The CAPM: A Reformulation under Conditions of Risks and Ambiguity

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ABSTRACT

By means of an information-enhanced basic model of financial markets, the classical CAPM has been reformulated in two fundamental respects. The first is to have the beta-pricing basis extended to include two more market factors; the second is to have the pricing model allow for the tradeoff relation between market ambiguity and ambiguity aversion in the context of market incompleteness. As a result, the reformulated CAPM captures not only *systematic* information associated with market factors but also *systemic* information associated with the state of market structure. Hence, it is *relatively more efficient* in terms of information.

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I. Introduction

Compared with the capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965) and Black (1972), the distinct part of this article is that the CAPM is reformulated under conditions of *risks and ambiguity*. The change of conditions form *risk* to *risks and ambiguity* means some alternative basic model of financial markets. Consequently, market equilibrium and market equilibrium valuation of capital assets can be conceived in an alternative framework of analysis or with an alternative information space. Should the alternative basic model of financial markets be *relatively more efficient*, a better market valuation system of capital assets could be derived from it, suggesting at least one market-tractable capital asset pricing model.

Take information inclusiveness or information completeness as a measure of model efficiency, we could define the notion of *relative efficiency* between alternative assumings of the basic model of financial markets. A basic model of financial markets is said to be *relatively more efficient* if and only if its information inclusiveness or its information space is more compactible than its alternative. Along the lines, we could define also the notion of *absolute efficiency* of a basic model of financial markets. A basic model of financial markets is said to be *absolutely efficient* if and only if it *is* the real-world financial markets. Since the basic model of financial markets can never be identical to the real-world financial markets, the market equilibrium valuation of capital assets from any basic model of financial markets is a fundamental valuation that is relatively "correct" only. Anyhow, two fundamental valuations can be compared by means of the notion of relative efficiency or relative correctness.

The CAPM of Sharpe (1964) and Lintner (1965), as well as its Black's version (1972), took on its empirical limitations as early as in the late 1970s. Basu (1977) reported the price-earnings-ratio effect, noting that firms with low price-earnings ratios have higher sample returns and firms with high price-earnings ratios have lower mean returns than would be the case if the market portfolio was mean-variance efficient. Benz (1981) documented the size effect which is that low market capitalization firms have higher sample mean returns than would be expected if the market portfolio was mean-variance efficient. Afterwards, the so-called anomalies literature has burgeoned in terms of theoretical puzzles[1] and empirical anomalies[2]. At last, four decades after its birth, Fama and French (2004) made the conclusion that the failure of the CAPM in empirical tests implies that most applications of the model are invalid.

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[1] Such as the *equity premium* puzzle of Mehra and Prescott (1985); the *risk-free-rate* puzzle of Weil (1989); the *volatility* puzzle by Shiller (1981), Hansen and Singleton (1983); and the *predictability* puzzle by De Bond and Thaler (1985), Jegadeesh and Titman (1995).

[2] Fama and French (1992, 1993) find that beta cannot explain the difference in return between portfolios formed on the basis of the ratio of book value of equity to market value of equity. Firms with high book-market ratio have higher average returns than is predicted by the CAPM. This and other empirical anomalies were late organized into Fama's well-known three-factor model.

This article agrees with Fama and French in respect to the invalidity of the CAPM in its applications, but departs from their conjecture about the causes of the CAPM's problems. In the author's view, it is neither "many simplifying assumptions" nor "difficulties in implementing valid tests" that lead to the invalidity of the CAPM in its applications. It is the *relative inefficiency* of the neoclassical fundamental valuation of capital assets that causes the CAPM's problems. Hence, in contrast to the prevailing empirical approaches, according to the author, the better approach to the CAPM's problems is that we establish some alternative fundamental valuation of capital assets which is relatively more efficient. To achieve this objective, we should begin with an alternative basic model of financial markets which is relatively more efficient as compared with the existing neoclassical one.[1]

Let us return to the beginning issue of the shift of conditions. It is known that Sharpe's original theory of market equilibrium was developed under conditions of *probabilized* risk. Due to this conceptual limiting of future uncertainty, Sharpe's basic model of financial markets suffered the consequences of information insufficiency in at least two respects. First, the basic model is taken as containing one simple field–measurable stochastic process such that the mean-variance analysis of market equilibrium is limited to one market factor of systematic risk. Technology of production was chosen the sole market factor of systematic risk while other possible factors of systematic risk were ignored. Once market equilibrium valuation of capital assets becomes the focus of attention, this type of information losses would produce severe analytical biases. In the second respect, market structure information is missing from Sharpe's basic model of financial markets. In the context of neoclassical economics, Sharpe's model of financial markets was implicitly assumed of market completeness or effective completeness. On the one hand, the complete market structure reassures us about the absence of unprobabilized uncertainty, strengthening the model's defining of probabilized risk. On the other hand, with the absence of the general case of market incompleteness, market structure information is omitted from Sharpe's basic model of financial markets. This type of information losses has long prevented us from framing a proper insight of *systemic* properties of market equilibrium or market equilibrium valuation. Up to this point, it becomes clear the intention behind the shift of conditions for our market equilibrium valuation. It remains aiming at a market equilibrium theory of capital asset prices, only it should be derived from a relatively more efficient basic model of financial markets. The validity of the idea rests on, of course, the necessity of enhancing information in the basic model of financial markets.

[1]Hansen and Richard (1987) once suggested that we think of the analysis of asset pricing models as proceeding in two steps. The first is to derive alternative pricing functions from more primitive assumptions on the underlying economic environment; the second is to deduce the restrictions that these alternative pricing functions imply for the population movements of time series data on asset payoffs and prices. If their alternative pricing functions were to understand as alternative fundamental valuations, the present investigation could be regarded as an effort at the first step.

We examine first market factor information in a basic model of financial markets. Empirical evidence has indicated that the CAPM beta does not fully explain the cross section of expected asset returns. This evidence suggests that additional factors may be required to characterize the behavior of expected returns. Theoretical arguments also suggest that more than one factor is needed. As an alternative to the CAPM, Rose (1976) developed the Arbitrage Pricing Theory (APT), which allows for several sources of risk. In an intertemporal context, Merton (1973) delivered a multifactor model with the market portfolio serving as one factor and state variables serving as additional factors. Nonetheless, those technical-level improvements can hardly solve the problem. The real problem here is that one must answer the question of "which ones and how many?" According to Hansen and Richard (1987), the problem falls into the category of the first-step analysis of a capital asset pricing model, which is associated with more primitive assumptions on the underlying economic environment. If it is the case, the solution to the problem, i.e., the answer to the question, should be implied in some market-factor enhanced basic model of financial markets.

Consider carefully the typical analytical state of a basic model of financial markets, i.e., the general market equilibrium of the basic model of financial markets. It would be realized that our understanding of the general market equilibrium had actually changed in fundamental ways. In the early days of economics, the economic environment assumed for asset market equilibrium was Say's Law, technology of production was hence deemed to be the sole engine behind market equilibrium of capital assets. However, the "marginalist revolution" of 1870s, and afterwards, Marshall (1890) and Keynes (1936), suggested to us that the balance of agents' mentality was also the necessary economic condition for market equilibrium. Preference of agents should therefore be assumed as well as an engine behind market equilibrium of capital assets. Besides, the fact that general market equilibrium of an economy requires the balance of the cost of using the markets, i.e., the balance of transaction costs, was discovered by Coase (1937, 1960). Consequently, market microstructure by means of transaction costs should be also regarded as an active determinant of market equilibrium of capital assets. So, with only a little retrospective reflection, one would realize that economic ideas underlying general market equilibrium in a basic model of financial markets have changed substantially in regard to market factors. The state of market equilibrium requires at least three dimensions of market balance, which are: the balance of commodity, the balance of mentality, and the balance of transaction costs. Market factors which underlie those dimensions of balance could be identified as technology of production, preference of agents, and market microstructure. Moreover, in terms of market participants, market equilibrium in a basic model of financial markets could be recognized as a consequence of the interaction between sellers, buyers, and market-makers. Each set of market participants creates its own space in market equilibrium on the basis of their respective market factors.

With market-factor information being enhanced in the basic model of financial markets, we have actually put forward our proposal to the problem of additional market factors. Thereby, in the sequel we explain the second-stage issue of enhancing market-structure information.

Recall the three equivalent criteria put forth by Harrison and Kreps (1979) for a viable market equilibrium valuation of capital assets, or in their own term, for a viable price system. These are: i) the *viability* property;[1] ii) the *extension* property;[2] and iii) no free lunches property.[3] Obviously, the classical CAPM belongs to such a viable market equilibrium valuation of capital assets. It possesses one and thus all the properties of a viable market equilibrium valuation. In terms of the basic model of financial markets, however, satisfying these criteria of viability is not without cost. It restricts our assumption of the market structure of the basic model of financial markets to a special case only. That is, it requires the basic model of financial markets to be complete or effectively complete. As this special case is not the market system in which investors actually trade,[4] it is realized finally that the basic model of financial markets which implies the classical CAPM has been suffering information losses with respect to its assumption of market structure.

Market incompleteness causes model uncertainty or unprobabilized uncertainty of the market system [note that it is distinct from the notion of "parameter uncertainty"]. If we define this type of future uncertainty as "*market ambiguity*", then the presence of market ambiguity indicates that the basic model of financial markets has been enhanced to the general case, which includes market structure information. As a consequence, we can no longer take for granted that there exists a risk-free asset in the basic model of financial markets. Except for the special case of complete markets, there is no logical basis to assume that a risk-free asset (or portfolio) should be implied in the basic model of financial markets. The complication, however, suggests an alternative approach to the long-standing problem of pricing zero-beta fund,[5] for agents in the basic model of financial

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[1] It requires that the equilibrium valuation of capital assets be derived from a viable model of financial markets. A model of financial markets is said to be *viable* if some investor from certain preference class can find an (constrained) optimal investment strategy.

