Unified China and Divided Europe

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Abstract

This paper studies the causes and consequences of political centralization and fragmentation in China and Europe. We argue that the severe and unidirectional threat of external invasion fostered political centralization in China while Europe faced a wider variety of smaller external threats and remained politically fragmented. We test our hypothesis using data on the frequency of nomadic attacks and the number of regimes in China. Our model allows us to explore the economic consequences of political centralization and fragmentation. Political centralization in China led to lower taxation and hence faster population growth during peacetime than in Europe. But it also meant that China was relatively fragile in the event of an external invasion. Our results are consistent with historical evidence of warfare, capital city location, tax levels, and population growth in both China and Europe.

Keywords: China; Europe; Great Divergence; Political Fragmentation; Political Centralization

JEL Codes: H2, H4, H56; N30; N33; N35; N40; N43; N45

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1 Introduction

Since Montesquieu, scholars have attributed Europe’s success to its political fragmentation (Montesquieu, 1989; Jones, 2003; Mokyr, 1990; Diamond, 1997). Nevertheless, throughout most of history, the most economically developed region of the world was China, which was often a unified empire. This contrast poses a puzzle that has important implications for our understanding of the origins of modern economic growth: Why was Europe perennially fragmented after the collapse of Rome, whereas political centralization was an equilibrium for most of Chinese history? Can this fundamental difference in political institutions account for important disparities in Chinese and European growth patterns?

This paper proposes a unified framework to (a) provide an explanation for the different political equilibria in China and Europe; (b) explore the economic consequences of political centralization and fragmentation. Our model predicts when and where empires are viable based on the nature and intensity of the external threats that they face. It also highlights the different implications that political centralization and fragmentation would have on the locations of capital cities, levels of taxation, and population growth.

Our model focuses on the role of geography in determining China’s recurring unification and Europe’s enduring fragmentation. Historically, China faced a severe, unidirectional threat from the Eurasian steppe. Europe confronted several smaller threats from Scandinavia, Central Asia, the Middle East, and North Africa. We show that if multitasking is inefficient, empires will not be viable in Europe and political fragmentation will be the norm. On the other hand, empires were more likely to emerge and survive in China because the nomadic threat threatened the survival of small states more than larger ones. The different equilibria have important economic consequences: political centralization allowed China to avoid wasteful interstate competition and thereby enjoy faster economic and population growth during peacetime. However, the presence of multiple states to protect different parts of the continent meant that Europe was relatively more robust to both known threats and unexpected negative shocks.

To test the mechanisms identified in our model, we use time series analysis to show that an increase in the frequency of nomadic attacks on China is associated with a decrease in the number of regimes in historical China. Our estimates suggest that each additional nomadic attack on China reduces the number of regimes by a factor of two.

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1 We use ‘China’ and ‘China proper’ interchangeably throughout the paper. China proper refers to the middle and lower reaches of the Yellow River and the river basins of the Yangzi, Huai, and Xi rivers and their tributaries, or approximately the landmass between the Mongolian Steppe and the South China Sea (See Figure 14a). The bulk of this landmass is less than 750 meters above sea level. Fertile alluvial soil and rich water resources promoted early development of agriculture in this area. We equate Europe with Europe west of the Hajnal line that runs from Trieste to St. Petersburg. Consistent with our theory, Eastern Europe, which historically faced significant threats from the steppe, did see the rise of larger empires.
attack per decade increased the probability of political unification in China by about 6.0% in the long run. We go on to use the model to explain several notable features of Chinese and European history such as the location choice of imperial capitals, army size, and different levels of taxation at either end of Eurasia. Finally we provide evidence suggesting that political centralization led to greater volatility of Chinese population growth and discuss the likely implications of this prediction for economic growth.

This paper is related to a range of literatures. Our theoretical framework builds on the research on the size of nations originated by Friedman (1977) and Alesina and Spolaore (1997, 2003). In particular, our emphasis on the importance of external threats is related to the insights of Alesina and Spolaore (2005) who study the role of war in shaping political boundaries. In examining the causes of political fragmentation and centralization in China and Europe, we build on earlier work that points to the role of geography such as Diamond (1997) and on many historians who stress how the threat of nomadic invasion from the steppe shaped Chinese history (Lattimore, 1940; Grousset, 1970; Huang, 1988; Barfield, 1989; Gat, 2006).

Numerous economists, historians, political scientists, and sociologists have argued that political fragmentation in Europe had important consequences, including leading to the growth of economic and political freedom (Montesquieu, 1989); helping preserve the existence of independent city states and permitting the rise of a merchant class (Pirenne, 1925; Hicks, 1969; Jones, 2003; Hall, 1985; Rosenberg and Birdzell, 1986); encouraging experiments in political structures and investments in state capacity (Baechler, 1975; Cowen, 1990; Tilly, 1990; Hoffman, 2012; Gennaioli and Voth, 2013); intensifying warfare and therefore increasing urbanization and incomes (Voigtländer and Voth, 2013b); and fostering innovation and scientific development (Diamond, 1997; Mokyr, 2007; Lagerlof, 2014).

Baechler observed that ‘political anarchy’ in Europe gave rise to experimentation in different state forms (Baechler, 1975, 74). Cowen (1990) argues that interstate competition in Europe provided an incentive for early modern states to develop capital markets and pro-market policies. Tilly (1990) studies the role capital-intensive city states played in shaping the emergence of nation states in Europe. Hoffman (2012) uses a tournament model to explain how interstate competition led to military innovation in the early modern Europe. Gennaioli and Voth (2013) show the military revolution induced investments in state capacity in some, but not all, European states. Voigtländer and Voth (2013b) argue that political fragmentation interacted with the Black Death so as to shift Europe into a higher income state-steady Malthusian equilibrium.

Diamond argues that ‘Europe’s geographic balkanization resulted in dozens or hundreds of independent, competing statelets and centers of innovation’ whereas in China ‘a decision by one despot could and repeatedly did halt innovation’ (Diamond, 1997, 414–415). Mokyr notes that ‘many of the most influential and innovative intellectuals took advantage of ...the competitive ‘states system’. In different ways, Paracelsus, Comenius, Descartes, Hobbes, and Bayle, to name but a few, survived through strategic moves across national boundaries. They were able to flee persecutors, and while this imposed no-doubt considerable hardship, they survived and prospered’ (Mokyr, 2007, 24). Lagerlof (2014) develops a growth model that emphasizes the benefits to scale in innovation under political unification and a greater incentive to innovate under political fragmentation. He calibrates the model to the initial conditions of China and Europe and shows that there are parameter values in which political fragmentation can give rise to the emergence of sustained growth in Europe.
was persistently fragmented, we complement this literature and recent research that emphasizes other aspects of Europe’s possible advantages in the Great Divergence such as the higher age of first marriage than the rest of the world (Voigtländer and Voth, 2013a); public provision of poor relief verses reliance on clans as was the case in China (Greif et al., 2012); institutions that were less reliant on religion (Rubin, 2011); greater human capital (Kelly et al., 2013), or higher social status for entrepreneurs and inventors (McCloskey, 2010). Finally, our analysis is related to the rise of state capacity in Europe (Dincecco, 2009; Dincecco and Katz, 2014; Johnson and Koyama, 2013, 2014a,b).

Our study of the consequences of political centralization and fragmentation is related to Rosenthal and Wong (2011) who examine the costs and benefits of political centralization and fragmentation in China and Europe. They argue that political fragmentation led to more frequent warfare in medieval and early modern Europe, which imposed high costs but also set in motion processes that would give Europe an advantage in producing an industrial revolution; in particular, it lent an urban bias to the development of manufacturing which led to more capital intensive forms of production.

Like Rosenthal and Wong, we emphasize that political fragmentation was costly, but we depart from their view that the advantages of political fragmentation only accrued after 1750 (Rosenthal and Wong, 2011, 33).\(^5\) We develop a different argument based on the observation that the costs of political collapse and external invasion were particularly high in China. That is, we argue that the Chinese empire could indeed have been more conducive to Smithian economic growth during stable periods as Rosenthal and Wong claim, but note that it was also less robust to external shocks. Our model shows that population reversals are more likely in empires, but between negative shocks, population in an empire may expand faster than in a system of competing states. We argue that this greater volatility of economic development, output, and population was the major barrier to sustained economic growth in China before 1800.

The framework that we introduce has important implications for our understanding of the origins of economic growth. Since more people means more ideas, growth theory often contains a scale-effect that implies that larger economies should be the first to experience modern economic growth (Kremer, 1993; Jones, 2001). However, as Aiyar et al. (2008) point out, as long as technological knowledge is embodied primarily in humans (instead of, for example, stored in computers)—as was the case in the premodern world—the effect of population change on the stock of knowledge is asymmetric: technological knowledge grows slowly with population growth, but regresses swiftly when the population contracts. Pairing this insight with our theory suggests

\(^5\)Their argument neglects evidence that Chinese per capita income was higher in 1100 than in 1700 (Broadberry, 2013). Furthermore, it is tied to a specific argument that sees the Industrial Revolution as the product of a specific ratio of factor prices (Allen, 2009).
that because China was more centralized and more vulnerable to negative population shocks, it experienced more frequent interruptions in cumulative innovation. In other words, the start-stop nature of China’s growth diminished its chances of escaping the Malthusian trap, while the European economy was able to expand gradually to the point where the transition from stagnation to growth was triggered (as in theories of unified growth, for instance, Galor and Weil 2000; Galor 2011). In this respect, we offer a new interpretation that reconciles a big puzzle in the history of economic growth: why China, the most populous economy in the world for much of recorded history, was capable of coming ‘within a hair’s breadth of industrializing in the fourteenth century’ (Jones, 2003, 160), but swiftly and permanently lost its technological lead after the prolonged and devastating wars of the Mongol conquests.

Finally, our model also sheds light on the optimal location of capital cities. Economic theory generally predicts that capital cities should be centrally located to maximize tax revenue or improve governance (e.g., Alesina and Spolaore 2003; Campante et al. 2014). This is confirmed by empirical studies showing that isolated national or subnational capital cities are associated with greater corruption (Olsson and Hansson, 2011; Campante and Do, 2014). We show that that if the effectiveness of public goods provision (military defense in our example) differs with the location of provision, it may be optimal to establish the political center of the empire away from its economic or population center. This helps explain why the Romans moved their their capital city from Rome to Constantinople in AD 330, and why Beijing, a city on the northern periphery of China proper, was China’s political center for more than six centuries.

The rest of the paper is structured as follows. Section 2 provides historical evidence that characterizes (i) the extent to which China was politically unified and Europe fragmented throughout their respective histories, and (ii) the degree to which both China and Europe were threatened by external invasions. In Section 3 we introduce a model of political centralization and decentralization. Section 4 provide empirical evidence to support our hypothesis that a severe threat from the Eurasian steppe discouraged political fragmentation in China. In Section 5, we show that our model provides a coherent framework that can help to make sense of the choice of capital cities, differential levels of taxation, and population growth patterns in historical China and Europe. Section 6 concludes.

## 2 The Puzzle: Unified China and Divided Europe

Why was China politically unified for much of its history whereas Europe has been politically fragmented since the end of the Roman Empire? Chinese historical records indicate that less than 80 states ruled over parts or all of China between AD 0 and 1800 (Wilkinson, 2012). Nussli
China’s first unification preceded Rome’s dominance of the Mediterranean. The Chinese built a unitary state as early as the third century BC (Elvin, 1973; Needham, 1995; Fukuyama, 2011). Moreover, the Chinese empire outlasted Rome. Although individual dynasties rose and fell, China as an empire survived until 1912. Between AD 1 and 1800, the landmass between the Mongolian steppe and the South China Sea was ruled by one single authority for 1007 years (Ko and Sng, 2013). Every period of political unity was followed by reunification—in China, Humpty Dumpty could always be put back together again. This phenomenon is captured in the famous opening lines of the classic Chinese novel *Romance of the Three Kingdoms*: ‘The empire, long divided, must unite; long united, must divide. Thus it has ever been.’

In comparison, Europe after the fall of the Roman Empire was characterized by persistent political fragmentation—no subsequent empire was able to unify a large part of the continent for more than a few decades. The number of states in Europe increased from 37 in 600 AD to 61 in 900 and by 1300 there were 114 independent political entities. The level of political fragmentation in Europe remained high during the early modern period.

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*The Nussli (2011) data does not capture all political entities in Europe since that number is unknown—there may have been as many as 1000 sovereign states within the Holy Roman Empire alone—but it does record the majority of large and small political entities (Abramson, 2013). By contrast, the Chinese dynastic tables are well known and the potential for disagreement is immaterial for our purposes. We count only sovereign states. Including vassal states would further strengthen the argument.*
Through the years, scholars have offered various theories to explain China’s tendency toward unification and Europe’s persistent fragmentation, ranging from culture and language (e.g., logographic versus phonogramatic writing systems) to topography (e.g., the presence or absence of internal mountain barriers. See Diamond 1997, 322–333, for a detailed discussion). In this paper, we put forth a different geographical explanation. We argue that in order to understand why China has typically been unified whereas Europe has been fragmented, we need to assess the threats and challenges that they faced given their geography. Europe was threatened by Goths, Sarmatians, Vandals, Huns, Avars, Bulgars, Arabs, Berbers, Magyars, Vikings, Pechnegs, Cumans, Mongols, and Turks. Similarly, settled populations in China contended with a range of steppe nomads and semi-nomadic people: Xiongnu, Juanjuan, Uygurs, Khitan, Jurchen, Mongols, and Manchus (Grousset, 1970; Barfield, 1989; Chaliand, 2005). But, for largely geographical reasons, China’s external threats were more severe.

The history of violent conflicts between the Chinese and the nomads has been extensively documented and discussed in the literature (for example, see Barfield 1989; Di Cosmo 2002a). As Lattimore (1940) pointed out seven decades ago, the geography of Eurasia created a natural divide between the river basins of China and the Mongolian Steppe. In the Chinese river basins, fertile alluvial soil, sufficient rainfall, and moderate temperature encouraged the early development of intensive agriculture. In the steppe, pastoralism emerged as an adaptation to the arid environment. Given the ecological fragility of the steppe, where droughts often led to extensive and catastrophic deaths among animal herds, there was a tendency for the steppe nomads to invade their settled neighbors for food during periods of cold temperature. This set the stage for recurring military engagements between the steppe inhabitants and the Chinese in history (Bai and Kung, 2011).

While Eastern Europe, too, was vulnerable to incursions from the Eurasian steppe, Western Europe was relatively protected from nomadic invasions due to its forests and mountain ranges, and because the semi-pastoral lands of Hungary and Ukraine provided a buffer against nomadic

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7We do not claim that only geography mattered. In fact, different factors—geography, culture, and others—likely reinforced each other. For example, as Di Cosmo (2002a) points out, the recurring conflicts between the steppe inhabitants and the agrarian Chinese helped forge a common cultural identity among the early competing states in ancient China. In the conclusion, we also discuss how our framework complements Charles Tilly’s argument that independent city states along the corridor between southern England and northern Italy prevented the emergence of large empires in Europe (Tilly, 1990).

8See Appendix A.1 for a list of all major nomadic invasions of both China and Europe.

