

The Distribution of Information, the Market for Financial News, and the Cost of Capital*

Paul Marmora[†]

November 2014

Abstract

Recent empirical evidence suggests that stocks exhibiting a greater degree of asymmetric information among investors earn higher returns. By incorporating a market for financial news, this paper presents a rational expectations model consistent with this observation. When private information about a firm is highly concentrated within a small segment of the population, few individuals expect to hold a large enough stake in the firm to warrant purchasing a copy of firm-specific news. Given increasing returns to scale in news production, these few individuals find a copy of news prohibitively expensive to purchase, which prevents them from learning more about the firm and therefore raises their required risk premium. This result hinges crucially on the existence of competitive information markets, which suggests that the financial news media plays an important role in determining how the cost of capital varies with the inequality of information across investors.

*I am grateful to Dimitrios Diamantaras, Moritz Ritter, Oleg Rytchkov, and Charles Swanson for very helpful and insightful comments. All remaining errors are my own.

[†]Department of Economics, Temple University, 848 Ritter Annex, Philadelphia, PA 19122. E-mail: paul.marmora@temple.edu.

1 Introduction

A long-standing question in the finance and accounting literature is how asymmetric information among investors affects a firm's cost of capital. Although a growing body of empirical evidence suggests that stocks with a greater degree of information asymmetry earn higher returns (e.g., Easley et al., 2002; Botosan et al., 2004; Kelly and Ljungqvist, 2013; Peng He et al., 2013), there is no clear consensus in the literature regarding how a less equal distribution of information across investors results in a higher risk premium. Several papers argue that asymmetric information creates a new form of systematic risk because investors command a higher premium for trading against those who are better informed (e.g., O'Hara, 2003; Easley and O'Hara, 2004; Hughes et al., 2007). However, it appears that in rational expectations models with price-taking agents, an asset's expected risk premium is only affected by the average quality of information that agents possess, not by how this quality is distributed across agents (Admati, 1985; Wang, 1993; Lambert et al., 2012).

In this paper, I explore an alternative mechanism whereby differences in the quality of information across investors can lead to higher expected returns, even if these differences are not directly incorporated into stock prices. Specifically, I argue that firms with a less equal distribution of information across investors are generally more expensive to acquire *additional* information on. This deters investors from learning more about these firms and therefore raises the risk premium that an average investor commands for holding their shares.

The intuition is tied to differences in the cost and benefit of news provision. On one hand, news production involves a high fixed cost of investigation and a near 0 cost of replication, so that the cost of acquiring firm-specific news depends on *how many* people purchase a copy—the more copies purchased, the smaller the share of the fixed cost each purchaser must bear. On the other hand, the benefit of acquiring firm-specific news depends on *how much* a person expects to invest—one piece of news can be used to evaluate multiple shares

of a company's stock. When a firm's private information is highly concentrated within a small fraction of the population, few individuals expect to hold enough outstanding shares to warrant purchasing additional firm-specific news. As a result, each prospective purchaser must incur a larger share of the fixed cost of investigation to obtain a copy, which makes it prohibitively expensive to learn more about the firm. This raises the average level of investors' uncertainty when the portfolio decision is made, which raises the firm's expected cost of capital.

To capture this intuition, I develop a three-period noisy rational expectations model with a continuum of risk-averse agents that competitively trade one risky asset. The total quality, or precision, of prior signals about the risky asset's payoff is heterogeneously distributed across the population. Before agents decide how much to invest, they each have an opportunity to purchase an additional news signal about the asset's payoff. I make two assumptions on the form of this news signal. First, agents purchase this piece of news on a competitive information market characterized by increasing returns to scale in production. Second, an agent's ability to interpret the news signal is increasing in the precision of their initial information.

In this setting, I find that a higher inequality in the distribution of initial (prior) precision reduces the average (posterior) precision of agents' beliefs at the time of investment, which increases the asset's expected risk premium. Notably, this result is contingent on both information markets and increasing returns to learning. In fact, when the cost of acquiring the news signal is exogenously fixed, I reach exactly the opposite conclusion: a sufficiently high inequality of initial information actually decreases excess returns. The reason is that when initial information about a firm is highly concentrated, a small fraction of agents expect to hold so many shares that they are willing to acquire news even when the expected return per share is extremely low, which results in a smaller equilibrium risk premium.

These results have several implications. First, they suggest that whether the cost of cap-

ital rises or falls with a higher inequality of information in competitive equity markets rests crucially on the extent to which individuals are able to share in the cost of news production. For example, in financial markets with few media outlets, a more equal distribution of information across many investors should be associated with higher costs of capital, as no single investor expects to hold enough shares to pay the entire fixed cost of producing additional information themselves. On the other hand, a more equal distribution of information should be associated with a lower cost of capital in markets with a sophisticated news industry because stocks that many investors expect to hold will be cheaper to acquire news on.

Second, the model has implications for the reporting preferences of the financial press. Indeed, one way to interpret the mechanism driving the main result is that widely-circulated, low-cost news providers are more likely to cover stocks with a more equal distribution of information, i.e., stocks in which many investors each expect to hold a smaller number of shares. This interpretation is consistent with evidence that the mass media is more likely to cover stocks primarily owned by individual investors (e.g., Fang and Peress, 2009; Solomon, 2012). Because a more equal distribution of information is associated with more extensive media coverage, the negative relation between the equality of information and the cost of capital predicted by the model is also consistent with the growing empirical literature linking greater media coverage to higher asset prices (e.g., Huberman and Regev, 2001; Fang and Peress, 2009; Tetlock, 2010, 2011; Engelberg et al., 2012).

This paper is related to several strands of research. First, it contributes to the recent debate about whether information asymmetry among investors affects the cost of capital in perfectly competitive markets.¹ O'Hara (2003), Easley and O'Hara (2004), and Hughes et al. (2007) conclude that firms with a greater degree of asymmetric information have higher costs of capital. However, Lambert et al. (2012) point out that these theoretical results are

¹Lambert et al. (2012) shows that asymmetric information can have a separate effect on the cost of capital, but only in models with imperfect competition, like Kyle (1985).

entirely driven by changes to the average precision of information, not by changes to how this precision is distributed across the population. Importantly, this debate is concerned with how an exogenous distribution *directly* affects returns, whereas the distributional effects I document arise *indirectly* via the learning decision. That is, my results imply that asymmetric information increases the risk premium, but only because it lowers the total amount of additional information produced, thereby lowering average posterior precision. The end result is that firms with a less equal distribution of precision experience a higher cost of capital, even though, consistent with Lambert et al. (2012), the risk premium is only affected by the average posterior precision of beliefs.

The learning decision that agents face blends two different approaches used in the literature. In the first, agents learn by acquiring additional noisy signals about future payoffs subject to an exogenous cost (Grossman and Stiglitz, 1980; Verrechia, 1982). In my model, the precision of an agent’s acquired signal is increasing in the precision of his prior beliefs, a type of increasing returns to learning that is also present in the learning constraints motivated by the information-theoretic concept of entropy, such as those used in Peng and Xiong (2006), Van Nieuwerburgh and Veldkamp (2009), Mondria and Wu (2010), and Kacperczyk et al. (2014).² In the second approach, an agent can sell noisy signals about future payoffs to other agents (Admati and Pfleiderer, 1986, 1990; Allen, 1990). Within this strand of research, my model is most closely related to Veldkamp (2006), who uses the same cost structure and competitive information market as the one used here to explain periodic “media frenzies” during times of heightened market volatility. However, in her paper, all agents are ex-ante homogenous and acquire news signals with equal precision, so that her framework cannot

²As Sims (2006) asserts, the entropy-based learning technology is better suited to address the cost of “processing” readily available information, not the cost associated with how information is made available in the first place. Most finance-oriented papers in this literature assume that investors can reduce all asset uncertainty to arbitrarily low levels if endowed with sufficient information-processing capacity, which, in light of Sim’s criticism, implies that every investor is exogenously supplied access to payoff information on all stocks.

address how the cost of capital is affected by the distribution of information.

Hong and Stein (2007) provide an alternative mechanism through which news suppliers can affect asset returns. They argue that media coverage captures the attention of investors with heterogeneous beliefs, which, according to Miller (1977), drives asset prices up in the presence of short-sales constraints. In contrast, the mechanism presented here does not rely on short-sales constraints: risk-averse investors are aware that all assets exist and are free to take any position on them, but are simply less willing to hold an asset whose payoff they are more uncertain of.³

Finally, the analysis builds on Peress (2010), who shows that expanding a firm's investor base without raising capital induces incumbent shareholders to conduct less research because they each expect to hold a smaller stake in the firm. His results are driven by the same scaling effects to the benefit of learning responsible for the results here, but there are two crucial differences. First, while Peress (2010) exogenously limits the size of an asset's investor base (as in Merton (1987)), in this paper an asset's ownership structure is determined endogenously from the distribution of prior information. More importantly, I include a competitive market for news, which partially offsets the risk-sharing effect he documents. That is, those who invest in a firm with a small shareholder base each expects to hold more risk, but they must also bear a higher cost of producing news.

The rest of the paper is organized as follows. Section 2 develops a noisy rational expectations model with both heterogeneous agents and endogenous information acquisition. Section 3 characterizes the model's equilibrium. Section 4 presents the model's main results. Section 5 reviews existing empirical literature that supports the model's predictions. Section 6 concludes and offers direction for future research.

³In heterogeneous noisy rational expectations models, each agent perceives a different risk-return tradeoff based on their individual information sets. Consequently, unlike the homogenous information setting of the CAPM, agents differ in their assessment of what constitutes an optimally diversified portfolio.

2 Model Setup

2.1 Timeline and Assets

In this paper, I analyze how the initial distribution of asset information across investors affects how much additional asset-specific news is produced. To do so, I develop a general equilibrium noisy rational expectations model with a continuum of heterogeneously informed agents of measure one and two assets: one risky asset (stock) and one riskless asset (bond).

The riskless asset has a price and payoff normalized to one and is in perfectly elastic supply. The per-capita supply of the risky asset is $\bar{x} + x$, where $x \sim N(0, \sigma_x^2)$. Random supply, usually attributed to the existence of noise traders or liquidity needs, prevents the risky asset price p , which is determined in equilibrium, from perfectly revealing all aggregate information.

The static model is divided into three periods. In period one, each agent chooses whether to purchase an additional piece of news about the risky asset that can be used to reduce payoff uncertainty. In period two, each agent chooses their optimal portfolio after observing the realization of both acquired news and the risky asset price. In period three, agents receive their payoffs.