[2] It requires that the equilibrium valuation of capital assets could be extended to the entire space of contingent payoffs, namely, R^S .

[3] It requires that any contingent payoff $x \in R^S$ be priced by arbitrage, i.e., there is a unique cost for x that is consistent with the market equilibrium valuation.

[4] As noted by Keynes (1936) long time ago with a more broad framework.

[5] Without the premise of incomplete markets, the pricing or even the presence of the zero-beta fund has long been a puzzle. In retrospect, given von Neumann-Morgenstern preference, Black (1972) and Merton (1973) have had actually confused the pricing of the zero-beta fund with the pricing of some unknown exogenous pricing factor, so that they maintained that their respective market equilibrium valuations were optimal. Along the lines, Hansen and Richard (1987) studied the effect of conditioning information omission on the mean-variance frontier of equilibrium valuation. Unfortunately, these brilliant insights of information omission did not develop into a theory of the zero-beta fund for the authors' focus was placed on deducing testable restrictions. On the other hand, the relevancy that efficient (optimal) CAPM valuation depends on the existence of a risk-free asset was finally recognized by some GEI (General Equilibrium with Incomplete Asset Markets) scholars. Nielsen (1988, 1990)

markets would have attitude towards the phenomenon of market ambiguity.

It looks indeed there could be an information-enhanced basic model of financial markets. With it as the point of departure, one can develop an alternative market equilibrium theory of capital asset prices, which is relatively more efficient. Nonetheless, it is important to note that, the failure of the CAPM, or more appropriately, the relative *inefficiency* of the CAPM as a model of information pricing does not mean that it is also a failure as a methodological model of market equilibrium valuation. In fact, except for the enhanced information space, the present study takes much advantage of the CAPM as a model of methodology.

As a model of methodology, the CAPM is established on two basic methodological principles. One is the tradeoff relation between expected return and probabilized risk. The other is the two-fund separation between the fund of risky assets and the fund of risk-free assets (or in Black's version, the fund of zero-beta assets). It can be argued that these two principles are actually the necessary conditions for any market equilibrium valuation of capital assets. In this sense, the novel basic model of financial markets achieves only that the two methodological principles be applied more efficiently to a market equilibrium valuation of capital assets. For instance, in respect to the tradeoff relation of the first principle, our understanding of probabilized risk is not necessarily limited to one dimension of market balance. As well, the zero-beta fund in the two-fund separation principle could be properly priced without the conditioning restriction of the special case of complete markets. A distinct tradeoff relation between market ambiguity and ambiguity compensation can therefore be induced into market equilibrium analysis.

Once it is accepted that the CAPM is not simply a model of information pricing but also a model of methodology, it would be realized that the CAPM can be reformulated with a distict information space, or more primitively, under a basic model of financial markets which is reletively more efficient. This motivates the author for the present study. The rest of the article proceeds as follows. Section II is employed to describe our understanding of the basic model of financial markets. In Section III, the first-stage reformulation of the CAPM is carried out with market-factor information enhanced in the basic model of financial markets. In Section IV, the second-stage reformulation of the CAPM is carried on with both market-factor and market-structure information enhanced in the basic model of financial markets. Section V presents a summary review of the development of theoretical insights which underlies our alternative understanding of the basic model of financial markets. Section VI concludes.

II. The Basic Model of Financial Markets

showed that uniqueness of equilibrium in the CAPM depends on the presence of a riskless asset, and without it, the possibility of satiation may lead to non-existence of market equilibrium. Geanakoplos and Shubik (1990) argued the technical difficulty of maintaining a riskless asset in the classical CAPM in a GEI the context.

Denote a single-period basic model of financial markets by $\xi(\succeq, w, M)$, where $\succeq$ represents the class of preferences with which agents in the model make decisions; w represents the initial endowment of wealth measured by the units of account; and M the asset payoff matrix of the basic model of financial markets. If there are totally I agents in the basic model of financial markets, then

$$\succeq = \{\succeq^i\}, \quad i = 1, \cdots, I \text{ ;}$$

and

$$w = \{w_i\}, \quad i = 1, \cdots, I, \text{ with } \sum_{i=1}^{I} w_i \geq 0 .$$

A traded asset $j (= 1, \ldots, J)$ is represented by an income or wealth vector $x_j \in R^S$, denoting the promised payoff each state on date 1. The market structure of the basic model of financial markets is determined by the collection of traded assets implied in the $S \times J$ payoff matrix M.[1]

With the relative relation of the markets of asset J against the states of nature S, we can define *strong* and *weak* conditions of complete markets for the basic model of financial markets. The strong condition, as in Arrow-Debreu's context, requires that the basic model of financial markets has a complete market structure if and only if the payoff matrix M has rank S, or equivalently, if and only if the number of independent markets of asset J is larger or equal to the number of states of nature S. This is the concept of complete markets associated with unconditional Pareto optimality of market equilibrium. Diamond (1967) recognized, however, that unconditional Pareto optimality is not a useful concept for judging the efficacy of a model of financial markets. He adopted the notion of *constrained* Pareto optimality which can be achieved by the model of financial markets which is effectively complete. The model of financial markets is said to be *effectively* complete if every Pareto-optimal transfer of wealth is attainable through the market set of the model. So, the concept of effectively complete markets permits of the cases that the number of asset markets less than the number of states of nature, namely $J < S$, and is hence the weak condition of market completeness.[2] If neither strong nor weak condition is assumed for a basic model of financial markets, then the model is said to be *general* in terms of market structure, with complete markets as its special case. Nevertheless, in order to present our basic model of financial markets in a natural manner, we should explain first the market-factor information of the model, and then the market-structure information. In the rest of this section we follow these two steps accordingly.

[1] In principle, the number of future states of nature S can be either finite or infinite. For simplicity, we take the case of finite states.

[2] In terms of *viable* market equilibrium valuation this weak condition is consistent with the *extension* property by Harrison and Kreps (1979).

As the beginning of the first step, we assume that there is a probability space (Ω, F, P) and defined on it the basic model of financial markets $\xi(\succeq, w, M)$. In light of this probabilistic definition, our basic model of financial markets exhibits as a stochastic market equilibrium system. So it is noted, associated with the class of preferences $\succeq$ in the model it is found the market balance of mentality underlain by the stochastic variable "preference of agents"; associated with the initial endowment of wealth w it is found the market balance of commodity underlain by the stochastic variable "technology of production"; and associated with the asset payoff matrix M in the model it is found the balance of transaction costs determined by the stochastic variable "market microstructure". As the realization of a marketed general equilibrium requires the synchrony of the three dimensions of market balance, these stochastic variables are therefore equally *substantial* to the realization of market equilibrium. It follows that these three stochastic variables are all field-measurable random variables in the basic model of financial markets.

Now let K, L, N be field-measurable Hilbert linear spaces of square integrable random variables, where $k \in K$ and $k(s) \in R$ is the amount of commodity claimed by a portfolio of assets if state s occurs; $l \in L$ and $l(s) \in R$ is the cost needed for consummating a trade of the portfolio if state s occurs; and $n \in N$ with $n(s) \in R$ is the measure of market sentiment associated with the portfolio when state is s. Hence, we have three probabilistically-defined linear spaces as follows:

$$K \equiv K^2(\Omega, F, P), \quad L \equiv L^2(\Omega, F, P), \text{ and } \quad N \equiv N^2(\Omega, F, P).$$

Those are market-factor information spaces of, respectively, technology of production, market microstructure, and preference of agents for the basic model of financial market. Thus, we may call K the information space of technology; L the information space of market microstructure; N the information space of preference, corresponding to technological risk, transactional risk, and psychological risk in the basic model of financial markets, respectively. Technically, as those information spaces are deemed to be orthogonal to each other, the market-factor information as a whole in the basic model of financial markets can therefore be represented as an orthogonal direct sum of the those one-dimension field-measurable linear spaces. It is

$$H \equiv H^2(\Omega, F, P) = K \oplus L \oplus N$$

defined on (Ω, F, P).[1] The orthogonal direct sum of K, L and N means

$$K \cap L = \{\mathbf{0}\}, \quad K \cap N = \{\mathbf{0}\}, \quad L \cap N = \{\mathbf{0}\},$$

so that if $h \in H$ we have

$$H = \{h = k + l + n \mid k \in K, l \in L, n \in N\}.$$

Besides, in order to guarantee the complete market-factor information space is non-negatively

[1] In order to avoid unnecessary vagueness in the argument, we have taken a relatively strong requirement on the structure of the complete information space H. Actually, *orthogonality* is not a necessary condition.

defined, we also assume

$$H \equiv \{h \in H : P(h \geq 0) = 1, \text{ and } \; P(h > 0) > 0\}.$$

On the basis of the market-factor information space H, we can define a market-finance or wealth-transfer space for the basic model of financial markets. It is the set of all possible wealth transfers that can be attained by trading in the basic model of financial markets. Denoted by $\langle W \rangle$, the market-finance space is generated by the $(S+1) \times J$ matrix W with date 0 as state 0 and is a subspace of R^{S+1}. The structure of the market-finance space is therefore the asset payoff matrix allowing for the date 0's costs $q = (q_1, \ldots, q_J)$. Thus, the market-finance space could be formulated as

$$\langle W \rangle = l^2(\Omega, F, P) = \left\langle \begin{matrix} -q \\ M \end{matrix} \right\rangle = \{\tau \in R^{S+1} \,\big|\, \tau = W\lambda, \quad \lambda \in R^J\},$$

where λ is the portfolio of assets purchased by an investor, and τ is the vector of wealth transfers across the states induced by the portfolio. It is important to note that the market-finance space $\langle W \rangle$ is a sufficient condition for the market-factor information space H as well as for its subspaces K, L and N, because it is simply the direct functional representation of the basic model of financial markets $\xi(\succeq, w, M)$. Specifically, a random variable $\tau \in \langle W \rangle$ should be treated as a strictly increasing transformation of $h \in H$, namely: $\tau = \tau(h) : H \to \langle W \rangle$. As such, given the market-factor information space H, the market-finance space $\langle W \rangle$ could be interpreted as the preference space over which investors make their choice decisions. For any $\tau \in \langle W \rangle$ induced by a portfolio λ, an investor would compare it with others in order to maximize the utility of the wealth transfer. Moreover, from the linearity property of the market-factor information space H, it follows that the transformation function $\tau(\cdot)$ is also linearly separable with k, l, n, which means

$$\tau(h) = \tau_k(k) + \tau_l(l) + \tau_n(n) + \varepsilon,$$

where $\tau_k(k)$, $\tau_l(l)$, $\tau_n(n)$ are functions of their respective categories of information, and ε is the noise.