9Lieberman (2009) distinguishes between China, which lies in the exposed zones of inner Asia, and the protected rimland of Europe and Southeast Asia. He notes that ‘For centuries nonpareil equestrian skills, an ethos focused on hunting and warfare, proficiency with the short double-reflex bow (which allowed volleys from horseback), tactical flexibility, a ruthlessness and stamina demanded by an unforgiving environment, remarkable mobility, and a far larger percentage of men trained for war than in settled societies’ meant that settled society faced a perennial threat from the horsemen of the steppe.
The Hunnic invasions of the fifth century, the Magyar invasions of the ninth century, and the Mongol invasions of the thirteenth century pose partial exceptions to this. Nevertheless, it is well established that the military superiority of all of these invaders declined dramatically once they entered Europe due to the absence of the vast tracts of grassland required to maintain the high ratio of horses to men that nomadic armies typically relied upon for their effectiveness (Di Cosmo, 2002b). By contrast, China’s more compact geography and a sharper transition from the steppe to heavily settled lands meant that steppe invasions posed a more extensive threat to its populated agricultural communities and urban centers. Figure 2 illustrates the distance of cities in China and Europe from the Eurasian steppe. As it makes clear, Guangzhou, the southernmost major Chinese city, was almost as close to the steppe as Vienna, the easternmost major western European city.

Although the sedentary Chinese were more populous by far, before the advent of effective gunpowder weapons the steppe nomads often held the upper hand in military engagements as their expertise in horses allowed them to develop powerful and mobile cavalry units (Barfield, 1989; Gat, 2006). And when the odds were not in the nomads’ favor, the Eurasian steppe provided a safe haven for them to retreat into. Since they could reach the Black Sea from Mongolia in a matter of weeks through the undifferentiated ‘highway of grass’ (Frachetti, 2008, 7), they effectively enjoyed an ‘indefinite margin of retreat’—no matter how badly they were defeated in battle, they could never be conquered in war (Lattimore, 1940). Until Russia’s expansion into Central Asia in the seventeenth and eighteenth centuries denied the nomads this traditional escape route, the steppe threat was a recurring problem that the Chinese could not
permanently resolve (Perdue, 2005). Their best hope for security was the successful containment of the nomadic threat at the frontier—hence the construction of the first Great Wall immediately after the first unification of China under the Qin dynasty in 221 BC. The project was repeated time and again by successive dynasties at great cost to keep the ‘barbarians’ at bay.

It is therefore of no surprise that the threat of steppe nomads played a decisive role in Chinese history and the evolution of its political economy. Of the ten dynasties that ruled a unified or mostly unified China, three fell to nomadic invaders (Jin, Song, and Ming) and two were set up by nomadic conquerors (Yuan and Qing). The fall of two others (Qin and Sui) could be traced to the overzealous attitude of their rulers toward securing China’s northern border, which placed an unbearable strain on the peasants and led to widespread revolts. The remaining three, the Former Han, Later Han, and Tang, built their respective golden ages upon the temporary subjugation of the steppe.

Many scholars have recognized the importance of the steppe nomads to state formation in ancient China (Lattimore, 1940; Huang, 1988; Lieberman, 2009; Turchin, 2009; Ma, 2012; Deng, 2012). We build on this literature by highlighting another important element in the nature of this threat that has been overlooked so far: while—as the literature has pointed out—the severity of the nomadic problem provided the centripetal force that pushed the Chinese regions toward political centralization, it was also crucial that the external threats confronting China happened to be unidirectional and there were no major threats from other fronts that would have increased the appeal of a more flexible politically decentralized system.

Before 1800, all major invasions of China came from the north. We argue that this was geographically determined: as Figure 3 illustrates, China was shielded from the south and the west by the Himalayas, the Tibetan plateau, and the tropical rain forests of Indochina. By contrast, Europe was less isolated from other parts of Eurasia and consequently prospective European empires typically faced enemies on multiple fronts: Vikings from the north, Muslims from the south, Magyars, Mongols, and Ottomans and others from the east (Appendix A.2).

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10During the Warring States period (475–221 BC) when China was divided into about a dozen competing kingdoms, the three that bordered the steppe—Qin, Zhao, and Yan—built long walls that were later linked up by Qin to form the first Great Wall of China after its successful unification of China.

11Lewis (1991, 4–5) notes that the ‘high desiccated plateaus and deserts . . . dominated the terrain of Mongolia and Turkestan . . . formed barriers between it and the Indic and Islamic worlds to the west and south. While it was possible to reach Burma over difficult mountain passes leading there from the upper Yangtze valley, the most practicable routes to the west were by way of Kansu, Inner Mongolia, and eastern Turkestan and then on to Khorasan and southern Russia.’

12To be precise, what matters for our theory is the existence of a potential threat. Thus Europe continued to face a potential threat of invasion from North Africa due to geographical proximity throughout this period, despite the fact that the majority of actual invasions from North Africa occurred before 1300. Portugal and Spain faced a threat from North Africa through the sixteenth and seventeenth centuries, but they typically dealt with it through offensive action (i.e., Charles's V conquest of Tunis 1535—a response to raids by Barbarossa along the southern Italian coast—and Sebastian I's invasion of Morocco which ended in his defeat at the Battle of Alcácer
Figure 3: Although China was exposed to steppe invasion from its north, huge mountain ranges to its west, thick forests to its south, and the vast Pacific Ocean to its east meant that it was otherwise relatively isolated. By contrast, Europe was connected to the rest of Eurasia and Africa in multiple directions.

Figure 4a, which is derived from Peter Brecke’s Conflict Catalog Dataset (Brecke, 1999), shows that while warfare was much more common in Europe between 1400 and 1800, a majority of China’s military conflicts between 1400 and 1800 were with nomads. Meanwhile, according to Figure 4b, external invasions and major wars had a bigger impact in China than in Europe. In fact, it is clear that the most violent wars of the preindustrial period occurred in Asia and particularly in China.\footnote{All data on deaths from warfare in the preindustrial period are highly speculative, but for our purposes what is important is the order of magnitude rather than the precise numbers reported.} Only two wars with estimated death tolls in excess of 5 million are recorded for Europe compared with five for China.\footnote{The majority of deaths in preindustrial wars did not occur in the battlefield but were the result of disease and pressure on food supplies (see Voigtländer and Voth 2013b, 781 for a discussion).} Wars in China such as the An Lushan Rebellion, the Mongol invasions, and the Ming-Manchu transition were extremely costly conflicts, because they involved the collapses or near collapses of entire empires. Warfare in Europe was endemic, but rarely resulted in large scale socio-economic collapse. The only European war that matched the death tolls of the worst conflicts in Chinese history was the Thirty Years War, which was a German civil war into which the other European powers were drawn. In the next section, we show how political centralization in China and fragmentation in Europe could help explain these patterns.
3 MODEL

Building on the preceding discussion, we develop a model to explore the consequences of the severe one-sided threat that China faced in contrast to the weaker multi-sided threat faced by states in Europe. To keep our analysis tractable, we consider a continent, which may represent China or Europe, as a Hoteling’s linear city of unit length.\textsuperscript{15} The continent faces external threats that can be one or two-sided. In particular, the two-sided threat in this one-dimensional model is analogous to a multi-sided threat in the multi-dimensional real world.

One or more regimes may exist in the continent. Each regime (a) chooses its capital city (represented by a point along the linear line), (b) taxes its population, and (c) builds the military to resist the external threat and to compete with other states for territory and population. For a regime to remain viable, two conditions have to be met: (i) the regime has to offer military protection to at least $0 \leq \delta \leq 1$ fraction of its subjects; (ii) its net tax revenue has to be non-negative. If a regime that rules an entire continent is not viable, political centralization will not be stable in this continent. If a continent cannot support more than one viable regime, political fragmentation will be unstable. Our central concern is the stability of political centralization or fragmentation in a continent given the nature of external threats that it confronts.

\textsuperscript{15}We refer to both Europe and China as ‘continents’ for convenience.
3.1 Setup

We model a continent as a line $[0,1]$ with a unit mass of individuals uniformly distributed along this line. An individual at $x \in [0,1]$ is endowed with income $y + y$ where $y$ is taxable. For now, we fix the level of taxation at $y$; we will endogenize it later.

The continent is divided into $s \in \{1, 2, 3, \ldots\}$ connected, mutually exclusive intervals each ruled by a separate political authority or regime. We take $s$ as given and do not model how regimes arise. Historically, the emergence of a regime is often fraught with stochastic elements—the birth of a military genius; policy errors made by the incumbent ruler; climatic change; and so on—that are difficult to capture in a model. Instead, we are concerned with the viability of each regime in a continent of $s$ regimes: we ask, for any given $s$, is this political configuration stable and likely to persist given the continent’s external environment?

Of interest is the comparison between $s = 1$, which corresponds to political centralization or empire, and $s \geq 2$, which we will refer to as interstate competition or political fragmentation. Because of our choice of a linear city model, external threats could threaten a continent from at most two fronts, for clarity of exposition we focus on the corresponding case of $s = 2$ to represent $s \geq 2$. All subsequent results are qualitatively similar if we replace $s = 2$ with $s > 2$ as the standard with which to compare against $s = 1$. Furthermore, for convenience and because we are primarily interested in analyzing two comparable regimes, we will treat $l$ and $r$ as identical and focus on the symmetric equilibrium when deriving the results.\footnote{In particular, if one of the two regimes rules a much larger interval than the other one, it may be more appropriate to use ‘empire’ instead of ‘interstate competition’ to describe the political reality of the continent.}

The continent faces threats from outside. We model these external threats as emanating either from both frontiers of the continent (at $x = 0$ and $x = 1$) or just from one frontier of the continent (at $x = 0$ only without loss of generality). Whether the threat is one-sided or two-sided depends on the continent’s geographical environment which is exogenously determined. An external threat, if realized, causes gross damage $\lambda(\Lambda, 0) > 0$ at the frontier(s) of the continent (Figures 5 and 6). The damage can spread further into the continent: if a point is $t$ distance away from the frontier, the gross damage is $\lambda(\Lambda, t)$ where $\lambda_1 > 0$, and $\lambda_2 < 0$. The negative derivative of $\lambda$ with respect to $t$ implies that the threat decreases in strength as it moves inland.

A regime may invest in the military to mitigate the damage caused by the external threat. The cost of military investment is convex. For a given cost parameter $\theta$, for regime $i \in \{e, l, r\}$ to provide a military investment of $M_i \geq 0$, it costs $c(M_i, \theta)$, which is a continuously differentiable function and $c(0, \theta) = 0$, $c_1 > 0$, $c_{11} > 0$, $c_2 > 0$, and $c_{12} > 0$. An example of a functional form that satisfies these conditions is $c(M_i, \theta) = \theta M_i^2$.

Like the external threat, the strength of the military is not constant across space; the military
is strongest at its center of deployment, but deteriorates over distance. Let $G_i$ denote regime $i$’s center of military deployment—referred to here as $i$’s capital city.\(^\text{17}\) For notational convenience, $G_e$ and $G_l$ are measured from 0 while $G_r$ is measured from 1. As illustrated in Figure 7, for a location that is $t$ distance away from $G_i$, regime $i$’s **military strength** on that location is given by $m(M_i,t)$, where $m_1 > 0$, and $m_2 < 0$.

We assume that the strength of the military deteriorates over distance (i.e., $m_2 < 0$) for two reasons: logistics and agency costs. It is well recognized that historically, the projection of military power was constrained by logistics (van Creveld, 2004). The Roman historian Keith Hopkins highlights the challenges that premodern transportation technologies imposed on political and military control by describing the Roman Empire as ‘several months wide—and larger in winter than in summer’ (Morris and Scheidel, 2009, 186). Shen Kuo (1031–1095), a polymath and ‘one of the greatest scientific minds in Chinese history’ (Needham, 1969, 27), estimated that in his time a soldier would need one porter for supplies to march 18 days, while three porters would be required for a 31-day campaign as the existing porter now needed someone else to carry his supplies (Shen, 2011).

In addition, military effectiveness may also deteriorate over distance due to the cost of controlling armies from afar or, specifically, from agency problems. If the ruler assumes direct control of the military as the commander-in-chief, his ability to react swiftly to events on the ground is constrained by slowness of communications. Alternatively, the ruler may delegate some of his troops to military generals stationed outside the capital city, but—as a safeguard against potentially disloyal commanders—keep the elite forces under close control. This will also generate the pattern of falling military effectiveness over distance.\(^\text{18}\)

The military can block the external threat from spreading inland. Specifically, consider a one-sided threat initiated at $x = 0$, if the military strength of regime $i \in \{e,l\}$ at $x$ is no less

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\(^{17}\) As a matter of interest, the capital was known as *jing-shi* in Chinese, or literally the peak (*jing*) and the military (*shi*).

\(^{18}\) Historically, we observe that the rulers of China and Rome were increasingly unwilling to delegate large armies to their subordinates and preferred to keep their armies under close control by stationing themselves near the frontiers. See Section 5.1 for more discussion.
than the gross damage of the external threat at that location, then x and any location to its right is said to be protected, that is, individuals at these locations suffers zero damage from the threat. Formally, a location x is protected by regime i if there exists 0 ≤ x′ ≤ x such that \( m(M_i, |G_i - x'|) - \lambda(\Lambda, x') \geq 0 \). Let \( \mathbb{D}_i \) denote the set of locations that is protected by regime i, or \( \mathbb{D}_i \equiv \{ x \in [0, 1] : x \text{ is protected by regime } i \} \). If x is not protected by regime i, or \( x \notin \mathbb{D}_i \), then \( \kappa_i \), the net damage at x, is the gross damage caused by the threat minus the strength of regime i at x, i.e., \( \kappa_i(x) \equiv \lambda(\Lambda, x) - m(M_i, |G_i - x|) \). For threats initiated at \( x = 1 \), we define protection, the set \( \mathbb{D}_i \), and the net damage \( \kappa_i \) in a similar fashion.

We assume that if a regime fails to provide protection to \( \delta \) fraction of its population, then a revolution occurs and the regime receives a negative payoff. This assumption, common in models of political economy, captures the idea that a regime that disregards the welfare of its subjects will be overthrown by a revolution, but revolutions involve overcoming collective-action problems and therefore require support from a threshold population of \( 1 - \delta \) to be successful (Tullock, 1971; Alesina and Spolaore, 2003; Acemoglu and Robinson, 2006). It is also consistent with the Confucian belief that the legitimacy of a government is contingent upon its ability to protect the people from harm and tax reasonably so that the people can maintain a constant means of livelihood. A government that loses this ability loses its legitimacy, or its ‘mandate from heaven’, and the people would therefore be entitled to depose it (Mencius, 2004).

Besides resisting external threats, in the absence of political centralization investing in the military also helps competing regimes determine their borders. When \( s = 2 \) and the continent’s taxable output \( y \) is shared between two regimes, l and r. Let \( b \) represent the border of the two regimes. Without loss of generality, we restrict the locations of capitals to \( G_l + G_r \leq 1 \), that is, regime l is always to the left of regime r. The border \( b \) is determined by the condition \( m(M_l, b - G_l) = m(M_r, (1 - b) - G_r) \). In other words, \( b \) is the location where the military
strengths of both regimes are equal, as illustrated in Figure 8. The net revenues for regimes \( l \) and \( r \) are \( V_l = by - c(M_l) \) and \( V_r = (1 - b) y - c(M_r) \). By comparison, when an empire (\( e \)) controls the whole continent, its net tax revenue is \( V_e = y - c(M_e) \).

3.2 Equilibrium

Consider the optimization problem facing a single regime or empire (\( e \)). Regime \( e \) first decides the location of capital \( G_e \in [0, 1] \) and then decides military investment \( M_e \geq 0 \) to maximize the net revenue \( V_e = y - c(M_e) \). Since this is a two-stage decision process, we employ backward induction to solve the model.\(^\text{20}\)

Consider \( x^*(\Lambda) \in [0, 1] \) such that \( \lambda(\Lambda, x^*(\Lambda)) = 0 \). In other words, \( x^*(\Lambda) \) is the leftmost location where the gross damage caused by the threat emanating from the left is zero. If such \( x^*(\Lambda) \) does not exist, let \( x^*(\Lambda) \equiv 1 \). Consider \( \Lambda_I \) and \( \bar{\Lambda}_I \) such that \( x^*(\Lambda_I) = 1 - \delta/2 \) and \( x^*(\bar{\Lambda}_I) = 1 - \delta \) respectively. Let \( \hat{\delta} \) denote the fraction of the continent that is protected from the external threat in equilibrium (i.e. \( \hat{\delta} = |D_e| \) when \( s = 1 \) and \( \hat{\delta} = |D_l| + |D_r| \) when \( s = 2 \)).

