A Model of Monetary Policy and Risk Premia

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Abstract

We present a dynamic heterogeneous-agent asset pricing model in which monetary policy affects the risk premium component of the cost of capital. Risk tolerant agents (banks) borrow from risk averse agents (depositors) and invest in risky assets subject to a reserve requirement. By varying the nominal interest rate, the central bank affects the spread banks pay for external funding (i.e., leverage), a link that we show has strong empirical support. Lower nominal rates result in increased leverage, lower risk premia and overall cost of capital, and higher volatility. The effects of policy shocks are amplified via bank balance sheet effects. We use the model to implement dynamic interventions such as a "Greenspan put" and forward guidance, and analyze their impact on asset prices and volatility.

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I. Introduction

In the textbook model (e.g. Woodford 2003), monetary policy impacts the economy by inducing changes in the risk-free interest rate. Yet, a growing body of evidence shows that monetary policy also has a large impact on the *risk premium* component of the cost of capital.[1] Furthermore, many central bank interventions can be usefully interpreted as directly targeting risk premia. These include interventions undertaken during the financial crisis such as large-scale asset purchases, "Operation Twist", across-the-board asset guarantees, and lender-of-last-resort operations, all of which specifically target the prices of risky assets. Monetary policy may also influence risk premia in normal times; an active debate centers on whether a "Greenspan put" in the late 1990s or abnormally low rates in the mid-2000s encourage excessive leverage and "reaching for yield".[2]

The link between monetary policy and risk premia works at least in part through financial institutions (Adrian and Shin 2010). For this reason, it appears in the banking literature alternatively as the bank lending channel (Bernanke and Blinder 1992, Kashyap and Stein 1994), the credit channel (Bernanke and Gertler 1995), and financial stability policy more broadly (Stein 2012). Yet risk premia are the subject of asset pricing.

In this paper, we provide a dynamic asset pricing model of the risk premium channel of monetary policy. The central bank varies the nominal interest rate in order to regulate the effective risk aversion of the marginal investor in the economy. It does so by influencing financial institutions' cost of leverage. Lowering the nominal interest rate reduces the cost of leverage, which increases risk taking and decreases risk premia.

Specifically, we model an endowment economy populated by two types of agents, those with low risk aversion and those with high risk aversion. We think of the relatively risk tolerant agents as pooling their wealth in the form of the net worth (i.e., equity) of financial intermediaries, which we identify as banks. Because banks invest on behalf of the risk tolerant agents, in equilibrium they take leverage. They do so by borrowing from the relatively risk averse agents, or taking deposits. Our view of banks and deposits is purposely simplified, abstracting from other functions such as screening and monitoring in order to focus on risk taking and risk premia.

The central bank requires banks to hold a fraction of the deposits that they raise as reserves.[3] Reserves are a liability of the central bank and they enter circulation via open

[1] Bernanke and Kuttner (2005) document that monetary policy surprises have a powerful impact on stock prices, and show that this is induced primarily by changes in risk premia, with very little of the effect coming directly from changes in the risk-free rate. Hanson and Stein (2012), and Gertler and Karadi (2013) extend these results to long-term Treasury bonds and credit spreads, respectively.

[2] See for example Blinder and Reis (2005), Rajan (2011), and Yellen (2011).

[3] We do not micro-found the reserve requirement, but this can be done in several ways. An important

market operations. Aside from this constraint, the model is frictionless. In particular, there are no nominal rigidities, which allows us to focus exclusively on the risk premium channel. The single state variable of the model is the share of bank capital in total wealth.

The difference between the return on reserves and the return on risk-free bonds represents the opportunity cost of holding reserves, and hence the cost of taking leverage. This difference equals the nominal interest rate. Hence, the central bank regulates banks' demand for leverage by inducing changes in the nominal rate. An increase in the nominal rate represents an increase in the cost of leverage and so it reduces banks' demand for leverage. As banks are the risk tolerant investors in the economy, this causes aggregate risk taking to fall and the economy's effective risk aversion to rise, driving up the equilibrium risk premium.

The solution to the model shows that the nominal rate equals the shadow price of banks' leverage constraint–the reserves requirement. When the reserve requirement binds strongly, and banks' demand for leverage is tightly constrained, the nominal rate is high. When the reserves requirement is slack and banks' demand for leverage is satiated, the nominal rate is zero. A zero nominal rate therefore implies that further easing cannot increase banks' risk taking. Indeed, any further attempt to lower the nominal rate results in banks holding excess reserves. As a result, the nominal rate in the model is bounded below by zero.

Our model allows the central bank to specify the nominal interest rate policy as a function of the state variable, the net worth share of the banking sector. We solve for the dynamics of reserves required for the central bank to support its target nominal rate. The solution shows that the nominal rate depends on the dynamics of total reserves, not on their quantity. The reason is that the return to holding reserves does not depend on their level, but on their growth rate over time.[4] We take reserves to be the numeraire in the model, so inflation is the endogenous change in the price of consumption in units of reserves, or minus the capital gain on reserves.

We show that banks' optimization problem can be rewritten as an unconstrained portfolio-choice problem by replacing the interest rate on deposits or risk-free bonds with the Fed Funds rate, the rate banks charge to lend to each other in the interbank market. The literature refers to the spread between these two rates as the external finance spread (e.g., Bernanke and Gertler 1995) because it represents the difference between the rate paid to

rationale for reserves is that deposit insurance severs the link between banks' risk taking and the rate they pay on deposits. This creates a role for the government in regulating banks' risk taking. A reserve requirement provides a way of doing so with the added advantage that the tightness of bank funding is visible to regulators as a market price (Stein 2012). In turn, the standard rationale for government-run deposit insurance is the need to avoid inefficient bank runs (Diamond and Dybvig 1983).

[4] One way to see this is to consider a one-time doubling of total reserves. This would halve the value of each unit of reserves (i.e. double the price level), but it would not affect the holding return of reserves going forward, and so it would leave the nominal rate unchanged.

borrow a dollar externally and the rate earned on a dollar that is "inside" the bank. Monetary policy can therefore be viewed as governing bank leverage by altering the external finance spread.

A novel prediction of our model is that the external finance spread is proportional to the nominal rate. Figure 1 shows the corresponding empirical relationship. It plots 20-week moving averages of the Fed Funds rate and the Fed Funds-TBill spread for the period July 1980 to May 2008. The sample average Fed Funds-TBill spread is 0.57%, which is large since Fed Funds are overnight and extremely safe.[5] The relationship between this spread and the level of the Fed Funds rate is remarkably tight and nearly proportional. The raw correlation is 86%, and the two series track each other closely both in the cycle and in the trend. This evidence shows that there is a strong relationship between the nominal interest rate and bank funding conditions, which is the essential mechanism underpinning our model.

The model's asset pricing implications all follow from the interaction of the external finance spread with the nominal rate in combination with heterogeneity in agents' risk tolerances. We emphasize that any channel that gives rise to the observed relationship between the external finance spread and the nominal rate will induce the same asset-pricing dynamics. Such channels can originate with frictions on either the asset or liabilities side of bank balance sheets that impose a cost on taking leverage. In the body of the paper we model the reserves requirement, an asset-side cost.

The appendix presents a version of the model where the leverage cost arises instead on the liabilities side. In that version, deposits provide households with liquidity services and therefore pay a low rate, but must also be secured with collateral. The spread banks earn on deposits is controlled by the nominal rate, which therefore governs the tradeoff banks face between funding cost and leverage, just as in the main model.[6]

To analyze the full implications of the model, we solve for the equilibrium using projection methods. To demonstrate the impact of monetary policy, we compare prices and quantities between a high nominal rate and a low nominal rate regime. We show that when nominal rates are high, bank leverage is low, the Sharpe ratio and risk premium of the endowment claim are high, and the valuation of the endowment claim is low. We also show that volatility is decreasing in the nominal rate. Volatility in the model is endogenously stochastic; it depends on banks' net worth. Because low interest rates increase bank leverage, they also increase the volatility of the state variable, the volatility of discount rates, and hence the

[5]By comparison, the credit spread (Moody's Baa versus Aaa long-term bonds) averages 1.07% over the same period.

[6]Driscoll and Judson (2013) show empirically that deposit rates are "sticky", meaning that they do not move one-for-one with the nominal interest rate, and hence the spread banks earn on deposits is driven by the nominal rate.

volatility of returns.

We further examine two dynamic interest rate policies. The first policy studies the impact of forward guidance. Under forward guidance, the central bank commits to keeping nominal rates low even after the economy recovers (the wealth share of the banking sector rises above some threshold). We show that by reducing future discount rates through forward guidance, the central bank is able to induce an additional increase in prices even when current nominal rates are at the zero lower bound.

The second policy captures a "Greenspan put" by decreasing nominal rates as bank net worth falls. This policy stabilizes prices locally by boosting bank leverage. However, as leverage rises and eventually becomes satiated, further negative shocks cause prices to fall rapidly. At this point, volatility is significantly higher than it would have been otherwise. Thus, in our framework a Greenspan put can support valuations in the short run at the expense of greater instability in the long run.

Finally, we extend the model to allow the central bank to deviate from its expected nominal rate policy. We show that policy shocks lead to a second round amplification effect on risk premia and other equilibrium quantities. This effect is akin to a financial accelerator: when nominal rates fall unexpectedly, the assets on bank balance sheets rise more than their liabilities, which raises banks' net worth and enables them to expand their balance sheets, pushing risk premia down even further.

The rest of this paper is organized as follows: Section II reviews the literature, Section III presents the model, Section IV characterizes the equilibrium, Section V presents results for a benchmark economy, Section VI examines the effects of dynamic policies, Section VII introduces an extension with policy shocks, and Section VIII concludes.

II. Related literature

Our paper is related to the literature on the bank lending channel of monetary policy initiated by Bernanke (1983) and formalized by Bernanke and Blinder (1988) and Kashyap and Stein (1994). The bank lending channel relies on an imperfect substitutability between bank loans and unintermediated bonds so that a contraction in bank lending affects the overall availability of funding and spills over to the macroeconomy. The transmission runs through bank reserves: a drop in the supply of reserves forces banks to shrink their balance sheets.

Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) develop the broader balance sheet channel of monetary policy, which emphasizes the impact of policy shocks on the net worth of borrowers, and by extension their ability to raise capital and invest. Jermann and Quadrini (2012) and Christiano, Motto, and

Rostagno (2014) incorporate balance sheet frictions inside a DSGE model and find that they can account for a large proportion of the observed macroeconomic fluctuations.

Against the backdrop of the financial crisis, recent models shift attention from firms to financial intermediaries (e.g. Adrian and Shin 2010, Gertler and Kiyotaki 2010, Cúrdia and Woodford 2009, Adrian and Boyarchenko 2012, Brunnermeier and Sannikov 2013, He and Krishnamurthy 2013). In these models, a maturity or liquidity mismatch between intermediary assets and liabilities causes interest rate shocks to affect intermediary net worth, driving the supply of credit.

Our contribution to these literatures is to develop an asset pricing framework in which monetary policy directly influences the risk premium component of the cost of capital. We model an economy populated by agents with different levels of risk aversion, which gives rise to a credit market as in Dumas (1989), Wang (1996) and Longstaff and Wang (2012). Risk tolerant agents deploy their wealth in levered portfolios that we interpret as banks. They raise funds by selling bonds to risk averse households, or depositors. The key friction is a cost on leverage. Our model is thus related to models in which margin requirements lead to incomplete risk sharing (e.g. Gromb and Vayanos 2002, Brunnermeier and Pedersen 2009, Gârleanu and Pedersen 2011, Ashcraft, Garleanu, and Pedersen 2011). Geanakoplos (2003) derives this type of market incompleteness endogenously, and Geanakoplos (2009) emphasizes that the resulting variation in leverage has a large impact on asset prices. An important distinction of our model is that the tightness of the leverage constraint depends on monetary policy through the nominal interest rate.

Stein (1998, 2012) also studies the ability of the central bank to control bank leverage. In Stein (2012), leverage entails a negative externality resulting from fire sales. Reserves function as "pollution permits" whose price, the nominal rate, provides a market-based signal that enables regulators to maintain financial stability. In our framework the central bank controls the price of reserves by regulating their dynamics, which allows it to influence the external finance premium faced by banks. The external finance premium in turn affects bank leverage and risk premia.

In contrast to the literature, our model does not require any nominal price rigidities.[7] Our asset pricing framework allows us to focus exclusively on risk taking. This differs from other papers on monetary policy and bank balance sheets including Stein (2012), Adrian and Shin (2010), and Dell'Ariccia, Laeven, and Marquez (2011).

Our paper is also related to the literature on the role of government liabilities as a source

[7] In this sense our model represents a counter-example to Kashyap and Stein's (1994) conjecture that absent nominal price rigidities, "there can be no real effects of monetary policy through either the lending channel or the conventional money channel."

of liquidity for the financial sector (Lucas 1990, Woodford 1990, Gertler and Karadi 2011, Caballero and Farhi 2013, Krishnamurthy and Vissing-Jorgensen 2012, Greenwood, Hanson, and Stein 2012). In our model, policy accommodation provides liquidity to banks and "crowds in" investment in risky assets similarly to the role of government debt in Woodford (1990).

On the empirical side, Bernanke and Blinder (1992) and Bernanke and Gertler (1995) are early papers that find support for the bank lending and balance sheet channels of monetary policy. Bernanke and Blinder (1992) show that monetary tightening as reflected in a shock to the Fed Funds rate, leads banks to shrink their balance sheets. Nagel (2014) shows that the Fed Funds rate is closely related to spreads between non-deposit bank liabilities and TBills. Kashyap, Stein, and Wilcox (1993) show that bank funding is sensitive to policy shocks, and Kashyap and Stein (2000) find that this is especially true of smaller banks. More recently, Jiménez, Ongena, Peydró, and Saurina Salas (2011) and Landier, Sraer, and Thesmar (2013) provide further corroborating evidence on the links between monetary policy and bank balance sheets.

Our model generates a positive relationship between nominal interest rates and risk premia, a phenomenon sometimes referred to as "reaching for yield". A fast-growing literature finds support for this relationship. In a key paper, Bernanke and Kuttner (2005) document that surprise rate hikes induce large negative stock returns. Using a VAR decomposition, they find that this effect is largely due to increases in expected excess returns, and that very little is directly attributable to changes in expected real interest rates. Bekaert, Hoerova, and Lo Duca (2013) use the VIX index in a similar analysis, finding that tightening shocks increase investor risk aversion. Hanson and Stein (2012) show that policy shocks affect long-term Treasury bond premia, while Gertler and Karadi (2013) find similar results for credit spreads.

III. Model

We model an infinite-horizon exchange economy in continuous time $t \geq 0$, with aggregate endowment D_t that follows a geometric Brownian motion:

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D dB_t. \tag{1}$$

The economy is populated by a continuum of agents with total mass one. There are two types of agents, A and B. Both types have recursive preferences as in Duffie and Epstein (1992), the continuous-time analog to the discrete-time formulation of Epstein and Zin (1989).[8]

To ensure stationarity, we assume that agents die at a rate κ. New agents are also born at a rate κ with a fraction $\overline{\omega}$ as type A and $1-\overline{\omega}$ as type B. Gârleanu and Panageas (2008) show that under these assumptions, κ simply increases agents' effective rate of time preference and hence the lifetime utility V_0^i of an agent of type $i = A, B$ is given by the recursion

$$V_0^i = E_0\left[\int_0^\infty f^i\left(C_t^i, V_t^i\right) dt\right] \tag{2}$$

$$f^i\left(C_t^i, V_t^i\right) = \left(\frac{1-\gamma^i}{1-1/\psi^i}\right) V_t^i \left[\left(\frac{C_t^i}{\left[\left(1-\gamma^i\right) V_t^i\right]^{1/(1-\gamma^i)}}\right)^{1-1/\psi^i} - (\rho+\kappa)\right]. \tag{3}$$

The felicity function f^i is an aggregator over current consumption and future utility. The parameters ψ^i and γ^i, $i = A, B$, denote agents' elasticity of intertemporal substitution (EIS) and relative risk aversion (RRA).

Without loss of generality, we assume that A agents are more risk tolerant, $\gamma^A < \gamma^B$. We view these agents as pooling their wealth into the net worth (i.e. equity capital) of the "banks" in the economy (or more generally the financial sector). We abstract from other aspects of financial intermediation and adopt this simplified view in order to focus on risk taking.[9] We therefore often refer to the A agents as the banks and their wealth as the equity capital of the banking sector.

Let W_t^i denote the total wealth of type-i agents at time t. We denote the wealth share of A agents by ω_t:

$$\omega_t = \frac{W_t^A}{W_t^A + W_t^B}. \tag{4}$$

We show below that ω_t summarizes the state of the economy. To derive its dynamics, we assume that the wealth of agents who die is bequeathed to the newly born on an even per-capita basis. We can then write the law of motion of ω_t as

$$d\omega_t = \kappa\left(\overline{\omega} - \omega_t\right) dt + \omega_t\left(1-\omega_t\right)\left[\mu_\omega\left(\omega_t\right) dt + \sigma_\omega\left(\omega_t\right) dB_t\right]. \tag{5}$$

The evolution of ω_t has an exogenous component due to demographic turnover that ensures stationarity, and an endogenous component (in brackets) due to differences in the rates of

[8] These preferences allow us to vary the elasticity of intertemporal substitution (EIS) independently of the risk aversion coefficient. An EIS greater than one ensures that valuations are decreasing in risk aversion.

[9] We note that in our setup it is not necessary to impose a restriction on equity issuance as in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2013).

saving and the portfolio choices of the two agent types.

Agents trade a claim on the aggregate endowment. The price of this claim is P_t, its dividend yield is $F(\omega_t) = D_t/P_t$, and its return process is

$$dR_t = \frac{dP_t + D_t dt}{P_t} = \mu(\omega_t)\, dt + \sigma(\omega_t)\, dB_t. \tag{6}$$

Agents also trade instantaneous risk-free bonds, i.e. deposits, that pay the endogenously-determined real interest rate $r(\omega_t)$.

