Informed Trading, Forced Trades and Amplification Mechanisms*

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Abstract

We analyze non-fundamental, yet rational, asset price deviations and their evolutions when traders are exposed to *forced-trades* (e.g. fire-sales). The first objective of this study is to provide a *generalized, information-based* framework for examining various types of asset price deviations stemming from traders' funding constraints. Second, our study explains the determinants of *persistent* price deviations (e.g. crashes) following high-leverage periods. We do so within an information-setting of double uncertainty (in the asset value and in the fraction of forced-trades) and further dynamically modeling the mechanism producing the forced-trades in the context of margin trading. Our model predicts rational under-(over-) shooting in the prices when the fraction of unobserved forced-trades is initially under-(over-) estimated by the market maker. We demonstrate that, when the constrained traders are the informed traders, rather than the uninformed traders, then due to an informational positive feedback mechanism the mispricing exhibits persistence, as opposed to reversal. Finally, we discuss policy issues that could mitigate the destabilizing effects.

JEL classification: G01, D83, G14, G12, G18

Keywords: Asset price anomalies, Price deviations and reversals, Positive feedback mechanisms, Margin trading, Financial stability, Regulatory transparency

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1 Introduction

It is well documented that speculating informed traders who bid away pricing inefficiencies possibly face limits to their actions because new trades and existing positions often require capital while traders' capital is limited. This obstacle to price discovery also exists for traders who have trade needs independent of the asset's fundamentals while possibly providing liquidity to the markets. One way for traders to circumvent this capital constraint is to rely on margin trading, i.e. using the security as collateral and borrowing partially against it. However, the difference between the security's price and its collateral value denoted as the margin or haircut, must be financed with the traders own capital. Margin trading is a very common way for making use of some limited but readily available leverage. One should note that, margin trading comes with its additional risks and costs: Initial and maintenance margin requirements[1]. Under maintenance margin requirements, adverse price changes can trigger margin calls and result in *forced trades*. Margin trading has often been blamed during the recent years for playing a major role in generating severe asset price declines due to these *forced trades*.

Even though most trading is conducted by fund managers or banks' proprietary trading desks, the capital used for trading mainly belongs to their investors. Adverse price changes can also trigger withdrawal of these investors' capital, another source of the *forced trades* occurring in the markets.

Ben-David, Franzoni, and Moussawi (2012) show that hedge funds have significantly reduced their equity holdings during the recent financial crisis, where the margin calls/risks (42%) and the customer redemptions (50%) were the primary drivers of these selloffs. By means of forced trades, their study proves the importance and relevance of forced trades, both via margin trading and investors' redemptions, especially during periods of financial distress. Ultimately, the amount of forced trades in a given market can be misestimated due to opacity and financial turmoil. When the *forced trades* cannot be well differentiated from the trades that are actually associated with the change in the asset's fundamentals, we will observe (even non-fundamental) price deviations that can cause further price deviations. This describes the price related positive feedback mechanism, i.e. a mechanism that

[1]The initial margin requirement is the amount required to be collateralized in order to open a new position, therefore can prevent traders from opening new positions. The maintenance requirement is the minimum amount to be kept collateralized over time in order to preserve the open position, thus can make the trader to close existing positions. On instruments determined to be especially risky, the broker may set the maintenance requirement as high as the initial requirement, and sometimes in addition as variable.

can lead a small initial price movement to induce a relatively large aggregate price effect within a relatively short period. This often happens during periods of financial turmoil.

The first objective of this study is to provide a generalized, information-based framework for examining various types of asset price deviations stemming from traders' funding constraints. This framework allows further investigation of their dynamic interactions with the market liquidity and the price volatility. The price deviations can take numerous forms regarding their evolutions (their directions, magnitudes and persistence). We do so within an information setting of double uncertainty; one uncertainty dimension regarding the liquidation value of the asset and the other one regarding the level of forced trades.

Second, using the same information setting and further modeling the mechanism driving forced trades dynamically[2], our study aims to explain the determinants of *persistent* price deviations, such as crashes, that are observed in the financial markets following high-leverage periods. This observation is well documented within the recent literature since the 2007-08 financial crisis.

This study brings together two strands of literature, namely the one on dynamic information-based market microstructure and the one on financial positive feedback mechanisms, via forced trades. We develop a parsimonious model derived from the Glosten and Milgrom (1985) model by adding forced trades[3] and further allowing the fraction of these traders in the markets to be under-(over-) estimated. In this context, we analyze the inference problem of the market makers by performing various Monte Carlo simulations of trade sequences using the analytically derived competitive bid and ask prices. We analyze the evolutions of the Bid, Ask, Trade Prices, Mid-Quotes and the updated posterior probabilities for the asset value and for the proportion of *constrained* traders. Meanwhile, we are particulary interested in the convergence and deviations of the trading prices. We find that, when the level of forced trades are estimated correctly, the mechanisms behind them per se, such as margin trading or investors' redemption, do not create any biases in the trade prices. The observed under-(over-)shoots are related to the under-(over-)estimation of the fraction of forced trades, not simply to the probabilistic structure in the proportion of forced trades. Notice that, the direction of the price deviation depends also on the

[2]for example, allowing margin calls to be triggered overtime with adverse price changes.

[3]In our model, there exist *constrained* traders who have to conduct forced trades, regardless of their initial intentions and information, in the direction of reducing their asset positions. Note that, the market maker cannot observe explicitly whether the picked trader is *constrained* or *unconstrained* at that period. The first part of our analysis covers forced trades in general, while later we focus particularly on margin trading with margin calls and forced trades arising dynamically as a result of adverse price changes.

direction of the forced trades; for instance in the context of margin trading, for assets in which the *constrained* traders have net short positions, rather than long positions, the consequences are reversed. The price diversion is due to the fact that, while in reality the *constrained* traders are often selling (buying) in the first periods, the market confuses these on average with informed selling (buying), which then drives the prices downwards (upwards). Over time, either the correct fraction of *constrained* traders is learned and the mispricing is corrected quickly, or that learning is very slow (inhibited) and the mispricing lasts longer (persists). The model predicts longer lasting deviations when the (erroneously estimated) *constrained* trades are mainly undertaken by the informed traders. To understand the intuition behind this result, first notice that the expected (by the market maker) fraction of informed traders determines the impact size of a single trade, consequently the speed of price convergence and correction. Second, suppose that the under-estimated *constrained* traders are mainly informed traders. Then, it is initially expected in the markets that there exist more information-based trades than in reality. In such a trade environment, the impact of a single trade is higher. Consequently, the initial price deviation occurs rather quickly. However, over time the markets realize that the information-carrying trades are not that many, as a result the correction process and the reversal is slow. The results are consistent with the empirical finding by Cella, Ellul, and Giannetti (2013) that stocks held mostly by hedge funds (when represented in our model as informed traders) experience more severe price drops and slower price reversals during episodes of market turmoil than those mostly held by mutual funds (when represented in our model as uninformed traders).

Next, we show that there exists an informational positive feedback mechanism that amplifies these deviations. We analyze it in the context of margin trading. It is important to explain this specific downward spiral clearly. When the prices move adversely in the markets, which could be just uncertainty driven and random, they can trigger (further) margin calls. As mentioned previously, on average, this does not induce further price movements in our market-setting with only margin trading, i.e. with no misestimation of the proportion of *constrained* traders. In other words, within such a setting the well-known positive price feedback mechanism does not operate. However, when in addition the fraction of margin called traders is erroneously estimated, we show that a feedback loop can form: The initial adverse price movement triggers (further) margin calls, and this results in *more severe misestimation*, which then induces further adverse price changes. Eventually, that triggers further margin calls and so on.

The remainder of the paper is organized as follows. The next section presents the literature the paper is related and contributes to. Section 3 gives, together with the *basic* model, the details of our *extended* model and its analysis in depth; allowing the *uninformed* traders or the *informed* traders to be constrained. In this section, we specify the static and the dynamic versions of the *extended* model, where for the latter the fraction of constrained traders evolves dynamically and depending on previous price changes. Section 4 discusses the numerical simulation results for the *basic* model, as well as for the static and the dynamic versions of the *extended* model presented in Section 3. In Section 5, we check the robustness of our results and discuss the policy implications of our findings. Finally, section 6 concludes.

2 Related Literature

A noteworthy strand of literature focuses on financial positive price feedback mechanisms, especially those that operate during crises. The flash crash in May 2010 that impacted the Dow Jones Index adversely almost by 10 percent in few hours, the short-squeeze event of October 2008 that temporarily drove the shares of Volkswagen AG (VOW) to approximately five times of its current price in less than two days, the quant hedge fund crisis in August 2007, the crisis following the Russian default in 1998 and the resulting bailout of Long Term Capital Management as well as the crash of 1987 are common examples of such events to which this literature refers[4]. During such periods of financial turmoil, market liquidity is fragile and prices are driven severely by other forces rather than by movements in fundamentals. For instance, Gennotte and Leland (1990) first point out that, in the absence of significant news, hedging strategies were blamed for the stock market crash of October 1987. Then, they show how a relatively small amount of selling could cause such a large a price drop, i.e. market crashes. To do so, they develop a rational expectations model in which prices play an important role in shaping expectations and show that market crashes could occur in illiquid markets even with relatively little hedging.

[4]Gorton (2008), Brunnermeier (2009), and Krishnamurthy (2010) review the recent crisis and amplification mechanisms. While mainly providing a literature survey around a model, Krishnamurthy (2010) classifies the factors that many observers point to. Broadly speaking, we can categorize these factors as balance sheet amplifiers (e.g., margining and corresponding margin requirements, limited funding and capital, leverage, correlated existing positions due to widespread trading strategies, tight credit conditions, pro-cyclical regulatory requirements on capital adequacy) and information amplifiers (e.g., ambiguity, wide bid-ask spreads, opacity, complexity).

The information-based approach in market microstructure literature has greatly enhanced our understanding of market behavior, by providing an insight on how information can significantly affect quotes and spreads. If some traders have superior information about the underlying value of an asset, their trades could reveal what this underlying value is; and thus affect the behavior of prices. On the contrary, how prices can in turn affect the behavior of traders has not yet been well explored. Neither highly referred sequential trade models such as Glosten and Milgrom (1985) nor batch trading models such as Kyle (1985) allow traders to learn from the movement of prices anything new, i.e., that is not already in their information set. In actual asset markets however, developing technical trading strategies according to price patterns or even simply reacting with respect to price changes is very common. This issue is also mentioned within the views of O'Hara (2001) on remaining puzzles concerning the trading process in the market microstructure literature. Moreover, sudden price changes and liquidity dry-ups which are observed within the mentioned models of positive price feedback mechanisms, do not arise endogenously within these information-based market microstructure models due to missing components and connections. Extending them in this direction proves to be necessary. For instance, Cohen and Shin (2003) examines tick-by-tick trading data and finds in the US Treasury market that, trades and price movements appear likely to exhibit positive price feedback at short horizons, particularly during periods of market stress. They suggest that "The standard analytical approach to the microstructure of financial markets, which focuses on the ways in which the information possessed by informed traders becomes incorporated into market prices through order flow, should be complemented by an account of how price changes affect trading decisions".

In the literature that focuses on financial positive price feedback mechanisms, our paper relates to the work by Brunnermeier and Pedersen (2009). Within a static setting and by allowing margins to increase with price volatility and by defining liquidity as the distance between the price and the fundamental value, the authors point out that such forced trades can become an important component of the mentioned positive price feedback mechanisms and that under specific circumstances margin requirements can easily become destabilizing. Their model as well as the following literature have different research objectives than us. One of the aims of our study is to explain the determinants of *persistent* price deviations, such as crashes, following high-leverage periods, including the situations with many investors utilizing their margin accounts but lacking cushion. The other objective of our study is to provide a generalized, information-based framework for examining various types of asset price deviations stemming from traders funding constraints, and for investigating their dynamic interactions with the market liquidity (bid-ask spread) and price volatility. Notice that, the framework which our paper sets up owns a dynamic setting. Moreover, it also complements this strand of literature with informational aspects; with Bayesian learning, which is key to extracting information from order flows in the information-based market microstructure models.

Our paper relates to several other studies in the market microstructure literature with informational aspects and short-run mispricing. We rely on the framework proposed by Jacklin, Kleidon, and Pfeiderer (1992), since these authors also use an extended version of Glosten and Milgrom (1985) model. By adding exogenously defined uninformed portfolio insurance traders, and further allowing their fraction among all traders to be under-estimated, their model exhibits rational asset price deviations with reversals. In our model, it is not the correlated trading strategies that come into play but the funding constraints and the forced trades. Moreover, our model allows also the informed traders, in addition to the uninformed traders, to be constrained, and in turn to generate different results as mentioned previously. Finally and importantly, modeling the margin trading mechanism dynamically and depending on previous price changes allows us to analyze the margin trading in depth. Deriving from Jacklin, Kleidon, and Pfeiderer (1992), Avery and Zemsky (1998) show also that two dimensional uncertainty is vital for significant short-run mispricing. However, instead of forced trades, they focus on endogenous rational herding behavior and for that they incorporate further a third dimension of uncertainty by letting the informed traders receive only (heterogenous) signals on the asset value. Park and Sabourian (2011) generalize this finding and provide clear conditions on how to achieve similar results including the one of Avery and Zemsky (1998). With respect to the analysis of constraints in the market microstructure literature, an important contribution is provided by Diamond and Verrecchia (1987). Relying on the basic structure of Glosten and Milgrom (1985) model with one dimension of uncertainty and constraining the ability of traders to short sell, they show that constraining short sales reduces the informational efficiency of prices but does not bias them upward. A closely related model is developed by Colliard (2013). The author uses the standard Glosten and Milgrom (1985) model and adds exogenously defined uninformed positive feedback traders, and further allow the proportion of such trades to be under-(over-) estimated. His model leads to rational asset price deviations with reversals as covered in Jacklin, Kleidon, and Pfeiderer (1992). More importantly, he adds to this setting an additional *information-supply* category of traders.

These traders have private information on whether the market is under pressure due to fire sales. Different to our study, the main objective of Colliard (2013) is to analyze the existence (absence) of *information-supply* type of traders and its effect on price efficiency. Another important difference is that, in our model both the informed traders as well as the uninformed traders can be constrained. This generates different outcomes including an informational positive feedback mechanism. Furthermore, in our model, we assign traders initial asset positions and capital and let these vary over time. That allows for a robust analysis of margin trading itself, and of the interplay between the price deviations and liquidity. It also permits to differentiate if a sell trade is a normal sell or a short-sell, which is crucial if one wants to study short-sell bans.

3 Model and Analysis

3.1 The Basic Model

The core structure of the model, labeled the *basic* model, is the discrete-time, finite-horizon, sequential trade framework used by Glosten and Milgrom (1985)[5]. Within a quote-driven market setting, trading takes place sequentially, with only one trader at each period being allowed to trade with the market maker. At each trade session, a single share of an asset is traded.

The asset being traded has an eventual (at time T) liquidation value, given by the random variable V[6]. Before the trade begins, either good or bad news arrive to the market. If it is good news, then V is realized as V_H, and if it is bad news V is realized as V_L. Traders are of two types, either informed or uninformed[7]. The informed traders know privately the realized value of V. The uninformed traders and the market makers know initially only the prior distribution of V; it is assumed to be V_H with probability δ^H, and V_L otherwise.

There is an infinite number of each type of market participant, i.e. market makers, informed and uninformed traders. All of them are assumed to be risk-neutral and acting

[5]This is a seminal framework for addressing issues related to the adjustment of prices to information, meanwhile allowing the inference problem of the market makers and the quote process to be analyzed explicitly. Moreover, it illustrates that a bid-ask spread can arise purely due to information asymmetry.

[6]Time T may be interpreted as the time at which no trader has an informational advantage, therefore the bid and ask prices for the asset being equal to V.

[7]Broker-dealers, which are usually subsidiaries of commercial banks, investment banks or investment companies, might represent the market makers in our model, whereas hedge funds or banks' proprietary trading desks might represent the informed traders.

perfectly competitively[8]. The infinitely many assumption for informed traders rules out their strategic behavior[9].

Market makers have the responsibility to quote bid and ask prices at each period and should be able to stand behind them. While doing so, to protect themselves as much as possible against the informed traders, they need to make inference about the realized value of V based on the information carried by trades arriving to the market over time. Market makers face no inventory costs or constraints and, under perfect competition, earn zero expected profits from each trade.

The time interval is divided into T discrete intervals, with arbitrary lengths between them. Each period, the trade is conducted between the market maker and a randomly picked trader among the existing population of traders in the market. Whether this trader is an informed or uninformed is not explicitly observed by the market maker. The probability that this picked trader is an informed trader is μ. This probabilistic setting, together with the trade size limit, rule out the possibility of instantaneous revelation of information when the market maker trades with an informed trader.

A share is underpriced to a trader if its ask price is less than its perceived value[10] and overpriced if its bid price is above its perceived value. Since informed traders speculate for profit and know the realized value of V, their decision to buy or sell depends on this value relative to the bid and ask quotes. As shown later, the bid and ask prices will always be within the (V_L,V_H) interval. Consequently, an informed trader will always be willing to buy if good news have arrived, and will always be willing to sell if bad news have arrived to the market.

In contrast, uninformed traders' trade decision is assumed not to depend normally[11] on neither quotes nor the liquidation value. The uninformed traders normally trade for reasons exogenous to our model[12] and accept making expected losses eventually. Note that,

[8] Some of these assumptions for some of the market participants are not necessary; for instance, instead of infinitely many, only two market makers having a Bertrand competition would be enough to bring the bid and ask prices to their perfect competition values.

[9] which could be rational under a wide range of conditions; for instance a strategy that involves disguising the information could have lead to higher profits. Indeed, since an informed trader's turn to trade a second time almost surely will not arrive, delaying or disguising trades could have been optimal only if the model were infinite-horizon and the private information infinitely long-lived.

[10] The conditional expectation of the liquidation value given trader's information.

[11] Without taking the frictions due to forced trades yet into account.

[12] The reasons might include immediate consumption needs, risk management requirements and alternative outside investment opportunities.

uninformed traders do not perform better, i.e. decrease the probable losses they incur, by updating their believes over time. They are sometimes called also liquidity traders within this study, because it is the uninformed traders who provide the liquidity to the informed. The picked uninformed trader is assumed to be willing to buy with a probability of γ^B and sell otherwise, with a probability of γ^S $(= 1 - \gamma^B)$. Due to the exogenous modeling, there is no sense to assume different values for γ^B and γ^S, therefore we take them as equal.

Fig.1 illustrates the probability structure of the *basic* model. Moreover, it indicates all possible outcomes and features of a trade; i.e. whether it is a Buy or a Sell, plus the type and motivation of the trader conducting it.

[Insert Fig.1 here]

We formalize the Buy or Sell motive of uninformed/informed traders, and bid and ask price quotation of the market makers as follows: The traders can either invest in the asset and defer consumption to T, or they do not invest and consume at t. The liquidation value V is paid and consumed in the distant future T and market participants discount future consumption by the factor ρ. So the present value of future consumption at period $t = 1, 2, ..., T$[13] is given by $Z_t = \rho\mathbb{E}[V|F_t]$, where F_t represents the particular market participant's information at the beginning of period t[14]. Z_t determines how much a market participant is willing to pay to buy or how much she is willing to accept to sell a single unit of the asset. We assume that, absent a shock, $\rho = 1$ for all the market participants. We model uninformed trader's motivation as an exogenous shock to an individual's time preference; they receive one of two possible shocks: $\rho = +\infty$ with probability γ^B, which implies that the uninformed trader has a very high present valuation of of future consumption and therefore is willing to buy the asset as a means of deferring consumption, or $\rho = 0$ with probability γ^S, which implies that the uninformed trader has very little present valuation of of future consumption and therefore is willing to sell the asset to satisfy consumption today. As mentioned previously, γ^S is assumed to be equal to γ^B.