So, as far as market-factor information is concerned, the market-finance space is simply a dimensional extension of the neoclassical preference space. Thus, following Harrison and Kreps (1979), from the extended preference space we could as well derive a viable market equilibrium valuation of capital assets, i.e., a viable price system for the basic model of financial markets. Nevertheless, it ought to be noted that the neoclassical implication of market completeness keeps the premise for the derivation. Otherwise, we can't guarantee the existence of a risk-free transfer of wealth $\overline{\tau}$ in the market-finance space as the link of certainty between the present and the future.

It is therefore the time we turn our attention to the second-step issue of our basic model of financial markets, which is to generalize the model in terms of market-structure information. We begin with the argument that market (effective) completeness is the necessary condition that guarantees the existence of a risk-free transfer of wealth $\bar{\tau} \in \langle W \rangle$ in the basic model of financial markets $\xi(\succeq, w, M)$. For convenience, instead of the market-finance space per se, we examine the simplified form of the market-finance space, i.e., the date-1 subspace of the market-finance space $\langle M \rangle \subset \langle W \rangle$. In other words, we argue the point by means of the asset structure of the asset payoffs span $\langle M \rangle$.

It is known that the return to a certain investment strategy $\bar{r} \in \langle M \rangle$ can be written as $\bar{r} = \alpha \mathbf{1}$ for some $\alpha \in R$, where $\mathbf{1} = (1, \ldots, 1) \in R^S$. Let

$$\langle \mathbf{1} \rangle = \left\{ \bar{r} \in \langle M \rangle \middle| \bar{r} = \alpha \mathbf{1},\, \alpha \in R \right\} = \left\{ \bar{r} \in \langle M \rangle \middle| \mathrm{var}(\bar{r}) = 0 \right\}$$

be the one-dimensional subspace of $\langle M \rangle$ consisting of all non-random returns. Its orthogonal complement $\langle \mathbf{1} \rangle^{\perp}$ consists of all the returns on purely random investment strategies

$$\langle \mathbf{1} \rangle^{\perp} = \left\{ \tilde{r} \in \langle M \rangle \middle| E(\mathbf{1}, \tilde{r}) = 0 \right\} = \left\{ \tilde{r} \in \langle M \rangle \middle| E(\tilde{r}) = 0 \right\}.$$

Thus, the asset payoffs span $\langle M \rangle$ can be expressed as an orthogonal decomposition of the asset span on investment strategies. That is

$$\langle M \rangle = \langle \mathbf{1} \rangle \oplus \langle \mathbf{1} \rangle^{\perp}.$$

It follows that the return on any investment strategy $r \in \langle M \rangle$ is composed of a certain part $\bar{r}$ and a purely random part $\tilde{r}$, i.e.,

$$r = \bar{r} + \tilde{r},$$

where

$$\bar{r} = E(r)\mathbf{1} \in \langle \mathbf{1} \rangle, \quad \tilde{r} = r - E(r)\mathbf{1} \in \langle \mathbf{1} \rangle^{\perp}.$$

The importance of market (effective) completeness is that it renders the non-random subspace $\langle \mathbf{1} \rangle$ a set of sole element. When asset structure of the asset payoffs span $\langle M \rangle$ is complete, $\mathbf{1} = (1, \ldots, 1) \in R^S$ is *completely* determined in the asset payoffs span such that $\langle 1 \rangle \subset \langle M \rangle$ has only one element. When asset structure of the asset payoffs span $\langle M \rangle$ is effectively complete, $\mathbf{1} = (1, \ldots, 1) \in R^S$ is *uniquely* determined in the asset payoffs span such that $\langle 1 \rangle \subset \langle M \rangle$ has only one element. In either case, the marketed risk-free return r_f exhibits as the equilibrium market realization of the sole element of the non-random subspace $\langle \mathbf{1} \rangle$ for the asset payoffs span $\langle M \rangle$. As date-0 is certain to investors, the unique existence of the risk-free return $r_f \in \langle M \rangle$ is equivalent to the unique existence of the risk-free wealth transfer $\bar{\tau} \in \langle W \rangle$ in the basic model of financial markets $\xi(\succeq, w, M)$.

So long as the present and the future could be linked by a *unique* risk-free transfer of wealth $\bar{\tau}$, the basic model of financial markets $\xi(\succeq, w, M)$ would be construed as an instrument of certainty and agents in the model would not perceive any model uncertainty or ambiguity. This happens to be the neoclassical understanding of the basic model of financial markets, anyhow. The state of market structure under the circumstances is not information and therefore is irrelevant to market equilibrium analysis or market equilibrium valuation. Nevertheless, once the premise of market (effective) completeness is relaxed to the general case of market incompleteness, not only the ambiguity caused by market incompleteness but also the attitudes taken by agents in the basic model of financial markets towards this ambiguity become a crucial part of information for both market equilibrium analysis and market equilibrium valuation. A marketed risk-free return $r_f \in \langle M \rangle$ or a marketed risk-free wealth transfer $\overline{\tau} \in \langle W \rangle$ may no longer exist in the basic model of financial markets.

Taking this market-structure information into consideration, we would denote our *generalized* basic model of financial markets by $\xi(\succeq^*, w, M^*)$ as contrasted with its special case $\xi(\succeq, w, M)$. In this general denotation, M^* is the asset payoff matrix allowing for incomplete asset structure; $\succeq^*$ is the class of agents' preferences with attitude toward ambiguity, meaning that in addition to risk aversion agents may as well have ambiguity aversion.

Hitherto we have completed the two-step enhancement of the information space of the basic model of financial markets. As a result, it comes to light a much more general market framework for our understanding of market equilibrium and market equilibrium valuation of capital assets. Equipped with this deepened understanding, we could derive from the enhanced basic model of financial markets respectively two empirically tractable capital asset pricing models, one is for the special case of without ambiguity and ambiguity aversion, and the other is the general case of with ambiguity and ambiguity aversion.

III. Reformulated CAPM without Impacts of Ambiguity

As soon as our basic model of financial markets meets the weak condition of market completeness, it degenerates into its special case which is much the same as the neoclassical one except for enhanced market-factor information. When it is the case, the positive properties of the neoclassical market equilibrium would maintain. As a result, we could see, the two Fundamental Theorems of welfare economics remain valid in the factor-information enhanced market equilibrium analysis, and the three equivalent viability criteria of Harrison and Kreps (1979) remain the benchmark for a viable market equilibrium valuation. Moreover, at least one market equilibrium valuation of capital assets or price system could be found that meets the benchmark. Thus, with $\xi(\succeq, w, M)$ instead of $\xi(\succeq^*, w, M^*)$, we see that, to derive a market-tractable pricing model of capital assets from the only factor-information enhanced basic model is equivalent to reformulating the CAPM without concerning impacts of ambiguity and ambiguity aversion.

As the factor-information enhanced basic model maintains all the positive properties of the neoclassical market equilibrium, it follows from Harrison and Kreps (1979) that at least one viable price system (market equilibrium valuation of capital assets) could be derived from the basic model of financial markets $\xi(\succeq, w, M)$. Denote the viable price system by $(\langle M \rangle, \pi)$ and with it as the point of departure, our job for the rest of this section is to develop an empirically tractable asset pricing model. It is the reformulated CAPM without market ambiguity and agents' ambiguity aversion.

Suppose that there are $J-$number assets traded on the market-finance space $\langle W \rangle$ with date-0 prices $q \in R^J$ and date-1 payoffs matrix M that satisfies $E[|M|^2] < \infty$. We write

$$\langle M \rangle = \{x \in R_+^S : x = M\lambda,\ \forall \lambda \in R^J\} \quad \text{and} \quad \langle M \rangle \subseteq R_+^S$$

the asset payoffs span of all attainable marketed payoffs. In order to preclude prices of investment strategies from being zero, we assume in addition that the set $\langle M \rangle \cap R_+^S \setminus \{0\}$ is nonempty and $x \in \langle M \rangle \cap R_+^S \setminus \{0\}$. With this deletion of the zero, we could conveniently define on the asset payoffs span a linear and strictly positive payoff pricing functional $\phi : \langle M \rangle \to R$ as

$$\phi(x) \equiv \{\lambda q : x \in \langle M \rangle, \forall \lambda \in R^J\}.$$

