When Formulas Fail: On the Variability of the Exponential Growth Bias

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Abstract

This paper analyzes whether individuals who have been well trained to apply exponential discounting and compound interest calculations in simple savings scenarios are also successful in dealing with exponential effects in more complicated debt scenarios. In an experimental study, involving 251 German undergraduate business students, we find strong evidence of an "amortization bias", a tendency to linearly estimate the remaining balance on a loan at various points of a debt payoff schedule. Interestingly, the same group of subjects shows almost no exponential growth bias for simple savings questions. We develop a measure for the exponential growth bias that naturally extends over different tasks and parameter settings and provide first within-subject evidence for the variability of the bias in different areas of application. Our findings indicate that teaching individuals a formula for calculating compound interest does not help to develop a general or intuitive grasp of exponential effects in more complicated settings. This discouraging insight adds complexity to the discussion about effective ways to support individuals in overcoming the bias.

JEL Classification: D14

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I. Introduction

Exponential growth bias (EGB), defined by Stango and Zinman (2009) as "the tendency to linearize exponential functions when calculating them intuitively", has been demonstrated in various domains (Keren 1983; Wagenaar and Sagaria 1975; Wagenaar and Timmers 1979).[1] Lately, EGB has been extensively analyzed in the context of household finance, including the underestimation of compound savings growth and annual interest rates on credit cards (Almenberg 2011; McKenzie and Liersch 2011; Stango and Zinman 2009). EGB has also been demonstrated within an experimental setting in the savings domain, where Eisenstein and Hoch (2007) find that 90% of respondents underestimate compound growth in savings questions. Levy and Tasoff (2014) also find "substantial" EGB in their incentivized experiment, noting that "subjects are largely unaware of their bias and undervalue assistance." The ramifications of EGB, within the context of household finance, are stated by Stango and Zinman (2009), who find that households with higher EGB tend to "borrow more, save less, and favor shorter maturities" compared to the less biased households.

Given the immense consequences of these decisions, one could come up with the suggestion to simply teach people about the effects of compounding interest in order to decrease the bias. However, it has never been tested whether understanding the basic concept of exponential growth in a savings scenario or memorizing the general compound interest formula would also remove judgment errors in other scenarios in which exponential effects are relevant. The goal of our research is to shrink this gap and to provide first within subject insights on the variability of exponential growth bias across different domains. To this end, we confront a special group of subjects, who have been extensively trained to deal with simple exponential effects in the savings domain, with some practical household finance questions from the debt domain.

[1] Because of this "tendency towards linearity", EGB is sometimes also referred to as "linear bias" in academic literature.

Recently, due to the numerous borrowing options provided by banks, many consumers have been placed in a position of deciding how much they can borrow and at which terms. This makes the basic understanding of loans and how they are amortized paramount. Consequently, the wrong heuristic/decision can potentially lead to unsustainable debt and higher costs of borrowing. According to Lusardi (2011), one in five Americans used one of these high-cost borrowing methods (payday loan, pawn shops, etc.) between 2005 and 2009.[2]

We are not the first to explicitly study the extent of EGB in the debt domain. Stango and Zinman (2009) find that consumers demonstrate a payment/interest bias in which people show a systematic tendency to underestimate the interest rate of a particular loan, given the principal amount, monthly payment and maturity. Soll (2013) finds that people underestimate the time it takes to eliminate a debt based on a known monthly payment. To further extend the body of evidence on the neglect of exponential effects in the debt domain we decided to use a different debt related task for our within subject analysis. We ask our subjects to estimate the remaining balance on a loan at various points of a debt payoff schedule and observe a systematic bias which we call “amortization bias”.

In the debt domain, the formal derivation of the remaining balance *B* after *n* (≤*N*) payments amounts to:

$$B = A \cdot \left[1 - \frac{(1+i)^n - 1}{(1+i)^N - 1}\right] \quad (3)$$

if the overall loan is to be paid back in *N* equal installments on *A*, the initial balance of a loan with an interest rate *i*. The equation shows that the calculation of the remaining balance is driven by simple exponential effects: the principal reduction is just a quotient of two

[2] It has also been shown that those possessing greater “debt literacy” - those who understand the basic concepts of debt - are more likely to avoid using these high-cost methods of borrowing and pay their credit cards in full (Lusardi and Tufano 2009).

compound interest terms.[3] However, the connection is not as obvious as in a simple savings scenario, where a present value PV is just multiplied by an exponential factor $(1+i)^t$ to obtain the final value *FV* of investing *t* periods at an interest rate of *i*:

$$FV = PV \cdot (1+i)^t \tag{4}$$

This makes the comparison of the two scenarios particularly interesting. From a technical perspective, the calculation in the amortization scenario (3) is not much more complicated than the calculation in the simple savings scenario (4). But while it can be expected that many educated individuals have learned formula (4) and are able to apply it in a simple savings scenarios, this is much less likely for the amortization task in the debt domain. Here, most individuals will have to rely on their intuition or a heuristic. One goal of our research is to explore whether understanding compound interest in a simple savings scenario provides individuals with a sufficient intuitive grasp of exponential effects to also deal well with slightly more complicated judgment tasks that are driven by exponential growth effects, too.

II. Background and Hypotheses

To tackle our research question, we choose an experimental design feature that distinguishes our work from previous research. We use a very specific subject pool. Our experimental subjects are fourth-semester bachelor students (sophomores), who were enrolled in the mandatory Corporate Finance class for the Bachelor of Science Business program at the University of Muenster in Germany. This group of students is well trained, as part of their formal education, in how to perform discounting and compound savings calculations. In fact, all students participating in this experiment have previously taken a mandatory course, Financial Mathematics, in their first semester that extensively teaches and tests these concepts. We can expect that most of these participants will be able to provide accurate

[3] For the full derivation of equations (1), (2), and (3), see Appendix 1.

answers when they are confronted with simple savings questions, like how an initial investment grows over some years (prospective scenario) or what one-time investment is needed to reach a savings goal after some years (retrospective scenario). In these cases, answers only rely on a correct application of the exponential formula $FV = PV * (1 + i)^t$ (prospective scenario) and $PV = FV/(1 + i)^t$ (retrospective scenario).

We are interested whether this extensive learning provides a general and intuitive grasp on exponential effects that extends to other scenarios or if it is simply a solid application of a previously learned formula. In order to answer this research question in a controlled environment, we confront our experimental subjects with two types of household finance estimation tasks: (a) estimations in a simple savings scenario, where the compound interest formula can be directly applied, and (b) estimations in a more complex debt amortization scenario, which is also driven by exponential effect, though the actual exponential amortization equation derived in equation (3) is less obvious. This study is the first, to the best of our knowledge, to compare bias strength within subject for two different types of household finance decisions that are both based on exponential effects. To structure our within-subject analysis, we provide the following two hypotheses:

H1 (No exponential growth bias in the savings domain): Due to their previous training, the participants in this experiment will provide accurate estimates for the simple savings questions. More explicitly, the majority of participants will not show an exponential growth bias in the savings domain.

H2 (Amortization bias in the debt domain): In the debt domain, these same participants will be systematically biased and provide answers that lean towards a linear estimate of the remaining balance on a loan at various points of their debt payoff schedule.

III. Measuring Bias Size

One of the major challenges for the type of research we want to do is that we have to compare bias strength across tasks and for different parameters. Therefore, we need a measure for the exponential growth bias that naturally extends over different tasks and can be calibrated in a meaningful way. Wagenaar and Timmers (1979) and Stango and Zinman (2009) have suggested measuring the strength of the exponential growth bias in the savings domain by inserting a parameter θ into the relation between present and final value to make it:

$$FV = \mathrm{PV} \cdot (1 + i)^{(1-\theta)t} \tag{5}$$

An unbiased answer is given for $\theta = 0$. Participants with a θ greater than zero show a typical exponential growth bias and provide a future value *FV* that underestimates the effects of compound interest for a given *PV*, *t*, and *i*.

This measure is able to distinguish between individuals who are unbiased and those who show an exponential growth bias (or a reverse bias). It can also rank subjects' answers by bias size for a given scenario of *i* and *t*. We also like the general approach of attaching the bias measure to the accumulation factor *f* in the equation

$$FV = \mathrm{PV} \cdot f_{i,t}(\theta) \tag{6}$$

and to measure the bias as $\theta = {f_{i,t}}^{-1}\left(\frac{FV}{PV}\right)$. Such an approach seems promising for an extension of the concept to other domains and tasks in which exponential components are the key drivers of the calculations (as we have seen in equation (3) for our amortization problem). If bias size is attached to the distortion of these exponential components, we have a simple and canonical way of relating bias size to each other across scenarios and tasks even though the absolute response scales can be very different.

Unfortunately, the very specific factor $f_{i,t}(\theta) = (1+i)^{(1-\theta)t}$, used in the previous literature, turns out to be problematic if we want to compare the bias size for different parameter combinations within task and even more so across different tasks.