**Proposition 1** (Empire). When the threat is two-sided:

1. If \( \Lambda \leq \Lambda_I \), the regime locates the capital city at \( G_e \in [0, 1] \), makes zero military investment, and \( \hat{\delta} \geq \delta \);

2. There exists \( \Lambda_{II} > \Lambda_I \) such that if \( \Lambda > \Lambda_{II} \), the regime locates the capital city at the center of the continent, spends a non-zero amount on the military to confront the threat emanating from both frontiers, and \( \hat{\delta} = \delta \);

3. If \( \Lambda_I < \Lambda \leq \Lambda_{II} \), the regime locates the capital city closer to one frontier than the other, invests a non-zero amount on the military to confront the threat emanating from the frontier that its capital city is closer to, and \( \hat{\delta} = \delta \);

When the threat is one-sided:

4. If \( \Lambda \leq \bar{\Lambda}_I \), the regime locates the capital city at \( G_e \in [0, 1] \), makes zero military investment, and \( \hat{\delta} \geq \delta \);

5. If \( \Lambda > \bar{\Lambda}_I \), the regime locates the capital city at \( G_e^* = 1 - \delta \) and spends a non-zero amount on the military to confront the threat emanating from \( x = 0 \), and \( \hat{\delta} = \delta \).

\(^{19}\)For a complete treatment of the determination of \( b \) in all cases, see Appendix A.3.

\(^{20}\)Proofs of the propositions are provided in Appendix A.4–A.6.
In Cases 1 and 4 above, the threat is weak enough for the empire to ignore it completely. This is because the sole motivation for the empire to invest in the military is to keep $\delta$ fraction of its population protected, so as to prevent a revolution. Hence, if the threat does not affect more than $1 - \delta$ of the population, the regime merely ignores it. In all other cases, the threat is meaningful and the empire builds a military to meet the threshold of protecting $\delta$ of the population.

Now consider the two-stage game with interstate competition ($s = 2$). Regimes $l$ and $r$ first simultaneously choose the location of their capital cities $G_l \in [0, 1]$ and $G_r \in [0, 1]$. After knowing the locations of each other, the two regimes simultaneously make military investments $M_l \geq 0$ and $M_r \geq 0$. This is a complete information game and we employ subgame-perfect equilibrium as the solution concept.

**Proposition 2** (Political Fragmentation). *When the threat is two-sided:*

1. There exists a threshold threat level $\Lambda_{III}$ such that if $\Lambda \leq \Lambda_{III}$, the revolution constraints do not bind and $\hat{\delta} \geq \delta$. The equilibrium military investments and location of capitals are the same as in the case when $\Lambda = 0$.

2. Otherwise, the revolution constraints bind and $\hat{\delta} = \delta$.

With or without the external threat, regimes in a competitive state system have to invest in the military to gain territories. Case 1 of Proposition 2 states that unless the external threat is severe (i.e. $\Lambda > \Lambda_{III}$), regimes do not have to make additional military investments to protect their populations as their existing military capacity—built up as a result of competition among themselves—already meets this requirement.

### 3.3 Implications for Political Centralization or Fragmentation

We now derive the implications of this simple model. First, Propositions 1 and 2 suggest that political centralization and fragmentation have different strengths and weaknesses. Since an empire invests in the military only to protect itself against external threats, while in a competitive state system regimes will invest in the military even in the absence of external threats so as to gain and maintain territorial control, interstate competition may lead to over-investment in the military. From a static perspective, there are Pareto gains to be reaped if competitive regimes coordinate among themselves to reduce their military spendings:

**Corollary 1** (Wastefulness of interstate competition). *In the absence of external threats, military investment is zero under an empire but strictly positive under interstate competition.*
However if the external threat is meaningful in that it induces an empire to spend a non-zero amount on the military to deal with it, then the empire will only provide protection to a fraction $\delta$ of the population so as to satisfy the revolution constraint (Figure 9). By contrast, in a competitive state system ($s = 2$), the competition-induced over-investment in the military may result in a larger-than-$\delta$ fraction of the continent being defended from external threats (Figure 10). Hence:

**Corollary 2 (Robustness of interstate competition).** When external threats are meaningful, interstate competition protects a weakly bigger interval of the continent than an empire does.

Proposition 1 also suggests that the choice of an empire’s capital city is influenced by the nature of the external threats that it confronts. If the costs of tax collection do not vary with distance, in the absence of external threats it does not matter where the empire’s capital city is located. If such costs are linear or convex, it can be shown that the empire’s capital city should always be at the center of the continent, i.e., $G^*_e = 0.5$. However, if the empire faces a meaningful one-sided threat (emanating from the left), it will locate its capital city at $G^*_e = 1 - \delta$ to contain the threat. In this case, centrality of the capital city is no longer optimal unless $\delta = 0.5$. The higher is $\delta$, the closer the capital city is to the frontier where the threat originates. Hence:

**Corollary 3 (Locational choice of capital city).** When the external threat is meaningful and one-sided, it is not optimal for an empire to locate its capital city at the center of the continent, i.e., $G^*_e \neq 0.5$.

Theoretical and empirical studies generally argue that capital cities should be centrally located to maximize tax revenue or improve governance (Alesina and Spolaore 2003; Olsson and Hansson 2011; Campante et al. 2014; Campante and Do 2014). Corollary 3 suggests that if the state is expected by its subjects to provide public goods, as long as the effectiveness of public goods provision (military defense in this case) differs with the location of provision, separating the political center of the empire from its economic or population center could be optimal. This is true whether or not the costs of tax collection vary with distance. In Section 5.1, we provide a historical discussion in light of this prediction.

Next, we use this simple setup to show that whether or not a continent is politically centralized or fragmented is shaped by the nature of the external threats that it faces.

As earlier discussed, we define a regime as viable if its equilibrium net revenue is non-negative. When $s = 1$ and regime $e$ is not viable, then political centralization or empire is not a stable outcome, that is, even if an empire emerges, it is not sustainable in the long run. Likewise, when $s = 2$ and if one of the two regimes is not viable, political fragmentation will be unstable. We
are concerned with the stability of political centralization or fragmentation given the nature of the external threat that it confronts.

Trivially, if an external threat is small enough so that it will not lead to a revolution when military investment is zero, then the threat has no impact on the viability of regimes. In this case, whether the continent is politically centralized or fragmented would not matter. Both outcomes are stable. In the remaining section, we focus on meaningful threats that cannot be completely ignored.

Specifically, we compare the scenario of a meaningful one-sided threat ($\Lambda > \bar{\Lambda}_I$) with that of a two-sided threat. For the two-sided threat, we focus on the range $\Lambda_I < \Lambda \leq \Lambda_{III}$, where the threat is meaningful but not severe enough so that the military investments of regimes in interstate competition are driven by the arms race among themselves and not by the external threat. For easy reference, we call this a moderate threat. The two scenarios are analogous to the Chinese and the European cases respectively.

**Proposition 3 (Net revenue comparison).** Under a one-sided threat, the net revenue of regime $e$ is always larger than the sum of net revenues of regimes $l$ and $r$. Under a moderate two-sided threat, the net revenue of regime $e$ is decreasing in $\theta$ but the sum of net revenues for regimes $l$ and $r$ are increasing in $\theta$.

Under a one-sided threat, as the strength of the threat ($\Lambda$) increases, the net revenue of regime $e$ and the sum of net revenues of regimes $l$ and $r$ both decrease. According to Proposition 3, there exists some threshold level $\hat{\Lambda}$ such that when $\Lambda = \hat{\Lambda}$, the sum of net revenues of regimes $l$ and $r$ is zero while the net revenue of regime $e$ is still strictly positive. Meanwhile, under a moderate, two-sided threat, as the cost of military investment ($\theta$) increases, the net revenue of $e$ will decrease rapidly relative to the sum of net revenues of $l$ and $r$. Hence, Proposition 3 gives rise to Corollaries 4 and 5 below:

**Corollary 4 (Stability under one-sided, severe threat).** When the external threat is one-sided and severe, political centralization is more likely to be stable than political fragmentation.
Corollary 5 (Stability under two-sided, moderate threat). When the external threat is moderate and two-sided, political centralization is less likely to be stable than political fragmentation if \( \theta \) is high.

3.4 Taxation

Now let us endogenize taxation. Previously, the amount of taxes paid in the continent was always equal to per capita income \( y \). Suppose regime \( i \) has the option of reducing the tax burden of its people by \( R_i \geq 0 \). Lowering taxes eases the revolution constraint (as it helps keep the population content) so that an individual at \( x \) does not engage in revolution if:

\[
R_i + m(M_i, |G_i - x|) - \lambda(\Lambda, x) \geq 0.
\]

When \( s = 1 \), as long as the threat level is meaningful enough for the empire to spend a non-zero amount on the military, the revolution constraint will always bind in equilibrium regardless of whether the threats are one-sided or two-sided. We show in Appendix A.7 that if \( \theta \) is sufficiently high, that is, if building a military is costly, the empire will opt to provide some tax reimbursement instead of relying solely on building the military to satisfy the revolution constraint.

By contrast, when \( s = 2 \), the revolution constraint will not bind in equilibrium unless the threat is severe. In this case, regimes \( l \) and \( r \) will set \( R_l = R_r = 0 \).

Consider the two scenarios that we are examining: if an empire emerges in the face of a severe one-sided threat, the effective level of taxation will be \( y - R_e \), where \( R_e \geq 0 \); if interstate competition prevails under a moderate two-sided threat, the level of taxation will remain at \( y \). More generally, we can state:

Corollary 6 (Taxation). Taxation is weakly lower under political centralization than under political fragmentation.

3.5 Population Dynamics and Long-run Growth

Until now, we have assumed that external threats are always present. Consider a dynamic model where in each period, the external threat is realized with some positive probability. Each individual lives for one period and inelastically supplies labor to produce \( y + y \), where \( y \) is not taxable and \( y \) is taxed. For individual \( x \) under regime \( i \), the disposable income is \( \bar{y} = y + R_i - \kappa_i(x) \) where \( R_i \) is the tax reimbursed by regime \( i \) and \( \kappa_i(x) \) is the net damage caused by the stochastic shock. Each individual chooses between private consumption \( c \) and producing \( n \) offspring to maximize her utility \( u(c, n) \) subject to the budget constraint \( \rho n + c \leq \bar{y} \), where \( \rho \) represents
Figure 11: Under political centralization, there is a positive level of tax reduction, i.e. $R_e^* > 0$. Under political decentralization, tax reduction is zero.

Figure 12: Under political decentralization, the fraction of the population protected from invasion is at least $\delta$. Under political centralization, it is at most $\delta$.

the cost of raising a child. We assume that $c$ and $n$ are complements and $u$ is increasing and concave in both arguments. Standard optimization implies that the optimal number of children is $n = g(\bar{y})$ where $g' > 0$.

For simplicity, we assume that individuals redistribute themselves uniformly over the continent at the beginning of each period. Population growth is therefore given by:

$$N = \int_0^1 ndx = \int_0^1 g(\bar{y}) \, dx.$$ 

Let $N_E$ and $N_F$ denote population growths in continent $E$ and continent $F$ respectively. The two continents are identical except that continent $E$ is ruled by an empire ($s = 1$) and faces a severe one-sided threat of size $\Lambda_E$, while continent $F$ is politically fragmented ($s = 2$) and faces a moderate two-sided threat of size $\Lambda_F$.

When the external threat is not realized, the populations in the two continents grow to $N_E = g(\bar{y} + R_e)$ and $N_F = g(\bar{y})$ respectively. Since $N_E > N_F$, population grows faster under the empire.

However, the converse may be true if the external threat is realized. In this case, realized population growth under empire and political fragmentation is given by, respectively:

$$N_E = \int_0^1 g(\bar{y} + R_e - 1_{x \notin D_e}[\lambda(\Lambda_E, x) - m(M_e, |G_e - x|)]) \, dx;$$

$$N_F = \int_0^1 g(\bar{y} - 1_{x < b, x \notin D_l}[\lambda(\Lambda_F, x) - m(M_l, |G_l - x|)]$$

$$- 1_{x > b, x \notin D_r}[\lambda(\Lambda_F, 1 - x) - m(M_r, |G_r - (1 - x)|)]) \, dx.$$ 

Now $N_E < N_F$ if $\Lambda_E$ is sufficiently large with respect to $\Lambda_F$.

For the purpose of illustration, let $u(c, n) = c^{1-\gamma}n^\gamma$. It can be shown that when the shock is
realized, the population growth under empire and under fragmentation are given by:

\[
N_E = \gamma \rho \left\{ (y + R_e) - \int_{x \notin \mathbb{D}_e} \lambda(\Lambda_E, x) - m(M_e, |G_e - x|) \, dx \right\}; \\
N_F = \gamma \rho \left\{ y - 2 \int_{x < b, x \notin \mathbb{D}_l} \lambda(\Lambda_F, x) - m(M_l, |G_l - x|) \, dx \right\},
\]

where \( \text{Area} \langle E \rangle \) and \( \text{Area} \langle F \rangle \) are illustrated in Figures 11 and 12.

Given the nature of the threats and the political configurations in the two Continents, \( \text{Area} \langle E \rangle \) is likely to be larger than \( \text{Area} \langle F \rangle \) for two reasons: First, \( \Lambda_E > \Lambda_F \); Second, as we have shown in Corollary 2, the empire offers protection to only \( \delta \) fraction of continent \( E \) (and less than \( \delta \) if tax reduction is offered), while the fraction of continent \( F \) that is protected is always weakly larger than \( \delta \) due to the presence of interstate competition. If \( \text{Area} \langle F \rangle < \text{Area} \langle E \rangle - R_E \), it follows that \( N_E < N_F \):

**Corollary 7 (Population Change). If the external threat is not realized, population grows faster under political centralization. If the external threat is realized, a population contraction is more likely under political centralization than under political fragmentation.**

Corollary 7 suggests that population growth is usually faster under an empire. However, in the event of an exogenous shock, the fall in population may be less severe under political fragmentation. In other words, population growth is likely to be more volatile under political centralization relative to political fragmentation. See Appendix A.8 for a numerical example of Corollary 7.

In interpreting our model, we have focused on external invasions. More generally, however, negative shocks could also stem from unforeseen political collapses and peasant rebellions in addition to invasions from outside. The central point we emphasize is that interstate competition results in a greater proportion of territory being protected than is the case under political centralization.

### 4 Threats and Regime Size: Empirical Evidence

The model predicts that the stability of political centralization and fragmentation is unaffected by external threats when these threats are sufficiently small; an environment with moderate external threats originating from multiple fronts favors interstate competition; a unidirectional threat promotes political unification, especially if the threat is severe. To test these predictions,
we first investigate the empirical relationship between the frequency of nomadic attacks and the number of regimes in China proper. Subsequently, we examine historical evidence from Europe for consistency with the predictions.

4.1 Empirical Evidence from China

Data Sources and Definition of Variables  In a recent paper, Bai and Kung (2011) show that nomadic incursions into China proper were correlated with exogenous variations in rainfall as climatic disasters such as droughts often triggered subsistence crises that drove the inhabitants of the ecologically fragile steppe to invade their settled neighbors for food. We make use of their data set and empirical strategy to check if there was a relationship between the frequency of nomadic attacks and the number of regimes in China proper. This helps to ensure that our empirical evidence is robust and is not subjected to selective adoption to suit our purpose.