A. *Deposits and reserves*

The difference in risk aversion between agents leads to the emergence of a credit market (Longstaff and Wang 2012). In particular, optimal risk sharing implies that the risk-averse B agents lend to the risk-tolerant A agents using the instantaneous risk-free bonds. Continuing with our interpretation of A agents as banks, we think of these bonds as deposits. To be clear, in addition to deposits, B agents can also hold the risky endowment claim directly.

The central bank regulates deposit taking through a reserve requirement. In particular, banks must hold reserves of no less than a fixed proportion of their deposits. Holding reserves is costly due to foregone interest.[10] Reserves are issued only by the central bank though they can be traded freely in a secondary market.

A reserve requirement can be motivated in several ways. For example, it provides a lever for regulating deposit creation, and deposit creation entails an externality in the context of deposit insurance. Deposit insurance itself can be optimal if deposits are susceptible to destructive bank runs as in Diamond and Dybvig (1983). A reserve requirement is particularly well-suited to regulating the externalities associated with deposit creation because it takes advantage of a price mechanism. The central bank monitors the cost of lending and borrowing reserves in interbank markets and responds to fluctuations in that cost by conducting open market operations.[11]

Rather than directly modeling the externalities associated with deposit creation, we take reserve requirements as given and study their implications for risk-taking. Formally, let $w_{S,t}$ be the banks' portfolio weight in the risky endowment claim. Of this, $\max\{w_{S,t} - 1, 0\}$ must be deposit-financed (borrowed). Let $w_{M,t}$ be the banks' portfolio weight in reserves. The

[10] Interest on reserves can be easily incorporated into the model as we discuss below.

[11] Stein (2012) draws the analogy with the market for pollution permits. Reserves regulate the supply of deposits based on a tradeoff between the private value of monetary services and the externality due to fire sales following crashes. In general, any negative externality associated with deposit creation introduces a role for reserves.

reserve requirement imposes the constraint

$$w_{M,t} \geq \max\left[\lambda\sigma_t^2\left(w_{S,t}-1\right),0\right]. \tag{7}$$

Banks must hold reserves in proportion to their deposits, if any, and reserves cannot be held short. The parameter λ controls the reserve requirement, $\lambda\sigma_t^2$. Scaling by σ_t^2 simplifies the resulting expressions, but is not essential.[12] When $\lambda = 0$, there is no reserve requirement and asset markets are complete. This case might correspond to a frictionless economy in which deposit-related externalities and the need to regulate them do not arise.

Let M_t denote the total quantity of reserves and Π_t the total real value of reserves in units of the endowment. It will be useful to define

$$G\left(\omega_t\right) = \frac{\Pi_t}{P_t} \tag{8}$$

as the total wealth share of reserves. Furthermore, let $\pi_t = \Pi_t/M_t$ denote the consumption value of each dollar of reserves. We take reserves to be the numeraire, so π_t is the inverse price level.[13] It follows that the realized rate of inflation is $-d\pi_t/\pi_t$. We assume that the central bank sets the path of reserves dM_t/M_t so that inflation is locally deterministic,

$$-\frac{d\pi_t}{\pi_t} = i\left(\omega_t\right)dt. \tag{9}$$

Locally deterministic inflation simplifies the exposition of the model and is arguably realistic. In Section IV.B, we show precisely how the central bank implements (9) by adjusting the drift of reserves growth and its exposure to the endowment shock.

Next, we define the nominal interest rate:

$$n\left(\omega_t\right) = r\left(\omega_t\right) + i\left(\omega_t\right). \tag{10}$$

We treat $n\left(\omega_t\right)$ as the central bank's policy instrument and solve for the path of reserves that implements it. We write this policy as a function of ω_t since it summarizes the state of the economy. Agents have rational expectations so they know this function. In Section VII, we also consider policy shocks, which may take the nominal rate away from its benchmark rule.

[12] In the absence of this scaling, the tightness of the reserves requirements varies inversely with the variance of the return on the endowment claim. This adds a degree of complication to the expressions which is inconsequential.

[13] In practice, reserves are fungible with currency, which serves as numeraire. Since our focus is on risk taking, we abstract from introducing a transactions medium such as currency.

The central bank controls the supply of reserves via open market operations, sales and purchases of bonds in exchange for reserves at prevailing market prices. Let B_t be the central bank's total holdings of bonds (hence the private sector as a whole holds $-B_t$). If we think of reserves as the central bank's liability, then its net worth is $B_t - \Pi_t$. Since open market operations are conducted at prevailing market prices, they do not change this net worth. However, the central bank earns a stream of "seignorage" profits on its portfolio, which is given by the sum of the interest income it earns on its bonds and the depreciation of its reserve liabilities, which is given by realized inflation. Thus, total seignorage is

$$B_t r(\omega_t)\,dt - \Pi_t \frac{d\pi_t}{\pi_t} \quad = \quad \Pi_t n(\omega_t)\,dt. \tag{11}$$

As we show below, no-arbitrage requires $n(\omega_t) \geq 0$ so seignorage is never negative. To close the model, we assume the central bank pays out its seignorage profits, which keeps its net worth at zero. To keep this refund from changing the wealth distribution, we assume it gets distributed to all agents in the economy in proportion to their wealth.

B. Optimization

We begin with the Hamilton-Jacobi-Bellman (HJB) equation of an agent in our economy. Let $V^i\left(W_t^h, \omega_t\right)$ denote the value function of agent h of type $i = A, B$. Also let c_t^h, $w_{S,t}^h$, and $w_{M,t}^h$ be the agent's consumption-wealth ratio, endowment claim portfolio weight, and reserves portfolio weight (the remaining weight is held in bonds). The HJB equation is

$$0 = \max_{c_t^h, w_{S,t}^h, w_{M,t}^h} f^i\left(c_t^h W_t^h, V^i\left(W_t^h, \omega_t\right)\right)dt + E\left[dV^i\left(W_t^h, \omega_t\right)\right] \tag{12}$$

subject to the agent's wealth dynamics[14] and reserve requirement

$$\begin{aligned} \frac{dW_t^h}{W_t^h} \quad &= \quad \Bigg(r(\omega_t) - c_t^h + w_{S,t}^h\left[\mu(\omega_t) - r(\omega_t)\right] + w_{M,t}^h\left[\frac{d\pi_t}{\pi_t} - r(\omega_t)\right] && (13)\\ &\qquad + G(\omega_t)\,n(\omega_t)\Bigg)dt + w_{S,t}^h \sigma(\omega_t)\,dB_t \\ w_{M,t}^h \quad &\geq \quad 0 && (14)\\ w_{M,t}^h \quad &\geq \quad \lambda\sigma^2(\omega_t)\left(w_{S,t}^h - 1\right). && (15) \end{aligned}$$

[14] These are the wealth dynamics should the agent manage to cheat death over the next instant. The agent accounts for the possibility of death directly in the felicity function (3).

The diffusive component of wealth depends only on the weight of the risky claim and not on the reserves holdings, which are locally risk-free. In the drift term, $G(\omega_t)\, n(\omega_t)$ represents the stream of seignorage refund payments.[15] By (9) and (10), the excess return on reserves, $d\pi_t/\pi_t - r(\omega_t)$, equals $-n(\omega_t)$, the negative of the nominal rate. Hence, reserves are costly when the nominal rate is positive.[16] Not that the reserve requirement consists of two parts: the shorting restriction that prevents agents from increasing the effective supply of reserves on their own, and the constraint on deposit taking.

The homogeneity of preferences implies that the consumption and portfolio policies are independent of wealth, so we can write them as functions of agent type only. Finally, denote the aggregated consumption-wealth ratio of type i agents by $c^i(\omega_t) = \int_i c^h(\omega_t) \frac{W^h}{W^i} dh$ for $i = A, B$, and similarly for the portfolio policies $w_S^i(\omega_t)$ and $w_M^i(\omega_t)$.

C. Equilibrium conditions

In equilibrium, the markets for goods (i.e. consumption), the endowment claim, and reserves must clear. The bond (deposit) market clears by Walras' law. Since the public's net bond holdings are minus the value of reserves, aggregate wealth equals the value of the endowment claim, $W_t^A + W_t^B = P_t$. The three market-clearing conditions can therefore be written as

$$\omega_t c^A(\omega_t) + (1-\omega_t)\, c^B(\omega_t) = F(\omega_t) \tag{16}$$

$$\omega_t w_S^A(\omega_t) + (1-\omega_t)\, w_S^B(\omega_t) = 1 \tag{17}$$

$$\omega_t w_M^A(\omega_t) + (1-\omega_t)\, w_M^B(\omega_t) = G(\omega_t)\,. \tag{18}$$

All three conditions are normalized by total wealth. The first equation gives the goods-market clearing condition, the second gives the market-clearing condition for the endowment claim, and the third gives the market-clearing condition for reserves.

IV. Analysis

In this section we derive the equations that characterize the equilibrium. These equations do not permit closed-form solutions. However, we are able to derive analytical expressions

[15]Recall from (11) that total seignorage is $\Pi_t n(\omega_t)$ and that it gets refunded in proportion to wealth, so an agent with wealth W_t^h gets $\Pi_t n(\omega_t) \frac{W_t^h}{P_t} = G(\omega_t)\, n(\omega_t)\, W_t^h$.

[16]Paying interest on reserves would partially offset this cost. In the end, what matters is the difference between the nominal rate and the interest rate on reserves, which represents the net cost of holding reserves. Interest on reserves could serve as a separate policy tool for achieving financial stability while maintaining price stability in the presence of nominal price rigidities, see Kashyap and Stein (2012).

that highlight key mechanisms. In the next section, we provide a full analysis of the model's implications by applying numerical methods.

A. *The value function and the demand for leverage*

For simplicity of notation, we drop agent, type, and time subscripts though it should be understood that they apply. Let $\theta_\lambda V_W W \geq 0$ and $\theta_0 V_W W \geq 0$ be the Lagrange multipliers on the reserves and non-negativity constraints. By Ito's lemma we can rewrite the HJB equation as the Lagrangian

$$\begin{aligned} 0 \;=\; & \max_{c,w_S,w_M} \; f\left(cW,V\right)+V_W W\left[r-c+w_S\left(\mu-r\right)-w_M n+Gn\right] \\ & +V_\omega\Big[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\mu_\omega\Big]+V_{W\omega}W\omega\left(1-\omega\right)w_S\sigma_\omega\sigma+\frac{1}{2}V_{WW}W^2\left(w_S\sigma\right)^2 \\ & +\frac{1}{2}V_{\omega\omega}\omega^2\left(1-\omega\right)^2\sigma_\omega^2+\theta_\lambda V_W W\left[w_M-\lambda\sigma^2\left(w_S-1\right)\right]+\theta_0 V_W W w_M. \end{aligned} \tag{19}$$

The following proposition gives the form of the value function up to an an unknown function of the wealth distribution $J(\omega)$ together with the equation that characterizes it.

Proposition 1. *Each agent's value function has the form*

$$V\left(W,\omega\right) \;=\; \left(\frac{W^{1-\gamma}}{1-\gamma}\right)J\left(\omega\right)^{\frac{1-\gamma}{1-\psi}}. \tag{20}$$

The unknown function $J(\omega)$ gives the agent's consumption-wealth ratio, $c^* = J$ and solves the second-order ordinary differential equation

$$\begin{aligned} \rho+\kappa \;=\; & 1/\psi J+\left(1-1/\psi\right)\left(r+\lambda\sigma^2\theta_\lambda+Gn\right)-1/\psi\frac{J_\omega}{J}\Big[\kappa\left(\overline{\omega}-\omega\right) \\ & +\omega\left(1-\omega\right)\mu_\omega\Big]-\frac{1/\psi}{2}\left[\left(\frac{\psi-\gamma}{1-\psi}\right)\left(\frac{J_\omega}{J}\right)^2+\frac{J_{\omega\omega}}{J}\right]\omega^2\left(1-\omega\right)^2\sigma_\omega^2 \\ & +\frac{1}{2}\left(\frac{1-1/\psi}{\gamma}\right)\left[\frac{\mu-r}{\sigma^2}-\lambda\theta_\lambda+\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega}{J}\omega\left(1-\omega\right)\frac{\sigma_\omega}{\sigma}\right]^2\sigma^2 \end{aligned} \tag{21}$$

if $\gamma^B-\gamma^A \geq \lambda n$, with $\theta_\lambda = n$ if the agent is of type A and $\theta_\lambda = 0$ if the agent is of type B. If instead $\gamma^B-\gamma^A < \lambda n$, J solves

$$\rho+\kappa \;=\; 1/\psi J+\left(1-1/\psi\right)\left(\mu-\frac{\gamma}{2}\sigma^2\right)-1/\psi\frac{J_\omega}{J}\Big[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\mu_\omega\Big]. \tag{22}$$

Proof of Proposition 1. The proof is contained in Appendix A. □

The function J is type-specific but not agent-specific since it does not depend on wealth. Instead, it depends solely on the wealth distribution ω. As a result, ω is a sufficient statistic for asset valuations and other equilibrium quantities.

Using the value functions, we can solve for agents' portfolio demands. Proposition 2 below provides the conditions under which banks take leverage (by issuing deposits), and characterizes their demand for the risky endowment claim as it depends on the central bank's nominal rate policy.

Proposition 2. *Banks take leverage/deposits ($w_S^A > 1$) if and only if*

$$\gamma^B - \gamma^A > \lambda n. \tag{23}$$

In this case, banks' portfolio holdings of the endowment claim are given by

$$w_S^A = \frac{1}{\gamma^A}\left[\frac{\mu - r}{\sigma^2} - \lambda n + \left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A}\omega\left(1-\omega\right)\frac{\sigma_\omega}{\sigma}\right]. \tag{24}$$

Proof of Proposition 2. The proof is contained in Appendix A. □

Equation (24) has three parts. The first term, $(\mu - r)/\sigma^2$, is the standard "myopic" mean-variance tradeoff for the endowment claim. It shows that banks take more leverage when there is a higher return premium per unit of risk. The third term, which depends on J_ω^A, represents the intertemporal hedging component of banks' demand for the risky asset. This component determines how much banks adjust their current risk taking to hedge future changes in investment opportunities. The investment opportunity set is stochastic because of variation in aggregate risk aversion that is induced by changes in the relative wealth ω of the risk-tolerant and risk-averse agents.

The term $-\lambda n$ in equation (24) gives the direct impact of the nominal rate on bank leverage, which we summarize in the following corollary.

Corollary 1. *All else equal, an increase in the nominal interest rate reduces bank leverage.*

For every dollar of deposit funding, banks must increase their reserves holdings by the reserve requirement. Since the excess return on reserves is the negative of the nominal rate, holding reserves is costly. An increase in the nominal interest rate raises the effective cost of deposits and results in less leverage.

Using (24), we can see that an increase in the nominal rate works like an increase in banks' effective risk aversion. This in turn raises the economy's aggregate risk aversion and hence also the risk premium.

Proposition 2 also shows that banks lever up only if agents' risk aversions differ sufficiently to overcome the cost of leverage. The difference in risk aversions multiplied by the return variance, $\left(\gamma^A - \gamma^B\right)\sigma^2$, measures the risk premium earned by banks on their first dollar of leverage. This premium reflects the gains from risk sharing. For banks to take leverage, it must be greater than the cost of leverage which is given by the nominal rate n multiplied by the reserve requirement $\lambda\sigma^2$.

Corollary 2. *If* $\lambda n \geq \gamma^B - \gamma^A$ *then* $w_S^A = w_S^B = 1$.

If the cost of leverage exceeds the difference in risk aversions then banks do not raise deposits and the two groups remain in "financial autarky".

B. The external finance spread and the Fed Funds rate

Next, we relate the external finance spread and the Fed Funds rate inside our model. The Fed Funds market is a short-term (mostly overnight) uncollateralized lending market for banks in the US.[17] The rate that prevails in this market, the Fed Funds rate, has emerged as a key target for monetary policy. Unlike deposits, Fed Funds loans are not subject to reserve requirements. In equilibrium, banks must be indifferent between raising a dollar of funding in the form of deposits or Fed Funds. The real Fed Funds rate (FFr) must therefore equal the real deposit (or risk-free bond) rate plus the cost of the reserve requirement:

$$FFr_t \quad = \quad r\left(\omega_t\right) + \lambda\sigma_t^2 n\left(\omega_t\right). \tag{25}$$

The term $\lambda\sigma^2 n$ captures the spread between the Fed Funds rate and the rate on deposits or TBills. This spread represents the external finance spread, since it is the difference between the value of a dollar inside the banking system versus outside. In the literature (Bernanke and Gertler 1995), this term is similarly used to refer to the gap between the cost of banks' marginal sources of funding and the rate on risk-free deposits or short-term TBills.

We highlight the importance of the external finance spread in the model by rewriting equation (24) for banks' optimal leverage/risky claim holdings as follows:

$$w_S^A \quad = \quad \frac{1}{\gamma^A}\left[\frac{\mu - FFr}{\sigma^2} + \left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A}\omega\left(1-\omega\right)\frac{\sigma_\omega}{\sigma}\right]. \tag{26}$$

[17] The Fed Funds market represents a substantial source of overnight funding for large US money-center banks. The other significant source of interbank uncollateralized dollar funding is the Eurodollar market. The prevailing rate in that market, LIBOR, tracks the Fed Funds rate very closely (Kuo, Skeie, and Vickery 2010).

This shows that banks' constrained leverage can be recast as an unconstrained optimal portfolio decision if the real interest rate is replaced by the cost of external financing, which exceeds the risk-free rate by the external finance spread.

Equation (26) shows that changes in the nominal rate affect bank leverage by changing the external finance spread. When the central bank increases the nominal rate, the external finance spread widens, reducing banks' demand for leverage. In turn, this increases in effective aggregate risk aversion and the price of risk.

