Emergent Superstar Cities

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Abstracts

An important insight in Gyourko, Mayer and Sinai (*American Economic Journal: Economic Policy* 5(4), 2013), that rising aggregate demand, rather than diverging local productivity, accounts for the widening house price dispersion across US cities after WWII, rests on the assumption of idiosyncratic location preferences and asymmetric housing supply elasticity across cities. Under such assumptions, cities with inelastic housing supply are "superstar" cities—they get more expensive, hence more exclusive to high-income households, as aggregate demand increases. We sharpen and extend this insight by presenting a model where "superstar" cities emerge from the interaction between increasing returns to local demand for differentiated non-traded services and non-homothetic preferences, instead of idiosyncratic location preferences and asymmetric housing supply elasticity. We consider an economy with heterogeneous workers differentiated by skill level, who earn income from employment either in the traded-good sector, where worker productivity depends on skill but not location, or in the non-traded-service sector, where worker productivity depends on local demand but not skill. A fixed cost is required for each variety of local service, giving rise to increasing return to local demand, which is income elastic. In equilibrium, high-skill workers share the location with a greater variety of local services and higher land rent, middle-skill workers prefer the location with less variety of local services and lower land rent, low-skill workers, who specialize in non-traded sector, are indifferent between locations. The model can also account for skill dispersion within cities, rising non-traded sector employment share, and a U-shaped welfare change across skill spectrum, as a result of increased skill disparity in the economy.

Key words: skill disparity, income sorting; house price dispersion; increasing return; taste for variety.

JEL classification: J3 R1 R3

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1. Introduction

House price dispersion across US metropolitan areas has widened considerably since World War II. Gyourko, Mayer and Sinai (2013) offer a fundamental insight that the widened dispersion can be a result of aggregate demand increase rather than local productivity divergence. They show that, when idiosyncratic tastes for locations are uncorrelated with income, asymmetric housing supply elasticity across cities is sufficient for aggregate to drive house price dispersion. In particular, cities with inelastic housing supply are “superstar” cities—they become more expensive, hence more exclusive to high-income households, as aggregate demand rises. We sharpen and extend this insight by presenting a model where “superstar” cities emerge from the interaction between increasing returns to local demand for differentiated non-traded services and non-homothetic consumer preferences, instead of idiosyncratic location preferences and asymmetric housing supply elasticity.

We consider an economy where heterogeneous workers, differentiated by skill level, are perfectly mobile and earn income from employment either in the traded-good sector or in the non-traded-service sector. Worker productivity in the former sector depends on skill but not location, whereas in the latter sector it depends on local demand but not skill. A fixed cost is required for each variety of local service, giving rise to increasing return to local demand. Workers derive their utility from the consumption of a numeraire traded good, housing, and differentiated non-traded services. The preferences are non-homothetic such that the demand for local service variety is income elastic. The equilibrium is characterized by worker choices of employment occupation and residential location, wage rates for non-traded service workers and land rent differential across cities clear the labor and housing markets. In equilibrium, low-skill workers choose non-traded service occupation according to comparative advantage. In addition, high-skill traded-sector workers share the location with a greater variety of local services and higher land rent, middle-skill traded-sector workers choose the location with less variety of local services and lower land rent, low-skill non-traded service workers are indifferent between locations, and worker utility is convex, non-decreasing, in skill level. Increasing population skill disparity by raising the share of high-skill workers in the economy has the effect of elevating the demand for the variety of non-traded services, enabling the high-skill city to offer greater variety of local services and thus become more attractive and more expensive.

Besides predicting widening house price dispersion as population skill disparity increases, the model accounts for several additional important features: 1) more expensive cities tend to be larger in population and also have a wider skill spectrum, 2) non-traded sector employment share increases with population skill disparity, and 3) increased population skill disparity produces U-shaped welfare changes across skill spectrum. While the evidence on the first two features is readily available in the literature, the last feature is broadly perceived but not fully appreciated. Our model predicts that increased population skill disparity actually benefits non-traded service workers, who have relatively low skills, and hurt traded-sector workers, who generally have high skills. The middle-skill traded-sector workers tend to suffer most. This happens because the increased skill disparity, as a result of rising share of high-skill workers in the economy, elevates the demand for non-traded services, raising the wage cost in the non-traded sector. The middle-skill workers, who do not benefit from the rising wage in the non-traded sector, suffer the most because they are hurt not only by the rising labor cost of non-traded services but also by getting pushed to smaller cities to have less variety of non-traded services to enjoy.

Our model is rooted in the tradition of the new economic geography literature (Fujita, Krugman, & Venables, 2001; Krugman, 1991) by emphasizing the role of increasing return at the city level in sustaining asymmetric spatial equilibrium. We focus on the increasing return with respect to local consumer amenity instead of that with respect to traded-sector productivity. Incorporating the latter is equivalent to augmenting the skill disparity, which reinforces the asymmetric equilibrium driven by the consumer amenity benefit. Imperfectly elastic housing supply is necessary to prevent the degenerate equilibrium with only one populated city. But relying not on asymmetric housing supply elasticity to drive asymmetric spatial equilibrium is important. Housing supply elasticity is not totally exogenous and can be altered by local land use regulations (Hilber & Robert-Nicoud, 2013; Molloy & Gyourko, 2014). Moreover, restricting housing supply does not necessary give a city any advantage in attracting high-skill workers; indeed, doing so can hurt the city's attractiveness by limiting the local demand size and hence the variety of non-traded services.

The key premises of our model are consistent with empirical evidence. Increasing returns to local demand density for consumer amenities are documented by Couture (2013) and Schiff (2014). Handbury and Weinstein (2012) examine barcode data and find larger metropolitan areas in US offer a larger variety of grocery goods and lower grocery retail price index. Glaeser, Kolko and Saiz (2001) also documented that large cities in Europe and US outperformed their smaller counterparts with respect to consumption benefits. The assumption of non-homothetic preferences is supported by the finding of increasing willingness to pay with skill level for non-traded amenities offered by large cities in Lee (2010) and Fu and Liao (2014).

Our model predicts a wider skill spectrum in the larger, more skilled cities, as these cities employ disproportionally more low-skill non-traded service workers. This prediction is consistent with the stylized fact that both high-skill and low-skill workers disproportionately sort into large cities (Combes, et al., 2012; Eeckhout, Pinheiro, & Schmidheiny, 2010). Davis and Dingel (2012) also assume that non-traded service sector requires no formal skills and hence employ low-skill workers. Empirical evidence show that the presence of high-skill workers improves employment outcomes for low-skill workers, especially for those employed in non-traded service sector. Moretti (2010), for example, finds that one additional skilled job in the traded sector generates 2.5 jobs in local goods and services sector in U.S. cities. Additional evidence can be found in Moretti and Thulin (2013), Manning (2004), and Kaplanis (2010).

Skill sorting across cities is extensively documented in the literature (Bacolod, Blum, & Strange, 2009; Combes, et al., 2012; Henderson, 1974). Most studies focus on productive advantages of skill sorting, such as skill complementarity in production (Baum-Snow & Pavan, 2012, 2013; Berry & Glaeser, 2005; Combes, Duranton, & Gobillon, 2008; Giannetti, 2001, 2003; Glaeser & Resseger, 2010; Matano & Naticchioni, 2012; Mion & Naticchioni, 2009), learning externalities (Davis & Dingel, 2012), and sharing of intermediate inputs (Davis & Henderson, 2008; Hendricks, 2011). Behrens et al (2010), Venables (2011) and Davis and Dingel (2012) are recent examples that provide micro foundation for asymmetric spatial equilibrium and skill sorting across symmetric locations driven by agglomeration economies in traded-good production. Our present paper is in the same spirit as these examples but focuses instead on agglomeration economies with respect to non-traded service supply and consumption benefits. Adamson et al.