2.2 Preferences

Agent i , with risk aversion ρ , makes information and portfolio decisions to maximize mean-variance utility over terminal wealth, $W_{i,3}$:

$$U_{i,1} = E_{i,1}[\rho E_{i,2}(W_{i,3}) - \frac{\rho^2}{2} \text{Var}_{i,2}(W_{i,3})]. \quad (1)$$

$E_{i,1}[\cdot]$ is an individual's expectation in period one, before he observes prices and acquired signals. $E_{i,2}[\cdot]$ and $\text{Var}_{i,2}[\cdot]$ are the expectation and variance of terminal wealth in period

two, after individuals have observed signal realizations but before the investment decision has been made.

Given that terminal wealth is normally distributed, equation (1) is functionally equivalent to:

$$U_{i,1} = E_{i,1}[\log (E_{i,2}[e^{-\rho W_{i,3}}])]. \quad (2)$$

This formulation of utility, whose axiomatic foundations are first provided in Kreps and Porteus (1978), represents a preference for early resolution of uncertainty, as in Epstein and Zin (1989). That is, investors with these preferences are not adverse to the risk resolved in period two: after private signals are realized but before payoffs are known. This property allows for the benefit of information to rise in the scale of investment, a feature that is absent in the case of standard CARA preferences (Peress, 2010).

2.3 The Distribution of Initial Information

The risky asset payoff f is not known with certainty, but each agent is endowed with some private prior knowledge about it. Formally, agent i is endowed with an independent private signal: $\tilde{f}_i \sim N(f, \tilde{\tau}_i^{-1})$.

In order to isolate how the initial distribution of information affects the learning decision, I assume that the prior precision of agent i is:

$$\tilde{\tau}_i = \alpha \bar{\tau} i^{\alpha-1}, \quad (3)$$

where $\bar{\tau} = \int_0^1 \tilde{\tau}_i di$ is the average quality of initial information and $\alpha \geq 1$. Given this assumption, α can readily be interpreted as the “inequality” in the distribution of initial information. Specifically, a higher α corresponds to a greater concentration of prior precision

among a smaller percentage of the population. Consistent with this interpretation, it can be shown that α is directly proportional to standard measures of inequality of a distribution used in other literatures, such as a Gini coefficient, often used to assess the concentration of income in a given population.⁴

Figure 1 plots the distribution of precision for different values of α . The dotted line corresponds to the distribution when $\alpha = 1$, where all agents are equally familiar about the risky asset's payoff. The dashed and solid lines correspond to the distribution with higher values of α , where a smaller segment of agents hold a greater proportion of the risky asset's prior information.

This inequality in the precision of initial information could originate from many sources. For example, a large body of theoretical research argues that the quality of corporate disclosure, which is known to differ widely across firms (Sengupta, 1998), can affect the degree of information asymmetry among shareholders (Diamond and Verrecchia, 1991; Kim and Verrecchia, 1997). Alternatively, many empirical studies have found that investors exhibit a strong preference for local firms (Coval and Moskowitz, 1999; Huberman, 2001; Grinnblatt and Keloharju, 2001), which is usually attributed to information advantages emanating from their geographic proximity (Brennen and Cao, 1997; Van Nieuwerburgh and Veldkamp, 2009; Mondria and Wu, 2010). Information differences across investors could also arise due to firm-specific characteristics that affect how quickly its news diffuses across the population, such as how much media coverage the firm has previously received (Peress, 2014).

⁴In this context, the Gini coefficient is equal to $1 - \frac{2}{\alpha+1}$. For an elementary exposition on the Gini coefficient and how it is used to measure income inequality, see Sen (1973).

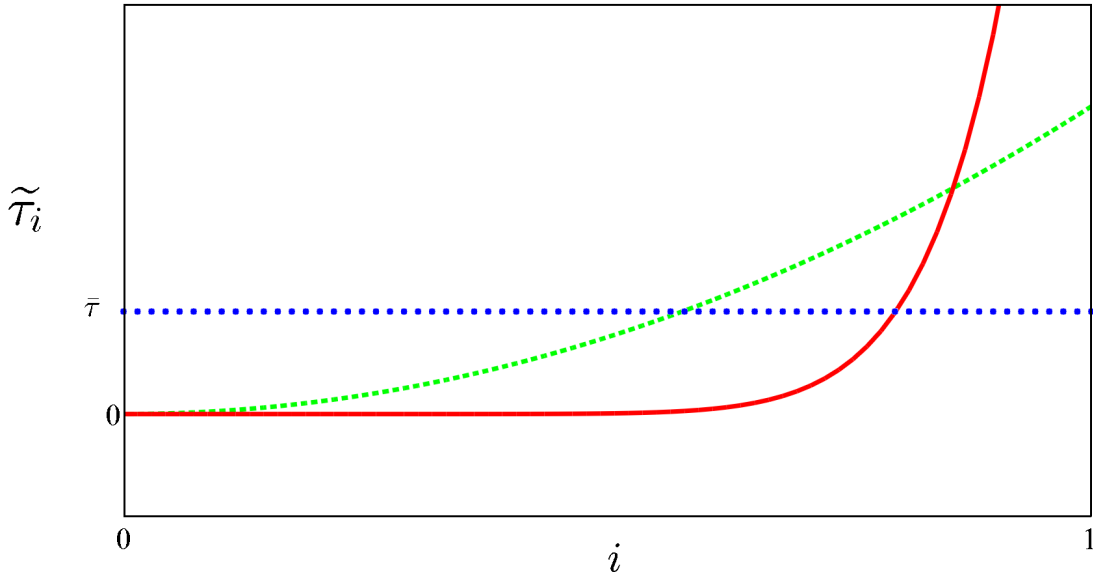


Figure 1: **The distribution of prior precision with varying levels of inequality.** This figure presents how the distribution of prior precision varies with α . The dotted line corresponds to $\alpha = 1$, where the distribution of precision is uniformly distributed across agents. The dashed curve and solid curve correspond to $\alpha = 3$ and $\alpha = 20$, respectively, whereby prior precision is more concentrated within a smaller fraction of the population. $\bar{\tau} = 1$.

2.4 News Production

In the first period, each agent decides whether to acquire an additional news signal, which can be used to reduce payoff uncertainty. Let χ be the per-capita fixed cost of discovering news about the risky asset's payoff, which can be interpreted as the cost of hiring a reporter that covers an upcoming announcement. Following Veldkamp (2006), I introduce competitive information markets by assuming that once news is discovered, a copy can be sold to others at no marginal cost.⁵ Importantly, an agent can freely enter the market for news even after other agents have announced the prices they will charge. In other words, the market for news is perfectly contestable. One way to ensure this contestability would be to assume that there are two sub-periods: in the first sub-period, each agent announces the price they

⁵High fixed costs and low marginal costs appear to be a fundamental aspect of the news industry (Hamilton, 2004).

will charge on the news item, and in the second sub-period, agents decide whether to enter the market by paying the fixed cost. This is a natural assumption for information markets, where prices are often set well before media outlets decide whether to report on a particular story.

Let d_j be an indicator variable equal to 1 if agent j discovers news and let $l_i(c_j, c_{-j}) = 1$ if agent i chooses to purchase news for price c_j , given other announced prices c_{-j} . Then, agent j chooses c_j and d_j to maximize profit:

$$\pi_j = \max_{d_j, c_j} d_j \left(c_j \int_0^1 l_i(c_j, c_{-j}) di - \chi \right), \quad (4)$$

where π_j enters agent j 's utility function through his terminal wealth, $W_{j,3}$. Pricing and entry decisions are a sub-game perfect Nash equilibrium. Since agents are strictly better off buying news at the lowest possible price, I economize on notation by denoting l_i as an indicator for whether agent i purchases news at the lowest offered price, c . Once purchased, a copy of news cannot be resold to other agents.

If agent i buys a copy of news (i.e., $l_i = 1$), the signal agent i observes is:

$$s_i = f + e_i, \quad (5)$$

where $e_i \sim N(0, \eta_i^{-1})$ is noise in interpretation. Importantly, I assume that η_i is equal to:

$$\eta_i = k\tilde{\tau}_i, \quad (6)$$

where k can be interpreted as an agent's "capacity" to process financial information. A similar form of signal precision can be explicitly derived by the entropy-motivated learning constraints used in recent asset-pricing literature, such as Van Nieuwerburgh and Veldkamp

(2009), Mondria and Wu (2010), and Kacperczyk et al. (2014).⁶ However, unlike those models, I assume that before an agent can allocate any of their processing-capacity k towards learning about an asset, they must first pay a separate cost to gain access to its news.⁷

While agents must pay for access to the news signal, all agents receive a free public signal about f from the price itself. Agents do not know the realization of this price signal before period two, but they can infer its precision in period one by knowing the learning decisions of other investors.⁸

2.5 Portfolio Selection

In period two, individuals use their prior beliefs \tilde{f}_i , their private signal s_i (if news was purchased), and price p to update posterior beliefs, which are then used to choose portfolios subject to a budget constraint. Let $q_{i,0}$ denote agent i 's demand for the riskless asset and let q_i denote agent i 's demand for the risky asset. If $W_{i,0}$ is agent i 's initial wealth, then the budget constraint is:

$$W_{i,0} = q_{i,0} + q_i p + l_i c. \quad (7)$$

Terminal wealth $W_{i,3}$ is equal to:

$$W_{i,3} = q_{i,0} + q_i f + \pi_i. \quad (8)$$

⁶The assumption that investors' ability to interpret asset news is increasing in their prior precision is supported by evidence that portfolio managers who specialize in select stocks tend to earn higher returns (Ivkovic et al., 2008).

⁷Requiring agents to pay a separate cost to acquire news addresses the point raised by Sims (2006) that entropy-based learning constraints are not well-suited to address "costly investigation", which includes the production of news.

⁸By not incorporating the price signal into (6), I am implicitly assuming that no processing-capacity is required to process the information in price. Mondria (2010) shows that subjecting the price signal to entropy constraints generates multiple equilibria. To avoid this issue, one can assume that investors are endowed with enough capacity to process the price signal, as in Van Nieuwerburgh and Veldkamp (2009).

Combining (7) and (8), agent i 's terminal wealth can be written as:

$$W_{i,3} = W_{i,0} + q_i(f - p) + \pi_i - l_i c. \quad (9)$$

2.6 Equilibrium Definition

A rational expectations equilibrium consists of agent decisions (l_i, q_i, c_i, d_i) , asset price p , news price c , and exogenous asset supply $\bar{x} + x$ such that:

- Given prices, each agent i chooses asset demand q_i and whether or not to purchase news l_i to maximize (1) subject to (6) and (9).
- Information supply entry decisions d_i and pricing strategies c_i are a sub-game perfect Nash equilibrium that maximize (4).
- Asset markets clear

$$\int_0^1 q_i di = \bar{x} + x.$$
- Rational expectations: Beliefs about payoffs, prices, and the optimal asset demands are consistent with their true distribution.

