# Entrepreneurial Vision, Information, and Cash

Arnoud Boot* Vladimir Vladimirov†

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(new version coming soon)

**Abstract**

We analyze the merits of delaying profitable investments, where such delays allow for more internal funding and keeping a higher stake in the firm. We show a self-reinforcing mechanism where entrepreneurial firms with better investment opportunities also invest sooner. Despite hoarding less cash, such firms might, nevertheless, have high cash holdings when they mature. Furthermore, we show that private firms hoard less than public firms. We also highlight the contrasting implications of different types of financing frictions for hoarding. Cross-sectional predictions emerge with differences in vision or incentive problems, but not, as commonly assumed, with information asymmetry or private benefits.

*University of Amsterdam, Amsterdam Center for Law & Economics (ACLE) and Finance Group. E-mail: A.W.A.Boot@uva.nl.

†University of Amsterdam, Finance Group. E-mail: Vladimirov@uva.nl. We thank Heitor Almeida, Mike Burkart, Ran Duchin, Andrea Gamba, Simon Gervais, Ron Giammarino, Radhakrishnan Gopalan, Alexander Gorbenko, Sebastian Gryglewicz, Roman Inderst, Filippo Ippolito, Ilan Kremer, Evgeny Lyandres, Andrey Malenko, Vojislav Maksimovic, Todd Milbourn, Erwarn Morellec, Boris Nikolov, Adriano Rampini, Rob Siemans, Xunhua Su, and conference and seminar particpants at the Edinburgh Corporate Finance Conference 2014, Frontiers of Finance 2014, Jerusalem Finance Conference 2014, SIFR Conference on The Financial Economics of Innovation and Entrepreneurship, Entrepreneurial Finance and Innovation Conference, and the University of Amsterdam for helpful comments and suggestions.

# 1 Introduction

The importance of cash hoarding as a way to reduce the exposure to potentiality prohibitively costly outside financing has been recognized to have first-order importance by both the empirical and theoretical literature (Opler et al., 1999; Bolton et al., 2011). However, the overwhelming abundance of cash in large prominent firms, such as Apple, sometimes tends to shift attention away from the heterogeneity of motives for cash hoarding across different types of firms. In this paper, we analyze a slice of the cash phenomenon by addressing several open questions regarding cash hoarding in growth firms.

One open question is whether growth firms should delay investment in order to hoard cash and reduce dependence on outside funding. And if they do so, is it true that firms with better investment opportunities should hoard more so that their owners keep a higher stake in the firm or should they rather avoid delay and finance growth externally. As a consequence, should we expect firms with better prospects to actually grow more slowly or should we, instead, observe a self-reinforcing effect where, by hoarding less and delaying less, they quickly outgrow their lesser-potential peers. Another question is how the decision to be public or private interacts with hoarding incentives and whether this could explain why private firms hoard less cash than comparable public firms (Gao et al., 2013). At a more basic level, we have only a limited understanding of how the nature of different frictions between outside financiers and insiders affect cash hoarding, and to what extent such frictions in growth firms can be brought into connection with the increase in cash holdings over the last decade (Bates et al., 2009).

We address these questions in a simple dynamic model of cash hoarding in which a firm wants to make something "big and bold." In our baseline setting, entrepreneurs and financiers may have a different vision about the future. Such differences may lead to disagreement about the optimal course of action and make external financing costly. Note that with differences in vision we do not refer to asymmetric information but to a different way of looking at the world. Hence, vision is not something that can be proved right or wrong, and financiers may not be willing to finance entrepreneurial visions with which they disagree. Hoarding cash seems, thus, a natural alternative for visionary–entrepreneurial–businesses to gain some leeway or elbow room. After deriving our main results, we compare our insights to those emerging from other financing frictions.

An important cost of cash hoarding is that it may lead to delays in investment. We show, first, that in an environment with potentially greater gains to entrepreneurship–i.e., with more profitable investment opportunities on average–shorter delays are optimal. Intuitively, firms with better opportunities are willing to depend more on external funding,

because the cost of delay is increasing in the attractiveness of the opportunities. Thus, firms with a higher growth potential also choose to expand more quickly. Furthermore, being more successful, these firms might end up being cash-rich as they mature despite pursuing a low-cash strategy in their growth phase.

Stark implications come up when relating cash hoarding to a firm's choice between public and private ownership. Private ownership may allow for a greater alignment in vision with financiers. However, it is not without costs as the liquidity benefits of public ownership might be substantial. Trading off these effects, the paper shows that when the choice between public and private ownership is endogenous, private firms hoard less cash than public firms. The reason is that firms choose private ownership when alignment is of paramount importance for obtaining cheaper financing. That is, firms that choose to be private are those that benefit most from alignment, because they prefer raising more external financing and hoarding less cash. Our analysis implies that this is more likely to be the case for visionary firms with better investment opportunities.

The second contribution of the paper is to contrast how the nature of different frictions affects cash hoarding. On the one hand, our intuition and results are robust: We can derive the same qualitative results in a setting in which financing is costly due to incentive problems stemming from a lower stake in the firm ala Holmstrom and Tirole (1997). On the other hand, other typical frictions, such as information asymmetry and private benefits, are surprisingly powerless in yielding cross-sectional predictions. The reason in the case of private benefits is simple: Delaying investment is just costly and does not resolve the problem that the private benefits accrue only to the manager. Thus, a manager that values an investment opportunity higher because of private benefits would not delay investment to hoard cash.

Similarly, in a setting with asymmetric information, firms would hoard the same amount of cash regardless of the profitability of their investment opportunities. Separation with differences in hoarding across types is not possible, and the only pooling equilibrium that survives refining out-of-equilibrium beliefs with D1 is with zero cash hoarding. The reason is that firms with better projects are most hurt by delays caused by hoarding. Thus, they seek to minimize hoarding, but this strategy can be easily mimicked by firms with less lucrative investment opportunities. This causes a rat race to zero hoarding by all.

In light of this striking difference between information asymmetry and differences in vision, it is reassuring that our disagreement results are robust when differences in vision and information asymmetry are jointly present. Then, the positive hoarding levels induced by disagreement effectively give some slack to inducing separation of types. Firms with good investment opportunities can now separate from lower quality types by choosing to

hoard less cash than such types.

We extend our model along several dimensions. We allow firms to hoard cash before the investment opportunity arrives. Now all firms hoard cash *before* arrival, but considering both the pre- and post-arrival hoarding, the key insight that firms with better opportunities hoard less cash continues to hold. Similarly, considering the asymmetric information setting, we continue to find that firms cannot separate: firms choose to hoard cash until arrival of the investment opportunity, and then immediately invest upon arrival. In a separate extension, we show that our insights are robust also when differences in vision and the profitability of the investment opportunity vary over time.

Furthermore, we analyze how the type of financing and payouts interact with cash hoarding. We obtain that firms hoard less cash to co-finance new investments when they have access to debt financing than when they issue more-information-sensitive securities, such as equity. Intuitively, the cost of financing caused by disagreement (and, thus, the necessity to hoard cash) matters less when financing is in a less "disagreement-sensitive" security. Furthermore, we show that dividend policy and share repurchases play no role when fresh capital is needed for new investments. Paying out cash only increases the dependence on external financing, and is, thus, counterproductive.

Our results give rise to a rich set of empirical implications. We predict that from the subset of firms with (profitable) investment opportunities, firms with lower cash-to-assets ratios will be more profitable. Furthermore, the long-term performance of growth firms that make new investments with a higher proportion of outside financing should be better. In terms of dynamics, we expect that firms participating, for example, in merger waves in the beginning of the cycle should on average perform better than firms that join later with a higher proportion of internal funds.[1] More broadly, an important insight of our analysis is that there is a self-reinforcing mechanism, in which firms with better opportunities also invest more quickly, hoarding in the process less cash. Then, as these firms mature, they will have a stronger cash flow stream, making it likely that they end up with large cash holdings despite their original low-cash strategy. Interestingly, our, at first sight, counter-intuitive insight that private firms hoard less cash than public firms (entrepreneurial firms with better opportunities remain private and hoard less cash) is supported by recent empirical evidence (Gao et al., 2013; Farre-Mensa, 2014).

When relating cash hoarding to profitability, we emphasize how important it is to control for measures of disagreement. We predict that firms that invest in unfamiliar busi-

[1]The empirical evidence is supportive of our results. Hoarding cash goes hand in hand with high q–i.e., growth opportunities–(Bates et al., 2009) and the long-term performance of firms making large investments (such as takeovers) is lower for firms with higher cash-to-assets ratios (Harford, 1999).

nesses with a higher dispersion of analyst forecasts should hoard more cash. This may also shed some light on how growth firms contribute to the steady increase in corporate cash holdings in recent years (Bates et al., 2009). In particular, the changing nature of growth firms towards becoming more disagreement-prone (by being more R&D intensive and having more intangible assets, which are difficult to value) may have led to more cash hoarding (Falato et al., 2013).[2] Notably, when we contrast this with the more standard setting in which the only friction is information asymmetry, these results cannot be obtained; cash hoarding is not effective in overcoming information asymmetry. Dividend policy and share repurchases are even less effective when the firm has a net funding need. Thus, growth firms should not return cash to shareholders if it is needed for new investments. Finally, we also predict that firms with higher debt capacity hoard less cash.

Our paper mainly relates to the fast growing literature on cash. Firms hoard cash because they may be unable to frictionlessly raise financing for new investments. Agency conflicts are one important such friction. According to the free cash flow hypothesis of Jensen (1986), entrenched managers hoard cash and then invest it rather than paying it out to shareholders even when there are poor investment opportunities. More recently, DeMarzo et al. (2012) take a different perspective and show that accumulating cash reserves can be part of the optimal contract in a dynamic agency framework. Intuitively, hoarding cash as a way of deferring compensation can provide incentives to exert effort (see also Biais et al., 2007). The evidence for hoarding cash for agency reasons is rather mixed, however.[3]

Firms may also hoard cash as a precautionary measure. Bolton et al. (2011) show that firms will keep a positive balance even if this necessitates costly external financing, since the marginal benefit of avoiding to seize operations is very high. In such cases, firms with stronger cash flow streams need to hoard less cash (Acharya et al., 2012). Related, Almeida et al. (2004) show that financially constrained firms save more cash out of cash flows.[4]

[2]Falato et al. (2013) argue that the inability to offer intangible assets as collateral has reduced the access to debt financing. We argue that the changing nature of firms may have made firms more disagreement-prone, which has made firms less willing to depend on outside financing *in general*. This helps explain why firms have shifted towards hoarding more cash rather than towards issuing more equity.

[3]Dittmar and Mahrt-Smith (2007) and Pinkowitz et al. (2006) show that cash is worth less when agency problems between inside and outside shareholders are greater, and Nikolov and Whited (2013) identify low managerial ownership as a key factor. In contrast, Opler et al. (1999) and Bates et al. (2009) find no evidence relating agency problems to cash holdings.

[4]In Hugonnier et al. (2012), a firm would hoard cash, because it may not be able to find a financier. Furthermore, in the recent literature on risk management, Acharya et al. (2012) show that firms with high aggregate risk exposure prefer cash to credit lines, and Rampini and Viswanathan (2010) argue that the opportunity cost of risk management is higher for constrained firms. In contrast to this literature, in which cash and credit lines help to avoid liquidity problems, our paper asks whether growth firms would actually delay a big investment to hoard cash and reduce their dependence on external financing (i.e., also

The existing evidence seems to strongly support the precautionary motive for hoarding cash (Opler et al., 1999; Bates et al., 2009). We also substantiate this motive when we allow for hoarding prior to the arrival of the investment opportunity.

An important insight from our model is that disagreement and the lumpy nature of investment opportunities may render investment impossible without cash participation by the owner-manager, leading to delays in investment. This approach closely relates our paper to Boyle and Guthrie (2003) and Bolton et al. (2013) who analyze how optimal investment timing changes with firms' cash holdings when external financing is costly. An important difference to these papers is that we endogenize and contrast the different types of frictions that make external financing costly, which helps us derive a number of novel predictions, including implications for cash hoarding in public versus private firms.