Figure 1 plots the empirical relationship between the level of the Fed Funds rate (solid line, left axis) and the Fed Funds-TBill spread (dashed line, right axis) for the period July 1980 to May 2008. The beginning of this period corresponds roughly to the abolition of Regulation Q, which limited the rate banks could pay on deposits, while the end corresponds roughly to the beginning of the financial crisis, which temporarily introduced credit risk into the Fed Funds market. The figure plots 20-week moving averages of these series.

The figure displays a remarkably tight relationship between the two series over this 28-year period. Indeed, the correlation is 86%. Moreover, the Fed Funds-TBill spread closely tracks both the trend and the cycles in the Fed Funds rate over this period. The evidence shows a relationship between the levels of interest rates and bank funding costs and therefore presents a challenge to models driven solely by interest rate shocks.

The average Fed Funds rate over this period is 6.25%, while the average Fed Funds-TBill spread is 0.57%. The sensitivity of the Fed Funds-TBill spread to the Fed Funds rate, estimated via OLS regression, is 0.14. In the model, this value corresponds to the reserve requirement. In practice, the reserve requirement on net transaction accounts in the US is 10%.

The relationship between the nominal rate and the external finance spread is more general than the reserves-based approach employed here. Broadly speaking, it can be induced by both asset- and liabilities-side frictions. The reserve requirement represents an asset-side friction. In Appendix C, we present a version of the model in which a liabilities-side friction, a tradeoff between cheap deposit funding and leverage, generates this relationship.

C. Reserves value and implementation dynamics

Recall that reserves are locally risk-free yet their excess return, $-n$, is negative in equilibrium. The reason for this is that reserves give banks the right to take leverage, which we can think of as a latent dividend stream. Its value is given by the Lagrange multiplier on banks' reserve requirement, θ_X^A, which in equilibrium equals the nominal rate n. At the same time, the risk-adjusted real return on any asset must equal r. Hence the capital gain on reserves,

$d\pi/\pi = -idt$, must adjust so that:

$$r \;=\; n - i. \tag{27}$$

This is Fisher's equation. Interpreted through the lens of asset pricing, it states that the real risk-free rate r equals the capital gain on reserves $-i$ plus the latent dividend stream n.

The following proposition solves for the value of reserves G and the law of motion for their quantity M that supports the central bank's nominal interest rate rule.

Proposition 3. *The value of reserves as a share of aggregate wealth is given by*

$$G(\omega_t) \;=\; \omega_t \lambda \sigma_t^2 (w_{S,t}^A - 1). \tag{28}$$

Under the central bank's nominal rate rule $n\left(\omega_t\right)$, the quantity of reserves M_t must follow the law of motion

$$\frac{dM_t}{M_t} \;=\; \left[n\left(\omega_t\right) - r\left(\omega_t\right)\right] dt + \frac{d\Pi_t}{\Pi_t} \tag{29}$$

$$\;=\; \left[n\left(\omega_t\right) - r\left(\omega_t\right)\right] dt + \frac{dG\left(\omega_t\right)}{G\left(\omega_t\right)} + \frac{dP_t}{P_t} + \frac{dG\left(\omega_t\right)}{G\left(\omega_t\right)} \frac{dP_t}{P_t}. \tag{30}$$

Proof of Proposition 3. The proof is contained in Appendix A. □

The dynamics of the quantity of reserves in equation (29) are given as a function of the central bank's policy $n(\omega)$, and two endogenous quantities, the total value of reserves $\Pi(\omega)$, and the real rate $r(\omega)$. The central bank adjusts the growth rate of reserves to achieve the target while responding to underlying shocks.

Note that the growth rate of reserves is stochastic even though realized inflation is locally deterministic. To attain the nominal rate $n\left(\omega\right)$, the central bank must influence the rate of return on reserves, which depends on the state of the economy ω. In particular, to maintain a stable nominal rate, the quantity of reserves must keep up with aggregate wealth P and demand for reserves G.

Equation (29) also implies the following corollary.

Corollary 3. *The nominal interest rate depends on the growth rate of reserves, not their level, which is not separately identified.*

This result follows directly from equation (29), which shows that $n(\omega)$ is related to the growth of M, not the level. A specific value of M pins down the price level ($\Pi = M\pi$), but the nominal rate depends only on the growth rate of M. Thus, the model features neutrality with respect to the quantity of reserves.

Description	Parameter	Value
Risk aversion A	γ^A	1.5
Risk aversion B	γ^B	15
EIS	ψ^A, ψ^B	3.5
Reserve requirement	$\lambda\sigma_D^2$	0.1
Endowment growth	μ_D	0.02
Endowment volatility	σ_D	0.02
Time preference	ρ	0.01
Death rate	κ	0.01
Type-A share of population	$\overline{\omega}$	0.10
Nominal Rate 1	n_1	0%
Nominal Rate 2	n_2	5%

Table I: **Parameter values.** This table lists the benchmark parameter values used to illustrate the results of the model.

V. Results

To further examine the impact of monetary policy on the economy, we choose values for the model parameters, specify a nominal rate policy, and solve for the resulting equilibrium. Since the model does not permit a closed-form solution, we solve it numerically. This requires solving the HJB equations of the two types of agents simultaneously. We do this using Chebyshev collocation, which produces a global solution. Fernández-Villaverde et al. (2012) emphasize the importance of obtaining global solutions for understanding the nonlinearities inherent in the effects of monetary policy.

A. Parameters

Table I displays our benchmark parameter values. We set the risk aversions of the two agents at 1.5 for type A and 15 for type B in order to generate a substantial demand for risk sharing.

We set the elasticity of intertemporal substitution (EIS) to 3.5 for all agents so that the two types differ only in risk aversion.[18] An EIS value greater than one implies that an increase in effective risk aversion, for example generated by a rise in the nominal interest rate, results in a decrease in the equilibrium wealth-consumption ratio. Thus, as rates rise,

[18]Campbell (1999) estimates an EIS less than one based on a regression of aggregate consumption growth on the real interest rate. Running this regression within our model would produce an estimate that is even lower—in fact zero—as consumption growth is i.i.d. Our model provides an example where this regression is misspecified due to limited risk sharing.

prices fall.

We pick the reserve requirement parameter λ so that $\lambda\sigma_D^2 = 0.1$. Since σ_D^2 is similar in magnitude to σ_t^2, the reserve requirement is about 10%. In practice, this is the reserve requirement for net transactions accounts in the US.

We set the endowment growth rate and volatility to 2%, consistent with standard estimates for US aggregate consumption growth and volatility. We set agents' time preference parameter ρ and death rate κ both to 0.01, which leads to real interest rates near 2%. To stabilize banks' wealth share ω at moderate levels, we set the population share of A agents $\overline{\omega}$ to 10%.

We compare equilibria across two nominal rate policies. In the first policy, the nominal rate is identically zero. In this case, holding reserves is costless, so the model is equivalent to a frictionless one with no reserve requirement. This case represents a useful frictionless benchmark. In the second policy, the nominal rate is at 5%, making reserves costly and constraining leverage. While the model allows for much more complex policy rules, we restrict attention to these simple cases in order to convey the main intuition. We consider dynamic policies later in the paper.

B. Portfolio Choice

Figure 2 shows the impact of higher nominal rates on the holdings of risky claims by banks (top panel) and depositors (bottom panel). The plots show portfolio weights across different values of the wealth distribution ω under policy $n_1 = 0\%$ (blue triangles) and policy $n_2 = 5\%$ (red squares).

As the nominal rate rises, bank leverage falls at every value of the wealth distribution. The drop is larger when banks' wealth is relatively small (low ω). When ω is close to zero, banks' risky asset holdings decrease from around 10 times their net worth to less than 2 times. At moderate levels of ω between 0.2 and 0.4 where the economy spends most of its time, banks' holdings of risky assets decrease from between 2 and 4 times net worth under n_1 to slightly above 1 under n_2, so that a near complete deleveraging takes place.

As the bottom panel of Figure 2 shows, depositor holdings of the risky asset offset the decrease in bank holdings. For instance, when ω is between 0.2 and 0.4, depositors hold 40% of their wealth in risky claims under $n_1 = 0\%$, rising to almost 100% under $n_2 = 5\%$. The shift in the allocation of risk in the direction of the more risk averse depositors is tantamount to increasing the effective risk aversion of the representative investor.

The relationship between the portfolio weight and the wealth share ω in Figure 2 is a result of market clearing. When ω is close to either zero or one, a single type of agent dominates the economy, which reduces the opportunity for risk sharing. Agents of the remaining type must hold all their wealth in the endowment claim, whereas agents of the disappearing type can be satisfied with a vanishingly small amount of borrowing and lending. Thus, when ω is near zero, prices are set by depositors, causing banks to take high leverage as long as the nominal rate is not too high. By contrast, when ω is near one, banks set prices, making risky claims unattractive to depositors unless a high nominal rate keeps the risk premium high.

We see that under $n_2 = 5\%$, reserves are sufficiently costly so that banks take almost no leverage. At even higher levels of n, the economy enters financial autarky (see Corollary 2): the credit market shuts down and both types of agents hold all their wealth in the risky endowment claim at all levels of ω. This is why under $n_2 = 5\%$ portfolio demand is relatively flat in ω for both types of agents.

C. The price of risk and the risk premium

Figure 3 shows how the Sharpe ratio (top panel) and risk premium (bottom panel) of the endowment claim change with the interest rate policy. As noted above, the effective risk aversion in the economy is higher at the higher nominal rate, and this is indeed reflected in a higher Sharpe ratio. At moderate levels of ω between 0.2 and 0.4, the price of risk goes up by a factor of between two and four in going from the low-rate policy $n_1 = 0\%$ to the high-rate policy $n_2 = 5\%$. The effect is even stronger at higher levels of ω, rising to an almost ten-fold increase near $\omega = 1$.

The upper value of the Sharpe ratio near 0.3 is due to the high risk aversion of depositors. When rates are high and depositors are required to hold almost 100% of their wealth in risky claims, the price of risk approaches $\gamma^B \sigma_D$, its value in an economy inhabited solely by the more risk averse agents.

The bottom panel of Figure 3 shows that the increase in the risk premium largely tracks the increase in the Sharpe ratio. At ω between 0.2 and 0.4, the risk premium rises from 0.15–0.3% under $n_1 = 0\%$ to near 0.6% under $n_2 = 5\%$. The small differences in the shapes of the risk premium and Sharpe ratio curves are due to changes in the volatility of the endowment claim induced by the two policies.

D. Volatility

Figure 4 plots the volatility of returns. Although cash flow volatility is constant, return volatility is time varying. Moreover, it exceeds cash flow volatility in a hump-shaped pattern. Under the low-rate policy $n_1 = 0\%$, return volatility peaks near $\omega = 0.2$ at about 2.8%, which is 40% higher than fundamental volatility.

The excess volatility of returns is a result of changes in discount rates. For a given nominal rate policy, the aggregate discount rate is determined by a weighted average of the risky-asset demands of the two agent types. The weights depend on ω. At moderate values of ω, banks take significant leverage and at the same time command enough wealth to affect prices. As a result, in this region endowment shocks have a large effect on banks' wealth share ω. This makes aggregate risk aversion and the discount rate volatile, which in turn makes prices volatile. By contrast, when either type of agent dominates the economy, returns do not change the risk aversion of the representative investor by much and there is little variation in discount rates. Return volatility is then close to fundamental volatility.

Note that excess volatility is much lower under the high-rate policy $n_2 = 5\%$. This is because bank leverage is reduced so that shocks do little to change the wealth distribution, and by extension discount rates. Hence, Figure 4 shows that a low interest rate policy is associated with greater endogenous risk. This result illustrates the potential role for monetary policy in promoting financial stability.

We note that return volatility is higher than fundamental volatility because discount rates are "counter-cyclical". The presence of leverage implies that a positive endowment shock disproportionately raises the net worth of banks, which lowers effective aggregate risk aversion and the discount rate. As a result, endowment shocks and discount rate shocks reinforce each other, amplifying realized returns.

E. The real interest rate

Figure 5 plots the equilibrium real interest rate under the two nominal rate policies. The real rate is lower under the high nominal rate policy $n_2 = 5\%$ than under $n_1 = 0\%$. The difference between the real rates under the two policies is greatest near $\omega = 1$. Recall that the same pattern holds for depositors' portfolio holdings in Figure 2. At high nominal rates, depositors retain a large amount of risk and so the real rate is lower.

It may seem surprising that the increase in the nominal rate has opposing effects on the risk premium and risk free rate. Yet, this is a direct consequence of the higher nominal rate increasing aggregate risk aversion. Higher risk aversion increases both risk prices and precautionary savings. The risk premium rises and the hence the (real) interest rate falls.[19]

We note that the real rate effect can in principle be reversed without affecting the risk

[19]The same result obtains in homogeneous economies in a comparative static with respect to risk aversion. Specifically, in a homogeneous economy with RRA γ and EIS ψ, we have $\frac{\partial}{\partial\gamma}(\mu - r) = \sigma^2 > 0$ and $\frac{\partial}{\partial\gamma} r = -\frac{1}{2}\sigma^2(1 + 1/\psi) < 0$.

premium effect, which is our main focus. For example, in the version of the model developed in Appendix C, this can happen due to depositors' preference for liquidity. Introducing nominal price rigidities would also cause the real and nominal rate to move in tandem.

F. Valuations

Figure 6 plots the wealth-consumption ratio under the two policies. Although a higher nominal rate has opposing effects on the risk premium and real risk-free rate, it has an unambiguous net impact on the value of the endowment claim. For all values of ω, the valuation ratio is higher under the low-rate policy n_1. The effect is strongest near the middle of the state-space where the value of the endowment claim is about 15% higher under n_1.

The sign of the net impact of nominal rates on valuations is a function of the EIS. When the EIS is greater than one, greater risk aversion reduces demand for assets causing valuations to fall. In this case the rise in the risk premium exceeds the fall in the interest rate. In contrast, when the EIS is less than one, the opposite occurs and valuations actually rise in risk aversion.

While the higher nominal rate uniformly decreases valuations, the size of the impact is non-monotonic in ω. In particular, it is highest at intermediate values of ω, when the wealth shares of both banks and depositors are substantial. In this region, the deleveraging induced by high nominal rates has a large impact on the allocation of risk: it causes demand for risky assets and the supply of deposits to shrink substantially. In contrast, when ω is near zero, a reduction in leverage has little effect on allocations since banks hold few assets. Similarly, when ω is close to one, the supply of deposits is low regardless of the nominal rate. Thus, the effect of monetary policy on valuations is largest when aggregate risk sharing (measured either by aggregate leverage or aggregate deposits), is at its greatest extent.

G. Wealth distribution

While the nominal rate has no effect on aggregate leverage when ω equals zero or one, it still has an effect on the price of the endowment claim, as Figure 6 shows. This is due to the impact that the nominal rate has on the dynamics of the wealth distribution. At higher nominal rates, banks take less risk and their wealth tends to grow more slowly. As a result, the stationary distribution for their wealth share ω centers around a lower value. This is shown in Figure 7, which depicts this stationary distribution under the two nominal rate policies obtained by solving the associated forward Kolmogorov equation.

Since a higher nominal rate diminishes the expected future size of banks, it increases the expected aggregate risk aversion of the economy, and hence also discount rates. This dynamic effect on prices via expected future risk aversion is in addition to the local, direct effect of higher nominal rates on risk taking. Below, we further explore the dynamic asset price effects of changes in interest rates by looking at policy shocks and forward guidance.

H. Reserves

Figure 8 plots the ratio of the value of reserves to total wealth (G) under each policy. The wealth share of reserves is very small under the high nominal rate policy $n_2 = 5\%$, and for most values of ω it is much greater under $n_1 = 0\%$. Since higher nominal rates make holding reserves more costly, banks hold less reserves (and take less leverage). Indeed, if the interest rate is high enough as to induce financial autarky (Corollary 2) reserves holdings fall to zero.

In Figure 8, the increase in equilibrium reserves holdings in moving from policy n_2 to a zero-interest rate policy is large. This occurs because under a zero nominal rate there is no cost to holding reserves as they have the same rate of return as bonds.

Figure 8 further shows that reserves holdings depend on the relative size of bank wealth ω. The relationship is non-monotonic. Holding the nominal rate fixed, equilibrium reserves holdings at first increase in banks' wealth, and then start to decrease. This shows that aggregate reserves can both increase and decrease independently of any change in the stance of monetary policy as measured by the nominal rate.

The non-monotonic relationship between banks' wealth and reserve holdings tracks the level of aggregate leverage in the economy, $\omega\left(w_S^A - 1\right)$. When bank wealth ω is small, aggregate leverage is small even though per dollar banks are highly levered ($w_S^A - 1$ is high). As banks' wealth increases, their per-dollar leverage decreases but it does so less rapidly at first as the risk premium remains high. Aggregate leverage therefore increases. As bank wealth continues to rise, however, the drop in the risk premium causes per-dollar leverage to decline faster, and therefore aggregate leverage falls.

VI. Understanding dynamic policies

We now analyze two applications in which dynamic policies play a central role. The first is one in which the central bank has already lowered the nominal rate to zero, and yet wishes to further support asset prices. In the literature this is often referred to as "hitting the zero lower bound". We show how the central bank can use "forward guidance", lowering investors' expectations of future nominal interest rates, to further support asset prices.

Our second application implements and interprets a policy that captures the notion of a

"Greenspan put". Under a Greenspan put, the central bank responds to negative shocks by decreasing nominal interest rates in an effort to stabilize asset prices.

A. The zero lower bound and forward guidance

A zero lower bound arises endogenously in our model. Mathematically speaking, the nominal rate must be nonnegative because it equals the Lagrange multiplier on the reserve holdings constraint. The intuition is that when the nominal rate is at zero, banks are satiated in their demand for risk, as shown in Proposition 2. Their weight in the risky asset then equals its unconstrained optimum, and they have no desire to increase risk taking any further.

If the central bank did try to decrease the nominal rate below zero, banks would borrow deposits to invest in reserves, rather than in risky assets. Since reserves are riskless, this combination would represent an arbitrage. The resulting demand for reserves would force the nominal rate back up to zero. This asymmetry between positive and negative nominal rates reflects the fact that the reserves requirement forces banks to hold a minimum amount of reserves, but does not prevent them from holding excess reserves.