The decision of each type of trader arriving to the market at period t depends on the bid (B_t) and ask (A_t) prices announced for period t. The trader sells if $Z_t < B_t$ and the trader

[13]Note that, setting the intervals with arbitrary lengths between them, i.e. $t = t_1 < t_2 < ... < T$, is more general. But, it does not affect our results if we assume $t = 1, 2, ..., T$ to simplify the notation.

[14]While F_t^{MM} for the market maker and F_t^U for the uninformed traders are assumed to be the same, the F_t^I for the informed traders includes the information on the realized value of V.

buys if $Z_t > A_t$. The $B_t \leq Z_t \leq A_t$ case does not occur for any of the traders, as we show while deriving the quotes.

The market makers have a discount factor $\rho = 1$ as mentioned, quote B_t and A_t according to,

$$B_t = \mathbb{E}[V|F_t^{MM}; s]$$

and

$$A_t = \mathbb{E}[V|F_t^{MM}; b].$$

This is because, B_t is quoted for current period's Sell (trade), and A_t is quoted for current period's Buy (trade).

Note that, whether the current period's trade is a Sell or a Buy depends on Z_t of the picked trader relative to A_t or B_t,

$$B_t = \mathbb{E}[V|F_t^{MM}; s] = \mathbb{E}[V|F_t^{MM}; Z_t < B_t]$$

and

$$A_t = \mathbb{E}[V|F_t^{MM}; b] = \mathbb{E}[V|F_t^{MM}; Z_t > A_t].$$

This means that, first of all, due to the randomness of the picked trader, Z_t is also random from the market maker's point of view while calculating the bid and ask prices. Second, the bid price equation above in its general form has B_t on both sides[15], which normally complicates the derivations. However, due to our way of modeling, derivations can be simplified.

First note that, Z_t takes on the values V_L or V_H for the informed traders and it is either 0 or $+\infty$ for the uninformed traders. And, the bid and ask prices will always be within the (V_H,V_L) interval. Therefore, whether the current period's trade is a Sell or a Buy does not depend on the exact values of the bid and ask prices, and the bid and ask price equations can be simplified as,

$$B_t = \mathbb{E}[V|F_t^{MM}; Z_t \leq V_L]$$

[15]The same is valid also for A_t.

and

$$A_t = \mathbb{E}[V|F_t^{MM}; Z_t \geq V_H].$$

Second, whether $Z_t \leq V_L$ or $Z_t \geq V_H$, depends[16] on the *basic* model standard parameters μ, γ^B, γ^S and δ^H. The fact that none of these variables depend on A_t or B_t, allow the bid and ask price derivations to be simple.

The ask price can be derived[17] via,

$$A_t = \mathbb{E}[V|F_t^{MM}; b] = V_H * P(V_H|F_t^{MM}; b) + V_L * P(V_L|F_t^{MM}; b)$$

$$\overbrace{P(V_H|F_t^{MM}; b)}^{Posterior} = \frac{\overbrace{P(V_H|F_t^{MM})}^{Prior} * P(b|V_H)}{\underbrace{P(V_H|F_t^{MM}) * P(b|V_H) + P(V_L|F_t^{MM}) * P(b|V_L)}_{P(b)}}$$

$$P(V_L|F_t^{MM}; b) = 1 - P(V_H|F_t^{MM}; b)$$

At this point we verify that the bid and ask prices are always within the (V_H,V_L) interval. Moreover, the $B_t \leq Z_t \leq A_t$ case does not occur.

The $P(b|V = V_H)$ and $P(b|V = V_L)$ are functions of the *basic* model parameters,

Probabilities of observing a Buy[18], under the two states of the world are,

$$\begin{aligned} P(b|V = V_H) &= \mu + (1 - \mu) * \gamma_B \\ P(b|V = V_L) &= (1 - \mu) * \gamma_B \end{aligned}$$

Note that, the bid and ask price processes are Markovian, i.e. one can derive them for the current period by only observing the previous period, without knowing the earlier states and their past trajectories.

This setting is the same as in Glosten and Milgrom (1985), which is a standard sequential trading model with one dimension of uncertainty. First, we predict that the convergence[19]

[16] due to the randomness in the trader picking process

[17] we show the derivation of the ask price in details, and the bid price derivation is analogous.

[18] Probabilities of observing a Sell is simply equal to (1−Probability of observing a Buy), since these are the only two outcomes of a trade.

[19] when the bid and ask prices for the asset are equal to V

will always occur, as long as there exist both informed and uninformed traders together in the market[20]. Second, we predict that the trade prices have the martingale property, i.e. its value at any t is equal to the conditional expectation of its value at a future time given the information available at t and we do not observe any biases in the trade prices.

3.2 The Extended Model

Within the *basic* model, which is the same as in Glosten and Milgrom (1985), none of the traders are subject to any funding constraints that could limit their actions. Consequently, we do not observe any forced trades. In the *extended* model, however, we can observe forced trades by traders who, for instance, experience investors' redemptions or who trade on margins to loosen their funding constraints and face (maintenance) margin requirements[21]. In this model, at each period $t = 1, 2, ..., T$, only a proportion $(1 - \varepsilon_t)$ of the traders, that we label as *unconstrained*, are not subject to forced trades. The rest of the traders at that period, a proportion (ε_t) of the traders, whom we label as *constrained*, have to conduct forced trades[22]. Thus, regardless of their initial intentions, the picked *constrained* traders have to conduct forced trades in the direction of reducing their asset positions[23], which might or might not be in the intended directions. Note that, whether a trader is *constrained* or *unconstrained* can vary over time; from one period to the next, some of the *constrained* traders possibly become *unconstrained*, and vice versa.

To extend the *basic* model formalization of the market makers' bid and ask price quotation,

[20] If there are no uninformed traders in the market, then the bid-ask spread has to be set so high that no informed trader has anymore the possibility to make profits and has no more incentives to trade. Whereas, if there are no informed traders in the market, information on the actual value of V cannot be disseminated via trades.

[21] When a trader buys an asset, she can use it as collateral and borrow against it, which is called margin trading. But, she cannot borrow the entire price. The difference between the asset's price and collateral value, denoted as the margin or haircut, must be financed with the trader's own capital. This difference has to stay above a maintenance margin requirement, the purpose of which is to protect the financier/broker against an adverse change in the value of the asset to the point that the trader can no longer cover the loan. Margin trading allows traders enjoy an amount of leverage, but it comes with a cost; when maintenance margin requirements are not met, triggered margin calls can force the trader to rewind her asset position. Even in some cases, margin trading is inevitable; for instance, short-selling does not free up capital and requires capital in the form of a margin. Note that, in this study, we focus mainly on the maintenance margin requirements and show later the robustness of our results with respect to the initial margin requirements, which is not implemented in our model.

[22] for instance, they might be receiving margin calls and have to conduct forced trades to meet the (maintenance) margin requirements.

[23] sell if they have net long positions, and buy if they have net short positions in the asset

we model *constrained* traders' motivation[24], via altered time preferences; when an *unconstrained* trader becomes *constrained*, ρ becomes 0 for traders with a net long positions in the asset, and it becomes $+\infty$ for traders with a net short positions in the asset.

We assume that, similarly to the inability to differentiate the informed traders from the uninformed traders, the market maker cannot observe explicitly whether the picked trader is *constrained* or *unconstrained* at that period.

We now extend the derivation of the quotes. Remember that, whether the current period's trade is a Sell or a Buy depends on the Z_t of the picked trader relative to the A_t or B_t,

$$B_t = \mathbb{E}[V|F_t^{MM}; s] = \mathbb{E}[V|F_t^{MM}; Z_t < B_t]$$

and

$$A_t = \mathbb{E}[V|F_t^{MM}; b] = \mathbb{E}[V|F_t^{MM}; Z_t > A_t].$$

For the *extended* model, first note that, Z_t takes on the values V_L or V_H for the *unconstrained* informed traders and it is either 0 or $+\infty$ for the uninformed traders or any *constrained* trader. And, the bid and ask prices will always be within the (V_L,V_H) interval. Therefore, whether the current period's trade is a Sell or a Buy does also not depend on the exact values of the bid and ask prices in the *extended* model, and the bid and ask price equations can be simplified as,

$$B_t = \mathbb{E}[V|F_t^{MM}; Z_t \leq V_L]$$

and

$$A_t = \mathbb{E}[V|F_t^{MM}; Z_t \geq V_H].$$

Second, whether $Z_t \leq V_L$ or $Z_t \geq V_H$, depends[25] on the *basic* model standard parameters (μ, γ^B, γ^S, δ^H), and on the *extended* model state variable ε_t, which represents the proportion of the *constrained* traders over time. Note that, for the moment we assume that[26], ε_t does not depend on A_t or B_t and that allows the bid and ask price derivations to be simple.

In practice, the market makers are able to gather some information on ε_t. For instance, regarding the margin trading, they can obtain information via publicly available regulatory

[24] for both uninformed and informed traders

[25] due to the randomness in the trader picking process

[26] and later we model it such that

reports[27]. But, first of all, this data is aggregated, i.e. not totally transparent to infer the exact proportion (ε_t) of the traders who are receiving margin calls. Second, these reports are made available to the public only monthly, therefore with a delay. Last, even though, the market makers could try to acquire further information on ε_t via their own broker-dealer relationships, actually this is not legally allowed and that does not cover the information by the other brokerage houses. Similar to the margin calls, fund investors' redemptions also induce forced trades in the financial markets. Especially for hedge funds, these customer redemptions are highly opaque[28]. Ben-David, Franzoni, and Moussawi (2012) show that hedge funds have significantly reduced their equity holdings during the recent financial crisis, where the margin calls/risks (42%) and the customer redemptions (50%) were the primary drivers of these selloffs. By means of forced trades, their study proves the importance and relevance of forced trades, both via margin trading and investors' redemptions, especially during the periods of financial distress. Ultimately, the amount of forced trades in a given market is uncertain and can be misestimated due to the mentioned opacity and financial turmoil.

Thus, as an important ingredient of our model, we allow for uncertainty in the level of *constrained* traders. Before the trades begin, it is the case that, ε_t starts initially either at a higher value of ε_0^H with probability δ^{ε^H} or at a lower value of ε_0^L otherwise. How ε_t is initially realized and its evolution is not observed by any market participant. However, the corresponding probability structure is known and updated over time. Note that, when ε_t starts with a high value we denote its evolution over time with ε_t^H, and otherwise with ε_t^L.

Due to double uncertainty, now we have four possible states of the world: $\{(V = V_H, \varepsilon_t = \varepsilon_t^H), (V = V_H, \varepsilon_t = \varepsilon_t^L), (V = V_L, \varepsilon_t = \varepsilon_t^H), (V = V_L, \varepsilon_t = \varepsilon_t^L)\}$. Then, following Jacklin, Kleidon, and Pfeiderer (1992) we derive the ask price[29] accordingly,

$$\begin{aligned} A_t = \mathbb{E}[V|F_t^{MM}; b] &= V_H * [P(V_H, \varepsilon_t^H|F_t^{MM}; b) + P(V_H, \varepsilon_t^L|F_t^{MM}; b)] \\ &+ V_L * [P(V_L, \varepsilon_t^H|F_t^{MM}; b) + P(V_L, \varepsilon_t^L|F_t^{MM}; b)] \end{aligned}$$

[27]For instance, pursuant to FINRA Rule 4521, FINRA member firms carrying margin accounts for customers are required to submit, the total of all debit balances in securities margin accounts and the total of all free credit balances in all cash accounts and all securities margin accounts. Then the data is compiled in aggregate form and made available publicly.

[28]even more opaque than margin trading, because currently regulatory reports on hedge funds, including the Form 13F, are very limited in content and these are publicly available with much longer delays.

[29]We show the derivation of the ask price in details. The bid price derivation is analogous.

$$\overbrace{P(V_H, \varepsilon_t^H | F_t^{MM}; b)}^{Posterior} = \frac{\overbrace{P(V_H, \varepsilon_t^H | F_t^{MM})}^{Prior} * P(b|V_H, \varepsilon_t^H)}{\begin{array}{c} P(V_H, \varepsilon_t^H | F_t^{MM}) * P(b|V_H, \varepsilon_t^H) + P(V_H, \varepsilon_t^L | F_t^{MM}) * P(b|V_H, \varepsilon_t^L) \\ + \underbrace{P(V_L, \varepsilon_t^H | F_t^{MM}) * P(b|V_L, \varepsilon_t^H) + P(V_L, \varepsilon_t^L | F_t^{MM}) * P(b|V_L, \varepsilon_t^L)}_{P(b)} \end{array}}$$

Note that, $P(V_H, \varepsilon_t^L | F_t^{MM}; b)$, $P(V_L, \varepsilon_t^H | F_t^{MM}; b)$ and $P(V_L, \varepsilon_t^L | F_t^{MM}; b)$ can be derived similarly by replacing V_H with V_L, and/or ε_t^H with ε_t^L.

Also for the *extended* model we verify that the bid and ask prices are always within the (V_H,V_L) interval. Moreover, the $B_t \leq Z_t \leq A_t$ case does not occur.

Within the next subsections, we specify the *extended* model further for four separate settings: (i) Constantly Constrained Uninformed Traders, (ii) Constantly Constrained Informed Traders, (iii) Dynamically Constrained Uninformed Traders, (iv) Dynamically Constrained Informed Traders. Note that, for the constantly constrained traders settings our analysis relies on forced trades in general. Whereas, for the dynamically constrained traders settings we model the mechanism that generates the forced trades in the context of margin trading. And, for each of the four settings, the quote derivation up to this point is the same, while the remaining derivations of the probabilities of observing a Buy are different.

3.2.1 Constantly Constrained Uninformed Traders

In this subsection, we start with a setting in which a constant[30] $\varepsilon_t = \varepsilon^U$ portion of the uninformed traders is *constrained* and they have net long positions in the asset, while none of the informed traders are *constrained*[31].

Fig.2 illustrates the probability structure of the *extended* model in the *constantly constrained uninformed traders* setting. In addition, the figure indicates all possible outcomes and features of a trade; i.e. whether it is a Buy or a Sell, plus the type and motivation of the trader conducting it. The uninformed traders could be forced either to sell or to

[30] over time

[31] In the next subsections, we let the portion of *constrained* traders (ε_t) vary endogenously over time. We also allow the informed traders to be *constrained*, as well as let the traders have net short positions in the asset.

buy. For instance, in the context of margin trading if the uninformed traders with net long asset positions receive margin calls, they will be forced to sell the asset.

[Insert Fig.2 here]

For this setting, before the trades begin, ε^U takes either the value $\varepsilon^{U,H}$ with probability $\delta^{\varepsilon^{U,H}}$ or the value $\varepsilon^{U,L}$ otherwise. The realized value of the ε^U is not observed by any market participant. However, its initial probability structure is known and updated over time.

One can think of a constant ε_t [32] in the context of margin trading such that there exist two clusters of uninformed traders regarding their margin accounts; a group (ε^U) whose margin accounts are initially so negative that, any possible asset price movements over time cannot take them out of the margin call state, and the remaining group $(1-\varepsilon^U)$ whose margin accounts are sufficiently high that any possible asset price over time will not induce them to receive margin calls.

The probabilities of observing a Buy[33], under the four possible states of the world are,

$$\begin{aligned}
P(b|V=V_H,\varepsilon^U=\varepsilon^{U,H}) &= \mu+(1-\mu)*(1-\varepsilon^{U,H})*\gamma_B \\
P(b|V=V_H,\varepsilon^U=\varepsilon^{U,L}) &= \mu+(1-\mu)*(1-\varepsilon^{U,L})*\gamma_B \\
P(b|V=V_L,\varepsilon^U=\varepsilon^{U,H}) &= (1-\mu)*(1-\varepsilon^{U,H})*\gamma_B \\
P(b|V=V_L,\varepsilon^U=\varepsilon^{U,L}) &= (1-\mu)*(1-\varepsilon^{U,L})*\gamma_B
\end{aligned}$$

The probabilities of observing a Buy are functions of the *basic* model parameters as well as the *extended* model state variable ε_t[34].

Note that, the bid and ask price processes are still Markovian, i.e. one can derive them for the current period by only observing the previous period, without knowing the earlier states.

[32] that the portion of *constrained* uninformed traders (ε^U) is constant over time

[33] Probabilities of observing a Sell is simply equal to (1−Probability of observing a Buy), since these are the only two outcomes of a trade.

[34] In this setting $\varepsilon_t=\varepsilon^U$, i.e. it is constant

3.2.2 Constantly Constrained Informed Traders

In this subsection, analogous to the previous subsection, we start with a setting in which a constant[35] $\varepsilon_t = \varepsilon^I$ portion of the informed traders is *constrained* and these traders have net long positions in the asset, while none of the uninformed traders are *constrained*.

For this setting, before the trades begin, ε^I takes either the value $\varepsilon^{I,H}$ with probability $\delta^{\varepsilon^{I,H}}$ or the value $\varepsilon^{I,L}$ otherwise. The realized value of the ε^I is not observed by any market participant. However, its initial probability structure is known and updated over time.

The probabilities of observing a Buy[36], under the four possible states of the world are,

$$
\begin{aligned}
P(b|V = V_H, \varepsilon^I = \varepsilon^{I,H}) &= \mu * (1 - \varepsilon^{I,H}) + (1 - \mu) * \gamma_B \\
P(b|V = V_H, \varepsilon^I = \varepsilon^{I,L}) &= \mu * (1 - \varepsilon^{I,L}) + (1 - \mu) * \gamma_B \\
P(b|V = V_L, \varepsilon^I = \varepsilon^{I,H}) &= (1 - \mu) * \gamma_B \\
P(b|V = V_L, \varepsilon^I = \varepsilon^{I,L}) &= (1 - \mu) * \gamma_B
\end{aligned}
$$

The probabilities of observing a Buy are functions of the *basic* model parameters as well as the *extended* model state variable ε_t[37].

Note that, the bid and ask price processes are still Markovian, i.e. one can derive them for the current period by only observing the previous period, without knowing the earlier states.

3.2.3 Dynamically Constrained Uninformed Traders

For the dynamically constrained traders[38] settings we start with modeling the mechanism that generates the forced trades in the context of margin trading; the forced trades are due to margin calls. And, it is the model's state variables, particularly the latest mid-quote, that determine dynamically whether and for which traders margin calls are triggered.

To formulate margin trading for the traders, first, we denote with N_t^j the number of assets possessed, by trader $j = 1, 2, ...$, at the beginning of period $t = 1, 2, ..., T$. Second, we

[35] over time

[36] Probabilities of observing a Sell is simply equal to (1−Probability of observing a Buy), since these are the only two outcomes of a trade.

[37] In this setting $\varepsilon_t = \varepsilon^I$, i.e. it is constant

[38] both uninformed and informed

denote with X_t^j the trader's margin account value at the beginning of the period. And, at each period, the sum of the trader's margin account value (X_t^j) and the value of her asset position ($N_t^j * M_{t-1}$) gives us the value of the trader's capital, denoted by Y_t^j.