In terms of methodology, our paper builds on the standard real options framework of McDonald and Siegel (1986).[5] By allowing discretion in the timing of the investment, we are the first ones to combine the real option framework with the literature on disagreement and to discuss the resulting implications for cash hoarding.

Our paper is organized as follows. Section 2 presents the model. Section 3 contains the main analysis relating disagreement to cash hoarding, and endogenizing the choice between private ownership and going public. Section 4 contains the comparison between the disagreement/visionary model and a model based on asymmetric information. In section 5, we analyze the various extensions. Section 6 discusses the empirical implications, and Section 7 concludes.

# 2 Model

Our baseline model considers a firm run by an owner-manager (henceforth, manager) that has an existing asset in place producing stochastic cash flows. If these cash flows are not invested, they accumulate in the form of cash reserves according to

$$dw_t = \mu w_t dt + \sigma w_t dZ_t,\ w_0 > 0. \tag{1}$$

As is standard in such models, $\mu$ and $\sigma > 0$ are constant and $(Z_t)_{t\geq 0}$ is a standard Brownian motion (e.g., Morellec and Schürhoff, 2011). Our key assumption is that $\mu < r$, where $r$ is the constant discount rate used by all. This assumption, which is shared with most of

on credit lines, which are anyway not common for growth firms–Sufi, 2009).

[5]Related to our finance application are Grenadier and Malenko (2011) who analyze real options signaling games. Morellec and Schürhoff (2011) discuss financial contracting, and Bouvard (2012) discusses real options financing under asymmetric information.

the literature, implies that hoarding cash within the firm is costly to insiders. We assume that all parties are risk neutral and protected by limited liability.

The manager has a profitable investment opportunity, requiring a cash outlay of $K$, which she initially does not have at hand. However, the manager has discretion over the timing of this investment. Our model is, thus, an adaptation of the standard real options framework (e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994), but differs from this framework in one important aspect: the firm is cash-constrained and may not be able to invest in a positive NPV project even if the manager can raise capital from a financier in a competitive capital market. We make this more precise in what follows.

**Vision and Disagreement** At $t = 0$, the manager and the financier publicly observe a signal about the profitability of the project. If the signal is good and the firm invests, its discounted expected cash flows become $\theta X_G$. If the signal is bad and the firm invests, its expected discounted cash flows become $\theta X_B$. The common parameter $\theta$ can be seen as a publicly observable indicator of the attractiveness of the firm's (or industry's) growth prospects.[6] We assume that $K > \theta X_B$ for all $\theta$, so that a bad signal translates into a negative NPV project. Furthermore, $\theta X_G > K$ at least for some $\theta$, so that investing after a good signal in such cases increases value. Note that the firm produces cash flows at an expected rate $\mu < r$ before the investment. Thus, the opportunity cost associated with hoarding cash is paying out $w_t$ together with effectively closing operations.[7] This would be a realistic assumption for many entrepreneurial firms for which the investment opportunities are the main component of valuation. All features of our model are common knowledge, and the cash flows and the level of cash are costlessly verifiable.

The main feature of our vision- (or disagreement-) based model is that although both parties observe the same signal, they may interpret it differently. More specifically, whatever inference is made by the manager, the financier believes that the inference is correct only with probability $\rho \in (0, 1)$. The agreement parameter $\rho$ is common knowledge and might depend on the nature of the investment opportunity and/or the nature of the business.[8] In our baseline model, $\rho$ is constant over time, but we relax this assumption in

[6] In Section 4 where we compare the vision setup to the information asymmetry specification, we let $\theta$ be private information. For now, it is observable and known to all. In Section 5.6, we show how our results could also emerge from an alternative setting in which financing is costly due to incentive problems ala Holstrom and Tirole (1997).

[7] Note that paying out all cash $w_t$ deprives the firm from the ability to produce cash flows (cf. (1)).

[8] Disagreement in a corporate finance context is usually motivated with heterogenous priors in the sense of Kurz (1994a,b)–e.g., Boot et al., (2008). However, it can also arise due to overconfidence on the part of either management or financiers (Bernardo and Welch, 2001; Daniel et al., 1998) and excessive pessimism (Coval and Thakor, 1998) or optimism (Manove and Padilla, 1999).

Section 5.7.

In what follows, we only need to consider the case in which the manager observes a good signal (i.e. infers that the project is good). This is because if the manager observes a bad signal and the financier (fully) agrees that the project is bad, it is never undertaken and the cash at hand is paid out. If the financier (fully) disagrees and believes that the project is good, it may be profitable for the manager to undertake the investment opportunity because she would be able to finance it at very favorable terms. In that case, the manager will not use her available cash, but pay it out, and fully finance the project with external financing.[9] Hence, when the manager receives a bad signal, there will not be any hoarding, and the cash at hand is paid out regardless of whether the financier agrees or disagrees.

The objective of our model is to analyze the firm's cash hoarding and investment decisions when there are differences in vision. We also analyze how these decisions interact with the firm's endogenous choice between public and private ownership. In addition, we contrast the resulting insights with those from a model based on asymmetric information. To streamline the exposition, we will add more structure to the baseline model where needed.

**Outside Financing** At the time at which the manager raises capital to make the investment, she is facing a competitive capital market. We model this by allowing the manager to make a take-it-or-leave-it offer. Since the offer is open to all financiers, they compete away all rents and only require breaking even. The manager sells equity to raise $K - w$ from the investment outlay $K$.[10] The financier's equity stake $\alpha$ must satisfy

$$\begin{aligned} K - w &= \alpha\theta\left(\rho X_G + (1-\rho) X_B\right) \\ &= \alpha\theta W^F \end{aligned} \tag{2}$$

where $W^F := \rho X_G + (1-\rho) X_B$. $\theta W^F$ stands for the financier's assessment of the firm's value, capturing that the financier shares the manager's assessment with probability $\rho$, and disagrees with probability $(1-\rho)$. From (2), the equity share that needs to be promised to the financier is

$$\alpha = \frac{K - w}{\theta W^F}. \tag{3}$$

[9] The manager's expected profit from fully externally financing the investment with equity would then be $\left(1 - \frac{K}{\theta X_G}\right)\theta X_B > 0$.

[10] We discuss optimal financial contracting in Section 5.3.

Define $\theta W^M := \theta X_G$ as the manager's assessment of the firm's value. The manager's net expected payoff at the point in time in which she raises $K - w_t$ and co-invests $w_t$ is:

$$V(w_t, \theta) := \left(1 - \frac{K - w_t}{\theta W^F}\right) \theta W^M - w_t. \tag{4}$$

Inspecting $V(w_t, \theta)$, we obtain:

**Lemma 1** *The manager's expected payoff $V(w_t, \theta)$ from raising $K - w_t$ and investing $K$ is increasing in her co-investment $w_t$, in the agreement parameter $\rho$, and in the profitability parameter $\theta$.*

# 3 Differences in Vision

## 3.1 Cash Hoarding and Investment

We will now derive how the lack of alignment between management and financiers–inducing potential disagreement–affects investment decisions, and in turn impacts the amount of cash the firm hoards prior to undertaking the investment. We solve for the value of the real option to invest using standard dynamic programming methods (Dixit and Pindyck, 1994). The problem is that of finding the optimal stopping rule $w^*$ (i.e., the level of cash holdings at which the investment is made) that maximizes the value of the option to invest $U$, while trading-off the benefit of reducing the funding cost against the time value of money lost from investing later, where

$$U(w_t, w^*, \theta) = \max E\left[\frac{1}{1 + rdt}\left[U(w_t + dw_t, w^*, \theta)\right]\right]. \tag{5}$$

Applying Ito's lemma, we obtain

$$rU = \mu w \frac{\partial U}{\partial w} + \frac{1}{2}\sigma^2 w^2 \frac{\partial^2 U}{\partial w^2}.$$

This equation is solved subject to the following boundary conditions. First, the manager's expected payoff at the time of investment should be equal to her payoff from investment: $U(w_t, w^*, \theta)|_{w_t = w^*} = V(w_t, \theta)|_{w_t = w^*}$. Second, the manager chooses the investment trigger so as to maximize her value at the endogenous investment threshold: $\frac{\partial}{\partial w^*} U(w_t, w^*, \theta)|_{w_t = w^*} = 0$. Finally, the option to hoard cash becomes worthless as the value of cash tends to zero: $\lim_{w_t \to 0} U(w_t, w^*, \theta) = \max\left[0, \left(1 - \frac{K}{\theta W^F}\right) \theta W^M\right]$. Indeed, if

the existing business falters ($w_t \to 0$), then it almost surely does not recover (cf. (1)), and the manager can invest only if she raises all financing externally.

Suppose, first, that disagreement is sufficiently strong such that $K > \theta W^F$. Then, solving this optimization problem yields the following expression for the manager's expected payoff

$$U(w_t, w^*, \theta) = \left( \left( 1 - \frac{K - w^*}{\theta W^F} \right) \theta W^M - w^* \right) \left( \frac{w_t}{w^*} \right)^{\beta} \tag{6}$$

where $\beta$ is the positive root of $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$, and $\mu < r$ implies that $\beta > 1$. Intuitively, the first term in (6) is the manager's expected payoff from investing at $w^*$, while the second term could be interpreted as the probability of reaching the cash level $w^*$ and investing. Trading off the marginal cost (due to $\mu < r$) and benefit of delay, the value maximizing investment threshold $w_\theta^{VM}$ is given by

$$w_\theta^{VM} = \frac{\beta}{\beta - 1} \left( K - \theta W^F \right) \frac{W^M}{(W^M - W^F)}. \tag{7}$$

If, instead, both parties believe that also in the presence of disagreement the investment opportunity is NPV positive ($K < \theta W^F$), they both believe that there is additional surplus to be obtained from avoiding costly delay and financing the project immediately. Hence, there would be no cash hoarding.

**Proposition 1** *If disagreement is sufficiently strong ($K > \theta W^F$), it is optimal for the manager to hoard cash prior to making the investment. The optimal cash level is given by (7). This threshold is decreasing in $\theta$ and $\rho$–i.e., there is less cash hoarding when the investment opportunity is better and when there is more agreement.*

What this proposition points at is that the costs of delaying via hoarding are higher when the opportunities are better, implying that the manager hoards less cash. Furthermore, the cost of delay weighs more when there is less disagreement. Thus, the manager will hoard less cash when there is less misalignment in vision with the financier.

The simplicity of this result should not conceal the importance its implications. One such implication is that firms with better investment opportunities also choose to expand more quickly. This leads to a self-reinforcing mechanism leading to an accelerated divergence over time between firms with good and bad investment opportunities. Another implication is that mature firms with strong cash holdings due to their success actually pursue low-cash strategies in their earlier growth phase (see for details Section 6).

## 3.2 Cash Hoarding in Public vs. Private Firms

We now add some structure to analyze the differences in cash hoarding between public and private firms. For this purpose, we explore the following trade-off between public and private ownership. Financiers in private firms hold illiquid claims; this illiquidity makes financing more expensive for these firms. However, a private firm might more easily avoid disagreement. For example, private financiers are often closely involved with firms in which they invest. They also have access to sensitive information needed to better understand the business and align vision–information that the manager might be unwilling to disclose to the market and, thus, to its competitors (Bhattacharya and Ritter, 1983; Maksimovic and Pichler, 2001). A related benefit is that the financiers in private firms are typically not dispersed, which helps to alleviate free riding and makes coordination between different classes of financiers easier (Brunner and Krahnen, 2008).

