Labor Time, Commodity, and State Money: Complimentary Approaches to Marxian Value Theory

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Abstract: The New Interpretation of Marxian value theory has a well-developed theory of money and price but is not a response to important critiques of Marxian value theory. Conversely the approach developed by Roberts and others, sometimes referred to as a 'single system' interpretation, is a comprehensive theory of value and price of production on a labor time standard, but is underdeveloped with regards to monetary phenomena. Both approaches use the principle of the conservation of value and the corollary that the price system distributes labor time. These are complimentary approaches and integrating them provides a labor theory that rigorously addresses value and price in an economy using commodity or state money. Roberts's contribution is not well understood, and this paper clarifies aspects of the theory that are consistently misinterpreted. It concludes that modern approaches to Marxian value theory are characterized by large areas of agreement and convergence rather than rivalry.

Keywords: Marxian value theory, transformation problem, New Interpretation, single system, money.

JEL Codes: B510, D460, E420

Several different approaches to Marxian value theory emerged in response to the critiques of the 1960's and 1970's. Since these were motivated by serious challenges to previous interpretations of a labor theory of value, this literature often emphasizes what is novel or different about a particular approach rather than how it is similar or complimentary to others. This development makes the literature on Marxian value theory since the early 1980's seem disparate and fragmented, characterized by rivalry rather than convergence.

This paper takes a different perspective. It identifies similarities and complementaries between two different approaches and develops a synthetic interpretation. Specifically, it considers the New Interpretation (NI), originally associated with the work of Gérard Duménil (1980) and Duncan Foley (1982, 1986), and an approach that has come to be referred to–somewhat inaccurately–as a "single system interpretation" (SSI). The SSI was initially presented by Bruce Roberts (1981) and subsequently elaborated by Wolff, Roberts and Callari (1982), Wolff, Callari and Roberts (1984), and Roberts (1987, 1997, 2009).[1] These two approaches have characteristics that make them quite different, but should be seen as complimentary rather than rival. Together these they provide a means to integrate both monetary and labor time magnitudes in a single consistent framework of competitive prices and values.

The NI deals exclusively with macroeconomic aggregates and emphasizes the role of money. It is a set of propositions about a labor theory of value in terms of macroeconomic aggregates. These propositions represent a very thin or minimal theory that any viable interpretation of a labor theory of value should

[1] In dealing with single-system value theory I only consider this version, which, in light of what came later, has come to be called the 'simultanaest' version. The Temporal Single System Interpretation (TSSI) is a later development that, because of its distinctive emphasis on sequential periods, is not easily compared with other non-temporal theories. Since the objective of this paper is to identify similarities and complimentarities between the NI and the SSI, discussing the TSSI is beyond its scope.

satisfy. Its primary weakness is that as a theory of aggregates, rather than a theory of individual commodity values or prices, it is not a response to the significant criticisms of Marxian value theory of the type leveled by Bortkiewicz (1952, (1906/7); 1949 (1907)), Steedman (1977), and others. The neo-Ricardian critique is well-known. Its primary assertion is that the conditions of production and distribution (the real wage) specified in physical terms are sufficient to determine the rate of profit and production prices, and, therefore, the quantity of labor "embodied" in commodities plays no role in their determination. Furthermore, this line of critique asserts, while it is also possible to derive a vector of embodied labor times from a given set of economic data, there is no systematic relationship between these labor times and competitive prices except in special cases. From this perspective Marxian value theory is at best a "dual system", one system for labor values and a different one for prices, and since prices are the relevant variables in a capitalist economy the value system is redundant. In this view Marxian value theory is also internally inconsistent because the two aggregate equalities that Marx states must hold between values and prices of production–the aggregate sum of prices equal to the aggregate sum of values and the aggregate sum of profits equal to the aggregate sum of surplus value–cannot be shown to simultaneously hold in this dual system.[2]

When initially developed the NI was often understood to be a solution to the problems so clearly articulated by critics of Marxian value theory, but this is no longer true. The NI does provide a way of defining a labor theory of value so that Marx's two aggregate equalities are satisfied in some form, but it has important limitations. The most obvious is that it applies only to macroeconomic aggregates, which is certainly less than what Marx endeavored to do

[2] ". . ., the sum of the profits in all spheres of production must equal the sum of the surplus-values, and the sum of the prices of production of the total social product equal the sum of its value (Marx 1976b, 173)."

in his treatment of values and prices of production in volume three of *Capital*. And despite some assertions to the contrary[3], the NI also appears to accept that Marxian theory is a dual system of values and prices, with each determined separately and independently of one another. There is no concept of "value" in the NI beyond the embodied labor coefficients that Pasinetti (1977, 76) calls "vertically integrated labor coefficients", which have figured prominently in the critique of a labor theory of value. Indeed, Foley (2000, 20) argues that the central insight of the NI involves the monetary expression of labor time (MELT), which dispenses with any need for a separate accounting based upon embodied labor coefficients at all (see also Mohun 2004, 77), but since the MELT is typically defined using them it does not fully dispense with them.

The NI is then only a very partial response to the criticisms raised in the debates over Marxian value theory, and leaves the "transformation problem" intact. Mohun, a prominent exponent of the NI, emphasizes this unambiguously:

> The (NI) is not in itself a 'solution' to anything. It is rather an approach to the labour theory of value that provides an *ex post* accounting system that is theoretically coherent, and compatible with accounting practices in capitalist society. As an accounting system, no relations of determination are expressed. In particular, that profits are an exact measure of unpaid labour is *not* a deduction from more primitive assumptions. Rather, the labour theory of value is itself *defined* so that profits are an exact measure of unpaid labour.

[3]Foley (1997, 493; 2000, 31-32, 35) classifies the NI as a type of single system approach because he takes the defining characteristics to be that money magnitudes are forms or expressions of value, and therefore it is possible to convert them to labor times by using a coefficient for the monetary expression of labor time. The TSSI does have these properties but they are not why its creators labeled it a 'single system' approach, nor are they what it has in common with the early versions of the SSI.