Nevertheless, the central bank can influence asset prices by changing the course of expected future interest rates, i.e. forward guidance. This is illustrated in Figure 9. The top panel plots two nominal rate policies, a benchmark policy and a forward guidance policy. Consider a situation in which bank capital has fallen to a low level as in a financial crisis, and as a result the central bank has lowered the nominal rate to zero. Under the benchmark policy $n_{fg,2}$ (red squares), investors believe the central bank will increase the nominal rate as soon as bank capital has recovered to a value of $\omega = 0.25$. In contrast, under the forward guidance policy $n_{fg,1}$ (blue triangles), the central bank commits to delaying the increase in the nominal rate until $\omega = 0.3$. Hence, under forward guidance, rates are expected to remain low for a prolonged period.

The bottom panel of Figure 9 plots the ratio of the prices of the endowment claim under the two policies, $P_{fg,1}/P_{fg,2}$. Consider the region where ω is less than 0.25, so the central bank has hit the zero lower bound under both policies. The plot shows that the central bank is nevertheless able to induce an increase in asset prices by guiding down expectations of future rates under policy $n_{fg,1}$. Indeed, forward guidance has a substantial impact of on the current price of the endowment claim. For example, for $\omega = 0.25$ the price of the endowment claim is around 4% higher under the forward guidance policy $n_{fg,1}$ than under the benchmark policy $n_{fg,2}$.

Guiding future nominal rates down increases prices by inducing a decrease in future discount rates. Investors expect that assets will be worth more in the future, and they are therefore willing to pay more for them today. Note that this effect is purely dynamic, it does not work by changing the cost of taking leverage today since this cost is already zero.

Finally, note that prices remain higher under forward guidance even at values of $\omega > 0.3$, where rates under the two policies are the same. This happens because investors take into account the positive impact of forward guidance on valuations when bank capital is low.

B. *Greenspan put*

As our second application of a dynamic policy, we implement a "Greenspan put".[20] We interpret a Greenspan put as a policy that reduces nominal interest rates in the event of a large enough sequence of negative shocks. Specifically, we consider the simple example of a constant-rate benchmark, $n_{gp,1}(\omega_t) = 0.04$, versus a Greenspan put alternative:

$$n_{gp,2}(\omega_t) \quad = \quad \min\left\{0.05, \frac{0.05}{0.3}\omega_t\right\}. \tag{31}$$

Under the Greenspan put policy, the nominal rate rises from 0% at $\omega = 0$ at a constant slope until it reaches 5% at $\omega = 0.3$, and then levels off. This implies that a sequence of negative shocks that pushes the bank capital share ω below 0.3 triggers progressive rate cuts. The level of the constant benchmark $n_{gp,1}$ is set so that the two policies have similar unconditional average nominal rates (integrated against the stationary distribution of ω).

The top left panel of Figure 10 plots the two policies, while the top right panel displays the valuation ratio of the endowment claim. The Greenspan put policy $n_{gp,2}$ results in lower prices when bank capital ω is high, as it implements a higher nominal rate. However, when ω approaches the cutoff 0.3 from above, the valuation under $n_{gp,2}$ approaches that under the benchmark $n_{gp,1}$. This occurs because of the nearing prospect of lower nominal rates. As ω falls below 0.3, the valuation under $n_{gp,2}$ flattens out and even mildly increases, whereas it falls under $n_{gp,1}$. In this way, the central bank is supporting asset prices by cutting nominal rates, which increases bank leverage. As ω continues to fall however, there is little room for further increasing leverage. Valuations can no longer be supported, and they start to fall steeply. By the time ω nears zero, prices are nearly equal under the two policies. The Greenspan put policy therefore has the effect of stabilizing prices in a moderate downturn but it cannot forestall a severe price decline in a highly adverse scenario.

The bottom left panel of Figure 10 plots the risk premium. When bank capital is high,

[20] The term dates to the late 1990s when critics faulted Federal Reserve chairman Alan Greenspan for "encouraging excessive risk taking by creating what came to be called 'the Greenspan put', that is, the belief that the Fed would, if necessary, support the economy and therefore the stock market" (Blinder and Reis 2005).

the higher nominal rates of the Greenspan put policy result in a higher risk premium. As ω declines towards 0.3, the stabilization effect of the policy results in lower risk premia. Once ω falls below 0.3, the risk premium drops precipitously as a result of the aggressive rate cutting. However, when ω nears zero, prices are set to fall even more steeply than under the benchmark policy, so the risk premium under the Greenspan put eventually exceeds that under the benchmark.

The bottom right panel of Figure 10 plots volatility. The pattern here is striking. Under the Greenspan put policy $n_{gp,2}$, volatility is lower when ω is high. This is due to the higher nominal rate in this region, which suppresses risk taking and stabilizes aggregate risk aversion. As ω declines towards 0.3, it dips further as the prospect of intervention keeps prices from falling. Below 0.3, the "put" goes "into the money" and the rate cutting kicks in, causing volatility to fall even further, briefly dipping below fundamental volatility. In this way, the Greenspan put is able to reduce volatility in moderate downturns. However, if ω declines even further, the temporary support runs out, and volatility increases sharply. The spike in volatility is due to the high level of leverage built up as a result of the Greenspan put policy.

The results in Figure 10 convey the basic tradeoff that underlies the Greenspan put. On one hand, it achieves short-run stability by boosting leverage in moderate downturns. However, that same leverage build-up leads to instability should the downturn prove severe. Moreover, greater leverage raises the likelihood that bank capital will fall to the low levels associated with a severe downturn. These results formalize the concern that the Greenspan put has short-term benefits and long-term costs (see Blinder and Reis 2005).

VII. Policy shocks

Under the baseline model, the interest-rate policy gives the nominal rate as a function of the single state variable ω. Consequently, there is no independent monetary policy shock. For this reason, we have thus far compared different policies across equilibria. In this section, we extend the model to incorporate an independent shock to the interest rate policy.

We model the monetary policy shock as exogenous. The central bank "surprises the market" by raising or lowering nominal rates independently of the endowment shock. A policy shock has two effects. The first is direct: it changes banks' cost of taking leverage. This effect is present in the baseline comparison across policies. The second, indirect effect, is that a surprise rate change impacts prices and causes the wealth distribution to change. This is called a balance sheet effect in the literature. It produces second-round effects on prices that amplify the direct impact of the rate change. In this way, the extended model features a dynamic that is akin to the financial accelerator.

A. *Model extension*

We extend the policy rule with two objectives in mind. First, we want to allow for a policy shock that is independent of the endowment shock. This shock takes the nominal rate away from the benchmark policy rule that agents know. This benchmark rule is a function of ω as under the baseline model. At the same time, we do not want the nominal rate to stray too far, so that the benchmark remains meaningful.

To that end, let $n_b(\omega_t) \in [\underline{n}, \overline{n}]$ be the benchmark policy rule and let the nominal rate n_t follow the process

$$dn_t = -\kappa_n \Big[n_t - n_b(\omega_t) \Big] dt + (n_t - \underline{n})(\overline{n} - n_t)\, \sigma_n dB_t^n, \tag{32}$$

where dB^n are the policy shocks, which we assume are independent of the endowment shocks dB. The nominal rate reverts towards the benchmark $n_b(\omega_t)$ at the rate κ_n. The structure of the diffusive component implies that n is bounded below by $\underline{n}$ and above by $\overline{n}$.

Because shocks to n are persistent, equilibrium now depends on two state variables, the wealth share ω and the nominal rate n. Hence, we rewrite all endogenous quantities as functions of the two state variables, and we maintain the same notation for the shock exposures with the understanding that they represent 2×1 vectors whose first and second components correspond to the endowment shock dB and the policy shock dB^n. The rest of the model, including the reserve requirement, is unchanged.

We now state the form of the agent's value function and optimal portfolio choice under the extended model, leaving the full derivation to Appendix B.

Proposition 4. *The agents' value functions are given by*

$$V(W, \omega, n) = \left(\frac{W^{1-\gamma}}{1-\gamma} \right) J(\omega, n)^{\frac{1-\gamma}{1-\psi}}, \tag{33}$$

where $J(\omega, n)$ represents the agents' optimal consumption-wealth ratio, $c^* = J$. Banks take leverage ($w_S^A > 1$) if and only if

$$\begin{aligned} \lambda n &< \gamma^B - \gamma^A \\ &\quad - \left[\left(\frac{1-\gamma^A}{1-\psi^A} \right) \frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B} \right) \frac{J_n^B}{J^B} \right] \left[\frac{\frac{F_n}{F} (n - \underline{n})^2 (\overline{n} - n)^2 \sigma_n^2}{\sigma_D^2 + \left(\frac{F_n}{F} \right)^2 (n - \underline{n})^2 (\overline{n} - n)^2 \sigma_n^2} \right]. \end{aligned} \tag{34}$$

In this case, their holdings of the endowment claim are given by

$$w_S^A = \frac{1}{\gamma}\left\{\frac{\mu - r}{\sigma'\sigma} - \lambda n + \left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\omega\left(1-\omega\right)\left(\frac{\sigma'_\omega\sigma}{\sigma'\sigma}\right) + \frac{J_n}{J}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\left(\frac{\sigma_2\sigma_n}{\sigma'\sigma}\right)\right]\right\}. \tag{35}$$

Proof. The proof is contained in Appendix B. □

The solution to the extended model follows that of the benchmark model. We note two differences. First, the portfolio demand (35) includes a hedging term for policy shocks. Second, the boundary of the region in which banks take leverage in $n \times \omega$ space (equation 34) depends on the difference in hedging demands between the two agent types. The reason is that, even though ω becomes locally deterministic in the no-leverage region, n does not.

B. *Results*

To examine the model with policy shocks, we set $n_b = 0.03$, $\underline{n} = 0.00$, $\overline{n} = 0.06$, $\kappa_n = 0.01$, and $\sigma_n = 5$.[21] A high persistence (low κ_n) increases their impact of policy shocks on valuations, while the boundaries $\underline{n}$ and $\overline{n}$ keep the nominal rate from drifting far away from its benchmark.[22]

Figure 11 shows the impact of a policy shock that raises the nominal interest rate from 1% to 4%, at different values of ω. As in the benchmark case, the increase in the nominal rate leads to a higher risk premium, lower real interest rate, and lower valuation of the endowment claim. Since the shock is highly persistent, the effects are similar to the changes observed in the benchmark model across the high and low nominal rate policies.

Figure 11 further shows that policy shocks have a second-round effect on prices. This occurs because in changing prices, the interest rate shock causes a change in banks' capital share. When the nominal rate is low, banks employ high levels of leverage. As a result, the fall in the value of the endowment claim resulting from the surprise increase in the nominal rate causes banks to lose capital disproportionately. The top left panel shows this fall in banks' capital share. The effect is greatest at moderate levels of ω where aggregate bank leverage is highest.

The top right panel shows that this bank balance sheet effect amplifies the first-round fall in prices. The dashed red line isolates the direct effect of the policy shock on the valuation

[21] Thus, the volatility of the nominal rate shock is 0.45% near the benchmark, the point at which it peaks.
[22] Figure B.1 in Appendix B plots the joint stationary density of n and ω under the model to give a sense of the distribution of these state variables.

ratio, calculated by holding ω constant. The total effect, given by the solid red line, also incorporates the additional price impact induced by the change in the bank capital share. By reducing banks' capital, surprise rate hikes reduce their risk-bearing capacity, causing prices to fall further. Hence, the total effect is always greater than the direct effect. This amplification resembles the financial accelerator of Bernanke, Gertler, and Gilchrist (1999), except it is driven by policy shocks.

The bottom panels of Figure 11 decompose the valuation effect of the policy shock into its risk premium and real interest rate components. Looking at the risk premium, the direct effect is large, while the indirect effect is small. This is because under these parameters, at the higher nominal rate bank leverage is low and therefore nearly flat in ω. Turning to the real interest rate, the direct effect of the policy shock is negative, as in the baseline model. However, because the shock shifts the wealth distribution towards the risk averse depositors who have a strong precautionary motive, the indirect effect actually raises the real rate. In this way, the policy shock dampens the fall in the real interest rate while amplifying the rise in the risk premium, leading to a greater fall in prices.

Finally, we note that policy shocks in our model have state-contingent magnitudes. Specifically, a rate hike leads to greater amplification than a rate drop because banks take less leverage at higher rates, so their capital share is less affected.

VIII. Conclusion

Contemporary monetary policy is substantially concerned with the functioning of the financial system and with valuations in the markets for risky assets. Through their effects on financial institutions, central bank interventions drive not only the level of interest rates in the economy, but also the level of risk premia.

We present a dynamic asset pricing framework that enables us to study the relationship between monetary policy and risk premia. The nominal interest rate positively affects the external finance spread that banks pay to obtain leverage, a relationship with strong empirical support. Lower rates lead to greater leverage, and hence lower risk premia. They also lead to higher volatility.

We develop two dynamic applications of our framework, forward guidance at the zero lower bound, and a "Greenspan put". A zero lower bound arises endogenously, reflecting satiation in banks' risk taking. Nevertheless, the central bank can support asset prices further by guiding down expectations of the path of future interest rates. A Greenspan put can stabilize prices locally by boosting leverage, but it leads to greater instability in the face of large shocks.

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Appendix A: Baseline model

Proof of Proposition 1. Conjecture that V has the form in (20). After substituting for V and f from (3), wealth drops out of the HJB equation (19):

$$\begin{aligned} 0 \;=\; & \max_{c,w_S,w_M} \left(\frac{1-\gamma}{1-1/\psi}\right)\left[\left(\frac{c}{J^{\frac{1}{1-\psi}}}\right)^{1-1/\psi} - (\rho+\kappa)\right] \\ & + (1-\gamma)\left[r - c + w_S(\mu - r) - \frac{\gamma}{2}(w_S\sigma)^2 - w_M n + Gn\right] \\ & + \left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\left[\kappa(\overline{\omega}-\omega) + \omega(1-\omega)\mu_\omega\right] + (1-\gamma)\frac{J_\omega}{J}\omega(1-\omega)w_S\sigma_\omega\sigma\right] \\ & + \frac{1}{2}\left(\frac{1-\gamma}{1-\psi}\right)\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)^2 + \frac{J_{\omega\omega}}{J}\right]\omega^2(1-\omega)^2\sigma_\omega^2 \\ & + (1-\gamma)\theta_\lambda\left[w_M - \lambda\sigma^2(w_S - 1)\right] + (1-\gamma)\theta_0 w_M. \end{aligned} \tag{A.1}$$

The FOC for consumption gives

$$c \;=\; J. \tag{A.2}$$

Substituting and rearranging,

$$\begin{aligned} (\rho+\kappa)\left(\frac{1-\gamma}{1-1/\psi}\right) \;=\; & \max_{w_S,w_M} (1-\gamma)\left(\frac{1/\psi}{1-1/\psi}\right)J + (1-\gamma)\left[r + w_S(\mu-r)\right. \\ & \left. - \frac{\gamma}{2}(w_S\sigma)^2 - w_M n + Gn\right] + \left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\left[\kappa(\overline{\omega}-\omega)\right.\right. \\ & \left.\left. + (1-\gamma)\frac{J_\omega}{J}\omega(1-\omega)w_S\sigma_\omega\sigma\right] \right. \\ & + \frac{1}{2}\left(\frac{1-\gamma}{1-\psi}\right)\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)^2 + \frac{J_{\omega\omega}}{J}\right]\omega^2(1-\omega)^2\sigma_\omega^2 \\ & + (1-\gamma)\theta_\lambda\left[w_M - \lambda\sigma^2(w_S-1)\right] + (1-\gamma)\theta_0 w_M. \end{aligned} \tag{A.3}$$

Portfolio demand is characterized by

$$w_S \;=\; \frac{1}{\gamma}\left[\frac{\mu - r}{\sigma^2} - \lambda\theta_\lambda + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega}{J}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right] \tag{A.4}$$

$$n \;=\; \theta_0^B + \theta_\lambda. \tag{A.5}$$

Let

$$\underline{w}_S \;=\; \frac{1}{\gamma}\left[\frac{\mu - r}{\sigma^2} - \lambda n + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega}{J}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right] \tag{A.6}$$

$$\overline{w}_S \;=\; \frac{1}{\gamma}\left[\frac{\mu - r}{\sigma^2} + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega}{J}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right]. \tag{A.7}$$

There are three possible cases:

$$w_S \quad = \quad \begin{cases} \overline{w}_S & \text{if} \quad \overline{w}_S \leq 1 \\ 1 & \text{if} \quad \underline{w}_S \leq 1 < \overline{w}_S \\ \underline{w}_S & \text{if} \quad 1 < \underline{w}_S. \end{cases} \tag{A.8}$$

The corresponding multipliers are

$$\{\theta_0, \theta_\lambda\} \quad = \quad \begin{cases} \{n, 0\} & \text{if} \quad \overline{w}_S \leq 1 \\ \left\{\frac{\gamma}{\lambda}\left(1 - \underline{w}_S\right), \frac{\gamma}{\lambda}\left(\overline{w}_S - 1\right)\right\} & \text{if} \quad \underline{w}_S \leq 1 < \overline{w}_S \\ \{0, n\} & \text{if} \quad 1 < \underline{w}_S. \end{cases} \tag{A.9}$$

Substituting into the HJB equation and simplifying,

$$\begin{aligned} \rho + \kappa \quad = \quad & 1/\psi J + (1 - 1/\psi)\left(r + \lambda\sigma^2\theta_\lambda + Gn\right) - 1/\psi \Bigg(\frac{J_\omega}{J}\left[\kappa\left(\overline{\omega} - \omega\right)\right. \\ & \left. + \omega\left(1-\omega\right)\mu_\omega\right] + \frac{1}{2}\left[\left(\frac{\psi-\gamma}{1-\psi}\right)\left(\frac{J_\omega}{J}\right)^2 + \frac{J_{\omega\omega}}{J}\right]\omega^2\left(1-\omega\right)^2\sigma_\omega^2\Bigg) \\ & + \frac{1}{2}\left(\frac{1-1/\psi}{\gamma}\right)\left[\frac{\mu - r}{\sigma^2} - \lambda\theta_\lambda + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega}{J}\omega\left(1-\omega\right)\frac{\sigma_\omega}{\sigma}\right]^2\sigma^2. \end{aligned} \tag{A.10}$$