The problem arises because for this specific $f_{i,t}(\theta)$ the typical underestimation of the final value of a long-term investment is not modeled as a (partial) neglect of the higher order compound interest components of the total interest but as a distortion of the perceived investment time. The naïve investor who completely neglects compound interest and believes that a \$100 endowment will grow to \$120 over five years at a 4% interest rate is assigned a θ of 0.07, because he behaves like an investor who fully appreciates compound interest but collapsed the relevant time to a period of $\left((1-.07)\cdot\ 5\right) = 4.65$ years. The same naïve investor would get assigned a θ of 0.42 when confronted with a 20 year investment at 10% interest rate, because his naïve estimate of the final value, \$300, is obtained for an investment of $(0.58\ \cdot 20) =\ 11.6$ years while perfectly appreciating all compound interest components.

A measure that assigns such different θ values to an individual who follows a consistent and canonical strategy (of completely ignoring compound interest components), doesn't seem to be very suitable for our purpose and for any research that considers the exponential growth bias to be a personal trait rather than a scenario dependent bias.[4]

Therefore, we extend the set of basic properties that have been claimed by Stango and Zinman (2009) and are fulfilled by their approach $f_{i,t}(\theta) = (1+i)^{(1-\theta)t}$ and propose that a convincing measure should furthermore have the following properties: [5]

[4] Levy and Tasoff (2014) point towards an even more obvious problem of the approach: an exponential growth biased individual (θ>0) would be predicted to also misjudge the return in a one-period setting, i.e. the one-year return of an investment with (annual) interest rate i would be estimated to be smaller than i.

[5] A different set of properties for a suitable growth bias measure is suggested by Levy and Tasoff (2014). They do not search for a measure that extends over different scenarios and domains, however, but analyze a framework, in which interest rates can vary over time.

(a) It should be calibrated not only for the perfect exponential decision maker (i.e. have the property ${f_{i,t}}^{-1}((1+i)^t) = 0$ but also for a completely naïve decision maker who fully ignores the compound interest components. An intuitive calibration would be to claim: ${f_{i,t}}^{-1}(1+t\cdot i) = 1$.

(b) It should be able to assign a θ-value to any meaningful estimate, i.e. ${f_{i,t}}^{-1}$ has to be defined on the complete set of meaningful answers, and it should be monotonic.

(c) It should have properties (a) and (b) not only for the standard savings scenario from equation (6), but also for other domains and tasks, in particular the debt amortization scenario we consider in this research.

In Appendix 2, we discuss various potential measures with respect to these properties and find a simple geometric mixture of the linear and the exponential return, i.e. an accumulation function $\check{f}_{i,t}(\theta) = (t\cdot i)^{(\theta)} \cdot ((1+i)^t - 1)^{(1-\theta)} + 1$ to be appropriate.

It has the desired properties:

$$\check{f}_{i,t}{}^{-1}((1+i)^t) = 0 \tag{7}$$

and
$$\check{f}_{i,t}{}^{-1}(1+t\cdot i) = 1 \tag{8}$$

and is able to assign a bias size θ to any answer *FV*>*PV* in the savings domain.[6]

It also behaves nicely in the amortization scenario. If we follow the general approach of attaching the bias measurement to the exponential components and generalize the amortization equation (3) to become

$$B = A \cdot \left[1 - \frac{\check{f}_{i,n}(\theta) - 1}{\check{f}_{i,N}(\theta) - 1}\right] \tag{9}$$

[6] We would not consider it a sensible answer if an investor estimated that a $10,000 endowment PV, invested at an interest rate of i=5% for 3 years, would "grow" to a final value FV of $9,900.

we can write:

$$B = A \cdot \breve{g}_{i,n,N}(\theta) \tag{10}$$

with

$$\breve{g}_{i,n,N}(\theta) = 1 - \frac{\check{f}_{i,n}(\theta) - 1}{\check{f}_{i,N}(\theta) - 1} \tag{11}$$

The derived function $\breve{g}_{i,n,N}(\theta)$ that is used to determine the bias size in the amortization scenario as: $\theta = \breve{g}_{i,n,N}{}^{-1}\left(\frac{B}{A}\right)$ has the same nice calibration properties as $\check{f}_{i,t}(\theta)$. It assigns a bias of 0 to a perfect exponential estimate, i.e.

$$\breve{g}_{i,n,N}{}^{-1}\left(1 - \frac{(1+i)^n - 1}{(1+i)^N - 1}\right) = 0 \tag{12}$$

and a bias size of 1 to a completely naïve debtor who assumes the remaining balance to decrease linearly in time:

$$\breve{g}_{i,n,N}{}^{-1}\left(1 - \frac{n}{N}\right) = 1 \tag{13}$$

The function $\breve{g}_{i,n,N}(\theta)$ is furthermore monotonic and can assign a bias size θ to any answer $B<A$ in the debt domain.[7]

IV. Experimental Setup

Participants

This experiment involved 251 undergraduate business students who were enrolled in a Corporate Finance class at the University of Muenster. The median participant age was 23 years old, ranging from 19-27. All students, with the exception of nine, were from Germany. A total of 128 males and 123 females participated. The experiment was conducted in a computer lab and was fully set up in English, although a German translation of the main

[7] And again we would not consider it a sensible answer if an investor estimated that a debt amount of $200,000 in a 20-year amortization scheme has "decreased" to an outstanding balance of $210,000 after 5 years.

questions was also provided in the experiment.[8] The overall study was not only aiming for an answer to the aforementioned research questions but also looked at possibilities to de-bias participants in the debt domain. Therefore, the study consisted of three stages. Upon completing various debt and savings questions in stage 1, participants were exposed to different task-specific tutorials. Afterwards, they were asked the same savings and debt questions again, both immediately after taking the tutorial (stage 2) and three weeks later (stage 3). In this paper we will only consider the results from stage 1. Findings from stages 2 and 3 regarding the effectiveness of the different tutorials are discussed in Foltice and Langer (2014).

Incentives

Each participant was given a base amount of €15 (~$19) for showing up for the first two stages, which lasted 90-120 minutes. Additional variable payouts of €20, €40, and €60 were given to three randomly chosen participants out of the 20-25 subjects in each session. The additional payouts were determined by the overall average accuracy (absolute error %) of all questions in stages 1 and 2, compared to the other chosen participants in the group. Subjects were informed upfront that their expected payment was monotonic in the accuracy of their estimates. Given the complexity of the mechanism we refrained from providing more details about the payment structure.

Procedure

Every participant received a total of sixteen questions, shown in figure 1, consisting of eight debt questions and eight savings questions (screenshots of the experiment are shown in supplement A). The savings questions were further divided into four prospective savings

[8] Since part of the Bachelors program in Muenster is taught in English, we can safely assume that the German participants had little trouble understanding the English instructions.

questions and four retrospective savings questions.[9] The order of the debt and savings questions were randomized, with half of the participants receiving savings questions first and the other half receiving the debt questions first.[10] The eight debt questions consisted of four mid-term loans, which were 10-year loans with an initial balance of $20,000, and four long-term loans, which were 30-year loans with an initial balance of $200,000. A two-by-two matrix was chosen for the mid-term and long-term loans (6% and 10% yearly interest rate/ 50% and 75% time remaining on the loan).

After completing the questions, each participant took a five-question basic financial literacy test (van Rooij, Lusardi and Alessie 2011) and a ten-question numeracy quiz (Lipkus, Samsa and Rimer 2001).[11]

Figure 1: Savings and Debt Question Vectors : Details of the eight savings and debt questions given to each participant in all three stages of the experiment.

Savings Questions (8)		
Prospective (4)	Yearly Interest Rate	Time (in Years)
Initial balance of $10,000	7% 12%	12 36
Retrospective (4)	Yearly Interest Rate	Time (in Years)
Savings goal of $100,000	7% 12%	12 36

Debt Questions (8)		
Mid Term (4)	Yearly Interest Rate	Time Remaining on Loan (%)
Initial 10-year loan of $20,000	6% 10%	75% 50%
Long Term (4)	Yearly Interest Rate	Time Remaining on Loan (%)
Initial 30-year loan of $200,000	6% 10%	75% 50%

Unlike previous experiments examining exponential growth bias in the savings domain, we allow the use of calculators during the experiment. This design decision is driven by the

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[9] We used a question structure in the savings domain that is consistent with Eisenstein and Hoch (2007). The prospective savings questions started with an initial savings amount of $10,000, while the retrospective savings questions sought to "have $100,000 in your savings account x years from today." A two-by-two matrix was chosen for the prospective and retrospective questions, consisting of 7% and 12% yearly interest lasting 12 and 36 years.

[10] In the savings domain, the prospective and retrospective questions were grouped together.

[11] The 240 participants analyzed in the savings domain correctly answered 91.7% of all questions in the five question financial literacy quiz and correctly answered 92.4% of all questions on the 10 question numeracy quiz.

observation that in 2012 about 85% of all adults own a cell phone and 91% own an electronic device (Pew Research Center 2011). Therefore, if someone is making a savings or debt estimate such as the ones asked in this experiment, there is a very high probability that there is an accessible calculator nearby.[12] This experimental design separates the conceptual difficulties decision makers have with understanding exponential growth from pure computational disabilities that could be overcome through the use of a calculator. In other words, if a participant happens to know the correct heuristic/equation for accurately answering a question but is prohibited from using a calculator in the experiment, we would run the risk of misspecifying the causes and real-world relevance of the bias.[13]

V. Results

Savings Domain

In the savings domain, four of the 251 participants were immediately eliminated from the data set for completing the eight savings questions in a median time of five seconds or less. We additionally eliminated all completely insensible answers, i.e. answers less (more) than the initial balance (savings goal) in the prospective (retrospective) questions. Seven additional subjects were completely eliminated from the savings data set due to more than three insensible answers. Table 1 examines the results on individual question level using all relevant answers from the 240 remaining participants in the dataset. Across all eight savings questions, 89.7% were answered correctly.[14] The median θ is 0 for each of the eight individual questions.