The state variables X_t^j and N_t^j for the picked trader j are updated after each period's trade according to,

- If the picked trader buys the asset, then

$$N_t^j = N_{t-1}^j + 1$$

  and

$$X_t^j = X_{t-1}^j - A_{t-1}.$$

- If the picked trader sells the asset, then

$$N_t^j = N_{t-1}^j - 1$$

  and

$$X_t^j = X_{t-1}^j + B_{t-1}.$$

A margin call is triggered for any trader j in the market, if:

$$X_t^j < (|N_t^j| * m - N_t^j) * M_{t-1},$$

or alternatively:

$$Y_t^j < |N_t^j| * m * M_{t-1},$$

where m is the margin rate[39] and M_{t-1} is the latest mid-quote[40] and defined as $(A_{t-1} + B_{t-1})/2$. Since there exist no B_0 bid and A_0 ask quotes, M_0 is assumed to be equal to $\mathbb{E}[V|F_0^{MM}] = \delta^H * V_H + (1 - \delta^H) * V_L$.

[39]In this study, we mainly deal with (maintenance) margin requirements on stocks, but our framework also allows analyzing margin trading for other asset classes. $0 < m < 1$, and under the rules of FINRA and the exchanges, as a general matter, the minimum level for m is 25%. Brokers are free and often set it as at around 50%.

[40]Even though using it is a common practice, actually the mid-quote does not have an economical meaning. Instead of the mid-quote, one can use the last transaction price as well as the last quotes; the bid price for long positions, and the ask price for the short positions.

To clarify the specified details of margin trading and its related margin requirements within a simple setting, a numerical example is given in the Appendix.

Note that, the ratio of *constrained* traders at each period, ε_t, is determined according to the portion of the traders for whom margin calls are triggered at the beginning of that period according to the equations above.

A margin called trader is able to exit the margin call state, either by reducing her position[41] when she is given the opportunity to trade, or when the prices change over time in her favor. But, since the number of trade periods is finite, while we have infinitely many traders, finite number of trades over time will not significantly alter the distributions of the number of traders' assets and of their margin account values, $f(N_t)$ and $f(X_t)$ respectively[42]. Therefore, a trader picked at any trade period will almost surely have $N_t^j = N_0^j$ assets and her margin account value will almost surely be $X_t^j = X_0^j$, i.e. same as their initial values[43]. Consequently, what drives ε_t significantly in our model is only the fact that M_{t-1} is changing over time.

We assume that the market maker knows only the initial probability distributions of the number of traders' assets and of their margin account values across the traders population, i.e. $f(N_0)$ and $f(X_0)$ respectively.

To keep it simple, we assume also that all of traders start with the same number of assets, $N_0^j = N_0$ and $f(N_0)$ is actually deterministic. But, we allow traders to have different initial values of their margin accounts, which allows for heterogeneity regarding receiving margin calls.

Remember that, while deriving the bid and ask quotes previously, we assume that ε_t does not depend on A_t or B_t and that allows the bid and ask price derivations to be simple. Now in the context of margin trading, we do not assume it, but we model margin trading such that, within our infinitely many traders setting, ε_t depends only on the *extended* model parameters m, N_0^j, X_0^j and the state variable M_{t-1}. The fact that none of these variables depend on A_t or B_t, still allow the bid and ask price derivations to be simple[44].

[41]hence increasing the value of her margin account

[42]even though the state variables N_t^j and X_t^j for the picked trader j are updated after each period's trade as described above

[43]This means also that, from the market maker's point of view, regarding the quoting process, the effective probability distributions of N_t^j and X_t^j, $f(N_t)$ and $f(X_t)$ respectively, are constant over time.

[44]In practice, for margin requirement calculations mostly last mid-quote, and sometimes last transaction prices or quotes, are used. If $M_t = (A_t + B_t)/2$ were used instead of M_{t-1}, bid and ask price derivation complications would remain.

Note that, we allow for uncertainty in the level of *constrained* traders. Before the trades begin, it is the case that, ε_t starts initially either with a higher value of ε_0^H with probability δ^{ε^H} or with a lower value of ε_0^L otherwise. This is attained by allowing two possible, and different, initial probability distributions for the traders' margin accounts values: $f^H(X_0)$ and $f^L(X_0)$. The realized initial $f(X_0)$ distribution, and consequently the realized ε_0, is not observed by any market participant. However, its probability structure is known and updated over time. In our way of modeling, from the market maker's point of view, regarding the quoting process, the distribution $f(X_t)$ is effectively constant over time. Thus, learning the realized initial $f(X_0)$ distribution means learning also the effective distribution of $f(X_t)$ over time.

Last, the probabilities of observing a Buy, as well as a Sell, are the same[45] as in the previous subsection of constantly constrained uninformed traders, except that now $\varepsilon_t^{U,H}$ and $\varepsilon_t^{U,L}$ vary over time.

3.2.4 Dynamically Constrained Informed Traders

In this subsection, the modeling is analogous to the previous subsection, where instead of the uninformed traders, informed traders are constrained. Note that, the probabilities of observing a Buy, as well as a Sell, are the same[46] as in the previous subsection of constantly constrained informed traders, except that now $\varepsilon_t^{I,H}$ and $\varepsilon_t^{I,L}$ vary over time.

3.2.5 Predictions of the Extended Model

Within the Results section further, we perform various Monte Carlo simulations of trades over T periods, by using the derived bid and ask prices, in order to analyze the inference problem of the market makers and the quote process. We analyze the evolutions of the Bid, Ask, Trade Prices, Mid-Quotes and the updated posterior probabilities for V and ε_t. We are particulary concerned about the convergence[47] and any deviation[48] regarding the trade prices.

First of all, we have an *extended* version of the Glosten and Milgrom (1985) model; it is extended by allowing a portion of the traders to be *constrained*, as well as extended

[45] under all the four possible states of the world
[46] under all the four possible states of the world
[47] when the bid and ask prices for the asset are equal to V
[48] a bias on average

with allowing a second dimension of uncertainty, that is in the level of *constrained* traders. First, we predict that the convergence can become slower or quicker than before, but the (smooth) convergence will still occur, as long as in the market there exist together both *unconstrained* informed and another type of trader who are trading not for profit reasons[49]. Second, due to double uncertainty and possible confusions by the market makers, we predict that under some circumstances the trade prices might exhibit biases. In that case, we analyze qualitatively and quantitatively the evolutions of these deviations, such as their directions, persistence and recoveries.

Next, we have the four settings. We predict for the setting (ii) Constantly Constrained Informed Traders, compared to the setting (i) Constantly Constrained Uninformed Traders, that the effects will be different and probably more pronounced. Because, if some uninformed traders have to conduct forced trades, it will not be a big change in the sense that still their trades will not contain any information on V, but only there will be some shift in the portion of traders in the market that are willing to buy rather than to sell, independent of this liquidation value. However, if some informed traders are forced to trade, then they will not be able to disseminate their private information in the markets, and they will even confuse the market makers. Regarding the settings (iii) and (iv) with Dynamically Constrained Traders, we are actually allowing for contagion of margin calls and forced trades among traders, i.e. the forced trades of some traders can result in further forced trades of other traders. In such an environment, we predict that a positive feedback mechanism is involved and that can exacerbate the mentioned price deviations.

4 The Numerical Simulation Results

Having the bid and ask prices derived, we now perform the various Monte Carlo simulations of trades over T periods, as described, and analyze the outcomes, such as the evolutions of the Bid, Ask, Trade Prices, Mid-Quotes and the updated posterior probabilities for V and ε_t. For each simulation, we set $V_H = \$150$, $V_L = \$50$, $\delta^H = 0.5$, $\gamma^B = \gamma^S = 0.5$, $\mu = 1/3$.

And, for the dynamically constrained traders settings, where we model the mechanism that generates the forced trades in the context of margin trading, we set $m = 0.5$, $N_0 = 2$. Note that, without loss of generality, by choosing $N_0 = 1/m = 2$, the margin call triggering

[49] such as *unconstrained* uninformed traders or any type of *constrained* traders

rule[50] becomes:

$$X_t^j < (2 * 0.5 - 2) * M_{t-1},$$

$$X_t^j < -M_{t-1}.$$

This means that, any trader whose initial margin account value is less than $-M_{t-1}$ is *constrained* for period t and receive a margin call, while the rest of the traders are *unconstrained*. Selecting N_0 in this way, keeps the dynamics of margin calls directly anchored to the mid-quote and keeps our analysis simple.

We perform $n = 10,000$ runs to calculate averages[51]. Within these $10,000$ runs, we have both $V = V_H$ and $V = V_L$ cases. We average all of these simulation results to obtain our *overall* results, unconditional on the realized value of V.

As a *base* setting, we start with analyzing the *basic* model, where in the markets $\varepsilon_t = 0$ with certainty, i.e. there are no forced trades at all.

4.1 Unconstrained Traders

By setting $\varepsilon_t = 0$ the *extended* model collapses into the *basic* model. First of all, for the *base* setting, we show in the Graph (i) of Fig.3, the *overall* results for the average Bid, Ask, Trade Prices and Mid-Quotes. The uppermost red dashed line is the average ask prices, while the lowermost blue dashed line is the average bid prices and the middle light-blue dotted line is the average mid-quotes. Finally, the pink solid line, which is our main concern, is the average trade prices. This setting with no forced trades is the same as in Glosten and Milgrom (1985), which is a standard sequential trading model with one dimension of uncertainty. We observe that trade prices have the martingale property, i.e. their values at any t are equal to the conditional expectation of their values at a future time given the information available at t. In the Graph (ii) of Fig.3, we illustrate the same of Graph (i) for the bad-news case, where we observe the smooth and exponential convergence of prices to V_L over time. Similarly, in the Graph (iii), we illustrate the

[50]for traders who have not yet traded by period t, which in fact correspond to almost all of the traders within our infinitely many traders setting.

[51]The number of runs (n) should be set high enough to let the average results reflect the expected values accurately enough.

same for the good-news case. Graph (*iv*) depicts for the good-news case, the posterior probability for $V = V_H$. In line with Graph (*iii*), we observe in the Graph (*iv*) how the posterior probability of $V = V_H$ starts from $\delta^H = 0.5$ and converges over time smoothly and exponentially to unity, its correct value.

The Buy/Sell probabilities under V_H and V_L, as expected by the market maker and in reality are given within Table 1.

under	Expected (B/S)	Reality (B/S)
V_H	0.667/0.333	0.667/0.333
V_L	0.333/0.667	0.333/0.667

Table 1: Buy/Sell probabilities under V_H and V_L, as expected by the Market Maker and in reality. Unconstrained Traders Setting.

This table indicates that the market maker expects under V_H more Buys (0.667) than Sells (0.333), while she expects under V_L less Buys (0.333) than Sells (0.667). Her expectations are correct and reflects the reality. Suppose, in reality it is V_H. Then, the market experiences on average more Buys (0.667) than Sells (0.333). Observing more Buys than Sells, the market maker correctly infers that the liquidation value of the asset is probably V_H. This smooth inference process reflects itself in monotonous convergence of the trade prices, as observed in the Graphs (*iii*) and (*iv*) of Fig.3.

Note that, the convergence[52] occurs in this setting at around $t = 50$.

[Insert Fig.3 here]

4.2 Constantly Constrained Uninformed Traders

Within this setting, we analyze two scenarios: (i) the level of *constrained* uninformed traders is high ($\varepsilon^U = \varepsilon^{U,H}$) and it is initially very much underestimated by the market maker (i.e. $\mathbb{E}[\varepsilon^U|F_0] << \varepsilon^{U,H}$), (ii) the level of *constrained* uninformed traders is high ($\varepsilon^U = \varepsilon^{U,H}$) and this is known ($\delta^{\varepsilon^{U,H}} = 1$). Note that, the first scenario (i), which is the one that contains our main findings, is the average of simulation runs considering only the cases where the level of *constrained* uninformed traders is actually high, i.e. simulation runs conditional on $\varepsilon^U = \varepsilon^{U,H}$. The second scenario is for comparison purposes.

[52]the bid and ask prices for the asset being within the %1 neighborhood of the correct V

To attain scenario (i) we set the parameters $\varepsilon^{U,L} = 1/3$, $\varepsilon^{U,H} = 0.9$ and $\delta^{\varepsilon^{U,H}} = 0.1$. To attain scenario (ii) we set the parameters $\varepsilon^{U,H} = 0.9$ and $\delta^{\varepsilon^{U,H}} = 1$.

First of all, we show the *overall* results for the two scenarios (i) and (ii) in Fig.4 and Fig.5 respectively.

[Insert Fig.4 here]

Under the scenario (i), illustrated in Fig.4, we observe an undershoot that drives the prices down to \$75 within roughly the first 20 periods, and this does not correct much before the 250th period. In order to be able to compare different undershoots observed in the trade prices, we quantify them by defining three dimensions of an undershoot: (i) the magnitude of the undershoot (the difference between its minimum and its initial value)[53], (ii) the time it takes to reach the minimum, and (iii) the recovery rate of the undershoot (the slope of the recovery from the minimum to correction level[54]). For instance for the undershoot in Fig.4, the magnitude of the undershoot is \$24.75, it takes 21 periods to reach the minimum and the recovery rate is $23.76/227 = 0.105$.

The trade prices in Fig.5 are almost the same[55] as in Fig.3 - Graph (i) of the *base* setting with no forced trades. From Fig.5 we conclude that, the forced trades themselves, even at high levels, do not create any biases in the trade prices, i.e. the trade price process is still martingale straight along \$100.

[Insert Fig.5 here]

By comparing Fig.4 with Fig.5, we suggest that the undershoot in the scenario (i) is related to the double uncertainty[56] and to the underestimation of the level of *constrained* uninformed traders. For scenario (i), while in reality it is the *constrained* uninformed traders who are often selling in the first periods, the market on average confuses them with informed selling, which then drives the prices downwards. Over time, the correct level of *constrained* uninformed traders is learned and the mispricing is corrected. For

[53] Note that, the maximum possible magnitude of an undershoot is the difference between V_L and the initial value, which is \$100.

[54] we define the correction level as the %1 neighborhood of the correct value of the prices, which is its initial value due to its normally martingale property.

[55] in the sense of exhibiting martingale property

[56] the one in the liquidation value of the asset and the other one in the level of *constrained* uninformed traders

this scenario, the Buy/Sell probabilities under V_H and V_L in reality ($\varepsilon^U = \varepsilon^{U,H} = 0.90$) and as initially expected by the market maker ($\mathbb{E}[\varepsilon^U|F_0] = 0.39$) are given within Table 2. This table indicates that initially the market maker expects under V_H more or less

under	Expected (B/S)	Reality (B/S)
V_H	0.537/0.463	0.367/0.633
V_L	0.203/0.797	0.033/0.967

Table 2: Buy/Sell probabilities under V_H and V_L, as initially expected by the Market Maker and in reality. Constantly Constrained Uninformed Traders Setting and Scenario (i).

equally probable Buys (0.537) and Sells (0.463), while she expects under V_L much more Sells (0.797) than Buys (0.203). Suppose, in reality it is V_H. Then, the market apparently experiences on average more Sells (0.633) than Buys (0.367). Observing more Sells than Buys, the market maker mistakenly starts thinking that the liquidation value of the asset is probably V_L, even though it is V_H; this reflects itself in a slower convergence of, if not an undershoot in, the trade prices. Fig.6, having the results for the average Bid, Ask, Trade Prices and Mid-Quotes under good-news case and scenario (i), depicts this phenomenon. Note that, the fact that 0.367 [57] in Table 2 is almost equally distanced to 0.537 [58] as to 0.203 [59] makes the convergence of prices to V_H initially delayed. If 0.367 were clearly closer to 0.537 than to 0.203, the convergence would start at the very first periods. Next, suppose in reality it is V_L. Then, the market experiences on average much more Sells (0.967) than Buys (0.033). Observing more Sells than Buys, the market maker again starts thinking, but this time correctly, that the liquidation value of the asset is probably V_L. And, observing more Sells than Buys, even more compared to the expected levels, makes the prices converge to the liquidation value V_L only faster; i.e. what the initial confusion induces is in the correct direction of convergence. Overall, the delayed convergence within the V_H case and the faster convergence within the V_L case, when averaged, result in a clear undershoot as depicted in Fig.4.

[Insert Fig.6 here]

Another perspective is brought to the situation, when the inference processes regarding the double uncertainty are analyzed in details. In Fig.7, we show for the good-news case

[57] the Buy probability in reality, i.e. $V = V_H$ and $\varepsilon^U = \varepsilon^{U,H}$

[58] the Buy probability expected under V_H

[59] the Buy probability expected under V_L

and scenario (i), the posterior probabilities for $V = V_H$ and $\varepsilon^U = \varepsilon^{U,H}$. First, the pink solid line (probability of good-news) does not improve initially, since the green dashed line (probability of high amount of forced trades) fails to start converging to unity, its correct value. In the *basic* model, the convergence of the pink solid line is smooth and at an exponential rate starting from the first periods. Second, the green dashed line is observed to stay away from unity for a long time[60]. This situation does not allow the learning on the correct liquidation value V to be effective.

[Insert Fig.7 here]

Note that, for this setting we set $T = 250$ [61]. This value for T is high enough[62], because, as shown in Fig.4, the mispricing correction, as well as the smooth convergence, are both completed (just) before $t = 250$. We set $T = 250$ also for the next settings to be able to compare the convergence and correction outcomes of these settings relative to this one.

It is important to point out that, having a constant level of *constrained* uninformed traders within the static[63] *extended* model induces effectively a static shift from Buys to Sells of uninformed traders. This is equivalent to, by means of the model parameters, having the *basic* model with particularly chosen $\gamma^S > \gamma^B$. This means first of all that, our results in this subsection are not limited to forced trades. Therefore, this subsection's results are valid also for various other mechanisms; any other mechanism that induces an equivalent uncertainty and shift in the levels of γ^S/γ^B will generate the same observations. Second, this subsection's results are not the product of any positive feedback mechanisms. This is because, within this static forced trades setting, the adverse price changes do not result in any further forced trades and price movements.

We finally show in Fig.8 the *overall* results for the scenario (i) in which the *constrained* uninformed trader have net short, rather than long positions, i.e. $N_0 = -2$ while the rest of the parameters are the same. We see the consequences are reversed. For the

[60] This is the case because the initially expected amount of forced trades is far from its correct value and the convergence occurs slowly.

[61] T is interpreted as the time at which no trader has an informational advantage, i.e. the bid and ask prices for the asset being equal to V. This can happen either smoothly via information dissemination by informed traders before or at period T, or suddenly via a public announcement of the private information at period T.

[62] but not too high

[63] static, because we have constantly *constrained* traders and adverse price changes do not endogenously alter the amount of forced trades in the market

overshoot in Fig.8, the magnitude of the overshoot is \$25.15, it takes 17 periods to reach the maximum and the recovery rate is $24.16/227 = 0.106$. Note that, overestimation instead of underestimation would also result in similar reversed outcomes, i.e. overshoots.