Given that $\langle M \rangle$ is at least effectively complete, it follows from the equivalent viable criteria of the price system $(\langle M \rangle, \pi)$, we know that $\phi(x) = \pi(x)$. In addition, due to the fact that the market-finance space $\langle W \rangle$ as well as the asset payoffs span $\langle M \rangle$ is technically assumed a Hilbert linear space, from the Riesz-Frechet representation theorem, it follows that it can be found a unique pricing kernel $d_\pi \in \langle M \rangle$ which defines the payoff pricing functional π on the asset payoffs span $\langle M \rangle$ such that

$$\pi(x) = E[d_\pi x], \quad \forall x \in \langle M \rangle. \tag{3.1}$$

Let Θ be the priced subspace of $\langle M \rangle$ spanned by the non-random expectations and the *identified* market pricing factors, and $\Theta^\perp$ the zero-price orthogonal complement. The asset payoffs span $\langle M \rangle$ is hence decomposed as $\langle M \rangle = \Theta + \Theta^\perp$ with $\Theta \cap \Theta^\perp = \{0\}$. For the payoff of any investment strategy $x \in \langle M \rangle$ there exist a unique vector $x^\Theta \in \Theta$ and $\varepsilon \in \Theta^\perp$ such that

$$x = x^\Theta + \varepsilon. \tag{3.2}$$

Although nothing looks new in regard to its form, we must note that the information space underlying the form is no longer that of the neoclassical basic model of financial markets. It has been shifted to the market-factor enhanced information space of $H = K \oplus L \oplus N$, on which we defined the market-finance space $\langle W \rangle$. As the asset payoffs span $\langle M \rangle$ is the reduced form of the market-finance space, it follows that the priced subspace $\Theta \subset \langle M \rangle$ can be spanned by an orthogonal information system $\{\mathbf{1}, f_K, f_L, f_N\}$. The information of non-random expectations in the system is apparent. The information of random market factors, represented in the system by normalized zero-expectation factors f_K, f_L, f_N, comes from each subspace of H, i.e., from K, L, and N. Thereby, from (3.2) it is known that the payoff x can be expressed as

$$x = x^{\Theta} + \varepsilon = E[x]\cdot\mathbf{1} + b_{xK} f_K + b_{xL} f_L + b_{xN} f_N + \varepsilon_x , \qquad (3.3)$$

where $E[x] = E[\mathbf{1}\cdot x]\big/E[\mathbf{1}^2]$, $b_{xK} = E[f_K x]\big/E[f_K{}^2]$, $b_{xL} = E[f_L x]\big/E[f_L{}^2]$, and $b_{xN} = E[f_N x]\big/E[f_N{}^2]$ are factor-loadings of the contingent payoff x on respective factors.

Now let us apply the unique payoff pricing functional (3.1) to (3.3). It yields

$$\begin{aligned}\pi(x) &= E[x]E[d_\pi] + b_{xK}E[d_\pi f_K] + b_{xL}E[d_\pi f_L] + b_{xN}E[d_\pi f_N] + E[d_\pi \varepsilon_x] \\ &= E[x]\tau + b_{xK}\tau_K + b_{xL}\tau_L + b_{xN}\tau_N , \qquad \forall x \in \langle M \rangle \qquad (3.4)\end{aligned}$$

where $E[d_\pi] = \pi(\mathbf{1}) = \tau$, $E[d_\pi \varepsilon] = 0$, and $E[d_\pi f_\bullet] = \pi(f_\bullet) = \tau_\bullet$ for $\bullet = K, L, N$.

Due to the fact that asset prices are nonzero such that we can write the payoff x of an investment strategy j (either a portfolio or an asset) in terms of its return r_j, that is

$$r_j = E[r_j] + \beta_{jK} f_K + \beta_{jL} f_L + \beta_{jN} f_N + \xi_j , \qquad (3.5)$$

where $r_j = x/\pi(x)$, $\beta_{j\bullet} = b_{x\bullet}/\pi(x)$ for $\bullet = K, L, N$ and $\xi_j = \varepsilon_x/\pi(x)$.

Along the lines, by applying the payoff pricing functional π to both sides of (3.5) and rearranging the terms, we obtain the exact pricing relation

$$E[r_j] = \lambda_0 + \beta_{jK}\lambda_K + \beta_{jL}\lambda_L + \beta_{jN}\lambda_N \qquad (3.6)$$

with $\lambda_0 = 1/E[d_\pi]$ and $\lambda_\bullet = -E[d_\pi f_\bullet]/E[d_\pi]$ for $\bullet = K, L, N$.

In order to be applicable to empirical data, this beta-representation of the pricing functional relation of the viable price system $(\langle M \rangle, \pi)$ needs to be specified one-step further. From the financial markets in practice we can draw for each normalized zero-expectation risky factor a mimicking portfolio, which is completely correlated with the market factor under concern. We may call these mimicking portfolios factor-equivalent or factor-reference portfolios.[1] Let $R^* = (R_K^*, R_L^*, R_N^*)$, where R_K^* denotes the factor-mimicking return that is completely correlated with the stochastic variable of technology of production, f_K; R_L^* the factor-mimicking return that is completely correlated with the stochastic variable of market microstructure, f_L; and R_N^* the factor-mimicking return that is completely correlated with the stochastic variable of preference of agents, f_N. The property of complete correlation implies that the covariance matrix of the system $\{\mathbf{1}, R_K^*, R_L^*, R_N^*\}$ is exactly the same as that of the information system $\{\mathbf{1}, f_K, f_L, f_N\}$. Therefore, according to Huberman and Kandel (1987), when the minimum-variance frontier of $R^* = (R_K^*, R_L^*, R_N^*)$ intersects the minimum-variance frontier of traded assets $r = (r_1, \ldots, r_J)$, the exact linear relation (3.6) holds and its parameters $\lambda_0, \lambda_K, \lambda_L, \lambda_N$ stand for respectively the zero-beta return, premium on bearing technological risk, premium on bearing transactional risk, and premium on bearing psychological risk.

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[1] Refer to also either Connor (1984) or Ingersoll (1984) or Lu (2010).

On the basis of the above argument it can be shown that the exact factor pricing relation (3.6) can be further specified as

$$E(r_j) = \lambda_0 + \beta_{jK}[E(R_K^*) - \lambda_0] + \beta_{jL}[E(R_L^*) - \lambda_0] + \beta_{jN}[E(R_N^*) - \lambda_0]. \quad (3.7)$$

Moreover, as the risk-free return r_f is a marketed return and is uniquely implied in the asset payoffs span, so that we have $\lambda_0 = r_f$. The market-tractable capital asset pricing model without market ambiguity and agents' ambiguity aversion is therefore

$$E(r_j) - r_f = \beta_{jK}[E(R_K^*) - r_f] + \beta_{jL}[E(R_L^*) - r_f] + \beta_{jN}[E(R_N^*) - r_f]. \quad (3.8)$$

It is worth mentioning that, in contrast to Fama-French's Three-factor Model (1993), which is a factor-enhanced market model of asset returns, this *Three-Beta Capital Asset Pricing Model* (Three-Beta CAPM) is a multi-factor theory of market equilibrium valuation of capital assets, derived from the basic model of financial markets, or more exactly, from the special case of the basic model, $\xi(\succeq, w, M)$. According to the theory, the classical Single-Beta CAPM is a degeneracy of the Three-Beta CAPM since it is the case when the information space of market factors, H, collapses into the information space of commodity only, K.

IV. Reformulated CAPM with Impacts of Ambiguity

When satisfying the weak condition of market completeness, the basic model of financial markets, $\xi(\succeq, w, M)$, as well as its market-finance space, $\langle W \rangle$, is reckoned to be certain in sense of the absence of ambiguity or model uncertainty. In terms of market equilibrium valuation of capital assets, this absence of ambiguity means not only the existence of a *viable* price system $(\langle M \rangle, \pi)$ but also that the no-arbitrage equilibrium equation

$$q = \pi_1 M \quad (4.1)$$

holds with a unique vector of state prices $\pi_1 \in R^S$. However, when the assumption of satisfying the weak condition of market completeness is dropped, we would face the general case of the basic model of financial markets, $\xi(\succeq^*, w, M^*)$, from which ambiguity or model uncertainty endogenously arises. Consequently, we may no longer be able to find out a viable price system $(\langle M \rangle, \pi)$ from the market-finance space $\langle W \rangle$ for the vector of state prices $\pi_1 \in R^S$ is no longer uniquely determined in the market-finance space $\langle W \rangle$. There is a subset of many vectors of state prices, and each potentially satisfies the no-arbitrage equilibrium equation (4.1).

In the general case of the basic model of financial markets $\xi(\succeq^*, w, M^*)$, the market-finance space $\langle W \rangle$ is incomplete such that $\dim\langle W \rangle = \dim\langle M \rangle = J < S$. In consequence, we have many instead of a unique vector of state prices associated with a no-arbitrage equilibrium transfer of wealth $\tau^* \in \langle W \rangle$. For if (τ^*, π^*) is a no-arbitrage equilibrium with $\pi^* = (1, \pi_1^*)$, then (τ^*, π) is also a no-arbitrage equilibrium for any $\pi = (1, \pi_1)$ with π_1 lying in the subset

$$D = \{\pi_1 \in R_{++}^S \,|\, \pi_1 M = \pi_1^* M\}. \quad (4.2)$$

If $rank\, M = J$, then $\dim D = S - J$, which means, for the market-finance space $\langle W \rangle$, the dimension of indeterminacy in state prices is $S - J$.

The typical property of the indeterminacy in state prices is that the market-finance space $\langle W \rangle$ implies at least $S - J$ viable price systems but none of which could be realized as a marketed one. Market ambiguity caused by the indeterminacy in state prices prevents any potential viable price system from becoming a real one in the basic model of financial markets $\xi(\succeq^*, w, M^*)$. As each potential viable price system implied in the basic model is associated with a risk-free transfer of wealth $\bar{\tau} \in \langle W \rangle$, or equivalently, with a risk-free return $\bar{r}$, the indeterminacy in viable price systems suggests a collection of potential risk-free returns. Of course, none of those risk-free returns could become actually-marketed return in the basic model of financial markets $\xi(\succeq^*, w, M^*)$. Therefore, in contrast to the viable price system $(\langle M \rangle, \pi)$ derived from the special case of the basic model financial markets, $\xi(\succeq, w, M)$; the market equilibrium valuation of capital assets $(\langle M^* \rangle, \pi^*)$ derived from the general case of the basic model of financial markets, $\xi(\succeq^*, w, M^*)$, could not be a *viable* price system with respect to Harrison-Kreps's criteria as it does not imply a marketed risk-free return.

Then, how are the positive properties of the market valuation system $(\langle M^* \rangle, \pi^*)$? Given the indeterminacy of the market-finance space $\langle W \rangle$ or the asset payoffs span $\langle M^* \rangle$ caused by market incompleteness, we could think about the matter from the perspective of a representative agent in the basic model of financial markets $\xi(\succeq^*, w, M^*)$.