> This means that the (NI) is a very general one, and remains valid whatever prices happen to be (2004, 77).

The NI has been criticized by other Marxists as well. Shaikh and Tonak (1994, 179) critique the NI as a version of Adam Smith's "labor commanded" theory of value rather than a Marxian theory of value in which it is the labor in production that adds value to commodities. They argue that the NI definition of the value of labor power is simply the labor commanded in exchange by the wage and surplus value is simply the labor commanded in exchange by profit income. Given the NI's reliance on macroeconomic aggregates and monetary magnitudes it is not difficult to see why they would draw that conclusion. Indeed Foley (1982, 44) anticipated that critique in his initial statement of the NI and tried to address it, but it persists. Saad-Filho (1996) and Moseley (2000) have both criticized the NI for redefining the aggregate equalities that Marx states must hold between the price measure and value measure of the social product, as applying to the net product rather than the gross or total product. Likewise Mohun (2004) suggests that the NI lacks generality because it satisfies the price-value equality only for the net product and thus excludes constant capital from the aggregate identity. Saad-Filho also criticizes the NI for its reliance on empirical market prices to define the value of money and thereby all value quantities. In the NI the effects of circulation on market prices, such as supply and demand imbalances, will impact monetary quantities and this is then transferred back onto the value quantities through the value of money. The performance of labor time in production then loses explanatory priority to these circulation issues in the NI.

Given that the NI is only a very partial response to the criticisms of Marxian value theory, the question remains if other approaches respond in a more satisfactory way. The SSI does because it is a theoretically consistent theory of the determination of commodity values and competitive prices in which both of Marx's aggregate equalities are satisfied. A unique aspect of the SSI that makes this possible is that all of the primary variables–values, prices of production, surplus value, and profit–are denominated in units of socially necessary abstract labor time. This labor time basis of the SSI is one of its defining characteristics, but this has generally been misunderstood. This use of labor magnitudes is not surprising for a Marxian theory, but it does raise questions about the role of money (commodity or state) and money-denominated magnitudes in this approach. These monetary issues, which are central to the NI, are underdeveloped in the SSI, and hence represent an important element of complementarity between the two theories if they can be show to be compatible.

Since the NI is a theory of aggregates there are many theories of value and price that could be consistent with it. A basic objective in this paper is to demonstrate that the SSI is one of them, and that, at least in this respect, they are complimentary rather than competing theories. An important difference between the NI and the SSI is that while the NI uses the value of money and the MELT to eliminate the need for a comprehensive labor time accounting, the SSI provides a distinct concept of value that does not give rise to the problems associated with embodied labor coefficients. Thus the SSI provides the direct counter to the neo-Ricardian critiques of Marxian value theory that the NI lacks, while also not being subject to the criticisms from within Marxism that are leveled against to the NI.

There have been at least two previous attempts to consider the similarity or compatibility of the NI and the SSI by Foley (2000, 30-34) and Mohun (2004, 79-83). Foley discusses the SSI under the heading of the Temporal Single System Interpretation (TSSI), and by subsuming it under the temporal approach fails to grasp the important differences between them. He also makes the significant error of mistaking the dimension of the prices of production in the SSI, assuming money-denominated prices (as in the TSSI and NI) instead of labor time prices of production. Mohun also makes this same mistake regarding the labor time dimension of the SSI price of production. This dimensional error may seem a minor detail in the comparison of these two theories, but both the coherence and innovativeness of the SSI, as well as an important point of fidelity to Marx's own approach to values and prices of production, actually rely on this point. Leaving this issue uncorrected in the literature promises to lead to significant confusion among readers (as it did at one time for the author of this paper).

The remainder of the paper is divided into three sections. The first presents a basic outline of the NI that draws heavily from Mohun's (1994) exceptionally clear and concise presentation. The second section develops an outline of the SSI that relies heavily on Roberts (1997). This second section also considers similarities and differences between the SSI and the NI, clarifies some points of confusion over the TSSI, and introduces monetary variables into the SSI by drawing on the NI theory of money. A final section offers concluding remarks.

The New Interpretation

A basic insight of Marxian value theory is that something is created in production and subsequently preserved in the circulation process. Foley (2012 [1983]) refers to a "law of the conservation of value" in which value (socially necessary abstract labor time) is created in production and conserved in exchange. This is central to the NI because it links labor time to the subsequent expression of value through the price system. Duménil (1984-85) refers to value as a "social substance" created in production as the result of new labor being incorporated with existing commodities, which is then distributed through the price system to the newly-produced commodities. He argues,

> The act of pricing does not create the social substance, but merely distributes it. . . The price system can only arrange the distribution between individuals and among classes . . . This is the core of Marx's theory (1983-84, 436).

And further,

> Whatever the competitive regime, the price-form always corresponds to the same process of the expression of social labor time in the body of another commodity or in a symbol of value (1983-84, 440).

Likewise, Mohun proposes that a price system is a method of distributing labor time to individual commodities and that "prices are always . . . representations or forms of value, or abstract labour (Mohun 1994, 404)."

Mohun (1994) concisely describes this relationship between labor time, commodities, and prices in the NI using two equations. The first defines the relation between the total labor performed in the economy and the gross and net product:

$$L = \mathbf{lx} = \boldsymbol{\lambda}\mathbf{y} \tag{1}$$

L, a scalar, is the total hours of socially-necessary labor time in production[4]; $\mathbf{l} > \mathbf{0}$ is the (row) vector of direct labor inputs per unit output; $\boldsymbol{\lambda} > \mathbf{0}$ is the (row) vector of "labour embodied per unit of commodity output", or, using

[4] Throughout this paper hours of labor time are understood to be hours of socially necessary abstract labor time.