(2004) and Gottlieb and Glaeser (2006) also highlight the consumption benefits of skill sorting; but they assume exogenous distribution of consumer amenities.

Our model is presented in section 2. The sorting equilibrium is characterized in section 3. Section 4 provides an algorithm that searches for equilibrium solutions. Numerical examples are shown in section 5. Section 6 concludes.

2. The Model

We consider an economy with two cities at symmetric locations. The economy has a population of perfectly mobile workers with heterogeneous skill levels. They consume housing in one of the two cities, a numeraire traded good, and a bundle of differentiated non-traded services. They have a taste for variety of non-traded services and their utility function is non-homothetic such that the income elasticity of demand for the non-traded services is greater than unity. The productivity of traded-good producers equals to their skill level but is independent of location, whereas the productivity of non-traded-service producers is independent of their skill level but is subject to increasing return with respect to local demand (market thickness). The housing supply in each city is imperfectly elastic so that housing price dispersion widens as housing consumptions in two cities diverge. In such a setting, we show that the relatively low-skill workers will choose to specialize in producing non-traded services and cities will specialize with respect to different diversity of local services to cater to different income segments. The city that offers a greater diversity of local services (low non-traded-service price) and a higher compensating housing price—the superstar city—caters to high skill workers, who have greater willingness to pay for local service diversity, and also attracts a greater proportion of low-skill workers to provide non-traded services.

2.1. Consumption

Workers derive their utility from the consumption of a traded good, X, composite non-traded services S, and housing, H. Previous studies have shown that the income elasticity of demand for housing expenditure is less than 1 (Albouy, 2008; E. L. Glaeser, Kahn, & Rappaport, 2008;

Moretti, 2013). We assume that both housing and the traded good are necessity goods, thus income elasticity of demand for non-traded services is greater than 1. This is a key assumption that drives spatial sorting of skills in our model.

Consumers' preference is defined by the indirect utility function,

(1) $$V(I,G,P)=\frac{1}{\varepsilon}\left(\frac{I}{G}\right)^{\varepsilon}-\frac{\beta}{\gamma}\left(\frac{P}{G}\right)^{\gamma},$$

where I is individual income and P is the composite index of housing price P_h and the traded good price P_X:

(2) $$P=\alpha^{-\alpha}\left(1-\alpha\right)^{\alpha-1}P_h^{\alpha}P_X^{1-\alpha}.$$

The Cobb-Douglas form of the composite price index implies that expenditure share of housing is a constant α of expenditure on housing and the traded good; $0<\alpha<1$. We use the traded good as numéraire good, thus setting P_X to 1. $0<\varepsilon<1$ measures the degree of non-homotheticity of the utility. If $\varepsilon=0$, the utility is homothetic. $\gamma>0$ will define the price elasticity of housing and traded-good consumption.

We let workers have a taste for variety of non-traded services, defining the composite price index of the non-traded services, G, as,

(3) $$G=\left(\int_0^n p\left(i\right)^{1-\sigma}di\right)^{1/\left(1-\sigma\right)},$$

where $p\left(i\right)$ is the price for variety i, n is the range of varieties produced and $\sigma>1$ is the elasticity of substitution between any two varieties.

By Roy's identity, the demand for traded good *X* and that for housing *H* by a worker are given by, respectively:

(4) $$q_X=-\frac{\partial V/\partial P_X}{\partial V/\partial I}=\left(1-\alpha\right)\beta I\left(\frac{G}{I}\right)^{\varepsilon}\left(\frac{P}{G}\right)^{\gamma}$$ and

(5) $q_h = -\dfrac{\partial V / \partial P_h}{\partial V / \partial I} = \alpha\beta I\left(\dfrac{G}{I}\right)^{\varepsilon}\left(\dfrac{P}{G}\right)^{\gamma}\dfrac{1}{P_h}.$

The income elasticity of demand for housing and the traded good is $1-\varepsilon$. The price elasticity for housing is $\alpha\gamma - 1$. The demand for non-traded-service variety i by a worker is given by

(6) $q_{S,i} = -\dfrac{\partial V / \partial p(i)}{\partial V / \partial I} = \dfrac{I}{G}\left(\dfrac{G}{p(i)}\right)^{\sigma}\left[1-\beta\left(\dfrac{G}{I}\right)^{\varepsilon}\left(\dfrac{P}{G}\right)^{\gamma}\right]$

Note that income need to be sufficiently high to generate both a positive demand for non-traded services and a positive utility. We must have $\gamma \geq \varepsilon$ so that positive demand for non-traded services guarantees positive utility and the demands for the traded good and housing are non-increasing in the composite price index for non-traded services.

2.2. Production

Non-traded Service Sector

Non-traded services are produced by labor independent of skill and the production technology is identical for all varieties in all locations. Each worker supplies one unit of labor. The supply of each variety of non-traded services requires a fixed cost of F units of labor. The fixed cost can be in the form of research and development, setting up necessary equipment and shops, or obtaining necessary business licenses. In addition, each unit of service output also requires a constant marginal labor input c. Producing a quantity $z(i)$ of any variety thus requires l units of labor input:

(7) $l = F + cz(i).$

Given a wage rate w for labor in non-traded service sector and price $p(i)$, the profit $\pi(i)$ for reach service variety is given by:

(8) $\pi(i) = p(i)z(i) - w[F + cz(i)].$

Given a constant price elasticity of demand, profit maximization entails a constant mark-up pricing over the marginal cost of production:

(9) $p\left(i\right)^{*} = \dfrac{\sigma}{\sigma - 1} cw$.

Free entry drives the profit to zero,

(10) $\pi\left(i\right)^{*} = w\left(\dfrac{1}{\sigma - 1} cz\left(i\right)^{*} - F\right) = 0$.

Thus, the equilibrium output for each variety is given by

(11) $z\left(i\right)^{*} = F\left(\sigma - 1\right) / c$,

which requires a labor input of

(12) $l^{*} = F\sigma$.

We choose the unit of measure for labor input such that $c = \left(\sigma - 1\right)/\sigma$. Thus,

(13) $z\left(i\right)^{*} = F\sigma$,

(14) $p\left(i\right)^{*} = w$.

Traded Good Sector

Work productivity in the traded-good sector benefits from formal training that produce skills. We define skill level such that each worker's productivity (employment income) in the traded-good sector equals his skill level, indicated by index *b*. The distribution of *b* in the worker population is described by a density function, $k(b)$, on a finite support $\left[\underline{b},\ \overline{b}\right]$. Workers are free to choose employment in any sector. Comparative advantage *a la* Roy (1951) allocates low-skill workers to the sector where skill does not benefit productivity. Specifically, given the wage rate *w* in the non-traded service sector, workers with skill level below w will choose employment in the non-traded sector and those with $b > w$ will choose employment in the traded-good sector.

Housing Sector

Following Behrens et al (2010) and Davis and Dingle (2012), we adopt a most stripped-down representation of the housing sector. Housing service is produced by capital only. A standard monocentric city urban form (Alonso, 1964; Muth, 1969) entails a constant cost of housing service (including commuting cost and land rent) throughout the city as land rent varies by location to compensate differential commuting cost. That cost of housing service in location *j*, denoted by $P_{h,j}$, must increase with city size in terms of total quantity of housing space consumed $Q_{h,j}$; the rate of increase, however, will depend on the city's housing supply elasticity, which regulates the residential density. We assume $P_{h,j} = \theta Q_{h,j}^{\rho}$, where parameters $\theta, \rho > 0$ and ρ represents the inverse of housing supply elasticity, which is invariant across locations.