Bai and Kung’s data span 2,060 years (from 220 BC to AD 1839) and are drawn from four sources: A Chronology of Warfare in Dynastic China (China’s Military History Editorial Committee, 2003), A Compendium of Historical Materials on Natural Disasters in Chinese Agriculture (Zhang et al., 1994), A Concise Narrative of Irrigation History of the Yellow River (Editorial Committee of Irrigation History of the Yellow River, 1982), and the Handbook of the Annals of China’s Dynasties (Gu, 1995). Of these sources, the first three have been widely used in related research and are considered reliable sources while the fourth contains general historical information that can be easily verified.\footnote{A detailed check of the accuracy of the data on Sino-nomadic conflicts is available in Bai and Kung (2011, Table A.2.)}

As listed in Table 1, the decadal variables Bai and Kung constructed include: (i) the frequency of nomadic attacks on China’s Central Plain (Henan, Shanxi, Shaanxi, Hebei); (ii) two precipitation variables that measure the extent of severe droughts and floods in the Central Plain; (iii) other climatic control variables (snow and other low temperature disasters, temperature); (iv) historical dummy variables to control for the three periods in Chinese history when nomadic regimes ruled the Central Plain; and (v) a time trend. Drawing from Wei (2011), we add two variables required for this exercise: a dummy variable that takes the value of 0 if China was politically unified in a given decade and 1 otherwise (\textit{Fragmentation}); and a logged variable that counts the average number of regimes in China proper in a given decade (\#\textit{Regime}).

Baseline Estimation  As a baseline to investigate the effects of nomadic invasions on political fragmentation in China proper, we adopt a simple autoregressive distributed lag (ADL) model:
Table 1: List of Variables and Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>mean</th>
<th>s.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fragmentation  (y_{1t})</td>
<td>(=1) if more than 1 regime ruled China proper in decade (t)</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td>#Regimes</td>
<td>Average number of regimes in China proper in decade (t) (log)</td>
<td>0.39</td>
<td>0.54</td>
</tr>
<tr>
<td>Nomadic attacks  (z_{1t})</td>
<td>Frequency of attacks initiated by the nomads in decade (t)</td>
<td>2.53</td>
<td>3.50</td>
</tr>
<tr>
<td>Lower precipitation (z_{1t})</td>
<td>Share of years with records of drought disasters on the Central Plain in decade (t)</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>Higher precipitation (z_{1t})</td>
<td>Share of years with records of Yellow River levee breaches in decade (t)</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>Snow disasters (z_{1t})</td>
<td>Share of years with records of heavy snow on the Central Plain in decade (t)</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>Low temperature (z_{1t})</td>
<td>Share of years with records of low-temperature calamities (e.g., frost) on the Central Plain in decade (t)</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>Nomadic conquest 1 (w_{1t})</td>
<td>(=1) if the Central Plain was governed by the nomads (617–589)</td>
<td>0.13</td>
<td>0.33</td>
</tr>
<tr>
<td>Nomadic conquest 2 (w_{1t})</td>
<td>(=1) if the Central Plain was governed by the nomads (1126–1368)</td>
<td>0.12</td>
<td>0.32</td>
</tr>
<tr>
<td>Nomadic conquest 3 (w_{1t})</td>
<td>(=1) if the Central Plain was governed by the nomads (1644–1839)</td>
<td>0.10</td>
<td>0.29</td>
</tr>
<tr>
<td>Time trend (w_{1t})</td>
<td>Decade: -22–183 (219 BC–1839)</td>
<td>80.5</td>
<td>39.6</td>
</tr>
</tbody>
</table>

Sources: Bai and Kung (2011) and Wei (2011).

\[ y_{t} = \phi_{0} + \sum_{i=1}^{p} \phi_{i}y_{t-i} + \sum_{i=0}^{q} \mu_{i}x_{t-i} + \epsilon_{t}, \]  

where the dependent variable \(y_{t}\) is either the dummy variable *Fragmentation* or \#*Regime*, the logarithm of the number of regimes in China proper in decade \(t\). The explanatory variable \(x_{t}\) is the number of nomadic incursions into China proper in decade \(t\).

According to Corollary 4, an increase in the severity of nomadic threat favors political centralization. While this effect may not be immediate—in the short run, a spike in nomadic attacks could lead to the collapse of the central authority and the emergence of a host of succession states to fill up the political vacuum—Corollary 4 suggests that increased nomadic attacks should have a long run negative effect on the number of regimes in China proper. In other words, we expect \(\mu_{0} + \mu_{1} + \mu_{2} + \ldots + \mu_{q} < 0\).

The ADL model is appropriate for our purpose because of its flexibility. Furthermore, it generates unbiased long run estimates and valid t-statistics even in the presence of endogeneity (Harris and Sollis, 2003). To validate our use of the ADL methodology, we use the Augmented Dickey-Fuller test to ensure that all variables are stationary. To determine the appropriate number of lags, we follow the general-to-specific approach proposed by Ng and Perron (1995) to seek the values of \(p\) and \(q\) in equation 1 that minimize the Akaike Information Criterion (AIC).

We find that the AIC is minimized when \(p = 3\) and \(q = 1\), regardless of whether the dependent variable is *Fragmentation* or \#*Regime*. In the baseline linear probability model reported in column (a) of Table 2, where the dependent variable is *Fragmentation*, we find that the nomadic invasion variable and its lagged value are both statistically significant, but they carry opposite signs: an

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22When implementing the general-to-specific approach, we choose \(p = q = 10\) as the cut-off and check every combination of \(p \leq 10\) and \(q \leq 10\).
additional nomadic attack in decade $t$ is associated with a 1.2% increase in the probability of politically fragmentation in China in the same decade, but an attack in the previous decade (at $t − 1$) is associated with a larger 1.96% decrease in the probability of politically fragmentation in decade $t$. In line with Corollary 4, the relationship between nomadic invasions and political fragmentation is negative in the long run: each additional nomadic attack is associated with a decrease in the probability of politically fragmentation in China—or an increase in the probability of political unification—of 6.3% ($= \frac{0.012−0.0196}{1−0.906+0.283−0.256}$). Given that China experienced an average of 2.5 nomadic attacks per decade, the relationship between nomadic invasions and its political unity is clearly a significant one.\footnote{For this estimation and all subsequent ones that we report, we have conducted the Durbin’s h-test to ensure that the errors are serially independent. In addition, we have also checked that the roots of the characteristic equation in each specification are all smaller than 1 and therefore the estimation model is ‘dynamically stable’.}

In column (b) of Table 2, we replace $\text{Fragmentation}$ with $\text{#Regime}$ as the dependent variable, and obtain consistent results: while nomadic attacks appear to have negligible effect on the
number of regimes in China proper, the lagged effect is significant: every additional nomadic attack is associated with a decrease in the number of regimes in China proper one decade later by 1.37%.

One may suspect that political fragmentation could leave China divided and weakened, and therefore increase the likelihood of nomadic attacks. In other words, there may exist an endogeneity problem in Equation 1. For our purpose, this source of endogeneity is not an issue. If indeed political fragmentation had the effect of increasing nomadic attacks, one can verify that the magnitude of estimates that we have derived so far are biased downwards and the true effect of nomadic invasions on China’s political unity would be even larger. Nonetheless, as a check in column (c) we repeat the time-series regression in column (b) but drop the non-lagged nomadic invasion variable, which is in any case statistically insignificant. This should mitigate some of the concerns over potential endogeneity since the remaining lagged nomadic invasion variable is determined before the present period. We find similar results after dropping the non-lagged nomadic invasion variable: the short run and long run effects are $-1.15\% = e^{-0.0116} - 1$ and $-8.53\% = e^{1.0116 - 0.066 + 0.343 - 0.147 - 1}$ respectively.

To see if the results would change under an alternative specification, in columns (d)–(f) we deviate from the classic ADL model and introduce the control variables as used in Bai and Kung (2011) into our estimation equation, which now becomes:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=0}^{q} \mu_i x_{t-i} + \sum_{i=0}^{q} \psi_1^i z_{1t-i} + \sum_{i=0}^{q} \psi_2^i z_{2t-i} + \pi W_t + \epsilon_t$$

where $z_{1t}$ and $z_{2t}$ are rainfall variables measuring droughts and floods and $W_t$ is a vector of seven other climatic and historical control variables as discussed previously (Table 1). In Table 2, columns (d), (e), and (f) replicate, respectively, the estimations in columns (a), (b), and (c) using the new specification. Since Bai and Kung (2011) detect a strong and robust relationship between the frequency of nomadic invasions and rainfall factors, our introduction of the rainfall variables and the full range of controls alongside the frequency of nomadic invasions leads to multicollinearity and increases the standard errors of the estimates. However, as columns (d)–(f) show, we obtain very similar and statistically significant coefficient estimates. This gives us confidence in the robustness of our results.

**VAR Estimation** Following Bai and Kung (2011), we also implement the following vector autoregression (VAR) as a robustness check:
Table 3: Robustness Checks: VAR Model

<table>
<thead>
<tr>
<th></th>
<th>(a1) Fragmentation</th>
<th>(a2) Nomadic attacks</th>
<th>(b1) #Regimes</th>
<th>(b2) Nomadic attacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fragmentation: Lag 1</td>
<td>0.893***</td>
<td>2.075***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0665)</td>
<td>(0.733)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fragmentation: Lag 2</td>
<td>-0.317***</td>
<td>-1.631*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0848)</td>
<td>(0.935)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fragmentation: Lag 3</td>
<td>0.225***</td>
<td>-0.377</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0656)</td>
<td>(0.723)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#Regimes: Lag 1</td>
<td>1.026***</td>
<td>1.904***</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.0660)</td>
<td>(0.721)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#Regimes: Lag 2</td>
<td>-0.343***</td>
<td>-1.330</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0882)</td>
<td>(0.963)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#Regimes: Lag 3</td>
<td>0.131**</td>
<td>-0.254</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0638)</td>
<td>(0.696)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomadic attacks: Lag 1</td>
<td>-0.0176***</td>
<td>0.321***</td>
<td>-0.0130**</td>
<td>0.333***</td>
</tr>
<tr>
<td></td>
<td>(0.00626)</td>
<td>(0.0690)</td>
<td>(0.00626)</td>
<td>(0.0684)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomadic attacks: Lag 2</td>
<td>0.00701</td>
<td>0.257***</td>
<td>0.00943</td>
<td>0.236***</td>
</tr>
<tr>
<td></td>
<td>(0.00657)</td>
<td>(0.0724)</td>
<td>(0.00654)</td>
<td>(0.0715)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomadic attacks: Lag 3</td>
<td>-0.00602</td>
<td>-0.0108</td>
<td>-0.00269</td>
<td>0.000803</td>
</tr>
<tr>
<td></td>
<td>(0.00638)</td>
<td>(0.0703)</td>
<td>(0.00632)</td>
<td>(0.0690)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower precipitation</td>
<td>0.0270</td>
<td>2.550**</td>
<td>-0.0450</td>
<td>2.570**</td>
</tr>
<tr>
<td></td>
<td>(0.0922)</td>
<td>(1.016)</td>
<td>(0.0933)</td>
<td>(1.019)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher precipitation</td>
<td>-0.173</td>
<td>-3.376**</td>
<td>-0.0561</td>
<td>-3.623***</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(1.357)</td>
<td>(0.126)</td>
<td>(1.381)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Snow disasters</td>
<td>-0.0189</td>
<td>1.853</td>
<td>0.0358</td>
<td>1.937</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
<td>(1.682)</td>
<td>(1.53)</td>
<td>(1.669)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low temperature disasters</td>
<td>-0.0421</td>
<td>-0.824</td>
<td>-0.111</td>
<td>-0.822</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
<td>(1.373)</td>
<td>(1.27)</td>
<td>(1.386)</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>-0.0166</td>
<td>-0.372</td>
<td>0.0176</td>
<td>-0.443*</td>
</tr>
<tr>
<td></td>
<td>(0.0225)</td>
<td>(0.248)</td>
<td>(0.0228)</td>
<td>(0.249)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomadic conquest 1</td>
<td>0.125*</td>
<td>-1.484**</td>
<td>0.142**</td>
<td>-1.744**</td>
</tr>
<tr>
<td></td>
<td>(0.0662)</td>
<td>(0.730)</td>
<td>(0.0665)</td>
<td>(0.726)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomadic conquest 2</td>
<td>0.0623</td>
<td>-1.173</td>
<td>0.0911</td>
<td>-1.335*</td>
</tr>
<tr>
<td></td>
<td>(0.0692)</td>
<td>(0.763)</td>
<td>(0.0693)</td>
<td>(0.757)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nomadic conquest 3</td>
<td>-0.159*</td>
<td>-3.339***</td>
<td>-0.0663</td>
<td>-3.357***</td>
</tr>
<tr>
<td></td>
<td>(0.0909)</td>
<td>(1.002)</td>
<td>(0.0911)</td>
<td>(0.995)</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Time trend</td>
<td>0.0007</td>
<td>0.008</td>
<td>0.0006</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.006)</td>
<td>(0.0006)</td>
<td>(0.006)</td>
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<tr>
<td>Observations</td>
<td>203</td>
<td>203</td>
<td>203</td>
<td>203</td>
</tr>
</tbody>
</table>

Constant terms not reported. Standard errors in parentheses: *** p < 0.01, ** p < 0.05, * p < 0.1.

As Table 3 illustrates, the estimates from the VAR model share the same order of magnitude with the results from the ADL estimations in Table 2. The coefficient estimate of Lag-1 nomadic conquest 1 is 0.125*, indicating a significant impact on the dependent variable. The negative coefficient estimates for Lag-2 and Lag-3 suggest a decreasing trend in the effect over time. The inclusion of interactions and lagged dependent variables allows for a more nuanced understanding of the relationships between variables. As such, the VAR model provides a robust framework for analyzing the complex dynamics involved in the study of nomadic conquests and regime changes.

The results also highlight the importance of considering different lags in the explanatory variables, as shown by the varying coefficients for Lag-1, Lag-2, and Lag-3. This approach helps to capture the temporal dynamics and potential feedback effects in the system. The inclusion of higher-order interactions further enhances the model’s explanatory power, allowing for a more comprehensive analysis of the data. Overall, the VAR model offers a valuable tool for researchers seeking to model and understand the complex interplay of factors influencing nomadic conquests and regime changes.
attack is -0.0176 in column (a1), compared with -0.0196 in column (a) of the previous table, and -0.0130 in column (b1), compared with -0.0137 in column (b) of the previous table. This lends further credence to the robustness of the estimates.

It is worth noting that the climatic variables are statistically insignificant in columns (a1) and (b1). Any effect of climate change on political unification appear to have worked through the nomadic invasion channel. Furthermore, our VAR estimation replicates Bai and Kung’s main finding that nomadic invasions were positively correlated with less rainfall and negatively correlated with more rainfall (columns a2 and b2). Despite introducing new dependent variables (\textit{Fragmentation} and \#\textit{Regime}), their results remain intact in our analysis. Indirectly, this provides another piece of evidence to support the validity of our empirical exercise.

4.2 \textit{Historical Evidence from Europe}

We are unable to replicate the empirical exercise for Europe because data on the number of regimes in Europe only exists on a per century basis. However, European historical patterns do conform to predictions of our theoretical model.