3 Solution

The model is solved through backward induction. First, each agent decides on their optimal asset demand given arbitrary signals and prices. Second, knowing how optimal asset demand relates to every possible signal/price combination, each agent makes their information choices given the information choices of other agents.

3.1 Optimal Asset Demand

In period two, agents use the public price signal and acquired private signals to update beliefs. Let $z(p)$ denote the public price signal and let ζ denote its precision. Then with normally distributed random variables, posterior beliefs are a weighted average of the signals observed:

$$\text{Var}_{i,2}(f|\tilde{f}_i, l_i s_i, p) = \hat{\tau}_i^{-1} = \frac{1}{\tilde{\tau}_i + l_i \eta_i + \zeta}, \quad (10)$$

$$E_{i,2}(f|\tilde{f}_i, l_i s_i, p) = \hat{f}_i = \frac{\tilde{\tau}_i \tilde{f}_i + l_i \eta_i s_i + \zeta z(p)}{\tilde{\tau}_i + l_i \eta_i + \zeta}, \quad (11)$$

They make their investment decisions to maximize period two utility:

$$U_{i,2} = \rho E_{i,2}(W_{i,3}) - \frac{\rho^2}{2} \text{Var}_{i,2}(W_{i,3}). \quad (12)$$

After substituting (9) into (12), the period two problem becomes:

$$U_{i,2} = \max_{q_i} W_{i,0} + \rho q_i (\hat{f}_i - p) - \frac{\rho^2 q_i^2}{2 \hat{\tau}_i} - l_i c + \pi_i. \quad (13)$$

Taking first order conditions with respect to q_i leads to optimal share-holdings:

$$q_i = \frac{\hat{\tau}_i}{\rho} (\hat{f}_i - p). \quad (14)$$

Asset demand is increasing in both posterior precision $\hat{\tau}_i$ and the expected excess return per share $\hat{f}_i - p$. Intuitively, the more certain I am about payoffs or the more my personal evaluation of the payoff is above the opportunity cost of purchasing a share, the more risk I will be willing to undertake.

3.2 Market Clearing Price

Aggregating optimal asset demand across agents and imposing the market-clearing condition determines the asset's equilibrium price. Following Hellwig (1980) and Admati (1985), price is a linear function of the risky asset's payoff.

Proposition 1. *Given information choices, there exists a unique linear rational expectations equilibrium. The equilibrium asset price is given by:*

$$p = f - \left(\frac{\rho \bar{x}}{\theta + \zeta} + \frac{\rho x}{\theta} \right), \quad (15)$$

where:

$$\theta = \bar{\tau} + \int_0^1 l_i \eta_i di, \quad (16)$$

$$\zeta = \left(\frac{\theta}{\sigma_x \rho} \right)^2. \quad (17)$$

Proof: See Appendix.

The term θ represents the average precision of priors and purchased signals, so that $\theta + \zeta$ is equal to the posterior precision of the “average agent”: $\int_0^1 \hat{\tau}_i di$. Substituting this term into equation (15) and taking expectations reveals that the expected excess return per share in period one is:

$$E_1(f - p) = \frac{\rho \bar{x}}{\int_0^1 \hat{\tau}_i di}. \quad (18)$$

The realization of price is only observed in period two, so this unconditional expected return is the same across all agents, and is equivalent to the expected return that an outside observer would estimate. Note that $E_1(f - p)$ is increasing in both risk aversion and average number

of outstanding shares, while it is decreasing in average posterior precision. The intuition is that, in order for the market to clear, asset prices must be lower to compensate an average investor for holding more risk. Since \bar{x} , ρ , and $\bar{\tau}$ are exogenous, equation (18) implies that a change in the distribution of initial information can only affect expected returns through the average precision of purchased signals, $\int_0^1 l_i \eta_i di$.

3.3 Optimal Information Acquisition

The Appendix shows that substituting optimal asset demand (14) into the period two objective function (13) and taking expectations over period two indirect utility yields the period one problem:

$$U_{i,1} = \max_{l_i, c_i, d_i} W_{i,0} + R(\theta) (\tilde{\tau}_i + l_i \eta_i + \zeta) - l_i c + \pi_i, \quad (19)$$

where:

$$R(\theta) = \frac{1}{2} \left(\left(\frac{\rho \sigma_x}{\theta} \right)^2 + \left(\frac{\rho \bar{x}}{\theta + \left(\frac{\theta}{\rho \sigma_x} \right)^2} \right)^2 \right). \quad (20)$$

Agents are price-takers, so they take the term $R(\theta)$ as fixed. The first step in solving an agent's period one problem is to identify how much suppliers will charge for a copy of news. The following proposition states that, given increasing returns to scale in information production, the equilibrium cost of news is declining in the number of purchasers.

Proposition 2. *In equilibrium, one agent supplies news to the entire market: $d_v^* = 1$ and $d_j^* = 0$ for all $j \neq v$. Furthermore, this agent will charge at average cost:*

$$c^* = \frac{\chi}{\lambda^*}, \quad (21)$$

where $\lambda^* = \int_0^1 l_i^* di$.

Proof: See Appendix.

The more agents who buy a copy of news, the smaller the share of the fixed cost of discovery each agent incurs. For this reason, high demand news is less expensive to purchase. A crucial aspect of Proposition 2 for the ensuing analysis is that equation (21) is only a function of how many agents purchase a copy and not how much each agent is willing to pay conditional on having purchased. The reason is tied to the assumption of free entry: if any supplier charges above average cost, another agent can discover news, charge slightly below the incumbent, and take the entire market.⁹

Because agents are not ex-ante identical, equilibrium news demand not only depends on how many agents purchase, but also depends on which agents are doing the purchasing. Before describing how λ^* is determined, the following Lemma greatly reduces the number of permissible allocations.

Lemma 1. *In equilibrium, agent i purchases news only if all agents with a higher prior precision do the same: $l_i^* = 1$ only if $l_v^* = 1$ for all $v > i$. Therefore:*

$$\theta(\lambda^*, \alpha) = \bar{\tau} + \int_{1-\lambda^*}^1 k\bar{\tau}\alpha i^{\alpha-1}. \quad (22)$$

Proof: See Appendix.

Two separate factors are responsible for Lemma 1. First, recall that agents who are initially more familiar with the risky asset can more accurately interpret additional asset news. A second effect, independent of any assumption placed on η_i , is that agents who are initially

⁹Although this result (i.e., the price of news declining in the number of purchasers) does rely on both the free entry assumption and the fixed-cost production technology, it is robust to alternative forms of competition, such as Cournot or monopolistically competitive frameworks (Veldkamp, 2006).

more familiar with the risky asset expect to hold more shares ex-ante. Since the benefit of information is rising in the expected scale of investment—one piece of news can be used to evaluate many shares—agents with a higher prior precision find adding one unit of precision more valuable. The combination of both these effects mean that, given a particular $R(\cdot)$ and c , an agents' willingness to purchase is strictly increasing in their prior precision, which in turn means that if a certain fraction of agents are purchasing, it must be the fraction who are most initially informed.

Given Proposition 2 and Lemma 1, equilibrium news production is fully characterized by λ^* .

Proposition 3. *Let $B(\lambda, \alpha)$ be the net benefit of news to the marginal purchaser:*

$$B(\lambda, \alpha) = R(\theta(\lambda, \alpha)) k\bar{\tau}\alpha(1 - \lambda)^{\alpha-1} - \frac{\chi}{\lambda}. \quad (23)$$

Then λ^ is defined by the following conditions:*

- *If $B(\lambda, \alpha) < 0$ for all $\lambda \in (0, 1]$, then $\lambda^* = 0$.*
- *If $B(1, \alpha) \geq 0$, then $\lambda^* = 1$.*
- *If $B(\lambda, \alpha) > 0$ for some $\lambda \in (0, 1)$ and $B(1, \alpha) < 0$, then:*

$$B(\lambda^*, \alpha) = 0, \quad (24)$$

$$\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*} < 0, \quad (25)$$

Proof: See Appendix.

There are three factors driving the net benefit of news to the marginal purchaser. First, the benefit of news is increasing in $R(\cdot)$, which is positively correlated with the risky asset's expected excess return per share. Also of note is that, for a given α , $R(\cdot)$ is strictly decreasing

in λ . This is the canonical strategic substitutability in information acquisition result of Grossman and Stiglitz (1980): the more agents who acquire news, the lower the expected return (or equivalently, the more informative price is). An additional feature here stemming from my assumption on η_i is that this negative effect is amplified if the investors who acquire news have a high prior precision because these investors expect to trade more aggressively on the news item, which makes price more informative and lowers the required risk premium further.

Second, the benefit of news to the marginal purchaser is decreasing in the marginal purchaser's signal precision $k\bar{\tau}\alpha(1-\lambda)^{\alpha-1}$, which is itself a decreasing function of λ . The latter relation is a direct consequence of Lemma 1: the more agents who purchase news, the lower the signal precision of the agent who derives the least benefit from doing so.

Finally, the benefit curve is decreasing in the price of news, which is decreasing in λ by Proposition 2. Overall, λ has three effects on the benefit curve: two negative and one positive. When λ is arbitrarily close to 0, the positive effect dominates, so that the benefit curve generally takes the shape illustrated in Figure 2. Note that while $B(\lambda, \alpha)$ crosses 0 at two different points in Figure 2, only the higher point can be an equilibrium for supply of information. Namely, at any point where $B(\lambda, \alpha) = 0$ and $\frac{\partial B(\lambda, \alpha)}{\partial \lambda} > 0$, any agent can enter the market for news, charge slightly below average cost and make a small profit. The same cannot be said for the point where $B(\lambda, \alpha) = 0$ and $\frac{\partial B(\lambda, \alpha)}{\partial \lambda} < 0$, making it the unique equilibrium.

4 Main Results

In this section, I present the main results, most of which are a direct implication of the following proposition.