3. Equilibrium

We first characterize equilibrium for the case of two *ex ante* identical locations, labeled city 1 and city 2 respectively. Individual workers choose a city to live, an occupation and the consumption bundle to maximize their own utility. In equilibrium, non-traded sector wage rates in each city, w_1 and w_2, housing prices, $P_{h,1}$ and $P_{h,2}$, and composite non-traded service prices G_1 and G_2, clear the market for non-traded service workers and housing in each city. Spatial equilibrium requires any advantage of lower composite non-traded service price to be compensated by a higher housing price such that a marginal worker will be indifferent between two cities.

To build the intuition for the basic properties of an asymmetric equilibrium, Figure 1 shows the utility offered by each city for workers at different skill levels. Without loss of generality, we assume city 1 to have a lower composite non-traded service price and higher housing price: $G_1 < G_2$ and $P_{h,1} > P_{h,2}$. The utility offer curve of city 1 is steeper, with a slope of $G_1^{-\varepsilon} / \varepsilon$. With a higher composite non-traded service price G_2, the slope of the utility offer curve of city 2 is smaller, $G_2^{-\varepsilon} / \varepsilon$. A lower housing price $P_{h,2}$ shifts the city 2 utility offer curve to the left and determines the cutoff skill level b_1, above which skill the traded-sector workers will live in city 1. The non-traded sector wage rate in city 2, w_2, determines the cutoff skill level $b_2 = w_2$, below which skill the workers are better off employed in the non-traded sector. Non-traded service workers enjoy the same utility level represented by the horizontal line in Figure 1 that intersects the utility offer curve of city 2 at b_2. The intersection of this horizontal line with the utility offer curve of city 1 determines the non-traded service wage rate in city 1, w_1, which compensates the non-traded service workers in city 1 for the housing price premium $P_{h,1} - P_{h,2}$. The equilibrium utility across the skill spectrum is thus convex and non-decreasing in skill level; it is constant for low-skill workers in the non-traded service sector, it then rises with skill level above the cutoff point b_2 along the city 2 utility offer curve until the cutoff point b_1, it then rises more steeply along the utility offer curve of city 1 above the cutoff skill level b_1.

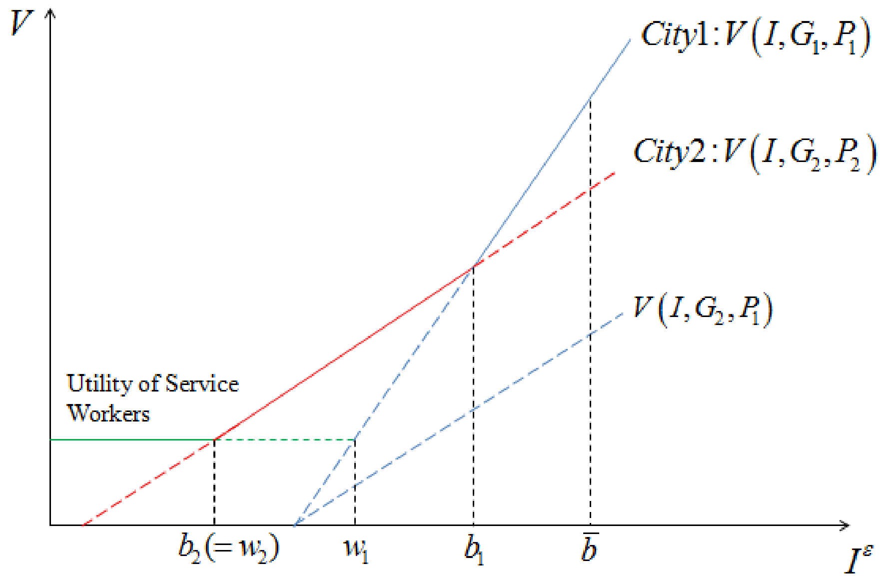

Figure 1. Utility offered by City 1 and City 2 at different skill levels

Although the exact positions of two utility offer curves must be determined in general equilibrium, it is clear from Figure 1 that, as long as the composite non-traded service price is lower in city 1, city 2 must offer a lower housing price in order to have any positive number of workers to populate it. And as long as the composite non-traded service prices offered by the two cities are different, traded-sector workers sort themselves perfectly by skill levels between the two cities. High-skill workers outbid middle-skill workers in the city offering a lower composite non-traded service price. We formalize this result in proposition I (the proofs is provided in Appendix).

Proposition I (skill sorting of traded workers)

In asymmetric equilibrium, cities offer different levels of composite non-traded service price and different housing prices that compensates the difference in composite non-traded service price. Moreover, high-skill traded-sector workers sort into the city with a low composite non-traded service price but a higher housing price (City 1); the middle-skill traded workers sort into the city with a high composite non-traded service price but a low housing price (City 2).

Figure 1 shows that, given the population mass L and skill distribution $k(b)$, consumer preferences, production technologies, and housing supply elasticity, the asymmetric equilibrium is fully characterized by the two skill cutoff levels b_1 and b_2. The equations (15) through (25) below define these two cutoff skill levels. Equation (15) defines b_1, such that the traded-sector workers with skill b_1 are indifferent between two cities:

(15) $$\frac{1}{\varepsilon}\left(\frac{b_1}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{b_1}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma}$$

Equation (16) defines the cutoff skill b_2, such that the workers with skill b_2 are indifferent between employment in the traded sector and employment in non-traded service sector,

(16) $$b_2 = w_2.$$

Equation (17) describes the condition for non-traded service workers to be indifferent between two cities:

(17) $$\frac{1}{\varepsilon}\left(\frac{w_1}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{w_2}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma}$$

The total population of non-traded service workers in the whole economy is $L\int_{\underline{b}}^{b_2} k(t)\,dt$. Let ϕ to denote the proportion of them who live in city 1. Equations (18) and (19) define the service price index in each city,

$$(18)\quad G_1 = \left[\frac{\phi L \int_{\underline{b}}^{b_2} k(t)\,dt}{F\sigma}\right]^{1/(1-\sigma)} w_1$$

$$(19)\quad G_2 = \left[\frac{(1-\phi) L \int_{\underline{b}}^{b_2} k(t)\,dt}{F\sigma}\right]^{1/(1-\sigma)} w_2$$

Equations (20) and (21) define the zero-profit conditions for non-traded service supply.

$$(20)\quad F\sigma = L\left(\frac{G_1}{w_1}\right)^{\sigma}\left\{\int_{b_1}^{\bar{b}} \frac{t}{G_1}\left[1-\beta\left(\frac{G_1}{t}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\right]dt + \phi K(b_2)\frac{w_1}{G_1}\left[1-\beta\left(\frac{G_1}{w_1}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\right]\right\}$$

$$(21)\quad F\sigma = L\left(\frac{G_2}{w_2}\right)^{\sigma}\left\{\int_{b_2}^{b_1} \frac{t}{G_1}\left[1-\beta\left(\frac{G_2}{t}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]dt + (1-\phi) K(b_2)\frac{w_2}{G_2}\left[1-\beta\left(\frac{G_2}{w_2}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]\right\}$$

On the right-hand side is the aggregate demand for individual variety in each city, which must equals to $F\sigma$, to assure that the producers earn zero profit.