Europe has historically been politically fragmented; the closest Europe came to be ruled by a unified political system was under the Roman Empire. The rise of Rome parallels the rise of the first empire in China (Scheidel, 2009). In terms of the model, one advantage Rome had over its rivals in the Hellenistic world was a lower $\theta$. Rome’s ability to project power and increase its resources of manpower was unequalled among European states in antiquity (Eckstein, 2011). Thus Rome was able to impose centralized rule upon much of Europe. Our model suggests that two factors can account for the decline of Rome: (1) over time, Rome’s military advantage (which is inversely related to $\theta_{Rome}$) declined relative to the military capacities of its rivals such as the Persian empire or the Germanic confederacies; and (2) these rising threats came from multiple directions along Rome’s long border.\textsuperscript{25} Like episodes of dynastic and imperial collapse in China, the fall of the western Empire was associated with political disintegration and economic collapse across Europe (Ward-Perkins, 2005). However, unlike in China, all subsequent attempts to rebuild the Roman Empire failed.

The most successful subsequent attempt to build a European-wide polity was the creation of a Frankish empire by the Carolingians. The Carolingian dynasty was established by Pippin III (r. 752–768) and under the reign of Charlemagne (r. 768-814) came to control an empire that spanned France, parts of Spain and much of Italy, Germany and central Europe (Collins, 1998; McKitterick, 2008; Costambeys et al., 2011).

Consistent with our model, the Carolingian empire was not long-lasting. It went into decline...
as the successors of Charlemagne struggled to deal with the external threats posed by the Magyars, the Vikings, and the Muslims from different fronts (Morrissey, 1997).\textsuperscript{26} The empire was divided in 843 and this marks an end of point of Carolingian empire (Figure 13).

In East Francia, a different dynasty, the Ottonians, came to power as a response to the repeated Magyar invasions and established the Holy Roman Empire. At its height in the eleventh century, it comprised the modern countries Austria, Czech Republic, Germany, Italy, the Low Countries, and Switzerland. However, the decline of the Holy Roman Empire began soon after the Magyars, its main foe, converted to Christianity and served instead as a bulwark against further barbarian invasions. Increasingly, emperors based in Germany found it difficult to control their Italian provinces and by the thirteenth century, the Holy Roman Empire was no more than a loose federation of German principalities.

Incidentally, the threats posed to the Europeans by the Vikings and the Muslims also receded after the eleventh century. One could argue that from then on, Western Europe no longer experienced meaningful multi-sided external threats. If this interpretation is correct, our model predicts that the status quo of political fragmentation would persist, and it did. The Mongol invasion of Europe in the thirteenth century was too brief to provide a sustained impetus toward European unification. However, the less dramatic but more sustained rise of the Ottoman empire after the fifteenth century serves as yet another test of our model and it provides further

\textsuperscript{26}As Spruyt observes ‘the breakup of central authority coincided with the increasing raids by Magyars, Saracens, and Vikings’ (Spruyt, 1994, 37). Viking raids began in the reign of Charlemagne but greatly increased in ambition after the death of Louis the Pious exposed the fragility of the Carolingian hegemony and hastened the emergence of localized regional power centers.
supporting evidence that our mechanisms are relevant. Iyigun (2008) shows that the external threat of invasion from the Ottomans between 1410 and 1700 reduced the frequency of interstate warfare in Europe. Indeed, a comparison of the political maps of central and eastern Europe of the fourteenth century and seventeenth century indicates that ‘a significant degree of political consolidation accompanied the Ottoman expansion in continental Europe’ (Iyigun, 2008, 1470).

5 APPLYING THE MODEL

We are now in a position to use our theory to offer new insights into the choice of capital city location, differential levels of taxation and military buildup, and differential patterns of population change at the two ends of Eurasia.

5.1 Locations of Capital Cities

Our model predicts that large empires will locate their capitals in order to respond to the threat of external invasion. There are numerous examples of empires changing capitals in order to protect their empires against external enemies; to substantiate our model we focus on examples from China and Europe.

Consistent with our model, for most of its history, China’s capital city was located in its northern or northwestern frontier instead of the populous Central Plain or Lower Yangzi Delta. For the 1,418 years between 221 BC and AD 1911 when China proper was under unified rule, Beijing and Changan (modern day Xi-an) served as its national capital for 634 years and 553 years respectively, or together 8.4 years out of every 10 years (Wilkinson, 2012).

Changan was China’s preeminent political center in the first millennium (Figure 14a). It was the capital city of the unified dynasties of Qin (221–206 BC), Former Han and Xin (202 BC–AD 23), Sui (581–618), and Tang (618–907). Figure 14a illustrates two salient characteristics of Changan’s geographical location that buttress our argument: (1) it was not the population or economic center of the empire;27 and (2) it shielded China’s populous Central Plain from nomadic invasions by virtue of its strategic location between the steppe and central-eastern China.

In the second millennium when China’s threat from the north shifted from Inner Asia eastward to the semi-nomadic lands of Manchuria, Beijing replaced Changan as the new political center of China. The emergence of Beijing was due to its proximity to the northern frontier—the Chinese thinker and political theorist Huang Zongxi (1610–1695) likened making Beijing the national

27During the Han dynasty, for example, only 3 million people, or around 4 percent of the Chinese population, resided in Guanzhong, the region where Changan was located. By contrast, the Guandong region in central-eastern China was home to 60 percent of the empire’s subjects.
**Figure 14:** Capital cities in China and the Roman Empire. Panel (a) depicts the Han dynasty’s capital city, Changan, and its most populous prefectures. Beijing replaced Changan as China’s preeminent political center in the second millennium. Panel (b) depicts the major cities of the Roman Empire. During the Tetrarchy period, there were four capitals: Trier, Milan, Sirmium, and Nicomedia. Constantinople and Ravenna were the capitals of the Eastern and Western Empires respectively. Carthage, Alexandria, and Antioch were the largest cities of the Roman Empire after Rome itself. Adapted from Herrmann (1966), Skinner et al. (2007), and Talbert (2000).

For the European case, our evidence comes from the Roman Empire—the single long-lasting empire to span much of the continent in European history. We find strong support for this prediction of the model. The Roman Republic and Empire expanded symmetrically from the city of Rome over several centuries to encompass the entirety of the Mediterranean and western Europe. Over time therefore, the location of the capital became less and less convenient from the point of view of military operations. This was not a major issue during the first century of the Empire, but as the severity of the external threats facing the empire grew from the mid-second century onward emperors spent less and less time in Rome and they eventually set up other capital cities in which to reside.\(^{28}\) Figure 14b depicts both the old capital of Rome and the capitals established during the latter part of the third century: Trier on the Rhine frontier, Milan at entrance of the Alps shielding Italy from invasion, Sirmium on the Danube frontier, and Nicomedia in the east. Consistent with the predictions of the model, therefore, it was the

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\(^{28}\) For example, Gallenius (r. 253–268) did not visit Rome until the fifth year of his reign while Diocletian (r. 284–306), who established his capital at Nicomedia in modern day Turkey, did not visit Rome until the twentieth year of his reign (Rees 2004, 28 and Goldsworthy 2009, 162).
threat of invasion that drove the choice of capital location for the Roman Empire.\textsuperscript{29}

Importantly, the choice of these new capital cities did not correspond to the largest cities. After Rome, which remained the biggest city in the empire through this period,\textsuperscript{30} the most populous cities in the empire were Alexandria with around 600,000 inhabitants, and Antioch and Carthage with between 300,000–500,000 people (Scheidel, 2013, 78). However, with the exception of Antioch, these cities were far from the frontiers and were not chosen as capital cities for this reason.\textsuperscript{31}

When the emperor Constantine (r. 306–338) established a new permanent capital at the small Greek city of Byzantine, renamed as Constantinople, he chose this location not because it had any economic significance at the time, but because it was close to both the eastern frontier of the empire and to the important Danube front where the empire faced some of its most determined enemies.\textsuperscript{32}

Interestingly, the lessons of Chinese and Roman history also provide ample evidence to support our assumption in the model that rules out the possibility of a state maintaining two or more comparable political-military centers. During the mid-Tang dynasty, the Xuanzong emperor (r. 712–756) implemented a polycentric political-military system and devolved much of the central government’s political authority to frontier military governors with the goal of improving military responsiveness and effectiveness. However, Xuanzong’s favorite and most powerful frontier governor, An Lushan, infamously revolted in 755 as the military might of An’s army fed popular suspicion of his political ambitions, which ironically compelled An to revolt. A similar development took place during the early Ming dynasty with the implementation of a de facto twin-capital system in which the emperor resided in the populous south and his uncle, the Prince of Yan, coordinated border defense in the strategic north. The arrangement again proved unstable and mutual suspicion led to the outbreak of a bloody civil war in 1399 with the Prince of Yan emerging as the eventual victor. So as to prevent history from repeating itself, the usurper moved the capital city from Nanjing (the ‘southern capital’) to Beijing (the ‘northern capital’) in order to maintain direct control of the large army along the northern border.

Similarly, the Roman Empire was not able to maintain multiple capitals within a single empire for a long period of time. The fourfold division of the empire inaugurated by Diocletian (r. 284–306), known as the Tetrarchy, did not last long beyond his retirement in 306 and led to a

\textsuperscript{29}Rees notes that ‘[t]he motivation for this move away from the ancient capital seems to have been strategic . . . the north Italian cities, Milan, Ravenna, and Aquileia, were closer to the main military areas than Rome was’ (Rees, 2004, 27).

\textsuperscript{30}Though its population declined from a peak of 1 million in the 2nd century to approximately 750,000 in the 4th century.

\textsuperscript{31}Antioch was in fact a major base to fight the Persians even if it was not officially a capital city. For example, the Emperor Julian (r. 361–363) spent nine months in Antioch preparing for his invasion of Persia.

\textsuperscript{32}For example, see Odahl (2004, 232) and Goldsworthy (2009, 186).
series of civil wars that only ended with Constantine’s reunification in 324. Civil wars reoccurred during periods of imperial division in 337–350, 360–361, 383–388, 392–395. By the end of the fourth century, the centrifugal forces affecting the empire led to the permanent division of the empire into East and West and from this point on the two empires coexisted as separate political entities until the fall of the Western Empire in 476.

5.2 Taxation and Army Size

Our model predicts that taxation would be higher in Europe relative to China (Corollary 6). This contradicts traditional comparative accounts of Europe and China, which complained that economic development in China was retarded by high levels of taxation (e.g. Jones 2003), but it is consistent with recent scholarship in economic history. Tax rates in Europe were high and especially so in the Dutch Republic and England after 1689 (Hoffman and Norberg, 1994; Bonney, 1999).33 In contrast, taxes were comparatively low in China. Karaman and Pamuk (2013) provide data on taxes revenues for a range of European countries. Table 4 depicts this data in conjunction with estimates of per capita and total tax revenue from China from Sng (2014). Tax revenue per capita in France was lower than in either the Dutch Republic or England, but it remained much higher than in China.34 The average European per capita level of taxation as measured in silver was roughly four times higher compared to China. As China was a net importer of silver, the value of silver in China might have been higher than in Europe. Following Ma (2013), we use the bare-bones subsistence basket constructed in Allen et al. (2011) to estimate the tax burden in Europe and China and obtain similar results. Clearly, as Corollary 6 suggests, taxation was lighter under politically centralized China than it was in fragmented Europe.35

By and large, the taxes raised by European states were spent on warfare.36 Scholars have

33This increase in tax revenue obviously implies that the tax revenues collected by the central state were lower before 1700. However, this does not mean that the total level of taxation was lower when one includes feudal dues, local taxes, and tithes to the Church. Much of the increase in tax revenue collected by centralized states came at the expense of these forms of taxation which were generally restricted and abolished in the seventeenth and eighteenth centuries. For a comparison for taxes in England and France in the seventeenth century see Johnson and Koyama (2014a).

34Johnson and Koyama (2014b) document the increase in tax revenues at a regional level in France throughout the seventeenth century.

35Our hypothesis complements a strand of the literature that attributes light taxation in China to a severe principal-agent problem in its government (Kiser and Tong, 1992; Ma, 2013; Sng, 2014; Sng and Moriguchi, 2014). Instead of focusing on agency costs in tax collection as this literature does, we look at agency costs in public goods provision (as argued in Section 3.1, agency costs in military control lead to the deterioration of military strength over distance) and derive similar results.

36In war years, over 75 percent of French revenue was spent on the military in the seventeenth century (Félix and Tallett, 2009, 155); in eighteenth century Britain, this figure varied between 61 and 74 per cent (Brewer, 1988, 32); while the peacetime military budget of Prussia during the eighteenth century accounted for 80 per cent of central government expenditure (Wilson, 2009, 119).
Table 4: Per capita tax revenue in grams of silver. European average tax revenue includes Venice, Austria, Russia, Prussia, and Poland-Lithuania in addition to England, France, Dutch Republic and Spain. Sources: Karaman and Pamuk (2013) and Sng (2014). In parentheses we include a comparison of per capita tax revenue as a proportion of ‘bare-bones’ subsistence in 1750 as measured by Allen et al. (2011).

long noted that Europe was the ‘seat of Mars’. Figure 4a confirms this impression for the period between 1400 and 1700. Furthermore, the proportion of individuals under arms in Europe was much higher than in China (Table 5).

This fact can be explained in terms of Corollary 1 which predicts that states under interstate competition will invest heavily in their military. Table 5 indicates that in the middle of the eighteenth century the percentage of individuals in the army in Europe was between 1 and 2 percent of the population. In contrast, in Qing China the proportion of the population in the army was at most 0.4 percent of the population.

The data in Table 5 reflects the army sizes of the major European military powers in the eighteenth century. Obtaining consistent data for earlier periods in Europe is extremely difficult. We are aware that the Military Revolution led to a dramatic rise in the size of the standing armies of the major European powers in the seventeenth and eighteenth centuries (Parker, 1976, 1988) and that the size of standing armies in Europe in earlier periods was much smaller. However, this does not mean that the proportion of the population under arms was smaller in the earlier centuries as endemic warfare compelled a large proportion of the population to serve as warriors at some point. In the medieval period while there were no professional armies, so to speak, a significant fraction of the eligible male population might be expected to service in the army.

37 For example ‘No other continent in recorded history fought so frequently, for such long periods, killing such a high proportion of its population’ (Voigtländer and Voth, 2013, 168). For a general discussion see Tilly (1990).

38 In China as in Europe, the majority of government revenue was spent on the military. The main difference was that China’s total military spending was much lower. As Vries (2012) observed, ‘with roughly twenty times as many inhabitants, China, in real terms, per year on average only spent roughly 1.8 times as much on the military as Britain did during the period from the 1760s to the 1820s. Per capita in real terms Britain thus spent more than ten times as much on its army and navy than China’ (Vries, 2012, 12).

39 In the early middle ages, Heather (2009, 59-60) notes that standard estimates of the proportion of warriors to non-warriors in the Germanic tribes was between one to four or one to five, implying that 20 to 25 percent of the entire population could be under arms during periods of war (which was endemic).
<table>
<thead>
<tr>
<th>Number of Soldiers</th>
<th>% of Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Britain</td>
<td>79,000</td>
</tr>
<tr>
<td>France</td>
<td>183,000</td>
</tr>
<tr>
<td>Dutch Republic</td>
<td>45,000 (1750s)</td>
</tr>
<tr>
<td>Spain</td>
<td>109,000 (1750s)</td>
</tr>
<tr>
<td>Austria</td>
<td>190,000 (1778)</td>
</tr>
<tr>
<td>Prussia</td>
<td>133,000 (1751)</td>
</tr>
<tr>
<td>Russia</td>
<td>408,000 (1780)</td>
</tr>
<tr>
<td>Saxony</td>
<td>23,000 (1750s)</td>
</tr>
<tr>
<td>Bavaria</td>
<td>15,000 (1750s)</td>
</tr>
<tr>
<td>Hanover</td>
<td>37,283 (1750s)</td>
</tr>
<tr>
<td>Piedmont</td>
<td>45,000 (1750s)</td>
</tr>
<tr>
<td>Sweden</td>
<td>35,000 (1756)</td>
</tr>
<tr>
<td>European total/average</td>
<td>1,252,000</td>
</tr>
<tr>
<td>China</td>
<td>800,000</td>
</tr>
</tbody>
</table>

Table 5: Size of standing armies in China and Europe in the mid-eighteenth century. Sources: Wang (1985), Gallenga (1855, 194), Sichart (1898), Duffy (1977, 212), Kennedy (1987), and Doyle (1992, 212–245); Population data from McEvedy and Jones (1978). Rasler and Thompson (1994) contains alternative army size estimates but these do not affect our comparison between China and Europe. Finally, we do not report European naval strength, but the Chinese navy was small in this period and is reflected in the figure that we report.