More formally, we assume that a private firm faces a lower degree of disagreement than a public firm: $\rho^{priv} > \rho^{pub}$. The downside, however, is that private ownership imposes a cost $L$ on the financier, which can reflect, for example, that the financier's investment is less liquid or the fixed costs of being more involved in the firm. The financier's participation constraint when the manager raises $K - w$ as a private firm is

$$\alpha\theta W^F\left(\rho^{priv}\right) = K - w + L,$$

where $W^F(\rho)$ makes explicit the dependence of $W^F$ on the agreement parameter $\rho$. Hence

$$\alpha = \frac{K - w + L}{\theta W^F\left(\rho^{priv}\right)} \tag{8}$$

and following the same steps as in (7), we obtain that the optimal levels of cash hoarding in a private and a public firm are

$$w_\theta^{priv} = \frac{\beta}{\beta - 1}\frac{\left(K + L - \theta W^F\left(\rho^{priv}\right)\right)}{\left(W^M - W^F\left(\rho^{priv}\right)\right)}W^M \tag{9}$$

$$w_\theta^{pub} = \frac{\beta}{\beta - 1}\frac{\left(K - \theta W^F\left(\rho^{pub}\right)\right)}{\left(W^M - W^F\left(\rho^{pub}\right)\right)}W^M. \tag{10}$$

Just comparing $w_\theta^{priv}$ and $w_\theta^{pub}$ does not give a clear idea if a private firm hoards more or less than a public firm because the decision to go public is endogenous and depends on $\theta$. Thus, we need to compare the manager's expected payoff in both cases given her optimal cash hoarding policies. Doing so, we obtain:

**Proposition 2** *(i) Firms that choose to remain private hoard less cash than they would hoard if they had chosen public ownership. (ii) Firms with better opportunities (high $\theta$) choose to stay private for higher levels of illiquidity costs $L$.*

For understanding part (i) of the proposition, observe that the time-value-cost of delay is the same regardless of the firm's public or private status. Hence, the endogenous choice of this status is driven by the differential effects of cash hoarding on the costs of outside financing. In a private firm, the benefit of less disagreement ($\rho^{priv} > \rho^{pub}$) is higher when the need for external financing is higher. The liquidity cost $L$, on the other hand, is constant. Hence, it is firms with higher external financing needs (and less hoarding) that go private.[11]

Another implication of Proposition 2 is that entrepreneurial firms with better investment opportunities are more likely to remain private. Since these firms are prepared to raise more external financing to avoid delay (Proposition 1), these firms benefit most from the higher agreement and alignment of vision with financiers under private ownership.

# 4 Comparing Differences in Vision to Asymmetric Information

What we have shown so far is that in an entrepreneurial/visionary set-up the potential disagreement between the manager and financiers has implications for cash hoarding and the choice of public versus private ownership. In this section, we establish how these implications compare to the role that cash hoarding plays in the more standard asymmetric information setting where firms have private information about their investment opportunities.[12]

A key insight that we develop is that in contrast to the visionary formulation, cash hoarding by delaying investment is not an equilibrium strategy in a model of asymmetric information. Hoarding cannot be part of a separating equilibrium, and the only pooling equilibrium that survives D1 is with no hoarding. This clearly distinguishes the private

[11] We can also assume that the illiquidity cost has a variable component $l$, which is proportional to the financier's stake in the firm. Then, to break even, the financier requires that $\alpha = \frac{K - w^*_{priv} + L}{\theta W^F(\rho^{priv})(1-l)}$. Observe, however, that $l$ has the same effect on the cost of financing as that of a lower level of agreement, which is also proportional to the amount raised from external financiers. Thus, increasing either of these costs shifts the indifference threshold between public and private ownership in the same fashion, making public ownership preferable for higher $\theta$. Also note that in the extreme case, in which there are only proportional illiquidity costs (i.e., $l > 0$, but $L = 0$), the choice between public and private ownership becomes trivial. It is then the same for all $\theta$, and would boil down to comparing $\theta W^F\left(\rho^{priv}\right)(1-l)$ and $\theta W^F\left(\rho^{pub}\right)$.

[12] A comparison to a setting with private benefits is relegated to Section 5.5.

information setting from the visionary model where hoarding/delaying was an integral part of the equilibrium. Furthermore, we show that while the asymmetric information formulation could have implications for the choice between public and private ownership, also in that context it does not lead to implications for cash hoarding.

## 4.1 Investment without Cash Hoarding

In the asymmetric information setting, we interpret $\theta$ as the privately known type of the firm (unknown to financiers). It is common knowledge that $\theta$ is drawn from a CDF $F$ on $[\underline{\theta}, \overline{\theta}]$. Let $\widehat{\theta}$ be the financier's belief about the now unobservable type $\theta$. In (3) we now have $\alpha = \frac{K-w}{\widehat{\theta} W^F}$ and (4)-(6) need to take into account $\widehat{\theta}$–i.e., we write $V\left(w_t, \widehat{\theta}, \theta\right)$ and $U\left(w_t, w^*, \widehat{\theta}, \theta\right)$. Our assumption that the manager has access to a competitive market for capital and can make a take-it-or-leave-it offer to the financier gives rise to a game of signaling, as the manager is privately informed about the firm's type.

A candidate for an equilibrium of the signaling game in which each type $\theta$ plays a pure strategy is a triple of functions $(w_\theta^*, \mu^*, \pi)$, where $w_\theta^*$ is the cash level, which manager of type $\theta$ chooses to hoard and co-invests in the investment opportunity; $\mu^*$ is the financier's posterior belief, which maps $w_\theta^*$ into the set of probability distributions over the type set $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$; and $\pi$ represents the financier's decision to finance the project, where $\pi : w_\theta^* \to [0, 1]$, with $\pi = 1$ corresponding to accepting and $\pi = 0$ corresponding to rejecting the offer. Our equilibrium concept is that of a Perfect Bayesian Equilibrium.

Summarizing, the manager maximizes (5) subject to the condition that the proposed contract is individually rational for a financier who makes zero profit (i.e., analogously to (3), $\alpha = \frac{K-w}{\widehat{\theta} W^F}$) and who uses Bayes rule to form his posterior beliefs $\mu^*$ to draw an inference $\widehat{\theta}$ about the firm's type. Note that since the expected cash flow of the investment is linear in $\theta$, we can use $\widehat{\theta} = \int_{\underline{\theta}}^{\overline{\theta}} \theta d\mu^*(\theta)$ to summarize the financier's beliefs about $\theta$.

The focus of the following discussion is on establishing why a separating equilibrium with cash hoarding does not exist when the only financing friction is information asymmetry (in Section 5.1, we follow the steps outlined here to analyze the existence of a separating equilibrium when information asymmetry and disagreement are present jointly). Note that such an equilibrium requires that the proposed contract be incentive compatible. More formally, suppose that there is a monotonic differentiable function $w_\theta^*(\cdot)$ such that outside financiers believe that the firm of type $\theta$ exercises (invests) at $w_\theta^*(\theta)$. Then, if the manager decides to exercise at $\widehat{w} \in w_\theta^*\left(\left[\underline{\theta}, \overline{\theta}\right]\right)$, outside financiers infer that the type is $w_\theta^{*-1}(\widehat{w})$ and

the manager's expected payoff is

$$U(w_t, \widehat{w}, w_\theta^{*-1}(\widehat{w}), \theta) = \left( \left( 1 - \frac{K - \widehat{w}}{w_\theta^{*-1}(\widehat{w}) W^F} \right) \theta W^M - \widehat{w} \right) \left( \frac{w_t}{\widehat{w}} \right)^\beta,$$

which generalizes (6). Since the exercise decision must be on the optimal path, $w_\theta^*$ solves:

$$w_\theta^* = \arg \max_{\widehat{w} \in w^*([\underline{\theta}, \overline{\theta}])} U(w_t, \widehat{w}, w^{*-1}(\widehat{w}), \theta) \tag{11}$$

where, assuming that a separating equilibrium exists, we evaluate the respective FOC at $w^{*-1}(\widehat{w}) = \theta$. Before analyzing whether a solution to this problem exists, we start by showing a useful result.

**Lemma 2** *Single crossing holds because*

$$\frac{\partial}{\partial \theta} \left( - \frac{\frac{\partial}{\partial w^*} U}{\frac{\partial}{\partial \widehat{\theta}} U} \right) > 0, \tag{12}$$

*where $\widehat{\theta}$ is the financier's inference about the firm's type $\theta$.*

The intuition for (12) is that, while hoarding helps to reduce the dependence on external financing, it is costly (as $r < \mu$) and firms with better investment opportunities face higher costs of delay than firms with worse investment opportunities. So at any level of hoarding and for any beliefs $\widehat{\theta}$, the better firms would gain more (or lose less) from reducing the hoarding. Hence, delaying is most costly for good types.[13]

Consider now a candidate for a separating equilibrium in which a high type tries to distinguish herself from low types by hoarding cash. When information asymmetry is the only financing friction, we have that $W^F = W^M$. In this case, a high type cannot distinguish herself from a lower type by not hoarding cash. Such a signal is not costly. Hence, a signal would be costly and, thus, potentially credible only if high types choose a higher cash level than low types. However, such a delay is costlier for a high type than for lower types and would be mimicked. Thus, it cannot lead to separation.[14]

**Proposition 3** *There is no separating equilibrium in which a manager hoards cash. In the unique pooling equilibrium of the financing game satisfying D1, the manager co-finances the investment only with her available cash at hand and does not hoard additional cash.*

[13] The single crossing property does *not* depend on our assumption that the manager's assessment of the firm's value is linear in $\theta$. It is sufficient that the firm's value is increasing in $\theta$.

[14] In Section 5.4 we show that there is no separation even if we allow for payouts (dividends or share repurchases)–i.e., negative hoarding.

For arbitrary out-of-equilibrium beliefs, there is a continuum of pooling equilibria in which the manager hoards cash. Not all equilibria are intuitive in the sense of D1, however. D1 requires that upon observing a deviation, the financier restricts his out-of-equilibrium beliefs only to the set of types who are most likely to have deviated. Since the highest type is always the one most likely to deviate from a pooling equilibrium with positive cash hoarding, the only equilibrium that survives D1 is that all types pool at $w^* = w_0$ provided this leads to a positive payoff and $\alpha = \frac{K-w_0}{\widehat{\theta W}} \leq 1$ (otherwise no financing and investment takes place).[15]

## 4.2 Cash Hoarding in Public vs. Private Firms with Asymmetric Information

We discuss now the choice between public and private ownership with asymmetric information and compare it to the analysis in the visionary model in Section 3.2. To make a balanced comparison to that setting, we assume that there is less information asymmetry between the investor and the manager in a private firm than in a public firm. Thereby, recall that our perspective is that, due to his closer involvement with the firm, a (sophisticated) private investor would be better able to understand the firm. Also the manager of a growth (innovative) firm might agree to disclose sensitive information in more detail to a private sophisticated or specialist investor than to the public market and, thus, its competitors.[16] This mimics the assumption of more agreement (higher $\rho$) in a private firm. As before, a private firm incurs illiquidity costs $L$. The outcome from solving such a setting is reminiscent of the visionary model in that firms with higher quality projects choose private ownership. However, also now, the asymmetric information setting does not lead to any predictions about the firm's cash hoarding policy.

With asymmetric information, the issue is the possibility of self-selection. Firms with better investment opportunities have most to gain from avoiding underpricing. Given the larger information asymmetry they face when going public, they are more willing to be

[15] There can be no other pooling equilibrium with $w^* > w_0$ because the highest type would then successfully deviate by undertaking the investment at a lower cash level $\widetilde{w} < w^*$. Analogously to the intuition behind the single crossing result (Lemma 2), the highest type is prepared to give up a larger equity share (while being better off than on the equilibrium path) than lower types. Hence, for any $w^* > w_0$, we can construct a deviation to a lower cash level, for which the highest type benefits and for which she is the type who is most likely to have deviated. By D1, the investor should limit his out-of-equilibrium beliefs to this type. For such beliefs, he profits from accepting, making such deviation indeed successful.

[16] Note that our argument is about specialist investors. Naturally, we are *not* arguing that general and unsophisticated investors have more information about private than about public firms (see Ferreira et al., 2014).

private and bear the cost of illiquidity. We, thus, have an equilibrium (when $L$ is not too large) in which firms with better investment opportunities go private, and lower quality firms go public. Clearly, if $L$ becomes prohibitively large, all firms would go public.