Pasinetti's term, the "vertically integrated labor coefficients"; $\mathbf{x} > \mathbf{0}$ is the (column) vector of gross outputs. If $\mathbf{A} \geqslant \mathbf{0}$ is the (square) matrix of dimensionless production coefficients, then $\mathbf{y} = (\mathbf{I} - \mathbf{A})\,\mathbf{x} \geqslant \mathbf{0}$ is the (column) vector of net outputs. Equation (1) defines the equality between labor performed during a time period and its expression in the value of the net product for the NI.

Mohun's second equation defines the relationship between the labor embodied in the net product and its monetary expression:

$$\boldsymbol{\lambda}\mathbf{y} = \mathbf{p}\mathbf{y}\lambda_m \tag{2}$$

where $\mathbf{p}$ is a given (row) vector of prices, denominated in units of "money" per unit commodity, and λ_m is the (scalar) "value of money", which is measured in labor time per unit money (see also Foley 1986, 14-15). Since $\boldsymbol{\lambda}, \mathbf{y}$ and $\mathbf{p}$ are all taken as data in this approach, equation (2) defines the value of money as, $\lambda_m = \frac{\boldsymbol{\lambda}\mathbf{y}}{\mathbf{p}\mathbf{y}}$. It is important to note here that prices in this system must be money prices, that is prices denominated in units of either a money commodity or state money per unit of the ith commodity.

Equation (2) expresses formally the law of the conservation of value by defining aggregate labor time $\boldsymbol{\lambda}\mathbf{y}$ as identically equal to the price measure of the net product $\mathbf{p}\mathbf{y}\lambda_m$. Since the dimension for the value of money parameter λ_m is labor time per unit money, this converts the money-denominated magnitude $\mathbf{p}\mathbf{y}$ into a quantity of labor time and creates dimensional homogeneity in (2). Foley (2000, 7) remarks that one of Marx's far-reaching and underappreciated insights is his synthesis of a labor theory of value with a theory of money. This synthesis also underlies (2), which establishes an equivalence between a quantity of labor time and a quantity of money through the concept of the value of money.

Note that while the unit for prices is defined in terms of money, money itself is not defined as either commodity or state money. Foley (1982, 39) passes over this point by stating that the relation characterized by Mohun in (2) will hold in the case of either commodity or state money. This means that the NI is a very general system of definitions and any number of price theories could be compatible with it. The vector $\mathbf{p}$ in (2) is then not a particular price vector, but instead is a set of possible price vectors. These could be associated with any number of different theories of price setting, and include prices denominated in terms of a *numeraire* commodity (money commodity) or state money. There is also at least one system in which λ_m is unnecessary to establish the equality between a quantity of labor time and a quantity of money. That is a system in which prices are denominated in units of abstract labor time per unit commodity. In that case the scalar product $\mathbf{py}$ is itself a measure of labor time and can be equated directly with the quantity of labor in the net product without the unit conversion factor λ_m to provide dimensional homogeneity. It is demonstrated below that the SSI is a system of this type.

Equations (1) and (2) establish the chain of equivalence,

$$L = \mathbf{lx} = \boldsymbol{\lambda}\mathbf{y} = \mathbf{py}\lambda_m \tag{3}$$

This formalizes the basic insight of the NI, which is that the net product of the economy is the same whether it is measured in terms of labor time (L, $\mathbf{lx}$, or $\boldsymbol{\lambda}\mathbf{y}$) or measured in commodity or state money prices ($\mathbf{py}\lambda_m$). This equivalence is possible because each term in (3) is a quantity of labor time, with the value of money parameter converting the price quantity to a labor time quantity.

For Mohun (1994) a third equation defining the value of labor power (VLP)

completes the NI approach to Marx's theory of value:

$$VLP = w\lambda_m \tag{4}$$

where w is the wage rate and has dimension money per hour. VLP is derived from (2) and (1) as,

$$w\lambda_m = \frac{w\mathbf{lx}}{\mathbf{py}} \tag{5}$$

Mohun describes this unconventional definition of VLP in the following way:

> The value of labor power is the share of wages in net output. This defines the value of labor power in terms of a share of aggregate money value added. Rather than the concrete labor embodied in commodities the workers consume, the value of labor power is a proportion of abstract labor performed (Mohun 1994, 403).

It can be shown that the NI satisfies Marx's two aggregate equalities, but only for macroeconomic aggregates and even this requires a partial reinterpretation of the equalities. The equality between the sum of the prices and the sum of the values in the economy is satisfied directly by (2), but only for the net product of the economy, not the total product. The aggregate equality between the sum of profits and the sum of surplus value for the gross product can be shown to follow from (1), (2), (4) and (5).[5]

[5] Proof: It is shown in (5) that $w\lambda_m$ is the wage share of net output. Designating aggregate money profits as Π, the profit share $(1 - w\lambda_m)$ is,

$$1 - w\lambda_m = \frac{\mathbf{py}{-}w\mathbf{lx}}{\mathbf{py}} = \frac{\Pi}{\mathbf{py}}$$

The ratio of the value of labor power to the profit share is,

$$\frac{1 - w\lambda_m}{w\lambda_m} = \frac{\Pi}{w\mathbf{lx}} = \frac{\Pi}{W}$$

where W is aggregate wages. The portion of aggregate labor time that is surplus is $S =$

Given that the NI is a theory of aggregates that does not respond comprehensively to most of the criticisms of Marxian value theory, the question remains if other work exists that does. The SSI does provide such a response. This literature emerged contemporaneously with the NI, but has received much less attention, at least in its original formulation.