Equation (22) through (25) define the clearing of housing markets in both cities.

$$(22)\quad Q_{h,1} = \alpha\beta L\int_{b_1}^{\bar{b}} t\left(\frac{G_1}{t}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\frac{1}{P_{h,1}}k(t)dt + \alpha\beta\phi LK(b_2)w_1\left(\frac{G_1}{w_1}\right)^{\varepsilon}\left(\frac{P_1}{G_1}\right)^{\gamma}\frac{1}{P_{h,1}}$$

$$(23)\quad Q_{h,2} = \alpha\beta L\int_{b_2}^{b_1} t\left(\frac{G_2}{t}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\frac{1}{P_{h,2}}k(t)dt + \alpha\beta(1-\phi) LK(b_2)w_2\left(\frac{G_2}{w_2}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\frac{1}{P_{h,2}}$$

$$(24)\quad P_{h,1} = \theta Q_{h,1}^{\rho}$$

$$(25)\quad P_{h,2} = \theta Q_{h,2}^{\rho}$$

In asymmetric equilibrium, our model predicts that non-traded sector employment in superstar city, *i.e.*, city 1 (with a low composite non-traded service price and a higher housing price), is always greater than that in city 2, as stated in the following proposition.

Proposition II (employment in non-traded service sector)

In asymmetric equilibrium, non-traded sector employment is larger in City 1 (with a lower composite non-traded service price and a higher housing price) than in City 2.

The proof is in Appendix. Intuitively, non-traded service workers in superstar city earn a lower wage than the cutoff traded sector workers, who are indifferent between the two cities. To compensate the low-skill service workers, who do not benefit very much from a low compositie non-traded service price, the superstar city must pay higher wage to compensate them for the higher housing price. Eventually, the population of low-skill workers, as well as the number of service varieties they produce, must grow to the extent that the service price index in the superstar city is lower despite the higher non-traded sector labor cost.

In summary, the asymmetric equilibrium emerging from the interaction between non-homothetic preferences and increasing return to local demand for non-traded consumer amenities, the supply of which employs low-skill workers, has richer implications beyond house price dispersion. The model also predicts the impact of aggregate skill distribution on income disparity within as well as between cities, on the employment of non-traded sector in the economy and across cities, and on the size distribution of cities. Since these predictions are based on structural parameters, such as income and price elasticity of demand, taste for variety, increasing return in non-traded service supply, housing supply elasticity, and skill distribution, the model can be calibrated to evaluate various counterfactuals, such as change in skill distribution and housin supply elasticity.

4. Algorithm

To illustrate the emergence of an assymmetric equilibrium, we provide an algorithm to find equilibrium cutoff skill levels b_1 and b_2. We adopt a bounded Pareto distribution to characterize the aggregate skill distribution. The Pareto distribution is a good approximation for income distribution observed in many countries, such as US. It's shape can be modified by a single paramter, a shape parameter ξ, which also determines inequality measures such as Gini coefficient. We adopt a support for the skill distribution from 1 to 100, to broadly reflect the reality of productivity spectrum across individuals in an economy like US. Thus the skill probability density function is given by $k(b)=\xi b^{-\xi-1}/\left(1-0.01^{\xi}\right)$, with $\xi > 1$, which has a mean value of approximately $\xi/(\xi-1)$ and a Gini coefficient of approximately $1/(2\xi-1)$.

The existence of the equilibrium can be demonstraed using a phase diagram for the two cutoff skill levels b_1 and b_2, as shown in Figure 2. The horizontal axis of the diagram is skill cutoff for service workers, b_2, and the verticle axis is the skill cutoff for traded-sector workers in city 1, b_1. Note that b_2 coincide with w_2, the non-traded sector wage rate in city 2. To determine how b_1 and b_2 will adjust when they deviate from the equilibrium levels, we construct two equilibrium curves. The first one traces the combination of b_1 and b_2 that clears the market for non-traded service employment. We refer to it as "zero excess employment demand" curve. The excess demand for non-traded service workers, or excess employment demand *EED*, is determined by the following equation:

$$EED = L\left(\frac{G_2}{w_2}\right)^{\sigma}\left\{\int_{b_2}^{b_1}\frac{t}{G_1}\left[1-\beta\left(\frac{G_2}{t}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]dt+\left(1-\phi\right)K(b_2)\frac{w_2}{G_2}\left[1-\beta\left(\frac{G_2}{w_2}\right)^{\varepsilon}\left(\frac{P_2}{G_2}\right)^{\gamma}\right]\right\}-F\sigma.$$

To compute *EDD* all equilibrium conditions described by Eq (15) through Eq (25), except Eq (15) and Eq (21), are satisfied. The zero excess employment demand curve is thus defined by *EED* = 0. It is shown as the steeper curve in Figure 2. To the left of this curve, the non-traded service wage rate is too low, such that the supply of workers to the non-traded sector falls short of the demand (*EED* >0). As a result, the non-traded sector wage rate, hence b_2, will rise.

The second equilibrium curve traces the combination of b_1 and b_2 that clears the housing market in city 1 and city 2. The housing market clearance requires the marginal traded-sector worker in city 1 to obtain the same utility that City 2 can offer, so as to be indifferent between the cities. We refer to this curve as "equal utility for marginal worker" curve. The utility difference between City 1 and City 2 for the marginal traded worker in City, denoted by *UDM*, is determined by:

$$UDM = \frac{1}{\varepsilon}\left(\frac{b_1}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{b_1}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma}.$$

Again, in compute *UDM*, all equilibrium conditions, Eq (15) through Eq (25), except Eq (15) and Eq (21), are satisfied. The equal marginal utility curve is thus defined by *MUD* = 0. It is shown as the flatter curve in Figure 2. Below this curve, the marginal traded-sector worker in city 1 will find city 1 too expensive (hence offering a lower utility than city 2, *MUD* < 0) and thus prefer to move to city 2. As the middle-skill marginal workers get pushed out of city 1, the skill cutoff for traded-sector workers in city 1 rises.

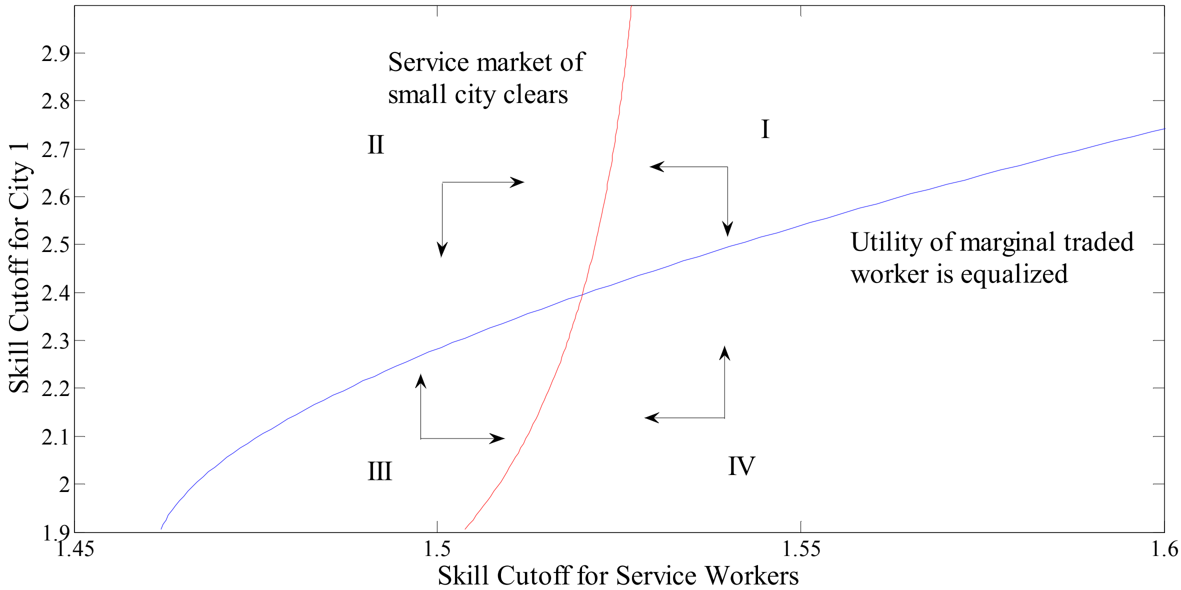

Figure 2: Phase digram for equilibrium skill cutoff points

Notes: The model parameters are $L = 4$, $\alpha = 0.6$, $\beta = 0.6$, $\varepsilon = 0.1$, $\sigma = 7$, $\gamma = 0.1$, $\theta = 0.2$, $\rho = 0.5$, $F = 0.001$ and $\xi = 13/6$ (which gives a skill Gini coefficient of approximately 0.3).