[Insert Fig.8 here]

4.3 Constantly Constrained Informed Traders

In this setting, we analyze again two scenarios: (i) the level of *constrained* informed traders is high ($\varepsilon^I = \varepsilon^{I,H}$) and it is initially very much underestimated by the market maker (i.e. $\mathbb{E}[\varepsilon^I|F_0] << \varepsilon^{I,H}$), (ii) the level of *constrained* informed traders is high ($\varepsilon^I = \varepsilon^{I,H}$) and this is known ($\delta^{\varepsilon^{I,H}} = 1$). Note that, the first scenario (i), which is the one that contains our main findings, is the average of simulation runs considering only the cases where the level of *constrained* informed traders is actually high, i.e. simulation runs conditional on $\varepsilon^I = \varepsilon^{I,H}$. The second scenario is for comparison purposes.

Similar to the previous subsection, to attain scenario (i) we set the parameters $\varepsilon^{I,L} = 1/3$, $\varepsilon^{I,H} = 0.9$ and $\delta^{\varepsilon^{I,H}} = 0.1$. To attain scenario (ii) we set the parameters $\varepsilon^{I,H} = 0.9$ and $\delta^{\varepsilon^{I,H}} = 1$.

Note that, as mentioned at the previous subsection, we set $T = 250$ to be able to compare the convergence and correction outcomes of this setting relative to previous subsection results[64].

First of all, we show the *overall* results for the two scenarios (i) and (ii) in Fig.9 and Fig.10 respectively.

[Insert Fig.9 here]

Under the scenario (i), in Fig.9, we observe an undershoot that drives the prices almost down to \$60 within roughly the first 100 periods, and this is almost not at all corrected by the 250th period. In order to be able to compare different persistent undershoots observed in the trade prices, we quantify them by defining three dimensions of a persistent undershoot: (i) the magnitude of the (remaining) deviation at $t = T$ (the difference

[64]This value for T was high enough, as shown in Fig.4, for the mispricing to get corrected in the previous subsection.

between its value at T and its initial value)[65], (ii) the persistence rate of the deviation after experiencing its maximum (the ratio of the magnitude of the deviation at $t = T$ to the magnitude of the maximum deviation), and (iii) the recovery rate of the deviation (the slope of the recovery from the time it is minimum till $t = T$). For instance for the undershoot in Fig.9, the magnitude of the (remaining) deviation is \$37.43, after the deviation is at its maximum at $t = 157$ it persists at a rate of 0.959 until $t = 250$ and its recovery rate is $1.54/93 = 0.017$.

The trade prices in Fig.10 are exhibiting the martingale property as in Fig.3 - Graph (i) of the *base* setting with no forced trades. From Fig.10 we again conclude that, the forced trades themselves, even at high levels, do not create any biases in the trade prices, i.e. the trade prices are still martingale straight along \$100.

[Insert Fig.10 here]

By comparing Fig.9 with Fig.10, we again suggest that the undershoot in the scenario (i) is related to the double uncertainty[66] and to the underestimation of the level of *constrained* informed traders. For the scenario (i), while in reality it is the *constrained* informed traders who are often (forced-) selling in the first periods, the market on average confuses this with informed[67] selling, which then drives the prices downwards. For this scenario, the Buy/Sell probabilities under V_H and V_L in reality ($\varepsilon^I = \varepsilon^{I,H} = 0.90$) and as initially expected by the market maker ($\mathbb{E}[\varepsilon^I|F_0] = 0.39$) are given within Table 3. This table indicates that

under	Expected (B/S)	Reality (B/S)
V_H	0.532/0.468	0.367/0.633
V_L	0.333/0.667	0.333/0.667

Table 3: Buy/Sell probabilities under V_H and V_L, as initially expected by the Market Maker and in reality. Constantly Constrained Informed Traders Setting and Scenario (i).

the market maker expects under V_L correctly the probabilities of Buys (0.333) and Sells (0.667) [68]. She expects under V_H more or less same percentages of Buys (0.532) and Sells

[65]Note that, the maximum possible magnitude of a deviation is the difference between V_L and the initial value, which is \$100.

[66]the one in the liquidation value of the asset and the other one in the level of *constrained* informed traders

[67]information-containing

[68]For the *constrained* informed traders setting, it is not a coincidence that the expected and in reality

(0.468). First suppose, in reality it is V_H. Then, the market apparently experiences on average less Buys (0.367) than it is expected under V_H. Observing less Buys, the market maker mistakenly starts thinking that the liquidation value of the asset is probably V_L, even though it is V_H; this reflects itself in an undershoot in the trade prices. Fig.11, having the results for the average Bid, Ask, Trade Prices and Mid-Quotes under good-news case and scenario (i), depicts this phenomenon. Note that, in Table 3, 0.367, i.e. the Buy probability in reality where $V = V_H$ and $\varepsilon^I = \varepsilon^{I,H}$, is very close to 0.333, i.e. the Buy probability expected under V_L, while it is far to 0.532, which is the Buy probability expected under V_H. This is why we achieve for the *constrained informed traders* setting a larger undershoot. In Fig.11, after the prices are driven downwards to their minimum value, the recovery is dramatically slow and we observe a persistent mispricing. This is mainly due to the fact that in reality the level of *unconstrained* informed traders, therefore the correct amount of information-containing trades is very low[69]. When, over time, the correct level of *unconstrained* informed traders is learned to be lower and lower than it is initially expected[70], the impact of a single trade decreases and the recovery of prices, that came mistakenly close to V_L, takes place slower and slower[71]. Fig.12, shows for the good-news case and scenario (i), the posterior probabilities for $V = V_H$ and $\varepsilon^I = \varepsilon^{I,H}$. Note that, the green dashed line (probability of high amount of forced trades) is very slowly converging to unity[72] and the pink solid line (probability of good-news) mistakenly goes down most of the periods. Fig.13 has the results for a single simulation of the Bid, Ask, Trade Prices and Mid-Quotes under good-news case and scenario (i). Even though it is the good-news case, we observe that at $t = 250$ the prices are close to V_L, rather than to V_H. This is a single but representative simulation result. Because, out of $10,000$ simulation runs[73], for $3,374$ good-news cases we observe that at $t = 250$ the prices are between \$50 and \$75. Next, suppose in reality it is V_L. In this case, there is no confusion in the markets resulting from whether the market experiences on average a different percentage of Buys (0.333) and Sells (0.667) than expected. However, Fig.14, having the results for the average Bid, Ask, Trade Prices and Mid-Quotes under bad-news case and scenario

Buy/Sell probabilities are the same for V_L, because *constrained* informed traders would sell anyway under V_L.

[69] See Fig.10 for the consequences when that is the case and there is no underestimation

[70] Note that, it is learned very slowly.

[71] When we allow for a greater T for the simulations, we see that the correction occurs not before $t = 5,000$.

[72] its correct value

[73] where around $5,000$ of them are good-news cases

(i), illustrates a very slow convergence rate between the $150th$ and the $250th$ periods; at $t = 250$ the average trade prices are still at around \$58. This is mainly due to the very low amount of information-containing trades, which is learned by the market by $t = 150$. Overall, when we average the V_H and V_L cases, we observe a persisting undershoot, as depicted in Fig.9.

[Insert Fig.11 here]

[Insert Fig.12 here]

[Insert Fig.13 here]

[Insert Fig.14 here]

It is again important to point out that, having a constant level of *constrained* informed traders within the static *extended* model induces effectively a static shift from informed traders to the uninformed traders; particularly from Buys of informed traders to Sells of uninformed traders. This is equivalent to, by means of the model parameters, having the *basic* model with particularly chosen $\gamma^S > \gamma^B$ and a lower μ. This means first of all that, our results in this subsection are not limited to forced trades. Therefore, this subsection's results are valid also for various other mechanisms; any other mechanism that induces an equivalent uncertainty and shift in the levels of γ^S/γ^B and μ will generate the same observations. Second, this subsection's results are also not the product of any positive feedback mechanisms. This is because, within this static forced trade setting, the adverse price changes do not result in any further forced trades and price movements. Next subsections, having dynamically *constrained* traders, provide deeper insights specific to forced trades, particularly to margin trading, as well as show how an informational positive feedback mechanism plays a role to amplify the mentioned price deviations.

4.4 Dynamically Constrained Uninformed Traders

First of all, we model the initial distribution of the traders' margin account values, $f^U(X_0)$, as uniform distribution. Fig.15 depicts for the cases when the uninformed traders are (i) highly constrained, $f^{U,H}(X_0) = U(-150, -94.4)$ and (ii) not highly constrained, $f^{U,L}(X_0) = U(-150, 0)$. The areas that are left to $-\$100$ correspond to $\varepsilon_0^{U,H}$ and $\varepsilon_0^{U,L}$ respectively. Therefore, ε_t^U starts initially either with a higher value equal to $\varepsilon_0^{U,H} = 0.9$ or with a lower value equal to $\varepsilon_0^{U,L} = 1/3$. These are the same fractions used within the previous subsections for the *constantly* constrained traders, so that the results can be compared better[74]. Over time, $-M_{t-1}$ changes within the $(-\$150, -\$50)$ range and the $\varepsilon_t^{U,H}$ and $\varepsilon_t^{U,L}$ are updated as the new areas of the distributions that are to the left of $-M_{t-1}$. For instance, if $-M_{t-1}$ is equal to $-\$120$ at some point of time, then there will be less margin calls in the market than initially and the effective $\varepsilon_t^{U,H}$ and $\varepsilon_t^{U,L}$ will be equal to 0.54 and 0.20 respectively[75].

[Insert Fig.15 here]

Within this setting, we analyze only the first scenario where the level of *constrained* uninformed traders is high ($\varepsilon_t^U = \varepsilon_t^{U,H}$) and it is initially very much underestimated by the market maker (i.e. $\mathbb{E}[\varepsilon_0^U|F_0] << \varepsilon_0^{U,H}$). To attain this, we again set $\delta^{\varepsilon_0^{U,H}}$ equal to 0.1. Note that, for this scenario we average the simulation results considering only the cases where the level of *constrained* uninformed traders is actually high, i.e. simulation results conditional on $\varepsilon_t^U = \varepsilon_t^{U,H}$.

We show the *overall* results in Fig.16. We observe an undershoot that drives the prices down to \$80 within roughly the first 10 periods, and this does not correct much before the 150th period. The magnitude of the undershoot is \$20.08, it takes 9 periods to reach the minimum and the recovery rate is 0.171. The observed undershoot is quite similar to the one observed in the *constantly* constrained uninformed traders setting. Both have magnitudes between $\$20 - \$25$ and get corrected before $t = 250$. In this setting the magnitude of the undershoot is smaller, which we explain next.

[Insert Fig.16 here]

The *overall* results are the averages of the results under the good-news case and the bad-news case. For the former the results are presented in Fig.17. Note that, for this

[74] Note that, if the level of margin calls were kept constant, for instance hypothetically by keeping them independent of the varying prices, then this dynamic setting would be equivalent to the previous constantly constrained uninformed traders setting.

[75] effective, from the market maker's point of view and regarding the quoting process

setting, the *initial* Buy/Sell probabilities under V_H and V_L, in reality and as expected by the market maker are the same as in the *constantly* constrained uninformed traders setting. Therefore, they are still represented by Table 2. The difference is that, in this setting the values in Table 2 vary over time with M_{t-1} moving. Since the mid-quotes have substantially different average trajectories under $V = V_H$ and $V = V_L$, we examine the Buy probabilities separately under these cases. The dynamics of the Buy probabilities under V_H and V_L, each as expected by the Market Maker, when $\varepsilon_t^U = \varepsilon_t^{U,H}$ and when $\varepsilon_t^U = \varepsilon_t^{U,L}$ under the good-news case is presented in Fig.18. Suppose, in reality it is V_H. In Fig.18, the three lines we are concerned are the pink solid line [76], the red dashed line [77] and the blue dashed line [78]. As mentioned in the *constantly* constrained traders settings, an important indicator is the position of the pink solid line between the red and the blue dashed lines. Initially it is almost equally distanced to them. Remember that, this was also the case for the *constantly* constrained uninformed traders setting as seen in Table 2 and it made the convergence of prices to V_H initially delayed. In this setting, however, as seen in In Fig.18, the pink solid line quickly and correctly converges to the the red dashed line. Consequently, here the convergence of prices to V_H is initially not delayed[79] and the magnitude of the undershoot in Fig.16 is smaller than the one in the *constantly* constrained uninformed traders setting.

[Insert Fig.17 here]

[Insert Fig.18 here]

In summary, even though the observed undershoot is smaller in magnitude, it is still quite similar to the one in the *constantly* constrained uninformed traders setting. And, we do not observe any positive feedback mechanisms that amplify the price deviations.

4.5 Dynamically Constrained Informed Traders

We model the initial distribution of the traders' margin account values, $f^I(X_0)$, exactly in the same way in the dynamically constrained *uninformed* traders setting, i.e. also as

[76] the Buy probability in reality, i.e. $V = V_H$ and $\varepsilon_t^U = \varepsilon_t^{U,H}$

[77] the Buy probability expected under V_H

[78] the Buy probability expected under V_L

[79] see Fig.17

uniform distribution and as given in Fig.15. Within this setting, we again analyze only the first scenario where the level of *constrained* informed traders is high ($\varepsilon_t^I = \varepsilon_t^{I,H}$) and it is initially very much underestimated by the market maker (i.e. $\mathbb{E}[\varepsilon_0^I|F_0] << \varepsilon_0^{I,H}$). To attain this, we once more set $\delta^{\varepsilon_0^{I,H}}$ equal to 0.1. Note that, for this scenario we average the simulation results considering only the cases where the level of *constrained* informed traders is actually high, i.e. simulation results conditional on $\varepsilon_t^I = \varepsilon_t^{I,H}$.

We show the *overall* results in Fig.19. We observe an undershoot that drives the prices down to \$70 within roughly the first 125 periods, and this is not at all corrected by the 250th period. The magnitude of the deviation at $t = 250$ is \$31.30, and there is no recovery. The observed undershoot seems quite similar to the one observed in the *constantly* constrained informed traders setting. Both have magnitudes between \$30 − \$40 by the 150th period and both highly persist afterwards. However, a very important difference is that in this setting there is absolutely no recovery, which we explain in the next paragraph. Note that, when we allow for a greater T for the *constantly* constrained informed traders setting simulations[80], we see that the correction actually occurs at around $t = 5,000$ despite the high persistence. Moreover, the mentioned saturation of the prices[81] in Fig.14 is also resolved when we allow for a greater $T = 5,000$. The *overall* results are the averages of the results under the good-news case and the bad-news case and the results for the former are presented in Fig.20. The *overall* undershoot in Fig.19 apparently stems from the undershoot seen in Fig.20.

[Insert Fig.19 here]

[Insert Fig.20 here]

First of all, in this setting out of $10,000$ simulation runs[82], for $3,776$ good-news cases we document that at $t = 250$ the prices are between \$50 and \$75 [83]. When we average these cases and the correctly converging cases together, we achieve the undershoot seen in Fig.20.

[80] see Fig.9

[81] For the constantly constrained informed traders setting under bad-news case we observe an extremely slow convergence after $t = 150$. The average trade prices at $t = 250$ are still at around \$58, i.e away from $V_L = \$50$.

[82] where around $5,000$ of them are good-news cases

[83] That is even worse than the situation in the *constantly* constrained informed traders setting which has $3,374$ good-news cases exhibiting such misconvergence.

Second, the Table 3 represents for this setting, the *initial* Buy/Sell probabilities under V_H and V_L, in reality and as expected by the market maker. Referring to this table and following the same arguments as done in the *constantly* constrained informed traders setting subsection, one can explain the reasons why we achieve in Fig.20 also a large undershoot in the trade prices[84] and why that persists[85]. However, this does not explain why there is no recovery at all. Note that, the values in Table 3 represents only the initial Buy/Sell probabilities. In this dynamic setting they vary over time with M_{t-1} moving. And again, since the mid-quotes have substantially different average trajectories under $V = V_H$ and $V = V_L$, we examine the Buy probabilities separately under these cases. The dynamics of the Buy probabilities under V_H and V_L, each as expected by the Market Maker, when $\varepsilon_t^I = \varepsilon_t^{I,H}$ and when $\varepsilon_t^I = \varepsilon_t^{I,L}$ under the bad-news case is presented in Fig.21. The three lines we are concerned are the red solid line [86], the red dashed line [87] and the blue dashed line [88]. When the prices move towards V_L over time the red solid line converges to the blue dashed line[89] and stays there. This is an important observation and it is because of a downward spiral affecting the prices. This downward spiral in this setting is a positive feedback mechanism that works as follows: When the prices move in the direction of V_L, the distance between the red solid line and the blue dashed line decreases. That means the inference that is made by observing the trades gets distorted. In turn this distortion increases the prices' tendency of moving towards V_L and that feedback forms the vicious cycle. This mechanism pushes the prices strongly towards V_L and makes them stay there. If the prices are moving towards V_L because it is the bad-news case, then this is not a problem. However, remember that, for the 3, 776 good-news cases the prices convergence mistakenly to V_L and stay there. We can conclude that the main underlying reason is this feedback mechanism.

[Insert Fig.21 here]

In short, the observed undershoot seems to be quite similar to the one in the *constantly* constrained informed traders setting, because none of them exhibit any clear recovery. But,

[84]Because 0.367, the initial Buy probability in reality where $V = V_H$ and $\varepsilon^I = \varepsilon^{I,H}$, is very close to 0.333, the initial Buy probability expected under V_L, while it is far to 0.532, which is the initial Buy probability expected under V_H.

[85]mainly due to the fact that in reality the correct amount of information-containing trades is very low

[86]the Buy probability in reality, i.e. $V = V_H$ and $\varepsilon_t^I = \varepsilon_t^{I,H}$

[87]the Buy probability expected under V_H

[88]the Buy probability expected under V_L

[89]Note that, the blue dashed line is not clearly seen because it overlaps with the black solid line.

due to the positive feedback mechanism that amplifies the price deviations and worsens the inference distortions, the undershoot does not recover at all in this dynamic setting.

Finally, in Fig.21, we see that the initial distance between the red solid line and the blue dashed line is already very close to each other. Even though this situation exacerbates the problem, it is not required for the feedback mechanism to occur. We show this in the following Robustness Analysis section.

5 Robustness Analysis

In the previous main results section, we performed various Monte Carlo simulations and made many conclusions. Now, we check whether these results are robust with respect to the selected model parameters. The most important ones are the level of *constrained* traders and the severity of the market makers' underestimation for it.

Remember that, for the *constantly* constrained traders settings we set the parameters $\varepsilon_0^L = 1/3$, $\varepsilon_0^H = 0.9$ and $\delta^{\varepsilon_0^H} = 0.1$ to attain scenario (i). Or, for the *dynamically* constrained traders settings, we modeled the initial distribution of the traders' margin account values, $f(X_0)$, as uniform distribution; particularly as $f^H(X_0) = U(-150, -94.4)$ and $f^L(X_0) = U(-150, 0)$. That means, for all of the settings, the initial level of *constrained* traders is actually very high and it is initially very much underestimated by the market maker ($\mathbb{E}[\varepsilon|F_0] = 0.39 << \varepsilon_0^H = 0.90$). Moreover, for the *dynamically* constrained traders settings, if $-M_{t-1}$ moves from $-\$100$ towards $-\$94.4$ [90], then the value of $1 - \varepsilon_t^H$ [91] decreases from 0.1 to 0 very quickly.

