\rho\left(q^*\right)p$	$\frac{1}{2}\rho\left(q^*\right)p$	$\frac{1}{2}\rho\left(q^*\right)p$	$\frac{1}{2}$
Σ				$\frac{1}{2}+\rho\left(q^*\right)(1+p)$		

Therefore the life expectancy at birth will be $e_0 = \frac{1}{2} + \rho\left(q^*\right)(1+p)$.

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Detailed calculations: The Lagrangian

$$\mathcal{L} = \ln c_t + \alpha \ln (n_t - n_0) + \gamma \ln q_t + p \ln \frac{Rs_t}{p} + \lambda \left(y - c_t - s_t - \frac{n_t}{\rho (q_t)} q_t \right)$$

First order conditions:

$$\begin{aligned} c_t &: \frac{1}{c_t} = \lambda \\ s_t &: \frac{p}{s_t} = \lambda \\ n_t &: \frac{\alpha}{n_t - n_0} = \lambda \frac{q_t}{\rho (q_t)} \\ q_t &: \frac{\gamma}{q_t} = \lambda n_t \frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t} \end{aligned}$$

which yield

$s_t : \frac{p}{s_t} = \frac{1}{c_t} \Longrightarrow$

$$\frac{s_t}{c_t} = p$$

$n_t : \alpha \frac{\rho(q_t)}{q_t} = \frac{n_t}{c_t} - \frac{n_0}{c_t} \Longrightarrow \frac{n_0}{c_t} = \frac{n_t}{c_t} - \alpha \frac{\rho(q_t)}{q_t} \Longrightarrow$

$$\frac{1}{c_t} = \frac{1}{n_0} \left[\frac{n_t}{c_t} - \alpha \frac{\rho (q_t)}{q_t} \right]$$

$q_t : \frac{\gamma}{q_t} = \frac{n_t}{c_t} \frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t} \Longrightarrow$

$$\frac{n_t}{c_t} = \frac{\gamma}{q_t} \left[\frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t} \right]^{-1}$$

Plug the above three into the adulthood budget constraint (divided by c_t)

$$1 + \frac{s_t}{c_t} + \frac{n_t}{c_t} \frac{q}{\rho (q_t)} = y \frac{1}{c_t}$$

$\Longrightarrow$

$$1 + p + \frac{n_t}{c_t} \frac{q_t}{\rho (q_t)} = \frac{y}{n_0} \left[\frac{n_t}{c_t} - \alpha \frac{\rho (q_t)}{q_t} \right]$$

$\Longrightarrow$

$$\begin{aligned} 1 + p + \frac{y}{n_0} \alpha \frac{\rho (q_t)}{q_t} &= \left[\frac{y}{n_0} - \frac{q_t}{\rho (q_t)} \right] \frac{n_t}{c_t} \\ 1 + p + \frac{y}{n_0} \alpha \frac{\rho (q_t)}{q_t} &= \left[\frac{y}{n_0} - \frac{q_t}{\rho (q_t)} \right] \frac{\gamma}{q_t} \left[\frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t} \right]^{-1} \end{aligned}$$

which determines the optimal level of child quality. Suppose that

$$\rho\left(q_t\right)=\rho_0\left(q_t\right)^{\mu},\ \rho_0,\mu>0$$

and ρ_0 is small enough such that $\rho\left(q_t\right)<1$. Then $\frac{\rho(q_t)}{q_t}=\rho_0\left(q_t\right)^{\mu-1}$, $\frac{q_t}{\rho(q_t)}=\frac{1}{\rho_0}\left(q_t\right)^{1-\mu}$, and $\frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t}=\frac{1}{\rho_0}\left(1-\mu\right)\left(q_t\right)^{-\mu}$. Thus we have

$$1+p+\frac{y}{n_0}\alpha\rho_0\left(q_t\right)^{\mu-1}=\left[\frac{y}{n_0}-\frac{1}{\rho_0}\left(q_t\right)^{1-\mu}\right]\frac{\gamma}{q_t}\frac{\rho_0}{1-\mu}\left(q_t\right)^{\mu}$$

$\Longrightarrow$

$$1+p+\frac{y}{n_0}\alpha\rho_0\left(q_t\right)^{\mu-1}=\frac{y}{n_0}\gamma\frac{\rho_0}{1-\mu}\left(q_t\right)^{\mu-1}-\frac{\gamma}{1-\mu}$$