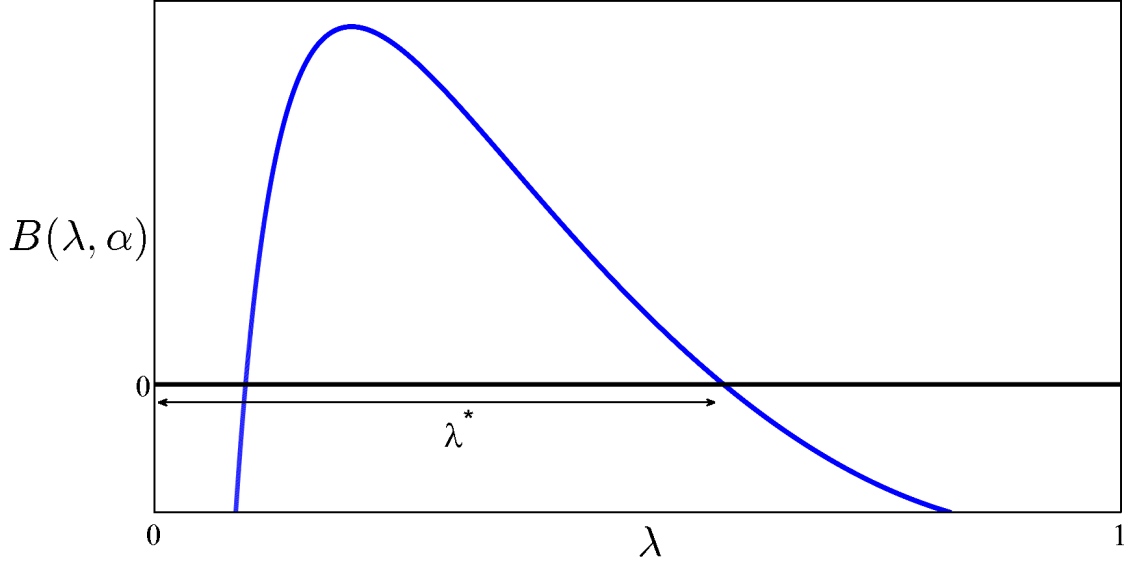


Figure 2: **Equilibrium news production.** This figure presents the benefit of news to the marginal purchaser as a function of λ . $k = 2$, $\rho = 2.5$, $\bar{\tau} = 1$, $\bar{x} = 2$, $\sigma_x^2 = 2$, $\chi = 1.1$, $\alpha = 3$.

Proposition 4. *Assume a positive fraction of agents purchase news in equilibrium. Then:*

$$\frac{\partial c^*}{\partial \alpha} > 0. \quad (26)$$

Proof: See Appendix.

For intuition, consider how an incremental increase in the level of inequality α affects the benefit of news to the marginal agent, holding the fraction of purchasers λ^* fixed. First, an increase in α redistributes prior precision from less informed agents to more informed agents. Due to increasing returns to learning, this redistribution increases average posterior precision for any value of λ admissible by Lemma 1, which lowers the expected return per share by equation (18). Second, an increase in α changes the marginal agent's signal precision η_i . Given an endogenous cost of news, this agent becomes less familiar with the risky asset, which reduces how aggressively he plans to trade on the story. Since the marginal agent was indifferent to purchasing before the change (by definition), and both the expected return

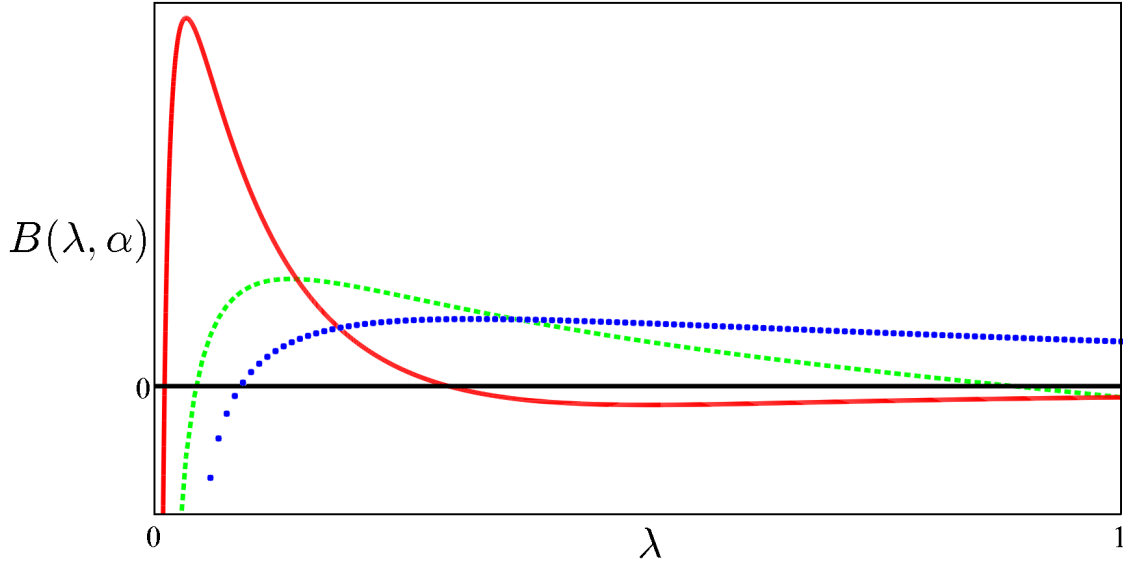


Figure 3: **The benefit of purchasing news with varying levels of inequality.** This figure presents how the benefit of news to the marginal purchaser varies with α . $k = .6$, $\rho = 2.5$, $\bar{\tau} = 1$, $\bar{x} = 2$, $\sigma_x^2 = 2$, $\chi = 1.1$, $\alpha(\text{dotted})=1$, $\alpha(\text{dashed})=2$, $\alpha(\text{solid})=8$.

per share and expected shareholdings are lower after the change, he is strictly better off not purchasing. This makes news more expensive for the other purchasers, who are now left with having to cover a larger share of the fixed cost of discovery. Figure 3 illustrates how the benefit curve is affected by changes to α .¹⁰ Note that higher values of α are associated with a smaller fraction of news purchasers and consequently higher news prices.

For the purposes here, Proposition 4 suggests that news markets reduce the incentive of rational investors to learn about stocks whose information is highly concentrated. This is most clearly evident by considering how the market for news alters the learning decision of a hypothetical investor whose prior precision is unaffected by α . Without information markets, an increase in α can only affect this hypothetical investor's willingness to acquire news via a change in the expected return per share. However, with information markets, the price of news is also increasing in α , meaning that the investor may be less inclined to

¹⁰In Figure 3, when $\alpha = 1$, $\lambda^* = 1$ and no agent is indifferent between purchasing and not purchasing. However, with an infinitesimally small increase in α , $\eta_0 = 0$, so that Proposition 4 still applies in this case.

purchase even if the expected return per share goes up, which gives rise to this paper's main result.

Proposition 5. *Assume a positive fraction of agents purchase news in equilibrium. Then:*

$$\frac{\partial E_1(f - p)^*}{\partial \alpha} > 0. \quad (27)$$

Proof: See Appendix.

From Proposition 4, an increase in the inequality of prior precision reduces the number of agents who expect to hold enough shares of the risky asset to warrant paying for additional asset-related news. As a result, the remaining prospective purchasers must bear a higher share of the fixed cost of discovery, which discourages them from buying and therefore increases the risky asset's expected return.

Importantly, the cross-sectional return pattern implied by Proposition 5, i.e., a higher concentration of prior information leads to higher excess returns, hinges crucially on *both* an endogenous news price arising from the information market *and* increasing returns to learning implied by my assumption on η_i . To emphasize how both features factor into the result, the next two propositions illustrate how excess returns vary with α when either feature is absent.

Proposition 6. *Assume there are no information markets and the price of news is exogenously fixed. Then there exists an $\bar{\alpha}$ such that if $\alpha > \bar{\alpha}$:*

$$\frac{\partial E_1(f - p)^*}{\partial \alpha} < 0. \quad (28)$$

Proof: See Appendix.

Proposition 7. *Assume signal precision is independent of prior precision: $\eta_i = k$. Then:*

$$\frac{\partial E_1(f - p)^*}{\partial \alpha} = 0. \quad (29)$$

Proof: See Appendix.

Proposition 6 states if the price of news is exogenously fixed and α is sufficiently high, one reaches the exact opposite conclusion: in equilibrium the excess return actually *decreases* in α . To understand why, recall that agent i 's terminal wealth from the risky asset is the return per share times the number of shares purchased, $q_i(f - p)$. With a low level of α , each agent holds a low amount of prior precision, and hence expects to hold a low number of shares. Consequently, agents are only willing to purchase news if each share expects to pay a relatively large amount. On the other hand, with a high level of α , a small fraction of agents expect to hold so many shares that they are willing to purchase news even if the payoff per share is low. Because agents are willing to keep purchasing news even at low returns, the equilibrium risk premium is smaller.

While this effect is still present when there are information markets, it is outweighed by an increasing cost of news. In other words, for high values of α and correspondingly low values of λ^* , the marginal agent still expects to hold a large stake in the risky asset, and is therefore willing to pay for information even at extremely low excess returns per share. Nevertheless, the high price of news brought on by low number of purchasers makes it prohibitively expensive to do so. Thus, the ability of agents to share the fixed cost of information production reverses how the risk premium moves with the concentration of prior precision.

The impact that an endogenous news price has on the relationship between equilibrium returns and the inequality of initial information is captured by Figure 4. In Figure 4a, the price of news is endogenous, i.e., $c = \frac{\chi}{\lambda^*}$, so that the risk premium is monotonically increasing

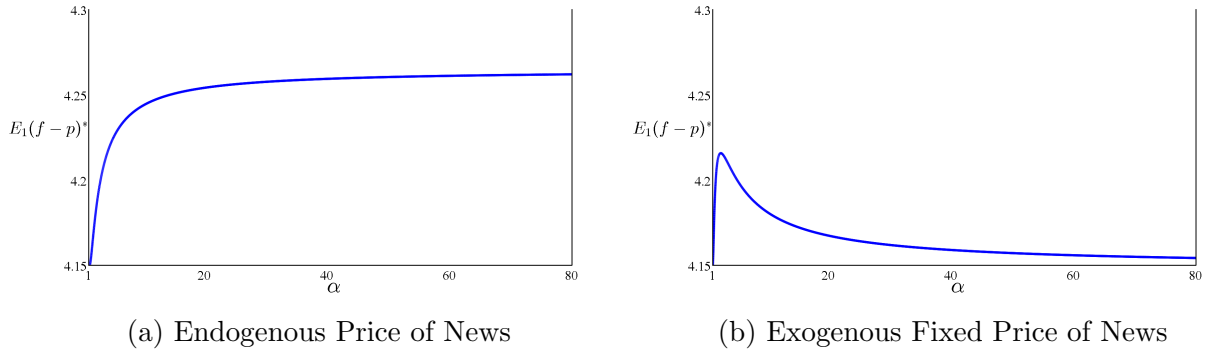


Figure 4: **Equilibrium returns with and without an endogenous price of news.** This figure presents how the equilibrium return per share $E_1(f - p)^*$ is affected by α with and without an endogenous price of news. $k = .2$, $\rho = 2.5$, $\bar{\tau} = 1$, $\bar{x} = 2$, $\sigma_x^2 = 2$, $\chi = 1.1$, $c(\text{figure a}) = \frac{\chi}{\lambda^*}$, $c(\text{figure b}) = \frac{\chi}{.5}$.