The two equilibrium curves, the zero excess employment demand curve and equal marginal utility curve, divide the phase diagram into four regions, as shown in Figure 2. In each region, b_1 and b_2 will change due to market adjustment, as indicated by the arrows. The phase diagram shows that b_1 and b_2 will converge to the intersection of the two equilibrium curves, which defines the equilibrium.

The phase diagram provides two important insights. First, there exists a unique asymmetric equilibrium, as long as the preference for consumer amenity variety is not too strong (σ not too small) in relation to housing supply elasticity ($1/\rho$ not too large). Otherwise, city 1 will end up attracting everyone and the equilibrium degenerates into a single-city outcome. Second, the asymmetric equilibrium is stable. Any deviation from the equilibrium skill cutoff combination b_1 and b_2 will be corrected by market adjustment.

It can be shown that in equilibrium, total income from traded sector exactly covers the aggregate housing expenditure in each city:

(26) $P_{h,1}Q_{h,1} = \alpha L \int_{b_1}^{\bar{b}} tk(t)\,dt$

(27) $P_{h,2}Q_{h,2} = \alpha L \int_{b_2}^{b_1} tk(t)\,dt$

These two equations are convenient for solving equilibrium, because together with Eq (24) and Eq (25) they determine housing prices and quantities based on two variables, b_1 and b_2, only. Thus, for any initial values b_1 and b_2, we can determined housing price and quantity in each city. Then we use Eq (17), Eq (18), Eq (19), and Eq (22) to solve two remaining unknowns, ϕ and w_1. Subsequently, we calculate *EED* and *MUD* and adjust b_1 and b_2 in the direction that reduces the magnitude of *EED* and *MUD*. We repeat these steps until *EED* and *MUD* converge to zero.

5. Numerical Simulations

Using the algorithm described above, we numerically solve the asymmetric equilibrium for a two-city economy. We present the baseline case in Table 1. In equilibrium City 1 is more populous than City 2, attracts top skill traded-sector workers, offers a lower composite price index of non-traded services, has a higher housing price, and employs proportionally more workers in the non-traded sector. Unlike Gyourko, Mayer and Sinai (2013), where the superstar city is exclusive to high income households, the equilibrium of our model entail the larger city

(City 1) to have a wider spectrum of skills as documented in Eeckhout et al. (2010).

Table 1. Two-city Asymmetric Equilibrium

Main features	City 1	City 2
Population	2.0583	1.9417
Traded workers' skill	2.3955-100	1.5199-2.3955
Non-traded service employment	1.4558	0.9295
Non-traded service wage	1.5855	1.5199
Composite price index of non-traded services	0.6514	0.6901
Housing price	0.4042	0.3588

Notes: The baseline case parameters are $L = 4$, $\alpha = 0.6$, $\beta = 0.6$, $\varepsilon = 0.1$, $\sigma = 7$, $\gamma = 0.1$, $\theta = 0.2$, $\rho = 0.5$, $F = 0.001$, and $\xi = 13/6$ (a skill Gini coefficient of approximately 0.3).

We next show how the equilibrium evolves as the aggregate skill disparity, indicated by the skill Gini coefficient, rises. We decrease the shape parameter of skill distribution such that its Gini coefficient increases from 0.3 to 0.6 (reflecting an increasing share of high-skill workers in the economy). The results are shown in Figure 3a through 3h. As the skill Gini coefficient increase to 0.6, city 1 grows even bigger and accounts for 62% of total population, as opposed to 51% when Gini coefficient is 0.3. During this process, the skill cutoff for the traded workers in city 1 also increases, indicating that middle-skill traded workers are pushed to City 2. As shown in Figure 3a, the skill cutoff for non-traded service sector also increases, indicating that least skilled traded-sector workers are switching to the non-traded sector to cater to an increasing demand for non-traded services.

The employment share of service workers increases in both cities. Figure 3b shows the gain in non-traded service employment in the economy as a whole rise about 13 percentage points (with a corresponding loss of employment share by the traded-good sector) as the skill Gini coefficient doubles from 0.3. Interestingly, since 1960 U.S. manufacturing employment share declined by about 15 percentage points (Baily & Bosworth, 2014) as income Gini coefficient rose from 0.35 to 0.45. The results of our model suggest that the loss of low-skill manufacturing jobs in the U.S. is not entirely due to competition from China; growing domestic demand for non-traded services would play an important role.

Figure 3c shows that, as the aggregate skill inequality increases, City 1 becomes more attractive in terms of the variety of local consumer amenities it can offer. The composition price index of non-traded services in City 1 over than in City 2 declines from 0.94 when the skill Gini is 0.3 to 0.90 as the skill Gini rise to 0.6. The housing price premium in City 1 increases from 12% to 32%. Figure 3d shows that house price dispersion and city population size dispersion both increases with aggregate skill inequality.