$\Longrightarrow\times\left(q_t\right)^{1-\mu}$

$$\left(1+p+\frac{\gamma}{1-\mu}\right)\left(q_t\right)^{1-\mu}=\frac{y}{n_0}\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)$$

$\Longrightarrow$

$$\left(q_t\right)^{1-\mu}=y\frac{\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}$$

To have a positive q, we need to assume that $\frac{\gamma}{1-\mu}-\alpha>0$ and $1+p+\frac{\gamma}{1-\mu}>0$, or $\frac{\gamma}{1-\mu}-\alpha<0$ and $1+p+\frac{\gamma}{1-\mu}<0$. Later positive consumption and number of children eliminates the second case.

$$q^*=\left[y\frac{\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}\right]^{\frac{1}{1-\mu}}.$$

Once q^* is obtained, then

$$\begin{aligned}\frac{n_t}{c_t} &= \frac{\gamma}{q^*}\left[\frac{d\left(\frac{q_t}{\rho(q_t)}\right)}{dq_t}\right]^{-1}\\ &= \frac{\gamma}{q^*}\frac{\rho_0}{1-\mu}\left(q_t\right)^{\mu}\end{aligned}$$

$$\begin{aligned}\frac{1}{c^*} &= \frac{1}{n_0}\left[\frac{n_t}{c_t}-\alpha\frac{\rho\left(q_t\right)}{q_t}\right]\\ &= \frac{1}{n_0}\left[\frac{\gamma}{q^*}\frac{\rho_0}{1-\mu}\left(q_t\right)^{\mu}-\alpha\rho_0\left(q_t\right)^{\mu-1}\right]=\frac{\rho_0}{n_0}\left(q_t\right)^{\mu-1}\left(\frac{\gamma}{1-\mu}-\alpha\right)\end{aligned}$$

$\Longrightarrow$

$$\begin{aligned}c^* &= \frac{n_0}{\rho_0}\left(q_t\right)^{1-\mu}\frac{1}{\left(\frac{\gamma}{1-\mu}-\alpha\right)}\\ &= \frac{n_0}{\rho_0}y\frac{\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}\frac{1}{\left(\frac{\gamma}{1-\mu}-\alpha\right)}=\frac{1}{1+p+\frac{\gamma}{1-\mu}}y.\end{aligned}$$

Then

$$\begin{aligned} n^* &= c^* \frac{\gamma}{q^*} \frac{\rho_0}{1-\mu} (q^*)^{\mu} \\ &= \frac{1}{1+p+\frac{\gamma}{1-\mu}} y \frac{\gamma \rho_0}{1-\mu} (q^*)^{\mu-1} \\ &= \frac{1}{1+p+\frac{\gamma}{1-\mu}} y \frac{\gamma \rho_0}{1-\mu} \frac{1}{y} \frac{n_0 \left(1+p+\frac{\gamma}{1-\mu}\right)}{\rho_0 \left(\frac{\gamma}{1-\mu}-\alpha\right)} \\ &= \frac{\frac{\gamma}{1-\mu}}{\left(\frac{\gamma}{1-\mu}-\alpha\right)} n_0 = \frac{\gamma}{\gamma - \alpha (1-\mu)} n_0 \end{aligned}$$

and

$$s^* = pc^*$$

and

$$d^* = \frac{Rs}{p} = Rc^*$$

Finally, the fertility rate

$$\frac{n^*}{\rho (q^*)} = \frac{n^*}{\rho_0} (q^*)^{-\mu}$$

Child mortality rate

$$1 - \rho (q^*) = 1 - \rho_0 (q^*)^{\mu}.$$

and life expectancy: see below. Note that we need to make sure that the mortality rate is between 0 and 1:

$$\begin{aligned} \rho_0 (q^*)^{\mu} &< 1 \\ \rho_0 \left[y \frac{\rho_0 \left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0 \left(1+p+\frac{\gamma}{1-\mu}\right)} \right]^{\frac{\mu}{1-\mu}} &< 1 \end{aligned}$$

$\Longrightarrow$

$$\begin{aligned} (\rho_0)^{\frac{1}{1-\mu}} \left[y \frac{\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0 \left(1+p+\frac{\gamma}{1-\mu}\right)} \right]^{\frac{\mu}{1-\mu}} &< 1 \\ \rho_0 \left[y \frac{\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0 \left(1+p+\frac{\gamma}{1-\mu}\right)} \right]^{\mu} &< 1 \end{aligned}$$