The market-clearing equation (17) for the endowment claim implies that only one type of agents, if any, takes leverage, so the equilibrium must be in one of the three cases,

$$w_S^A > 1, \qquad w_S^B < 1 \tag{A.11}$$
$$w_S^A = 1, \qquad w_S^B = 1 \tag{A.12}$$
$$w_S^A < 1, \qquad w_S^B > 1. \tag{A.13}$$

Substituting,

$$\left\{w_S^A, w_S^B\right\} \quad = \quad \begin{cases} \left\{\underline{w}_S^A, \overline{w}_S^B\right\} & \text{if} \quad \overline{w}_S^B \leq 1 < \underline{w}_S^A \\ \{1, 1\} & \text{if} \quad \underline{w}_S^A, \underline{w}_S^B \leq 1 < \overline{w}_S^A, \overline{w}_S^B, \\ \left\{\overline{w}_S^A, \underline{w}_S^B\right\} & \text{if} \quad \overline{w}_S^A \leq 1 < \underline{w}_S^B. \end{cases} \tag{A.14}$$

Call these three cases (i), (ii), and (iii). Under case (i),

$$w_S^A \quad = \quad \frac{1}{\gamma^A}\left[\frac{\mu - r}{\sigma^2} - \lambda n + \left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A}\omega\left(1-\omega\right)\left(\frac{\sigma_\omega}{\sigma}\right)\right] \tag{A.15}$$
$$w_S^B \quad = \quad \frac{1}{\gamma^B}\left[\frac{\mu - r}{\sigma^2} + \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\omega\left(1-\omega\right)\left(\frac{\sigma_\omega}{\sigma}\right)\right]. \tag{A.16}$$

Note that

$$\frac{\sigma_\omega}{\sigma} \quad = \quad \frac{1}{1-\omega}\left(w_S^A - 1\right). \tag{A.17}$$

Stock-market clearing gives

$$
\begin{aligned}
1 &= \omega w_S^A + (1-\omega)\, w_S^B && \text{(A.18)}\\
&= \omega w_S^A + (1-\omega)\,\frac{1}{\gamma^B}\Bigg\{\gamma^A w_S^A + \lambda n && \text{(A.19)}\\
&\quad - \left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\right]\omega\left(w_S^A - 1\right)\Bigg\}.
\end{aligned}
$$

This gives a linear equation for w_S^A in terms of exogenous and conjectured quantities. The solution is

$$
w_S^A = \frac{1 - \frac{1}{\gamma^B}(1-\omega)\lambda n - \frac{1}{\gamma^B}\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\right]\omega(1-\omega)}{\omega + (1-\omega)\frac{\gamma^A}{\gamma^B} - \frac{1}{\gamma^B}\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\right]\omega(1-\omega)}. \tag{A.20}
$$

We need to verify $w_S^A > 1$, which gives

$$
\lambda n < \gamma^B - \gamma^A. \tag{A.21}
$$

From here we can get $(\mu - r)/\sigma^2$, and σ_ω/σ. This also gives σ and hence σ_ω, and as a result, $\mu - r$. To get the drift of ω, apply Ito's Lemma to (4) and use $W^A + W^B = P$ to obtain

$$
\frac{d\omega}{\omega(1-\omega)} = \left(\frac{dW^A}{W^A} - \frac{dW^B}{W^B}\right) - \left(\frac{dW^A}{W^A} - \frac{dW^B}{W^B}\right)\left(\frac{dP}{P}\right). \tag{A.22}
$$

Substituting for the evolution of aggregate type-A and type-B wealth gives (4) and

$$
\mu_\omega = \left(w_S^A - w_S^B\right)(\mu - r) - \lambda\sigma^2\left(w_S^A - 1\right)n - \left(J^A - J^B\right) - \sigma_\omega\sigma. \tag{A.23}
$$

This can be plugged into the dynamics of returns to get μ:

$$
\begin{aligned}
dR &= \frac{dD/F}{D/F} + F dt && \text{(A.24)}\\
&= \frac{dD}{D} - \frac{dF}{F} - \left(\frac{dD}{D}\right)\left(\frac{dF}{F}\right) + \left(\frac{dF}{F}\right)^2 + F dt && \text{(A.25)}\\
\mu &= \mu_D + F - \frac{F_\omega}{F}\omega(1-\omega)\left(\mu_\omega + \sigma_\omega\sigma_D\right) && \text{(A.26)}\\
&\quad + \left[\left(\frac{F_\omega}{F}\right)^2 - \frac{1}{2}\frac{F_{\omega\omega}}{F}\right]\omega^2(1-\omega)^2\sigma_\omega^2\\
\sigma &= \sigma_D - \frac{F_\omega}{F}\omega(1-\omega)\,\sigma_\omega. && \text{(A.27)}
\end{aligned}
$$

From here, get r using $r = \mu - (\mu - r)$. Finally, plug the constraints $\theta_\lambda^A = n$ and $\theta_\lambda^B = 0$ into the HJB equations to verify the conjectures for J^A and J^B. To obtain the value of reserves, use the reserves-market clearing equation (18),

$$\left\{w_M^A, w_M^B\right\} = \left\{\frac{G}{\omega}, 0\right\}. \tag{A.28}$$

The binding leverage constraint pins down the value of reserves:

$$G = \omega\lambda\sigma^2\left(w_S^A - 1\right). \tag{A.29}$$

Under case (ii),

$$\left\{w_S^A, w_S^B\right\} = \{1, 1\} \tag{A.30}$$

$$\left\{w_M^A, w_M^B\right\} = \{0, 0\}. \tag{A.31}$$

The stock market clears and $G = 0$. From here, we get $\sigma_\omega = 0$ and so $\sigma = \sigma_D$. Next, use

$$\mu_\omega = -\left(J^A - J^B\right). \tag{A.32}$$

in the dynamics of returns (A.26) and (A.27) to get μ and σ. Substituting into the HJB equations and simplifying,

$$\rho + \kappa = 1/\psi J + (1 - 1/\psi)\left(\mu - \frac{\gamma}{2}\sigma^2\right) - 1/\psi\frac{J_\omega}{J}\left[\kappa\left(\overline{\omega} - \omega\right) + \omega\left(1 - \omega\right)\mu_\omega\right]. \tag{A.33}$$

This case requires

$$\lambda n > \left|\gamma^A - \gamma^B\right|. \tag{A.34}$$

The real interest rate lies inside a range between a lending and a borrowing rate.

Case (iii) is analogous to Case (i) with the roles reversed. It requires $\lambda n < \gamma^A - \gamma^B$, which is ruled out by assumption. This completes the proof. □

Proof of Proposition 2. Banks take leverage under case (i) in the proof of Proposition 1 above. This case From (A.21), requires $\lambda n < \gamma^B - \gamma^A$. Banks' portfolio demand is then given by (A.15). □

Proof of Proposition 3. Equation (28) follows from the fact that reserves are costly and therefore the reserve requirement binds, see (A.29). To obtain (29), apply Ito's Lemma to $\Pi_t = M_t\pi_t$ and use the fact that inflation $-d\pi_t/\pi_t = \iota(\omega_t)\,dt = [n(\omega_t) - r(\omega_t)]\,dt$ is locally deterministic (equations 9 and 10). Finally, apply Ito's Lemma to $\Pi_t = G_tP_t$ to obtain (30). □

Appendix B: Policy shocks extension

This Appendix contains the derivations for the model with policy shocks. Denote the dynamics of ω by

$$d\omega \quad = \quad \kappa\left(\overline{\omega}-\omega\right)dt+\omega\left(1-\omega\right)\Big[\mu_{\omega}\left(\omega,n\right)dt+\sigma_{\omega}\left(\omega,n\right)'dB\Big]. \tag{B.1}$$

Write the return process

$$\begin{aligned} dR &= \frac{dP+Ddt}{P} && \text{(B.2)}\\ &= \mu\left(\omega,n\right)dt+\sigma\left(\omega,n\right)'dB, && \text{(B.3)}\end{aligned}$$

the instantaneous real risk-free rate as $r=r\left(\omega,n\right)$, and the dividend yield as $F=F\left(\omega,n\right)$. Applying Ito's Lemma gives

$$\begin{aligned} \mu &= \mu_D+F-\frac{F_{\omega}}{F}\left[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\left(\mu_{\omega}+\sigma_{\omega}'\sigma_D\right)\right]-\frac{F_n}{F}\kappa_n\left(n-n_b\right)\\ &\quad+\left[\left(\frac{F_{\omega}}{F}\right)^2-\frac{1}{2}\frac{F_{\omega\omega}}{F}\right]\omega^2\left(1-\omega\right)^2\sigma_{\omega}'\sigma_{\omega}+\left[\left(\frac{F_n}{F}\right)^2-\frac{1}{2}\frac{F_{nn}}{F}\right]\left(n-\underline{n}\right)^2\left(\overline{n}-n\right)^2\sigma_n^2\\ &\quad+2\left[\left(\frac{F_{\omega}}{F}\right)\left(\frac{F_n}{F}\right)-\frac{1}{2}\frac{F_{\omega n}}{F}\right]\omega\left(1-\omega\right)\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_{\omega,2}\sigma_n && \text{(B.4)}\\ \sigma &= \begin{bmatrix}\sigma_D\\0\end{bmatrix}-\frac{F_{\omega}}{F}\omega\left(1-\omega\right)\sigma_{\omega}-\frac{F_n}{F}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\begin{bmatrix}0\\\sigma_n\end{bmatrix}. && \text{(B.5)}\end{aligned}$$

The reserve requirement is

$$w_M \quad \geq \quad \max\Big[\lambda\sigma'\sigma\left(w_S-1\right),0\Big]. \tag{B.6}$$

The wealth dynamics are as in the benchmark case.

Proof of Proposition 4. Dropping agent subscripts and applying Lagrange multipliers $\theta_{\lambda}V_WW$ and θ_0V_WW on the leverage and non-negativity constraints, the HJB equation is

$$\begin{aligned} 0 &= \max_{c,w_S,w_M} f\left(cW,V\right)+V_WW\left[r-c+w_S\left(\mu-r\right)-w_Mn+Gn\right] && \text{(B.7)}\\ &\quad+V_{\omega}\left[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\mu_{\omega}\right]+V_n\kappa_n\left(n-n_b\right)\\ &\quad+V_{W\omega}W\omega\left(1-\omega\right)w_S\sigma_{\omega}'\sigma+V_{Wn}Ww_S\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_2\sigma_n+\frac{1}{2}V_{WW}W^2w_S^2\sigma'\sigma\\ &\quad+\frac{1}{2}V_{\omega\omega}\omega^2\left(1-\omega\right)^2\sigma_{\omega}'\sigma_{\omega}+V_{\omega n}\omega\left(1-\omega\right)\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_n\sigma_{\omega,2}\\ &\quad+\frac{1}{2}V_{nn}\left(n-\underline{n}\right)^2\left(\overline{n}-n\right)^2\sigma_n^2+\theta_{\lambda}V_WW\left[w_M-\lambda\sigma'\sigma\left(w_S-1\right)\right]+\theta_0V_WWw_M.\end{aligned}$$

Conjecture that the value function has the form

$$V(W,\omega,n) = \left(\frac{W^{1-\gamma}}{1-\gamma}\right) J(\omega,n)^{\frac{1-\gamma}{1-\psi}}. \tag{B.8}$$

Then wealth drops out of the HJB equation:

$$\begin{aligned}
0 = & \max_{c,w_S,w_M} \frac{1}{1-1/\psi}\left[c^{1-1/\psi}J^{1/\psi}-(\rho+\kappa)\right]+r-c+w_S(\mu-r)-\frac{\gamma}{2}w_S^2\sigma'\sigma \\
& -w_M n+Gn-\frac{1/\psi}{1-1/\psi}\left[\frac{J_\omega}{J}\left[\kappa(\overline{\omega}-\omega)+\omega(1-\omega)\mu_\omega\right]+\frac{J_n}{J}\kappa_n(n-n_b)\right. \\
& \left.+(1-\gamma)\frac{J_\omega}{J}\omega(1-\omega)w_S\sigma_\omega'\sigma+(1-\gamma)\frac{J_n}{J}w_S(n-\underline{n})(\overline{n}-n)\sigma_2\sigma_n\right] \\
& -\frac{1}{2}\frac{1/\psi}{1-1/\psi}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)^2+\frac{J_{\omega\omega}}{J}\right]\omega^2(1-\omega)^2\sigma_\omega'\sigma_\omega \\
& -\frac{1/\psi}{1-1/\psi}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)\left(\frac{J_n}{J}\right)+\frac{J_{\omega n}}{J}\right]\omega(1-\omega)(n-\underline{n})(\overline{n}-n)\sigma_n\sigma_\omega \\
& -\frac{1}{2}\frac{1/\psi}{1-1/\psi}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_n}{J}\right)^2+\frac{J_{nn}}{J}\right](n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2 \\
& +\theta_\lambda\left[w_M-\lambda\sigma'\sigma(w_S-1)\right]+\theta_0 w_M.
\end{aligned} \tag{B.9}$$

The FOC for consumption gives

$$c = J. \tag{B.10}$$

Substituting and rearranging,

$$\begin{aligned}
0 = & \max_{w_S,w_M} \frac{1/\psi J-(\rho+\kappa)}{1-1/\psi}+r+w_S(\mu-r)-\frac{\gamma}{2}w_S^2\sigma'\sigma-w_M n+Gn \\
& -\frac{1/\psi}{1-1/\psi}\left[\frac{J_\omega}{J}\left[\kappa(\overline{\omega}-\omega)+\omega(1-\omega)\mu_\omega\right]+\frac{J_n}{J}\kappa_n(n-n_b)\right. \\
& \left.+(1-\gamma)\frac{J_\omega}{J}\omega(1-\omega)w_S\sigma_\omega'\sigma+(1-\gamma)\frac{J_n}{J}w_S(n-\underline{n})(\overline{n}-n)\sigma_2\sigma_n\right] \\
& -\frac{1}{2}\frac{1/\psi}{1-1/\psi}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)^2+\frac{J_{\omega\omega}}{J}\right]\omega^2(1-\omega)^2\sigma_\omega'\sigma_\omega \\
& -\frac{1/\psi}{1-1/\psi}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)\left(\frac{J_n}{J}\right)+\frac{J_{\omega n}}{J}\right]\omega(1-\omega)(n-\underline{n})(\overline{n}-n)\sigma_n\sigma_{\omega,2} \\
& -\frac{1}{2}\frac{1/\psi}{1-1/\psi}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_n}{J}\right)^2+\frac{J_{nn}}{J}\right](n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2 \\
& +\theta_\lambda\left[w_M-\lambda\sigma'\sigma(w_S-1)\right]+\theta_0 w_M.
\end{aligned} \tag{B.11}$$

Portfolio demand is characterized by

$$\begin{aligned} w_S &= \frac{1}{\gamma}\left\{\frac{\mu-r}{\sigma'\sigma}-\lambda\theta_\lambda\right. & \text{(B.12)} \\ &\quad\left.+\left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\omega\left(1-\omega\right)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right)+\frac{J_n}{J}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\left(\frac{\sigma_2\sigma_n}{\sigma'\sigma}\right)\right]\right\} \\ n &= \theta_0^B+\theta_\lambda. & \text{(B.13)} \end{aligned}$$

Let

$$\begin{aligned} \underline{w}_S &= \frac{1}{\gamma}\left\{\frac{\mu-r}{\sigma'\sigma}-\lambda n\right. & \text{(B.14)} \\ &\quad\left.+\left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\omega\left(1-\omega\right)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right)+\frac{J_n}{J}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\left(\frac{\sigma_2\sigma_n}{\sigma'\sigma}\right)\right]\right\} \\ \overline{w}_S &= \frac{1}{\gamma}\left\{\frac{\mu-r}{\sigma'\sigma}\right. & \text{(B.15)} \\ &\quad\left.+\left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\omega\left(1-\omega\right)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right)+\frac{J_n}{J}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\left(\frac{\sigma_2\sigma_n}{\sigma'\sigma}\right)\right]\right\}. \end{aligned}$$

There are three possible cases:

$$w_S = \begin{cases} \overline{w}_S & \text{if} \quad \overline{w}_S \le 1 \\ 1 & \text{if} \quad \underline{w}_S \le 1 < \overline{w}_S \\ \underline{w}_S & \text{if} \quad 1 < \underline{w}_S. \end{cases} \tag{B.16}$$

The corresponding multipliers are

$$\{\theta_0,\theta_\lambda\} = \begin{cases} \{n,0\} & \text{if} \quad \overline{w}_S \le 1 \\ \left\{\frac{\gamma}{\lambda}\left(1-\underline{w}_S\right),\frac{\gamma}{\lambda}\left(\overline{w}_S-1\right)\right\} & \text{if} \quad \underline{w}_S \le 1 < \overline{w}_S \\ \{0,n\} & \text{if} \quad 1 < \underline{w}_S. \end{cases} \tag{B.17}$$