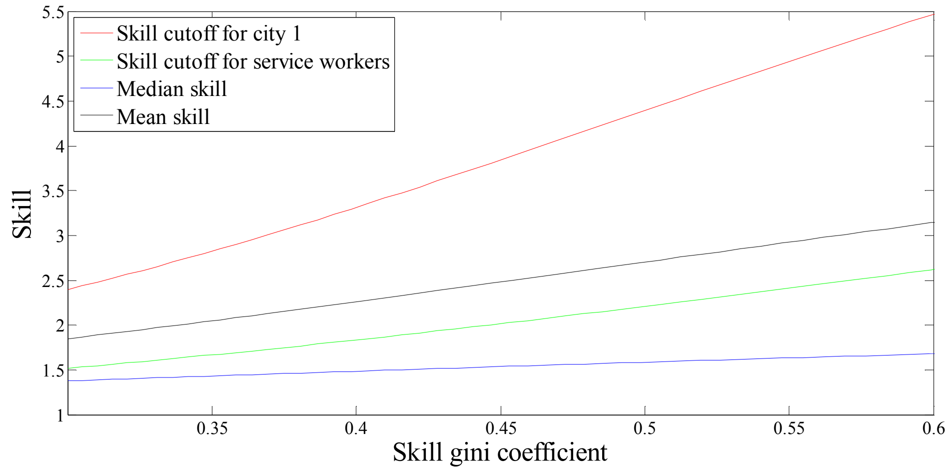

Figure 3a

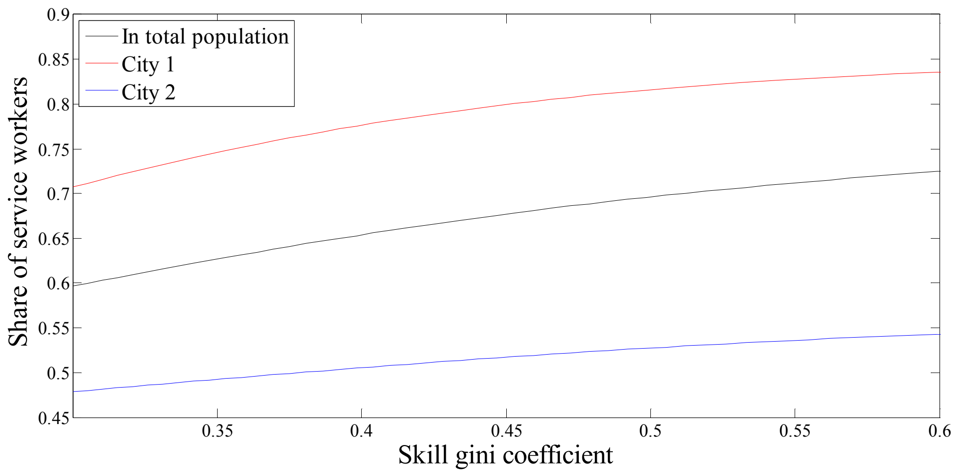

Figure 3b

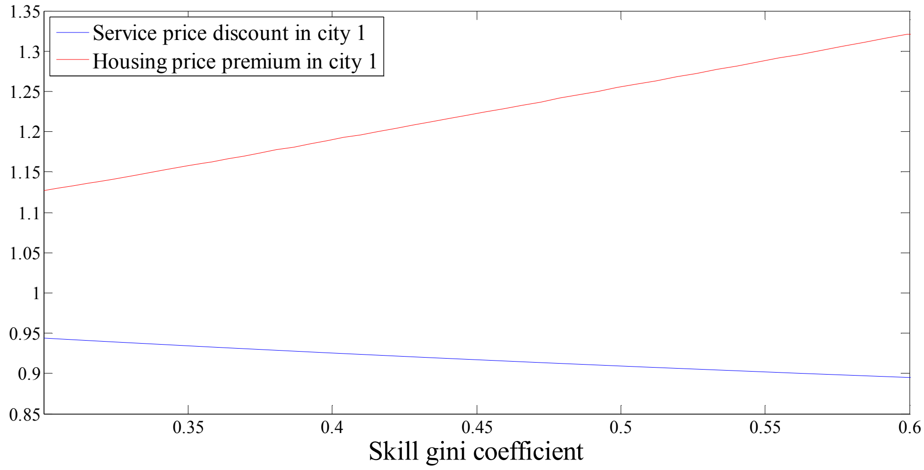

Figure 3c

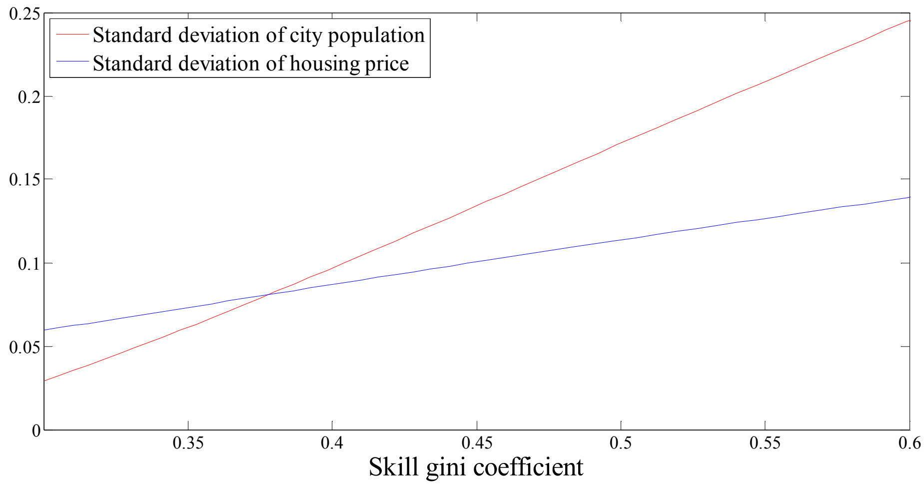

Figure 3d

To explore the welfare implications of aggregate skill inequality for workers at different skill levels, we depict the utility paths of workers at skill level 1, 2, 4 and 8, respectively, in figures 3e through 3h. The red lines in the figure represent the utility that workers can obtain working in the traded-good sector in city 1. The blue lines represent the utility offered by the traded-sector employment in city 2. The green lines represent the utility offered by non-traded service sector (in either city). Workers will choose the occupation and city that offer the highest utility.

As shown in Figure 3e, the bottom-skill workers will always be employed in the non-traded service sector. Their welfare increases as the aggregate skill inequality rises. Workers at skill level 2 is employed in the traded sector in City 2 when the aggregate skill inequality is low, as shown in Figure 3f. At a skill Gini coefficient of 0.45, these workers find it profitable to switch to employment in the non-traded service sector. Their welfare initially declines as the aggregate skill inequality rises and then improves with the aggregate skill inequality after they switch to employment in the non-traded sector.

Figures 3g and 3h display the welfare paths for workers at skill level 4 and 8, who always work in the traded sector. Their welfare always decline with aggregate skill inequality. The middle-skill traded workers enjoy City 1 when aggregate skill inequality is low and are pushed to City 2 when the aggregate skill inequality becomes sufficiently high. The high-skill traded workers always choose City 1.