$\Longrightarrow$

$$\rho_0 < \left[\frac{n_0}{y} \frac{1+p+\frac{\gamma}{1-\mu}}{\frac{\gamma}{1-\mu}-\alpha} \right]^{\mu} = \left[\frac{n_0}{y} \frac{\gamma + (1+p)(1-\mu) +}{\gamma - \alpha (1-\mu)} \right]^{\mu} = \bar{\rho}$$

Now the effect of an increase in income. Totally differentiate the following with respect to y

$$(q_t)^{1-\mu} = y\frac{\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}$$

$\Longrightarrow$

$$(1-\mu)(q^*)^{-\mu}\frac{dq^*}{dy} = \frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{1+p+\frac{\gamma}{1-\mu}}$$

$\Longrightarrow$

$$\frac{dq^*}{dy} = \frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}(q^*)^{\mu} > 0.$$

And

$$\frac{d^2q^*}{dy^2} = \frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\mu(q^*)^{\mu-1}\frac{dq^*}{dy} > 0.$$

Then child mortality

$$\frac{d\left[1-\rho(q^*)\right]}{dy} = -\rho_0\frac{d(q^*)^{\mu}}{dy} = -\rho_0\mu(q^*)^{\mu-1}\frac{dq^*}{dy} < 0.$$

And

$$\begin{aligned}
\frac{d^2\left[1-\rho(q^*)\right]}{dy^2} &= -\rho_0\mu(\mu-1)(q^*)^{\mu-2}\frac{dq^*}{dy}\frac{dq^*}{dy} - \rho_0\mu(q^*)^{\mu-1}\frac{d^2q^*}{dy^2} \\
&= \rho_0\frac{dq^*}{dy}\mu\left[(1-\mu)(q^*)^{2\mu-2}\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma} - (q^*)^{2\mu-2}\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\mu\right] \\
&= \rho_0\frac{dq^*}{dy}\mu\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}(q^*)^{2\mu-2}(1-2\mu) < 0 \text{ when } \mu > \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\frac{d^2\left[1-\rho(q^*)\right]}{dy^2} &= \rho_0\frac{dq^*}{dy}\mu\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}(q^*)^{2\mu-2}(1-2\mu) \\
&= \rho_0\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}(q^*)^{\mu}\mu\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\left[\frac{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}{y\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}\right]^2(1-2\mu) \\
&= \rho_0(q^*)^{\mu}\mu\frac{1}{(1-\mu)^2}\frac{1}{y^2}(1-2\mu) \\
\frac{d^2\left[1-\rho(q^*)\right]}{dy^2} &= \frac{\rho_0}{y^2}(q^*)^{\mu}\frac{(1-2\mu)\mu}{(1-\mu)^2}
\end{aligned}$$

To obtain life expectancy at birth in this economy, we need some work. First we define some variables as following:

x : age category, $x = 0, 1, 2, ...k$. Here there are three periods, corresponding to age 0, 1, and 2.

N_x : the number of individuals in the study population who survive to the beginning of age category x.

$l_x = N_x/N_0$: the proportion of individuals who survive to the beginning of age category x.

$L_x = (l_x + l_{x+1})/2$: the proportion of individuals surviving to the midpoint of age category x assuming survivorship is linear within age categories.

$T_x = T_{x-1} - L_{x-1}$, and $T_0 = \Sigma_x L_x$. Measures the total number of age categories left to be lived by all individuals who survive to the beginning of age category x.

$e_x = T_x/l_x$, $e_{k-1} = 0.5$. Life expectancy, which is the mean number of age categories remaining until death for individuals surviving to the beginning of age category x.

Based on the model information, we create the following table. We focus on a cohort of age-0 children normalizing the size of their parents to be 1.

	x	N_x	l_x	L_x	T_x	e_x
Child	0	$\frac{n^*}{\rho(q^*)}$	1	$\frac{1}{2}[1+\rho(q^*)]$	$\frac{1}{2}+\rho(q^*)(1+p)$	$\frac{1}{2}+\rho(q^*)(1+p)$
Adult	1	n^*	$\rho(q^*)$	$\frac{1}{2}\rho(q^*)(1+p)$	$\rho(q^*)\left(\frac{1}{2}+p\right)$	$\left(\frac{1}{2}+p\right)$
Old	2	pn^*	$\rho(q^*)p$	$\frac{1}{2}\rho(q^*)p$	$\frac{1}{2}\rho(q^*)p$	$\frac{1}{2}$
	3	0	0	0	0	
Σ				$\frac{1}{2}+\rho(q^*)(1+p)$		