First of all, the representative agent would perceive market ambiguity or model uncertainty besides traditionally probabilized market-factor risks. Let it be a D-dimension basic model of financial markets $\xi(\succeq^*, w, M^*)$. So, as defined by (4.2), the dimension of indeterminacy in state prices for its market-finance space $\langle W \rangle$ is $S - J$. Now, it is important to note that, in logic, each dimension of indeterminacy in state prices π_1 corresponds to a unique stochastic discount factor $d_\pi \in \langle M \rangle$ which defines a viable price system $(\langle M \rangle, \pi)$, and that each dimension of the market-finance space $\langle W \rangle$ in state prices π_1 is defined on a probability space (Ω, F, P). Along the line of logic, when the basic model $\xi(\succeq^*, w, M^*)$ is known with D-dimension of indeterminacy, it is understood that the agent in the model believes that the market-finance space $\langle W \rangle$ were defined on a probability space $(\Omega, F, \wp)$ with some *multiple probability priors*, where $\wp = \{P\}$ is the set of probability priors whose size is determined by the D-dimension of indeterminacy. As such, only in the special case when $\dim D = S - J = 0$ that we could have the probability space on which we define the basic model of financial markets with a *unique probability prior*. Under any other circumstance when $\dim D = S - J > 0$, the basic model of financial markets is defined on the probability space $(\Omega, F, \wp)$ with *multiple probability priors*.

Secondly, the representative agent in the model would have attitude towards market ambiguity besides her conventional attitude towards probabilized market-factor risks. Actually, it is in this sense that we have the class of preferences $\succeq$ redefined as $\succeq^*$ in the general basic model of financial markets. In retrospect, ever since the experimental relevance of the distinction between risk and ambiguity was formally discussed by Ellsberg (1961), a range of new preference classes that can accommodate for ambiguity have been elaborated.[1] Among those many preference classes, the *multiple prior preferences* (MPP) formulated by Gilboa and Schmeidler (1989) is typical of the features of the present concern, i.e., the features of the class of preferences $\succeq^*$. Let us apply it to our ambiguous market-finance space $\langle W \rangle$ such that the representative agent's preference $\succeq^*$ can be defined as:

For $\tau, \tau' \in \langle W \rangle$, $\tau \succeq^* \tau'$ if and only if

$$U(\tau_0) + \min_{P\in\wp} E_P[U(\tau_1)] \geq U(\tau_0') + \min_{P\in\wp} E_P[U(\tau_1')], \qquad (4.3)$$

or equivalently, in terms of returns (payoffs) in the asset span $\langle M^* \rangle$, it is written as

$$\text{for } r, r' \in \langle M \rangle,\ r \succeq^* r' \text{ if and only if } \min_{P\in\wp} E_P[U(r)] \geq \min_{P\in\wp} E_P[U(r')], \qquad (4.4)$$

where the probability measure P is chosen from $\wp$, a convex set whose size can be interpreted as representing the level of perceived ambiguity. The agent's attitude towards market ambiguity is displayed in the MPP by the use of the *min* operator, indicating that the agent considers the most unfavorable probability distribution to protect her against possible misfortunes caused by model uncertainty. This is the agent's aversion behavior towards market ambiguity.

Thirdly, with the class of preferences $\succeq^*$ formulated in terms of the MPP, it is seen that the market equilibrium valuation $(\langle M^* \rangle, \pi^*)$ comes from two sets of the agent's decisions. On the one hand, the tradeoff decisions between risk and expected return remain in the part of $E_P[U(\cdot)]$, the standard expected-utility operator to be maximized with a chosen probability measure P. On the other hand, agents' ambiguity decisions are represented by the part of $\min_{p\in\wp}$ which is the decision rule novel from that of risk decisions. The new decision rule suggests some different tradeoff relation between the present and the future, which is, the tradeoff between ambiguity bearing and ambiguity aversion. This reminds us of Tobin's (1958) *Two Fund Separation Theorem*, of which, the true implication might lie in the separation of decisions between the fund of risk and the fund of ambiguity.

Last but not least, the ambiguity tradeoff relation offers no condition of determinacy for the

[1] Two excellent reviewing works on the development of the literature, which models agents whose choices are consistent with settings of ambiguity and which explores its relevant implications for financial market outcomes, are contributed respectively by Epstein and Schneider (2010) and by Guidolin and Rinaldi (2013).

agent in the model with respect to both market equilibrium and market equilibrium valuation even though it remains compatible with the standard mean-variance paradigm. In actuality, however, so long as there is the no-arbitrage condition, market equilibrium as well as risk and ambiguity premium would be determined in the basic model of financial markets. So, in the general case of the basic model of financial markets, it is no-arbitrage rather than market equilibrium that provides the sense of determinacy for agents in the model. Consequently, the marketed price system of capital assets $(\langle M^* \rangle, \pi^*)$ is no longer a *viable* market equilibrium valuation.

Nevertheless, with the price system $(\langle M^* \rangle, \pi^*)$ as the point of departure, we can develop as well a market-tractable pricing model of capital assets. The key to the job is how we have the effects of market ambiguity incorporated into the restrictions implied by the methodology of the CAPM.

Consider a basic model of financial markets $\xi(\succeq^*, w, M^*)$, for which, the decision problem can be thought of as a two-player zero-sum game. One of the players is the representative agent who behaves as a MPP investor; the other is the malevolent Nature whose set of choices is all the possible market structures of the market-finance space for the basic model of financial markets. Denoted by Δ the set of all possible market structures, the subset D defined by (4.2) with $\dim D = S - J$ is hence an element of Δ, i.e., $D \in \Delta$. Once D is chosen by the malevolent Nature for the basic model of financial markets, the representative agent would accept it and respond to it on the basis of MPP, i.e., she follows min-max decision rules. Of course, the agent perceives different levels of ambiguity (model uncertainty) with different D-dimension basic models of financial markets and the same level of ambiguity with the same D-dimension model. Besides, as just referred to, the size of the set of the agent's probability priors $\wp$ is determined by the D-dimension of indeterminacy, chosen by the malevolent Nature.

Unless in the special case where the D-dimension model $\xi(\succeq^*, w, M^*)$ degenerates into $\xi(\succeq, w, M)$ with $\dim D = S - J = 0$ such that the set of probability priors $\wp$ has only one element, the above-section-arrived Three-Beta CAPM

$$E(r_j) - r_f = \beta_{jK}[E(R_K^*) - r_f] + \beta_{jL}[E(R_L^*) - r_f] + \beta_{jN}[E(R_N^*) - r_f]$$

would not be valid in respect to its capture of market-structure information. As the viable market equilibrium valuation $(\langle M \rangle, \pi)$ is a *marketed* price system in the special case of the basic model and due to the fact that the Three-Beta CAPM is its empirically-testable specification, the risk-free return r_f in the Three-Beta CAPM is therefore the *marketed* rate of interest or the *marketed* opportunity cost of money (cash) holding. However, in the general case of the basic model $\xi(\succeq^*, w, M^*)$ with $\dim D = S - J > 0$ such that the probability set $\wp$ has multiple priors, the *marketed* price system $(\langle M^* \rangle, \pi^*)$ is typically not viable as a result of the existence of market ambiguity and agents' ambiguity aversion. Consequently, the *viable* Three-Beta CAPM can not hold with the price system $(\langle M^*\rangle, \pi^*)$, not because of the *marketed* exact factor pricing but because of the *marketed* rate of interest.

As discussed in section II, the basic model of financial markets loses its certainty link between the present and the future if its implication of market completeness is dropped. Ambiguity-averse agents in the model recognize the situation such that as their behavior towards ambiguity is displayed liquidity preference or *marketed* rate of interest. Given this context, no *marketed* rate of interest could be ambiguity-free in the basic model of financial markets even though a set of risk-free returns is implied in the model in dimension of state prices.

Now imagine that a non-random vector $\mathbf{1} \in \langle M \rangle$ and a stochastic discount factor $d_\pi \in \langle M \rangle$ are marketed in the basic model of financial markets $\xi(\succeq^*, w, M^*)$. In the special case as the basic model degenerates into $\xi(\succeq, w, M)$, the *marketed* discount factor d_π defined on the probability space (Ω, F, P) with single probability measure P. The no-arbitrage equilibrium pricing of the *marketed* non-random vector is hence

$$\pi(\mathbf{1}) = E[d_\pi \cdot \mathbf{1}];$$

and the *marketed* rate of interest

$$r_M = 1/\pi(\mathbf{1}) = 1/E[d_\pi \cdot \mathbf{1}] = r_f .$$

By contrast, in the general case of the basic model, the *marketed* discount factor d_π is defined on the probability space $(\Omega, F, \wp)$ with multiple probability priors, $\wp = \{P\}$. Based on the MPP, the no-arbitrage equilibrium pricing of the *marketed* non-random vector becomes

$$\pi^*(\mathbf{1}) = \min_{P\in\wp} E_P[d_\pi \cdot \mathbf{1}].$$

Accordingly, the *marketed* rate of interest for the general case is

$$r_M^* = 1/\pi^*(\mathbf{1}) = 1/\min_{P\in\wp} E_P[d_\pi \cdot \mathbf{1}] = r^*,$$

where the *marketed* rate of interest r^* is not a risk-free return, or more accurately, not an ambiguity-free return.