Substituting into the HJB equation and simplifying,

$$\begin{aligned} \rho+\kappa &= 1/\psi J+\left(1-1/\psi\right)\left(r+\lambda\sigma'\sigma\theta_\lambda+Gn+\frac{\gamma}{2}w_S^2\sigma'\sigma\right) & \text{(B.18)} \\ &\quad -1/\psi\left[\frac{J_\omega}{J}\omega\left(1-\omega\right)\left[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\mu_\omega\right]+\frac{J_n}{J}\kappa_n\left(n-n_b\right)\right] \\ &\quad -\frac{1/\psi}{2}\left[\frac{J_{\omega\omega}}{J}\omega^2\left(1-\omega\right)^2\sigma_\omega'\sigma_\omega+2\frac{J_{\omega n}}{J}\omega\left(1-\omega\right)\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_n\sigma_{\omega,2}\right. \\ &\quad \left.+\frac{J_{nn}}{J}\left(n-\underline{n}\right)^2\left(\overline{n}-n\right)^2\sigma_n^2\right]-\frac{1/\psi}{2}\left(\frac{\psi-\gamma}{1-\psi}\right)\left[\left(\frac{J_\omega}{J}\right)\omega\left(1-\omega\right)\sigma_\omega\right. \\ &\quad \left.+\left(\frac{J_n}{J}\right)\left(n-\underline{n}\right)\left(\overline{n}-n\right)\begin{bmatrix}0\\ \sigma_n\end{bmatrix}\right]'\left[\left(\frac{J_\omega}{J}\right)\omega\left(1-\omega\right)\sigma_\omega+\left(\frac{J_n}{J}\right)\left(\overline{n}-n\right)\begin{bmatrix}0\\ \sigma_n\end{bmatrix}\right]. \end{aligned}$$

The markets for goods, stocks, and reserves must clear (the bond market clears by Walras' law):

$$\begin{aligned} \omega c^A + (1-\omega)\, c^B &= F && \text{(B.19)} \\ \omega w_S^A + (1-\omega)\, w_S^B &= 1 && \text{(B.20)} \\ \omega w_M^A + (1-\omega)\, w_M^B &= G. && \text{(B.21)} \end{aligned}$$

Market-clearing implies that only one type of agents, at most, takes leverage, so there are three possible cases in equilibrium:

$$\begin{aligned} w_S^A > 1, &\quad w_S^B < 1 && \text{(B.22)} \\ w_S^A = 1, &\quad w_S^B = 1 && \text{(B.23)} \\ w_S^A < 1, &\quad w_S^B > 1. && \text{(B.24)} \end{aligned}$$

Call these three cases (i), (ii), and (iii). Under case (i),

$$\begin{aligned} w_S^A &= \frac{1}{\gamma^A}\left\{\frac{\mu - r}{\sigma'\sigma} - \lambda n \right. && \text{(B.25)} \\ &\left. + \left(\frac{1-\gamma^A}{1-\psi^A}\right)\left[\frac{J_\omega^A}{J^A}\omega(1-\omega)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right) + \frac{J_n^A}{J^A}(n-\underline{n})(\overline{n}-n)\left(\frac{\sigma_2\sigma_n}{\sigma'\sigma}\right)\right]\right\} \\ w_S^B &= \frac{1}{\gamma^B}\left\{\frac{\mu - r}{\sigma'\sigma}\right. && \text{(B.26)} \\ &\left. + \left(\frac{1-\gamma^B}{1-\psi^B}\right)\left[\frac{J_\omega^B}{J^B}\omega(1-\omega)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right) + \frac{J_n^B}{J^B}(n-\underline{n})(\overline{n}-n)\left(\frac{\sigma_2\sigma_n}{\sigma'\sigma}\right)\right]\right\}. \end{aligned}$$

Note that

$$\begin{aligned} \frac{\sigma_\omega'\sigma}{\sigma'\sigma} &= \frac{1}{1-\omega}\left(w_S^A - 1\right) && \text{(B.27)} \\ \frac{\sigma_2\sigma_n}{\sigma'\sigma} &= \left[1 + \frac{F_\omega}{F}\omega\left(w_S^A - 1\right)\right]\frac{-\frac{F_n}{F}(n-\underline{n})(\overline{n}-n)\sigma_n^2}{\sigma_D^2 + \left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}. \end{aligned}$$

Stock-market clearing gives

$$\begin{aligned} 1 &= \omega w_S^A + (1-\omega)\, w_S^B && \text{(B.28)} \\ &= \omega w_S^A + (1-\omega)\frac{1}{\gamma^B}\left\{\gamma^A w_S^A + \lambda n \right. && \text{(B.29)} \\ &- \left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\right]\omega\left(w_S^A - 1\right) - \left[1 + \frac{F_\omega}{F}\omega\left(w_S^A - 1\right)\right] \\ &\left. \cdot \left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_n^B}{J^B}\right]\left[\frac{-\frac{F_n}{F}(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}{\sigma_D^2 + \left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}\right]\right\}. \end{aligned}$$

This gives a linear equation for w_S^A in terms of exogenous and conjectured quantities. The solution is

$$w_S^A = \frac{\begin{array}{c} 1-\frac{1}{\gamma^B}(1-\omega)\lambda n - \frac{1}{\gamma^B}\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\right]\omega(1-\omega) \\ -\frac{1}{\gamma^B}\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_n^B}{J^B}\right]\left[\frac{\frac{F_n}{F}\left[(1-\omega)-\frac{F_\omega}{F}\omega(1-\omega)\right](n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}{\sigma_D^2+\left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}\right] \end{array}}{\begin{array}{c} \omega+(1-\omega)\frac{\gamma^A}{\gamma^B} - \frac{1}{\gamma^B}\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_\omega^B}{J^B}\right]\omega(1-\omega) \\ -\frac{1}{\gamma^B}\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_n^B}{J^B}\right]\left[\frac{\frac{F_n}{F}\left[-\frac{F_\omega}{F}\omega(1-\omega)\right](n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}{\sigma_D^2+\left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}\right] \end{array}} \tag{B.30}$$

We need to verify $w_S^A > 1$:

$$\begin{aligned} \lambda n &< \gamma^B - \gamma^A \\ &\quad - \left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_n^B}{J^B}\right]\left[\frac{\frac{F_n}{F}(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}{\sigma_D^2+\left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}\right]. \end{aligned} \tag{B.31}$$

From here we can get $(\mu - r)/(\sigma'\sigma)$, $(\sigma_\omega'\sigma)/(\sigma'\sigma)$, and $(\sigma_2\sigma_n)/(\sigma'\sigma)$. This also gives σ and hence σ_ω, and as a result, $\mu - r$.

Next, calculate w_S^B (verify $w_S^B < 1$) and calculate

$$\mu_\omega = \left(w_S^A - w_S^B\right)(\mu - r) - \lambda\sigma'\sigma\left(w_S^A - 1\right)n - \left(J^A - J^B\right) - \sigma_\omega'\sigma. \tag{B.32}$$

This can be plugged into the dynamics of returns to get μ, which also gives r. Finally, plug the constraints $\theta_\lambda^B = n$ and $\theta_\lambda^A = 0$ into the HJB equations to verify the conjectures for J^A and J^B.

Money-market clearing gives

$$\left\{w_M^A, w_M^B\right\} = \left\{0, \frac{G}{1-\omega}\right\}. \tag{B.33}$$

The binding leverage constraint pins down the value of reserves:

$$G = (1-\omega)\lambda\sigma'\sigma\left(w_S^B - 1\right). \tag{B.34}$$

Under case (ii),

$$\left\{w_S^A, w_S^B\right\} = \{1, 1\} \tag{B.35}$$

$$\left\{w_M^A, w_M^B\right\} = \{0, 0\}. \tag{B.36}$$

The stock market clears and $G = 0$. From here, we get $\sigma_\omega = 0$ and so $\sigma = \sigma_D$. Next, use $\mu_\omega = -\left(J^A - J^B\right)$ in the dynamics of returns to get μ and σ. Substituting into the HJB equations and simplifying,

$$\begin{aligned}\rho+\kappa &= 1/\psi J+(1-1/\psi)\left[\mu+\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_n}{J}(n-\underline{n})(\overline{n}-n)\sigma_n\sigma_2-\frac{\gamma}{2}\sigma'\sigma\right] \\ &\quad -1/\psi\left[\frac{J_\omega}{J}\left[\kappa(\overline{\omega}-\omega)+\omega(1-\omega)\mu_\omega\right]+\frac{J_n}{J}\kappa_n(n-n_b)\right] \\ &\quad -\frac{1/\psi}{2}\left[\left(\frac{\psi-\gamma}{1-\psi}\right)\left(\frac{J_n}{J}\right)^2+\frac{J_{nn}}{J}\right](n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2.\end{aligned} \tag{B.37}$$

This case requires

$$\begin{aligned}\lambda n &> \Bigg|\gamma^A-\gamma^B \\ &\quad +\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_n^A}{J^A}-\left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_n^B}{J^B}\right]\left(\frac{\frac{F_n}{F}(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}{\sigma_D^2+\left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}\right)\Bigg|\end{aligned} \tag{B.38}$$

The real interest rate lies inside a range between a lending and a borrowing rate.

Case (*iii*) is analogous to Case (*i*) with the roles reversed. It requires

$$\begin{aligned}\lambda n &< \gamma^A-\gamma^B \\ &\quad +\left[\left(\frac{1-\gamma^A}{1-\psi^A}\right)\frac{J_n^A}{J^A}-\left(\frac{1-\gamma^B}{1-\psi^B}\right)\frac{J_n^B}{J^B}\right]\left[\frac{\frac{F_n}{F}(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}{\sigma_D^2+\left(\frac{F_n}{F}\right)^2(n-\underline{n})^2(\overline{n}-n)^2\sigma_n^2}\right].\end{aligned} \tag{B.39}$$

This completes the proof. □

We solve the model using Chebyshev collocation with complete polynomials up to order N in ω and n with $N=30$.

Appendix C: "Sticky deposits" and the external finance spread

Here we develop a parallel channel underlying the relationship between the nominal interest rate and banks' external finance spread based on a liabilities-side friction rather than the asset-side friction presented in the main body of the paper. Both channels induce the same equation linking banks demand for leverage to the nominal interest rate and therefore have the same implications for risk premia.

The setup parallels that in the benchmark model. We focus on the points of departure. The first is that B agents now have a preference for liquidity services, which they derive from bank deposits. Let w_M be the portfolio share of deposits. We modify the preferences of type-B agents to include a current period utility flow from deposits:

$$f^B = \left(\frac{1-\gamma^B}{1-1/\psi^B}\right) V \left[\left(\frac{C}{\left[\left(1-\gamma^B\right)V\right]^{1/(1-\gamma^B)}}\right)^{1-1/\psi^B} + \left(1-1/\psi^B\right)\chi\left(w_M^B\right) - (\rho+\kappa)\right]. \tag{C.1}$$

The utility from deposits is given by the function χ, which we assume is increasing, concave, and satisfies the Inada conditions. There are no other changes to preferences relative to the benchmark model. A preference for liquidity makes households willing to supply deposits at a low rate. For simplicity we leave agent A's preferences unchanged, but the model can be modified so that they also value liquidity.

Deposits are a long-lived asset that we take as numeraire. Each dollar of deposits is worth π_t units of consumption (the inverse price level). As in the main model, the central bank controls the inflation rate $-d\pi/\pi$ so that it is locally deterministic, $-d\pi_t/\pi_t = i\left(\omega_t\right)dt$. The real rate is r and the nominal rate is $n\left(\omega_t\right) = r\left(\omega_t\right) + i\left(\omega_t\right)$. Again, we think of n (or i) as an exogenous policy variable.

There are two key assumptions. The first is that the rate households earn on deposits is low and "sticky", meaning that it does not adjust one-for-one with market rates. Deposit stickiness has been documented extensively in the banking literature (e.g. Driscoll and Judson 2013). It implies that the spread between the deposit rate and and the nominal rate is increasing in the nominal rate. To keep things simple, we assume deposits earn zero nominal interest, although any positive fraction of the market rate would do.

Deposits are a particular debt liability of A agents, the banks, to be contrasted with non-deposits (such as Fed Funds). Deposits provide households with liquidity, whereas non-deposits do not. As a result, non-deposits have a higher equilibrium rate of return, making them expensive as a source of funding for banks.

The second assumption is that non-deposits require less collateral than deposits, which sets up a tradeoff between leverage and funding cost. To formalize this, consider a bank with risky assets (over equity) w_S^A and suppose that the fraction of these assets that is pledgeable as collateral is $\left(1-\alpha_S\right)w_S$, with $0 < \alpha_S < 1$. Further suppose that each dollar of deposits requires a dollar of pledgeable collateral, whereas a dollar of non-deposits requires only $1-\alpha_B$ dollars of pledgeable collateral, with $0 < \alpha_B < \alpha_S$. Letting w_M^A be the bank's deposit holdings (so $w_M^A < 0$ when the bank is issuing deposits), and letting w_B^A similarly be the bank's non-deposit or bond holdings, the bank must have enough collateral to pledge against its liabilities:

$$-w_{M,t}^A - \left(1-\alpha_B\right)w_{B,t}^A \leq \left(1-\alpha_S\right)w_{S,t}^A. \tag{C.2}$$

The left side of this equation gives the pledgeable collateral required for the bank's liabilities, and the right side gives its total pledgeable capacity.

Imperfect pledgeability of assets is a widespread assumption in the literature. It can be motivated by a lack of commitment as in Kiyotaki and Moore (1997), moral hazard as in Holmström and Tirole (1998), or an arbitrarily small probability of a crash as in Moreira and Savov (2014). The assumption that deposits require greater collateralization than non-deposits follows the information-sensitivity argument in Gorton and Pennacchi (1990). This argument stresses the idea that money-like instruments must be sufficiently collateralized to dissuade agents from acquiring private information that could undermine their liquidity.

The collateral constraint (C.2) can be rewritten in a way that parallels the reserve requirement (7) in the text. Using $w_M + w_B + w_S = 1$ in equation (C.2)) and rearranging gives

$$w_{M,t}^A \geq \left(\frac{\alpha_S - \alpha_B}{\alpha_B}\right) w_{S,t}^A - \frac{1-\alpha_B}{\alpha_B}. \tag{C.3}$$

If we then let $\alpha_S = (1 + \lambda\sigma_t^2)\,\alpha_B$, then the constraint becomes

$$w_{M,t}^A \geq \lambda\sigma_t^2 w_{S,t}^A - \frac{1-\alpha_B}{\alpha_B}. \tag{C.4}$$

This says that for each dollar of extra funding ($w_{S,t}^A$), banks must shrink their deposit funding ($w_{M,t}^A$) by $\lambda\sigma_t^2$ dollars.

Because the deposit rate is fixed, the amounts of deposits demanded by banks may not equal the amount supplied by depositors. In the reserves-based model, the central bank clears the money market. Here we instead model a new institution that fulfills this role, which we call C banks, making them as simple as possible.

C banks issue deposits equal to the discrepancy between deposit supply coming from B households, and deposit demand coming from A banks, $G(\omega_t) = \omega w_{M,t}^A + (1-\omega)\, w_{M,t}^B$. On the asset side, C banks make loans to A banks at the non-deposit rate. This means that C banks earn a spread $n_t G_t$. To minimize distortions, we assume this spread is refunded to all agents in proportion to their wealth. Our interpretation of C banks is that they fulfill the role of regional retail banks, funneling deposits to the risk-taking national banks via interbank lending.

The optimization problem of B agents is similar to that in the main model except for the inclusion of the demand for liquidity. The HJB equation of B agents is

$$0 = \max_{c, w_S, w_M} f^B(cW, V, w_M)\,dt + E\left[dV^B(W, \omega)\right] \tag{C.5}$$

subject to the wealth dynamics

$$\frac{dW}{W} = [r - c + w_S(\mu - r) - w_M n + Gn]\,dt + w_S \sigma dB, \tag{C.6}$$

where Gn is the refund from the C banks. The deposit-taking constraint does not bind in equilibrium for B agents, so we leave it out. Then, by Ito's Lemma, the Lagrangian is

$$\begin{aligned} 0 = \max_{c, w_S, w_M} & f^B(cW, V, w_M) + V_W^B W[r - c + w_S(\mu - r) - w_M n + Gn] \\ & + V_\omega^B[\kappa(\omega_0 - \omega) + \omega(1-\omega)\mu_\omega] + V_{W\omega}^B W\omega(1-\omega) w_S \sigma_\omega \sigma + \frac{1}{2} V_{WW}^B W^2 (w_S\sigma)^2 \\ & + \frac{1}{2} V_{\omega\omega}^B \omega^2 (1-\omega)^2 \sigma_\omega^2. \end{aligned} \tag{C.7}$$

Conjecture that the value function has the form

$$V^B(W,\omega) = \left(\frac{W^{1-\gamma}}{1-\gamma}\right) J^B(\omega)^{\frac{1-\gamma}{1-\psi}}. \tag{C.8}$$

Wealth drops out of the HJB equation. The FOC for consumption gives $c = J$. B agents' portfolio demand is characterized by

$$w_S^B = \frac{1}{\gamma}\left[\frac{\mu - r}{\sigma^2} + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega^B}{J^B}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right] \tag{C.9}$$

$$\chi'(w_M) = n. \tag{C.10}$$

The supply of deposits is given by $w_M = \chi'(n)^{-1}$ and is decreasing in n. Substituting into the HJB equation and simplifying gives

$$\begin{aligned}\rho + \kappa = {} & 1/\psi J^B + (1 - 1/\psi)\left[r + \chi\left(\chi'(n)^{-1}\right) - \chi'(n)^{-1} n + Gn\right] \\ & -1/\psi\left(\frac{J_\omega^B}{J^B}\left[\kappa(\omega_0 - \omega) + \omega(1-\omega)\mu_\omega\right]\right. \\ & \left. + \frac{1}{2}\left[\left(\frac{\psi-\gamma}{1-\psi}\right)\left(\frac{J_\omega^B}{J^B}\right)^2 + \frac{J_{\omega\omega}^B}{J^B}\right]\omega^2(1-\omega)^2\sigma_\omega^2\right) \\ & + \frac{1}{2}\left(\frac{1-1/\psi}{\gamma}\right)\left[\frac{\mu - r}{\sigma^2} + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega^B}{J^B}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right]^2\sigma^2.\end{aligned} \tag{C.11}$$