Therefore the life expectancy at birth will be

$$e_0 = \frac{1}{2} + \rho(q^*)(1+p)$$

Then

$$\begin{aligned}
\frac{de_0}{dy} &= (1+p)\rho'(q^*)\frac{dq^*}{dy} \\
\frac{de_0}{dy} &= (1+p)\rho_0\mu(q^*)^{\mu-1}\frac{dq^*}{dy} > 0.
\end{aligned}$$

Second order derivative.

$$\begin{aligned}
\frac{1}{(1+p)\rho_0\mu}\frac{d^2e_0}{dy^2} &= (\mu-1)(q^*)^{\mu-2}\frac{dq^*}{dy}\frac{dq^*}{dy} + (q^*)^{\mu-1}\frac{d^2q^*}{dy^2} \\
\frac{1}{(1+p)\rho_0\mu}\frac{d^2e_0}{dy^2} &= \frac{dq^*}{dy}\left[\begin{array}{c}(\mu-1)(q^*)^{2\mu-2}\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma} \\ +(q^*)^{2\mu-2}\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\mu\end{array}\right] \\
&= \frac{dq^*}{dy}(q^*)^{2\mu-2}\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}[2\mu-1]
\end{aligned}$$

$\Longrightarrow$

$$\begin{aligned}
\frac{d^2 e_0}{dy^2} &= \frac{dq^*}{dy}\left(q^*\right)^{2\mu-2}\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\left[2\mu-1\right](1+p)\rho_0\mu \\
&= \frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\left(q^*\right)^{\mu}\left[\frac{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}{y\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}\right]^2\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\left[2\mu-1\right](1+p)\rho_0\mu \\
&= \frac{1}{(1+p)(1-\mu)+\gamma}\left(q^*\right)^{\mu}\frac{\left(1+p+\frac{\gamma}{1-\mu}\right)}{y}\frac{\left(1+p+\frac{\gamma}{1-\mu}\right)}{y}\frac{1}{(1+p)(1-\mu)+\gamma}\left[2\mu-1\right](1+p)\rho_0\mu \\
&= \frac{1}{y^2(1-\mu)^2}\left(q^*\right)^{\mu}\left[2\mu-1\right](1+p)\rho_0\mu \\
&= \frac{(1+p)\rho_0}{y^2}\left(q^*\right)^{\mu}\frac{(2\mu-1)\mu}{(1-\mu)^2} > 0 \text{ when } \mu > \frac{1}{2}
\end{aligned}$$

To show the effect of income on fertility rate $n^*/\rho\left(q^*\right)$:

$$\begin{aligned}
\frac{n^*}{\rho\left(q^*\right)} &= \frac{n^*}{\rho_0\left(q^*\right)^{\mu}} = \frac{n^*}{\rho_0 q^*}\left(q^*\right)^{1-\mu} = \frac{\frac{\gamma}{\gamma-\alpha(1-\mu)}n_0}{\rho_0 q^*}y\frac{\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)} \\
&= \frac{\frac{\gamma}{\gamma-\alpha(1-\mu)}}{q^*}y\frac{\left(\frac{\gamma}{1-\mu}-\alpha\right)}{\left(1+p+\frac{\gamma}{1-\mu}\right)} = \frac{\gamma}{\gamma+(1+p)(1-\mu)}\frac{y}{q^*}
\end{aligned}$$

$$\frac{d\frac{n^*}{\rho(q^*)}}{dy} = \frac{\gamma}{\gamma+(1+p)(1-\mu)}\frac{d\frac{y}{q^*}}{dy}$$

$$\begin{aligned}
\frac{d\frac{y}{q^*}}{dy} &= \frac{1}{\left(q^*\right)^2}\left[q^* - y\frac{dq^*}{dy}\right] \\
&= \frac{1}{q^*}\left[1 - y\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\left(q^*\right)^{\mu-1}\right] \\
&= \frac{1}{q^*}\left[1 - y\frac{\rho_0}{n_0}\frac{\frac{\gamma}{1-\mu}-\alpha}{(1+p)(1-\mu)+\gamma}\frac{1}{y}\frac{n_0\left(1+p+\frac{\gamma}{1-\mu}\right)}{\rho_0\left(\frac{\gamma}{1-\mu}-\alpha\right)}\right] \\
&= \frac{1}{q^*}\left[1 - \frac{1}{1-\mu}\right] = -\frac{\mu}{1-\mu}\frac{1}{q^*}
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{d\frac{n^*}{\rho(q^*)}}{dy} &= \frac{\gamma}{\gamma+(1+p)(1-\mu)}\frac{d\frac{y}{q^*}}{dy} \\
&= -\frac{\gamma}{\gamma+(1+p)(1-\mu)}\frac{\mu}{1-\mu}\frac{1}{q^*} < 0.
\end{aligned}$$