[12] This validity was confirmed at a presentation of this paper, when we witnessed numerous accessible calculators after allowing participants to use "whatever device in the room" to calculate their estimates.
[13] Non-programmable, scientific calculators were provided to each participant in the experimental lab.
[14] A "correct answer" is defined as answers within $1 greater or less than the answer generated by using the compound interest formula.

Table 1. Overall Group Level Descriptive Statistics for Savings Questions - sorted by question. Note: the Prospective question starts with an initial amount of $10,000 and the retrospective question asks how much money one needs today in order to achieve the savings goal of $100,000 in *x* years. Interest rate and years are listed below, respectively.

Results Summary

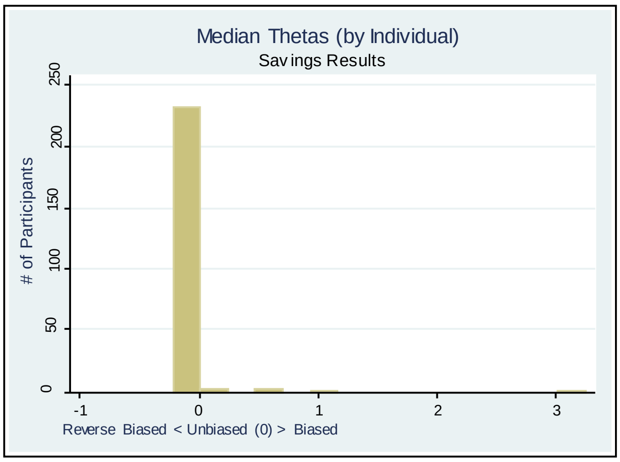
Question	N	Median θ	Min	Max	Correct Answers°	Mean	Median	Correct
Prospective; 0.07; 12	229	0.00	-3.8	2.60	92.2%	$ 22,807	$ 22,522	$ 22,522
Prospective; 0.07; 36	233	0.00	-0.47	3.37	92.7%	$114,235	$114,239	$114,239
Prospective; 0.12; 12	238	0.00	-0.93	4.20	92.4%	$ 38,352	$ 38,960	$ 38,960
Prospective; 0.12; 36	238	0.00	-0.2	3.13	88.2%	$562,293	$591,356	$591,356
All Prospective	**938**	**0.00**	**-3.80**	**4.20**	**91.4%**			
Retrospective; 0.07; 12	237	0.00	-7.12	27.36	86.1%	$ 43,884	$ 44,401	$ 44,401
Retrospective; 0.07; 36	232	0.00	-2.75	1.05	87.5%	$ 8,599	$ 8,754	$ 8,754
Retrospective; 0.12; 12	238	0.00	-5.05	16.05	87.4%	$ 26,686	$ 25,668	$ 25,668
Retrospective; 0.12; 36	230	0.00	-0.05	0.15	90.9%	$ 1,711	$ 1,691	$ 1,691
All Retrospective	**937**	**0.00**	**-7.12**	**27.36**	**87.9%**			
All Savings	**1875**	**0.00**	**-7.12**	**27.36**	**89.7%**			

° Answers within $1 +/- of the answer using the compound interest formula

The results shown in table 2 shed light on the exponential growth bias at the participant level, where the median bias "θ" is derived from all sensible answers given by a specific individual. 95% of all participants (228 out of 240) produced a median bias of exactly 0. Only nine participants exhibited a median θ greater than zero, while the remaining three subjects had a θ of less than zero. Only nine participants provided zero correct answers. Obviously, the median bias of all individual participants was again 0. Based on a binomial probability test for H1, we conclude that the majority of participants shows no exponential growth bias as the null hypothesis can be rejected at the 99% confidence level.

Table 2. Individual Descriptive Statistics and Histograms for Savings Questions - sorted by particpants median θ for all eight savings questions.

θ by Individual -Savings Domain	
Count	240
Median θ	0.00
Mean	0.02
St. Dev.	0.23
Minimum	-0.22
Maximum	3.25
*Median θ < 0; Reverse Biased	3
*Median θ = 0; Unbiased	228
*Median θ > 0; Biased	9



* Refers to the number of participants in each category

Presumably, these very precise estimates can be attributed to students knowing and computing the correct formula in the savings domain learned in their first semester Financial Mathematics course.[15] This is a testament to the intelligence of the students and provides encouraging evidence that extensively learning the actual equation can provide durable positive effects for subjects, at least in such simple savings scenarios.

Debt Domain

For the debt domain analysis, we again started with all 251 participants and immediately eliminated four individuals who completed all eight debt questions in a median time of five seconds or less. Again, estimates that did not fall within the sensible range of answers (i.e. answers less than zero or greater than or equal to the initial balance of the loan) were also eliminated from the analysis. In order to analyze the results at a participant level, all participants who provided less than three valid answers out of the eight debt questions were completely eliminated. As a consequence, we are able to only analyze about 68% (170 out of the 251) of all participants at the participant level. This unusually high elimination percentage was attributed to the fact that for a lot of eliminated answers the participant

[15] We estimate that this learning occurred roughly 18 months before the experiment.

obviously attempted to apply a variation of the compound growth equation to the debt questions, resulting in estimates that were well above the initial loan balance. The equation that we assume participants have used is written formally as $B = A(1+i)^N \cdot \left[\frac{N-n}{N}\right]$.[16] These participants apparently took what they knew - the compound savings equation - to determine the growth of the debt over time before they adjusted the resulting amount to the proportion of time left. In fact, based on the estimates given, we found that 70% of the eliminated participants relied on this erroneous logic to derive at least one estimate in the debt domain. This provides a first indication that knowing one compound savings equation doesn't necessarily equip an individual to make educated estimates across different exponential tasks. In some instances, it might even interfere with their ability to provide a reasonable intuitive estimate.

In table 3, we first examine the results for each of the eight debt questions and find statistically significant evidence of a strong amortization bias. In fact, the median bias for all debt questions was the naïve (linear) bias of 1. Biases appear to be slightly stronger for the long term debt questions; though both mid-term and long-term questions produce an overall median bias θ of 1.00 (both are significantly different from 0 at the 99% level.) Additionally, we see 91.3% of all answers in the long-term debt domain (64.3% in the mid-term questions) underestimate the actual answer.

[16] It has been casually mentioned that although German students are generally very strong in math and statistics, they often rely on what we refer to as "Formulaic Dependency" over common sense or intuition. Formulaic dependency is the tendency to solely rely on a formula to derive an answer, whether it makes sense intuitively or not.

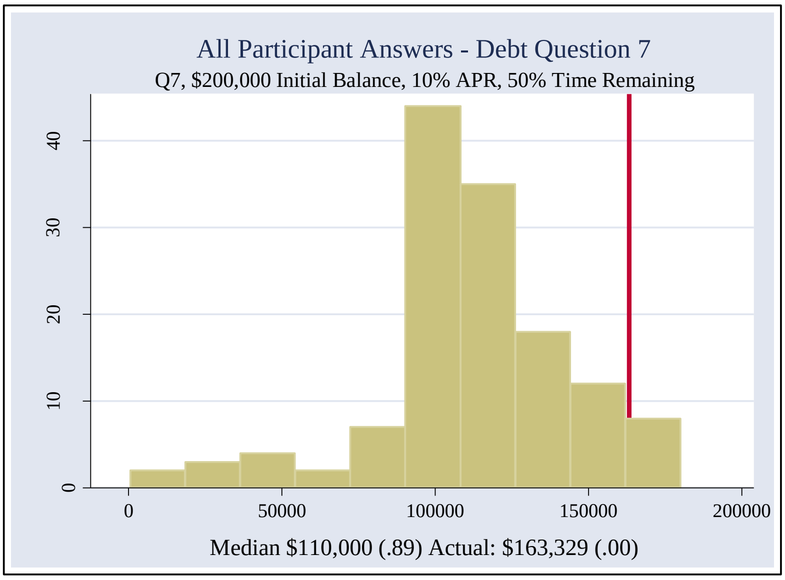
Table 3. Overall Debt Results Summary - sorted by Question. MT = mid-term, original loan amount of $20,000; LT = long-term, original loan amount of $200,000. 0.06 and 0.10 indicate the annual interest rate on the loan. 50% and 75% indicate the remaining time on the loan (as a percentage).