With r^* in lieu of r_f being treated as the *marketed* opportunity cost of money (cash) holding, the *generalized* Three-Beta CAPM would emerge from the general case of the basic model of financial markets $\xi(\succeq^*, w, M^*)$. According to the Two-Fund Separation Theorem and substituting r^* for r_f, the reformulated CAPM *without impacts of ambiguity,* i.e., equation (3.8), is generalized to the reformulated CAPM *with impacts of ambiguity*, that is:

$$E(r_j) - r^* = \beta_{jk}[E(R_K^*) - r^*] + \beta_{jL}[E(R_L^*) - r^*] + \beta_{jN}[E(R_N^* - r^*)]. \qquad (4.5)$$

It is worth noting that equation (3.8) and equation (4.5) are respectively *market restrictions* of the price system $(\langle M\rangle,\pi)$ and the price system $(\langle M^*\rangle,\pi^*)$. The difference is that $(\langle M\rangle,\pi)$ is viable while $(\langle M^*\rangle,\pi^*)$ is not. Thus, whereas compared to each other, the difference between r^* and r_f in their market restrictions, denoted by $\alpha^A(=r^*-r_f)$, would provide us with several important implications. First, it suggests that zero-beta uncertainty and hedging behavior toward it, first noticed by Black (1972), should be coped with by means of a certain *systemic* approach rather than the usual *systematic* approach. That is, ambiguity-averse agents should be compensated for bearing market ambiguity or model uncertainty in their trading decisions. Subtract equation (3.8) from (4.5) and make some arrangement we obtain that

$$r^*-r_f=(\beta_{jK}+\beta_{jL}+\beta_{jN})[r^*-r_f],$$

and hence

$$\beta_{jK}+\beta_{jL}+\beta_{jN}=1.$$

This means that ambiguity premium $\alpha^A=r^*-r_f$ is irrelevant to the market portfolio of risky assets, and is determined exclusively by agents' attitude towards ambiguity and the level of market ambiguity. It also confirms the argument by Tobin (1958), that liquidity preference as behavior towards risk. Certainly, with our enhanced understanding of the basic model of financial markets, it should be interpreted as behavior towards ambiguity or systemic risk (i.e., model uncertainty). In addition, it may also imply that the old-days monetary approach is better than the relatively recent state-variable approach to the intertemporal investment decision problem. For instance, Keynes's liquidity preference theory could have been more convincing than Merton's (1973) insight of changing set of investment opportunities. In principle, ambiguity in the future arises from within the basic model of financial markets and hence is the problem of the systemic malfunction of the system of financial markets.

V. A Summary Review of Insights to the Reframing of the BMFM

The term of 'the basic model of financial markets (BMFM)' appeared first in Black's (1986) paper on the possible implications of the word 'noise' in financial markets. Black said, "In my basic model of financial markets, noise is contrasted with information. People sometimes trade on information in the usual way. They are correct in expecting to make profits from these trades. On the other hand, people sometimes trade on noise as if it were information. If they expect to make profits from noise trading, they are 'incorrect'."[1] Black's statement of the BMFM consists of two lines of division. One is the line between *information* and *noise*; the other is the line between *correct* trading and *incorrect* trading. As the second line derives from the first, an implication we may deduce from the statement is that different drawings of line between information and noise would invoke different acceptances of *correct* trading, and hence, different definitions of *correct*

[1] *The Journal of Finance, vol. XLI, No*. 3, p. 529.

market equilibrium valuation or fundamental value of capital assets. As a matter of fact, after the establishment of the neoclassical finance, marked typically by the birth of the CAPM, besides the emergence of theoretical puzzles and empirical anomalies since the late 1970s, there have been also streams of theoretical insights which are oriented to the necessity of redrawing the line between information and noise. So, it is worth a summary review of those streams of insights.

The neoclassical view on the BMFM is basically developed from the Arrow-Debreu general equilibrium model of an exchange economy. The typical features of this neoclassical view can be characterized as:

i) the BMFM is a closed system of stochastic process of technology of production;[1]

ii) the BMFM meets at least the *weak* condition of market completeness;[2]

iii) the BMFM is a closed system of the class of risk-averse investors.[3]

These typical features determine the neoclassical conception of market factor and market structure of the BMFM, i.e., the information space of the BMFM. Confined to thus determined information space, it is easy to see, the empirical failure of the neoclassical market equilibrium valuation, typically of the CAPM, is not the failure of the neoclassical BMFM as a methodology of market equilibrium valuation but the failure of the neoclassical BMFM as an information space for market equilibrium valuation of capital assets. So it is clear, amendments to those typical features are logical requirements for any real progress in one's understanding of market equilibrium and market equilibrium valuation of capital assets. Fortunately, in retrospect, we could see actually a historical revelation of the amending process to those typical features, which would have ended in an alternative view on the BMFM. So, along the line of the revelation of the amending process, we carry on our summary review of the streams of theoretical insights. We start with the first typical feature which is related to our understanding of market factors.

It is a neoclassical convention that a pure exchange economy with technological uncertainty is taken directly as the analytical setting of the BMFM. The market equilibrium of the BMFM is hence one dimension of balance, i.e., the balance of commodity. The technology of production, which decides the physical state of the exchange economy, is the only dynamic market factor that underlies the market equilibrium of the BMFM. As there are no other dimension of balance or underlying reasons behind market equilibrium, the pure exchange economy with technological

[1] See for example, Lucas (1978) reckoned that equilibrium asset prices as a function of the physical state of an exchange economy so that they will fluctuate as "Productivity in each unit fluctuates stochastically through time".

[2] For it is believed that the basic model of financial markets implies at least one price system that satisfies the three equivalent conditions of viability, in particular, satisfies the extension property of a viable price system which is assumed for the basic model whether it is a finite or an infinite system. See Harrison and Kreps (1979) and Kreps (1981).

[3] Arrow (1964) qualified his earlier statement (1953) that "the competitive allocation of risk-bearing is guaranteed to be viable only if the individuals have attitudes of risk-aversion" as sufficient rather necessary condition for the existence of a competitive equilibrium.

uncertainty is thus not only necessary but also sufficient condition for the neoclassical BMFM with respect to the information of market factors. In consequence, market equilibrium valuation of capital assets, i.e., equilibrium capital asset prices in the exchange economy, is determined by stochastic outcomes of the technology of production, the sole dynamic market factor of the exchange economy. The problem is, however, the physical process of the exchange economy can hardly cover the diversified array of unrelated causal elements in the real-world financial markets. Hence, the discrepancy between theory and markets has come to light.

In attempts to account for the discrepancy, two strands of literature have emerged for the past few decades. These are the behavioral strand of literature and the frictional strand of literature. Though complementary by nature, the behavioral strand of literature has appeared more ambitious and appealing than the frictional one has. Nonetheless, both strands adhere to the neoclassical conception of fundamental value, meaning that both believe the neoclassical line of division between *information* and *noise* and the line between *correct* market equilibrium valuation and *incorrect* market equilibrium valuation.

Behavioral literature holds that the discrepancy can be plausibly understood using models in which some agents are not fully rational. The market process of asset prices deviates from fundamental values due to the fact that less rational traders trade on noise rather than on information in financial markets. The real-world market process of asset prices is hence inefficient since it consists of a *correct* market valuation and an *incorrect* market valuation of capital assets. Moreover, the market mispricing cannot be gotten rid of, not only because of there are agents who are not fully rational but also because of that rational agents can hardly undo the effects caused by less rational traders. So, as noted by Barberis and Thaler (2003), the field of behavioral finance has two building blocks: psychology and limits to arbitrage.[1] As a consequence, financial markets are inevitably inefficient [Shleifer (2000)] and noise traders could create their own space in the financial markets [De Long, Shleifer, Summers and Waldmann (1990)].

While the behavioral literature attributes the mispricing of market equilibrium valuation mainly to psychology-based noise, a less influential strand of literature which prefers to argue the problem on the basis of market frictions appeared early in the beginning of 1980s [e.g., Garman and Ohlson (1981), Prisman (1986)]. Though it seems as if only a complementary argument to the behavioral thesis of ‘limits to arbitrage’, market frictions are nonetheless caused by market transaction costs and enter into market equilibrium as certain market phenomena, accounting for deviations from the fundamental value. In contrast to the behavioral approach, the frictional approach maintains

[1] According to Barberis and Thaler (2003), “psychology” catalogues the kinds of deviations from full rationality we might expect to see; and “limits to arbitrage” argues that it can be difficult for rational traders to undo the dislocations caused by noise traders.

that market deviations from the fundamental value are result of agents' rational decisions. They are not construed as mispricing but as the discrepancy between market practice and market fundamental valuation [e.g., He and Modest (1995), Luttmer (1996)], and thus can be dealt with by technical means [Jouini and Kallal (1995)]. Meanwhile, the relevancy of market frictions to market liquidity premium has also been noted by the frictional approach since Kyle (1985) and Constantinides (1986). Nevertheless, due to that transaction costs in the frictional literature are usually assumed proportional to trade, and market frictions caused by proportional transaction costs are basically stationary so that they can produce only the second-order effects on market equilibrium and market equilibrium prices of capital assets.

The literature of the discrepancy between the neoclassical theory and empirical markets did deepen our understanding of financial markets. For example, it has led us to think certain market-factor information might have been omitted from the neoclassical market equilibrium analysis. Nevertheless, several logical weaknesses have remained in the literature. First, all of the theses on deviations from the fundamental value are founded on some second-order concepts of the neoclassical BMFM. The behavioral literature bases its arguments on the notion of 'noise' and the frictional literature on the notion of 'frictions'. Consequently, both strands of the literature are at best the empirical systematic-qualification to the neoclassical fundamental valuation. Second, without allowing for the balance of mentality in market equilibrium, the way the behavioral literature brings 'psychology' to the neoclassical BMFM is somewhat artificial. It has to assume two distinct classes of investors: one is the class of rational investors who behave as normal neoclassical agents and the other is the class of psychological investors who trade on noise only. Psychology is therefore an *ad hoc* annex to the neoclassical BMFM. Third, without allowing for the balance of transaction costs in market equilibrium, the frictional literature is limited to the level of empirical adaptations to the neoclassical fundamental valuation. It can hardly develop into a theory of market causal elements that substantially accounts for a part of market equilibrium and market equilibrium valuation. Last, but methodologically the fatal flaw, is that both strands of the literature adhere to the neoclassical fundamental valuation, which means that both accept that the pure exchange economy with technological uncertainty be a sufficient condition for the BMFM and therefore a sufficient information space for market equilibrium valuation of capital assets.