A "Single System" Interpretation

Price of Production

In the NI the price system is the means whereby labor time, the "social substance" created in production, is distributed to commodities. This is also true in the SSI, but in a much more direct way than the NI. To see this, assume the simplest case of an economy with n single-product industries, a single annual production period, and no fixed capital. These assumptions are not necessary for this approach, but they greatly simplify exposition.[6]

Let $\mathbf{b} \geqslant \mathbf{0}$ be a (column) vector containing the wage bundle of commodities per unit labor. The (square) matrix $\mathbf{M}$ of advances by capitalists in physical terms is then,

$$\mathbf{M}= \mathbf{A} + \mathbf{bl} \tag{6}$$

$L(1 - w\lambda_m)$ and the portion that constitutes variable capital is $V = Lw\lambda_m$. Thus,

$$\frac{S}{V} = \frac{L(1 - w\lambda_m)}{Lw\lambda_m}$$

Cancelling L in the numerator and denominator of this expression, and using λ_m to create dimensional homogeneity between the labor-denominated terms and the money denominated ones, it is possible to write,

$$\frac{S}{V} = \frac{\Pi\lambda_m}{W\lambda_m}$$

or

$$S = \Pi\lambda_m$$

□

This is Mohun's (1994, 403-404) proof of the equality between aggregate surplus and aggregate profit.

[6] Roberts (1997, appendix two) analyzes joint production in the SSI.

The elements of $\mathbf{M}$ are, like the elements of $\mathbf{A}$, dimensionless, and in the simplest case this matrix is assumed to be irreducible. The vector of prices of production (competitive prices) $\mathbf{p}$ is given by,

$$\mathbf{pM}(1+r) = \mathbf{p} \tag{7}$$

where r is the (scalar) rate of profit. For convenience define $\lambda^M = \frac{1}{1+r} \Rightarrow 1+r = \frac{1}{\lambda^M}$. Substituting this into (7) and rewriting this system gives,

$$\mathbf{p}[\mathbf{M} - \lambda^M \mathbf{I}] = 0 \tag{8}$$

The scalar λ^M is an eigenvalue of $\mathbf{M}$, and assume both that it is the maximum eigenvalue of $\mathbf{M}$ and that the economy under consideration is viable, that is it is capable of producing positive profits, which requires that $\lambda^M < 1$ and ensures $r = (1/\lambda^M) - 1 > 0$. Since (8) is homogenous the system must have at least one degree of freedom and requires an additional equation to be determinate.

The formulation of the price of production system in (8) is similar to the standard Sraffian price system. The primary difference being that it incorporates Marx's assumption that the wage goods should be included in the capital advanced, and hence uses $\mathbf{M}$ rather than $\mathbf{A}$. This is identical to the Marxian price of production system presented by Pasinetti (1977, 126-127), but the similarities between these two systems end here. Pasinetti states that the prices of production in this system are determined up to the choice of *numeraire*, and "adding whatever further equation we choose in order to define a *numeraire* for the price system, we can determine all the 'prices of production' in terms of the chosen *numeraire* (1977, 127)." Following Pasinetti's approach leads directly to *numeraire* or commodity money prices, with price p_j denominated in units of the *numeraire* commodity per unit commodity j.

The SSI proceeds differently, and readers accustomed to completing the system (8) by establishing a *numeraire* commodity and adding a normalization condition should take note that this is not how this system is made determinate. Instead the SSI imposes the following condition to eliminate the degree of freedom in (8) and establish the scale for the price vector:

$$\mathbf{lx} = \widetilde{\mathbf{p}}\mathbf{y} \tag{9}$$

The vector $\widetilde{\mathbf{p}}$ provides a unique solution for the system of equations (8) − (9), but it is an unusual price vector. Since $\mathbf{l}, \mathbf{x}$ and $\mathbf{y}$ are given quantities, $\widetilde{\mathbf{p}}$ is absolutely determined. Furthermore, setting the price of the net product $\widetilde{\mathbf{p}}\mathbf{y}$ equal to total labor performed $\mathbf{lx}$ requires that the prices of production be measured in units of labor time per unit commodity. Thus the prices in $\widetilde{\mathbf{p}}$ can be referred to as "labor time prices of production". This is a distinctive aspect of the SSI that seems to be widely overlooked or misunderstood. In effect, instead of choosing one of the n produced commodities as the standard for price, the SSI uses labor time and arrives at prices that are determined absolutely. These prices are the part of the total labor time that must be allocated to each commodity in order to ensure a uniform rate of profit in a system of labor time prices of production. This price theory is congruent with Duménil's (1983-84, 440) description of Marxist production prices as "re-alloted labor time", as well as with Marx's treatment in volume three of *Capital*.[7]

It is a basic tenet of the NI that a price system distributes the total labor performed during the period to the net product of that period. This is stated formally in equation (2) of the NI, and this is also precisely what (9) achieves in

[7] Consider Marx's approach to prices of production in chapter IX of volume three of *Capital* (1976b). What Marx keeps constant in his controversial 'transformation' of values (labor times) to competitive prices is the total labor time, and each of his prices represent re-allocated parts of this total labor time. They are prices denominated in hours of labor time, not money.

the SSI. The resulting price vector is a set of labor-denominated prices that also satisfy the requirement for an equalized rate of profit among producers. But it is important to note that because these prices are denominated in hours of labor per unit commodity the two scalar quantities $\mathbf{lx}$ and $\widetilde{\mathbf{p}}\mathbf{y}$ can be directly equated because both are quantities of labor time, and there is no need to multiply the price of the net product by a value of money parameter to ensure dimensional homogeneity.[8] Thus the SSI also establishes equivalence between aggregate labor time and the aggregate price of the net product, but it does this in a system of labor-denominated prices that need no unit conversion factor. In this case the concept of a "value of money" associated directly with $\widetilde{\mathbf{p}}$ makes little sense.[9] The parameter λ_m in the NI provides dimensional homogeneity in (2) by converting the quantity of money $\mathbf{py}$ into a quantity of labor time that can be equated with the quantity of hours $\boldsymbol{\lambda}\mathbf{y}$. The vector $\widetilde{\mathbf{p}}$, on the other hand, is measured in hours per unit commodity and thus $\widetilde{\mathbf{p}}\mathbf{y}$ is a quantity of labor time that can directly be equated with the quantity $\mathbf{lx}$. But while the value of money is not useful in conjunction with $\widetilde{\mathbf{p}}$, this price vector is consistent with the basic principle that the price system distributes the labor time in the economy to the aggregate net product. This principle is common to both the NI and the SSI and was arrived at independently by Roberts (1981) as well as by Duménil (1980) and Foley (1982).