10 Appendix B

The data and the variables. List of countries

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Table 1

Percentage change in GDP per capita, mortality rate and life expectancy by regions and income, 1994 - 2014

Region/Income	GDP	Mortality rate		Life expectancy	
	per capita	$<$ 1yr	$<$ 5 yr	Female	Male
Region					
East Asia & Pacific	60	-39	-34	7	6
Europe & Central Asia	52	-57	-54	6	8
Latin America & Caribbean	29	-43	-41	5	7
Middle East & North Africa	109	-61	-58	8	7
Sub-Saharan Africa	73	-48	-41	18	18
Income					
Low income	65	-53	–46	22	23
Low middle income	32	-44	–39	7	7
Middle income	52	-40	-37	5	6

Source: Author's calculations and 2015 World Development Indicators.

GDP per capita is 2011 US dollars PPP.

Mortality rate is the number of deaths 1000 live births.

Table 2

Summary Statistics

	Full sample		SSA sample		Non-SSA sample	
Variables	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.
ln (under 1yr mortality rate)	3.60	0.72	4.20	0.49	3.27	0.61
ln (under 5yr mortality rate)	3.87	0.84	4.63	0.58	3.45	0.65
ln (life expectancy, female)	4.20	0.16	4.03	0.14	4.29	0.07
ln (life expectancy, male)	4.13	0.14	3.99	0.13	4.21	0.07
ln (GDP per capita)	8.32	0.93	7.65	0.91	8.70	0.71
ln (school enrollment, female)	4.27	1.02	4.12	1.11	4.36	0.95
ln (school enrollment, male)	4.34	1.01	4.25	1.12	4.40	0.95
ln (health expenditure)	5.08	1.04	4.40	0.97	5.45	0.87
ln (HIV prevalence, female)	0.59	0.69	1.09	0.78	0.20	0.20
ln (HIV prevalence, male)	0.45	0.50	0.76	0.60	0.21	0.18

Table 3

Direct effect GDP per capita on under 1-year mortality rate

VARIABLES	(1)	(2)	(3)
ln (Mortality rate), lagged	0.7048***	0.6689***	0.6835***
	(0.000)	(0.000)	(0.000)
SSA	0.5663***	0.5494***	0.4588***
	(0.000)	(0.000)	(0.000)
ln (GDP per capita), *gdpc*	0.1289***	0.1668***	0.1439***
	(0.000)	(0.001)	(0.000)
$gdpc \times gdpc$	-1.5245***	-1.9551***	-1.5956***
	(0.000)	(0.000)	(0.000)
ln (Education, female)		-0.0011**	0.0006**
		(0.032)	(0.043)
ln (Health expenditure)		-0.0250***	-0.0247***
		(0.000)	(0.000)
ln (HIV prevalence, female)			0.0515***
			(0.000)
1997-1999	-0.0225***	-0.0221***	-0.0280***
	(0.000)	(0.000)	(0.000)
2000-2002	-0.0626***	-0.0589***	-0.0738***
	(0.000)	(0.000)	(0.000)
2003-2005	-0.1071***	-0.0938***	-0.1157***
	(0.000)	(0.000)	(0.000)
2006-2008	-0.1436***	-0.1231***	-0.1441***
	(0.000)	(0.000)	(0.000)
2009-2011	-0.1737***	-0.1507***	-0.1674***
	(0.000)	(0.000)	(0.000)
2012-2014	-0.1848***	-0.1628***	-0.1737***
	(0.000)	(0.000)	(0.000)
Constant	3.0445***	3.9040***	3.2669***
	(0.000)	(0.000)	(0.000)
Hansen J test (p-value)[1]	0.0025	0.2021	0.1367
Serial correlation test (p-value)[2]	0.4142	0.6980	0.2190
No. of observations	808	808	624
No. of countries, n	128	128	98
No of Instruments, i	86	81	82
Instrument ratio, $r = n/i$	1.49	1.58	1.19

P-values in parentheses *** p<0.01, ** p<0.05, * p<0.1

[1] The null hypothesis is that the instruments are not correlated with the residuals.