These weaknesses suggest that inductive method may not be the proper approach toward the problem of the empirical gap between market data and neoclassical fundamental valuation of capital assets. Some alternative method is therefore needed to tackle the problem. The deductive amendment to the first typical feature of the neoclassical BMFM is evidently the choice. As soon as market equilibrium is recognized as consisting of muli-dimension balances, market-factor information would be enhanced to include more systematic causal elements. In terms of the balance of commodity in market equilibrium, the BMFM is now appreciated in regard to its physical state as ‘a pure exchange economy of technology’. Similarly, in terms of the balance of mentality in market equilibrium, the BMFM is appreciated in regard to its subjective state as ‘a pure exchange economy of preference’. And, in terms of the balance of transaction costs, the BMFM is appreciated in regard to its operation of market microstructure as ‘a pure exchange economy of transaction costs’. Thus, in contrast to that of the inductive method, the deductive approach holds that each dimension of the BMFM is necessary condition for the BMFM and the BMFM is sufficient condition for each dimension of the BMFM. Only when a pure exchange economy is thought of with all these necessary dimensions, could it be interpreted as a sufficient condition for the BMFM, and vice versa. The BMFM per se is now construed as a substantial equilibrium system of markets, the three dimensions of technology, preference, and market transaction costs are hence the underlying market forces for market equilibrium. In addition, due to the fact that market equilibrium of the BMFM requires the simultaneity of the three dimensions of balance, the dynamic stochastic process of the BMFM is thus determined by three uncorrelated stochastic processes. So, the deductive method to the problem of empirical gap is based on an extended information space implying three rather than one market factors. Moreover, each market factor underlies a F -measurable *stochastic variable* in the BMFM.

With this result, our discussion on the insights that would lead to the amendment to the first typical of the neoclassical BMFM comes to an end. In the sequel, we turn attention to the insights which would enhance our understanding in respect to the market structure of a BMFM. It relates to the amendments to the second and third typical features of the classical view on the BMFM.

Doubts about the validity of the postulate of market completeness were first raised with the study of external effects in a competitive market economy. Meade (1952) noted that externalities can be seen as being inherently tied to the absence of certain competitive markets. This argument of missing markets was later substantially extended to the model of general equilibrium. Arrow (1969, 1985) pointed out that the validity of the two Fundamental Theorems of welfare economics rests on: a) the existence of all relevant markets (including those of externalities), namely, the hypothesis of completeness or universality of markets, and b) the convexity of household indifference maps and firm production possibility sets. The first is the necessary condition while the second the sufficient for these Theorems. In other words, so long as the assumption of convexity holds, there exists a competitive equilibrium even if markets are incomplete. This means the generic existence of a general equilibrium economy with incomplete market structure.[1]

[1] As this insight can be borrowed to apply to our BMFM with dimensions, it is seen that we could have accepted actually a less restrictive BMFM. Each dimension of an exchange economy with incomplete market structure

A class of more recent and more direct theoretical advances associated with the amendment to the second typical feature comes from the relaxing of the condition for the general market equilibrium of an economy, i.e., the general equilibrium with incomplete asset markets. The study has developed into the theory of general equilibrium with incomplete asset markets (GEI).[1] As compared with the Arrow-Debreu model, the major finds from the theory of GEI are in effect three: a) it has been proved that there exist generically general equilibria for a GEI model; b) there is an indeterminacy in the general equilibrium due to the incompleteness of asset markets (the equilibrium manifold); c) equilibrium allocations in a GEI model are generically constrained inefficient or sub-optimal. Being the crux of the theory of GEI, the BMFM with incomplete market structure would possess, of course, all the three properties. However, as far as the present study is concerned, the property of equilibrium indeterminacy or equilibrium manifold is of particular interest. It is because that the property indicates the connection between the issue of market structure and the issue of model uncertainty in the study of general market equilibrium. As a result, the concept of *systemic* uncertainty comes to our purview, either with respect to the model of an entire economy or with respect to the sub-model of the economy such as the BMFM.[2]

By recognizing the market structure effects on market (general) equilibria, we have amended the second typical feature of the neoclassical BMFM. It remains only one step further to complete the amending process to the classical view on the BMFM. We need to revise the remaining typical feature which asserts that the BMFM is a system of the class of risk-averse investors.

Because of incompleteness of the market structure, now it is seen, beside market-factor uncertainty (or systematic risks) there is also market ambiguity (or systemic risk) arising from within the BMFM. So, besides risk-aversion, investors should also have attitudes toward market ambiguity.

In terms of economic insights, the literature of ambiguity aversion has actually some very early predecessors. Knight (1921) and Keynes (1921) were evidently among those who are aware of the difference between the concepts of probability-measurable risk and probability-unmeasurable uncertainty although the rationale behind the distinction may not have been clearly specified. Due to the flimsiness of the basis of knowledge about the future, Keynes (1937) once explained, that

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remains the necessary condition for the BMFM. And conversely, the BMFM with incomplete market structure maintains the sufficient condition for each dimension of the exchange economy. Along the line of thought, it becomes evident that different forms of market failure in general market equilibrium, not necessarily limited to the case of market externalities, are inherently tied to the absence of certain competitive markets, one way or another.

[1] The study of GEI was originated by Diamond (1967) and Radner (1972) who revised Arrow-Debreu’s general equilibrium model by introducing explicitly a financial (asset) market with incomplete market structure. The study brought forth a few important theoretical results which are distinct to those of the Arrow-Debreu model. For a general introduction of the positive analyses of the model or the related literature and recent developments, please refer to Geanakoplos (1990), Mas-Colell (1991), Magill and Quinzii (2008), and Balasko (2009).

[2] It should be emphasized here that model uncertainty (or model ambiguity) caused by the market missing is completely different from parameter uncertainty which is caused by model misspecifications.

the economic models with which agents expect the future could hardly be determinate. In order to deal with this sort of future indeterminacy, agents in Keynes's *General Theory* (1936) are framed with liquidity preference. This concept was developed to a higher level as Tobin (1958) defined agents' liquidity preference as behavior towards 'risk'[1] and argued the two-fund separation theorem in portfolio decisions.[2] Nevertheless, all those early insights had not been directly related to the later development of the literature of ambiguity and ambiguity aversion.

Similar to the early insights, the literature of ambiguity aversion came to light with the separation of the concept of uncertainty from that of risk or known probability outcomes. But it argued the thesis more technically on the basis of Ellsberg's (1961) paradox, which distinguished the definition of ambiguity (model uncertainty) from that of risk (known probability events). Under the circumstances of ambiguity, agents may plausibly consider a set of probability distributions instead of a unique prior for the future outcomes. Thanks to this seminal idea which is technically convenient, it has emerged an enhanced understanding of agents' attitude towards the future uncertainty. Apart from the preference over the risky events, there is also agents' preference over future ambiguity. Along the lines of argument, we have seen for recent decades a burgeoning of the literature of ambiguity aversion, with its theoretical results employed particularly in the model of financial markets.[3]

Drawn from both the early insights and the recent developments of the literature of ambiguity aversion, it can be concluded that as a descriptive theory the preference of agents over future outcomes involves not only decisions on known probability events (i.e., systematic risks) but also decisions on market ambiguity or model uncertainty (i.e., systemic risk). Once this conclusion is accepted as the amendment to the third typical feature of the neoclassical view of financial markets, we would end the amending process to the neoclassical BMFM. It results in an alternative and extended view on the BMFM, which is characterized with the following three new typical features:

i) the BMFM is a closed system of stochastic processes of technology of production, market transaction costs, and preference of agents;

ii) the BMFM is generically a closed system of incomplete markets;

iii) the BMFM is a closed system of the class of risk-averse as well as

[1] It is important to note here that the term "risk" in Tobin's context is purely monetary assets related uncertainty.

[2] Two-fund separation is also termed as Tobin's separation theorem. Tobin (1958) showed that an investor's optimal portfolio allocation is the result of a two-stage process: the first is that the investor decides in what proportions to purchase the available risky assets, and then decides how to divide his total investment between the fund of risky assets and the fund of a riskless asset. The allocation of wealth across different risky assets is irrelevant to investors' risk-preference which affects only the second stage of decision.

[3] The pivot of the literature was contributed by authors such as Ellsberg (1961), Schmeidler (1989), Gilboa and Schmeidler (1989), Hansen and Sargent (2001), Chen and Epstein (2002), Epstein and Schneider (2003, 2010), Klibanoff, Marinacci and Mukerji (2005), and Maccheroni, Marinacci and Rustichini (2006).

ambiguity-averse investors.

Compared with the neoclassical view stated at the beginning of this section, these amended typical features of the BMFM imply an information space more accommodating to the empirical data of asset prices, with respect to both the *systematic* risks caused by market factors and the *systemic* risk caused by market structure by means of market ambiguity or model uncertainty. In fact, these amended typical features render the neoclassical view the special case of our new understanding of the BMFM.

VI. Conclusion

By means of an information enhanced basic model of financial markets, the classical CAPM has been reformulated in two stages. The first is to have its factor-pricing basis extended to include two more market factors, namely, preference of agents and market microstructure besides technology of production; the second is to have the pricing model allow for the tradeoff relation between market ambiguity and ambiguity aversion. As a result, the reformulated CAPM captures not only market information associated with *systematic* market factors but also market information associated with the state of *systemic* market structure. In addition, as the reformulated CAPM is developed from an information-enhanced basic model of financial markets, it represents typically a *relatively more efficient* market equilibrium valuation of capital assets.

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REFERENCES

Arrow, Kenneth J. 1964. “The Role of Securities in the Optimal Allocation of Risk-bearing.” *Review of Economic Studies* 31: 91-96.