The rate of profit in this system is determined by the eigenvalue equation (8), and thus by the conditions of production and distribution, $\mathbf{A}, \mathbf{b}, \mathbf{l}$ and λ^M. But it is also possible to show this as the ratio of two labor time quantities by using $\widetilde{\mathbf{p}}$. The labor time in the gross product is defined equivalently as either

[8] Both Foley (2000, 31) and Mohun (2004, 80) mistake this basic point in their discussions of the SSI.

[9] In an economy using labor time as the money commodity the value of money would, of course, be unity and a "value of money" parameter would be dimensionless (the dimensions of hour per hour cancelling). So while it is possible to attribute a "value of money" to the system defined by (8) and (9) it makes little sense to do so.

$\widetilde{\mathbf{p}}\mathbf{x} \equiv (\widetilde{\mathbf{p}}\mathbf{A}\mathbf{x} + \widetilde{\mathbf{p}}\mathbf{b}\mathbf{l}\mathbf{x})(1 + r)$ or $\widetilde{\mathbf{p}}\mathbf{x} \equiv \widetilde{\mathbf{p}}\mathbf{y} + \widetilde{\mathbf{p}}\mathbf{A}\mathbf{x}$. Equating these two definitions, and then solving for r yields,

$$r = \frac{\widetilde{\mathbf{p}}\mathbf{y} - \widetilde{\mathbf{p}}\mathbf{b}\mathbf{l}\mathbf{x}}{\widetilde{\mathbf{p}}\mathbf{A}\mathbf{x} + \widetilde{\mathbf{p}}\mathbf{b}\mathbf{l}\mathbf{x}} \tag{10}$$

The numerator of this ratio is the share of the net product that accrues as profit. Since $\widetilde{\mathbf{p}}\mathbf{y} = \mathbf{l}\mathbf{x}$ (from (9)), this equals the total labor performed ($\mathbf{l}\mathbf{x}$) net of the labor necessary to produce the total wage bundle ($\widetilde{\mathbf{p}}\mathbf{b}\mathbf{l}\mathbf{x}$). This is labor time price of production measure of the aggregate surplus labor performed. The denominator is the sum of nonlabor production inputs ($\widetilde{\mathbf{p}}\mathbf{A}\mathbf{x}$) and the aggregate wage bundle ($\widetilde{\mathbf{p}}\mathbf{b}\mathbf{l}\mathbf{x}$), all evaluated as quantities of labor time. This is consistent with Marx's proposition (1967b, 42) that the rate of profit is the ratio of the unpaid labor (surplus labor) to paid labor (labor content of labor and nonlabor inputs). It is obvious from (10) that there is an inverse relation between the rate of profit and the real wage. Increasing the real wage bundle $\mathbf{b}$ reduces the profit rate by both reducing the profit in the numerator and increasing input cost in the denominator. This inverse relation between the rate of profit and the real wage can also be shown directly from the eigenvalue equation (8).[10]

For an economy using one of the n produced commodities as money, commodity money production prices in the SSI are determined by $\widetilde{\mathbf{p}}$. Choosing commodity k as the money commodity, these are defined as,

$$\overline{p}_j = \frac{\widetilde{p}_j}{\widetilde{p}_k} \tag{11}$$

where $\overline{p}_j$ is the commodity money price of commodity j, $\widetilde{p}_j$ is the price of

[10] Proof: By definition $r = (1/\lambda^M) - 1$ so r is monotonically decreasing in λ^M. By assumption $\mathbf{M}$ is an indecomposable, semipositive, square matrix, and λ^M its maximum eigenvalue. According to the Perron-Frobenius theorems for indecomposable semipositive matrices (see Kurz and Salvadori (1995, 517) Theorem A.3.5 (f)) λ^M is an increasing function of each of the elements of $\mathbf{M}$. By (6) increasing any element of $\mathbf{b}$ increases at least some of the elements of $\mathbf{M}$, and hence increases λ^M while reducing r. □

production of that commodity, and $\widetilde{p}_k$ is the price of production of the money commodity. These commodity money prices have dimension unit k per unit j, and represent the units of the money commodity k that exchange for one unit commodity j. Using $\overline{\mathbf{p}}$, the vector of commodity money prices, it is possible to determine the value of commodity money $\overline{\lambda}_m$ in the SSI. Substituting $\widetilde{\mathbf{p}} = \widetilde{p}_k \overline{\mathbf{p}}$ from (11) into (9) gives,

$$\mathbf{lx} = \widetilde{p}_k \overline{\mathbf{p}} \mathbf{y} \tag{12}$$

or,

$$\overline{\lambda}_m = \widetilde{p}_k = \frac{\mathbf{lx}}{\overline{\mathbf{p}}\mathbf{y}} \tag{13}$$

In this case total labor is defined by $\mathbf{lx}$ rather than $\boldsymbol{\lambda}\mathbf{y}$ (as in (2)). In the NI these quantities are equal according to (1), but the vertically integrated labor coefficients $\boldsymbol{\lambda}$ play no role in the SSI and hence cannot be used to define $\overline{\lambda}_m$.

The value of commodity money in the SSI is then conceptually similar to that of the NI but with an important difference. The value of commodity money in the SSI is simply the labor time price of production of the money commodity, both of which are denominated in hours of labor time per unit, and therefore $\overline{\lambda}_m = \widetilde{p}_k$. This is the amount of social labor time that one unit of the money commodity exchanges for in a system of competitive prices.