**Proposition 4** *With asymmetric information, neither public nor private firms hoard cash, but there is a threshold $\theta'$, such that all types above $\theta'$ operate as private, while all types below $\theta'$ operate as public firms.*

The crucial difference to our visionary-framework is that there is no equilibrium in which the manager hoards cash neither as a public nor as a private firm. This insight follows the same arguments as in Proposition 3. Thus, a framework introducing differences in vision (and disagreement) delivers new insights also when comparing cash hoarding in public versus private firms.

# 5 Extensions and Robustness

In this section we discuss several extensions. First, we analyze what happens to our vision-based model if we allow not only disagreement, but also information asymmetry (Section 5.1). Subsequently, we consider the possibility of cash hoarding prior to the arrival of the investment opportunity (Section 5.2). Section 5.3 discusses how cash hoarding and investment decisions are affected when the manager uses debt instead of equity financing. Section 5.4 analyzes dividends and share repurchases, Section 5.5 considers private benefits, and Section 5.6 extends our results to a setting in which the financing friction is an incentive problem ala Holmstrom and Tirole (1997). Finally, Section 5.7 discusses time-varying disagreement and expected profitability of the new project.

## 5.1 Hoarding and Investment with Differences in Vision and Private Information

In light of the differences in results depending on whether financing frictions stem from information asymmetry or differences in vision (disagreement), it is reassuring that our disagreement results are robust when we allow for differences in vision and asymmetric information to be present simultaneously. Specifically, to solve for the resulting signaling game, we make use again of the equilibrium notion and notation used in Section 4.1. The key results are that cash hoarding plays a role like in the setting with differences in vision only, and that resolving the information asymmetry might now be possible.

**Proposition 5** *In a setting combining differences in vision and asymmetric information, there is a unique separating equilibrium in which the manager separates with the amount of cash she uses to co-finance her investment. In this equilibrium, higher types hoard less cash. There is no pooling equilibrium that survives D1.*

We saw earlier that, when there is disagreement only, firms with better opportunities hoard less cash (Proposition 1). This difference in hoarding gives some slack for resolving asymmetric information when it is jointly present with disagreement. It allows firms with better opportunities to signal their types by hoarding *less* cash than firms with worse investment opportunities. This is credible since mimicking by speeding up investment and hoarding less is relatively more costly for lower types (Lemma 2).

Another difference to Proposition 3 is that all types will now hoard at least some cash before becoming indifferent between investing and waiting to invest. Since this threshold is lower for the better type, that type can always successfully deviate from a pooling equilibrium with positive cash hoarding. Hence, there is no pooling equilibrium that survives D1.

## 5.2 Cash Hoarding before Arrival of Investment Opportunity

Returning to our baseline vision-specification, we now extend our results to a setting in which at $t = 0$ the manager does not have an investment opportunity yet, but expects that such an opportunity may present itself at some future point in time. We assume that the time until this event follows an exponential distribution with parameter $\lambda$. The key insight from what follows is that our cross-sectional predictions from Sections 3 and 4 remain valid.

As we have shown, when there are differences in vision between the manager and the financier, a manager who is already presented with an investment opportunity hoards $w_\theta^*$ before investing (Proposition 1). The manager, therefore, delays a profitable investment if her available cash at hand is below $w_\theta^*$. To avoid such costly delay, she could start hoarding cash prior to the arrival of the investment opportunity. Thus, her decision problem before arrival is whether to pay out the available cash at hand or to set aside some cash and hoard more.

The advantage of paying out the available cash is that it can be invested more profitably outside the firm ($r > \mu$). However, setting aside cash and hoarding increases the value of the investment opportunity upon its arrival, and every additional dollar saved is more valuable than the previous one.[17] Thus, if the probability of arrival is sufficiently large,

[17] Not only does it allow to make the investment earlier, but it also reduces the time the already hoarded

the manager sets aside all cash and continues hoarding until the investment opportunity arrives or until she has the necessary capital to invest immediately upon arrival without external financing. Since firms with better investment opportunities hoard less cash after arrival (Proposition 1), our overall result is again that in total–considering pre-arrival and post-arrival hoarding–firms with better prospects hoard less.

**Proposition 6** *(i) The manager hoards cash if the probability of arrival is sufficiently high. Otherwise, she pays out all initially available cash and does not hoard neither pre- nor post-arrival. (ii) If the manager chooses to hoard cash, then, prior to the arrival of the investment opportunity, she sets aside all her initial cash and continues to hoard until the investment opportunity arrives (or until she becomes independent of external financing). Upon arrival of the investment opportunity, the manager follows Propositions 1-5.*

We can contrast this insight to that from a setting based on information asymmetry. Just as there is no separation with hoarding after the arrival of the investment opportunity (Proposition 3), there can be no separation when cash hoarding also takes place prior to arrival. First, consider the case in which the type of investment opportunity is unknown to the manager prior to arrival. In this case, the financing terms will be fair in expectation, so that the manager will not hoard.

Second, suppose that the manager knows the type of the project already prior to arrival. We have shown that the owner-manager invests immediately after arrival even if there is information asymmetry at this stage or she does not invest at all (Proposition 3). Hence, the benefit and the cost of hoarding an additional unit of cash before arrival are independent of the previously hoarded amount. Thus, the manager either prefers to hoard until arrival or not to hoard at all and pay out the available cash at hand. Hoarding to be able to make an investment, thus, occurs if the probability of arrival is sufficiently high. Taking the pre- and post-arrival results together, information asymmetry alone cannot predict why firms that hoard cash to finance an investment opportunity hoard different amounts depending on the profitability of this opportunity.

## 5.3 Cash Hoarding and Type of Financing

In the preceding analysis, we have assumed that the manager issues equity to finance the investment. This assumption is harmless in that it does not qualitatively change any of the preceding results, but–as we show–different types of securities may affect the level of

cash will remain locked-up in the firm prior to undertaking the investment–an effect which increases in that amount.

cash hoarding. The optimal financing contract would minimize the friction coming from disagreement.

The underlying driver for the optimal financial contract in a setting with disagreement is to offer a contract whose value is least dependent on disagreement. From the manager's perspective, the financier should be least sensitive to whether the project turns out to be good (as the manager believes) or bad (as the financier suspects could be the case). This is reminiscent of the intuition in the earlier asymmetric information literature (e.g., Nachman and Noe, 1994), in which the manager prefers to issue debt since, as a less information-sensitive security, it minimizes underpricing. While this analogy is interesting, recall that the setting with asymmetric information does not produce insights on hoarding. This is different in our vision setting, as now the type of financing does affect hoarding.

**Proposition 7** *The manager optimally hoards less cash to co-finance an investment if she has access to more debt financing.*

Being able to issue a security that makes the financier's payoff less sensitive to disagreement is intuitively similar to having less disagreement in the first place. Hence, all else equal, switching to such a security induces the manager to hoard less cash and raise more external financing for the investment. This is precisely the effect of having access to debt financing.

## 5.4 Share Repurchases and Dividend Policy

Our analysis so far has concentrated on cash hoarding. A related question is whether our model has something to say about payout decisions in the form of dividends and share repurchases, which both are elements of cash policy. The answer is straightforward: When outside financing is needed, payouts (which increase the funding need) should be minimized in both the vision-based and asymmetric information setups. That is, spending the available cash on share repurchases or dividend payments increases the firm's dependence on outside financing and is just costly. Thus, returning a dollar to shareholders in the form of repurchases or dividends is worth less to the manager than spending it on reducing this dependence.[18]

Observe also that with asymmetric information repurchases or dividends cannot help the firm signal its investment opportunities. Intuitively, for such a signaling mechanism to

[18]For a model of share repurchases in the presence of heterogenous beliefs in which the manager does not face a net funding need, see Bayar et al. (2013).

be credible, the manager must spend more on share repurchases (or dividends) than she receives from issuing new securities.[19]

**Proposition 8** *If the manager needs external financing for new investments, and there are differences in vision or information asymmetry about the profitability of the new investment, the firm should optimally not pay out cash.*

## 5.5 Private Benefits

The assumption that the manager and financiers have different visions should also be distinguished from a setting in which the difference in valuations is due to private benefits. Specifically, suppose that the manager and the financier agree that the expected cash flows are $\theta W^M$, but the manager additionally gains a private benefit $B$ from investing that cannot be shared with outsiders. In such a setting, the net expected payoff of the manager at the moment of investment equals (cf. (4)):

$$\begin{aligned} U_B(w,\theta) &= \left(1-\frac{K-w}{\theta W^M}\right)\theta W^M + B - w \\ &= \theta W^M + B - K. \end{aligned} \tag{13}$$

Observe that (13) does not depend on the amount of co-investment $w$. What this means is that hoarding cash to reduce the dependence on external financing is useless for the manager. Intuitively, both parties value the expected cash flows $\theta W^M$ in the same way, and a larger co-investment $w$ does not change the fact that the private benefit $B$ accrues only to the manager. This is the crucial difference to the case with disagreement. There, the manager's motive for co-investing $w$ (and, hence, hoarding) is that she has a higher valuation of the cash flows than the financier ($\theta W^F < \theta W^M$).[20]

The insight that private benefits do not lead to hoarding has actually been recognized in the recent empirical literature analyzing Jensen's (1986) free cash flow hypothesis. This literature claims that firms, suffering from the private benefits problem, have less cash as they spend their free cash flow more quickly on new investments (e.g., Dittmar and Mahrt-Smith, 2007; Harford et al., 2008).

[19] Signaling may be possible if the information asymmetry is not about the firm's investment opportunities, but about its assets in place (Bhattacharya, 1979).

[20] There could be a motive for cash hoarding only if the manager cannot raise financing even if she pledges all expected cash flows to the financier–i.e., if $\theta W^M < K - w_0$. In such a case, the manager will either not invest at all or will hoard cash until she can raise sufficient financing by pledging all cash flows to the financier; that is, essentially selling the firm.

## 5.6 Cash Hoarding and Incentives

An important issue for entrepreneurial firms is that external financing, which lowers the owner-manager's stake in the firm, could reduce her incentives to exert effort, increasing further the cost of external financing (Holmstrom and Tirole, 1997). We, thus, compare the model structure based on differences in vision to one based on incentives problems. Our results are robust also in this context.[21]

Specifically, suppose that conditional on being undertaken, the investment succeeds with probability $e$, in which case it yields $\theta W$, and fails with probability $1-e$, in which case it yields zero. The success probability reflects the effort exerted by the manager at cost $\frac{e^2}{2\nu}$ (assumed is that effort is undertaken after the investment). It is straightforward to show that there will be cash hoarding also in such a setting, and managers with better projects (high $\theta$) have incentives to hoard less cash. Indeed, assuming again equity financing, the manager's problem is to choose co-investment $w_\theta^*$ and the optimal level of effort $\widehat{e}$ that maximize her expected payoff

$$\left(\left(1-\frac{K-w_\theta^*}{e^*\theta W}\right)\widehat{e}\theta W - w_\theta^* - \frac{\widehat{e}^2}{2\nu}\right)\left(\frac{w}{w_\theta^*}\right)^\beta, \tag{14}$$

where $e^*$ is the equilibrium level of effort anticipated by the financier.

The conceptual difference to our baseline setting is that increasing the level of co-investment (and, thus, cash hoarding) not only increases the manager's stake in the firm, but also increases the equilibrium level of effort, and so the expected size of the firm. We verify in the Appendix that this additional complexity does not alter our qualitative results. Thus, our predictions also extend to a setting in which the main problem is not one of differences in vision–arguably more pertinent to young and growth firms–but where the main friction is that outsiders fear that insiders have insufficient incentives to exert effort. This is an interesting complementary explanation for cash hoarding.

## 5.7 Time-Varying Disagreement and Profitability

Our baseline specification assumes that differences in vision between the manager and financiers remain constant over time. However, one could argue that disagreement could vary and that the manager could try to time the market and raise financing when it is most beneficial to do so. Incorporating these features does not change our qualitative predictions.

[21] We thank Andrey Malenko for suggesting this discussion.