Question	N	Median θ	Min	Max	Mean	Median	Correct	Underestimated (%)
MT; 0.06; 50%	162	-0.01	-17.97	5.19	$ 11,782	$ 11,500	$ 11,485	49.4%
MT; 0.06; 75%	157	1.00***	-18.85	6.07	$ 14,026	$ 15,000	$ 16,061	69.4%
MT; 0.10; 50%	136	0.62***	-11.13	3.47	$ 11,602	$ 11,000	$ 12,439	66.9%
MT; 0.10; 75%	150	1.00***	-8.50	4.17	$ 13,562	$ 15,000	$ 16,688	72.7%
All Mid-Term	**605**	**1.00*** **	**-18.85**	**6.07**				**64.3%**
LT; 0.06; 50%	138	0.89***	-3.26	2.26	$107,014	$106,000	$142,098	85.5%
LT; 0.06; 75%	145	1.00***	-1.02	2.74	$126,825	$150,000	$177,440	88.3%
LT; 0.10; 50%	135	0.89***	-0.60	1.69	$112,076	$110,000	$163,329	94.1%
LT; 0.10; 75%	142	1.00***	-0.30	1.96	$134,341	$150,000	$188,210	97.1%
All Long-Term	**560**	**1.00*** **	**-3.26**	**2.74**				**91.3%**
All Debt	**1165**	**1.00*** **	**-18.85**	**6.07**				**77.2%**

Statistical Significance based on the Wilcoxon signed rank test - * 90%; ** 95%; *** 99%

Figure 2 provides an overall sample answer summary for one of the long-term debt questions, which asked for the remaining balance after 15 years on a 30 year, $200,000 loan with a 10% annual percentage rate (APR):[17]

Figure 2. Answers (in dollars) for all participants for debt question seven - by frequency. The red line indicates the correct answer of $163,329. θ is shown in parentheses.



At the participant level, we calculate the median θ for all relevant answers provided by each participant and find a similar conclusion to the question level results. Table 4 shows a statistically significant median individual θ of .96, only slightly better than the naïve bias

[17] All long debt answers in stage 1 are displayed in Appendix 3.

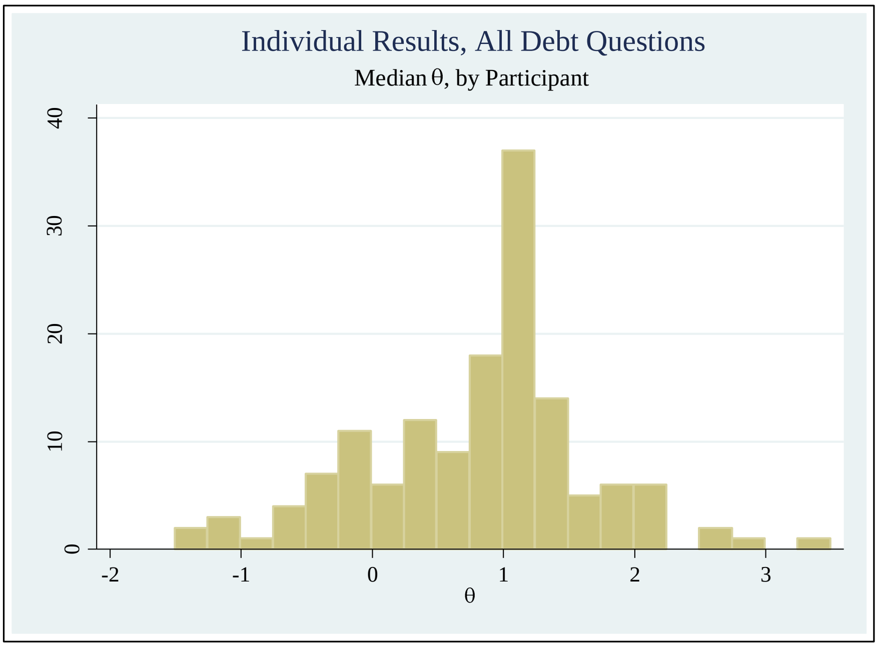
estimate. More than four fifths (81.2%) of all participants possessed a biased median θ greater than 0.

In summary, from both the question and participant level in the debt domain, we find support for H2 and can reject the null hypothesis that there is no amortization bias, with a 99% confidence level.

Table 4. Individual Results Summary - by participant. Individual results were recorded as the median θ for all relevant answers for each participant. Each particpant was required to have at least three relevant answers in each stage to be counted in the data set. A participant with a median θ>0 underestimated while those with a median θ<0 overestimated.

Group	N	Median θ	Min	Max	θ>0 (%)	θ<0 (%)
All Participants	**170**	**0.96*** **	**-1.51**	**3.49**	81.2%	18.8%

Statistical Significance of Wilcoxon signed rank test - * 90%; ** 95%; *** 99%



VI. Summary and Discussion

In this paper we analyze whether individuals who have been well-trained to apply exponential discounting and compound interest calculations in simple savings scenarios are also successful in dealing with exponential effects in more complicated debt scenarios. Thereby, we examine whether teaching individuals how to calculate compound interest helps them develop an intuitive grasp of exponential effects. The evidence that we generate in our experimental study with 251 business students is discouraging. Given their formal education, most students are able to provide exact answers for the simple exponential savings scenarios. At the same time they are strongly biased when facing more complicated amortization scenarios that also build on exponential effects but are less accessible to the application of a simple formula. In fact, in some cases the memorized formula seems to hinder the ability to make an intuitive estimate in the debt domain. We found an inordinate number of answers well above the initial balance as some students obviously attempted to use a variation of the previously learned compound savings equation in a domain where this formula fails to provide a sensible answer.

Our within-subject findings are novel and we believe that they are highly relevant. Exponential growth effects play a major role in many household finance decisions and the negative consequences of a systematic bias in such decision making can be tremendous (Stango and Zinman 2009). Our research contributes to the question of what could be an effective way to help people avoid the bias. The naïve hope that one could simply teach decision makers how to apply the exponential growth formula is undermined by our research insights. Even if individuals have been taught how to apply a formula, it does not provide them with an intuitive comprehension of exponential effects that helps in situations that are less accessible to formulas. It seems that task specific training would be necessary. Whether this could be better done by providing (simplified) formulas for the more complicated tasks or by feedback based learning is an important follow-up question. Foltice and Langer (2014) provide some first thoughts on such issues and also examine the stickiness of different types of training.

As a final note, we would like to reiterate that our study differs from previous experimental research through the fact that we allowed the use of pocket calculators. There is little doubt that this design choice is an important prerequisite for the impressive performance of our subjects in the savings domain. However, we think that the provision of pocket calculators increases the external validity of our experimental findings. If someone was to make a savings or debt estimate in real life, such as the ones elicited in this experiment, it is very likely that there is an accessible calculator nearby. From a pure research perspective, it would be nevertheless interesting to see how the results differed if no such technical support was provided. Such a design variation would allow a separation of the conceptual difficulties decision makers have with understanding exponential growth from pure computational disabilities that could be overcome through the use of a calculator.

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Appendix 1. Derivation of the equations (1), (2) and (3) from the main text (for payment per period P and remaining balance B).

For an overall loan amount A, a payment P per period, and an interest rate i, the remaining balance B_n after n periods can be derived recursively as:

$$B_n = B_{n-1} \cdot (1+i) - P.$$

Starting from $B_0 = A$

we then obtain $B_n = A \cdot (1+i)^n - P \cdot \sum_{t=0}^{n-1} (1+i)^t$

and $B_n = A \cdot (1+i)^n - P \cdot \frac{(1+i)^n - 1}{(1+i)-1}.$

This gives equation (2): $B_n = A \cdot (1+i)^n - \frac{P}{i} \cdot [(1+i)^n - 1].$

Equation (1) follows from the assumption that the loan is fully paid back after N periods, i.e.:

$$B_N = 0$$

This leads to: $A \cdot (1+i)^N - \frac{P}{i} \cdot [(1+i)^N - 1] = B_N = 0$

and solving for P gives: $P = A \cdot i \cdot \frac{(1+i)^N}{(1+i)^N - 1} = A \cdot i \cdot \frac{1}{1-(1+i)^{-N}}.$

To derive equation (3) we substitute (1) into (2) and obtain:

$$B_n = A \cdot (1+i)^n - \frac{P}{i} \cdot [(1+i)^n - 1] = A \cdot (1+i)^n - A \cdot \frac{[(1+i)^n - 1] \cdot (1+i)^N}{(1+i)^N - 1}$$

Some algebra gives:

$$B_n = A \cdot \frac{[(1+i)^N - 1] \cdot (1+i)^n}{(1+i)^N - 1} - A \cdot \frac{[(1+i)^n - 1] \cdot (1+i)^N}{(1+i)^N - 1}$$

$$= A \cdot \frac{(1+i)^N \cdot (1+i)^n - (1+i)^n}{(1+i)^N - 1} - A \cdot \frac{(1+i)^n \cdot (1+i)^N - (1+i)^N}{(1+i)^N - 1}$$

$$= A \cdot \frac{-(1+i)^n}{(1+i)^N - 1} - A \cdot \frac{-(1+i)^N}{(1+i)^N - 1} = A \cdot \frac{(1+i)^N - (1+i)^n}{(1+i)^N - 1}$$

$$= A \cdot \left[1 - \frac{(1+i)^n - 1}{(1+i)^N - 1}\right]$$

Appendix 2. Extended thoughts about an appropriate accumulation factor $\boldsymbol{f_{i,t}(\theta)}$

We have to consider two functions:

1. The function $f_{i,t}(\theta)$ that is the accumulation factor itself, at the same time it is the factor that solely determines the effects in the savings scenarios by the relation $FV = \mathrm{PV} \cdot f_{i,t}(\theta)$. We measure bias size by $\theta = {f_{i,t}}^{-1}\left(\frac{FV}{PV}\right)$ in the savings case.