*during earlier periods Dutch military strength was greater. The Dutch army was 93,000 in 1690.
*bBy 1786 Prussian army was 190,000.
*cSize of Hanoverian army fluctuated from 19,000 to 49,000 during the eighteenth century (Sichart, 1898).
*dThe Swedish army was much larger in earlier periods: 100,000 in 1710 (Kennedy, 1987).

during wartime while even in peacetime feudal lords maintained large numbers of armed retainers.

5.3 Population cycles in China and Europe.

Our model also offers new insights into the consequences of the persistence of political centralization in China and fragmentation in Europe. Corollary 7 predicts that population growth should be more variable under political centralization because political centralization is associated with lower taxes during peacetime but also greater vulnerability to external shocks. We provide evidence in support of this proposition by drawing on population data from China and Europe.

McEvedy and Jones (1978) provide comparable population estimates for the past two thousand years. Figure 15a presents their population estimates for China and Europe. It shows that the population growth of China was more variable than that of Europe. Figure 15b, which shows the percentage population change, confirms this finding. It is evident that the time series of Chinese population display greater variance. Interestingly, there is no visible difference in population variation at the two ends of Eurasia when they were ruled by empires (before AD 400) and when
they were fragmented (400–600), it is only after the consolidation of political centralization in China and fragmentation in Europe that the differences in population change patterns emerged. In addition, in Appendix A.9 we fit the population estimates with polynomials up to the sixth order and find that (i) it is easier to fit the European population estimates than it is to fit the Chinese population estimates because the latter are more scattered, and (ii) even if we set aside differences in the degree of goodness of fit, Europe’s fitted trend line is smoother than the Chinese one.

We use McEvedy and Jones (1978) because they provide estimates for both China and Europe over a long period of time. However, since they report data for every 50, 100, or 200 years, the resulting time series is necessarily smoother than would be the case if data was available at a higher frequency. In fact, this potential problem biases us against finding a difference between the population fluctuations in China and Europe as there are several well-known sharp declines in Chinese population that are either absent or moderated in the McEvedy and Jones (1978) data.

Figure 16 displays a higher frequency population series from Cao (2000).\footnote{We use the population estimates provided in Cao (2000) because of its coverage and relative accuracy. The plunges in China’s population depicted in Figure 15 would appear even more severe if we had used official historical statistics. For example, official historical records suggest that China’s population fell from more than 50 million to 7 million in the third century after the collapse of the Han dynasty. A substantial amount of this population ‘loss’ was likely due to the state’s inability to keep accurate records during times of crises instead of actual deaths. By contrast, Cao (2000) puts the late third-century estimate at 23 million instead of 7 million.} This data series is consistent with historical accounts which associate external invasions and political collapses with large declines in population. The fall of the Early and Later Han, Sui dynasties, the An
Lushan Rebellion, the fall of the Northern Song dynasty, the Mongol invasions, and the end of the Yuan and Ming dynasties are all visible in the figure.

For example, the Mongol invasions are associated with a sharp population collapse. Kuhn observes that ‘[p]opulation figures took another dramatic turn downward between 1223 and 1264, and by 1292 in the whole of China the population had decreased by roughly 30 million, or one third of the population, to 75 million. This was probably due to a combination of factors—warfare in north China, the Mongol invasions, and the bubonic plague or other epidemics. Whatever the causes, this was a decline in human population on a magnitude that the world has seldom seen’ (Kuhn, 2009, 75). The fall of the Yuan Dynasty is thought to have caused the population to fall again by approximately 23 percent. In contrast, historians of Europe report only one major Europe-wide collapse in population after the fall of the Roman Empire and this is the Black Death of the mid-fourteenth century.

5.4 Implications for Economic Growth

The start-stop nature of population growth and economic development in China that is predicted in our model and witnessed in history is potentially important in helping to account for China’s failure to achieve modern economic growth before 1800.

According to unified growth theory, population growth in the Malthusian era was associated with economic growth through its effect on (a) the supply of innovative ideas, (b) the demand for new technologies, (c) the rate of technological diffusion, (d) the division of labor, and (e) the scope for trade (Galor, 2005, 239). Once the stock of technological knowledge becomes large enough, however, there is an incentive for parents to invest in human capital for their children. As a result parents switch from investing in a large number of children with no or low amounts
of human capital to investing in a small-number of higher quality children thereby generating a
demographic transition and a shift to a modern growth regime in which a sustained increase in
per capita income can take place. This scale effect is deterministic; it predicts that the transition
to sustained economic growth is inevitable and—all else equal—economic growth should be
most likely to occur in the largest economy as that is where factors such as learning-by-doing,
technological diffusion, and the supply of innovative ideas should be largest.

This prediction is consistent with much of history; until around 1300 China was the most
innovative part of the world economy (Mokyr, 1990; Needham, 1995; Lin, 1995). However,
sustained economic growth did not begin in China even though it remained the world’s largest
economy until well into the 1800s.41 Growth theory often accounts for this puzzle by appealing
to differences in political or economic institutions and how they shape the incentive to innovate.
In this paper we suggest an alternative and complementary mechanism to explain why sustained
economic growth did not first take place in China. As a unified empire China was more vulnerable
to periodic collapses and population shocks as a direct or indirect consequence of the threat
of external invasion. This undermined the gradual accumulation of technological knowledge
that plays such an important role in generating the transition from stagnation to growth in the
theoretical growth literature.

We base this argument on Aiyar et al. (2008) who propose a model in which knowledge is
embedded in new intermediate goods. Negative shocks that cause the extent of the market to
contract, including a fall in the population, can cause the production of some intermediate goods
to be abandoned and this leads to a decline in knowledge or technological regress.42 Adding
this insight to our theory suggests that the higher variance of population growth in China—a
hitherto overlooked factor in the study of the Great Divergence—could help explain its slower
technological growth relative to Europe in the long run.

41 For actual GDP estimates, see Maddison (2003) and Broadberry et al. (2012); Broadberry (2013).
42 In a nutshell, their model studies an economy governed by Malthusian dynamics with population \( N = \ddot{N}(K) \)
with a number of intermediate goods \( F = \ddot{F}(K) \). \( F \) is a measure of technological sophistication as each
intermediate good \( g(l) \) where \( l \in [0,F_t] \) is used to produce final output. There is a fixed cost associated with
producing each intermediate good and for the production of each intermediate good to be viable it has to generate
enough revenue to cover this fixed cost.

Suppose there is a negative shock to the population: \( N_0 < \ddot{N} \). In this case, the number of varieties of
intermediate goods produced in equilibrium falls due to the decline in the size of the market to \( F^*(N_o) \). This fall
in the number of varieties also results in a decline in knowledge as technological knowledge is embodied in the
production of intermediate goods. Negative shocks to the population do not only lead to a fall in the rate of
technological progress as in standard models of long-run growth; negative shocks actually result in a decline in
the stock of knowledge.
6 Conclusion

The idea that Europe’s political and economic success is related to its political fragmentation goes back to the Enlightenment. Montesquieu noted that in contrast to Asia where strong nations are able to conquer and subdue their neighbors, in ‘Europe on the contrary, strong nations are opposed to the strong; and those who join each other have nearly the same courage. This is the reason of the weakness of Asia and of the strength of Europe; of the liberty of Europe, and of the slavery of Asia’ (Montesquieu, 1989, 266).

In this paper we have proposed a unified theory of the origins, persistence, and consequences of political centralization and fragmentation in China and Europe. We build on the argument that external threats were a powerful force for political unification in China, but were less of a factor in Europe. Our theory suggests that political centralization should indeed be stable in China, but not in Europe, and that this centralization was beneficial from a static perspective as it minimized costly interstate competition. However, we also show that in the event of an external invasion a centralized empire such as China was less robust than a decentralized state system.

Our theory provides a novel institutional channel through which geography could have shaped economic outcomes in Eurasia. It complements and enriches many existing explanations that have suggested that political fragmentation played an important role in the rise of Europe. For example, Tilly (1990) analyzed the factors that shaped Europe’s high level of fragmentation at the end of the middle ages and the forces that gave rise to the emergence of modern nation states by the nineteenth century. He argued that the ‘broad urban column that reached roughly southwest from the Italian peninsula to southern England dominated the map of fragmented sovereignty’ as the existence of capital-intensive city states along this corridor prevented the emergence of permanent large empires in continent Europe (Tilly, 1990, 133).

Tilly’s theory is important and influential but it does not explain the existence of independent city states in Europe in 1500—which he took as given. Indeed other scholars have suggested that independent city-states were able to flourish in this part of Europe precisely because of the breakup of the Carolingian empire and the failure of subsequent Holy Roman Emperors to reunify Charlemagne’s kingdom (Stasavage, 2011). Our theory complements Tilly’s hypothesis as we seek to explain why no long-lasting empire was sustainable after the Fall of the Roman

---

43 Similar arguments are also developed by Spruyt (1994) and Finer (1999).
44 John Hall expresses this argument well, noting that ‘the North Italian cities were themselves the creation of absence of a single centre of power in Europe’ and observing that ‘[h]ow much they owed to their freedom from interference and freedom to experiment is simply seen: once they became part of the Spanish mini-empire they contributed nothing new to European civilization’ (Hall, 1985, 136).
Empire and hence why it was possible for independent city states to survive in a fragmented Europe.

Other scholars have argued that decentralization gave Europe an edge in the Great Divergence because it led to greater innovation (Mokyr, 1990; Diamond, 1997; Lagerlof, 2014); support for merchants (Rosenberg and Birdzell, 1986) or political freedoms and representation (Hall, 1985). Recent work has also shown how the consequences of political fragmentation interacted with the Black Death to raise incomes and urbanization in Europe (Voigtländer and Voth, 2013b). Likewise, our theory complements these existing arguments, but we emphasize the significance of one previously neglected consequence of political centralization in China. There were periods of economic expansion, innovation, and population growth in China, but these were brought to a halt by external invasions and political crises. It was these population crises, we conjecture, that help to explain why China did not enter a period of sustained economic growth in the preindustrial era. In contrast, Europe’s polycentric system of states gave it the institutional robustness that was one of the preconditions for modern economic growth to occur.

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A Online Appendix

A.1 Location of Nomadic Invasions

<table>
<thead>
<tr>
<th>Phase</th>
<th>Century</th>
<th>Nomadic Peoples</th>
<th>W. Europe</th>
<th>Russia</th>
<th>China</th>
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<tr>
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<td>8th–2nd BC</td>
<td>Scythians</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>2nd BC</td>
<td>Xiongnu (Hsiung-nu)</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>2nd BC</td>
<td>Yuezhi (Yüeh-chih)</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3rd</td>
<td>4th</td>
<td>To-pa (Toba Turks)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>5th</td>
<td>Huns</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4th</td>
<td>6th</td>
<td>Tu-chueh (Göktürks)</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>6th</td>
<td>Ruren (Juanjuan)</td>
<td></td>
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<td>✓</td>
</tr>
<tr>
<td></td>
<td>7th</td>
<td>Avars</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>8th</td>
<td>Bulgars</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>7th</td>
<td>Khazar Turks</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td></td>
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<td>Magyars</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>9th</td>
<td>Uygurs</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>10th</td>
<td>Khitans</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>11th</td>
<td>Pechenegs and Kipchaks</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<td></td>
<td>12th</td>
<td>Jurchens</td>
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<td>✓</td>
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<td>Manchus</td>
<td>✓</td>
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</tr>
</tbody>
</table>

Table 6: Major waves of nomadic invasions. Source: Chaliand (2005). It is evident that China faced a greater threat from the steppe invaders that did Europe.
### A.2 The Multidirectional Threat in Europe

<table>
<thead>
<tr>
<th>Invader</th>
<th>Date of Invasion</th>
<th>Location of Invasion</th>
<th>Direction of Threat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huns</td>
<td>c. 370–450</td>
<td>Italy, France, Balkens</td>
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</tr>
<tr>
<td>Avars</td>
<td>580</td>
<td>South-Eastern Europe</td>
<td>East</td>
</tr>
<tr>
<td>Bulgars</td>
<td>c. 850</td>
<td>South-Eastern Europe</td>
<td>East</td>
</tr>
<tr>
<td>Arabs</td>
<td>711</td>
<td>Spain</td>
<td>South</td>
</tr>
<tr>
<td>Arabs</td>
<td>721</td>
<td>France</td>
<td>South</td>
</tr>
<tr>
<td>Arabs</td>
<td>732</td>
<td>France</td>
<td>South</td>
</tr>
<tr>
<td>Vikings*</td>
<td>793–1066</td>
<td>Britain</td>
<td>North</td>
</tr>
<tr>
<td>Vikings*</td>
<td>c. 810–1000</td>
<td>France</td>
<td>North</td>
</tr>
<tr>
<td>Vikings*</td>
<td>c. 810–1000</td>
<td>Low Countries</td>
<td>North</td>
</tr>
<tr>
<td>Arabs</td>
<td>831</td>
<td>Sicily</td>
<td>South</td>
</tr>
<tr>
<td>Arabs</td>
<td>840</td>
<td>Crete</td>
<td>South</td>
</tr>
<tr>
<td>Arabs</td>
<td>846</td>
<td>Italy</td>
<td>South</td>
</tr>
<tr>
<td>Magyars</td>
<td>907</td>
<td>Germany</td>
<td>East</td>
</tr>
<tr>
<td>Magyars</td>
<td>917</td>
<td>France</td>
<td>East</td>
</tr>
<tr>
<td>The Almohads</td>
<td>1172</td>
<td>Spain</td>
<td>South</td>
</tr>
<tr>
<td>Mongols</td>
<td>1240</td>
<td>Poland</td>
<td>East</td>
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<td>Mongols</td>
<td>1241</td>
<td>Hungary</td>
<td>East</td>
</tr>
<tr>
<td>Mongols</td>
<td>1241</td>
<td>Croatia</td>
<td>East</td>
</tr>
<tr>
<td>The Marinids</td>
<td>1340</td>
<td>Gibraltar</td>
<td>South</td>
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<tr>
<td>Ottomans</td>
<td>1371</td>
<td>Serbia</td>
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</tr>
<tr>
<td>Ottomans</td>
<td>1385</td>
<td>Albania</td>
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<td>1463</td>
<td>Bosnia</td>
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<td>Ottomans</td>
<td>1479</td>
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<td>South East</td>
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<td>1480</td>
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<td>Crimean Tatars</td>
<td>1480–1507</td>
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<td>1573</td>
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<td>1577, 1584, 1590</td>
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<td>East</td>
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<td>1669</td>
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</tr>
<tr>
<td>Ottomans</td>
<td>1683</td>
<td>Austria</td>
<td>South-East</td>
</tr>
</tbody>
</table>

#### Table 7: The Multi-directional Threat

A list of invasions of Europe from the Fall of the Roman Empire onwards. We list invasions attempts that failed as well as those that succeeded. *We count the Vikings as external invaders—due to their different religion and distinct culture they were seen as outsider invaders by contemporaries. But from the perspective of our model we consider later Swedish or Danish campaigns in Europe as instances of interstate competition.*
A.3 Definition

The gross damage \( t \) distance away from the frontier is \( \max \{\lambda(\Lambda, t), 0\} \), and the military strength \( t \) distance away from the capital city of regime \( i \) is \( \max\{m(M_i, t), 0\} \).