One straightforward way to model change in disagreement is to assume that disagreement could fully disappear at any given instant with some positive probability, and remains otherwise unchanged. Specifically, suppose that the time until such an event follows an exponential distribution with parameter $\lambda_\rho$. If disagreement disappears, the manager invests immediately as external financing seizes to be costly, and her expected payoff is $\theta W^M - K$. While such feature could create another motive for delaying investment, our qualitative results remain unchanged: Firms with better investment opportunities find it more costly to delay and, hence, hoard less cash.

The other issue is that the expected value of the investment opportunity could vary over time. Delay in investment could, then, occur for two reasons: delaying not only to hoard cash, but also to wait for the value of the investment opportunity to increase. Indeed, assuming that the NPV of the investment opportunity increases on average over time is a standard assumption in the related real options literature (e.g., Bolton et al., 2013). Our results remain robust also in such a setting. In particular, let the increase in NPV come from a lower investment outlay $K$. We now have that at any level of accumulated cash, a lower $K$ implies a lower need for external financing. This reduction in the need for external funding implies that the firm is willing to invest at a lower level of accumulated cash.[22] However this doesn't change our previous insight that firms with better investment opportunities (high $\theta$) delay and hoard less. We make all claims from this section more formal in the Appendix.

# 6 Empirical Implications

In what follows, we discuss several empirical implications of our model. We discuss in turn predictions related to the links between cash hoarding, the firm's investment opportunities and performance, the role played by the degree of disagreement $\rho$, the differences in terms of hoarding we expect between public and private firms, and the impact of the type of financing instruments on hoarding.

**Cash Hoarding, Growth Options, and Performance** The free cash flow theory of Jensen (1986) predicts that managers may invest in negative NPV projects rather than distributing cash to shareholders. Yet the empirical evidence on the relation between

[22] Observe that even though now there are two separate reasons to delay, the firm will not necessarily delay investment longer since a lower investment outlay $K$ implies also a lower need for hoarding cash. Furthermore, note that this insight does not depend on our assumption that the increase of NPV is due to $K$. The same result would obtain if the NPV would increase because of higher expected cash flows.

cash holdings and agency problems is rather mixed. On the negative side, Dittmar and Mahrt-Smith (2007) and Pinkowitz et al. (2006) show that cash is worth less when agency problems between inside and outside shareholders are greater, and Harford (1999) finds that cash-rich acquirers fare worse in takeovers. However, Opler et al. (1999) and Bates et al. (2009) are not able to support the importance of agency explanations in their respective studies. In particular, they show that firms with high Tobin's q hoard more cash than firms with low Tobin's q. Apparently, hoarding cash goes hand in hand with having a high valuation.

Our theory sheds some light on the above contradictory evidence. Only firms with growth prospects (growth options) will hoard cash in our model. Firms without investment opportunities will not do so. So having cash goes hand in hand with growth prospects and, hence, a high q as documented in Opler et al. (1999) and Bates et al. (2009). However, among the firms that have growth prospects, those with better investment opportunities (and hence better performance) will hoard less cash. This is in line with Harford's (1999) finding that cash-rich firms perform worse in takeovers, but is still consistent with the overall result that growth opportunities (and hence q) go hand in hand with cash. Hence, our model complements existing explanations by establishing a relation between cash hoarding policy, growth options, and performance. Our first main prediction is:

**Implication 1**: *(i) Growth firms hoard more cash than firms without growth prospects. (ii) From the subset of firms with growth prospects, firms with lower cash-to-assets ratios are more profitable than firms with higher ratios (i.e., more profitable opportunities go hand in hand with less hoarding).*[23] *(iii) Firms with more profitable investment opportunities co-finance new investments with a higher proportion of external funds.*

Our results complement precautionary based explanations (e.g., Bolton et al., 2011) by establishing a relation between the profitability of a firm's investment and its choice between internal and external financing (Proposition 6). An interesting insight from these results is that there is a self-reinforcing mechanism in which firms with more profitable opportunities also invest and grow more quickly. Furthermore, we can expect that as these firms mature and start generating cash flows at a higher rate than they need for new investments, they could end up with large cash holdings despite pursuing a low-cash strategy in their growth phase.

**Implication 2:** *Maturing growth firms with strong cash flows and, thus, potentially high cash holdings, previously hoard little cash while still in their active growth phase.*

Several recent empirical studies propose that firms with weak corporate governance

[23]The cash-to-assets-ratio decreases in the attractiveness of the opportunities (measured by the parameter $\theta$), as the optimal cash level is decreasing in $\theta$, while the value of the firm increases in $\theta$.

will hoard less cash, because they spend their free cash flow on new investments more quickly (Harford et al., 2008; Gao et al., 2013). Indeed, these papers find that weak corporate governance is correlated with weak performance. However, there is no clear relation between performance, governance, and excess cash holdings, though firms with better performance hoard less cash (Harford et al., 2008). Our preceding implication may explain why. Growth firms with better investment opportunities hoard less cash, delaying investment less and, thus, spending their free cash more quickly. This counteracts the effect that firms with agency problems spend their cash (more quickly) on bad projects. We can summarize this as follows.

**Implication 3:** *Firms with better investment opportunities hoard less cash and, thus, spend their free cash more quickly. This can help explain the negative relation between cash hoarding and performance and the lack of evidence relating performance, governance, and hoarding in the empirical literature focusing on agency problems.*

Another implication of our basic insight that firms hoard less cash and use less internal financing if their investment opportunities are better is that the stock price reaction to new investments financed with a higher proportion of external financing should be more positive. The following implications have not been tested to our knowledge.

**Implication 4**: *The stock price reaction to major investments by growth firms should be more positive if a higher fraction of these investments is financed from external sources.*

Our model could also be interpreted in the context of a firm expanding through acquisitions. Based on Implications 1 and 4, we expect:

**Implication 5**: *(i) Early acquirers in merger waves should have better long-run performance. (ii) The stock price reaction of acquirers financing a higher proportion of their* cash *payments in takeovers from external sources should be higher.*

Observe that the second part of the preceding implication is *not* related to the method of payment in takeovers, but to how the cash payment is financed.[24] Indeed, in the majority of takeovers paid in cash, the cash payment is externally financed (e.g., Martynova and Renneboog, 2009). For these cases, we predict that high-growth acquirers operating in more disagreement-prone industries (e.g., R&D intensive) that finance their *cash* payments in takeovers with a higher proportion of outside financing will do better.

**Cash Hoarding and Degree of Disagreement** The main focus of our implications above was that firms will hoard cash if disagreement plays a role and alignment is difficult.

[24] A large body of literature has investigated the method of payment in takeovers (for a recent survey see Betton et al., 2008). It has typically found that bidder announcement returns are higher when bidders pay in cash rather than equity.

However, the degree of disagreement is a separate factor affecting cash hoarding (Proposition 1). Controlling for it is imperative, as a firm with better investment opportunities, but facing very high disagreement, could end up hoarding more cash than a firm with poorer investment opportunities, but for which disagreement is less of an issue. We expect this to be reflected in the data in the following way.

**Implication 6**: *Growth firms facing more proxy fights, firms for which there is a higher dispersion of analyst forecasts, or whose non-voting shares trade at a larger discount relative to the voting shares, hoard more cash for new investments. This is also true for growth firms investing into unfamiliar businesses, more R&D intensive firms, or firms that may be reluctant to disclose sensitive information.*[25]

Bates et al. (2009) hypothesize that R&D intensive firms hoard more cash, since adverse shocks and financial distress are more costly in the presence of growth opportunities (see also Opler et al., 1999). They find this relationship despite controlling for other measures of growth opportunities, such as Tobin's q. Our analysis can help explain this: For the same growth opportunities (e.g., controlled for by Tobin's q), R&D intensive firms hoard more cash, because in such firms differences in vision–and, hence, disagreement–between entrepreneurs and their financiers will be more pronounced.

As the nature of growth firms changes across the board, we also predict that:

**Implication 7**: *As the composition of growth firms in the economy changes towards firms for which differences in vision and disagreement between entrepreneurs and financiers are more likely, the aggregate cash holdings in the economy will increase.*

Indeed, Bates et al. (2009) argue that the reason for more cash holdings in the economy is that the nature of firms has changed. In particular, they find that newly listed firms are more R&D intensive. Furthermore, Falato et al. (2013) find that firms have shifted towards holding more intangible assets. They argue that firms hoard more cash, because intangibles cannot be offered as collateral when raising debt.[26] Our model complements this argument. Since there is arguably more disagreement about the value of intangibles, our theory helps explain why growth firms choose to depend less on outside financing *in general*, potentially delaying investment, as opposed to shifting towards more equity issuance.

**Cash Hoarding in Public vs. Private Firms, Type of Financing, and Payouts**
Our theory also makes novel predictions regarding the cash holdings in public and private

[25] Recall that our cross-sectional results continue to hold when disagreement and information asymmetry are jointly present (Proposition 5), as it is likely to be the case in practice.

[26] In their model, firms do not switch to equity, because the issuance cost are prohibitively large, reducing the value of existing shareholders by more than the notional amount of equity issuance.

firms. In contrast to precautionary-based theories, which would predict that private firms should hoard more cash as they are more financially constrained, we show that private firms actually hoard less cash (cf. Proposition 2). Importantly, our theory takes into account that growth firms self-select whether they want to remain private or public. The following implication focuses on firms that have the necessary scale to operate both as a public and a private firm.

**Implication 8**: *A growth firm operating as a publicly held firm hoards more cash than when it chooses to be private.*

This surprising prediction finds strong support in a recent empirical study by Gao et al. (2013). They show that public firms hoard up to twice as much cash as comparable private firms. Furthermore, Asker et al. (2013) find that private firms not only have more cash, but they also invest more, have higher investment sensitivity, and have a higher return on assets. These findings are supportive of our prediction that entrepreneurial firms with better investment opportunities remain private and invest more quickly as they hoard less cash to co-finance their investments.

Our model also predicts that the type of financing that firms have access to will affect how much cash they need to hoard for new investments.

**Implication 9**: *Growth firms with higher debt capacity hoard less cash.*

It is interesting to discuss the implications of our analysis for firms that have all the cash they need to undertake new investments. For such firms we predict that the best use of cash would be to use it for new investment opportunities. Returning cash to shareholders in the form of dividends or share repurchases is justifiable only if it is not needed for new investments (cf. Proposition 8). It is strictly suboptimal in the case of differences in vision (disagreement), and also has little effect in the presence of information asymmetry. Indeed, a number of empirical studies have shown that the market reaction to dividend announcements is mixed (DeAngelo et al., 1996, 2000), implying that they are only a weak instrument for signaling quality. Similarly, the market does not react strongly to share repurchases announcements by firms even if they subsequently outperform (Ikenberry et al., 1995), which is also in line with our results that repurchases are not an effective instrument to signal growth opportunities (Proposition 8).

**Implication 10:** *Returning cash to shareholder is not efficient if it is needed for new investments regardless of whether the management and financiers have different visions or whether they have asymmetric information.*

**Managing Cash Reserves** A specific feature of our model is that the cash level changes stochastically over time. One implication of this assumption is that if the manager could

affect the volatility of this accumulation, she would choose higher volatility ($\sigma^2$) even if the expected growth rate ($\mu$) remains the same. Intuitively, the higher volatility increases the real option value of waiting to invest. This is especially relevant for firms investing their cash reserves in the capital market (before they need them for an in-house investment), which is common in practice.[27] Interestingly, this prediction finds support in a recent paper by Duchin et al. (2014) who find that high growth (high q) firms are more likely to invest their cash reserves in riskier assets.

**Implication 11**. *Growth firms hoarding cash to reduce their dependence on outside financing are more likely to park the existing cash in riskier assets.*

# 7 Conclusion

In a simple dynamic theory of optimal cash hoarding, we analyze whether a growth firm will choose to delay investments in order to hoard cash and depend less on outside finance. In our setting, firms hoard cash because they may view outside financing as prohibitively expensive due to disagreement with outside financiers about the firm's prospects. We find, first, that firms with better investment opportunities hoard less cash and finance a higher fraction of new investments with outside financing. The key reason is that they find it more costly to delay a more profitable opportunity. This is true despite the fact that their relative benefit of retaining a higher stake in the firm is also higher.