The magnitude of the effect of n on the intertemporal decision of B agents depends on $\chi\left(\chi'(n)^{-1}\right) - \chi'(n)^{-1} n$, which depends on the curvature of χ. If we use $\chi(w_M) = w_M^{1-\eta}/(1-\eta)$, then higher rates induce B agents to save less. This force pushes real and nominal rates in the same direction. The optimization problem of A agents is

$$0 = \max_{c, w_S, w_M} f^A(cW, V)\,dt + E\left[dV^A(W,\omega)\right] \tag{C.12}$$

subject to the wealth dynamics and the deposit-taking constraint

$$\frac{dW}{W} = \left[r - c + s(\mu - r) - mn + Gn\right]dt + s\sigma dB \tag{C.13}$$

$$w_M \geq \lambda\sigma^2 w_S - \frac{1-\alpha_B}{\alpha_B}. \tag{C.14}$$

Let $\theta V_W^A W \geq 0$ be the Lagrange multiplier on the constraint. Then, by Ito's Lemma, the

Lagrangian is

$$
\begin{aligned}
0 \quad = \quad & \max_{c, w_S} \; f^A (cW, V) + V_W^A W \left[r - c + w_S (\mu - r) - w_M n + Gn \right] \\
& + V_\omega^A \left[\kappa (\omega_0 - \omega) + \omega (1 - \omega) \mu_\omega \right] + V_{W\omega}^A W \omega (1 - \omega) w_S \sigma_\omega \sigma + \frac{1}{2} V_{WW}^A W^2 (w_S \sigma)^2 \\
& + \frac{1}{2} V_{\omega\omega} \omega^2 (1 - \omega)^2 \sigma_\omega^2 + \theta V_W^A W \left[m - \left(\lambda \sigma^2 w_S - \frac{1 - \alpha_B}{\alpha_B} \right) \right].
\end{aligned}
\tag{C.15}
$$

Conjecture that the value function has the form

$$
V^A (W, \omega) \quad = \quad \left(\frac{W^{1-\gamma}}{1 - \gamma} \right) J^A (\omega)^{\frac{1-\gamma}{1-\psi}}.
\tag{C.16}
$$

Wealth drops out of the HJB equation. The FOC for consumption gives $c = J$ and portfolio demand is characterized by

$$
\theta \quad = \quad n
\tag{C.17}
$$

$$
w_S^A \quad = \quad \frac{1}{\gamma} \left[\frac{\mu - r}{\sigma^2} - \lambda n + \left(\frac{1 - \gamma}{1 - \psi} \right) \frac{J_\omega^A}{J^A} \omega (1 - \omega) \frac{\sigma_\omega}{\sigma} \right].
\tag{C.18}
$$

This expression for banks demand for the risky endowment claim matches that in the benchmark model. Substituting into the HJB equation and simplifying,

$$
\begin{aligned}
\rho + \kappa \quad = \quad & 1/\psi J^A + (1 - 1/\psi) \left(r + Gn + \frac{1 - \alpha_B}{\alpha_B} n \right) - 1/\psi \left(\frac{J_\omega^A}{J^A} \left[\kappa (\omega_0 - \omega) + \omega (1 - \omega) \mu_\omega \right] \right. \\
& \left. + \frac{1}{2} \left[\left(\frac{\psi - \gamma}{1 - \psi} \right) \left(\frac{J_\omega^A}{J^A} \right)^2 + \frac{J_{\omega\omega}^A}{J^A} \right] \omega^2 (1 - \omega)^2 \sigma_\omega^2 \right) \\
& + \frac{1}{2} \left(\frac{1 - 1/\psi}{\gamma} \right) \left[\frac{\mu - r}{\sigma^2} - \lambda n + \left(\frac{1 - \gamma}{1 - \psi} \right) \frac{J_\omega^A}{J^A} \omega (1 - \omega) \frac{\sigma_\omega}{\sigma} \right]^2 \sigma^2.
\end{aligned}
\tag{C.19}
$$

The markets for goods, the endowment claim, and deposits must clear. Since all wealth is ultimately invested in the endowment claim, $W^A + W^B = P$.

$$
\omega c^A + (1 - \omega) c^B \quad = \quad F
\tag{C.20}
$$

$$
\omega w_S^A + (1 - \omega) w_S^B \quad = \quad 1
\tag{C.21}
$$

$$
\omega w_M^A + (1 - \omega) w_M^B \quad = \quad G.
\tag{C.22}
$$

The conditions for market clearing are therefore the same as in the benchmark model.

The solution follows that of the benchmark model with the above modifications to the HJB equations. The drift of ω now accounts for the fact that B agents now hold deposits:

$$
\mu_\omega \quad = \quad \left(s^A - s^B \right) (\mu - r) + \frac{1}{\omega} \chi' (n)^{-1} n - \left(J^A - J^B \right) - \sigma_\omega \sigma.
\tag{C.23}
$$

The final step is to verify that n is not too high, so that $w_S^A > 0$.

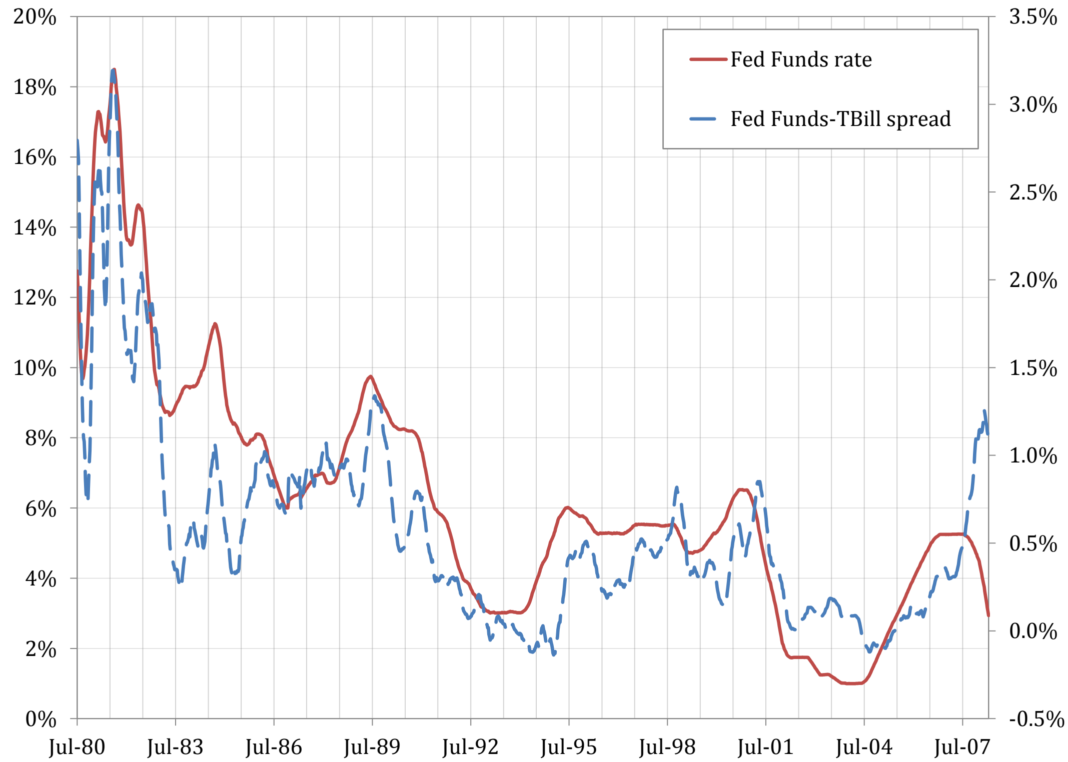

Figure 1: **Fed Funds-TBill spread vs. Fed Funds rate.** The figure plots the 20-week moving averages of the Fed Funds rate (solid red line) and the difference between the Fed Funds rate and the 3-month Treasury Bill rate (dashed blue line). The sample is 7/25/1980 to 5/9/2008.

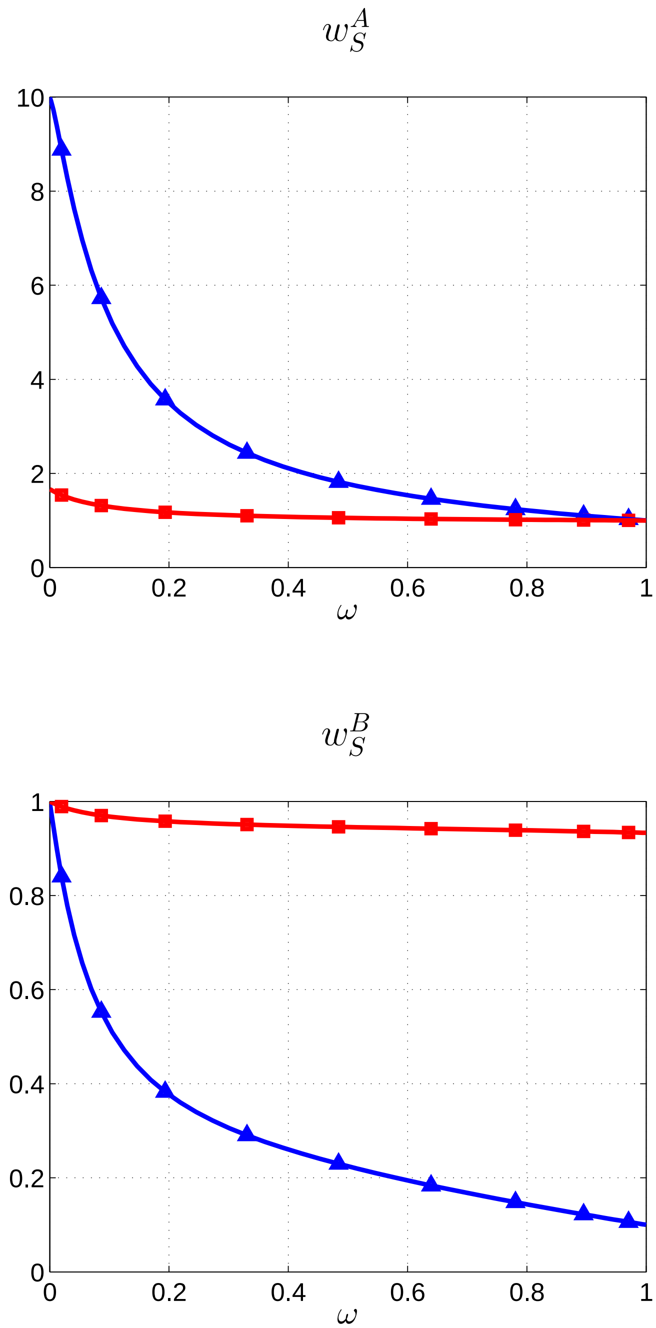

Figure 2: **Risk taking.** The figure plots the risky claim portfolio weight for agent A (top panel) and agent B (bottom panel) under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest-rate policies.

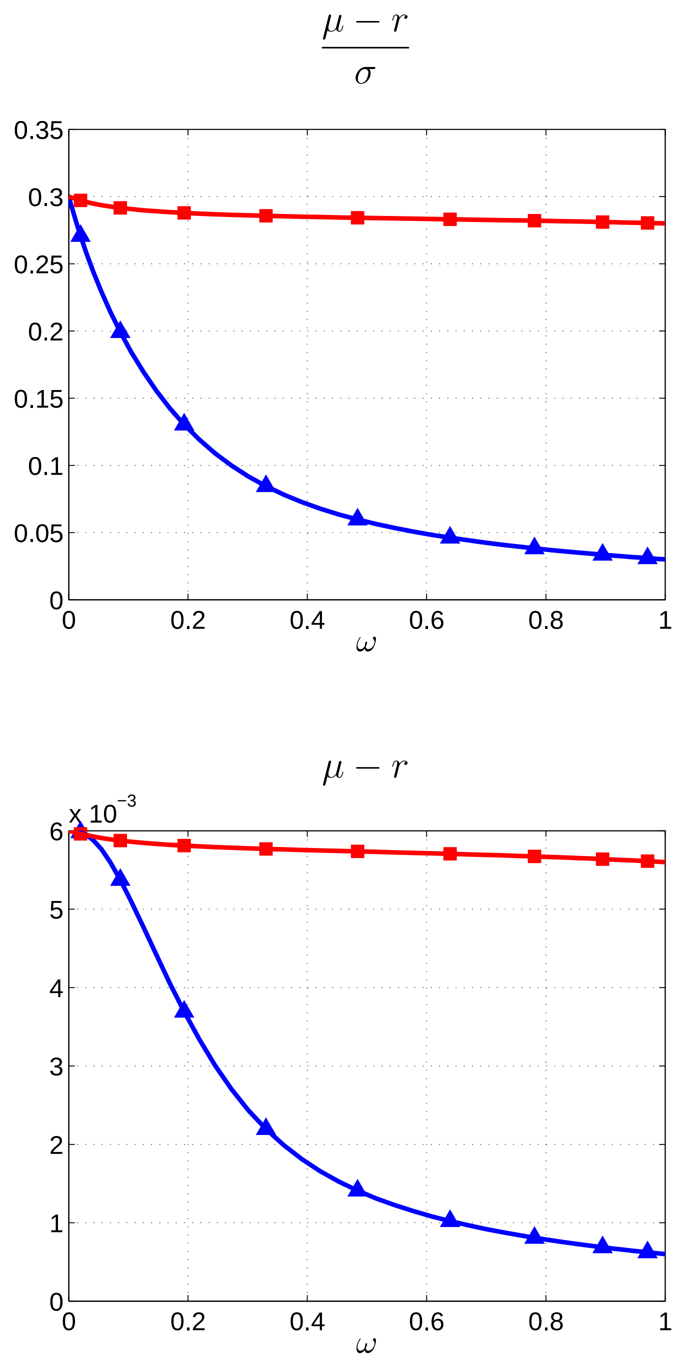

Figure 3: **The price of risk and the risk premium.** The figure plots the Sharpe ratio (top panel) and risk premium (bottom panel) of the endowment claim under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest rate policies.

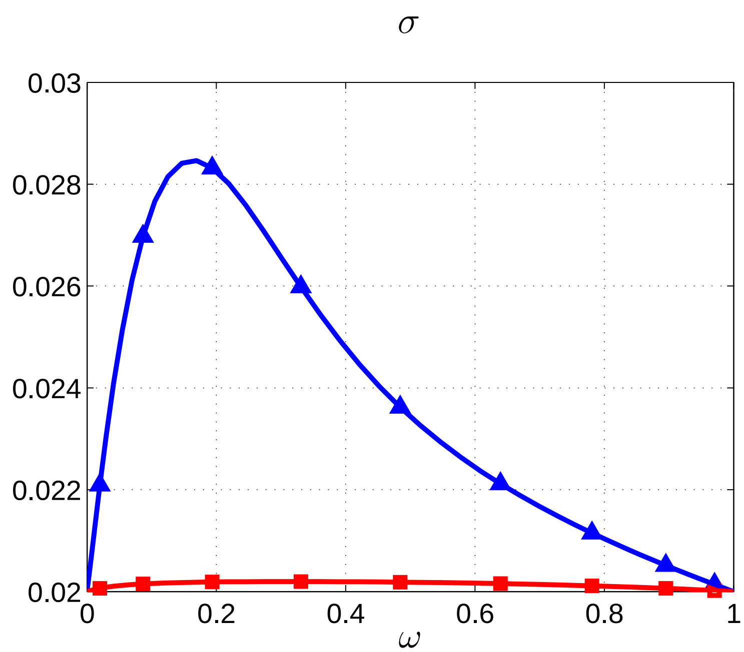

Figure 4: **Volatility.** The figure plots the volatility of the endowment claim under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest rate policies.

Risk-free rate (r)

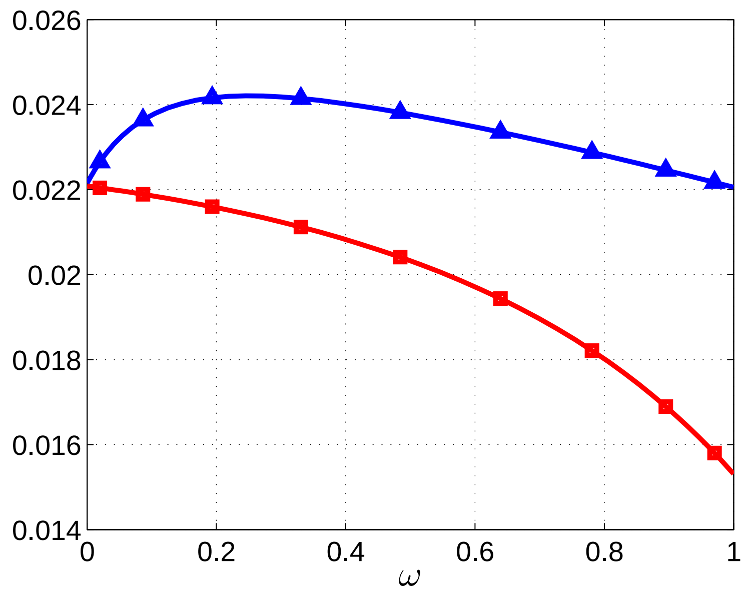

Figure 5: **Risk-free rate.** The figure plots the risk-free rate under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest rate policies.

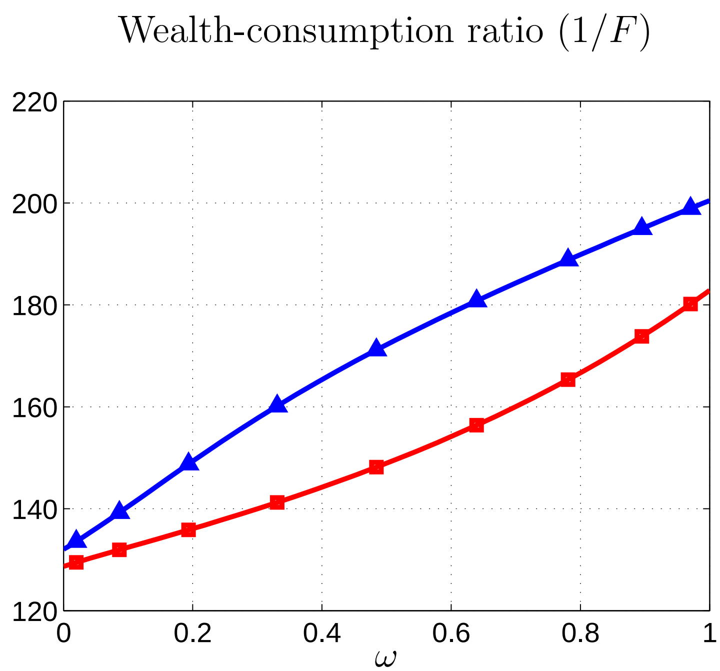

Figure 6: **Valuations.** The figure plots the wealth-consumption ratio under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest rate policies.