in α by Proposition 5. In contrast, Figure 4b presents the case where the price of news is exogenously fixed at $\frac{\chi}{.5}$, so that the risk premium monotonically decreases for a high enough level of α by Proposition 6.¹¹

Similarly, Proposition 7 reveals that when there are information markets but an agent's ability to process news is independent of his priors, the equilibrium expected return per share is independent of α . In fact, it can be shown that if there were multiple risky assets, in this case an agent's learning choice is completely independent of the quality of his initial information. The reason is that, on one hand, an investor can better diversify if he chooses to learn about a less familiar asset. On the other hand, as discussed in the previous section, an investor expects to hold a higher share of his portfolio in more familiar assets. Given mean-variance utility over wealth and a news signal independent of prior precision, these two effects completely offset.¹²

¹¹It is worth noting that Figure 4b contains an increasing portion for small values of α . Recall that an increase in α amounts to a redistribution of precision from less informed to more informed agents. At a high λ^* (brought on by a low α), the marginal agent is losing prior precision after an incremental increase in α , and hence more willing to forgo purchasing. Eventually, prior precision becomes so concentrated that the marginal purchaser is actually on the receiving end of this redistribution, which gives rise to Proposition 6. While Proposition 6 holds for any parameters, the increasing portion of Figure 4b only occurs for a low a exogenous news cost.

¹²The same would not be true in the case of agents with constant absolute risk aversion (CARA). For an in-depth discussion of alternative learning technologies and information preferences within a partial equilibrium setting, see Van Nieuwerburgh and Veldkamp (2010).

5 Empirical Implications

The model described in this paper makes testable predictions as to what kind of firms will be inexpensive for investors to learn about and how this affects a firm's information environment and cost of capital. In this section, I review some existing empirical literature that supports the model's conclusions.

5.1 Media Coverage and Individual Investors

A key insight of this model is that assets in which many investors each expect to hold a small fraction of outstanding shares will be cheaper to learn about than assets in which few investors each expect to hold a large fraction of outstanding shares. Inasmuch as mass media outlets charge cheaper prices and individual investors hold smaller portfolios than institutional investors do, this suggests that the mass media is more likely to cover stocks with high individual ownership.

This prediction is corroborated by several recent studies. In their analysis of the no-coverage premium, Fang and Peress (2009) also measure the determinants of media coverage for 4 nationally circulated daily newspapers: *New York Times*, *USA Today*, *Wall Street Journal*, and *Washington Post*, which together account for nearly 11% of daily circulation in the United States. They find that after controlling for other firm characteristics, most notably firm size and idiosyncratic volatility, a 1% increase in the fraction of shares owned by individual investors increases the annual number of articles published about a firm by .18. Similarly, Solomon (2012) reports that a 1% increase in the fraction of shares owned by institutions decreases the number of articles about a firm announcement by between 27% to 30% in the Factiva news archive. Both papers also find that firm size has a overwhelmingly positive effect on the probability of news coverage, lending further support to the notion that the media covers firms with a large shareholder base, even if each shareholder holds a

small stake in the firm.

This insight is also consistent with the growing body of evidence suggesting the mass media can influence the buying behavior of individual investors. Barber and Odean (2008) find that individual investors are much more likely to be net buyers of stocks in the news than those that are not. Engelberg et al. (2012) find that stocks receiving recommendations on the television show *Mad Money* experience large overnight price increases and subsequent reversals. Engelberg and Parsons (2011) find that local newspaper coverage increases the daily trade volume of local retail investors by anywhere between 8% to almost 50%. In the latter study, the authors exploit the exact timing of newspaper delivery relative to an observed spike in local trading, making it unlikely that media coverage and market reactions are both driven by some unobserved characteristic, such as how much a story peaks the public interest (Manela, 2014).

5.2 Asymmetric Attention and the Cross-Section of Returns

Apart from having consequences for the decisions of news suppliers, the model also makes a macro-level prediction concerning how the distribution of prior precision varies with the cost of capital. Although investors' information is not directly observable, a central tenet of the learning literature with inattentive agents is that prior precision should be highly correlated with the amount of attention paid to a stock (Van Nieuwerburgh and Veldkamp, 2009; Mondria and Wu, 2010). Therefore, one way to test the model's main result is to estimate whether stocks garnering a larger fraction of national attention from a smaller segment of the population earn higher excess returns.

Using a direct measure of investor attention first suggested by Da et al. (2011), Mondria and Wu (2012) find just that. Namely, they find that stocks with a greater degree of asymmetric attention, defined as the fraction of abnormal Google search volume for a stock generated by local investors, earn higher returns.

This finding is difficult to reconcile with models that do not include both information markets and increasing returns to learning. First, the distribution of information in and of itself should not affect the cost of capital in markets well-approximated by perfect competition (Lambert et al., 2012). Second, as per Proposition 6, a higher concentration of prior precision can actually decrease excess returns when there are no information markets and the cost of news is exogenous. Finally, models that incorporate information suppliers, such as Veldkamp (2006) and Admati and Pfleiderer (1986), typically assume that investors are ex-ante homogenous, and therefore do not address how the distribution of prior precision affects the cost of capital. Moreover, Proposition 7 shows that a heterogeneous distribution of prior precision and an information market alone cannot generate this relationship if all investors interpret a piece of news with equal precision.

6 Conclusion

The model presented in this paper predicts that firms with a higher fraction of its private information concentrated within a smaller fraction of the population will be more expensive to learn about, which deters investors from following these firms and increases their cost of capital. Although the model is static, in a dynamic setting with multiple risky assets I conjecture that this effect can persist even while new market participants who are equally familiar with all risky assets continuously enter the market. This is because new investors are more likely to be initially exposed to stocks whose news is cheaper to purchase. This initial exposure leads them to prefer learning about the same stocks in subsequent periods. The end result is that cross-sectional variation in expected returns can persist even in the absence of any cross-sectional persistence in volatility. A dynamic framework could also explain the puzzling observation that higher media coverage is associated with lower returns, despite the fact that media coverage appears to be a stable firm characteristic (Fang and Peress, 2009).

Another extension involves allowing investors the ability to purchase news of various qualities. Given the results documented here, a reasonable hypothesis is that the demand for news mimics the distribution of prior information: when prior information is concentrated, the market for news is represented by a small contingent of investors demanding high-quality news, whereas if prior information were uniformly distributed across the population, the market for news is represented by many investors demanding low-quality news.

Finally, this paper has an important implication for empirical efforts that use the amount of media coverage as a proxy for learning preferences: while media coverage may proxy for the information demands of many consumers, the direction of causality may indeed be reversed for a large segment of the population given increasing returns to scale in news production, especially if subscribers' demands are heterogeneous and the fixed cost of production is high. That is, it is not that media coverage reflects the learning preferences of all its subscribers, but rather subscribers' learning choices are restricted to the subset of stories covered substantially by the news.

References

- Admati, Anat (1985), “A Noisy Rational Expectations Equilibrium for Multi-Asset Securities.” *Econometrica*, 53(3), 629–658.
- Admati, Anat and Pauli Pfleiderer (1986), “A Monopolistic Market For Information.” *Journal of Economic Theory*, 39, 400–438.
- Admati, Anat and Pauli Pfleiderer (1990), “Direct and Indirect Sales of Information.” *Econometrica*, 58(4), 901–928.
- Allen, Franklin (1990), “The Market For Information and the Origin of Financial Intermediation.” *Journal of Financial Intermediation*, 1(1), 3–30.
- Barber, Brad and Terrance Odean (2008), “All that Glitters: The Effect of Attention on the Buying Behavior of Individual and Institutional Investors.” *Review of Financial Studies*, 21, 785–818.
- Botosan, Christine, Marlene Plumlee, and Yuan Xie (2004), “The Role of Information Precision in Determining the Cost of Equity Capital.” *Review of Accounting Studies*, 92, 233–259.
- Brennen, Michael and Henry Cao (1997), “International Portfolio Investment Flows.” *Journal of Finance*, 52(5), 1851–1879.
- Coval, Joshua and Tobias Moskowitz (1999), “Home Bias at Home: Local Equity Preference in Domestic Portfolios.” *Journal of Finance*, 54(6), 42045–2073.
- Da, Zhi, Joseph Engelberg, and Pengjie Gao (2011), “In Search of Attention.” *Journal of Finance*, 66(5), 1461–1499.

- Diamond, Douglas and Robert Verrecchia (1991), “Disclosure, Liquidity, and the Cost of Capital.” *Journal of Finance*, 46(4), 1325–1359.
- Easley, David and Maureen O’Hara (2004), “Information and the Cost of Capital.” *Journal of Finance*, 58(4), 1553–1583.
- Easley, David, Soeren Hvidkjaer, and Maureen O’Hara (2002), “Is Information Risk a Determinant of Asset Returns?” *Journal of Finance*, 57(5), 2185–2221.
- Engelberg, Joseph, Caroline Sasseville, and Jared Williams (2012), “Market Madness? the Case of Mad Money.” *Management Science*, 58(2), 351–364.
- Engelberg, Joseph and Christopher Parsons (2011), “The Causal Impact of Media in Financial Markets.” *Journal of Finance*, 66(1), 67–97.
- Epstein, Larry and Stanley Zin (1989), “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns; a Theoretical Framework.” *Econometrica*, 57(4), 937–969.
- Fang, Lily and Joel Peress (2009), “Media Coverage and the Cross-section of Stock Returns.” *Journal of Finance*, 64(5), 2021–2052.
- Grinnblatt, Mark and Matti Keloharju (2001), “How Distance, Language, and Culture Influences Stockholdings and Trades.” *Journal of Finance*, 56(3), 1053–1074.
- Grossman, Sanford and Joseph Stiglitz (1980), “On the Impossibility of Informationally Efficient Markets.” *The American Economic Review*, 70(3), 393–408.
- Hamilton, James T. (2004), *All the News That’s Fit To Sell: How the Market Transforms Information Into News*. Princeton University Press.