Other insights follow as well. We show that firms that choose to operate as private firms hoard less cash than they would if they operated as public firms. This is because firms choose to be private when they value the benefit from a higher alignment with financiers–the benefit of being private–more than the costs associated with offering less liquid securities. That is, firms choose to be private when they prefer financing a new investment with more external financing and less internally generated cash. This surprising prediction finds support in recent empirical work.

The second contribution of our paper is to provide for a better understanding of the effect of different financing frictions on cash hoarding. Our intuition and result translate also to a setting in which financing is costly due to incentive problems ala Holmstrom and Tirole (1997). However, an important insight is that restating the model in terms of information asymmetry or private benefits (instead of disagreement) yields only pooling outcomes in which investments are not delayed, and no cross-sectional predictions on cash hoarding ensue. The disagreement-based analysis, thus, helps enrich our understanding

[27] In this case $\mu < r$ could be interpreted as shareholders being more efficient at investing the cash on their own.

of cash policy. This is particularly interesting, because the relevance of differences in vision and disagreement has mushroomed in the modern era in which structural shifts–e.g. developments in information technology–impact firms and industries, yet for now leave substantial uncertainty about viable future business models. In this environment, choices have to be made that are far from routine, more visionary, trial-and-error based, and hence prone to disagreement.

Our results also provide new insights on the dynamics of firm development and cash holdings (as opposed to hoarding). Our analysis focuses on growth firms that are short on cash and operate in an uncertain environment. The ones with the best investment opportunities will choose to grow rapidly using outside funding, and relative to their lesser peers will be cash-poor. However, on average, they will be more profitable and successful. This implies that in the follow-up stage after they have established themselves–and possibly some uncertainty has been resolved–they may start earning cash at a higher rate than needed for investment and growth. High cash holdings are then a sign of past success. One could argue that the massive cash holdings of firms like Google, Microsoft, and Apple are a reflection of this. Our theory primarily focuses on the 'pre-abundance of cash' stages.[28]

The analysis in this paper could also help understand and explain broader issues. Our theory has some interesting implications for the industrial development in the uncertain disagreement-prone, visionary environment. An important implication of our analysis is that firms with better opportunities are not just better but also invest sooner (no delays in investment). This creates reinforcing effects allowing successful firms to differentiate themselves even faster. The result resembles an accelerated Darwinian survival process with 'winners taking it all'. This is in sharp contrast to the pooling outcome in an asymmetric information setting, in which the implied cross-subsidization slows down the Darwinian process.

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# Appendix A: Omitted Proofs

**Proof of Proposition 1.** Differentiating (6) with respect to $w^*$ we obtain

$$\frac{\partial}{\partial w^*}U(w_t,w^*,\theta)=\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\theta W^F}\right)\theta W^M-w^*\right)+\left(\frac{\theta W^M}{\theta W^F}-1\right)\right)\left(\frac{w_t}{w^*}\right)^{\beta}. \quad (15)$$

The first term shows the time-value-loss of waiting for $w$ to increase, while the second term shows the benefit from obtaining cheaper financing when increasing the co-investment. Note that if $\theta W^F > K$, the right-hand-side (RHS) is negative and hoarding cash is never optimal. Hence, the manager only hoards cash if the disagreement with the financier is sufficiently strong and $\theta W^F < K$. Then, the first order condition (FOC) yields:

$$w^*=\frac{\beta}{\beta-1}\frac{\left(K-\theta W^F\right)}{\left(\theta W^M-\theta W^F\right)}\theta W^M.$$

The second order condition is

$$\begin{aligned}\frac{\partial^2}{\partial\left(w^*\right)^2}U(w_t,w^*,\theta) &= \frac{\beta}{\left(w^*\right)^2}\left(\frac{\theta W^F-K}{\theta W^F}\right)\theta W^M\left(\frac{w_t}{w^*}\right)^{\beta}\\ &\quad -\frac{\beta}{w^*}\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\theta W^F}\right)\theta W^M-w^*\right)+\left(\frac{\theta W^M}{\theta W^F}-1\right)\right)\left(\frac{w_t}{w^*}\right)^{\beta}.\end{aligned}$$

At the internal optimum (when the FOC holds), which is the case when $\theta W^F < K$, the second line on the RHS is zero, and the expression is negative. Clearly, if the initial cash at hand is $w_0 > w^*$, the manager invests immediately. **Q.E.D.**

**Proof of Proposition 2.** (i) If a firm chooses to remain private, then a *necessary* condition for this to be more beneficial is that

$$\begin{aligned}
0 &< U^{priv}(w_t, w_\theta^{priv}, \theta) - U^{pub}(w_t, w_\theta^{pub}, \theta) \\
&= \left(\left(1 - \frac{K + L - w_\theta^{priv}}{\theta W^F\left(\rho^{priv}\right)}\right)\theta W^M - w_\theta^{priv}\right)\left(\frac{w_t}{w_\theta^{priv}}\right)^\beta \\
&\quad - \left(\left(1 - \frac{K - w_\theta^{pub}}{\theta W^F\left(\rho^{pub}\right)}\right)\theta W^M - w_\theta^{pub}\right)\left(\frac{w_t}{w_\theta^{pub}}\right)^\beta \\
&= \left(\frac{w_t}{w_\theta^{priv}}\right)^\beta \frac{\theta W^M}{(\beta-1)} \frac{K - \theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{pub}\right)} \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right) \qquad (16) \\
&\quad \times \left(\frac{\left(\theta W^M - \theta W^F\left(\rho^{priv}\right)\right)}{\left(\theta W^M - \theta W^F\left(\rho^{pub}\right)\right)} \frac{\theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{priv}\right)} - \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)^{\beta-1}\right) \\
&< \left(\frac{w_t}{w_\theta^{priv}}\right)^\beta \frac{\theta W^M}{(\beta-1)} \frac{K - \theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{pub}\right)} \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)\left(1 - \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)^{\beta-1}\right) \qquad (17)
\end{aligned}$$

where for the second equality (16) we plug in for $w_\theta^{priv}$ and $w_\theta^{priv}$ from (9) and (10). The inequality in the last line (17) follows from the fact that $W^F\left(\rho^{pub}\right) < W^F\left(\rho^{priv}\right)$, implying that also the term $\frac{\left(\theta W^M - \theta W^F\left(\rho^{priv}\right)\right)}{\left(\theta W^M - \theta W^F\left(\rho^{pub}\right)\right)} \frac{\theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{priv}\right)}$ in (16) is less than one. Thus, a necessary condition is that the last line (17) is positive, which is true only if the last term $1 - \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)^{\beta-1}$ is positive, implying that $w_\theta^{priv} < w_\theta^{pub}$.

(ii) From (16), the manager is just indifferent between private and public ownership if

$$\frac{W^M - W^F\left(\rho^{priv}\right)}{W^M - W^F\left(\rho^{pub}\right)} \frac{W^F\left(\rho^{pub}\right)}{W^F\left(\rho^{priv}\right)} = \left(\frac{K + L - \theta W^F\left(\rho^{priv}\right)}{K - \theta W^F\left(\rho^{pub}\right)} \frac{W^M - W^F\left(\rho^{pub}\right)}{W^M - W^F\left(\rho^{priv}\right)}\right)^{\beta-1} \qquad (18)$$

where we have pugged in for $w_\theta^{priv}$ and $w_\theta^{pub}$ from (9) and (10). Observe now that the manager strictly prefers staying private as $W^F\left(\rho^{priv}\right) > W^F\left(\rho^{pub}\right)$ for $L = 0$. Furthermore, the LHS of (18) is a constant, while the RHS increases in $L$. Thus, there is a unique threshold $\overline{L}_\theta$, above which the manager prefers going public.

We show next that the indifference threshold $\overline{L}_\theta$ must be increasing in $\theta$. Observe that the LHS of (18) is bounded between zero and one, while the second term on the RHS is greater than one. Hence, the first term on the RHS must be less than one, implying that $\overline{L}_\theta < \theta\left(W^F\left(\rho^{priv}\right) - W^F\left(\rho^{pub}\right)\right)$. Furthermore, at $L = \overline{L}_\theta$, the LHS is constant in $L$ and

$\theta$, while the RHS increases in $L$ and decreases in $\theta$ as:

$$\begin{aligned}\frac{\partial}{\partial\theta}\left(\frac{K+\overline{L}_\theta-\theta W^F\left(\rho^{priv}\right)}{K-\theta W^F\left(\rho^{pub}\right)}\right) &= \frac{K\left(W^F\left(\rho^{pub}\right)-W^F\left(\rho^{priv}\right)+W^F\left(\rho^{pub}\right)\frac{\overline{L}_\theta}{K}\right)}{\left(K-\theta W^F\left(\rho_{pub}\right)\right)^2}\\ &< \frac{W^F\left(\rho^{pub}\right)-W^F\left(\rho^{priv}\right)}{K-\theta W^F\left(\rho^{pub}\right)}<0\end{aligned}$$

where we have used that $\overline{L}_\theta<\theta\left(W^F\left(\rho^{priv}\right)-W^F\left(\rho^{pub}\right)\right)$. Thus, differentiating the RHS of (18) with respect to $\overline{L}_\theta$ and $\theta$, yields the result. **Q.E.D.**

**Proof of Lemma 2.** Let for this proof $W^M(\theta)$ and $W^F(\theta)$ denote more generically the manager's and the financier's assessment of the firm's value after investment (as a function of $\theta$), where in the main text we have $W^F(\theta)=\theta W^F$ and $W^M(\theta)=\theta W^M$. Single crossing holds as

$$\begin{aligned}&\frac{\partial}{\partial\theta}\left(-\frac{\frac{\partial}{\partial w^*}U\left(w_t,w^*,\widehat{\theta},\theta\right)}{\frac{\partial}{\partial\widehat{\theta}}U\left(w_t,w^*,\widehat{\theta},\theta\right)}\right)\\ =\ &\frac{\partial}{\partial\theta}\left(-\frac{\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{W^F(\widehat{\theta})}\right)W^M(\theta)-w^*\right)+\frac{W^M(\theta)}{W^F(\widehat{\theta})}-1\right)\left(\frac{w_t}{w^*}\right)^\beta}{\frac{K-w^*}{\left(W^F(\widehat{\theta})\right)^2}\frac{\partial}{\partial\widehat{\theta}}W^F(\widehat{\theta})W^M(\theta)\left(\frac{w_t}{w^*}\right)^\beta}\right)\\ =\ &\frac{1}{\frac{K-w^*}{\left(W^F(\widehat{\theta})\right)^2}\frac{\partial}{\partial\widehat{\theta}}W^F(\widehat{\theta})}\left(\frac{(\beta-1)\frac{\partial}{\partial\theta}W^M(\theta)}{\left(W^M(\theta)\right)^2}\right)>0.\end{aligned}\tag{19}$$

Thus, as claimed in footnote 13, for single crossing it is sufficient that the manager's and the financiers' assessments of firm value are increasing in $\theta$ (and not necessarily linear in $\theta$). **Q.E.D.**

**Proof of Proposition 3.** Take any two types $\theta_H>\theta_L$, which hoard cash $w_H$ and $w_L$ to signal their types, respectively. Absent disagreement, we have $W^F=W^M$, and the incentive constraint of types $\theta_L$ and $\theta_H$ not to mimic each other's strategies are

$$\left(\theta_L W^M-K\right)\left(\frac{w_t}{w_L}\right)^\beta \geq \left(\left(1-\frac{K-w_H}{\theta_H W^M}\right)\theta_L W^M-w_H\right)\left(\frac{w_t}{w_H}\right)^\beta \tag{20}$$

$$\left(\theta_H W^M-K\right)\left(\frac{w_t}{w_H}\right)^\beta \geq \left(\left(1-\frac{K-w_L}{\theta_L W^M}\right)\theta_H W^M-w_L\right)\left(\frac{w_t}{w_L}\right)^\beta \tag{21}$$

In what follows, we show that there can be no separating equilibrium in which high types hoard less (Claim 1) or more (Claim 2) cash than low types. Subsequently, we show that in the only pooling equilibrium that survives D1, the manager does not hoard cash (Claim 3).