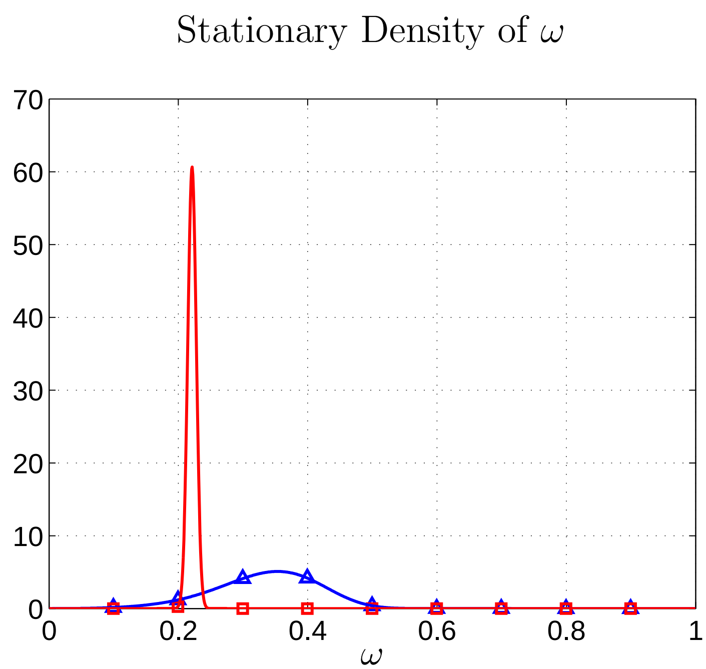

Figure 7: **The Stationary Density of Banks' Wealth Share (ω).** The figure plots the stationary density of ω, the share of wealth owned by banks, under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest rate policies.

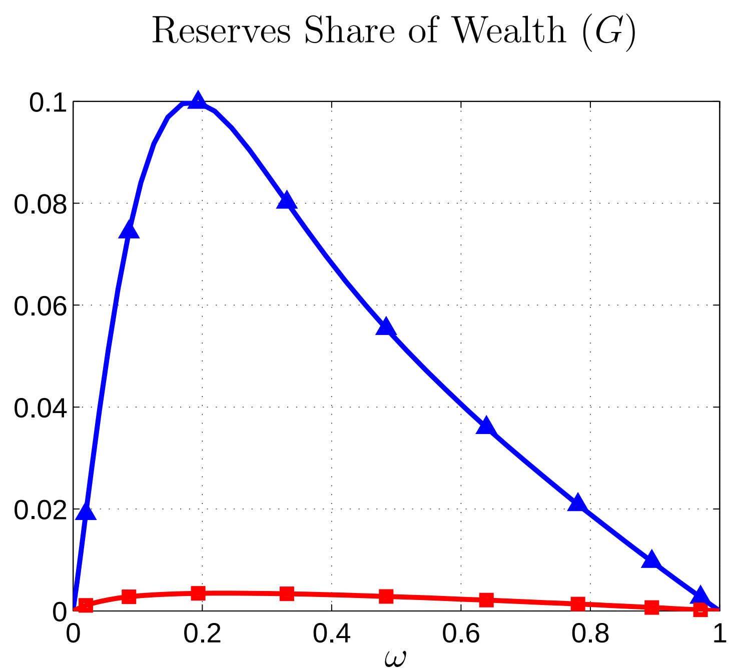

Figure 8: **Ratio of real reserves to wealth.** The figure plots the ratio of the real value of reserves to wealth G under the $\mathbf{n_1 = 0\%}$ (**blue** triangles) and $\mathbf{n_2 = 5\%}$ (**red** squares) interest rate policies.

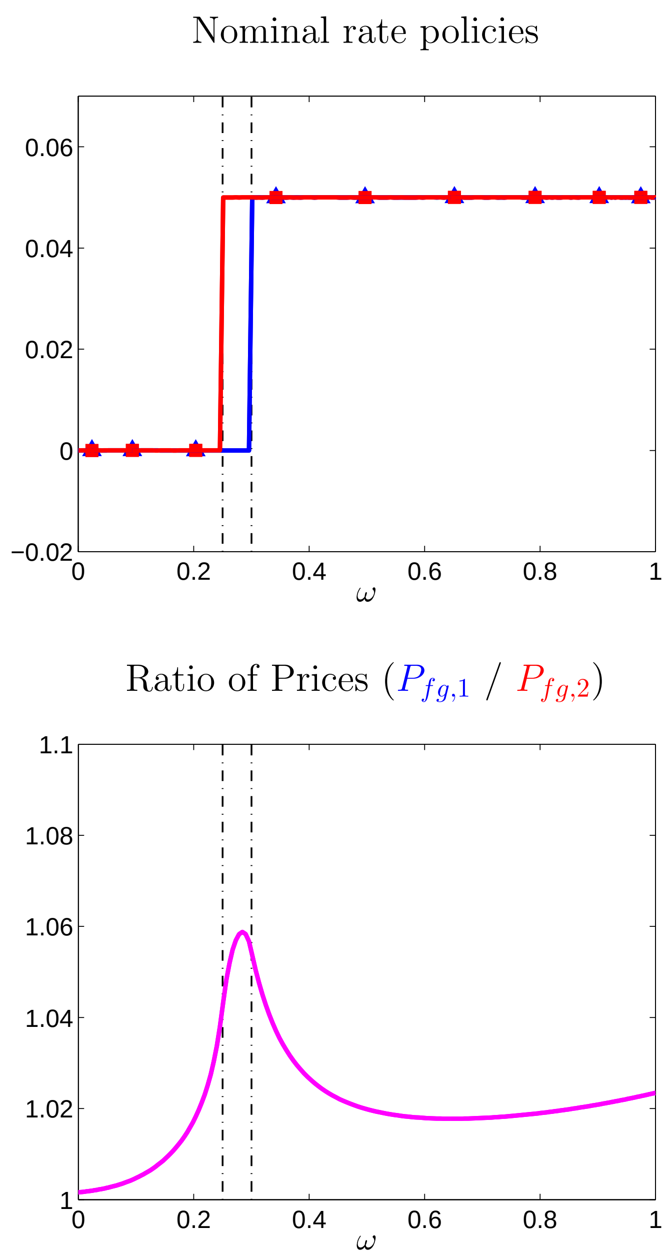

Figure 9: **Impact of forward guidance on prices.** The figure plots the impact of forward guidance on asset prices. The top panel plots the two nominal rate policies $n_{fg,1}$ (**blue** triangles) and $n_{fg,2}$ (**red** squares). The bottom panel plots the ratio of the price of the endowment claim for $n_{fg,1}$ relative to $n_{fg,2}$ ($P_{fg,1}$ / $P_{fg,2}$).

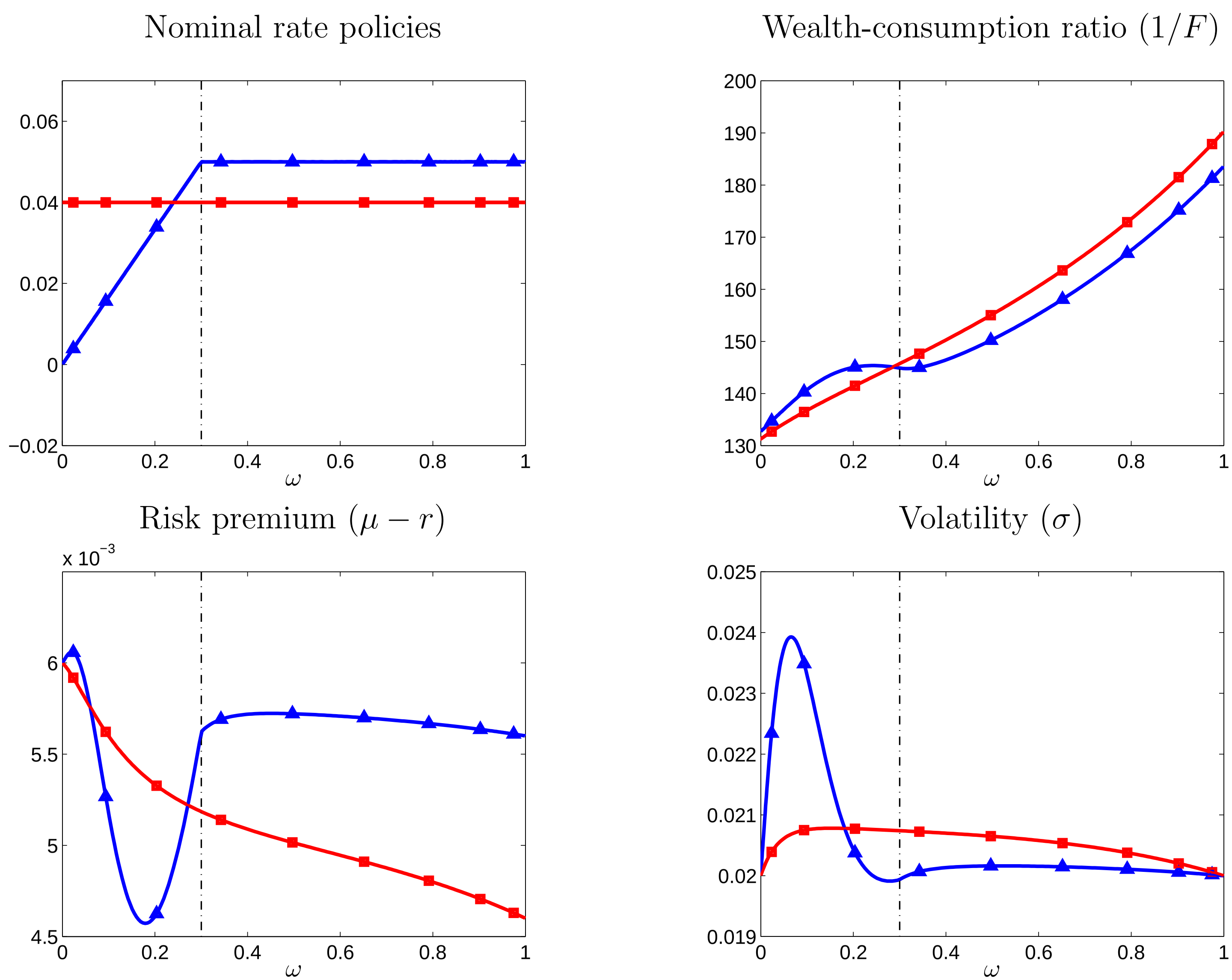

Figure 10: **Impact of Greenspan put policy on prices and volatility.** The figure plots the impact of a Greenspan put policy on prices, risk premia, and volatility. The top left panel plots the two nominal rate policies $n_{gp,1}$ (**blue** triangles) and $n_{gp,2}$ (**red** squares). The top right panel plots the wealth-consumption ratio $1/F$, the bottom left panel plots the risk premium $\mu - r$, and the bottom right panel plots volatility σ.

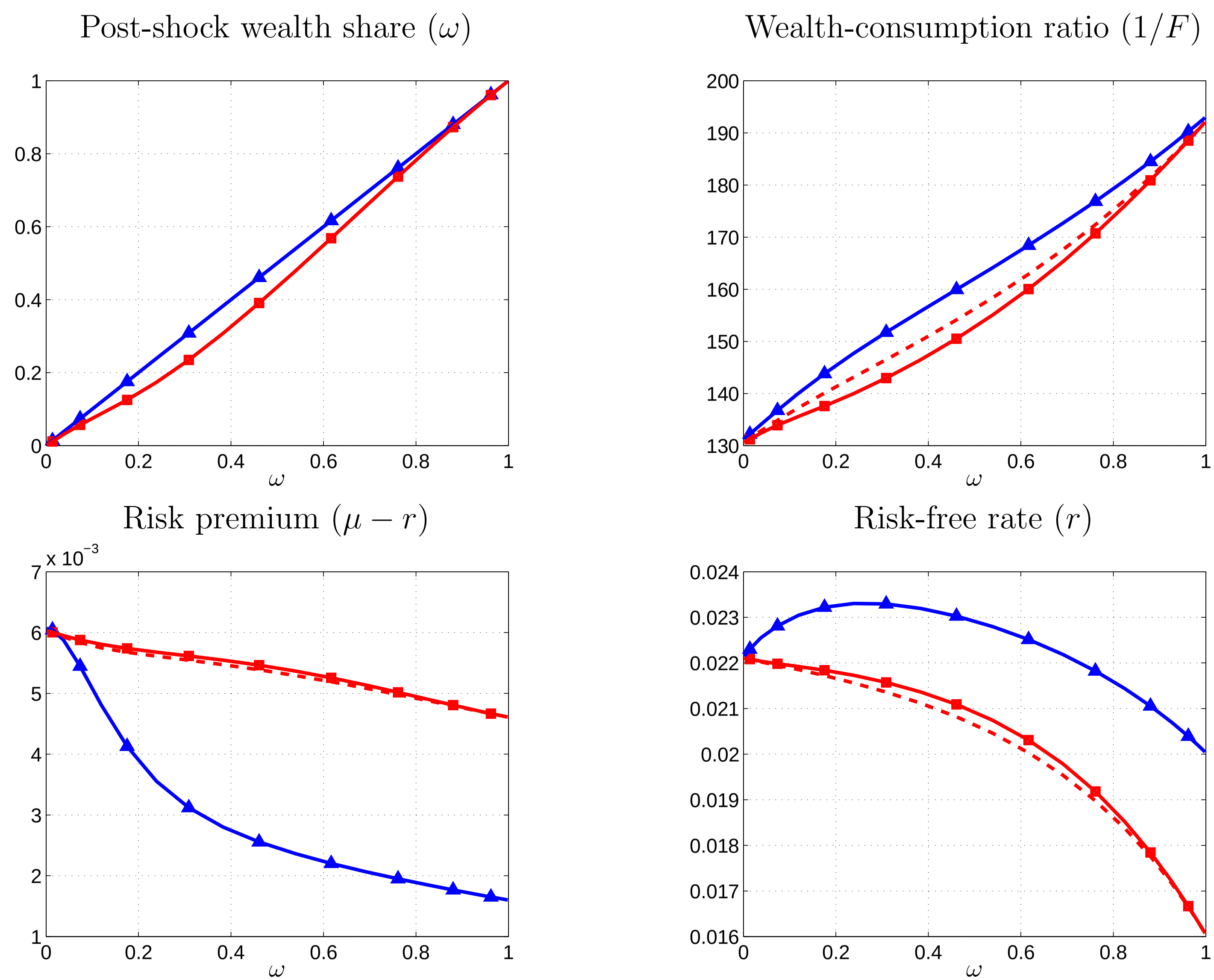

Figure 11: **Results from the model with policy shocks.** The figure plots the response of the wealth share ω, the wealth-consumption ratio $1/F$, the risk premium $\mu - r$, and the real rate r to a policy shock that raises the nominal interest rate n from 1% to 4%. Blue triangles represent initial values, red squares represent after-shock values. Dashed red lines correspond to the "direct effect" of the policy shock, which excludes the impact on the wealth share ω.

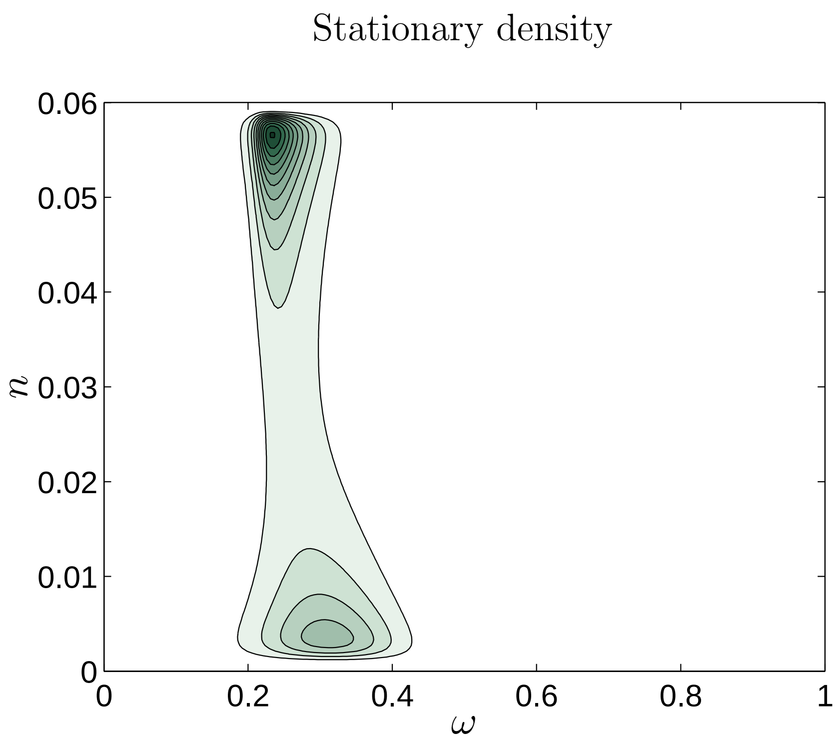

Figure B.1: **Stationary density: model with policy shocks.** Stationary density of the state variables ω and n in the extended model with policy shocks. We set $\underline{n} = 0.00$, $\overline{n} = 0.06$, $\kappa_n = 0.01$, and $\sigma_n = 5$. The density is obtained by solving the forward Kolmogorov equation of the system. The contour lines are at increments of 50.