- Hellwig, Martin (1980), “On the Aggregation of Information in Competitive Markets.” *Journal of Economic Theory*, 22(3), 477–498.
- Hong, Harrison and Jeremy Stein (2007), “Disagreement and the Stock Market.” *Journal of Economic Perspectives*, 21(2), 109–128.
- Huberman, Gur (2001), “Familiarity Breeds Investment.” *Review of Financial Studies*, 14(3), 659–680.
- Huberman, Gur and Tomer Regev (2001), “Contagious Speculation and a Cure for Cancer: A Nonevent that Made Stock Prices Soar.” *Journal of Finance*, 56(1), 387–396.
- Hughes, John, Jing Liu, and Jun Liu (2007), “Information Asymmetry, Diversification, and the Cost of Capital.” *Accounting Review*, 82(3), 702–729.
- Ivkovic, Zoran, Clemens Sialm, and Scott Weisbenner (2008), “Portfolio Concentration and the Performance of Individual Investors.” *Journal of Financial and Quantitative Analysis*, 43(3), 613–656.
- Kacperczyk, Marcin, Jaromir Nosal, and Luminita Stevens (2014), “Investor Sophistication and Capital Income Inequality.” Working Paper.
- Kelly, Bryan and Alexander Ljungqvist (2013), “Testing Asymmetric-Information Asset Pricing Models.” *Review of Financial Studies*, 26(6), 1443–1482.
- Kim, Oliver and Robert Verrecchia (1997), “Pre-Announcement and Event-Period Private Information.” *Journal of Accounting and Economics*, 24(3), 395–419.
- Kreps, David and Evan Porteus (1978), “Temporal Resolution of Uncertainty and Dynamic Choice Theory.” *Econometrica*, 46(1), 185–200.

- Kyle, Albert (1985), “Continuous Auctions and Insider Trading.” *Econometrica*, 53(6), 1315–1336.
- Lambert, Richard, Christian Leuz, and Robert Verrechia (2012), “Information Precision, Information Asymmetry, and the Cost of Capital.” *Review of Finance*, 16, 1–29.
- Manela, Asaf (2014), “The Value of Diffusing Information.” *Journal of Financial Economics*, 111(5), 181–199.
- Merton, Robert (1987), “A Simple Model of Capital Market Equilibrium with Incomplete Information.” *Journal of Finance*, 42(3), 483–510.
- Miller, Edward (1977), “Risk, Uncertainty, and Divergence of Opinion.” *Journal of Finance*, 32, 1151–1168.
- Mondria, Jordi (2010), “Portfolio Choice, Attention Allocation, and Price Comovement.” *Journal of Economic Theory*, 145(5), 1837–64.
- Mondria, Jordi and Thomas Wu (2010), “The Puzzling Evolution of the Home Bias, Information Processing, and Financial Openness.” *Journal of Economic Dynamics and Control*, 34(5), 876–896.
- Mondria, Jordi and Thomas Wu (2012), “Asymmetric Attention and Stock Returns.” Working Paper.
- O’Hara, Maureen (2003), “Presidential Address: Liquidity and Price Discovery.” *Journal of Finance*, 584, 1335–1354.
- Peng, Lin and Wei Xiong (2006), “Investor Attention, Overconfidence, and Category Learning.” *Journal of Financial Economics*, 80, 563–602.

- Peng He, William, Andrew Lepone, and Henry Leung (2013), “Information Asymmetry and the Cost of Equity Capital.” *International Review of Economics and Finance*, 27(6), 611–620.
- Peress, Joel (2010), “The Tradeoff between Risk Sharing and Information Production in Financial Markets.” *The Journal of Economic Theory*, 145(1), 124–155.
- Peress, Joel (2014), “The Media and the Diffusion of Information in Financial Markets: Evidence from Newspaper Strikes.” *Journal of Finance*.
- Sen, Amartya (1973), *On Economic Inequality*. Clarendon Press, Oxford.
- Sengupta, Partha (1998), “Corporate Disclosure Quality and the Cost of Debt.” *The Accounting Review*, 73(4), 459–474.
- Sims, Christopher (2006), “Rational Inattention: Beyond the Linear-Quadratic Case.” *American Economic Review Papers and Proceedings*, 96(2), 158–163.
- Solomon, David (2012), “Selective Publicity and Stock Prices.” *Journal of Finance*, 67(2), 599–638.
- Tetlock, Paul C (2010), “Does Public Financial News Resolve Asymmetric Information?” *Review of Financial Studies*, 23, 3520–557.
- Tetlock, Paul C (2011), “All the News That’s Fit to Reprint: Do Investors React to Stale Information?” *Review of Financial Studies*, 24(5), 1481–1512.
- Van Nieuwerburgh, Stign and Laura Veldkamp (2009), “Information Immobility and the Home Bias Puzzle.” *Journal of Finance*, 64(3).
- Van Nieuwerburgh, Stign and Laura Veldkamp (2010), “Information Acquisition and Under-Diversification.” *Review of Economic Studies*, 77(2).

Veldkamp, Laura (2006), “Media Frenzies in Markets for Financial Information.” *American Economic Review*, 96(3), 577–601.

Verrechia, Robert (1982), “Information Acquisition in a Noisy Rational Expectations Economy.” *Econometrica*, 50(6), 1415–1430.

Wang, Jiang (1993), “A Model of Intertemporal Asset Prices under Asymmetric Information.” *Review of Economic Studies*, 60, 249–282.

7 Appendix

7.1 Proof of Proposition 1

The proof is almost identical to the one found in Hellwig (1980) and Admati (1985), but with independent prior signals. First conjecture that p is of the form:

$$p = g_1 + g_2 f + g_3 x. \quad (30)$$

Next, use the market-clearing conditions to solve for g_1 , g_2 , and g_3 :

$$\bar{x} + x = \int_0^1 q_i di. \quad (31)$$

Substituting optimal asset demand (14) gives:

$$\bar{x} + x = \int_0^1 \frac{\hat{f}_i - p}{\rho \hat{\tau}_i^{-1}} di. \quad (32)$$

Substituting in equation (11) for investor i 's posterior mean \hat{f}_i gives:

$$\bar{x} + x = \int_0^1 \frac{\tilde{\tau}_i \tilde{f}_i + l_i \eta_i s_i + \zeta z(p)}{\rho \hat{\tau}_i^{-1} (\tilde{\tau}_i + l_i \eta_i + \zeta)} - \frac{p}{\rho \hat{\tau}_i^{-1}} di. \quad (33)$$

Since $\widehat{\tau}_i = \widetilde{\tau}_i + l_i \eta_i + \zeta$, this reduces to:

$$\begin{aligned} \rho(\bar{x} + x) &= \int_0^1 \widetilde{\tau}_i \widetilde{f}_i + l_i \eta_i s_i + \zeta z(p) - \frac{p}{\widetilde{\tau}_i} di \\ &= \int_0^1 \widetilde{\tau}_i \widetilde{f}_i + l_i \eta_i s_i di + \zeta z(p) - \int_0^1 p (\widetilde{\tau}_i + l_i \eta_i + \zeta) di. \end{aligned} \quad (34)$$

From (30), the public signal of f that each investor gleans from price is $\frac{p-g_1}{g_2}$. Substituting this in for $z(p)$ gives:

$$\rho(\bar{x} + x) = \int_0^1 \widetilde{\tau}_i \widetilde{f}_i + l_i \eta_i s_i di + \frac{\zeta p - \zeta g_1}{g_2} - \int_0^1 p (\widetilde{\tau}_i + l_i \eta_i + \zeta) di. \quad (35)$$

Since \widetilde{f}_i and s_i equal f in expectation:

$$\begin{aligned} \rho(\bar{x} + x) &= \int_0^1 \widetilde{\tau}_i + l_i \eta_i di f + \frac{\zeta p - \zeta g_1}{g_2} - \int_0^1 p (\widetilde{\tau}_i + l_i \eta_i + \zeta) di \\ &= \left(\bar{\tau} + \int_0^1 l_i \eta_i di \right) f - \frac{\zeta g_1}{g_2} + p \left(\frac{\zeta}{g_2} - \bar{\tau} - \int_0^1 l_i \eta_i di - \zeta \right). \end{aligned} \quad (36)$$

Let $\theta = \bar{\tau} + \int_0^1 l_i \eta_i di$, which is the average precision of the priors and private signals. Substituting θ into equation (36) and rearranging gives:

$$p \left(\frac{\zeta}{g_2} - \theta - \zeta \right) = \rho \bar{x} + \frac{\zeta g_1}{g_2} - \theta f + \rho x. \quad (37)$$

Next, solve for g_1 , g_2 , and g_3 by matching the coefficients of (30) with the coefficients of (37).

First, solving for g_2 :

$$g_2 = - \left(\frac{\zeta}{g_2} - \theta - \zeta \right)^{-1} \theta = 1. \quad (38)$$

Solving for g_3 :

$$g_3 = \left(\frac{\zeta}{g_2} - \theta - \zeta \right)^{-1} \rho = -\frac{\rho}{\theta}. \quad (39)$$

Solving for g_1 :

$$g_1 = \left(\frac{\zeta}{g_2} - \theta - \zeta \right)^{-1} \left(\rho \bar{x} + \frac{\zeta g_1}{g_2} \right) = -\frac{\rho \bar{x}}{\theta + \zeta}. \quad (40)$$

Substituting these coefficients into equation (30) yields:

$$p = f - \left(\frac{\rho \bar{x}}{\theta + \zeta} + \frac{\rho x}{\theta} \right), \quad (41)$$

which is exactly equation (15). Finally, each agent can invert this price function into a normally distributed signal about f , where the signal's precision is:

$$\zeta = \left(\frac{\theta}{\sigma_x \rho} \right)^2, \quad (42)$$

which completes the proof.