**Claim 1.** *There is no separating equilibrium in which a high type hoards less cash than lower types who also receive financing (i.e., $w_L \geq w_H$ cannot hold)*

Observe that (20) can be rewritten as

$$w_H \geq \frac{\theta_L \theta_H W^M \left(1 - \left(\frac{w_H}{w_L}\right)^\beta\right) - K\left(\theta_L - \theta_H \left(\frac{w_H}{w_L}\right)^\beta\right)}{(\theta_H - \theta_L)}$$

Note that the RHS decreases in $\left(\frac{w_H}{w_L}\right)^\beta$ if $K < \theta_L W^M$ (which holds as we show below). Arguing to a contradiction, suppose that $w_L \geq w_H$. Then, the inequality should be easiest to satisfy if $\left(\frac{w_H}{w_L}\right)^\beta$ is one. But if $\left(\frac{w_H}{w_L}\right)^\beta$ is one, the inequality boils down to showing that $w_H > K$. Hence, the manager can only separate as a higher type if she does not raise external financing. Thus, any candidate for a separating equilibrium with external financing must satisfy $w_L < w_H$.

It remains to show that if type $\theta_L$ receives financing, then $K < \theta_L W^M$. Suppose to the contrary that $K > \theta_L W^M$. Since the financier does not expect to break even on type $\theta_L$, $\theta_L$ could receive financing only if she is pooled with positive NPV types. Let $\theta_M$ be one such type–i.e., $K < \theta_M W^M$–where $\theta_M \in (\theta_L, \theta_H)$. By the argument from the previous paragraph $\theta_M$ must co-finance the investment with $w_M < w_H$. However, this leads to a contradiction, since $\theta_M$ is pooled with $\theta_L$ and, thus, $w_M = w_L > w_H$.

**Claim 2**. *There is no separating equilibrium in which a high type hoards more cash than lower types (i.e., $w_L < w_H$ cannot hold).*

Observe that (20) and (21) can be rewritten as

$$\begin{aligned} & \theta_L \theta_H W^M \left(1 - \left(\frac{w_L}{w_H}\right)^\beta\right) + (\theta_H - \theta_L) w_H \left(\frac{w_L}{w_H}\right)^\beta \geq K\left(\theta_H - \theta_L \left(\frac{w_L}{w_H}\right)^\beta\right) \\ \geq \quad & \theta_L \theta_H W^M \left(1 - \left(\frac{w_L}{w_H}\right)^\beta\right) + (\theta_H - \theta_L) w_L \end{aligned}$$

implying that a necessary condition for separation is $w_H \left(\frac{w_L}{w_H}\right)^\beta \geq w_L$, which is a contra-

diction to $w_L < w_H$ since $\beta > 1$.

**Claim 3.** *In the only pooling equilibrium that survives D1, the manager does not hoard cash.*

Suppose that there is a pooling equilibrium in which all types pool at a cash level $w^P > w_0$. $\widehat{\theta}$ is then simply $\widehat{\theta} = \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)$. We start by defining D1 in the context of this game. For use below, note that finding the most expensive financing contract (i.e., financier's response) $\widetilde{\alpha}(\theta)$ for which type $\theta$ is willing to deviate is equivalent to finding the worst out-of-equilibrium beliefs $\widetilde{\theta}(\theta)$ for which the investor still breaks even (i.e., $\widetilde{\alpha} = \frac{K-w}{\widetilde{\theta}W}$) and for which the manager is willing to deviate.

**Definition 1** *For every deviation $\widetilde{w}$, determine for every type the most "expensive" financing contract $\widetilde{\alpha}(\theta)$, respectively the worst out-of-equilibrium beliefs $\widetilde{\theta}(\theta)$, for which the deviation payoff $\widetilde{U}\left(w_t, \widetilde{w}, \widetilde{\theta}, \theta\right)$ is higher than the equilibrium expected payoff $U\left(w_t, w^P, \widehat{\theta}, \theta\right)$*

$$\widetilde{\theta}(\theta) = \arg\min_{\widetilde{\widetilde{\theta}}} \left\{ \widetilde{U}\left(w_t, \widetilde{w}, \widetilde{\widetilde{\theta}}, \theta\right) | \widetilde{U}\left(w_t, \widetilde{w}, \widetilde{\widetilde{\theta}}, \theta\right) \geq U\left(w_t, w^P, \widehat{\theta}, \theta\right) \right\}$$

*Then, D1 requires that the financier believe that the deviation comes from the types who find $\widetilde{w}$ attractive for the most expensive contract, respectively for the worst out-of-equilibrium beliefs $\Theta \in \arg\min_\theta \widetilde{\theta}(\theta)$.*

Suppose that we observe a downward deviation from $w^P$. In what follows we show that the type most likely to have deviated is the highest type. Observe, first, that when the investor breaks even, there is a type $\theta' \in \left(\underline{\theta}, \overline{\theta}\right)$ for whom $w^P$ coincides with $w_\theta^{VM}$, implying that

$$\frac{\partial}{\partial w^P}\widetilde{U}\left(w_t, w^P, \widetilde{\theta}, \theta\right) \begin{cases} < 0 \text{ for } \theta > \theta' \\ = 0 \text{ for } \theta = \theta' \\ > 0 \text{ for } \theta < \theta' \end{cases} \implies \frac{d\widetilde{\theta}}{dw^P} = -\frac{\frac{\partial}{\partial w^P}\widetilde{U}\left(w_t, w^P, \widetilde{\theta}, \theta\right)}{\frac{\partial}{\partial \widetilde{\theta}}\widetilde{U}\left(w_t, w^P, \widetilde{\theta}, \theta\right)} \begin{cases} > 0 \text{ for } \theta > \theta' \\ = 0 \text{ for } \theta = \theta'. \\ < 0 \text{ for } \theta < \theta' \end{cases}$$

Hence, to keep the same utility as on the equilibrium path following a *decrease* from $w^P$, we have to decrease $\widetilde{\theta}$ for $\theta > \theta'$. Moreover, since the marginal rate of substitution $\frac{d\widetilde{\theta}}{dw^P}$ increases in $\theta$ (analogously to Lemma 2), the change in $\widetilde{\theta}$ must be highest for the highest type. Hence, the higher the type, the higher the decrease in $\widetilde{\theta}$ (and so the higher $\widetilde{\alpha}(\theta)$) that the manager is prepared to tolerate following a deviation to $\widetilde{w} < w^P$.

We can, thus, construct a deviation contract $(\widetilde{\alpha}, \widetilde{w})$ with $\widetilde{w} < w^P$, such that only types $(\theta'', \overline{\theta}]$ (where $\theta'' \to \overline{\theta}$) find it profitable to deviate relative to their expected payoff

on the equilibrium path, and such that the financier makes a strictly positive expected profit when accepting for any out-of-equilibrium beliefs that place probability one on the deviation coming from this set of types.[29] By the definition above of D1 above, the deviation is successful for any refined out-of-equilibrium beliefs.

Finally, we show that we can support a pooling equilibrium with $w^P = w_0$ provided that $\frac{K-w_0}{\widehat{\theta}W^M} \leq 1$, where $\widehat{\theta}$ is the expectation over the set of types for which financing and investment yields more than $w_0$. Suppose that this set of types is just $\left[\underline{\theta}, \overline{\theta}\right]$ (the argument for other sets is analogous). Suppose that the financier observes a deviation to $\widetilde{w} > w_0$. By analogous arguments to above, lower types have a higher incentive for such a deviation relative to higher types. Suppose that there is an equity share $\alpha$ for which the deviation is profitable for types $[\underline{\theta}, \theta''')$. By D1 the financier should restrict his out-of-equilibrium beliefs to this set of types. In particular, he can believe that the deviation comes from type $\underline{\theta}$ with probability one. However, it is straightforward to show that for such beliefs the financier does not break even for a deviation that makes type $\underline{\theta}$ (the type most likely to have deviated) better off, implying that the financier would not accept the deviation.[30] **Q.E.D.**

**Proof of Proposition 4.** Consider the following candidate for an equilibrium. All types $\theta \geq \theta'$ raise all financing $(K - w_0)$ as private firms, while types $\theta < \theta'$ pool and raise all financing as public firms, where $\theta' := \int_{\underline{\theta}}^{\theta'} t \frac{dF(t)}{F(\theta')} \left(\frac{L}{K-w_0} + 1\right)$. Intuitively, the managers with the best investment opportunities remain private as the benefit of avoiding underpricing outweighs the illiquidity cost $L$. Instead, managers for which avoiding the illiquidity costs $L$ outweighs the potential cost of underpricing pool and go public. Clearly, if $L$ is very high, all firms would go public. In what follows we verify that this is an equilibrium.

Consider first the incentive constraint that private firms (types $\theta \geq \theta'$) do not prefer deviating to going public and being treated as type $\overline{\theta}_{pub} := \int_{\underline{\theta}}^{\theta'} t \frac{dF(t)}{F(\theta')}$.

$$\theta W^M - K - L \geq \left(1 - \frac{K - w_0}{\overline{\theta}_{pub} W^F}\right) \theta W^M - w_0.$$

This constraint boils down to $\theta \geq \overline{\theta}_{pub} \left(\frac{L}{K-w_0} + 1\right)$, which is satisfied for all $\theta \geq \theta' = \int_{\underline{\theta}}^{\theta'} t \frac{dF(t)}{F(\theta')} \left(\frac{L}{K-w_0} + 1\right)$.

Consider next the incentive constraint that public firms (types $\theta < \theta'$) do not deviate

[29] The latter is feasible, as in a pooling equilibrium types $(\theta'', \overline{\theta}]$ cross subsidize lower types, implying that the financier (who breaks even in expectation) actually makes a profit on these types.

[30] Recall that in a pooling equilibrium, type $\underline{\theta}$ is cross subsidized from higher types. Thus, the financier actually makes a loss on this type.

to going private, in which case their type is revealed and they incur $L$

$$\left(1-\frac{K-w_0}{\overline{\theta}_{pub}W^F}\right)\theta W^M - w_0 \geq \left(1-\frac{K-w_0+L}{\theta W^F}\right)\theta W^M - w_0,$$

which is satisfied if $\theta \leq \overline{\theta}_{pub}\left(\frac{L}{K-w_0}+1\right)$ or equivalently if $\theta \leq \theta'$.

Furthermore, note that private firms never choose to delay investment and hoard cash, since this does not reduce $L$, but (absent disagreement) it only delays the positive NPV investment. Similarly, public firms also do not hoard following the same reasoning as in Proposition 3. **Q.E.D.**

**Proof of Proposition 5.** We show first the existence of a separating equilibrium. Then we show that there is no pooling equilibrium that survives D1.

**Claim 1.** *There is a unique separating equilibrium*

To show existence of a separating equilibrium, we follow standard arguments.[31] Rewriting (11), we obtain

$$w^*_\theta = \arg\max_{\widehat{w}\in w^*\left(\left[\underline{\theta},\overline{\theta}\right]\right)}\left(\left(1-\frac{K-\widehat{w}}{w^{*-1}(\widehat{w})W^F}\right)\theta W^M-\widehat{w}\right)\left(\frac{w_t}{\widehat{w}}\right)^\beta$$

Taking the FOC and assuming that a separating equilibrium exists–i.e., $w^{*-1}(\widehat{w})=\theta$–we have

$$\frac{dw^*_\theta}{d\widehat{\theta}} = \frac{-\frac{\partial}{\partial\widehat{\theta}}U(w_t,w^*_{\widehat{\theta}},\widehat{\theta},\theta)|_{\widehat{\theta}=\theta}}{-\frac{\beta}{w^*_\theta}\left(\left(1-\frac{K-w^*_\theta}{\widehat{\theta}W^F}\right)\theta W^M-w^*_\theta\right)\left(\frac{w_t}{w^*_\theta}\right)^\beta+\left(\frac{\theta W^M}{\widehat{\theta}W^F}-1\right)\left(\frac{w_t}{w^*_\theta}\right)^\beta}. \tag{22}$$

To solve this equation we need the appropriate boundary condition. Since a high type has no incentive to mimic low types, we can set: $w^*_{\underline{\theta}} = w^{VM}_{\underline{\theta}}$, where $w^{VM}_\theta$ is given by expression (7). We can now apply Theorems 1-3 from Mailath (1987) to prove the proposition (in Appendix B we verify that the conditions for these theorems are satisfied). From these theorems it follows that there is a unique separating equilibrium in which $w^*_\theta$ is continuous and differentiable, satisfies (22), and $\frac{dw^*_\theta}{d\theta}<0$ ($\frac{dw^*_\theta}{d\theta}$ has the same sign as $\frac{\partial^2}{\partial\widehat{w}\partial\theta}U(w_t,\widehat{w},\widehat{\theta},\theta)$).