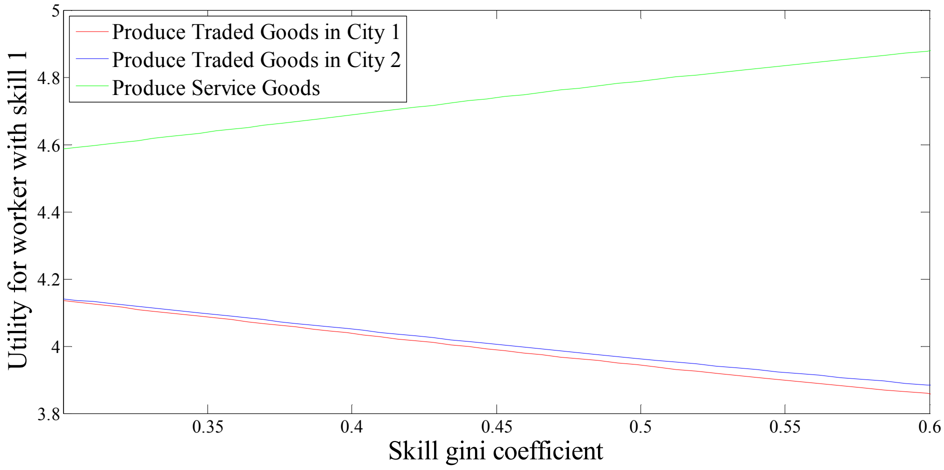

Figure 3e

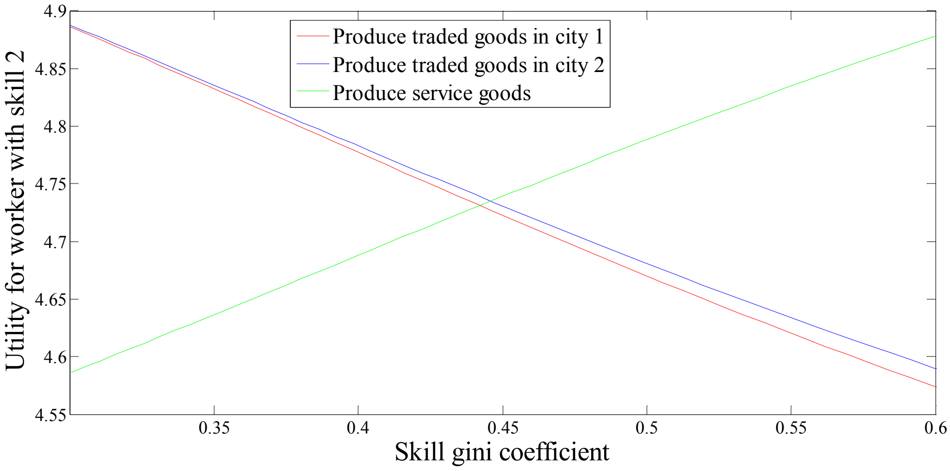

Figure 3f

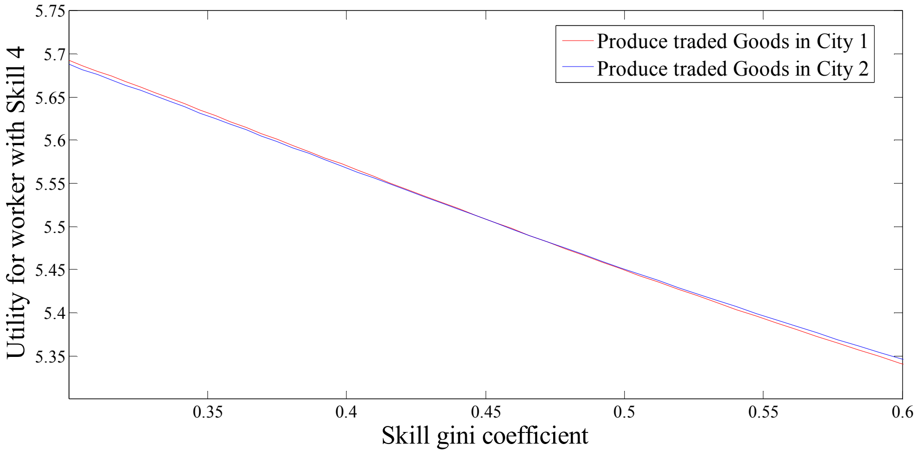

Figure 3g

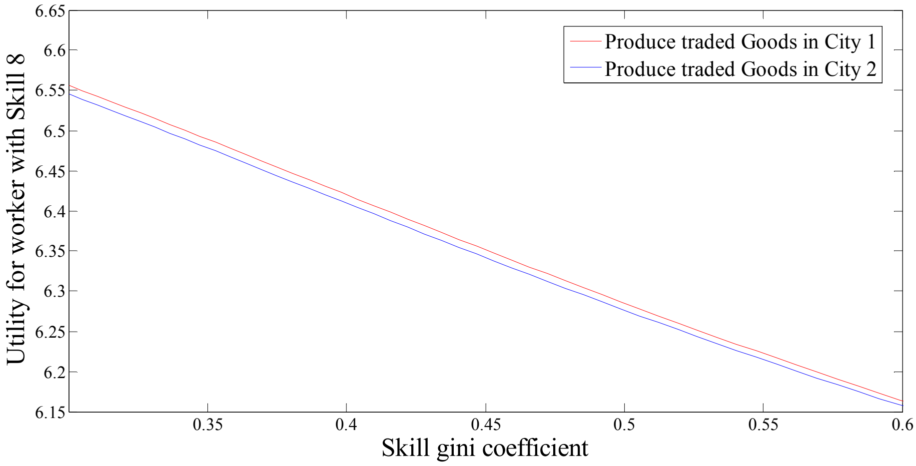

Figure 3h

In general, low-skill workers, who always choose employment in the non-traded sectors, always benefit from increased aggregate skill inequality, which elevates the demand for non-traded services. Low-skill traded sector workers are harmed by increased aggregate skill inequality initially but then benefit from it after they eventually move to the non-traded sector. Workers at middle and high skill levels, who never find non-traded service employment to their advantage, are always harmed by an increase in aggregate skill inequality, which pushes up the labor cost of non-traded services. The middle-skill workers tend to lose more because they eventually also get pushed out of City 1, which offer more attractive consumer amenities. Thus, the welfare impact of rising aggregate skill inequality across the skill spectrum is U shaped (more accurately, left-tilted L shaped), as shown in Figure 4.

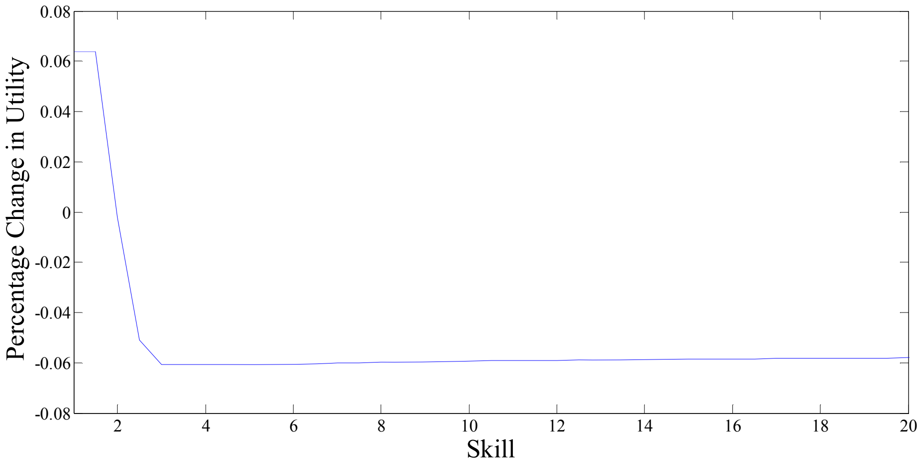

Figure 4. The welfare impact of rising aggregate skill Gini coefficient from 0.3 to 0.6

We further explore the effect of preference for consumer amenity variety and housing supply elasticity on spatial equilibrium outcomes. We examine two alternative scenarios. First, we reduce the consumer taste for non-traded service variety by increasing σ from 7 (baseline value) to 8. Second, we reduce the housing supply elasticity by increase ρ from 0.5 (baseline value) to 0.75 (corresponding to a median housing supply elasticity across US cities reported by Saiz (2010)). The results are presented in Table 2. Column 1 shows the simulation results for the baseline scenario. Each cell displays two numbers, corresponding to the outcome associated with a skill Gini coefficient of 0.3 (top number) and a skill Gini coefficient of 0.6 (bottom number), respectively. Column 2 and 3 show the simulation results for the alternative scenarios.

At a weaker preference for consumer amenity variety, the composite price of non-traded services becomes higher in both cities in comparison with the baseline case. City 1 becomes smaller, as the lower-skill traded-sector workers no longer find the consumer amenity benefit in City 1 sufficiently attractive to justify the higher housing price in City 1. As City 1 share of worker population decline, its advantage in offering better local consumer amenity also declined, reflected by the convergence of its composite non-traded service price towards that of City 2. Housing price dispersion between the cities also decrease. Non-traded sector employment does not change much, as the effect of reduced demand on productivity is compensated by increased production scale for each of the smaller set of non-traded services. The difference in non-traded sector employment between City 1 and City 2, however, narrows. Total housing expenditure in the economy (an income leakage to friction) does not change much, although City 1's share of that decreases. The welfare diminishes for everyone because of higher composite price of non-traded services. Moreover the welfare inequality diminishes as the higher composite price of non-traded services hurt high-income workers more than low-income workers. Finally, we find that widening aggregate skill inequality has a smaller impact on the dispersion of mean skill level, composite non-traded service price, housing price, and population size across the cities.