Under interstate competition, a location \( x \in [0, G_l] \) is controlled by regime \( l \) if regime \( l \) has a stronger military strength at its capital than regime \( r \), i.e., \( m(M_l, 0) \geq m(M_r, 1 - G_l - G_r) \). A location \( x \in [G_l, 1 - G_r] \) is controlled by regime \( l \) if regime \( l \) has a stronger military strength than regime \( r \) at \( x \), i.e., \( m(M_l, x - G_l) \geq m(M_r, (1 - x) - G_j) \). Locations controlled by regime \( r \) are defined in the same way. We restrict our attention to \( G_l + G_r \leq 1 \) so that the capital city of each regime always falls within the region that it controls.

The border \( b \) between two regimes is defined as the point between their capital cities where the regimes are equal in military strength. Formally, \( b \in [G_l, 1 - G_r] \) solves the following equation:

\[
m(M_l, b - G_l) = m(M_r, (1 - b) - G_r).
\]

If such a location does not exist, it implies that one of the two regimes does not control any subset of the line. In this case, we assume that \( b = 0 \) if \( m(M_l, 0) < m(M_r, 1 - G_r - G_l) \) and \( b = 1 \) if \( m(M_l, 1 - G_r - G_l) > m(M_r, 0) \). Summing up, let \( \hat{b} \) be defined by \( m(M_l, \hat{b} - G_l) = m(M_r, (1 - \hat{b}) - G_r) \), we have:

\[
b = \begin{cases} 
0 & \text{if } m(M_l, 0) < m(M_r, 1 - G_r - G_l), \\
1 & \text{if } m(M_l, 1 - G_r - G_l) > m(M_r, 0), \\
\hat{b} & \text{otherwise}.
\end{cases}
\]

Let \( \kappa_i(x) = \lambda(\Lambda, t) - m(M_i, t) \) denote the net damage of the external threat at \( x \) (controlled by regime \( i \)). A location \( x \in [0, 1] \) is protected by regime \( i \) from the external threat originated from \( x = 0 \) if there exists \( 0 \leq x' \leq x \) such that \( \kappa_i(x') \leq 0 \). If the external threat originates from \( x = 1 \), similar definition applies. Let \( \mathbb{D}_i = \{x \in [0, 1] : \text{x is controlled by i and } \kappa_i(x) \leq 0 \} \) denote the interval or the set of locations that is protected from the external threat by regime \( i \). Because of the revolution constraint, we assume that regime \( i \)'s net revenue is \( -\infty \) unless at least \( \delta \) fraction of the interval controlled by regime \( i \) is protected.

If the continent is ruled by one single regime, the net revenue of regime \( e \) is:

\[
V_e = \begin{cases} 
y - c(M_e) & \text{if } |\mathbb{D}_e| \geq \delta, \\
-\infty & \text{otherwise},
\end{cases}
\]

Under interstate competition, the net revenues of regimes \( l \) and \( r \) are given by:
\[
V_I = \begin{cases} 
by - c(M_I) & \text{if } b > 0, |\mathbb{D}_I| \geq b\delta \text{ and } G_I + G_r \leq 1, \\
-\infty & \text{otherwise,}
\end{cases}
\]

and

\[
V_r = \begin{cases} 
(1 - b)y - c(M_r) & \text{if } b < 1, |\mathbb{D}_r| \geq (1 - b)\delta \text{ and } G_I + G_r \leq 1, \\
-\infty & \text{otherwise,}
\end{cases}
\]

respectively.

A.4 Proposition 1

We restate Proposition 1 with technical details.

**Proposition 1.** When the threat is two-sided:

1. If \( \Lambda \leq \Lambda_I \), \( M_e^* = 0 \) and \( G_e^* \in [0, 1] \);

2. There exists \( \Lambda_{II} > \Lambda_I \) such that for all \( \Lambda > \Lambda_{II} \), \( G_e^* = \frac{1}{2} \) and \( M_e^* > 0 \);

3. If \( \Lambda_I < \Lambda \leq \Lambda_{II} \), \( G_e^* = 1 - x^*(\Lambda) - \delta \) and \( M_e^* > 0 \);

When the threat is one-sided:

4. If \( \Lambda \leq \bar{\Lambda}_I \), \( M_e^* = 0 \) and \( G_e^* \in [0, 1] \);

5. If \( \Lambda > \bar{\Lambda}_I \), \( G_e^* = 1 - \delta \) and \( M_e^* > 0 \).

**Proof.** First, consider a two-sided threat.

(1) If \( \Lambda \leq \Lambda_I \), the optimal military investment is zero as long as the capital is located between 0 and 1. Since regime \( e \)'s payoff is decreasing in its military investment, it is optimal to invest zero.

(2-3) Given that the two-sided threat is symmetric, there is no loss of generality if we assume \( G_e \leq 1/2 \). Since regime \( e \) always receives gross tax revenue \( y \), its optimization problem can be rewritten as:

\[
\min_{G_e, M_e} c(M_e)
\]
subject to
\[
\begin{align*}
\lambda (\Lambda, a) &\leq m (M_e, G_e - a), \\
\lambda (\Lambda, 1 - z) &\leq m (M_e, z - G_e), \\
z - a &\geq \delta, \text{ and} \\
a &\leq G_e \leq z
\end{align*}
\]

where \(a\) and \(z\) are, respectively, the leftmost and rightmost locations in the empire that suffer zero net damage when the military investment is \(M_e\).

If \(M_e = 0\) satisfies every inequality, then the optimal solution is a corner one and regime \(e\) invests zero in the military. This is the case when \(\Lambda \leq \Lambda_I\) (as discussed above).

Now consider the case when the solution is interior, i.e., \(M_e > 0\). Note that the first inequality must bind in equilibrium. Otherwise, regime \(e\) can increase its net tax revenue by reducing \(M_e\) and increasing \(G_e\). Hence, we have:

\[
\lambda (\Lambda, a) = m (M_e, G_e - a)
\]

Next consider the two cases when (i) the second inequality is binding, and (ii) when it does not bind.

Case (i): if \(\lambda (\Lambda, 1 - z) = m (M_e, z - G_e)\), it is because \(\Lambda\) exceeds a certain threshold value (since \(\partial \lambda / \partial \Lambda > 0\)). Let \(\Lambda_{II}\) denote this threshold. Note that in this case, the third inequality automatically binds, i.e., \(z = a + \delta\). Hence, we have:

\[
\lambda (\Lambda, a) = m (M_e, G_e - a), \text{ and} \\
\lambda (\Lambda, 1 - \delta - a) = m (M_e, \delta + a - G_e).
\]

Applying total differentiation:
\[
\begin{bmatrix}
\lambda_2 (\Lambda, a) + m_2 (M_e, G_e - a) & -m_1 (M_e, G_e - a) \\
-\lambda_2 (\Lambda, 1 - \delta - a) - m_2 (M_e, \delta + a - G_e) & -m_1 (M_e, \delta + a - G_e)
\end{bmatrix}
\begin{bmatrix}
da \\
dM_e
\end{bmatrix}
= \begin{bmatrix}
-\lambda_1 (\Lambda, a) \\
-\lambda_1 (\Lambda, 1 - \delta - a)
\end{bmatrix} d\Lambda + \begin{bmatrix}
m_2 (M_e, G_e - a) \\
-m_2 (M_e, \delta + a - G_e)
\end{bmatrix} dG_e
\]
It is easy to show that

\[
\Delta = \left| \begin{array}{cc}
\lambda_2 (\Lambda, a) + m_2 (M_e, G_e - a) & -m_1 (M_e, G_e - a) \\
-\lambda_2 (\Lambda, 1 - \delta - a) - m_2 (M_e, \delta + a - G_e) & -m_1 (M_e, \delta + a - G_e)
\end{array} \right| > 0 \text{ because } m_1 > 0, \lambda_2 < 0 \text{ and } m_2 < 0.
\]

Hence

\[
\frac{dM_e}{dG_e} = \frac{\left| \begin{array}{cc}
\lambda_2 (\Lambda, a) + m_2 (M_e, G_e - a) & -m_2 (M_e, G_e - a) \\
-\lambda_2 (\Lambda, 1 - \delta - a) - m_2 (M_e, \delta + a - G_e) & -m_2 (M_e, \delta + a - G_e)
\end{array} \right|}{\Delta} < 0 \text{ because } \lambda_2 < 0 \text{ and } m_2 < 0.
\]

Since \(\partial c/\partial M_e > 0\) and therefore \(\partial V_e/\partial M_e < 0\), we have

\[
\frac{dV_e}{dG_e} = \frac{\partial V_e}{\partial M_e} \frac{dM_e}{dG_e} > 0.
\]

Hence, to maximize its net tax revenue, regime \(e\) should locate its capital as close to 1/2 as possible. This implies that \(G_e^* = 1/2\), and \(a = \frac{1}{2} (1 - \delta)\). The optimal military spending \(M_e^*\) satisfies:

\[
\lambda \left( \Lambda, \frac{1}{2} (1 - \delta) \right) = m (M_e^*, \delta/2).
\]

Case (ii): if \(\lambda (\Lambda, 1 - z) < m (M_e, z - G_e)\), then \(x^*(\Lambda) < 1/2\), which in turn implies that:

\[
z = 1 - x^*(\Lambda).
\]

Otherwise, regime \(e\) can increase its net tax revenue by reducing \(M_e\) and \(G_e\) simultaneously. By the same reasoning, the third inequality must bind in equilibrium:

\[
G_e^* = a = z - \delta.
\]

Therefore, the optimal military spending \(M_e^*\) must satisfy:

\[
\lambda (\Lambda, 1 - x^*(\Lambda) - \delta) = m (M_e^*, 0).
\]

Cases (i) and (ii) denote point 2 and point 3 of Proposition 1 respectively. The threshold \(\Lambda_{II}\)
is the solution to the following system of equations:

\[ \lambda \left( \Lambda_{II}, \frac{1}{2} (1 - \delta) \right) = m(M_e, \delta/2), \quad \text{and} \]

\[ \lambda \left( \Lambda_{II}, 1 - x^*(\Lambda_{II}) - \delta \right) = m(M_e, 0). \]

Finally, we consider a one-sided threat.

(4) If \( \Lambda \leq \bar{\Lambda}_I \), then \( x^*(\Lambda) \leq 1 - \delta \) so that the fraction of protected area is no less than \( \delta \) even if there is no military investment. Since regime \( e \)'s payoff is decreasing in its military investment, the optimal military investment is zero and the capital city is located between 0 and 1.

(5) If \( x^*(\Lambda) > 1 - \delta \), the regime has to make a strictly positive military investment. Since military strength decreases over distance (\( \partial m/\partial t < 0 \)), regime \( e \) should locate its capital city at the point where the revolution constraint just binds. This implies that \( G^*_e = 1 - \delta \) and \( M^*_e \) solves \( \lambda(\Lambda, 1 - \delta) = m(M^*_e, 0) \).

A.5 Proposition 2

Before proving Proposition 2, it is useful to characterize the outcome of interstate competition in the absence of an external threat (\( \Lambda = 0 \)).

Lemma 1. When there is no external threat, given the locations of capital cities \( G_l \) and \( G_r \), the equilibrium military investments \( M^*_l \) and \( M^*_r \) satisfy:

\[
\frac{-m_1(M^*_l, b - G_l)}{m_2(M^*_l, b - G_l) + m_2(M^*_r, 1 - b - G_r)} y - \frac{\partial c(M^*_l)}{\partial M^*_l} = 0, \\
\frac{-m_1(M^*_r, 1 - b - G_r)}{m_2(M^*_l, b - G_l) + m_2(M^*_r, 1 - b - G_r)} y - \frac{\partial c(M^*_r)}{\partial M^*_r} = 0,
\]

and the equilibrium locations of capital cities \( G^*_l \) and \( G^*_r \) satisfy:

\[
\left( \frac{\partial b}{\partial G_l} + \frac{\partial b}{\partial M^*_l} \frac{\partial M^*_l}{\partial G_l} + \frac{\partial b}{\partial M^*_r} \frac{\partial M^*_r}{\partial G_l} \right) y - \frac{\partial c}{\partial M^*_l} \frac{\partial M^*_l}{\partial G_l} = 0, \\
- \left( \frac{\partial b}{\partial G_r} + \frac{\partial b}{\partial M^*_l} \frac{\partial M^*_l}{\partial G_r} + \frac{\partial b}{\partial M^*_r} \frac{\partial M^*_r}{\partial G_r} \right) y - \frac{\partial c}{\partial M^*_r} \frac{\partial M^*_r}{\partial G_r} = 0.
\]

Proof. Since the border \( b \) is determined by:

\[ m(M_l, b - G_l) = m(M_r, 1 - b - G_r), \]
By total differentiation, we have

\[ (m_2 (M_l, b - G_l) + m_2 (M_r, 1 - b - G_r)) \, db \]
\[ = -m_1 (M_l, b - G_l) \, dM_l + m_1 (M_r, 1 - b - G_r) \, dM_r \]
\[ + m_2 (M_l, b - G_l) \, dG_l - m_2 (M_r, 1 - b - G_r) \, dG_r, \]

so that

\[ \frac{\partial b}{\partial M_l} = \frac{-m_1 (M_l, b - G_l)}{m_2 (M_l, b - G_l) + m_2 (M_r, 1 - b - G_r)} > 0, \]

and

\[ \frac{\partial b}{\partial M_r} = \frac{m_1 (M_r, 1 - b - G_r)}{m_2 (M_l, b - G_l) + m_2 (M_r, 1 - b - G_r)} < 0, \]

since \( m_1 > 0 \) and \( m_2 < 0 \). In addition,

\[ \frac{\partial^2 b}{\partial M_l^2} = -\frac{(m_2 (M_l, b - G_l) + m_2 (M_r, G_r - b)) \, m_{11} (M_l, b - G_l) + m_1 (M_l, b - G_l) \, m_{21} (M_l, b - G_l)}{(m_2 (M_l, b - G_l) + m_2 (M_r, G_r - b))^2}, \]

which implies that the sufficient conditions for \( \frac{\partial^2 b}{\partial M_l^2} \leq 0 \) are \( m_{11} \leq 0 \) and \( m_{21} \leq 0 \). The same conditions would ensure that \( \frac{\partial^2 b}{\partial M_r^2} \geq 0 \).

Now, consider the second stage of interstate competition. Given \( G_l \) and \( G_r \), the optimization problems for regimes \( l \) and \( r \) are

\[ \max_{M_l} V_l = b (G_l, G_r, M_l, M_r) \, y - c (M_l), \text{ and} \]
\[ \max_{M_r} V_r = (1 - b (G_l, G_r, M_l, M_r)) \, y - c (M_r). \]

The respective FOCs are

\[ \frac{\partial b}{\partial M_l} \, y - \frac{\partial c}{\partial M_l} = 0, \text{ and} \]
\[ -\frac{\partial b}{\partial M_r} \, y - \frac{\partial c}{\partial M_r} = 0, \]
which implies that
\[
\frac{-m_1 (M_l, b - G_l)}{m_2 (M_l, b - G_l) + m_2 (M_r, 1 - b - G_r)}y - \frac{\partial c}{\partial M_l} = 0, \text{ and}
\]
\[
\frac{-m_1 (M_r, 1 - b - G_r)}{m_2 (M_l, b - G_l) + m_2 (M_r, 1 - b - G_r)}y - \frac{\partial c}{\partial M_r} = 0.
\]

Under our setup, it is never an equilibrium for regimes under interstate competition to invest zero in the military, i.e., it must be the case that \(M_l^* > 0\) and \(M_r^* > 0\). The SOCs are guaranteed if \(\partial^2 b/\partial M_l^2 \leq 0\) and \(\partial^2 b/\partial M_r^2 \geq 0\), since \(\partial^2 c/\partial^2 M_l < 0\) and \(\partial^2 c/\partial^2 M_r < 0\).