Suppose the initial level of *constrained* traders were not that high, and initially it was not very much underestimated by the market maker. Would our results be still the same and would our conclusions be still valid? To answer these questions, we now set $\varepsilon_0^L = 1/3$, $\varepsilon_0^H = 0.5$ and $\delta^{\varepsilon_0^H} = 0.1$ for the *constantly* constrained traders settings. And, we set $f^H(X_0) = U(-150, -50)$ and $f^L(X_0) = U(-150, 0)$ for the *dynamically* constrained traders settings. That means, for all of the settings, the initial level of *constrained* traders is moderate and initially it is mildly underestimated by the market maker ($\mathbb{E}[\varepsilon|F_0] = 0.35 < \varepsilon_0^H = 0.50$). Now, we re-run our simulations for each of the settings.

[90] the rightmost support point of $f^H(X_0)$

[91] the level of *unconstrained* traders

5.1 Constantly Constrained Uninformed Traders

First of all, we show the *overall* results in Fig.22. As in the severe underestimation case, we observe an undershoot in the prices that gets corrected, but it is milder. For the undershoot in Fig.22, the magnitude is only \$7.58, it takes 13 periods to reach this minimum and the recovery rate is 0.114, i.e. almost the same.

[Insert Fig.22 here]

Fig.23 has the results for the average Bid, Ask, Trade Prices and Mid-Quotes under the good-news case. We do not observe anymore a clear initial delay in the convergence of prices to V_H. That is why the undershoot *overall* is milder. The reason for that is because the Buy probability in reality[92] (0.500) is closer to the Buy probability expected under V_H (0.550), than to the Buy probability expected under V_L (0.217). And, that improves the convergence of prices to V_H.

[Insert Fig.23 here]

In summary, our main results and conclusions for the constantly constrained uninformed traders are robust.

5.2 Constantly Constrained Informed Traders

Fig.24 illustrates the *overall* results for the *constantly* constrained informed traders setting when the level of forced trades is moderate. For the undershoot in Fig.24, the magnitude is \$7.02, it takes 52 periods to reach this minimum and the recovery rate is 0.035. It is similar to the undershoot in Fig.22 with constantly and mildly constrained *uninformed* traders setting, but it exhibits a quite slower convergence rate. More importantly, unlike the undershoot generated within its severe underestimation case[93], here we observe an undershoot in the prices that gets corrected by $t = 250$.

[Insert Fig.24 here]

[92] i.e. $V = V_H$ and $\varepsilon^U = \varepsilon^{U,H}$
[93] see Fig.9

For this setting, the Buy/Sell probabilities under V_H and V_L in reality ($\varepsilon^I = \varepsilon^{I,H} = 0.50$) and as initially expected by the market maker ($\mathbb{E}[\varepsilon^I | F_0] = 0.35$) are given within Table 4.

under	Expected (B/S)	Reality (B/S)
V_H	0.550/0.450	0.500/0.500
V_L	0.333/0.667	0.333/0.667

Table 4: Buy/Sell probabilities under V_H and V_L, as initially expected by the Market Maker and in reality. Constantly Constrained Informed Traders Setting and Moderate level of Forced Trades.

Compared to Table 3, the Buy probability in reality[94] (0.500), is not anymore close to the Buy probability expected under V_L (0.333), but correctly closer to the Buy probability expected under V_H (0.550). This is why we do not achieve a large undershoot in Fig.24. Moreover, full recovery is achieved by $T = 250$, because in reality the level of *unconstrained* informed traders, therefore the correct amount of information-containing trades is not that low anymore.

Summed up briefly, for the *constantly* constrained informed traders setting the prices do not always exhibit a large and persisting undershoot, as depicted in Fig.9. For that, the Buy probability in reality should be closer to the Buy probability expected under V_L, than to the Buy probability expected under V_H. Additionally, the correct amount of information-containing trades should be low.

5.3 Dynamically Constrained Uninformed Traders

We show the *overall* results in Fig.25. Compared to the severe underestimation case, we observe almost an identical undershoot and correction in the prices. For the undershoot in Fig.25, the magnitude is \$7.75, it takes 12 periods to reach this minimum and the recovery rate is 0.124. Remember that, in the main results section, the undershoots generated under the *constantly* and *dynamically* constrained uninformed traders settings are similar to each other, too.

[Insert Fig.25 here]

To summarize, our main results and conclusions for the dynamically constrained uninformed traders are robust. And, we do not observe any positive feedback mechanisms that

[94] where $V = V_H$ and $\varepsilon^I = \varepsilon^{I,H}$

amplify the price deviations.

5.4 Dynamically Constrained Informed Traders

Fig.26 illustrates the *overall* results for the *dynamically* constrained informed traders setting when the level of forced trades is moderate. For the deviation in Fig.26, the magnitude is \$9.55, it takes 199 periods to reach this minimum and there is no recovery. The *overall* results are the averages of the results under the good-news case and the bad-news case and the results for the former are presented in Fig.27.

First of all, in this setting out of $10,000$ simulation runs[95], for $1,096$ good-news cases we document that at $t = 250$ the prices are between \$50 and \$75 [96]. When we average these cases and the correctly converging cases together, we achieve the undershoot seen in Fig.27.

[Insert Fig.26 here]

[Insert Fig.27 here]

Second, it has similar characteristics with the undershoot generated under its severe underestimation case[97]. As expected, here the level of forced trades is moderate and the magnitude of the undershoot is smaller. More importantly, compared to the undershoot within *constantly* and mildly constrained informed traders setting, which is illustrated in Fig.24, the undershoot is very different. In this case, it does not get corrected; it persists and stays at its minimum value. This is an important observation and it is due to the previously explained downward spiral, that is affecting the prices. To recap, the positive feedback mechanism works as follows: When the prices move in the direction of V_L, the inference that is made by observing the trades gets distorted. In turn this distortion increases the prices' tendency of moving towards V_L and that feedback forms the vicious cycle. This mechanism pushes the prices strongly towards V_L and makes them stay there. If the prices are moving towards V_L because it is the bad-news case, then this is not a

[95] where around $5,000$ of them are good-news cases

[96] As expected, that is better than the situation in its severe underestimation case, where the number is $3,776$.

[97] see Fig.19

problem. However, remember that, for the $1,096$ good-news cases the prices convergence mistakenly to V_L and stay there. And, the main underlying reason is this feedback mechanism. For the bad-news case, the dynamics of Buy probabilities are presented in Fig.28. When the prices move towards V_L over time the red solid line converges to the blue dashed line[98] and stays there. An important observation is, we see in Fig.28 that, even though the initial distance between the red solid line and the blue dashed line is not small, the vicious cycle is still generated. Therefore, a high distance exacerbates the problem, but is not necessary for the feedback mechanism to occur.

[Insert Fig.28 here]

Note that, an important component in generating the positive feedback mechanism is the well-explained[99] undershoot stemming from the underestimation of the level of *constrained* traders. The simulation results for scenario (ii), i.e. when the level of *constrained* informed traders is high and this is known, are not presented here but confirm this proposition.

Next, we comment on the robustness of the results with respect to our choice of the distribution supports. Remember that, for the highly constrained traders we set either $f^H(X_0) = U(-150, -94.4)$ within the main results section, or $f^H(X_0) = U(-150, -50)$ here. When the rightmost support point of the distribution is shifted from -50 towards -94.4, we showed that the problem is aggravated. In contrast, when the rightmost support point is shifted from -50 towards 0 or further, we find that[100] the no-recovery problem vanishes quickly.

In conclusion, we can summarize that the observed undershoot always get corrected under all[101] the constrained *uninformed* traders settings. The magnitude of this undershoot increases with the level of *constrained* traders and the severity of the market makers' underestimation for it. And, a downward spiral is not generated for the constrained *uninformed* traders settings; even for the dynamic settings. Next, compared to the constrained *uninformed* traders settings, for the *constantly* constrained *informed* traders settings, the undershoot is larger and more persistent[102]. However, when the level of forced trades and its underestimation are not severe, then the persistence rate is very low, and we observe

[98] Note that, the blue dashed line is not clearly seen because it overlaps with the black solid line.
[99] and that exists in all of the constrained traders settings
[100] The simulation results are not presented here.
[101] static or dynamic, severely constrained or mildly constrained
[102] It is persistent especially if the the correct amount of information-containing trades is very low.

that the undershoot indeed gets corrected. Finally, when the traders are *informed* and *dynamically* constrained, a positive feedback mechanism, that amplifies the price deviations and exacerbates the inference distortions, pushes the prices strongly towards V_L and makes them stay there.

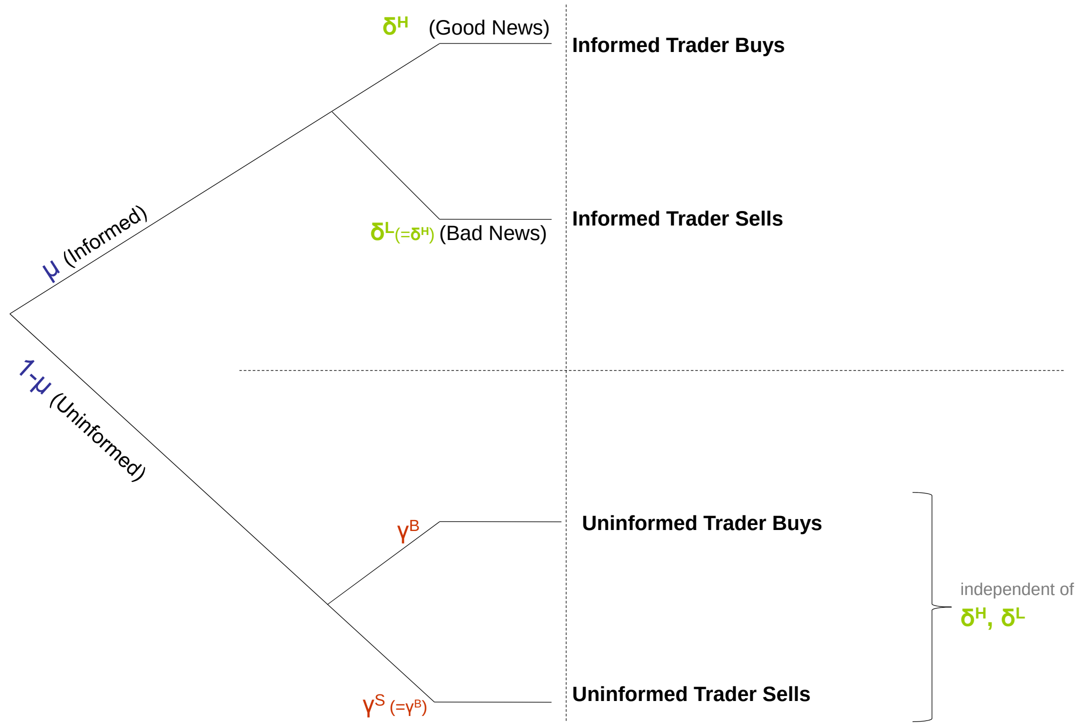

Figure 1: The probability structure of the *basic* model.

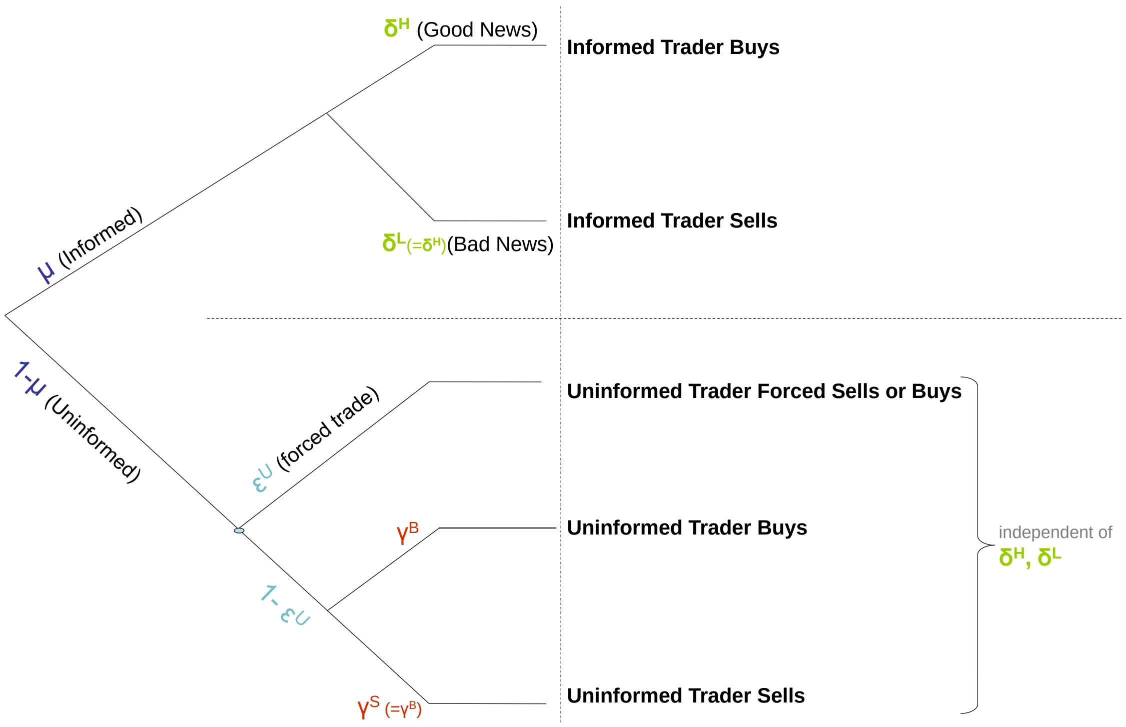

Figure 2: The probability structure of the *extended* model, *constantly constrained uninformed traders* setting.

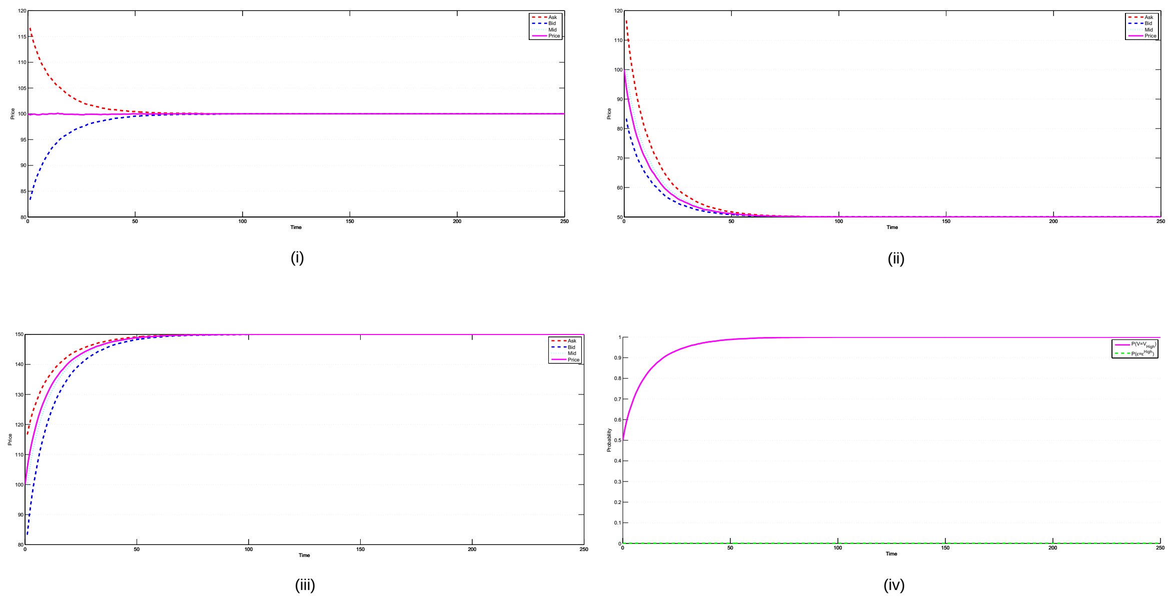

Figure 3: Average Bid, Ask, Trade Prices and Mid-Quotes. *Base* Setting, (i) *Overall* (both good- and bad- news) case, (ii) Bad-news case, (iii) Good-news case. (iv) Posterior Probability for $V = V_H$ under Good-news case.

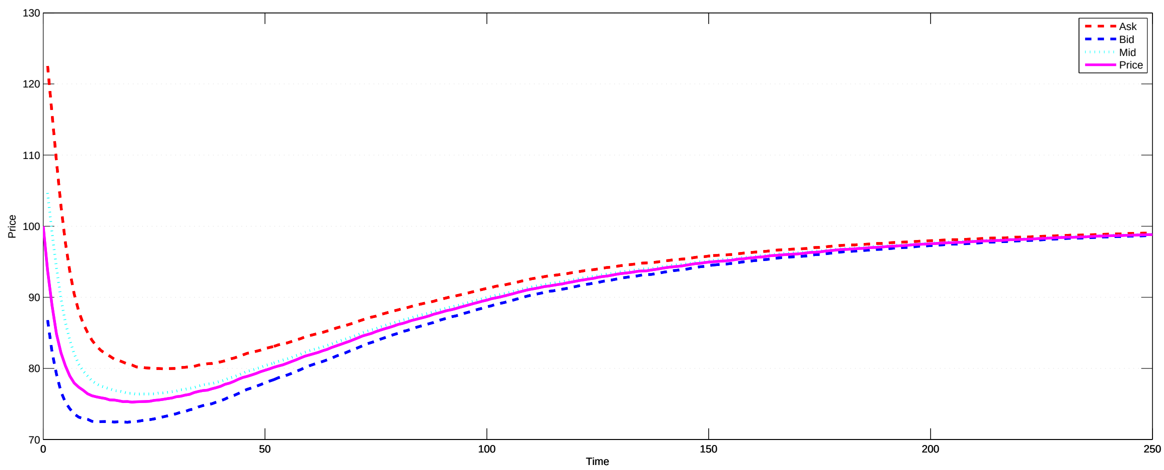

Figure 4: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Uninformed Traders Setting, Scenario (i) and *Overall* (both good- and bad- news) case.

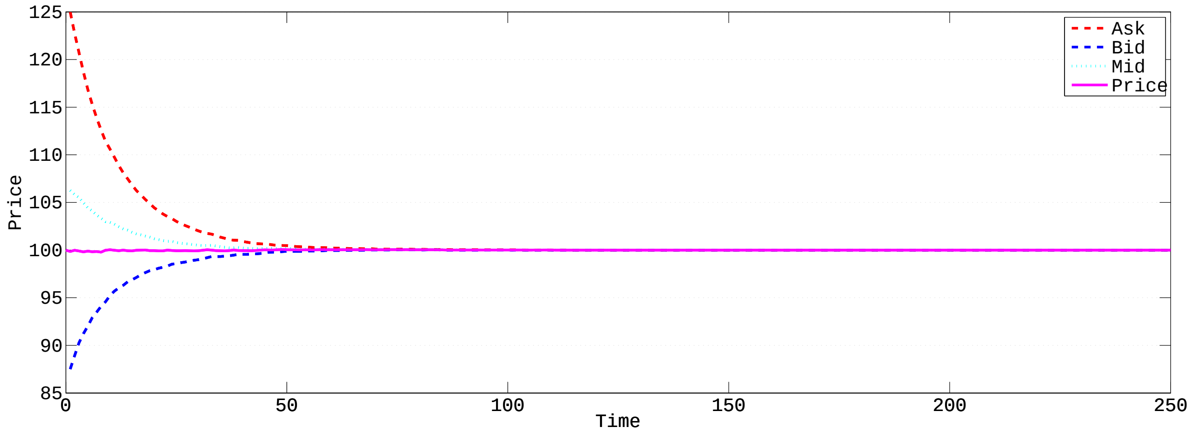

Figure 5: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Uninformed Traders Setting, Scenario (ii) and *Overall* (both good- and bad- news) case.