7.2 Deriving Period One Utility

The period two objective function is:

$$U_{i,2} = \rho E_{i,2}(W_{i,3}) - \frac{\rho^2}{2} \text{Var}_{i,2}(W_{i,3}), \quad (43)$$

subject to:

$$W_{i,3} = W_{i,0} + q_i(f - p) + \pi_i - cl_i. \quad (44)$$

Substituting the terminal wealth equation (44) into the period two utility function (43) gives:

$$U_{i,2} = \rho E_{i,2}(W_{i,0} + q_i(f - p) + \pi_i - cl_i) - \frac{\rho^2}{2} \text{Var}_{i,2}(W_{i,0} + q_i(f - p) + \pi_i - cl_i). \quad (45)$$

Prices are observed in period 2, $E_{i,2}(f) = \hat{f}_i$, and $\text{Var}_{i,2}(f) = \hat{\tau}_i^{-1}$. Distributing period two expectation and variance operators gives:

$$U_{i,2} = W_{i,0} + \rho q_i(\hat{f}_i - p) - \frac{\rho^2 q_i^2}{2\hat{\tau}_i} + \pi_i - cl_i. \quad (46)$$

Next, substitute in optimal asset demand (14) to find period two indirect utility and simplify:

$$\begin{aligned} U_{i,2} &= W_{i,0} + \rho \left(\frac{\hat{\tau}_i(\hat{f}_i - p)}{\rho} \right) (\hat{f}_i - p) - \frac{\rho^2}{2\hat{\tau}_i} \left(\frac{\hat{\tau}_i(\hat{f}_i - p)}{\rho} \right)^2 + \pi_i - cl_i \\ &= W_{i,0} + \frac{\hat{\tau}_i}{2} (\hat{f}_i - p)^2 + \pi_i - cl_i. \end{aligned} \quad (47)$$

Signal and price realizations are unknown to investors in period one, so expectations

must be taken over $(\widehat{f}_i - p)^2$ in order to derive period one utility:

$$E_{i,1}(U_{i,2}) = U_{i,1} = W_{i,0} + \frac{\widehat{\tau}_i}{2} E_{i,1} \left[\left(\widehat{f}_i - p \right)^2 \right] + \pi_i - cl_i. \quad (48)$$

Since $E(X^2) = Var(X) + E(X)^2$:

$$E_{i,1} \left[\left(\widehat{f}_i - p \right)^2 \right] = Var_{i,1}(\widehat{f}_i - p) + E_{i,1}(\widehat{f}_i - p)^2. \quad (49)$$

From equation (15):

$$E_{i,1}(\widehat{f}_i - p) = E_{i,1} \left(\widehat{f}_i - f - \frac{\rho \bar{x}}{\theta + \zeta} - \frac{\rho x}{\theta} \right). \quad (50)$$

Since $E_{i,1}(x) = 0$:

$$E_{i,1}(\widehat{f}_i - p) = -\frac{\rho \bar{x}}{\theta + \zeta}. \quad (51)$$

The period one expected return is the same across all agents. Next, using the Law of Total Variance:

$$\begin{aligned} Var_{i,1}(\widehat{f}_i - p) &= Var_{i,1}(f - p) - E_{i,1}(Var_{i,2}(\widehat{f}_i - p)) \\ &= Var_{i,1} \left(f - f - \frac{\rho \bar{x}}{\theta + \zeta} - \frac{\rho x}{\theta} \right) - E_{i,1}(Var_{i,2}(\widehat{f}_i - p)). \end{aligned} \quad (52)$$

The terms θ , \bar{x} , and ζ are known in period one, while p is known in period two, implying:

$$Var_{i,1}(\widehat{f}_i - p) = \frac{\rho^2 \sigma_x^2}{\theta^2} - \widehat{\tau}_i^{-1}. \quad (53)$$

Substituting equations (53) and (51) into equation (49), and substituting this equation into

the period one utility function (48) yields:

$$U_{i,1} = W_{i,0} + \frac{\widehat{\tau}_i}{2} \left(\frac{\rho^2 \sigma_x^2}{\theta^2} - \widehat{\tau}_i^{-1} + \left(\frac{\rho \bar{x}}{\theta + \zeta} \right)^2 \right) + \pi_i - cl_i. \quad (54)$$

Finally, dropping the constant term, substituting $\frac{\theta^2}{\rho^2 \sigma_x^2}$ in for ζ by equation (15), and substituting in the equation for posterior precision delivers the period one problem captured by equation (19):

$$U_{i,1} = W_{i,0} + \frac{1}{2} \left(\left(\frac{\rho \sigma_x}{\theta} \right)^2 + \left(\frac{\rho \bar{x}}{\theta + \left(\frac{\theta}{\rho \sigma_x} \right)^2} \right)^2 \right) (\widetilde{\tau}_i + l_i \eta_i + \zeta) + \pi_i - cl_i. \quad (55)$$

7.3 Proof of Proposition 2

The proof follows directly from the free entry assumption. If c^* is above average cost, another agent can enter the market for news, charge slightly below c^* , take the entire market and make a positive profit. If c^* is below average cost, any agent supplying news is making negative profit and thus strictly better off exiting the market. Finally, if c^* is priced at average cost and there is more than one supplier, then both suppliers must also be making negative profit and are strictly better off exiting.

7.4 Proof of Lemma 1

Lemma 1 is proven by contradiction. It will first be convenient to derive the (net) benefit of purchasing news to each agent, which is equal to expected utility conditional on purchasing minus expected utility conditional on not purchasing, given a particular cost c and average signal precision θ .

After substituting equation (10) into equation (19), expected utility conditional on $l_i = 1$

is:

$$U_{i,1} = W_{i,0} + R(\theta) (\tilde{\tau}_i + \eta_i + \zeta) + \pi_i - c. \quad (56)$$

The expected utility conditional on $l_i = 0$ is:

$$U_{i,1} = W_{i,0} + R(\theta) (\tilde{\tau}_i + \zeta) + \pi_i. \quad (57)$$

Taking the difference between equations (56) and (57) gives:

$$R(\theta)\eta_i - c. \quad (58)$$

Substituting in for η_i yields the net benefit of purchasing:

$$R(\theta)k\bar{\tau}\alpha i^{\alpha-1} - c. \quad (59)$$

Without loss of generality, assume that agent a purchases news in equilibrium, but agent b does not, where $b > a$. Then agent a must weakly prefer purchasing, whereas agent b must weakly prefer not purchasing:

$$R(\theta^*)k\bar{\tau}\alpha a^{\alpha-1} - c^* \geq 0, \quad (60)$$

$$R(\theta^*)k\bar{\tau}\alpha b^{\alpha-1} - c^* \leq 0. \quad (61)$$

Equations (60) and (61) imply that $a \geq b$, which is a contradiction. Therefore, an agent cannot purchase news in equilibrium unless all agents with a higher prior precision do the same. This in turn means that a given equilibrium fraction of purchasers λ^* must be the λ^*

agents with the highest prior precision, i.e., agents from $1 - \lambda^*$ to 1, so that:

$$\theta(\lambda^*, \alpha) = \bar{\tau} + \int_{1-\lambda^*}^1 \eta_i di. \quad (62)$$

Substituting in for η_i gives:

$$\theta(\lambda^*, \alpha) = \bar{\tau} + \int_{1-\lambda^*}^1 k\bar{\tau}\alpha i^{\alpha-1} di, \quad (63)$$

which completes the proof.

7.5 Proof of Proposition 3

First, given a particular $\lambda > 0$, define the (net) benefit of purchasing news to the agent who has the least to gain from doing so:

$$B(\lambda, \alpha) = R(\theta(\lambda, \alpha)) \eta_{1-\lambda} - \frac{\chi}{\lambda}, \quad (64)$$

where news is priced at average cost from Proposition 2 and the purchaser who benefits the least is agent $1 - \lambda$ from Lemma 1. Substituting in signal precision of agent $1 - \lambda$ gives:

$$B(\lambda, \alpha) = R(\theta(\lambda, \alpha)) k\bar{\tau}\alpha(1 - \lambda)^{\alpha-1} - \frac{\chi}{\lambda}. \quad (65)$$

There are three cases:

If $B(\lambda, \alpha) < 0$ for all $\lambda \in (0, 1]$: Assume $\lambda^* > 0$. Then all agents from $1 - \lambda^*$ to 1 must weakly prefer purchasing. However, the benefit of purchasing to agent $1 - \lambda$ is strictly negative for any $\lambda > 0$ by assumption. Therefore, $\lambda^* > 0$ cannot be an equilibrium. By the same logic, if $\lambda^* = 0$, no agent can enter the market for news and charge a price that makes them

positive profit. Therefore, $\lambda^* = 0$.

If $B(1, \alpha) \geq 0$: Assume $\lambda^* = 1$. Since the agent with the smallest prior precision (i.e., agent 0) weakly prefers to purchase, all other agents with a higher prior precision must as well. Furthermore, if any agent tries to enter the market for news and charge a small enough price to undermine the incumbent supplier, he must be charging less than χ and thus be making negative profit. Therefore $\lambda^* = 1$ is an equilibrium.

If $B(\lambda, \alpha) = 0$ for some $\lambda \in (0, 1)$: Assume that $\lambda^* \in (0, 1)$ and $B(\lambda^*, \alpha) = 0$. Then the marginal agent (agent $(1 - \lambda^*)$) is indifferent to purchasing by assumption, and all agents with high prior precisions strictly prefer doing so. Therefore, λ^* is an equilibrium for news demand. However, consider the case where $\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*} > 0$. In this case, an agent can enter the market for news, charge $\frac{\chi}{\lambda^* + e}$ for an arbitrarily small $e > 0$ and get at least $\lambda^* + e$ to purchase, making a positive profit. Therefore, only a point where $\frac{\partial B(\lambda, \alpha)}{\partial \lambda} \leq 0$ can be an interior equilibrium for both the supply and demand for news, which completes the proof.