[2] The null hypothesis is that the errors in the first difference regression exhibits no second order serial correlations.

Table 4

Direct effect of GDP per cpaita on under 5-years Mortality rate

VARIABLES	(1)	(2)	(3)
ln (Mortality rate), lagged	0.6520***	0.6259***	0.6193***
	(0.000)	(0.000)	(0.000)
SSA	0.7107***	0.6840***	0.5053***
	(0.000)	(0.000)	(0.000)
ln (GDP per capita), *gdpc*	0.3561***	0.4443***	0.5226***
	(0.000)	(0.000)	(0.000)
$gdpc \times gdpc$	-3.2270***	-4.1415***	-4.4521***
	(0.000)	(0.000)	(0.000)
ln (Education, female)		-0.0032***	-0.0024***
		(0.000)	(0.000)
ln (Health expenditure)		-0.0207***	-0.0382***
		(0.000)	(0.000)
ln (HIV prevalence, female)			0.1169***
			(0.000)
1997-1999	-0.0273***	-0.0239***	-0.0330***
	(0.000)	(0.000)	(0.000)
2000-2002	-0.0802***	-0.0736***	-0.0907***
	(0.000)	(0.000)	(0.000)
2003-2005	-0.1380***	-0.1227***	-0.1444***
	(0.000)	(0.000)	(0.000)
2006-2008	-0.1902***	-0.1667***	-0.1843***
	(0.000)	(0.000)	(0.000)
2009-2011	-0.2355***	-0.2085***	-0.2218***
	(0.000)	(0.000)	(0.000)
2012-2014	-0.2613***	-0.2339***	-0.2459***
	(0.000)	(0.000)	(0.000)
Constant	5.0064***	6.4339***	6.5641***
	(0.000)	(0.000)	(0.000)
Hansen J test (p-value)	0.0053	0.1566	0.0731
Serial correlation test (p-value)	0.7618	0.8401	0.5124
No. of observations	808	808	624
No. of countries, n	128	128	98
No of Instruments, i	86	81	82
Instrument ratio, $r = n/i$	1.49	1.58	1.19

Table 5
Direct effect of GDP per capita on adult female life expectancy

VARIABLES	(1)	(2)	(3)
ln(Life expectancy) lagged	0.7165***	0.6715***	0.6728***
	(0.000)	(0.000)	(0.000)
SSA	-0.1657***	-0.1698***	-0.0674***
	(0.000)	(0.000)	(0.000)
ln (GDP per capita), *gdpc*	-0.2095***	-0.1884***	-0.1543***
	(0.000)	(0.000)	(0.000)
$gdpc \times gdpc$	1.3194***	1.2270***	1.0938***
	(0.000)	(0.000)	(0.000)
ln (Education, female)		0.0029***	0.0024***
		(0.000)	(0.000)
ln (Health expenditure), *lhealth*		0.0306***	0.0361***
		(0.000)	(0.000)
$lhealth \times lhealth$		-0.0037***	-0.0029***
		(0.000)	(0.000)
ln (HIV prevalence, female)			-0.0492***
			(0.000)
1997-1999	0.0064***	0.0059***	0.0057***
	(0.000)	(0.000)	(0.000)
2000-2002	0.0131***	0.0126***	0.0077***
	(0.000)	(0.000)	(0.000)
2003-2005	0.0227***	0.0233***	0.0122***
	(0.000)	(0.000)	(0.000)
2006-2008	0.0364***	0.0375***	0.0190***
	(0.000)	(0.000)	(0.000)
2009-2011	0.0496***	0.0520***	0.0276***
	(0.000)	(0.000)	(0.000)
2012-2014	0.0500***	0.0559***	0.0280***
	(0.000)	(0.000)	(0.000)
Constant	0.1891***	0.3295***	0.2739***
	(0.000)	(0.000)	(0.005)
Hansen J test (p-value)	0.0052	0.0037	0.2060
Serial correlation test (p-value)	0.2415	0.1403	0.0746
No. of observations	785	785	622
No. of countries, n	127	127	98
No of Instruments, i	86	82	83
Instrument ratio, $r = n/i$	1.48	1.55	1.18