It is also important to note that while the NI is agnostic with respect to price setting, and asserts that (2) holds definitionally no matter how prices are determined (Foley 1982, 38), the SSI does have a specific theory of production prices. $\widetilde{\mathbf{p}}$ is determined by (8) and (9), and $\overline{\mathbf{p}}$ by adding (11). The SSI is then much more comprehensive than the NI because it incorporates a Marxian theory of price of production. Labor time production prices are determined endogenously from the conditions of production and distribution in a competitive economy rather than being exogenous. But clearly both $\widetilde{\mathbf{p}}$ and $\overline{\mathbf{p}}$ should be in the set of price vectors that are consistent with the NI because both sustain the fundamental proposition that value is created in production and preserved in exchange (the conservation of value principle), as well as the corollary that the price system simply distributes this value across the commodity output. A value of money parameter is a consequence of assuming an economy using a money commodity (or state money, as shown below), and hence is necessary for $\overline{\mathbf{p}}$ but not $\widetilde{\mathbf{p}}$.

In an economy using state money the result is somewhat different. The vector of state money production prices $\widehat{\mathbf{p}}$ (with dimension currency units per unit commodity) is,

$$\widehat{\mathbf{p}} = \frac{1}{\widehat{\lambda}_m}\widetilde{\mathbf{p}} \tag{14}$$

The scalar $\widehat{\lambda}_m$ is the value of state money and its reciprocal is the MELT. In this instance the MELT coefficient serves two purposes. The first is dimensional. The unit for this version of the MELT is units of state money per unit labor, and as such it converts the labor time production prices to a quantity denominated in currency units per unit commodity.

The second purpose of the MELT in this case is to scale the elements of $\widetilde{\mathbf{p}}$ relative to the elements of $\widehat{\mathbf{p}}$. State money has no intrinsic value, therefore $\widehat{\lambda}_m$ cannot be determined except in reference to an exogenously given vector of state money prices. The structure of relative prices is established by $\widetilde{\mathbf{p}}$ and is invariant with respect to changes in $\widehat{\lambda}_m$, but $\widehat{\lambda}_m$ itself depends on the vector of prices. The definition of $\widehat{\lambda}_m$ also follows from (2):

$$\mathbf{lx} = \widehat{\mathbf{p}}^o \mathbf{y} \widehat{\lambda}_m \tag{15}$$

or

$$\widehat{\lambda}_m = \frac{\mathbf{lx}}{\widehat{\mathbf{p}}^o\mathbf{y}} \tag{16}$$

In these two expressions $\widehat{\mathbf{p}}^o$ designates any arbitrary vector of state money prices. One possible choice of theoretical interest for $\widehat{\mathbf{p}}^o$ is a vector whose elements are identical to $\widetilde{\mathbf{p}}$ except for their dimension (one a vector of state money prices the other a vector of labor time prices of production). In that case $\mathbf{lx}$ is numerically (though not dimensionally) equal to $\widehat{\mathbf{p}}^o\mathbf{y}$, and $\widehat{\lambda}_m$ equals one hour of labor time per unit money (one currency unit equals one hour of labor). This gives $\widehat{\mathbf{p}}$ the same elements as $\widetilde{\mathbf{p}}$, while maintaining their different dimensions. Alternatively, $\widehat{\mathbf{p}}^o$ could be an estimate of an empirical price vector. The vector $\widetilde{\mathbf{p}}$ could also be estimated for this economy, as well as $\widehat{\lambda}_m$ and thereby $\widehat{\mathbf{p}}$. This provides a bridge between empirical phenomena in a state money using economy and labor time prices of production. Estimating the correlation between observed state money prices and labor time prices of production is one obvious research program made possible by this.

Value

Turning now to Marx's two aggregate equalities, the SSI proposes that these hold as postulates rather than proven theorems. These are defined as:

$$\mathbf{vx} \equiv \widetilde{\mathbf{p}}\mathbf{x} \tag{17}$$

$$\mathbf{sx} \equiv \boldsymbol{\pi}\mathbf{x} \tag{18}$$

The elements of the (row) vector $\mathbf{v}$ are commodity values for the n commodities; $\mathbf{s}$ is a (row) vector of surplus value per unit; and $\boldsymbol{\pi}$ is a (row) vector of profit per unit.

It is important to note that $\mathbf{v}, \mathbf{s}$ and $\boldsymbol{\pi}$ all have the same dimension as $\widetilde{\mathbf{p}}$, hours of labor time per unit. This makes it possible to write Marx's aggregate equalities as identical scalar products, with no need to convert $\widetilde{\mathbf{p}}$ or $\boldsymbol{\pi}$ from money to labor time magnitudes. However, Roberts is clear that these labor-time quantities have monetary analogs,

> As stated, both (17) and (18) are understood here as expressions in labor time terms; both could, of course, also be measured in terms of money, as Marx often does, once the monetary unit is defined and expressed as a particular magnitude of labor time, but the labor-time unit of account, which Marx (1967a, p. 38) establishes prior to considering money, is the principal focus of interest . . . . (1997, 486)

The vector $\boldsymbol{\pi}$ represents the profit per unit on the capital advanced evaluated at prices $\widetilde{\mathbf{p}}$ and marked-up at profit rate r, or $\boldsymbol{\pi} = \widetilde{\mathbf{p}}\mathbf{M}r$. An alternative expression for $\boldsymbol{\pi}$ can be derived using (8). Profit is the difference between the selling price of a commodity and the cost of inputs $\widetilde{\mathbf{p}}(\mathbf{I} - \mathbf{M})$. Equation (8) becomes $\widetilde{\mathbf{p}}\mathbf{M} = \widetilde{\mathbf{p}}\lambda^M$ when (9) is assumed, and from this,

$$\widetilde{\mathbf{p}}(\mathbf{I} - \mathbf{M}) = \widetilde{\mathbf{p}}(1 - \lambda^M)$$

or,

$$\boldsymbol{\pi} = \left(1 - \lambda^M\right) \widetilde{\mathbf{p}} \tag{19}$$