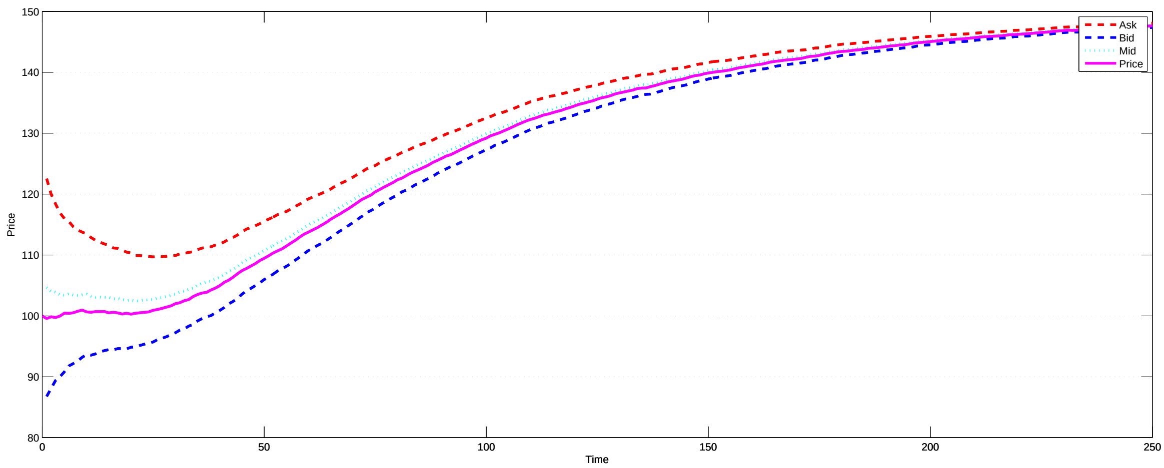

Figure 6: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Uninformed Traders Setting, Scenario (i) under Good-news case.

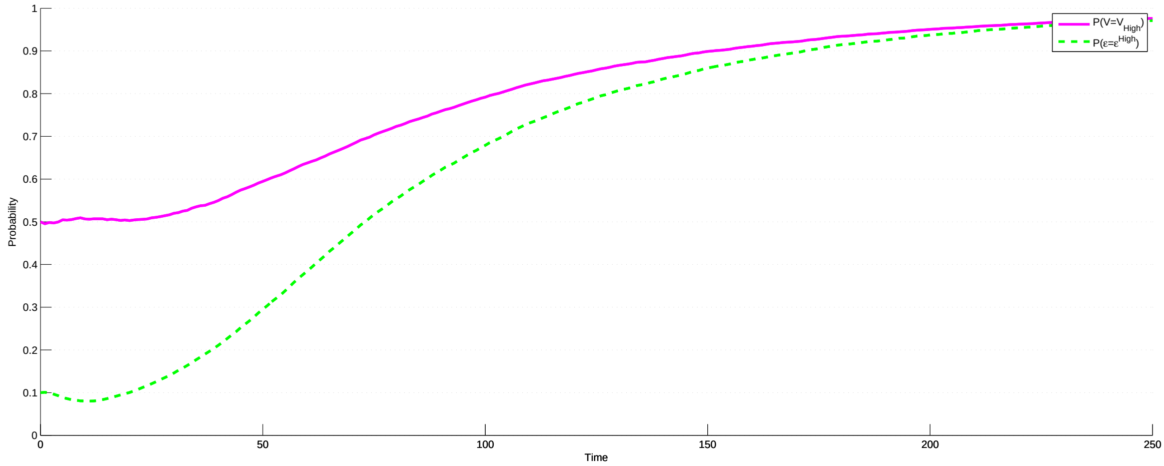

Figure 7: Posterior Probabilities for $V = V_H$ and $\varepsilon^U = \varepsilon^{U,H}$. Constantly Constrained Uninformed Traders Setting, Scenario (i) under Good-news case.

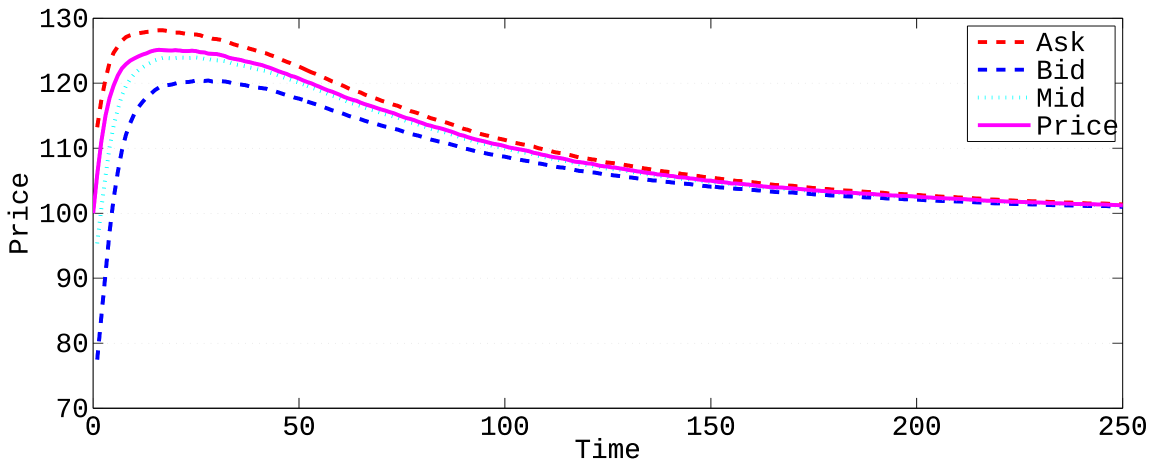

Figure 8: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Uninformed Traders Setting, Scenario (i), *Overall* (both good- and bad- news) case and *Short* position.

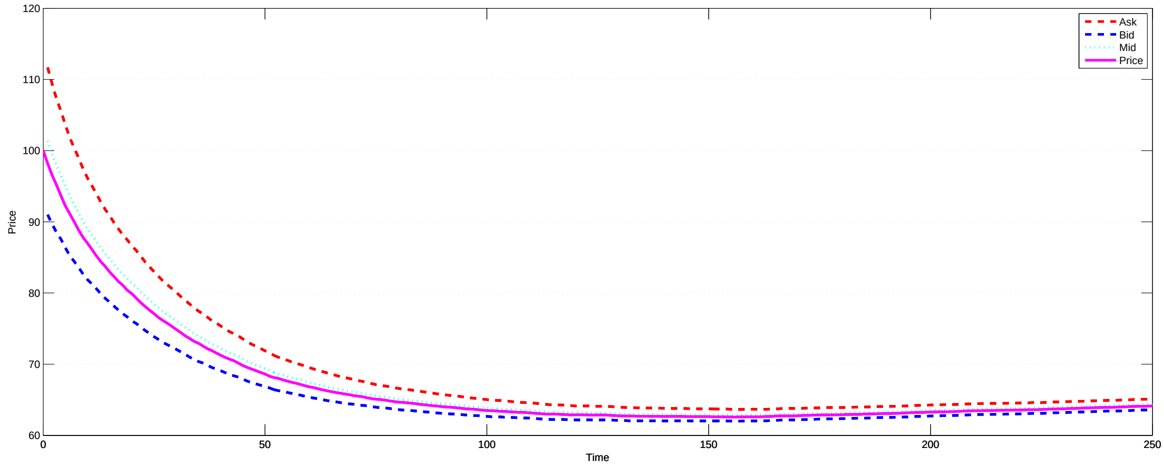

Figure 9: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Informed Traders Setting, Scenario (i) and *Overall* (both good- and bad- news) case.

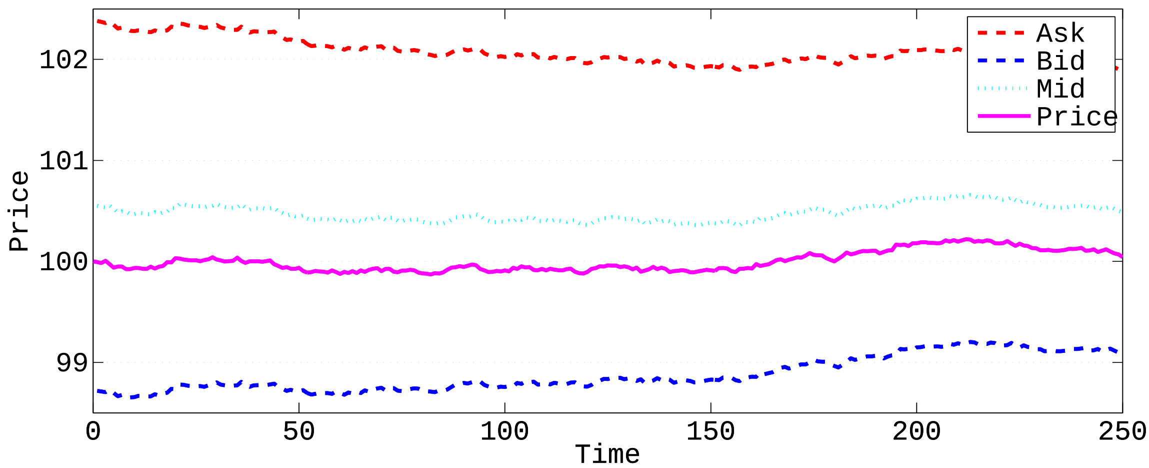

Figure 10: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Informed Traders Setting, Scenario (ii) and *Overall* (both good- and bad- news) case.

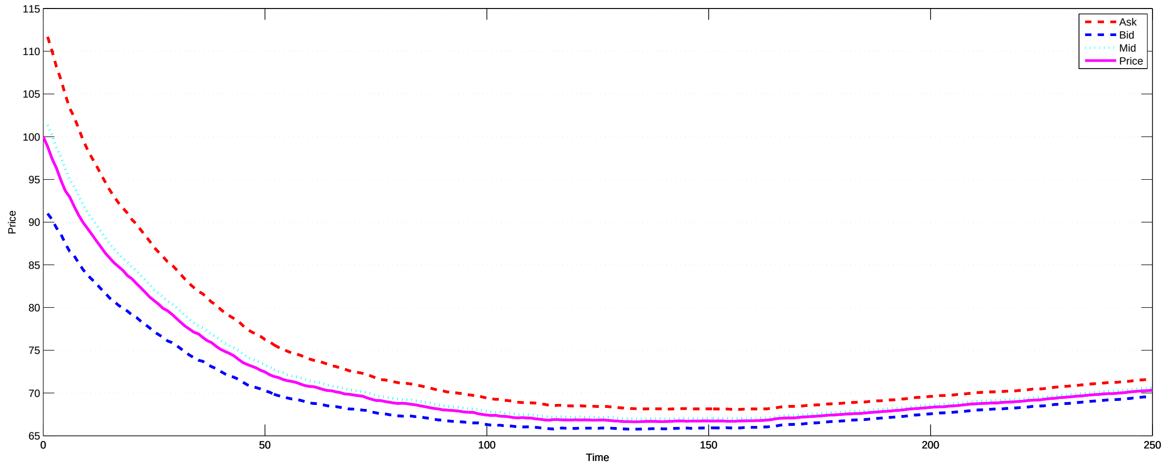

Figure 11: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Informed Traders Setting, Scenario (i) under Good-news case.

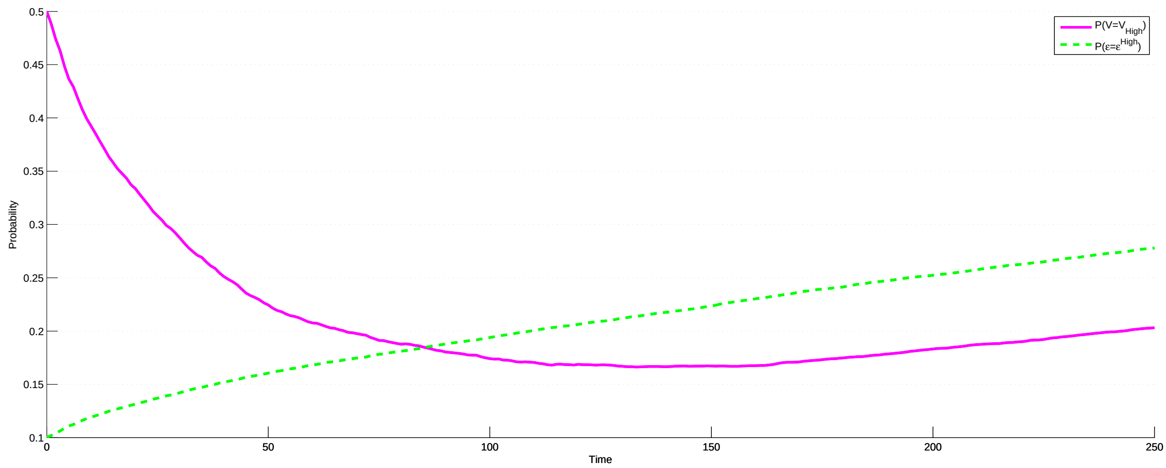

Figure 12: Posterior Probabilities for $V = V_H$ and $\varepsilon^U = \varepsilon^{I,H}$. Constantly Constrained Informed Traders Setting, Scenario (i) under Good-news case.

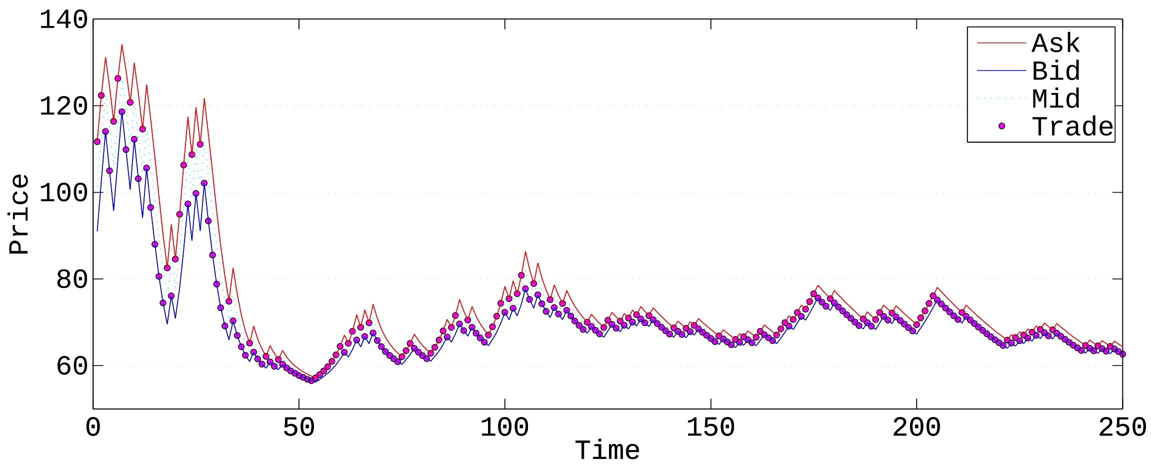

Figure 13: Single-Simulation Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Informed Traders Setting, Scenario (i) under Good-news case.

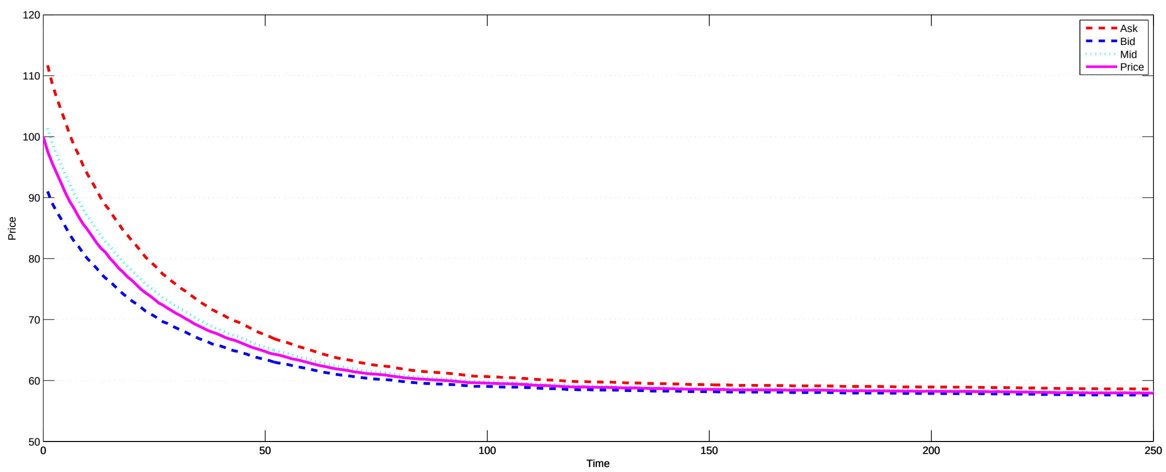

Figure 14: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Informed Traders Setting, Scenario (i) under Bad-news case.

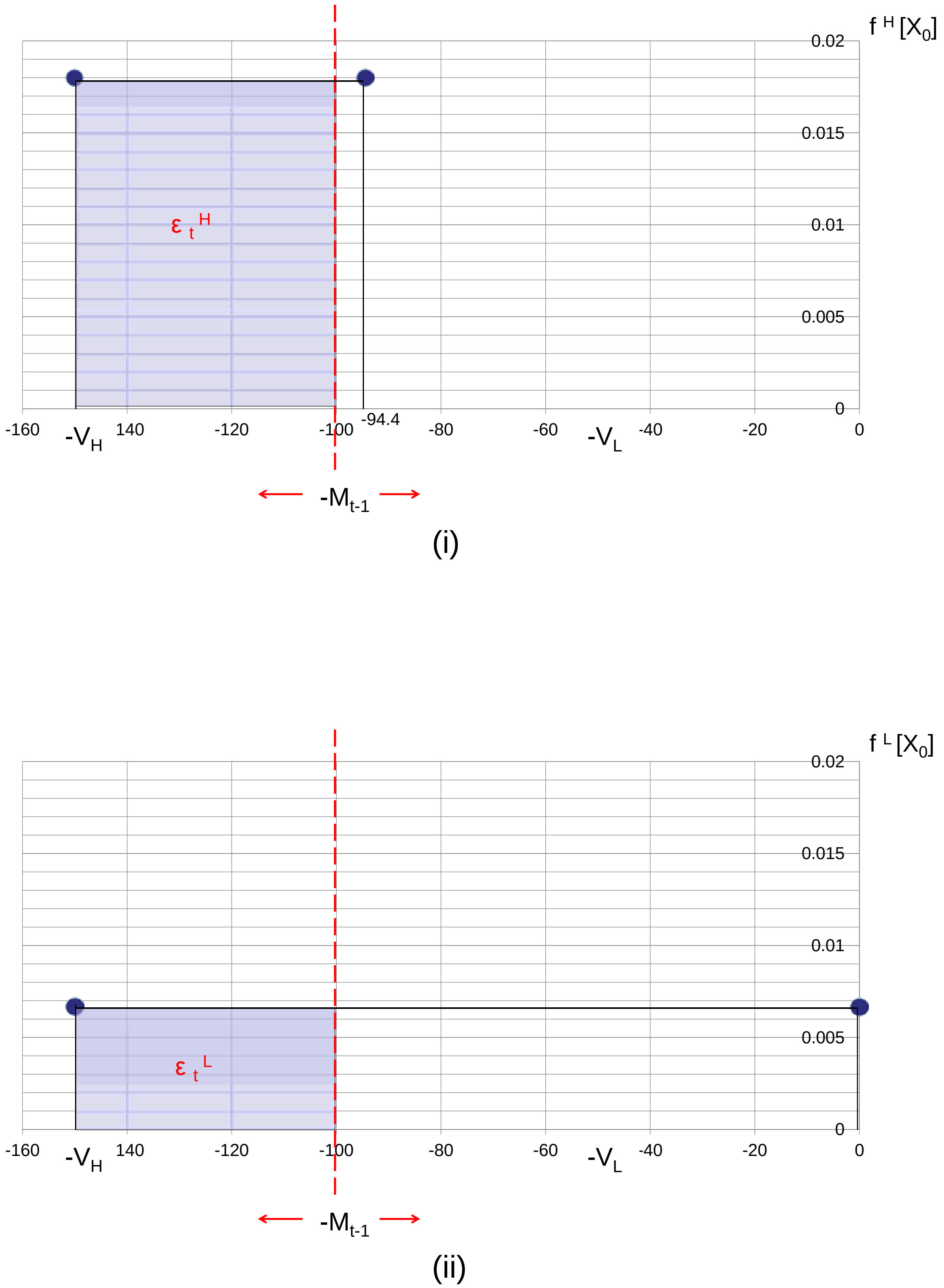

Figure 15: Initial Probability Distributions of the Uninformed/Informed Traders' Margin Account Values when the fraction of constrained traders is (i) High and (ii) Low.