Table 2: Comparative static analysis

Column	1	2	3
Scenario	Baseline	$\sigma = 8$	$\rho = 0.75$
Skill cutoff for traded-sector workers in City 1, b_1	2.3955 5.4655	2.5073 6.9131	2.5589 7.7438
Skill cutoff for non-traded service employment, b_2	1.5199 2.6190	1.5203 2.6252	1.5096 2.5855
City 2 mean skill	1.5429 2.4901	1.5568 2.5940	1.5609 2.6337
City 1 to City 2 mean skill ratio	1.3851 1.4241	1.3875 1.3952	1.3967 1.3920
Non-traded-sector employment share	0.5963 0.7245	0.5965 0.7254	0.5904 0.7197
Non-traded service employment in city 2	0.9295 0.8252	0.9972 1.0473	1.0249 1.1513
City 1 to City 2 ratio of non-traded-sector employment	1.5662 2.5120	1.3931 1.7707	1.3041 1.5007
City 1 share of total population	0.5145 0.6201	0.4837 0.5375	0.4647 0.4952
City 2 composite non-traded service price	0.6901 1.2275	0.7848 1.3708	0.6775 1.1624
City 1 to City 2 ratio of composite non-traded service price	0.9439 0.8953	0.9643 0.9449	0.9646 0.9511
City 2 housing price	0.3588 0.3979	0.3677 0.4314	0.4469 0.5653
City 1 to City 2 housing price ratio	1.1264 1.3211	1.0787 1.1635	1.0679 1.1239
Total housing expenditure	2.8060 5.2072	2.8041 5.1901	2.8280 5.2261
Housing expenditure in city 1	1.6510 3.6320	1.5608 3.1746	1.5222 2.9670
Housing expenditure in city 2	1.1550 1.5752	1.2432 2.0155	1.3058 2.2591
Utility of workers at different skill levels			
Skill=1	4.5859 4.8781	4.5216 4.8082	4.5089 4.7785

Skill=2	4.8878 4.5900	4.8194 4.8082	4.8188 4.7785
Skill=4	5.6924 5.3465	5.6122 5.2693	5.6232 5.2641
Skill=8	6.5564 6.1637	6.4630 6.0732	6.4866 6.0807
Skill=20	7.7943 7.3421	7.6823 7.2318	7.7237 7.2575

Notes: The baseline case parameters are $L=4$, $\alpha=0.6$, $\beta=0.6$, $\varepsilon=0.1$, $\sigma=7, \gamma=0.1$, $\theta=0.2$, $\rho=0.5$, $F=0.001$. The top number in each cell corresponds to the case of skill Gini=0.3, the bottom number, the case of skill Gini=0.6.

At a lower housing supply elasticity, $\rho=0.75$, City 1 becomes more exclusive to higher-skill traded-sector workers and its population share in the economy diminishes compared to the baseline case. The composite non-traded service price in City 2 decreases, and so does City 1's advantage in the composite price, as the demand for non-traded services is reduced by higher housing prices and housing expenditure leakage in the economy. The non-traded sector wage rate decreases, and so does the employment share of the non-traded service sector. Although the dispersion in mean skill level increases somewhat (at a low skill Gini coefficient), the dispersion in composite non-traded service price, housing price, and population size all decrease. Welfare for everyone is diminished due to higher housing expenditure (leakage). Again we note that widening aggregate skill inequality has a smaller impact on the dispersion of mean skill level, composite non-traded service price, housing price, and population size across the cities. This last result is in marked contrast with the result in Gyourko, Mayer and Sinai (2013), where more restrictive housing supply in a city helps strengthening its superstar-city status. In our case, more restrictive housing supply is an obstacle for the high-income city to take advantage of increasing returns in local consumer amenities.

6. Conclusion

We have presented a model to show asymmetric spatial equilibrium can emerge across perfectly symmetry locations in the presence of increasing returns in local consumer amenities and non-homothetic preferences for such amenities. Both premises are supported by empirical evidence recently documented in the literature. The model can account for widened housing price dispersion across cities solely by increased aggregate skill inequality (or increased share of high-skill workers) in the economy. A larger share of high-skill workers reinforces the increasing returns in local consumer amenities and income segregation among traded-sector workers across cities. The model helps sharpening an important insight in Gyourko, Mayer and Sinai (2013) but also clarifying the effect of local housing supply elasticity on asymmetric equilibrium outcome: restrictive housing supply may make the "superstar" city more exclusive but would moderate, rather than exacerbate, housing price dispersion across cities when aggregate skill inequality rises. This clarification has important policy implications—expanding housing supply in a "superstar" city can have unintended consequence of reinforcing its advantage in local consumer amenities and hence its high housing price.

More importantly, our model builds on a micro foundation that can be calibrated to quantify the contribution of aggregate skill inequality to housing price dispersion observed in a real economy. In addition, our model can also account for the rise of the employment share of non-traded service sector resulting from increased aggregate skill inequality, a significant feature of many economies like US. Related to the impact of aggregate skill inequality on employment structure, our model reveals that widening aggregate skill inequality can benefit low-skill workers due to increased demand for non-traded services, which low-skill workers generally have a comparative advantage in producing. Moreover, the welfare gain of the low-skill non-traded service workers is at the expense of high-skill traded-sector workers, who, although enjoying a greater variety of non-traded services in the presence of a larger share of high-skill workers in the economy, nevertheless have to pay higher labor cost for each variety of non-traded services.

Our model can be extended to incorporate local agglomeration economies in the traded-sector employment and to cases with more than two locations (to study more realistic housing price dispersion across cities).

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Appendix. Proofs of Propositions

Proof of Proposition I (skill sorting of traded workers)

First, given the indirect utility function Eq (1) and the assumption that worker productivity in the traded sector is independent of location, traded workers will always prefer living in the city with low housing price and low composite non-traded service price. Therefore, the city with high housing price and high composite non-traded service price will attract no traded workers and hence has no income to support non-traded service employment. Therefore, any equilibrium with positive population in both locations must have housing price differences across locations compensating the differences in composite non-traded service price.

Second, from the indirect utility function Eq (1), we have,

$$\text{(A1)} \quad \frac{\partial^2 V}{\partial I\, \partial G} = -\varepsilon I^{\varepsilon-1}\left(\frac{1}{G}\right)^{\varepsilon+1} < 0.$$

High income and low composite non-traded service price are complementary. If there exists a traded worker with skill level b^*, who is indifferent between two cities, i.e.,

$$\frac{1}{\varepsilon}\left(\frac{b^*}{G_1}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_1}{G_1}\right)^{\gamma} = \frac{1}{\varepsilon}\left(\frac{b^*}{G_2}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_2}{G_2}\right)^{\gamma},$$

then the single-crossing condition (A1) ensures that the traded workers with skill $b > b^*$ will all prefer the city with a lower composite non-traded service price and higher housing price.

Q.E.D

Proof of Proposition II (non-traded sector employment)

We prove this proposition in two steps:

Step 1. City 1 pays higher wage to the workers in non-traded service sector, i.e., $w_1 > w_2$.

Suppose that $w_1 \leq w_2$. Because $w_2 = b_2 < b_1$, proposition II says that non-traded service workers will strictly prefer city 2, and this conflicts with the condition that non-traded service workers are indifferent between two cities. Therefore, City 1 must pay higher wage to the non-traded service workers to make the non-traded service workers indifferent between two cities.

Step 2. City 1 must employ a larger number of non-traded service workers.

Because City 1 pays higher wage to the workers in non-traded service sector, from (18), the price of a single service variety is higher. To maintain a lower composite non-traded service price, City 1 must produce a greater variety of services. Therefore, City 1 must employ a larger number of non-traded service workers.

Q.E.D