7.6 Proof of Proposition 4

Assume that the equilibrium fraction of purchasers is positive ($\lambda^* > 0$). First, note that $\eta_0 = 0$ when $\alpha > 1$. Therefore, an equilibrium where all agents purchase ($\lambda^* = 1$) can only occur when $\alpha = 1$, which in turn implies that $\lambda^* < 1$ for any incremental increase in α . Next, note that the cost of news c^* is strictly decreasing in the equilibrium fraction of purchasers λ^* by Proposition 2. Given these two statements, proving Proposition 4 amounts to showing that:

$$\frac{\partial \lambda^*}{\partial \alpha} < 0, \tag{66}$$

when $\lambda^* \in (0, 1)$. Observe that:

$$\frac{d\theta(\lambda^*, \alpha)}{d\alpha} = \frac{\partial\theta(\lambda^*, \alpha)}{\partial\lambda^*} \frac{\partial\lambda^*}{\partial\alpha} + \frac{\partial\theta(\lambda^*, \alpha)}{\partial\alpha}, \quad (67)$$

which implies:

$$\frac{\partial\lambda^*}{\partial\alpha} = \frac{\frac{d\theta(\lambda^*, \alpha)}{d\alpha} - \frac{\partial\theta(\lambda^*, \alpha)}{\partial\alpha}}{\frac{\partial\theta(\lambda^*, \alpha)}{\partial\lambda^*}}. \quad (68)$$

Recall that $\theta(\lambda^*, \alpha) = \bar{\tau} + \int_{1-\lambda^*}^1 k\bar{\tau}\alpha i^{\alpha-1} di$, so by the Leibnitz rule:

$$\frac{\partial\theta(\lambda^*, \alpha)}{\partial\lambda^*} = \eta_{1-\lambda^*} > 0, \quad \frac{\partial\theta(\lambda^*, \alpha)}{\partial\alpha} = -k\bar{\tau}(1-\lambda^*)^\alpha \log(1-\lambda^*) > 0. \quad (69)$$

Therefore, equation (68) must be strictly negative if $\frac{d\theta(\lambda^*, \alpha)}{d\alpha} < 0$. From Proposition 3, if $\lambda^* \in (0, 1)$:

$$B(\lambda^*, \alpha) = R(\theta(\lambda, \alpha)) \eta_{1-\lambda^*} - \frac{\chi}{\lambda^*} = 0. \quad (70)$$

Taking the total derivative of both sides yields:

$$\frac{\partial B(\lambda^*, \alpha)}{\partial\lambda^*} \frac{\partial\lambda^*}{\partial\alpha} + \frac{\partial B(\lambda^*, \alpha)}{\partial\alpha} = 0. \quad (71)$$

which implies:

$$\frac{\partial\lambda^*}{\partial\alpha} = -\frac{\frac{\partial B(\lambda^*, \alpha)}{\partial\alpha}}{\frac{\partial B(\lambda^*, \alpha)}{\partial\lambda^*}}. \quad (72)$$

Next, note that:

$$\frac{\partial B(\lambda^*, \alpha)}{\partial \alpha} = \frac{\partial R(\theta(\lambda^*, \alpha))}{\partial \theta(\lambda^*, \alpha)} \frac{\partial \theta(\lambda^*, \alpha)}{\partial \lambda^*} \eta_{1-\lambda^*} + R(\theta(\lambda^*, \alpha)) \frac{\partial \eta_{1-\lambda^*}}{\partial \alpha}, \quad (73)$$

$$\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*} = \frac{\partial R(\theta(\lambda^*, \lambda^*))}{\partial \theta(\lambda^*, \alpha)} \frac{\partial \theta(\lambda^*, \alpha)}{\partial \lambda^*} \eta_{1-\lambda^*} + R(\theta(\lambda^*, \alpha)) \frac{\partial \eta_{1-\lambda^*}}{\partial \lambda^*} + \frac{\chi}{\lambda^{*2}}. \quad (74)$$

Substituting equations (72), (73), and (74) into equation (67) and canceling terms gives:

$$\frac{d\theta(\lambda^*, \alpha)}{d\alpha} = \frac{1}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} \left[-\frac{\partial \theta(\lambda^*, \alpha)}{\partial \lambda^*} \left(R(\theta(\lambda^*, \alpha)) \frac{\partial \eta_{1-\lambda^*}}{\partial \alpha} \right) + \frac{\partial \theta(\lambda^*, \alpha)}{\partial \alpha} \left(R(\theta(\lambda^*, \alpha)) \frac{\partial \eta_{1-\lambda^*}}{\partial \lambda^*} + \frac{\chi}{\lambda^{*2}} \right) \right]. \quad (75)$$

Since $R(\theta(\lambda, \alpha)) \eta_{1-\lambda^*} = \frac{\chi}{\lambda^*}$ in equilibrium:

$$\begin{aligned} \frac{d\theta(\lambda^*, \alpha)}{d\alpha} &= \frac{1}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} \left[-\frac{\partial \theta(\lambda^*, \alpha)}{\partial \lambda^*} \left(\frac{\chi}{\lambda^*} \frac{\partial \eta_{1-\lambda^*}}{\partial \alpha} \right) + \frac{\partial \theta(\lambda^*, \alpha)}{\partial \alpha} \left(\frac{\chi}{\lambda^*} \frac{\partial \eta_{1-\lambda^*}}{\partial \lambda^*} + \frac{\chi}{\lambda^{*2}} \right) \right] \\ &= \frac{\frac{\chi}{\lambda^*}}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} \left[-\frac{\partial \theta(\lambda^*, \alpha)}{\partial \lambda^*} \left(\frac{\frac{\partial \eta_{1-\lambda^*}}{\partial \alpha}}{\eta_{1-\lambda^*}} \right) + \frac{\partial \theta(\lambda^*, \alpha)}{\partial \alpha} \left(\frac{\frac{\partial \eta_{1-\lambda^*}}{\partial \lambda^*}}{\eta_{1-\lambda^*}} + \frac{\chi}{\lambda^*} \right) \right]. \end{aligned} \quad (76)$$

Next:

$$\begin{aligned} \left(\frac{\frac{\partial \eta_{1-\lambda^*}}{\partial \lambda^*}}{\eta_{1-\lambda^*}} + \frac{\chi}{\lambda^{*2}} \right) &= -\frac{(\alpha - 1)\bar{\tau}\alpha(1 - \lambda^*)^{\alpha-2}}{\bar{\tau}\alpha(1 - \lambda^*)^{\alpha-1}} + \frac{1}{\lambda^*} \\ &= \frac{1 - \alpha\lambda^*}{\lambda^*(1 - \lambda^*)}. \end{aligned} \quad (77)$$

Substituting the equation for $\frac{\partial \theta(\lambda^*, \alpha)}{\partial \lambda^*}$ and $\frac{\partial \theta(\lambda^*, \alpha)}{\partial \alpha}$ along with equation (77) into (76) gives:

$$\frac{d\theta(\lambda^*, \alpha)}{d\alpha} = \frac{\frac{\chi}{\lambda^*}}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} \left[-\eta_{1-\lambda^*} \left(\frac{\frac{\partial \eta_{1-\lambda^*}}{\partial \alpha}}{\eta_{1-\lambda^*}} \right) - k\bar{\tau}(1 - \lambda^*)^\alpha \log(1 - \lambda^*) \left(\frac{1 - \alpha\lambda^*}{\lambda^*(1 - \lambda^*)} \right) \right]. \quad (78)$$

Since $\frac{\partial \eta_{1-\lambda^*}}{\partial \alpha} = k\bar{\tau}(1-\lambda^*)^{\alpha-1}(1+\alpha \log(1-\lambda^*))$:

$$\begin{aligned} \frac{d\theta(\lambda^*, \alpha)}{d\alpha} &= \frac{\frac{\chi}{\lambda^*}}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} \left[-k\bar{\tau}(1-\lambda^*)^{\alpha-1}(1+\alpha \log(1-\lambda^*)) - k\bar{\tau}(1-\lambda^*)^\alpha \log(1-\lambda^*) \left(\frac{1-\alpha\lambda^*}{\lambda^*(1-\lambda^*)} \right) \right] \\ &= \frac{-\lambda^* k\bar{\tau}(1-\lambda^*)^{\alpha-1} \frac{\chi}{\lambda^*}}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} [\lambda^* + \log(1-\lambda^*)]. \end{aligned} \quad (79)$$

The term $\lambda^* + \log(1-\lambda^*)$ is strictly negative for all $\lambda^* \in (0, 1)$, and $\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}$ cannot be positive for any interior solution by Proposition 3. Therefore, $\frac{d\theta(\lambda^*, \alpha)}{d\alpha}$ is strictly negative, which implies $\frac{\partial \lambda^*}{\partial \alpha} < 0$ when $\lambda^* > 0$ by equation (68), thus completing the proof.

7.7 Proof of Proposition 5

Assume that the equilibrium fraction of purchasers is positive ($\lambda^* > 0$). From equation (18), the period one excess return per share $E_1(f-p)$ is:

$$E_1(f-p) = \frac{\rho \bar{x}}{\int_0^1 \widehat{\tau}_i di}, \quad (80)$$

where $\int_0^1 \widehat{\tau}_i di$ is strictly increasing in $\theta(\lambda^*, \alpha)$. Therefore, it is sufficient to show that:

$$\frac{d\theta(\lambda^*, \alpha)}{d\alpha} < 0, \quad (81)$$

when $\lambda^* > 0$. But this follows directly from equation (79). Therefore the equilibrium risk premium $E_1(f-p)^*$ is strictly increasing in α , which completes the proof.

7.8 Proof of Proposition 6

If the cost of news c is exogenously fixed, the net benefit of purchasing to the marginal agent is:

$$B(\lambda, \alpha) = R(\theta(\lambda^*, \alpha)) k\bar{\tau}\alpha(1 - \lambda^*)^{\alpha-1} - c, \quad (82)$$

which is now strictly decreasing in λ because the endogenous cost is absent. Since the period one excess return per share $E_1(f - p)^*$ is strictly decreasing in θ , it is sufficient to show that there exists a cutoff value $\bar{\alpha}$ such that:

$$\frac{d\theta(\lambda^*, \alpha)}{d\alpha} > 0, \quad (83)$$

for all $\alpha > \bar{\alpha}$. First, note that for a sufficiently high α , equation (82) is strictly positive when $\lambda = 0$, which in turn implies that $\lambda^* > 0$. Therefore, for a sufficiently high α , we can apply the characterization of an interior solution found in Proposition 3 and repeat the same steps as the proof of Proposition 4, which yields:

$$\begin{aligned} \frac{d\theta(\lambda^*, \alpha)}{d\alpha} &= \frac{c}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} \left[-k\eta_{1-\lambda^*} \left(\frac{\frac{\partial \eta_{1-\lambda^*}}{\partial \alpha}}{\eta_{1-\lambda^*}} \right) + k\bar{\tau}(1 - \lambda^*)^\alpha \log(1 - \lambda^*) \left(\frac{\alpha - 1}{1 - \lambda^*} \right) \right] \\ &= \frac{-k\bar{\tau}(1 - \lambda^*)^{\alpha-1}c}{\frac{\partial B(\lambda^*, \alpha)}{\partial \lambda^*}} [1 + \log(1 - \lambda^*)]. \end{aligned} \quad (84)$$

The term $1 + \log(1 - \lambda^*)$ is strictly positive for all λ^* less than 0.632. Since $\lambda^* \in (0, 1)$ for a sufficiently high α by the logic above, and λ^* is strictly decreasing for a sufficiently large increase in α by Proposition 4, $1 + (1 - \lambda^*)$ must be strictly positive above some cutoff α , which makes equation (84) strictly positive as well, thus completing the proof.

7.9 Proof of Proposition 7

Assume that the precision of news is independent of prior precision: $\eta_i = k$. In this case, the average precision of priors and purchased signals is not influenced by which agents are purchasing, so that:

$$\theta = \bar{\tau} + \int_0^{\lambda^*} k di, \quad (85)$$

which is independent of α . Since both θ and η_i are independent of α , a change in α cannot affect the equilibrium fraction of buyers, which completes the proof.