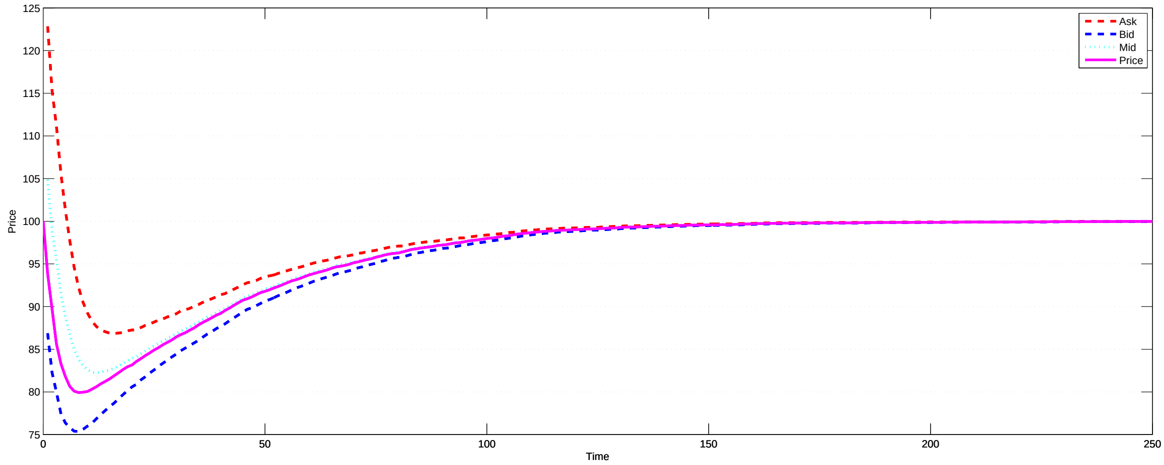

Figure 16: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Uninformed Traders Setting, *Overall* (both good- and bad- news) case.

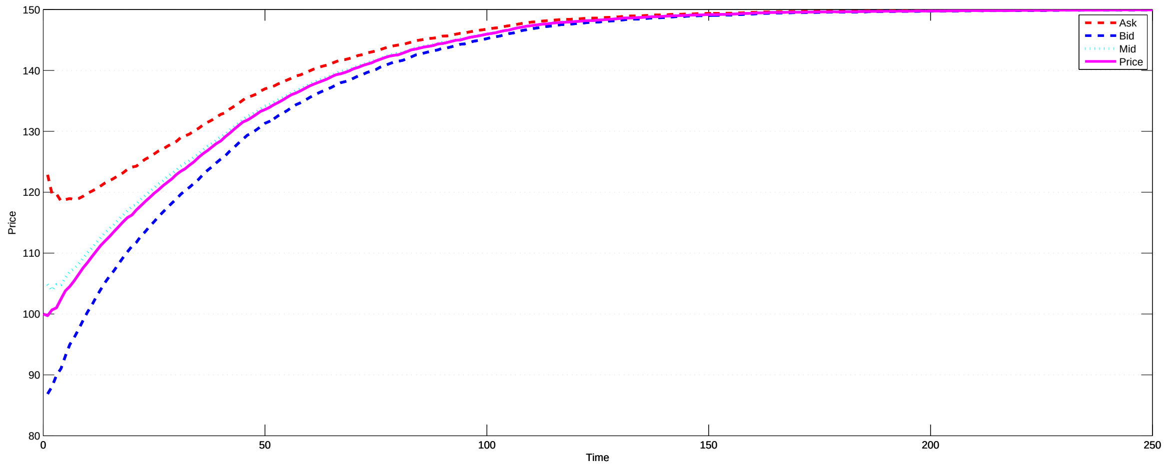

Figure 17: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Uninformed Traders Setting under Good-News case.

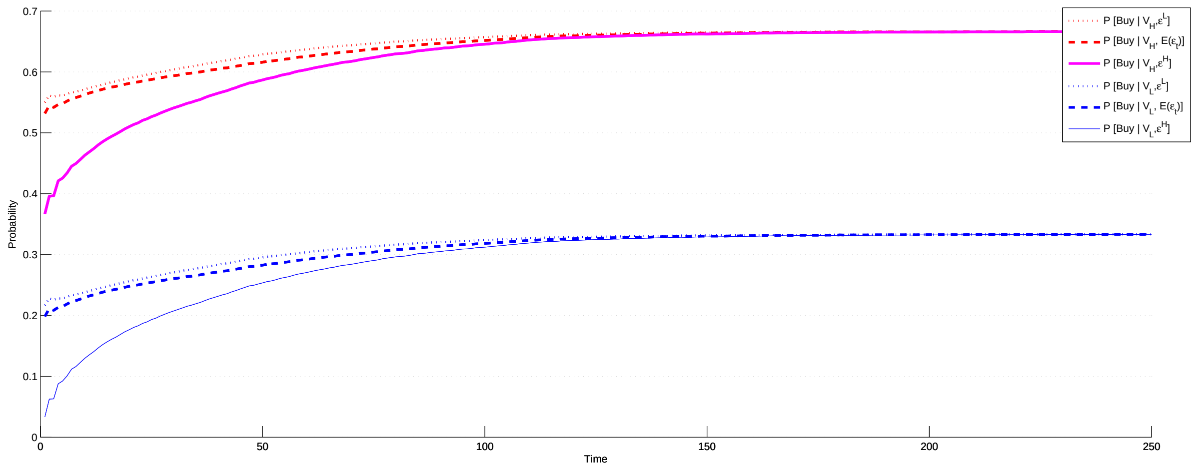

Figure 18: Buy probabilities under V_H and V_L, each as expected by the Market Maker, when $\varepsilon_t^U = \varepsilon_t^{U,H}$ and when $\varepsilon_t^U = \varepsilon_t^{U,L}$. Dynamically Constrained Uninformed Traders Setting under Good-News case

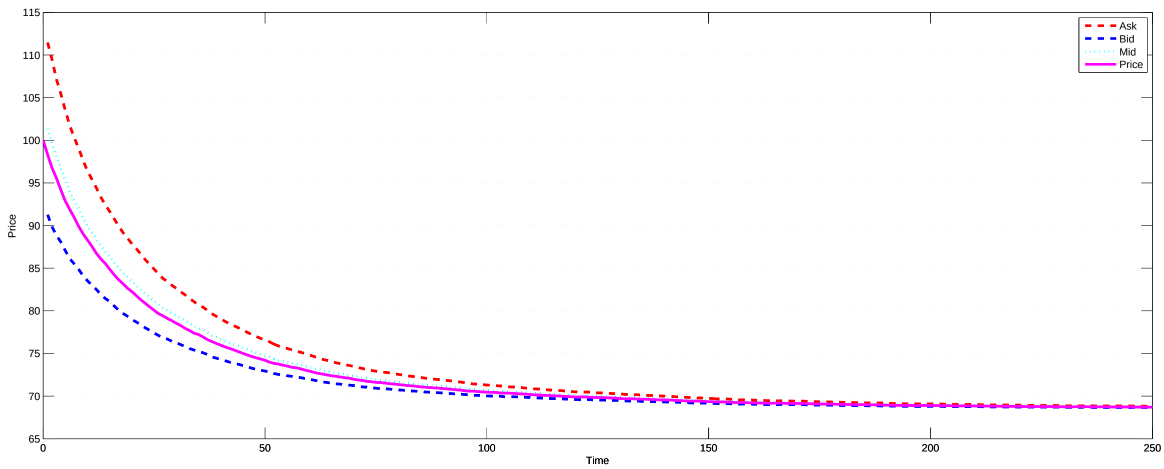

Figure 19: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Informed Traders Setting, *Overall* (both good- and bad- news) case.

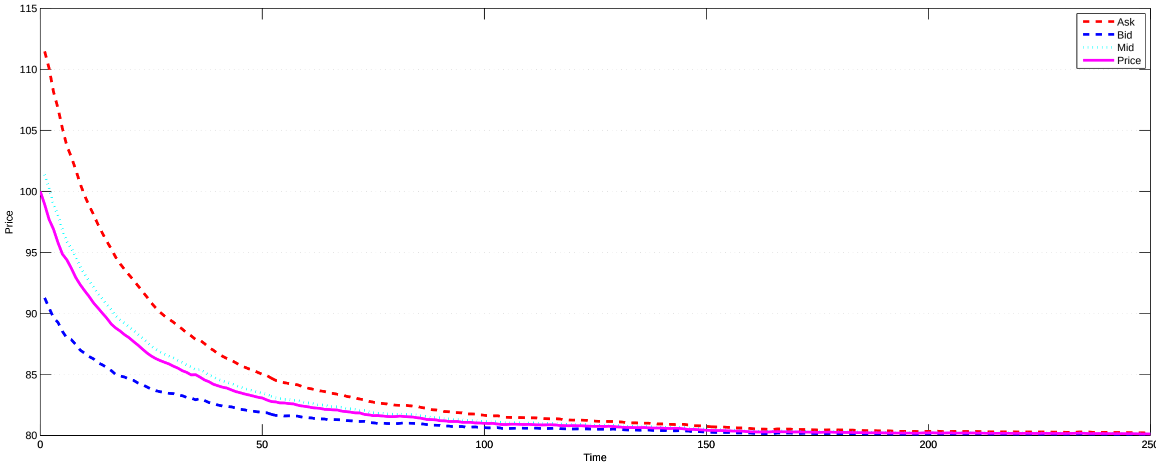

Figure 20: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Informed Traders Setting under Good-News case.

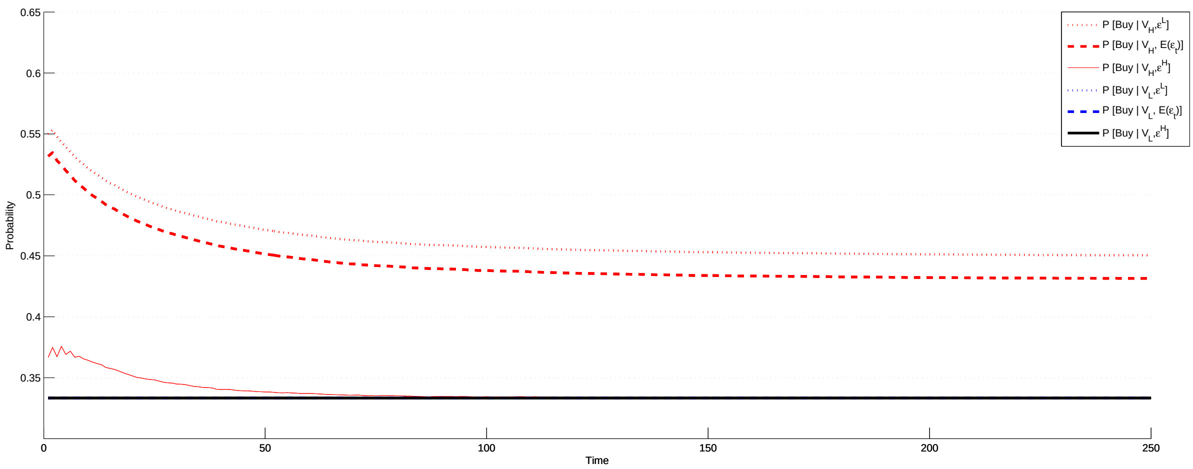

Figure 21: Buy probabilities under V_H and V_L, each as expected by the Market Maker, when $\varepsilon_t^I = \varepsilon_t^{I,H}$ and when $\varepsilon_t^I = \varepsilon_t^{I,L}$. Dynamically Constrained Informed Traders Setting under Bad-News case.

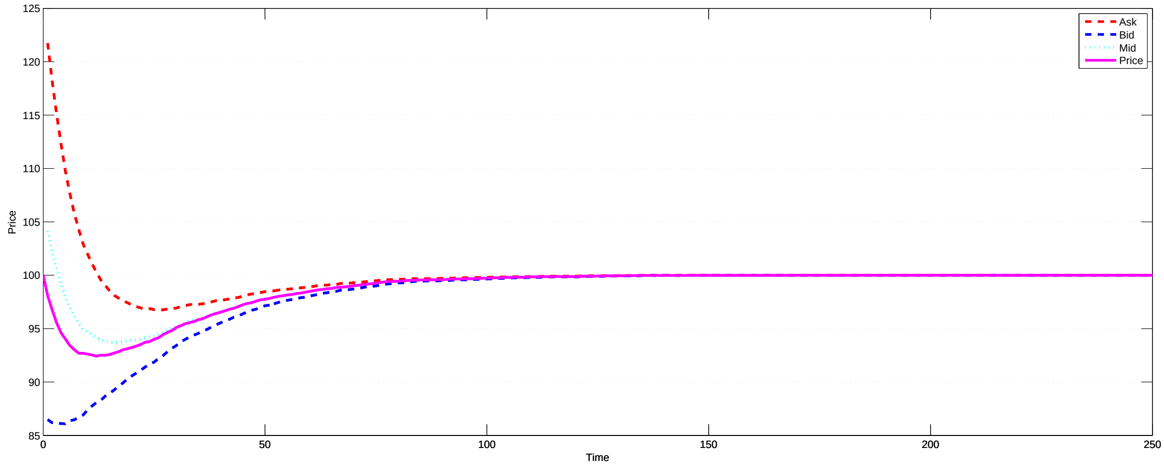

Figure 22: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Uninformed Traders Setting, Moderate level of Forced Trades and *Overall* (both good- and bad- news) case.

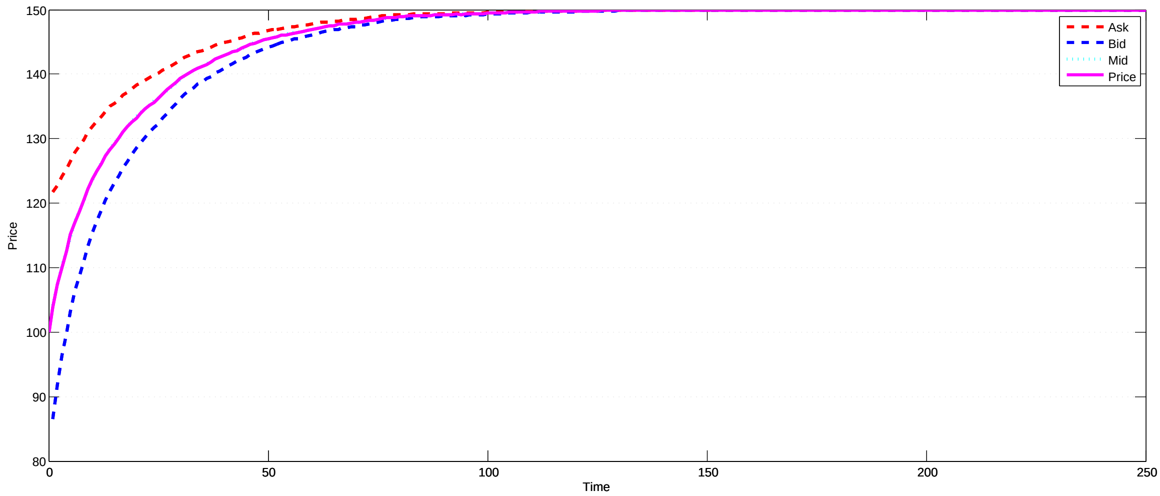

Figure 23: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Uninformed Traders Setting, Moderate level of Forced Trades and Good-news case.

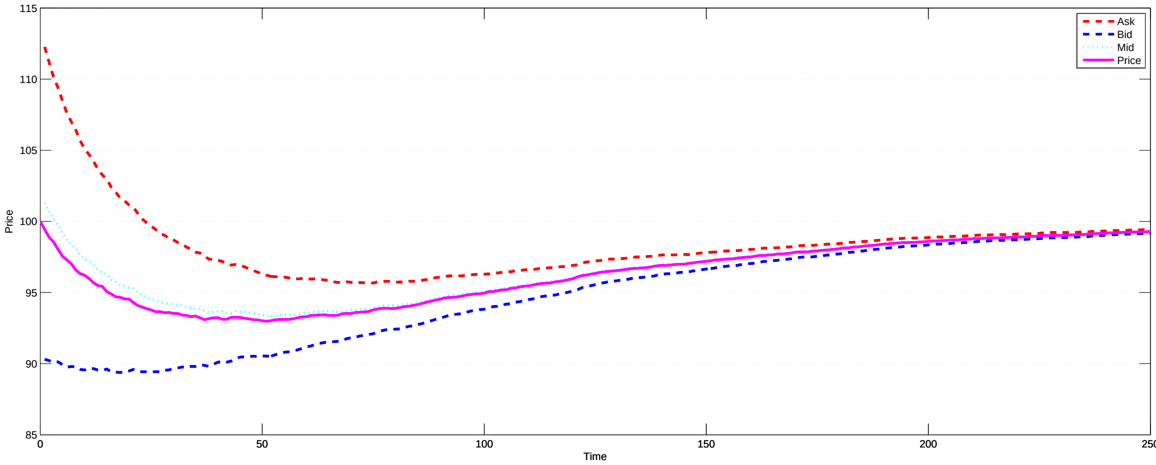

Figure 24: Average Bid, Ask, Trade Prices and Mid-Quotes. Constantly Constrained Informed Traders Setting, Moderate level of Forced Trades and *Overall* (both good- and bad- news) case.