Given some second-stage equilibrium military investments \(M_l^* (G_l, G_r)\) and \(M_r^* (G_l, G_r)\), consider the first stage of interstate competition where regimes \(l\) and \(r\) decide their capital city locations:
\[
\max_{G_l} V_l = b (G_l, G_r, M_l^* (G_l, G_r), M_r^* (G_l, G_r)) y - c (M_l^* (G_l, G_r)), \text{ and}
\]
\[
\max_{G_r} V_r = (1 - b (G_l, G_r, M_l^* (G_l, G_r), M_r^* (G_l, G_r))) y - c (M_r^* (G_l, G_r)).
\]

The respective FOCs are
\[
\left( \frac{\partial b}{\partial G_l} + \frac{\partial b}{\partial M_l^*} \frac{\partial M_l^*}{\partial G_l} + \frac{\partial b}{\partial M_r^*} \frac{\partial M_r^*}{\partial G_l} \right) y - \frac{\partial c}{\partial M_l^*} \frac{\partial M_l^*}{\partial G_l} = 0, \text{ and}
\]
\[
- \left( \frac{\partial b}{\partial G_r} + \frac{\partial b}{\partial M_l^*} \frac{\partial M_l^*}{\partial G_r} + \frac{\partial b}{\partial M_r^*} \frac{\partial M_r^*}{\partial G_r} \right) y - \frac{\partial c}{\partial M_r^*} \frac{\partial M_r^*}{\partial G_r} = 0.
\]

With the above results, we can restate Proposition 2.

**Proposition 2 (Political Fragmentation).** Let \(\hat{\delta}\) denote the fraction of the continent that is protected from the external threat in equilibrium (i.e. \(\hat{\delta} = |D_l| + |D_r|\)). When the threat is two-sided:

1. There exists a threshold threat level \(\Lambda_{III}\) such that if \(\Lambda \leq \Lambda_{III}\), the revolution constraints do not bind and \(\hat{\delta} \geq \delta\). The equilibrium military investments and location of capitals are the same as in the case when \(\Lambda = 0\).

2. Otherwise, the revolution constraints bind and \(\hat{\delta} = \delta\).

**Proof.** Consider the symmetric equilibrium: \(M_l^* = M_r^* = M^*, \ G_l^* = G_r^* = G^*\). Let \(\Lambda_{III}\) denote
the value of $\Lambda$ that solves:

$$\lambda \left( \Lambda_{III}, \frac{1}{2} (1 - \delta) \right) = m \left( M^*, G^* - \frac{1}{2} (1 - \delta) \right).$$

It is clear that if $\Lambda \leq \Lambda_{III}$, the revolution constraints of the two regimes do not bind, and vice versa. \hfill \Box

A.6 Proposition 3

**Proposition 3 (Viability).** Under a one-sided threat, the net tax revenue of regime $e$ is always larger than the sum of net tax revenues of regimes $l$ and $r$. If the threat is sufficiently large, regime $e$ is viable but regimes $l$ and $r$ are not. Under a moderate and two-sided threat, the net tax revenue of regime $e$ is decreasing in $\theta$ but the sum of net tax revenues for regimes $l$ and $r$ are increasing in $\theta$.

**Proof.** First, consider the case of a one-sided threat. Suppose that, contrary to Proposition 3, $V^*_e < V^*_l + V^*_r$,

then regime $e$ can mimic the choices of regime $l$, set $G_e = G^*_l$ and $M_e = M^*_l$, and obtain a payoff that is weakly greater than the sum of the net tax revenues of regimes $l$ and $r$, which is a contradiction. Hence, it must be the case that $V^*_e \geq V^*_l + V^*_r$.

In fact, the inequality has to be strict since regime $r$ makes a non-zero military investment. Note that both $V^*_e$ and $V^*_l + V^*_r$ are decreasing in $\Lambda$. As $\Lambda$ rises, for sufficiently large $\Lambda$, $V^*_l + V^*_r$ will turn negative while $V^*_e$ is still positive.

Next, consider the case of a moderate, two-sided threat ($\Lambda_I < \Lambda \leq \Lambda_{III}$).

For a centralized regime, these are two subcases to consider: if $\Lambda_I < \Lambda \leq \Lambda_{II}$,

$$\lambda (\Lambda, 1 - x^* (\Lambda) - \delta) = m (M^*_e, 0);$$

and if $\Lambda > \Lambda_{II}$,

$$\lambda \left( \Lambda, \frac{1}{2} (1 - \delta) \right) = m (M^*_e, \delta/2).$$

Given the two equations above, we know that as $\theta$ increases, $M^*_e$ does not change (since $\lambda$ and $m$ are independent of $\theta$). This implies that $c (M^*_e, \theta)$ increases with $\theta$, and $V$ decreases as a result.
For interstate competition, we focus on the symmetric equilibrium \((b = 1/2)\) and on regime \(l\) (since the case of regime \(r\) is identical by symmetry). Recall that in the second stage, the FOC is given by:
\[
\frac{\partial V_l}{\partial M_l} = \frac{-m_1(M_l, b - G_l)}{2m_2(M_l, b - G_l)y} - \frac{\partial c}{\partial M_l} = 0.
\]
Differentiating the condition with respect to \(\theta\) gives us:
\[
\frac{dM_l}{d\theta} = \frac{c_{12}}{\partial^2 V_l / \partial M^2_l} < 0,
\]
since \(\partial^2 V_l / \partial M^2_l < 0\) and \(c_{12} > 0\). Since \(b\) is a function of \(G_l, G_r, M_l,\) and \(M_r,\)
\[
\begin{align*}
\frac{dV_l}{d\theta} &= \frac{\partial b}{\partial G_l} \frac{\partial G_l}{\partial \theta} y + \frac{\partial b}{\partial G_r} \frac{\partial G_r}{\partial \theta} y + \frac{\partial b}{\partial M_l} \left( \frac{\partial M_l \partial G_l}{\partial \theta} + \frac{\partial M_l \partial G_r}{\partial \theta} \right) y \\
&\quad + \frac{\partial b}{\partial M_r} \left( \frac{\partial M_r \partial G_l}{\partial \theta} + \frac{\partial M_r \partial G_r}{\partial \theta} \right) y - \frac{\partial c}{\partial M_l} \left( \frac{\partial M_l \partial G_l}{\partial \theta} + \frac{\partial M_l \partial G_r}{\partial \theta} + \frac{\partial M_l}{\partial \theta} \right) \\
&= \left( \left( \frac{\partial b}{\partial G_l} + \frac{\partial b}{\partial M_l \partial G_l} + \frac{\partial b}{\partial M_r \partial G_l} \right) y - \frac{\partial c}{\partial M_l} \frac{\partial M_l}{\partial G_l} \right) \frac{\partial G_l}{\partial \beta} \\
&\quad + \left( \left( \frac{\partial b}{\partial G_r} + \frac{\partial b}{\partial M_l \partial G_r} + \frac{\partial b}{\partial M_r \partial G_r} \right) y - \frac{\partial c}{\partial M_l} \frac{\partial M_l}{\partial G_r} \right) \frac{\partial G_r}{\partial \beta} \\
&- \frac{\partial c}{\partial M_l} \frac{\partial M_l}{\partial \beta}. 
\end{align*}
\]
Since \(\frac{\partial M_r}{\partial G_r} = -\frac{\partial M_r}{\partial G_r}\), the first two terms in last expression are zero (From Lemma 1). This implies that:
\[
\frac{dV_l}{d\beta} = -\frac{\partial c}{\partial M_l} \frac{\partial M_l}{\partial \beta} \geq 0.
\]

\[\square\]

**A.7 Tax Reimbursement**

Claim. If \(\theta\) is sufficiently high, the empire provides a strictly positive tax reimbursement \((R_e > 0)\).

**Proof.** Consider the case of an empire facing a one-sided threat (for our purpose, it suffices to show that the claim is true for a one-sided threat. The proof for a two-sided threat is similar).
The optimization problem is given by:

\[
\begin{align*}
\max_{G_e,M_e,R_e} & \quad y - c(M_e) - R_e \\
\text{s.t.} & \quad R_e \geq 0; \\
& \quad m(M_e, G_e - (1 - \delta)) + R_e = \lambda(\Lambda, 1 - \delta); \\
& \quad m(M_e, |x - G_e|) \geq \lambda(\Lambda, x) \text{ for some } x \in [0, x^*(\Lambda)].
\end{align*}
\]

Let \( x^* = \arg \max_x m(M_e, |x - G_e|) - \lambda(\Lambda, x) \). Since \( \lambda_2 < 0 \), \( x^* \geq G_e \) (otherwise, the empire can increase its net tax revenue by increasing \( G_e \) and decreasing \( M_e \) or \( R_e \)).

If \( R_e^* = 0 \), then \( G_e = 1 - \delta \) since \( m_2 < 0 \). If \( G_e^* > 1 - \delta \), it must be the case that \( R_e^* > 0 \) (otherwise, if \( R_e^* = 0 \), the empire can increase its net tax revenue by decreasing \( G_e \) and \( M_e \) simultaneously). Therefore, it suffices to compare \( R_e^* = 0 \) and \( R_e^* > 0 \) when \( G_e = 1 - \delta \).

When \( G_e = 1 - \delta \), the Lagrangian optimization problem is:

\[
\max_{M_e,R_e} y - c(M_e) - R_e + \phi R_e + \gamma (m(M_e, 0) + R_e - \lambda(\Lambda, 1 - \delta))
\]

where \( \phi \) and \( \gamma \) are the Lagrangian multipliers. The first order conditions are given by:

\[
\begin{align*}
M_e : & \quad -c_M + \gamma m_1(M_e, 0) = 0, \\
R_e : & \quad -1 + \phi + \gamma = 0, \\
\phi : & \quad \phi R_e \geq 0 \text{ and either } \phi = 0 \text{ or } R_e = 0, \\
\gamma : & \quad \lambda(\Lambda, 1 - \delta) = m(M_e, 0) + R_e.
\end{align*}
\]

If \( R_e^* = 0 \),

\[
\begin{align*}
c_M(M_e^*) &= (1 - \phi^*) m_1(M_e^*, 0), \text{ and} \\
\lambda(\Lambda, 1 - \delta) &= m(M_e^*, 0).
\end{align*}
\]

Since \( M_e^* \) depends on \( \Lambda \) and \( \delta \) but is independent of \( \theta \), \( dM_e^*/d\theta = 0 \).

If \( R_e^{**} > 0 \), \( \phi^{**} = 0 \), and

\[
\begin{align*}
c_M(M_e^{**}) &= m_1(M_e^{**}, 0), \text{ and} \\
\lambda(\Lambda, 1 - \delta) &= m(M_e^{**}, 0) + R_e^{**}.
\end{align*}
\]

The above conditions imply that \( \frac{dM_e^{**}}{d\theta} = -\frac{c_{12}}{(c_{11} - m_{11})} < 0 \) since \( c_{12} > 0 \), \( c_{11} > 0 \), and \( m_{11} \leq 0 \).
Furthermore, note that:

\[ m(M_e^*, 0) = m(M_e^{**}, 0) + R_e^* > m(M_e^{**}, 0). \]

Since \( m_1 > 0 \), \( M_e^{**} < M_e^* \). Now, let

\[ \Psi = c(M_e^*) - c(M_e^{**}) - R_e^* = c(M_e^*) - c(M_e^{**}) - \lambda (\Lambda, 1 - \delta) + m(M_e^{**}, 0). \]

The empire should set \( R_e > 0 \) only if \( \Psi > 0 \). Now,

\[
\frac{d\Psi}{d\theta} = \frac{\partial c(M_e^*)}{\partial \theta} - \frac{\partial c(M_e^{**})}{\partial M_e^{**}} \frac{dM_e^{**}}{d\theta} - \frac{\partial c(M_e^*)}{\partial \theta} + \frac{\partial m(M_e^{**}, 0)}{\partial M_e^{**}} \frac{dM_e^{**}}{d\theta}
\]

\[ = \frac{\partial (c(M_e^*) - c(M_e^{**}))}{\partial \theta} - \frac{\partial (c(M_e^{**}) - m(M_e^{**}, 0))}{\partial M_e^{**}} \frac{dM_e^{**}}{d\theta}
\]

\[ = \frac{\partial (c(M_e^*) - c(M_e^{**}))}{\partial \theta} - \frac{\partial (c(M_e^{**}) - \lambda (\Lambda, 1 - \delta) + R_e^*)}{\partial M_e^{**}} \frac{dM_e^{**}}{d\theta}
\]

\[ = \frac{\partial (c(M_e^*) - c(M_e^{**}))}{\partial \theta} - \frac{\partial (c(M_e^{**}) + R_e^*)}{\partial M_e^{**}} \frac{dM_e^{**}}{d\theta}. \]

If \( \theta \) is sufficiently large, the first term above is positive because \( c_{M \theta} > 0 \) and \( M_e^* > M_e^{**} \), the second term is positive because \( \frac{\partial(c(M_e^{**})+R_e^*)}{\partial M_e^{**}} > 0 \) and \( \frac{dM_e^{**}}{d\theta} < 0 \), and \( \frac{d\Psi}{d\theta} > 0 \) as desired.

\[ A.8 \quad \text{A Numerical Example} \]

We illustrate Corollary 7 using a simple numerical example.

Suppose \( \lambda(\Lambda, t), m(M_i, t), c(M_i, \theta), \) and \( u(c, n) \) have the following functional forms: \( \lambda(\Lambda, t) = \Lambda - \alpha t, m(M_i, t) = M_i - \beta t^2, c(M_i, \theta) = \theta M_i^2, \) and \( u(c, n) = c^{1-\gamma} n^\gamma \). Let: \( \Lambda = 20, \alpha = 35, \beta = 100, \delta = 0.45, y = 1500, \) and \( \theta = 1 \). To show \( N_F > N_E, \) it suffices to demonstrate \( \text{Area} \langle E \rangle + R_e < \text{Area} \langle F \rangle \).

For continent \( E \), the capital city is located at \( 1 - \delta = 0.55 \), military investment is \( M = 1/(2\theta) = 0.5 \), and the tax rebate is \( R_e = -\frac{1}{2\theta} + (\Lambda - \alpha (1 - \delta)) = 0.25 \). Subsequently, \( \text{Area} \langle E \rangle + R_e = -5.4327 \).

For continent \( F \), the location of two capital cities are given by \( G_l = G_r = \frac{1}{2} - \frac{1}{2} \left( \frac{y}{49/3} \right)^{1/3} = 0.33264 \). Each regimes invests \( M = \frac{y^{2/3}}{47/3 + y^{2/3}} = 11.204 \). Subsequently, \( \text{Area} \langle F \rangle = 4.6374 \).

Since \( \text{Area} \langle E \rangle + R_e - \text{Area} \langle F \rangle = -5.4327 + 4.6374 = -0.7953 < 0 \), in this case the population falls more sharply under political centralization than under political fragmentation.
when the shock is realized.

A.9 Population Fluctuations

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t^2$</th>
<th>$t^3$</th>
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Constant terms are not reported. * significant at 10%, ** significant at 5%, *** significant at 1%.

**Table 8:** Fitting Year Polynomials to Chinese and European Population Data. Adjusted $R^2$ is higher for Europe than for China in each case.