2. The function $g_{i,n,N}(\theta)$ that determines the effects in the amortization scenario via the formula for the remaining balance: $B = A \cdot g_{i,n,N}(\theta)$. It holds: $g_{i,n,N}(\theta) = \frac{f_{i,N}(\theta) - f_{i,n}(\theta)}{f_{i,N}(\theta) - 1}$ if we replace all exponential terms by $f_{i,t}(\theta)$. We measure bias size by $\theta = {g_{i,t}}^{-1}\left(\frac{B}{A}\right)$ in this case.

The function $f_{i,t}(\theta)$ needs to have some "nice properties", not only to make it itself suitable for the savings scenario but also to make the derived $g_{i,n,N}(\theta)$ suitable for the amortization scenario. Both functions should be monotonic (in the relevant) range, they should be "well calibrated" and they should be able to assign θs to all reasonable answers for the given task.

We consider four different functional forms for the function $f_{i,t}(\theta)$:

A. The functional form $f_{i,t}(\theta) = (1+i)^{(1-\theta)t}$, previously used in the literature, can be considered inappropriate, because it is only calibrated for "perfect exponential" behavior but not for perfect linear behavior (perfect linear behavior results in different θ for different i and t.)

B. The functional form $\hat{f}_{i,t}(\theta) = \theta \cdot (1 + t \cdot i) + (1 - \theta) \cdot (1 + i)^t$ is more appropriate, because it is calibrated both on "perfect exponential" estimate (θ=0) and for perfect linear estimate (θ=1). It is also nice with respect to its "axiomatic foundation". It follows from a development of the exponential term as a sum: $(1 + i)^t = \sum_{j=0}^{t} \binom{t}{j} i^j = (1 + t \cdot i) + \sum_{j=2}^{t} \binom{t}{j} i^j$ and an underweighting of the higher order components: $\hat{f}_{i,t}(\theta) = (1 + t \cdot i) + (1 - \theta) \cdot \sum_{j=2}^{t} \binom{t}{j} i^j$.
The function $\hat{f}_{i,t}(\theta)$ has one problem, however. The derived $g_{i,n,N}(\theta)$ has the unattractive property that the function is not able to assign θs to all reasonable answers to the task. It can be shown that it holds: $\lim_{\theta\to\infty} g_{i,n,N}(\theta) = 1 - \frac{(1+i)^n - n\cdot i}{(1+i)^N - N\cdot i}$ <1. For example, if i=10%, N=20, and n=5, we have $\lim_{\theta\to\infty} g_{i,n,N}(\theta) = 1 - \frac{0.1105}{3.7275} = 0.97035$. Therefore, for A=$200,000, any B>$194,070 cannot be assigned a θ. The problematic range becomes larger for smaller i and for decreasing $\frac{N-n}{\mathrm{N}}$.

C. It can be shown that the problem arises because of the linearity of the function $\hat{f}_{i,t}(\theta)$ in θ. We need a function $f_{i,t}(\theta)$ that is convex in θ, to receive a reasonable range for the values of $g_{i,n,N}(\theta)$.

This leads to the function: $\dot{f}_{i,t}(\theta) = (1 + t \cdot i)^{\theta} \cdot (1 + i)^{(1-\theta)\cdot t}$. This function is interesting for various reasons:

i. It is just an extension of the previously used form (see A.)

ii. It has all the nice properties for $f_{i,t}(\theta)$ itself: it is calibrated for both the "perfect exponential" (θ=0) and the perfect linear estimate (θ=1).

iii. $f_{i,t}(\theta)$ assumes values in (0, ∞), which is no relevant restriction at the lower boundary, because answers FV of 0 or smaller would be considered "confused" anyway.

iv. It assumes reasonable values of $g_{i,n,N}(\theta)$ and maps to the complete range [-∞, 1]. Here, we have a restriction that answers of B ≥A could not be transformed into a θ, but this also seems to be a reasonable restriction (it can be argued again that B>A hints to a confused subject anyway).

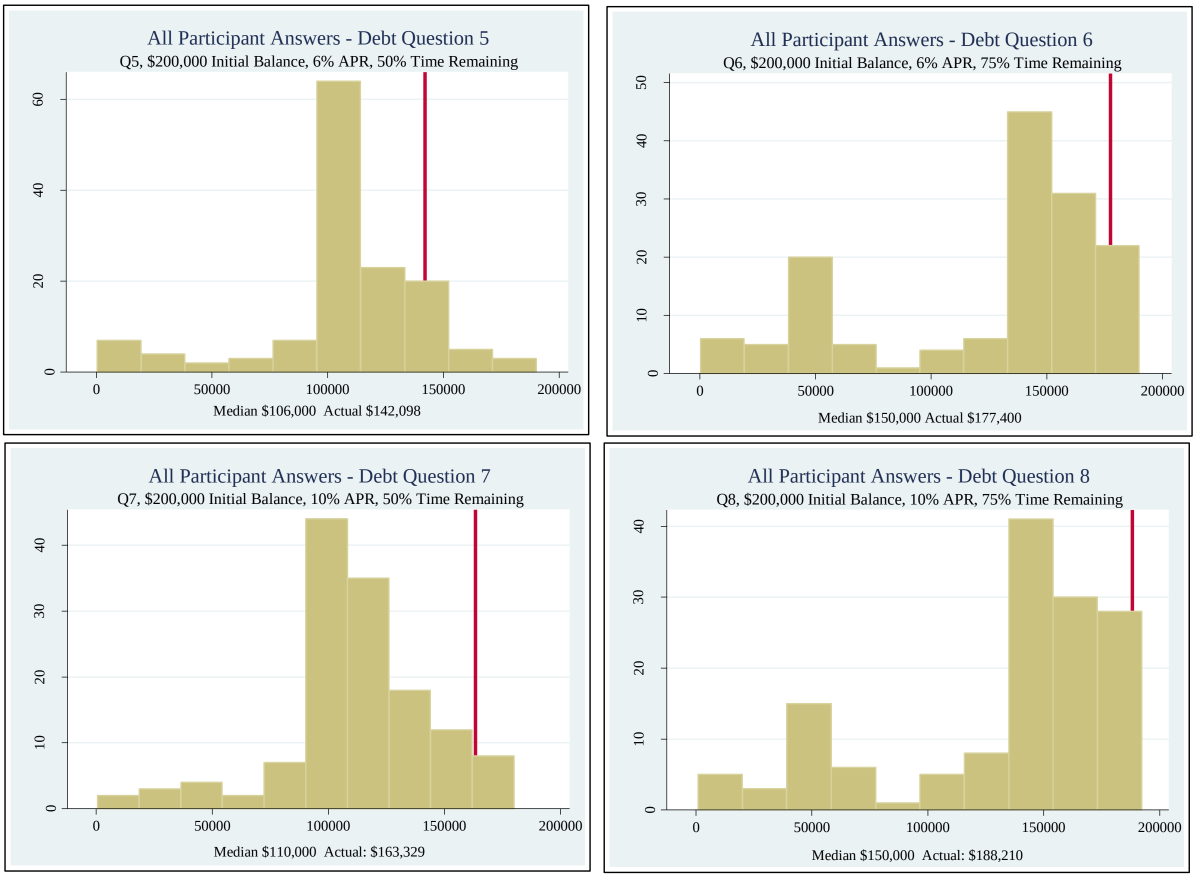
The only detriment of this measure is that has less of an intuitive component ("what part of the higher order interest is considered"). The function is also not continuous. It has a jump at $\frac{\ln(1+iN)}{\ln(1+iN)-N\cdot ln(1+i)}$. To give an example: for the parameters i=10%and N=20 no θ-values above 2.36 can be assumed.

An interesting insight is generated by writing the function $\dot{f}_{i,t}(\theta) = (1 + t \cdot i)^{(1-\theta)} \cdot (1 + i)^{t\cdot\theta}$ as $\dot{f}_{i,t}(\theta) = (1 + t \cdot i) \cdot e^{\ln\left(\frac{(1+i)^t}{1+ti}\right)\cdot\theta}$, because it shows that $\dot{f}_{i,t}(\theta)$ is simply an exponential function that is the only member of the functional family $f(\theta) = a \cdot e^{b\cdot\theta}$ that has the required properties:
$f(1) = (1 + i \cdot t)$ and $f(0) = (1 + i)^t$. This derivation leads to a further, even better suited functional type:

D. If we set up the functional family as $f(\theta) = a \cdot e^{b\cdot\theta} + 1$ and fit it to the two required data points, we have a more reasonable limit case. It holds that: $\lim_{\theta\to-\infty} a \cdot e^{b\cdot\theta} + 1 = 1$. Thereby we have an even more reasonable limit for sensible answers in the savings scenario. It is insensible to give an answer for FV that is not larger than PV. Any $f(\theta)$ ≤ 1 can be considered to be a confused answer [in the same way as any answer of B≥A can be considered a confused answer in the amortization case. If we fit the functional form to the two data points, we obtain:

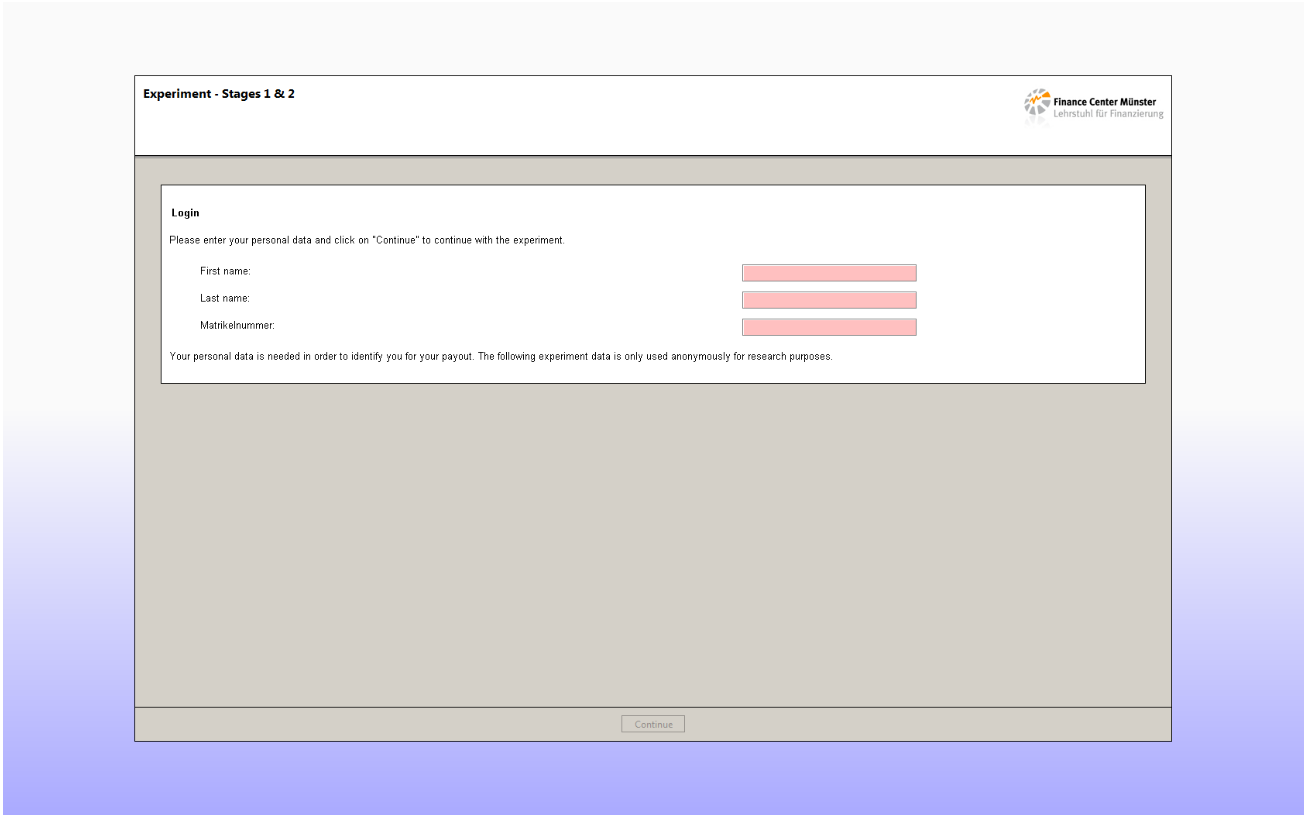
$$\check{f}_{i,t}(\theta) = (t \cdot i)^{\theta} \cdot ((1 + i)^t - 1)^{(1-\theta)} + 1$$

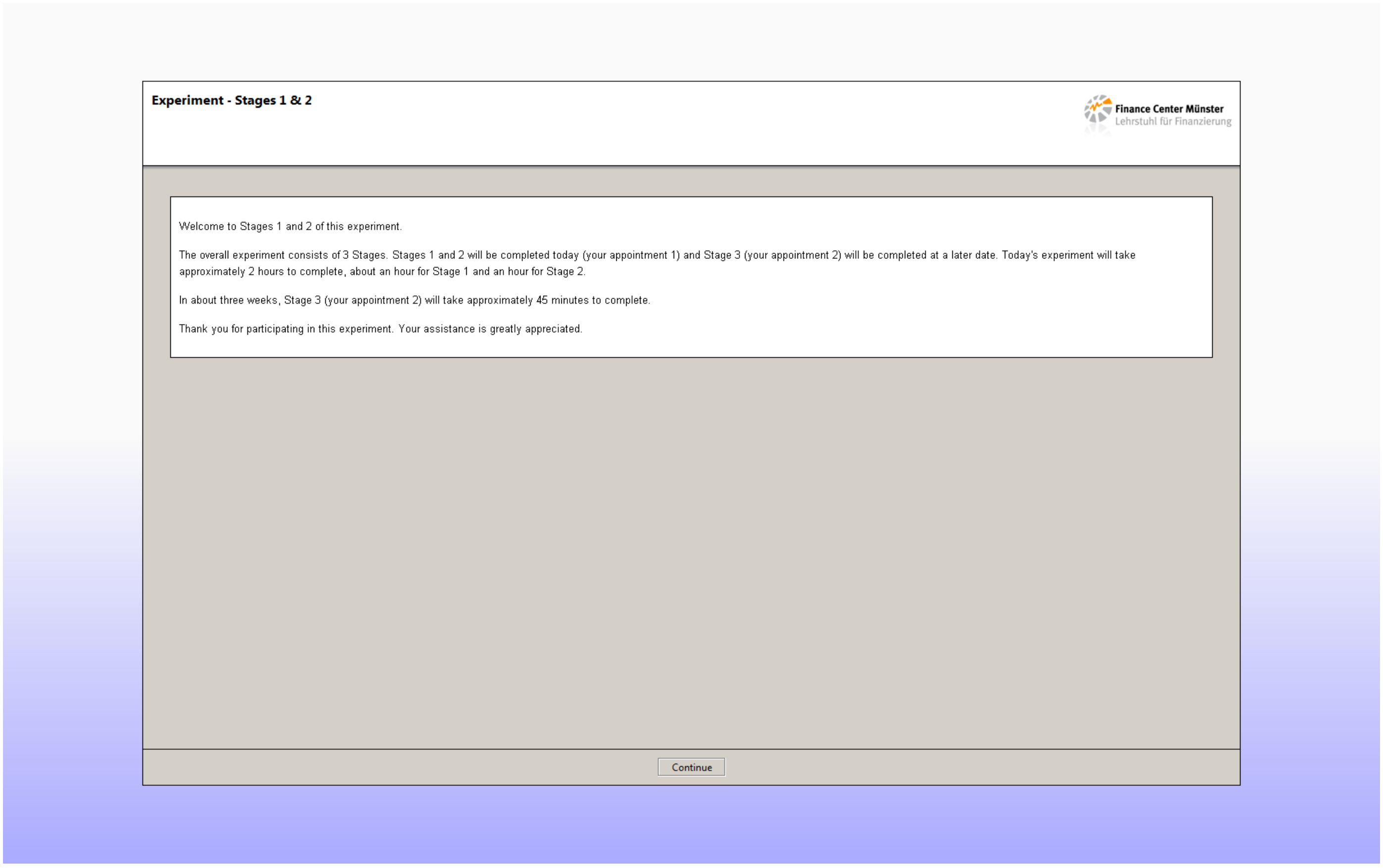
Appendix 3. Answers (in dollars) for all participants for all long term questions - by frequency. The red line indicates the correct answer. All questions start with an initial loan balance of $200,000