Arrow, Kenneth J. 1969. “The Organization of Economic Activity: Issues Pertinent to the Choice of Market versus Nonmarket Allocation.” In US Congress, Joint Economic Committee, *The Analysis and Evaluation of Public Expenditures: The PPB System* 1, Washington: GOP, 47-66.

Arrow, Kenneth J. 1985. “The Potentials and Limits of the Market in Resource Allocation.” In *Issues in Contemporary Microeconomics & Welfare*, ed. George R. Feiwel, 107-124. Albany: State University of New York Press.

Balasko, Yves. 2009. *The Equilibrium Manifold: postmodern developments in the theory of general equilibrium*. Cambridge, MA: The MIT Press.

Barberis, Nicholas R., and Richard H. Thaler. 2003. “A Survey of Behavioral Finance.” In *the*

Handbook of the Economics of Finance, ed. George Constantinides, Milt Harris and Rene Stulz, Chapter 18. Amsterdam, Netherlands: Elsevier North Holland.

Basu, S. 1977. “The Investment Performance of Common Stocks in Relation to Their Price to Earnings Ratio: A Test of the Efficient Market Hypothesis.” *Journal of Finance*, 32: 663-682.

Benz, R. 1981. “The Relation between Return and Market Value of Common Stocks.” *Journal of Financial Economics*, 9: 653-672.

Black, Fischer. 1972. “Capital Market Equilibrium with Restricted Borrowing.” *Journal of Business*, 45: 444-455.

Black, Fischer. 1986. “Noise.” *Journal of Finance*, 41: 529-543.

Chen, Z. and Larry G. Epstein. 2002. “Ambiguity, Risk and Asset Returns in Continuous Time.” *Econometrica*, 70: 1403-43.

Coase, R. H. 1937. “The Nature of the Firm.” *Economica*, 4(16): 386-405.

Coase, R. H. 1960. “The Problem of Social Cost.” *The Journal of Law & Economics*, 3: 1-44.

Connor, G. 1984. “A Unified Beta Pricing Theory.” *Journal of Economic Theory*, 34: 13-31.

Constantinides, George M. 1986. “Capital Market Equilibrium with Transaction Costs.” *Journal of Political Economy*, 94: 842-862.

De Bondt, W., and R. Thaler. 1985. “Does the Stock Market Overreact?” *Journal of Finance*, 40: 793-805.

De Long, Bradford J., Andrei Shleifer, Lawrence H. Summers and Robert J. Waldmann. 1990. “Noise Trader Risk in Financial Markets.” *Journal of Political Economy*, 98: 703-738.

Diamond, Peter A. 1967. “The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty.” *American Economic Review*, 57: 759-776.

Ellsberg, D. 1961. “Risk, Ambiguity and the Savage Axioms.” *Quarterly Journal of Economics*, 75: 643-669.

Epstein, Larry G., and Martine Schneider. 2003. “Recursive Multiple Priors.” *Journal of Economic Theory*, 113: 1-31.

Epstein, Larry G., and Martine Schneider. 2010. “Ambiguity and Asset Markets.” NBER Working paper 16181.

Fama, Eugene F. 1970. “Efficient Capital Markets: A Review of Theory and Empirical Work.” *Journal of Finance*, 25: 383-417.

Fama, Eugene F. 1991. “Efficient Capital Markets: II.” *Journal of Finance*, 46: 1575-1618.

Fama, Eugene F., and Kenneth R. French. 1993. “Common Risk Factors in the Returns on Stock and Bonds.” Journal of Financial Economics, 33: 3-36.

Fama, Eugene F., and Kenneth R. French. 2004. “The Capital Asset Pricing Model: Theory and

Evidence." *Journal of Economic Perspectives*, 18: 25-46.

Garman, Mark B., and James A. Ohlson. 1981. "Valuation of Risky Assets in Arbitrage-free Economies with Transactions Costs." *Journal of Financial Economics*, 9: 271-280.

Gilboa I., and D. Schmeilder. 1989. "Maximin Expected Utility with Non-unique Prior." *Journal of Mathematical Economics*, 18: 141-153.

Geanakoplos, John. 1990. "An Introduction to General Equilibrium with Incomplete Asset Markets." *Journal of Mathematical Economics*, 19: 1-38.

Geanakoplos, John, and Martin Shubik. 1990. "The Capital Asset Pricing Model as a General Equilibrium with Incomplete Markets." *The Geneva Papers on Risk and Insurance Theory*, 15: 55-71.

Guidolin, Massimo, and Francesca Rinaldi. 2013. "Ambiguity in Asset Pricing and Portfolio Choice: A Review of Literature." *Theory and Decision*, 74: 183-217..

Hansen, L. Peter, and S. F. Richard. 1987. "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models." *Econometrica*, 55: 587-613.

Hansen, L. Peter, and Thomas J. Sargent. 2001. "Robust Control and Model Uncertainty." *American Economic Review*, 91: 60-66.

Hansen, L. Peter, and K. Singleton. 1983. "Stochastic Consumption, Risk Aversion and the Temporal Behavior of Asset Returns." *Journal of Political Economy*, 91: 249-268.

Harrison, Michael J., and David M. Kreps. 1979. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory*, 20: 381-408.

He, Hua, and David M. Modest. 1995. "Market Frictions and Consumption-based Asset Pricing." *Journal of Political Economy*, 103: 94-117.

Huberman, G., and S. Kandel. 1987. "Mean-variance-spanning." *Journal of Finance*, 42: 873-888.

Ingersoll, Jonathan E. Jr. 1984. "Some Results in the Theory of Arbitrage Pricing." *Journal of Finance*, 39: 1021-39.

Jegadeesh, N., and S. Titman. 1993. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *Journal of Finance*, 48: 65-91.

Jouini, Elyes, and Hedi Kallal. 1995. "Martingales and Arbitrage in Securities Markets Transaction Costs." *Journal of Economic Theory*, 66: 178-197.

Keynes, John M. 1936. *The General Theory of Employment, Interest and Money*. London: Macmillan St. Martin's Press, 1973.

Keynes. John M. 1937. "The General Theory of Employment." *Quarterly Journal of Economics*, 51: 209-223.

Klibanoff, P., M. Marinacci, and S. Mukerji. 2005. “A Smooth Model of Decision Making under Ambiguity.” *Econometrica*, 73: 1849-92.

Knight, Frank H. 1921. *Risk, Uncertainty, and Profit*. Boston and New York: Houghton Mifflin.

Kreps, David M. 1981. “Arbitrage and Equilibrium in Economies with Infinitely Many Commodities.” *Journal of Mathematical Economics*. 8: 15-35.

Kyle, A. S. 1985. “Continuous Auctions and Insider Trading.” *Econometrica*, 53: 1315-1336.

Lintner, John. 1965. “The Valuation of Risk Assets and the Selection of Risky Investments in Stock portfolios and Capital Budgets.” *Review of Economics and Statistics*. 47: 13-37.

Lu, Christina T. 2000. “The Multiple F-measurable Variables Asset Pricing Model: Theory and Empirical Tests Based on China’s Stock Market.” PhD dissertation, Sun Yat-sen University (in Chinese).

Lu, Jialiu. 2008. “The Relatively Efficient Markets Hypothesis: Information Space and Refutable Models.” SSRN Working Paper Series, 1129145.

Lucas, Robert E. Jr. 1978. “Asset Prices in an Exchange Economy.” *Econometrica*, 46: 1429-1445.

Luttmer, E. G. J. 1996. “Asset Pricing in Economies with Frictions.” *Econometrica*, 64: 1439-1467.

Maccheroni, F., M. Marinacci, and A. Rustichini. 2006. “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences.” *Econometrica*, 74: 1447-98.

Magill, Michael, and Martine Quinzii, ed. 2008. *Incomplete Markets*, Vol. I, *Finite Horizon Economies*. Northampton, MA, USA: Edward Elgar Publishing, Inc.

Marshall, A. 1890. *Principles of Economics*. London: Macmillan.

Mas-Colell, Andreu. 1991. “Indeterminacy in Incomplete Market Economies.” *Economic Theory*, 1: 45-61.

Meade, James. 1952. “External Economies and Diseconomies in a Competitive Situation.” *Economic Journal*, 62: 54-67.

Mehra, R., and E. Prescott. 1985. “The Equity Premium: A Puzzle.” *Journal of Monetary Economics*, 15: 145-161.

Merton, Robert. C. 1973. “An Intertemporal Capital Asset Pricing Model.” *Econometrica*, 41: 867-887.

Modigliani, F., and M. H. Miller. 1958. “The Cost of Capital, Corporation Finance and the Theory of Investment.” *American Economic Review*, 48: 261-297.

Prisman, E. Z. 1986. “Valuation of Risky Assets in Arbitrage-free Economies with Frictions.” *Journal of Finance*, 41: 545-557.

Radner, Roy. 1972. “Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets.” *Econometrica*, 40: 289-303.

Rose, S. 1976. “The Arbitrage Theory of Capital Asset Pricing.” Journal of Economic Theory, 13: 341-360.

Schmeidler, D. 1989. “Subjective Probability and Expected Utility without Additivity.” *Econometrica*, 57: 571-587.

Sharpe, William F. 1964. “Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk.” *Journal of Finance*, 19: 425-442.

Shleifer, Andrei. 2000. *Inefficient Markets: An Introduction to Behavioral Finance*. New York: Oxford University Press.

Shiller, Robert. 1981. “Do Stock Prices Move too Much to be Justified by Subsequent Changes in Dividends?” *American Economic Review*, 71: 421-436.

Smith, Adam. 1776. *An inquiry into the nature and causes of the wealth of nations*. London: Oxford University Press.

Tobin, James. 1958. “Liquidity Preference as Behavior towards Risk.” *Review of Economic Studies*, 25: 65-86.

Weil, P. 1989. “The Equity Premium Puzzle and the Risk-free Rate Puzzle.” *Journal of Monetary Economics*, 24: 401-421.