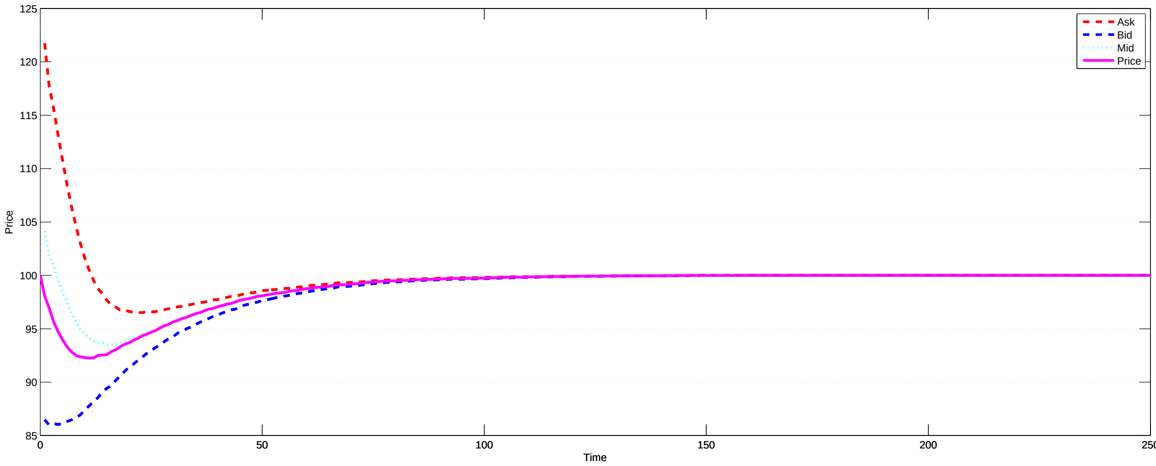

Figure 25: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Uninformed Traders Setting, Moderate level of Forced Trades and *Overall* (both good- and bad- news) case.

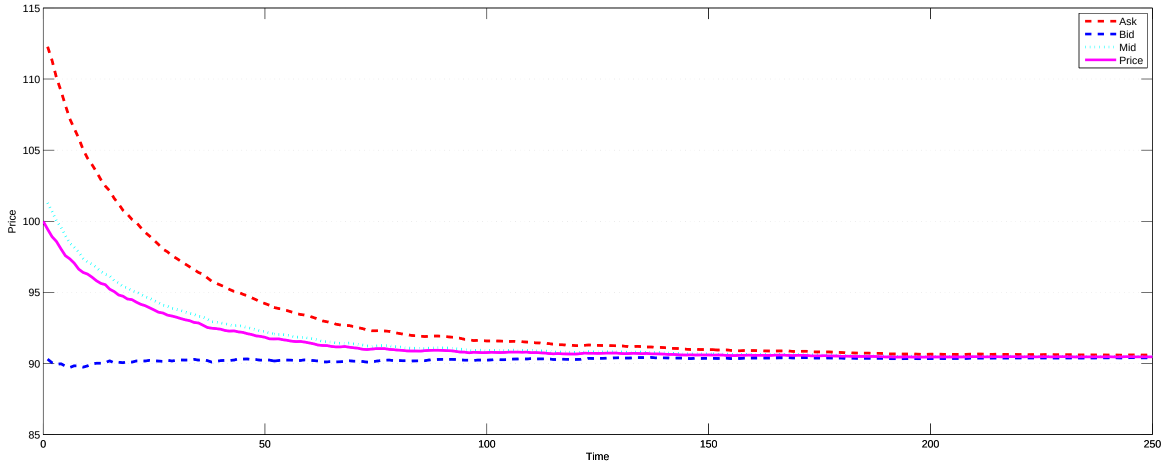

Figure 26: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Informed Traders Setting, Moderate level of Forced Trades and *Overall* (both good- and bad- news) case.

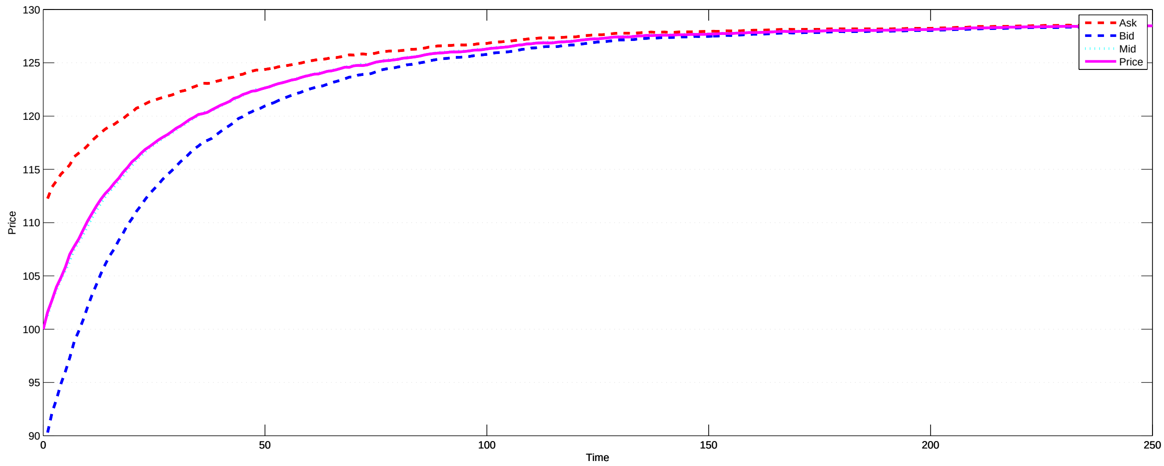

Figure 27: Average Bid, Ask, Trade Prices and Mid-Quotes. Dynamically Constrained Informed Traders Setting, Moderate level of Forced Trades under Good-News case.

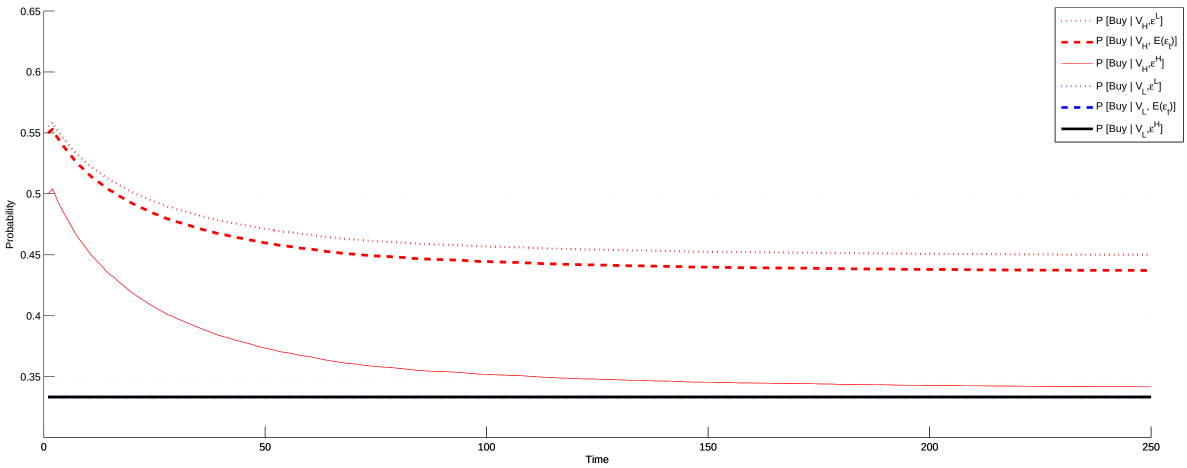

Figure 28: Buy probabilities under V_H and V_L, each as expected by the Market Maker, when $\varepsilon_t^I = \varepsilon_t^{I,H}$ and when $\varepsilon_t^I = \varepsilon_t^{I,L}$. Dynamically Constrained Informed Traders Setting and Moderate level of Forced Trades under Bad-News case.

5.5 Policy Implications

Margin trading, a mechanism for making use of some limited but readily available leverage, has often been blamed for playing a major role in generating severe asset price declines due to *forced trades*. Remember that, even though most trading is conducted by fund managers or banks' proprietary trading desks, the capital used for trading mainly belongs to their investors. Adverse price changes can also trigger withdrawal of these investors' capital, thus investors' redemptions constitute another source of *forced trades*. Ultimately, the amount of *forced trades* in a given market can be misestimated due to opacity and financial turmoil. When the *forced trades* cannot be well differentiated from the trades that are actually associated with the change in the asset's fundamentals, we will observe (even non-fundamental) price deviations that can in turn via a positive feedback mechanism cause further price deviations. In order to mitigate the destabilizing effects of *forced trades*, we propose to introduce higher transparency regarding the fraction of *forced trades* at the individual stock level. That may prevent non-fundamental asset price deviations and adverse feedback mechanisms. Note that, transparency will play an even more important role when the *constrained* traders are mainly informed traders, since in this case we have shown that these non-fundamental price deviations will be persistent. Below, we discuss transparency measures for margin trading and investors' redemptions respectively.

Regarding margin trading, FINRA[103] makes the following information publicly available via its monthly regulatory reports: Pursuant to FINRA Rule 4521, FINRA member brokerage firms carrying margin accounts for customers are required to submit the total of all debit balances in securities margin accounts and the total of all free credit balances in all cash accounts and all securities margin accounts. Then, the data is compiled into three aggregate numbers and made publicly available. We can infer from this data the tightness level of the margin constraints, but only for the overall market. This information is not granular enough to infer the exact proportion of the *forced trades* at the individual stock level. Even if this data were disclosed by FINRA at the margin accounts level, it would still not be possible to match them directly to the stocks, because often margin accounts have portfolio of stocks associated with them. Second, the frequency of reporting is on a monthly basis, and to be able to provide enough market transparency for market

[103]The Financial Industry Regulatory Authority (FINRA), formed by a consolidation of NYSE Regulation and NASD, is the largest independent regulator for all securities firms doing business in the United States. FINRA oversees about 4,000 brokerage firms, about 160,000 branch offices and approximately 650,000 registered securities representatives.

making activities this should become on a daily basis. Third, the tightness level of the margin constraints is indeed only a proxy for the *forced trades* in the market. As an improved solution, FINRA could collect from its member brokerage firms the information on whether the trade orders they submit to exchanges on behalf of their customers are due to margin calls or not. After aggregating this trade level information, FINRA could disclose for each stock the fraction of forced buys and forced sells separately, at least to market makers within quote-driven markets. Similarly to the biweekly reports of the short interest positions at the individual stock level by NASDAQ, FINRA could announce the margin call induced *forced trades*, ideally daily. Note that, brokerage firms might still utilize alternative trading systems (ATS), such as dark pools, crossing networks and ECNs, for *forced trades*. However, recently the Securities Exchange Commission (SEC) approved a rule change to require ATS to report to FINRA weekly volume information and number of securities transactions within the ATS by security, and the data is already made publicly available. These measures signal that in the future these alternative trading platforms will become more regulated and it will thus hopefully be possible to extend our recommended solution regarding *forced trades* to these platforms as well.

Concerning investors' redemptions, the information available to the public is very limited. Investors' redemptions can be tracked for funds by the changes in their assets under management (AUM). However, there exist various problems associated with this approach, especially for hedge funds. First, all hedge funds data is collected from hedge funds on a voluntary basis, and there is no single comprehensive and reliable source; there exist several hedge fund databases with little overlaps, such as TASS, CISDM, EUREKA and HFR. Second, since hedge funds have the rights to enforce "gates", actual investors' redemptions occur several months after the funds receive the redemption requests from their investors. Third, we are interested in the *forced trades* corresponding to investors' redemption requests. However, investors' redemptions do not necessarily lead to *forced trades* for stock positions. For instance, a fund might choose to close its other positions (in bonds, derivatives etc.) to meet its investors' redemptions. Thus, checking the change in AUM would not necessarily help extracting information on *forced trades* for stock positions. The mandatory 13F form, which is a reporting form filed quarterly by institutional investment managers pursuant to Section 13(f) of the Securities Exchange Act of 1934, constitutes publicly available information regarding the securities holdings of institutional investors. Stock trades by the institutional investors, such as hedge funds and mutual funds, can be extracted from the change in holdings between two consecutive quarters' reports. However, there are also some shortcomings with the 13F reports, because small institutional investors with less than \$100 million in U.S. equity do not need to fill in the 13F form. Furthermore, the institutional investors do not need to file any of their small positions (less than \$200,000 and 10,000 shares). And, short equity positions are fully missing in these reports. More importantly, even if the 13F reports perfectly reflected the most recent institutional holdings of stocks, the corresponding trades (changes in the holdings) would not necessarily all stem from *forced trades*. However, Ben-David, Franzoni, and Moussawi (2012) show that hedge funds have significantly reduced their equity holdings during the recent financial crisis, where the margin calls/risks (42%) and the customer redemptions (50%) were the primary drivers of these selloffs. Therefore, during financial distress periods the change in holdings becomes an important indicator of the *forced trades* in the financial markets and 13F filings should be taken into account by the market makers. We believe increased transparency from hedge funds, ideally with respect to their trading activities, should be demanded by regulators and investors. An important limitation that can be improved for the 13F reports is that, these (currently electronic) 13F forms could be gathered more frequently; i.e. frequently enough to support daily market making activities. Indeed, reporting frequency plays an even bigger role with respect to market transparency in highly volatile, turbulent markets.

6 Conclusion

This study brings together two strands of literature, namely the one on dynamic information-based market microstructure and the one on financial positive feedback mechanisms, via forced trades. The first objective of this study is to provide a generalized, information-based framework for examining various types of asset price deviations stemming from traders funding constraints.This framework allows further investigation of their dynamic interactions with the market liquidity and the price volatility. We do so within an information setting of double uncertainty; one uncertainty dimension regarding the liquidation value of the asset and the other one regarding the level of forced trades. We develop a model derived from the Glosten and Milgrom (1985) model by adding forced trades and further allowing the fraction of these traders in the markets to be under-(over-) estimated. In this context, we analyze the inference problem of the market makers by performing various Monte Carlo simulations of trade sequences using the analytically derived competitive bid and ask prices. We analyze the evolutions of the Bid, Ask, Trade Prices, Mid-Quotes and the updated posterior probabilities for the asset value and for the proportion of *constrained* traders. Meanwhile, we are particulary interested in the convergence and deviations of the trading prices. We find that, when the level of forced trades are estimated correctly, the mechanisms behind them, be it margin trading or investors' redemption, do not create any biases in the trade prices. The observed under-shoots are related to the under-estimation of the fraction of forced trades, not simply to the probabilistic structure in the proportion of forced trades. The price diversion is due to the fact that, while in reality the *constrained* traders are often selling in the first periods, the market confuses these trades on average with informed selling, which then drives the prices downwards. Over time, either the correct fraction of *constrained* traders is learned and the mispricing is corrected quickly as in the case of uninformed trades, or that learning is very slow (inhibited) and the mispricing lasts longer (persists) as in the case of informed trades.

Second, using the same information setting and further modeling the mechanism driving forced trades dynamically[104], our study explains *persistent* price deviations that are observed in the financial markets following high-leverage periods. In our model, in the context of margin trading, this occurs due to an informational positive feedback mechanism that amplifies the deviations when the fraction of forced trades undertaken by the informed traders is misestimated. This specific downward spiral can be explained as follows: When the prices move adversely in the markets, which could be just uncertainty driven and random, they can trigger (further) margin calls. When, in addition, the fraction of margin called traders is erroneously estimated, we show that a feedback loop can form: The initial adverse price movement triggers (further) margin calls, and this results in *more severe misestimation*, which then induces further adverse price changes. Eventually, that triggers further margin calls and so on.

The framework developed in our study could be extended along several dimensions. First of all, it could be used to rationalize numerous types of non-fundamental asset price deviations stemming from traders' funding constraints. For example, short-squeezes can be studied within our framework. These are situations in which heavily shorted stocks' prices move upwards causing many short sellers to close out their positions typically due to margin calls or due to previously placed stop-loss orders, and adding to the upward pressure on these stocks' prices. As an example, in October 2008 a short-squeeze temporarily drove the shares of Volkswagen AG (VOW) to approximately five times its current price

[104]for example, allowing margin calls to be triggered over time by adverse price changes.

in less than two days. Second, our framework can also be used to explain asset price bubbles (as overshoots). Furthermore, one can investigate the impact of short-sell bans and circuit breakers (e.g. up-tick rule) in the context of this paper. Finally, in the spirit of Brunnermeier and Pedersen (2009), we could analyze within our framework how the market liquidity (bid-ask spread) or the price volatility (extracted from the simulated price paths) dynamically interacts with these price deviations and with the involved informational positive feedback mechanism.

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7 Appendix

7.1 A simple margin trading example

Suppose, at the beginning of $t = 2$, the picked trader j, who has only some external capital but no asset position ($N_2^j = 0$), is willing to buy on margin, 1 unit of the XYZ stock, which currently has an ask price of $A_2 = \$120$ ($B_2 = \$100$ and $M_2 = \$110$). Initial margin requirements[105] obliges this customer to bring at least[106] \$60 of capital to purchase the stock. Let us assume that, she brings all her \$80 external capital to her margin account by the broker firm, receives a margin loan of \$40 from the broker firm and buys the stock. The broker firm keeps the stock as a collateral against this loan. At the beginning of $t = 3$, the value of her margin account $X_3^j = -\$40$, she owns $N_3^j = 1$ asset that worths $1 * M_2 = \$110$ and this corresponds to a trader's capital[107] $Y_3^j = -\$40 + \$110 = \$70$. Let us check whether the trader, due to maintenance margin requirements ($m = 0.5$), receives any margin calls for $t = 3$ or not: Since $Y_3^j = \$70$ is greater than \$55 ($|1| * 0.5 * \$110$), by a cushion of \$15, she does not receive any margin calls. Almost surely, our trader is not picked again to trade, and suppose that the stock price experiences declines over time, resulting in $M_8 = \$80$. At the beginning of $t = 9$, the value of her margin account is still $X_9^j = -\$40$, she owns $N_9^j = 1$ asset that worths \$80, and this corresponds to a capital of $Y_9^j = -\$40 + \$80 = \$40$. Since $Y_9^j = \$40$ is still not less than \$40 ($1 * 0.5 * \$80$), she still does not receive any margin calls; but note that, she does not have any cushion left by means of trader's capital within her margin account. Now, suppose $M_9 = \$60$. Then, at the beginning of $t = 10$, the value of her margin account is still $X_{10}^j = -\$40$, she owns $N_{10}^j = 1$ asset worths \$60, and this corresponds to a capital of $Y_{10}^j = -\$40 + \$60 = \$20$. Since $Y_{10}^j = \$20$ is now less than \$30 ($1 * 0.5 * \$60$), she does not satisfy anymore the maintenance margin requirements by the broker firm and receives a margin call; unless she can provide further external capital, that requires her to liquidate her long position in the asset at $t = 10$ or within the next trade round(s). Suppose that, she finds the opportunity to sell her asset at $t = 11$, when $B_{11} = \$45$. Then, at the beginning of $t = 12$, the value of her margin account is $X_{12}^j = -\$40 + \$45 = \$5$, she owns $N_{12}^j = 0$ asset worths \$0, and this corresponds to a capital of $Y_{12}^j = \$5$. Consequently, at $t = 12$ there will be no more margin calls issued.

[105] Once again, the initial margin requirement is not implemented in our model, but we check the robustness of our results with respect to it. Maintenance margin requirement is the main focus of this study.

[106] According to Reg. T, by Federal Reserve Board, the minimum initial margin requirement is 50%.

[107] also called the trader's equity within her margin account at the broker firm