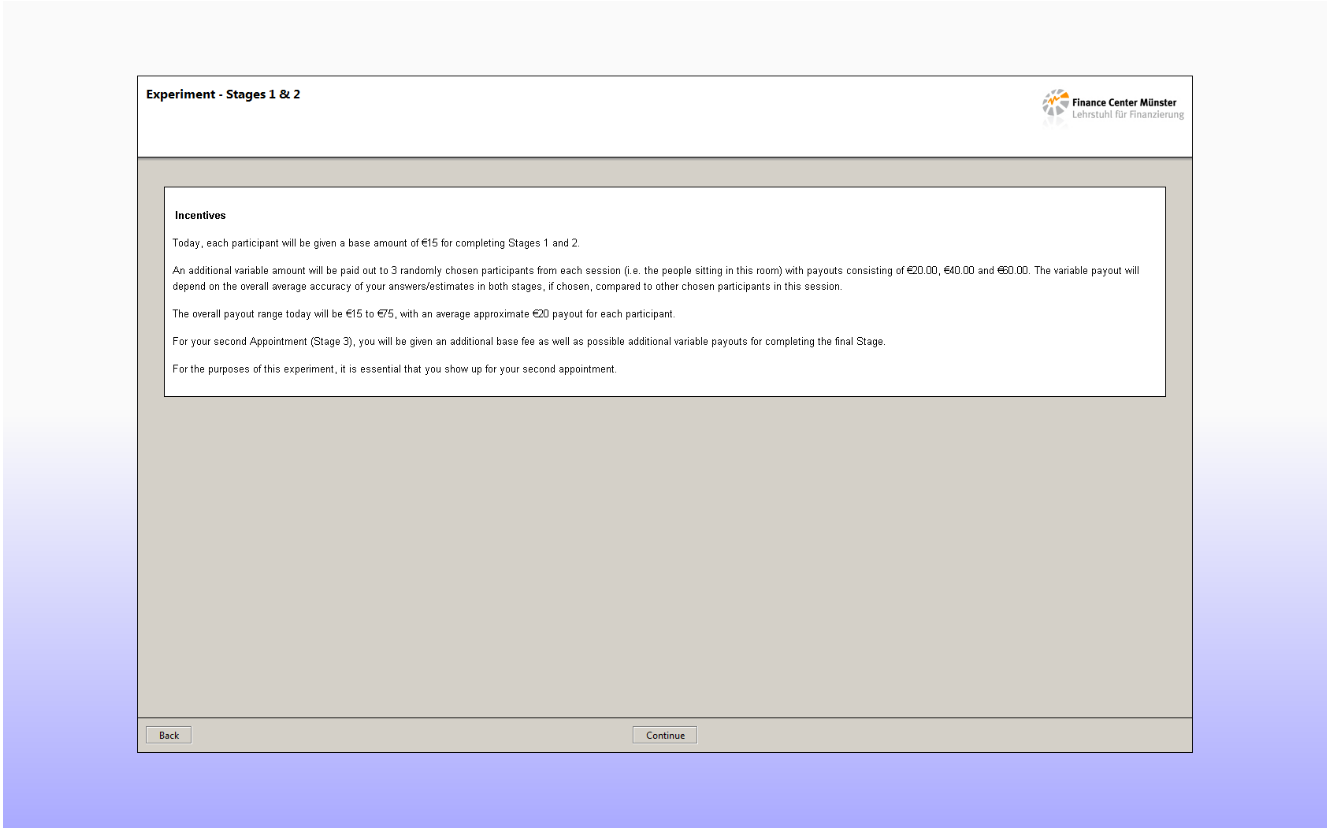
Supplemental A. Experiment Information

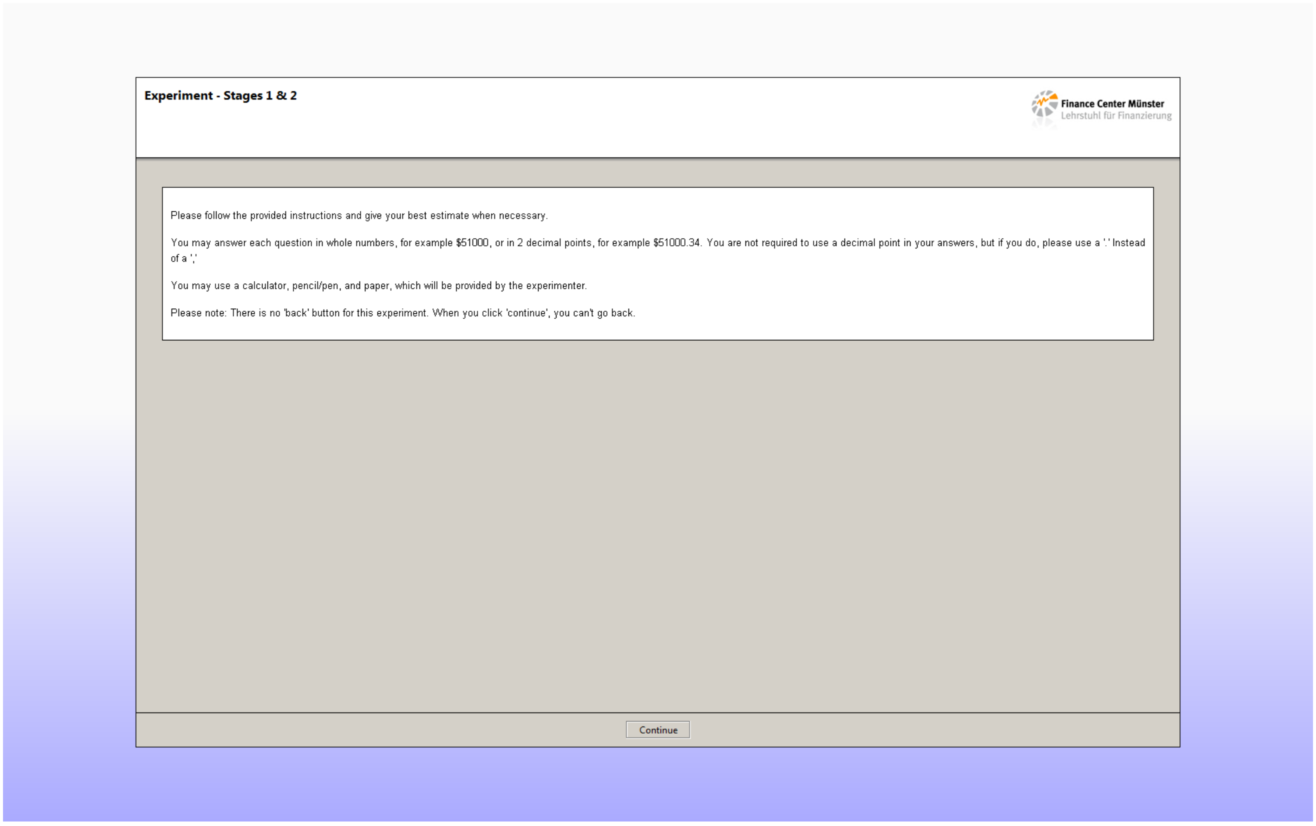
Experiment Introduction and Instructions





Experiment Introduction and Instructions (continued)





Introduction/Incentives and Instructions (text)

***Introduction** – Stages One and Two*

Welcome to Stages 1 and 2 of this experiment.

This experiment consists of 3 Stages. Stages 1 and 2 will be completed today and Stage 3 will be completed at a later date. It will take approximately 2 hours to complete Stages 1 and 2, about an hour for each Stage. Stage 3 will take approximately 45 minutes to complete.

Please follow the provided instructions and give your best estimate/guess when necessary.

You may use a calculator, pencil/pen, and paper, which will be provided by the experimenter.

Thank you for participating in this experiment. Your assistance is greatly appreciated.

***Incentives** - Stages one and two*

Today, each participant will be given a base amount of €15 for completing Stages 1 and 2.

An additional variable amount will be paid out to 3 randomly chosen participants from each session (i.e. the people sitting in this room) with payouts consisting of €20.00, €40.00 and €60.00. The variable payout will depend on the overall average accuracy of your answers/estimates in both stages, if chosen, compared to other chosen participants in this session.

The overall payout range today will be €15 to €75, with an average approximate €20 payout for each participant.

For your second Appointment (Stage 3), you will be given an additional base fee as well as possible additional variable payouts for completing the final Stage.

For the purposes of this experiment, it is essential that you show up for your second appointment.

***Final Instructions** – All Stages (1-3)*

Please follow the provided instructions and give your best estimate when necessary.

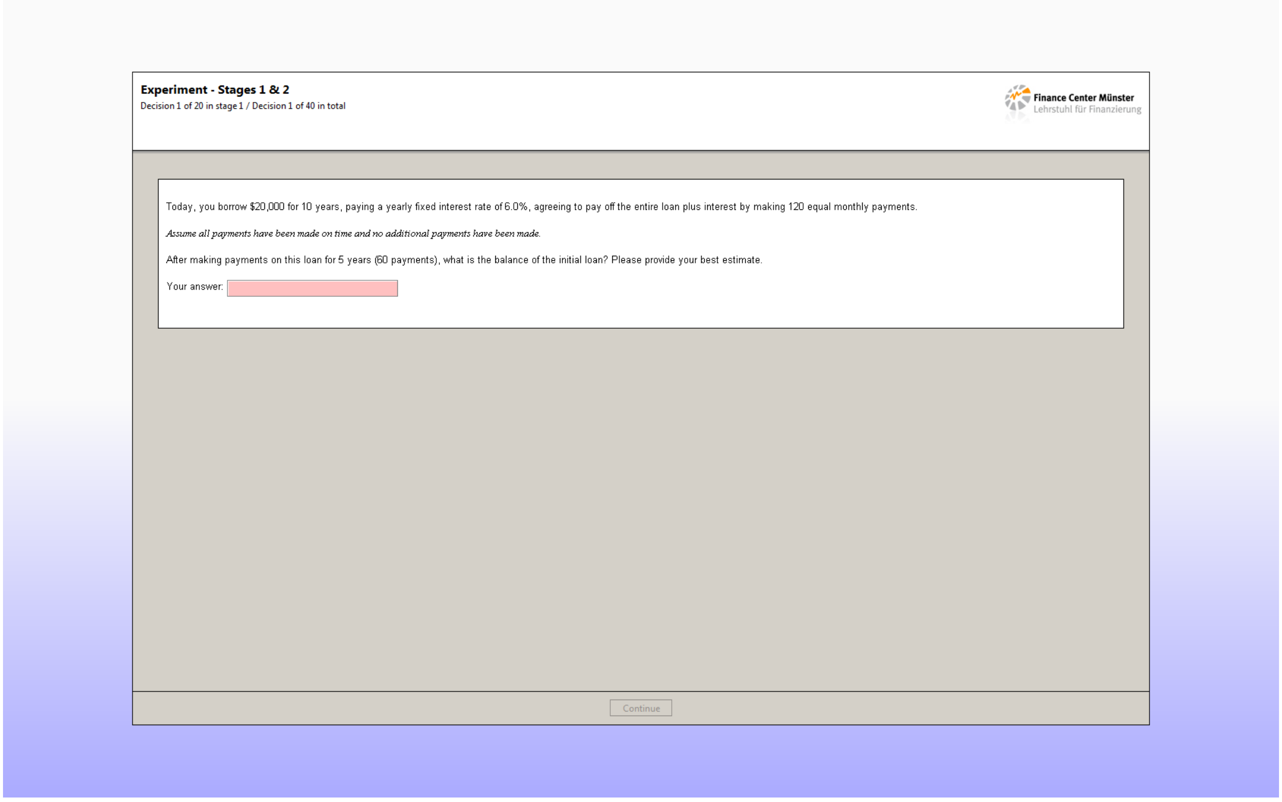
You may answer each question in whole numbers, for example $51000, or in 2 decimal points, for example $51000.34. You are not required to use a decimal point in your answers, but if you do, please use a '.' Instead of a ','

You may use a calculator, pencil/pen, and paper, which will be provided by the experimenter.