Table 6

Direct effect of GDP per capita on adult male life expectancy

VARIABLES	(1)	(2)	(3)
ln(Life expectancy) lagged	0.7262***	0.6801***	0.6787***
	(0.000)	(0.000)	(0.000)
SSA	-0.1583***	-0.1642***	-0.0707***
	(0.000)	(0.000)	(0.000)
ln (GDP per capita), *gdpc*	-0.1654***	-0.1432***	-0.0857***
	(0.000)	(0.000)	(0.000)
$gdpc \times gdpc$	0.9482***	0.8494***	0.5212***
	(0.000)	(0.000)	(0.000)
ln (Education, male)		0.0047***	0.0035***
		(0.000)	(0.000)
ln (Health expenditure), *lhealth*		0.0157***	0.0340***
		(0.000)	(0.000)
$lhealth \times lhealth$		-0.0023***	-0.0030***
		(0.000)	(0.000)
ln (HIV prevalence, male)			-0.0597***
			(0.000)
1997-1999	0.0101***	0.0084***	0.0088***
	(0.000)	(0.000)	(0.000)
2000-2002	0.0158***	0.0143***	0.0105***
	(0.000)	(0.000)	(0.000)
2003-2005	0.0243***	0.0246***	0.0152***
	(0.000)	(0.000)	(0.000)
2006-2008	0.0382***	0.0392***	0.0230***
	(0.000)	(0.000)	(0.000)
2009-2011	0.0515***	0.0536***	0.0316***
	(0.000)	(0.000)	(0.000)
2012-2014	0.0506***	0.0574***	0.0325***
	(0.000)	(0.000)	(0.000)
Constant	0.5423***	0.7222***	0.8730***
	(0.000)	(0.000)	(0.000)
Hansen J test (p-value)	0.0052	0.0037	0.1264
Serial correlation test (p-value)	0.2397	0.1387	0.0500
No. of observations	785	785	622
No. of countries, n	127	127	98
No of Instruments, i	86	82	83
Instrument ratio, $r = n/i$	1.48	1.55	1.18

Table 7

$\partial health/\partial gdpc = \hat{\alpha} + 2\hat{\beta} \times \overline{gdpc}$, evaluated at various values of $\overline{gdpc}$

Percentile of $\overline{gdpc}$	Value of $\overline{gdpc}$	Corresponding country	Under 1-year mortality $\hat{\alpha}$ = 0.1439; $\hat{\beta}$ =-1.5956	Life expectancy, female $\hat{\alpha}$ = -1.543; $\hat{\beta}$ =1.0938
10^{th}	7.062	Sierra Leone	-22.392***(0.000)	15.294***(0.000)
25^{th}	7.603	Haiti	-24.117***(0.000)	16.477***(0.000)
50^{th}	8.391	Bolivia	-26.633***(0.000)	18.202***(0.000)
75^{th}	9.117	Ecuador	-28.951***(0.000)	19.791***(0.000)
90^{th}	9.488	Romania	-30.134***(0.000)	20.602***(0.000)

Notes: $\overline{gdpc}$ is natural log of the average of GDP per capita from 1994-2014.

Table 8

The interaction effect of GDP per capita and SSA on mortality rate of children.