Roberts (1997, 2009) shows that $\mathbf{v}$ is determined by the conditions of production and distribution, that is $\mathbf{A}, \mathbf{l}, \mathbf{b}$ and λ^M along with the scale of output $\mathbf{x}$, but it is possible to derive a useful expression for this vector directly from

(17) and (18). Subtracting these two identities gives,

$$\mathbf{vx} - \mathbf{sx} = \widetilde{\mathbf{p}}\mathbf{x} - \boldsymbol{\pi}\mathbf{x}$$

Substituting $\widetilde{\mathbf{p}}\mathbf{Mx} = \widetilde{\mathbf{p}}\mathbf{x} - \boldsymbol{\pi}\mathbf{x}$ and rearranging gives,

$$\mathbf{vx} = [\widetilde{\mathbf{p}}\mathbf{M} + \mathbf{s}]\mathbf{x}$$

or,

$$\mathbf{v} = \widetilde{\mathbf{p}}\mathbf{M} + \mathbf{s} \tag{20}$$

Value $\mathbf{v}$ in the SSI is, thus, the sum of the wage and commodity inputs into production, evaluated in terms of their labor time price of production $\widetilde{\mathbf{p}}$, and surplus value $\mathbf{s}$. This is consistent with Marx's argument that in a competitive economy "commodity value = cost price + surplus value (1967b, 26)".

Equation (20) is the reason that this interpretation of Marxian value theory is referred to as a 'single system'. It shows a relation between values prices, rather than the strict separation of them required by dual system interpretations. Kliman and McGlone (1999) apply the term "single system" to this approach and remark that what the SSI and TSSI share in common is the idea that prices and values are interdependent, and hence form a single system. They present a definition of value that is similar to (20) in that it measures the constant capital advanced using prices rather than values.[11] Similarly Wolff, Roberts, Callari (1982) state,

> . . . the quantity of labor time in money form which each capitalist must actually advance to get his constant capital goods (their respective prices of production) becomes a constituent part of

[11] Their value equation differs in a number of ways from (20), not least of which is that it uses "actual market prices" rather than the labor time price of production vector $\widetilde{\mathbf{p}}$.

the value of the commodities produced with those constant capital goods (574. See also Wolff, Callari, Roberts 1984, 126).

The aspect of the TSSI and SSI that makes them both "single systems" is this determination of value by price. But it is also important to note that Roberts (1997) goes to great lengths to prove that $\widetilde{\mathbf{p}}$ is determined by $\mathbf{A}, \mathbf{l}, \mathbf{b}$ and λ^M, while $\mathbf{v}$ is determined by these same things plus $\mathbf{x}$. He emphasizes that $\mathbf{v}$ can be found from the data describing production and distribution without reference to prices at all (489), and furthermore that while competitive prices can be derived *without* reference to values (as shown above), they can also be derived *from* values using a linear operator that transforms $\mathbf{v}$ values into $\widetilde{\mathbf{p}}$ prices (491). This position is maintained in his later work (2009). These points are important because they make clear that it is labor time in production that adds value to commodities, not things occurring in circulation. Some version of equation (20) is present in all of the SSI literature, but Roberts's later work shows a change in interpretation away from the functional determination of $\mathbf{v}$ by $\widetilde{\mathbf{p}}$ in the early literature to a simple relation between them, with each determined independently by the conditions of production and distribution. So while it is appropriate to characterize the early SSI literature as describing values and prices in terms of a "single system", it appears to be inaccurate to refer to the later, fully-developed, statements of this approach in this way.

To complete the description of the basic quantities in the SSI, an expression for $\mathbf{s}$ can be derived by first noting that aggregate profit is identically equal to aggregate surplus value (according to (18)). This implies that the surplus value per unit commodity can be expressed as total profit distributed to the output commodities in proportion to the labor used in their production. The fraction of the total direct labor that commodity j embodies is $[(1/\mathbf{lx})l_j]$, and using this, the gross product vector $\mathbf{x}$, and (19), $\mathbf{s}$ is,

$$\mathbf{s} = (1 - \lambda^M)\widetilde{\mathbf{p}}\mathbf{x}\mathbf{l}(1/\mathbf{l}\mathbf{x}) \tag{21}$$

This is one way to describe $\mathbf{s}$ in the SSI. Like the definition of $\mathbf{v}$ this also relies on $\widetilde{\mathbf{p}}$, but this is a matter of convenience rather than necessity as $\mathbf{s}$ can also be determined without reference to price magnitudes.

Labor Time Equivalents and the Value of Labor Power

An expression for the SSI similar to the NI chain of equivalence (3) extends only to the various price measures of the net product,

$$L \equiv \mathbf{l}\mathbf{x} = \widetilde{\mathbf{p}}\mathbf{y} = \widetilde{p}_k\overline{\mathbf{p}}\mathbf{y} = \widehat{\lambda}_m\widehat{\mathbf{p}}\mathbf{y} \tag{22}$$

The equality $L \equiv \mathbf{l}\mathbf{x}$ is a definition; $\mathbf{l}\mathbf{x} = \widetilde{\mathbf{p}}\mathbf{y}$ is established by (9); $\mathbf{l}\mathbf{x} = \widetilde{p}_k\overline{\mathbf{p}}\mathbf{y}$ is established by (12); $\mathbf{l}\mathbf{x} = \widehat{\lambda}_m\widehat{\mathbf{p}}\mathbf{y}$ is established by (15). All of the terms in (22) are quantities of labor time and it clearly demonstrates the principle that a price system distributes the total labor performed during the period to the net product of that period is clearly present in the SSI in its labor time, commodity money, and state money price variants. It is also true that by definition $\mathbf{l}\mathbf{x} = \boldsymbol{\lambda}\mathbf{y}$, and hence $\boldsymbol{\lambda}\mathbf{y}$ could be added to the terms in (22) as in (3), but because $\boldsymbol{\lambda}$ plays no role in the SSI it would be specious to include it in (22). Unlike the NI chain of equivalence no measure of the value of the net product appears in (22). This is an important difference between the NI and the SSI, and it occurs because the SSI maintains that the value-price identity $\mathbf{v}\mathbf{x} \equiv \widetilde{\mathbf{p}}\mathbf{x}$ holds for the total product rather than the net product. Since $\mathbf{v} \neq \widetilde{\mathbf{p}}$ the value and price of any commodity bundle that is a subset of $\mathbf{x}$ will only be equal for commodity vectors that are scalar multiples of $\mathbf{x}$, and this will not generally be true of $\mathbf{y}$.