Please note: There is no 'back' button for this experiment. When you click 'continue', you can't go back.

Click 'continue' to begin the next/final stage.

Experiment Debt Question Format



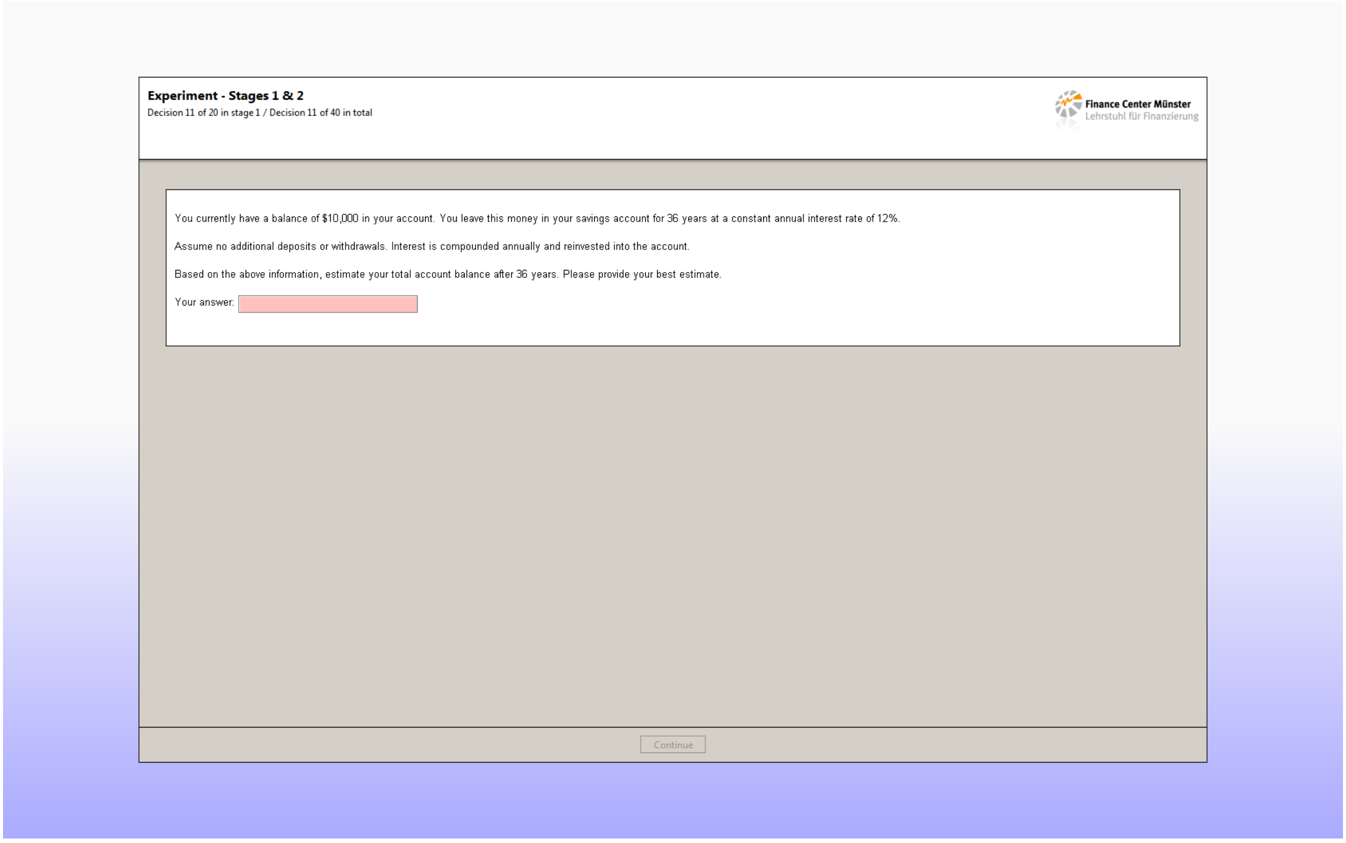
Text

Today, you borrow $______ for ____ years, paying a yearly fixed interest rate of _____%, agreeing to pay off the entire loan plus interest by making _____ equal monthly payments.

Assume all payments have been made on time and no additional payments have been made.

After making payments on this loan for ______ years (___ payments), what is the remaining balance of the initial loan? Please provide your best estimate.

Savings Question Format (Prospective)

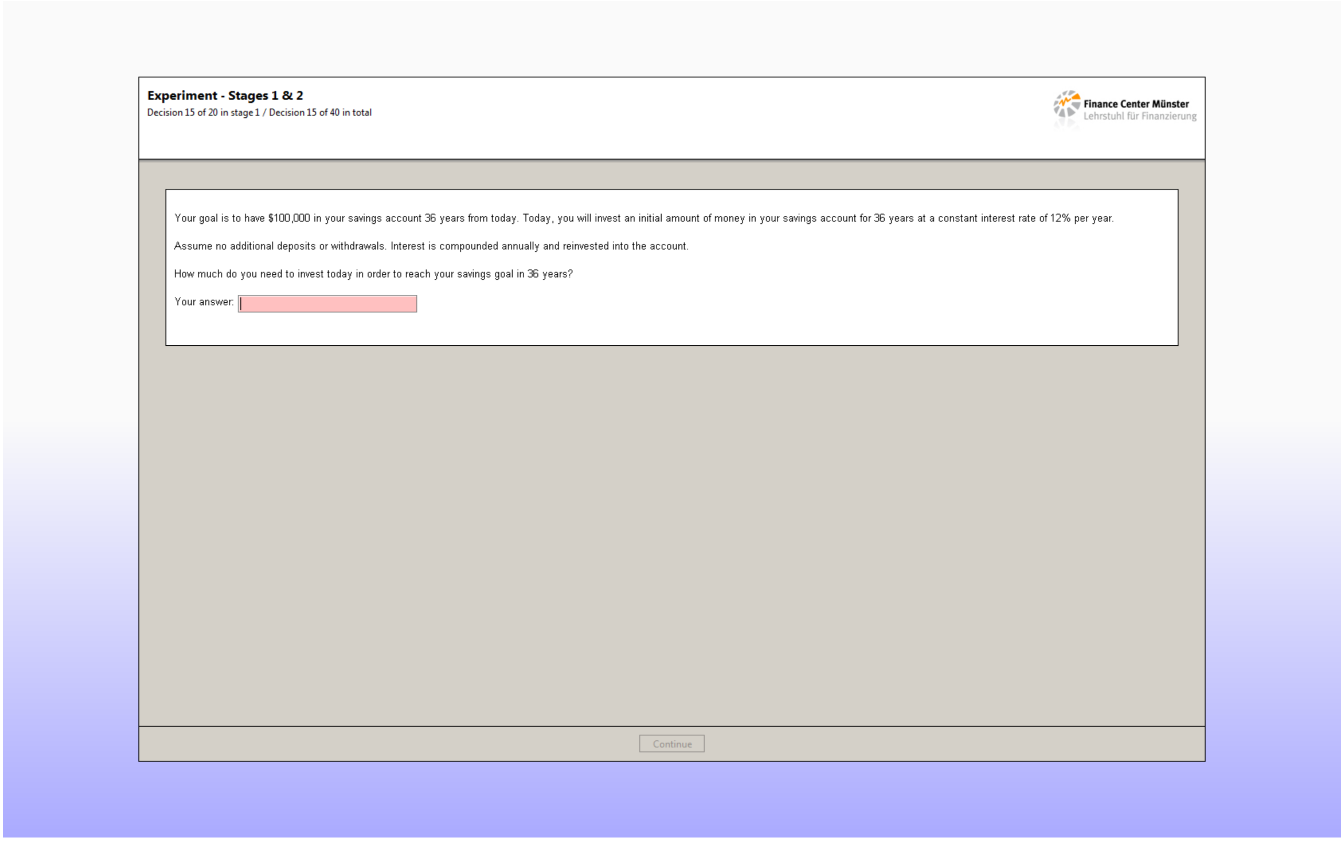


> You currently have a balance of $10,000 in your account. You leave this money in your savings account for __ years at a constant annual interest rate of _**%**.

Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account.

Based on the above information, estimate your total account balance after __ years. Please provide your best estimate.

Savings Question Format (Retrospective)



> Your goal is to have $100,000 in your savings account __ years from today. Today, you will invest an initial amount of money in your savings account for __ years at a constant interest rate of **_%** per year.

Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account.

How much do you need to invest today in order to reach your savings goal in __ years? Please provide your best estimate.

Information and Conclusion