	Under 1 year		Under 5years	
	No HIV	Include HIV	No HIV	Include HIV
VARIABLES	(1)	(2)	(3)	(4)
ln (Mortality rate), lagged	0.6458***	0.6577***	0.5974***	0.5985***
	(0.000)	(0.000)	(0.000)	(0.000)
SSA	-0.6287***	-0.4859***	-0.8973***	-0.6901***
	(0.000)	(0.000)	(0.000)	(0.000)
$SSA \times gdpc$	0.1415***	0.1128***	0.1868***	0.1430***
	(0.000)	(0.000)	(0.000)	(0.000)
ln (GDP per capita), $gdpc$	0.5221***	0.4018***	0.9671***	0.8643***
	(0.000)	(0.000)	(0.000)	(0.000)
$gdpc \times gdpc$	-5.1439***	-4.0552***	-8.8876***	-7.6618***
	(0.000)	(0.000)	(0.000)	(0.000)
ln (Education, female)	-0.0014**	0.0002	-0.0037***	-0.0026***
	(0.018)	(0.556)	(0.000)	(0.000)
ln (Health expenditure)	-0.0273***	-0.0265***	-0.0263***	-0.0432***
	(0.000)	(0.000)	(0.000)	(0.000)
ln (HIV prevalence, female)		0.0572***		0.1172***
		(0.000)		(0.000)
1997-1999	-0.0214***	-0.0265***	-0.0217***	-0.0309***
	(0.000)	(0.000)	(0.000)	(0.000)
2000-2002	-0.0582***	-0.0720***	-0.0677***	-0.0860***
	(0.000)	(0.000)	(0.000)	(0.000)
2003-2005	-0.0958***	-0.1161***	-0.1180***	-0.1414***
	(0.000)	(0.000)	(0.000)	(0.000)
2006-2008	-0.1267***	-0.1440***	-0.1628***	-0.1805***
	(0.000)	(0.000)	(0.000)	(0.000)
2009-2011	-0.1564***	-0.1695***	-0.2069***	-0.2194***
	(0.000)	(0.000)	(0.000)	(0.000)
2012-2014	-0.1715***	-0.1792***	-0.2352***	-0.2449***
	(0.000)	(0.000)	(0.000)	(0.000)
Constant	7.8243***	6.4614***	12.3139***	10.6506***
	(0.000)	(0.000)	(0.000)	(0.000)
Hansen J test (p-value)	0.2355	0.1384	0.1737	0.0484
Serial correlation test (p-value)	0.7888	0.1984	0.7512	0.4451
No. of observations	808	624	808	624
Number of countries, n	128	98	128	98
No of Instruments, i	82	83	82	83
Instrument ratio, $r = n/i$	1.56	1.18	1.56	1.18

Table 9

The interaction effect of GDP per capita and SSA on adult life expectancy.

	Female		Male	
	No HIV	Include HIV	No HIV	Include HIV
VARIABLES	(1)	(2)	(3)	(4)
ln (Life expectancy), lagged	0.6739***	0.6818***	0.6796***	0.6858***
	(0.000)	(0.000)	(0.000)	(0.000)
SSA	-0.2992***	-0.2752***	-0.2322***	-0.2564***
	(0.000)	(0.000)	(0.000)	(0.000)
$SSA \times gdpc$	0.0156***	0.0256***	0.0083***	0.0227***
	(0.000)	(0.000)	(0.000)	(0.000)
ln (GDP per capita), $gdpc$	-0.1485***	-0.0625***	-0.1261***	-0.0104
	(0.000)	(0.000)	(0.000)	(0.516)
$gdpc \times gdpc$	0.8409***	0.2285*	0.6789***	-0.2011
	(0.000)	(0.076)	(0.000)	(0.114)
ln (Education)	0.0027***	0.0019***	0.0046***	0.0030***
	(0.000)	(0.000)	(0.000)	(0.000)
ln (Health expenditure), lhealth	0.0307***	0.0387***	0.0155***	0.0361***
	(0.000)	(0.000)	(0.000)	(0.000)
lhealth × lhealth	-0.0038***	-0.0033***	-0.0024***	-0.0032***
	(0.000)	(0.000)	(0.000)	(0.000)
ln (HIV prevalence)		-0.0514***		-0.0617***
		(0.000)		(0.000)
1997-1999	0.0063***	0.0063***	0.0085***	0.0090***
	(0.000)	(0.000)	(0.000)	(0.000)
2000-2002	0.0134***	0.0093***	0.0146***	0.0115***
	(0.000)	(0.000)	(0.000)	(0.000)
2003-2005	0.0246***	0.0144***	0.0251***	0.0165***
	(0.000)	(0.000)	(0.000)	(0.000)
2006-2008	0.0394***	0.0217***	0.0400***	0.0245***
	(0.000)	(0.000)	(0.000)	(0.000)
2009-2011	0.0539***	0.0300***	0.0543***	0.0326***
	(0.000)	(0.000)	(0.000)	(0.000)
2012-2014	0.0578***	0.0300***	0.0582***	0.0334***
	(0.000)	(0.000)	(0.000)	(0.000)
Constant	0.8083***	1.3013***	0.9447***	1.7436***
	(0.000)	(0.000)	(0.000)	(0.000)
Hansen J test (p-value)	0.0040	0.2144	0.0041	0.2192
Serial correlation test (p-value)	0.1467	0.0797	0.1497	0.0652
No. of observations	785	622	785	622
No. of countries, n	127	98	127	98
No of Instruments, i	83	84	83	84
Instrument ratio, $r = n/i$	1.53	1.17	1.53	1.17