Finally, regarding the VLP, an equivalent expression to (5) using either commodity or state money prices can be derived for the SSI. With commodity money, for example, the VLP is,

$$VLP = \overline{w}\overline{\lambda}_m = \frac{\overline{w}\mathbf{lx}}{\overline{\mathbf{p}}\mathbf{y}} \tag{23}$$

As with the NI, this defines the value of labor power as the share of wages in net output. Here $\overline{w}$ is the wage rate as a quantity of commodity money per hour and $\overline{w} \equiv \widetilde{\mathbf{p}}\mathbf{b}\left(\frac{1}{\overline{\lambda}_m}\right)$. But, unlike the NI, in the SSI the value of labor power can be expressed directly as a quantity of labor time. The labor time expression for VLP using commodity money follows immediately from (23) and the definition of $\overline{w}$, or, equivalently, by substituting the definition of $\overline{w}$ and (9) into the third term of (23),

$$VLP = \widetilde{\mathbf{p}}\mathbf{b} \tag{24}$$

This is a much more orthodox interpretation of the value of labor power, which interprets it as the labor time associated with the commodity bundle advanced to workers per unit of labor power, but it is also consistent with the NI proposition that the VLP is the wage share of net output. The result is the same when using state money quantities $\widehat{\mathbf{p}}, \widehat{w}$ and $\widehat{\lambda}_m$. The value of labor power in terms of values $\mathbf{vb}$ will, in general, differ from the value of labor power in terms of labor time prices of production $\widetilde{\mathbf{p}}\mathbf{b}$ because of the differences between $\mathbf{v}$ and $\widetilde{\mathbf{p}}$. As with any commodity, or set of commodities, this value-price deviation (or difference between value and value-form) is a consequence of the re-allocation of labor time among commodities that occurs in exchange through the price system.

Conclusions

The NI is well known and fairly widely accepted despite its limitations; the SSI is less well known, and even less understood, despite its significant accomplishments. These should not be interpreted as competing approaches to Marxian value theory, but rather as complimentary approaches. The NI consists of a fairly small set of propositions. The law of the conservation of value implies that prices serve to distribute the value added by labor in production to the net product produced by that labor. This principle is one of the signal contributions of the NI, as is its emphasis on interpreting the theory of value in the context of a money-using economy. Both of these are clearly expressed in Duménil's and Foley's presentations, as well as in Mohun's formalization of the NI. But the NI is incomplete as a response to the traditional critiques of Marxian value theory. It lacks both a theory of prices and a theory of value beyond embodied labor coefficients. The NI can also be criticized on the grounds that it only satisfies Marx's aggregate identities by reinterpreting one as applying to the net rather than the gross product.

The SSI is a consistent theory of value and price for individual commodities that is also compatible with the basic propositions of the NI. Indeed the conservation of value and the principle that the price system distributes value added in production to the commodity output is present in the earliest presentation of the SSI. The unique labor-denominated price vector allows the SSI to develop a comprehensive price accounting using labor time as the fundamental unit of account. This aspect of the SSI seems to have been largely overlooked or mistaken by other interpreters. The reasons for this are not obvious, but it seems to involve some confusion with the TSSI, which assumes conventional money-denominated prices, or the unusual method of determining the price vector through equation (9) rather than with a *numeraire* commodity and normalization.

The SSI also provides a theory of value that is not subject to the well-known problems associated with vertically integrated labor coefficients, and satisfies both of Marx's aggregate equalities as stated by him. In short, the SSI provides a theoretically consistent Marxian theory of value and price of production, as well as an extensive response to the neo-Ricardian critique, which is something the NI is unable to do on its own. But the connection between the labor-denominated quantities in the SSI and monetary systems has not previously been developed, which made it difficult to use to interpret actual money-using capitalist economies. This paper demonstrates that integrating money, either commodity or state, into the SSI is possible. Further, it demonstrates that the basis for the money-denominated price vectors ($\overline{\mathbf{p}}$ and $\widehat{\mathbf{p}}$) is labor time manifesting through the price system in economies using commodity or state money. This treatment formalizes the relationship between commodities, commodity money, state money, and labor time in a much more comprehensive and consistent manner than has been achieved previously. Future work should undertake to apply this synthetic approach to empirical applications.

After surveying developments in recent decades on Marxian value theory Foley remarks "I see as a large degree of practical and operational agreement on the labor theory of value emerging (2000, 34)". This paper concurs with that assessment, though it has been necessary to correct some important misinterpretations about the SSI to see this clearly. But Foley's assessment that the central insight of the NI is that the MELT makes a separate accounting using embodied labor coefficients unnecessary, seems misguided. This would be true if embodied labor values were necessary for a comprehensive accounting using labor time, but this is not the case. The SSI provides a consistent method of doing this, and therefore the important contributions of the NI are found elsewhere, notably the way that it elaborates Marx's synthesis of the labor theory of value with a theory of money. Drawing from them both makes it clear that the fundamental Marxian labor time accounting provides a viable means to interpret the monetary phenomena that are characteristic of a capitalist economy.

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