[31] For a detailed general analysis on this point, see Mailath (1987) and for separation in the context of real options see Grenadier and Malenko (2011) and Morellec and Schürhoff (2011).

We now show that $w_\theta^* < w_\theta^{VM}$. To see this, rewrite (22) as

$$-\frac{\beta}{w_\theta^*}\left(\left(1-\frac{K-w_\theta^*}{w^{*-1}(\widehat{w})W^F}\right)\theta W^M - w_\theta^*\right)\left(\frac{w_t}{w_\theta^*}\right)^\beta + \left(\frac{\theta W^M}{w^{*-1}(\widehat{w})W^F}-1\right)\left(\frac{w_t}{w_\theta^*}\right)^\beta$$
$$= \frac{-\frac{\partial}{\partial\widehat{\theta}}U(w_t, w_{\widehat{\theta}}^*, \widehat{\theta}, \theta)}{\frac{dw_{\widehat{\theta}}^*}{d\widehat{\theta}}}. \tag{23}$$

Compare (23) to the optimality condition (15) in Proposition 1. The RHS of (23) is positive, while it is zero absent information asymmetry. Thus, taking into account that the LHS decreases in $w_\theta^*$, we must have $w_\theta^* < w_\theta^{VM}$.

**Claim 2.** *There is no pooling equilibrium that survives D1*

The difference to Proposition 3 is that the level of cash at which the manager is just indifferent between investing and not investing is decreasing in her type $\theta$. Hence, the only pooling equilibrium with $w^* > w_0$, for which we cannot construct a successful deviation for the highest type (in a pool) for any out-of-equilibrium beliefs satisfying D1 (which are trivial and place probability one on a downward deviation coming from this type) is at her indifference point. This contradicts the existence of a pooling equilibrium. **Q.E.D.**

**Proof of Proposition 6.** Observe first that the expected value of the investment opportunity upon its arrival is $U(w_{0,\theta}^*, w_\theta^*)$, where $U$ is given by (6) from Section 2 and $w_{0,\theta}^*$ is the cash level that the manager has hoarded before arrival. It is straightforward to verify that $U(w_{0,\theta}^*, w_\theta^*)$ is strictly increasing and convex in $w_{0,\theta}^*$ (cf. footnote 17). Suppose now that it is optimal to stop hoarding before arrival and before the manager has hoarded $K$ –i.e., $w_{0,\theta}^* < K$. We argue to a contradiction that this cannot be the case.

Suppose that before arrival, having reached $w_{0,\theta}^*$, the hoarded amount $w_t$ increases above $w_{0,\theta}^*$. Paying out $w_t - w_{0,\theta}^*$ cannot be optimal if hoarding until $w_{0,\theta}^*$ is optimal. First, the probability of arrival is the same at every instant. Second (given the convexity of $U$), the marginal increase in the option value $U$ that the manager would have after arrival is increasing in the hoarded amount before arrival. In contrast, paying out a unit of cash has the same value to the manager regardless of the previously hoarded amount. Hence, if hoarding dominates paying out for $w_t < w_{0,\theta}^*$, it is more beneficial also for $w_t > w_{0,\theta}^*$.

To determine whether the manager should start hoarding, we have to compare the expected payoff from hoarding as prescribed above with paying out $w_0$. Clearly, this expected payoff must be increasing in the probability of arrival $\lambda$. Hence, there is a threshold $\overline{\lambda}$, above which setting aside $w_0$ and hoarding is optimal. In this case, the manager hoards

until the arrival of the investment opportunity and, upon arrival, follows Propositions 1. Note that the manager will stop hoarding cash once she becomes independent of external financing.[32]

Next, we discuss the setting with information asymmetry.[33] Similarly to above, for any given beliefs of the financier, the optimal hoarding decision is not strictly quasi-concave (see also main text). It is either optimal to hoard cash until arrival (or becoming independent of external financing) or it is better to pay out all cash at hand $w_0$. Hence, there is no separating equilibrium in which different types hoard different amounts of cash before arrival (Mailath's condition 4) fails). For each type $\theta$ there is again a threshold $\overline{\lambda}_\theta$ (increasing in $\theta$) so that the manager hoards cash if $\lambda > \overline{\lambda}_\theta$. It is straightforward to show that in equilibrium, there is a cutoff type $\theta'$ determined by $\lambda = \overline{\lambda}_{\theta'}$ such that (for the corresponding financier's beliefs) all $\theta < \theta'$ pay out $w_0$ and do not hoard cash neither pre- nor post-arrival, while all $\theta \geq \theta'$ hoard cash until arrival and then follow Proposition 3.[34] Thus, information asymmetry alone cannot predict why firms that hoard cash to finance an investment opportunity hoard different amounts depending on the profitability of this opportunity. **Q.E.D.**

**Proof of Proposition 7.** To be able to compare debt and equity financing, we have to specify the cash flow generating process of the new project. Suppose that the good and the bad project are governed by a common cash flows generating process: $dx_t = \mu_x x_t dt + \sigma_x x_t dZ_t$ with $\mu_x, \sigma_x > 0$ and with $Z$ denoting a Brownian motion. Let the scrap value of the project be $S$. Ex ante, the initial value of this process $x_0$ is unknown, but the cumulative density function (cdf) over the possible realizations of $x_0$ for the good projects dominates that for the bad project in terms of FOSD. All of this is common knowledge. Observe that, after the investment is sunk and the initial value has been realized, the financier cannot infer whether the realization of $x_0$ is due to the project being good or bad.

Before investment, the manager's and the financier's assessments of the project's ex-

[32] To avoid the risk that the cash at hand falls below $K$, she may hoard slightly more than $K$ before starting to pay out. Furthermore, note that if the manager does not start hoarding, she pays out $w_0$ and then cannot invest upon arrival.

[33] The case combining information asymmetry and disagreement is analogous.

[34] It can also be shown that there is no separating equilibrium in which, before starting to hoard, different types separate by paying out different amounts of cash. Intuitively, such a signal is more expensive for higher types, as their investment opportunity is more valuable.

pected payoff are

$$W^i(\theta) := E^i\left[\frac{x_0}{r-\mu_x} + \frac{1}{1-\beta_2} S\left(\frac{x_0}{x_d}\right)^{\beta_2} |\theta\right] \text{ where } i = F, M \ , \tag{24}$$

where $E^M$ is conditional on the project being good, and $E^F$ assumes that it is good only with probability $\rho$. Furthermore, $\beta_2$ is the negative root of $\frac{1}{2}\sigma_x^2 y(y-1) + \mu_x y - r = 0$ and (24) takes into account that the project is optimally liquidated if $x_t$ falls below $x_d := \frac{\beta_2}{\beta_2 - 1} S(r - \mu_x)$ (see Morellec and Schürhoff (2011) for a similar derivation).[35]

Suppose now that the manager promises a small constant debt coupon payment $\varepsilon$ in addition to an equity share $\widetilde{\alpha}$. Furthermore, let the manager's share of the liquidation proceeds be $(1-\alpha)\left(S - \frac{\varepsilon}{r}\right) - \frac{\varepsilon}{r}$, implying that the financier is guaranteed $\frac{\varepsilon}{r}$ even in liquidation. Clearly, stipulating such a share is feasible for $S > 0$ and $\varepsilon$ sufficiently small. It is straightforward to check that it is optimal for the manager to liquidate the project at $x_d$ for such a sharing rule in liquidation (as it is optimal for pure equity financing). The equity share $\widetilde{\alpha}$ that satisfies the financier's participation constraint is $\widetilde{\alpha} = \frac{K - w - \frac{\varepsilon}{r}}{W^F(\theta)}$. By similar arguments to Proposition 1, we obtain that before investing, the manager hoards:

$$\widetilde{w}_\theta = \frac{\beta}{\beta - 1}\left(\left(K - W^F(\theta)\right)\frac{W^M(\theta)}{W^M(\theta) - W^F(\theta)} - \frac{\varepsilon}{r}\right) < w_\theta^{VM}.$$

**Q.E.D.**

**Proof of Proposition 8.** The proof focuses on the case with asymmetric information. Suppose that the manager owns only a fraction $\eta$ of the firm, and the remaining $\eta$ is owned by external shareholders. Consider a candidate for a separating equilibrium in which the highest type $\overline{\theta}$ repurchases a fraction $\eta - \eta_2$, so that after the repurchase she owns $(1 - \eta_2)$. Clearly, since delay is costly and it is not possible for high types to separate through cash hoarding, it is without loss of generality to focus on the "static" case in which the manager repurchases the shares and raises new financing in the same period. On a competitive market, the price $p$ at which type $\overline{\theta}$ repurchases a fraction $\eta - \eta_2$ of the firm must solve

$$\begin{aligned} \eta\left(\overline{\theta}W^M - K + w\right) &= \eta_2\left(\overline{\theta}W^M - K + w - p\right) + p \\ \Longrightarrow p &= \frac{\eta - \eta_2}{1 - \eta_2}\left(\overline{\theta}W^M - K + w\right) \end{aligned}$$

[35] Previously, we had $W^i(\theta) = \theta W^i$. However, recall that the results are valid as long as the assessments of firm's value $W^i(\theta)$ are increasing in $\theta$.

where we use that existing shareholders should be indifferent between selling and not selling their shares at the fair price. Hence, the incentive constraint of type $\underline{\theta}$ not to mimic type $\overline{\theta}$ is

$$(1-\eta)\left(\underline{\theta}W^M - K + w\right) \geq (1-\eta_2)\left(1 - \frac{K-(w-p)}{\overline{\theta}W^M}\right)\underline{\theta}W^M,$$

which, after plugging for $p$, boils down to $\frac{\left(\underline{\theta}-\overline{\theta}\right)}{\overline{\theta}}(K-w) \geq 0$, contradicting that $K > w$. Finally, note that if $K \leq w$, underpricing is irrelevant for the manager, as she can finance the project out of her own funds.

Next, we show by contradiction that there cannot exist an equilibrium in which the firm separates by paying out dividends. First, observe that if the manager does not separate with a dividend payment, she cannot do so through cash hoarding following such a payment (Proposition 3). Thus, the only candidate for a separating equilibrium is paying a dividend and then investing immediately. Then, the incentive constraint of any type $\theta_L < \theta_H$ not to mimic type $\theta_H$ is

$$\left(1 - \frac{K-w_L}{\theta_L W^F}\right)\theta_L W^M + (w_t - w_L) \geq \left(1 - \frac{K-w_H}{\theta_H W^F}\right)\theta_L W^M + (w_t - w_H), \qquad (25)$$

where $(w_t - w_i)$ is the dividend paid by type $\theta_i$. Using that without disagreement $W^M = W^F$, (25) can be rewritten as $w_H \geq K$, leading to the desired contradiction. **Q.E.D.**

**Proof: Cash Hoarding and Incentives (Section 5.6).** We briefly verify the claim that the incentives problem sketched in the main text leads to the same qualitative predictions as differences in vision. Suppose that for some given level of co-investment $w$ by the manager, the financier expects an equilibrium level of effort $e^*$, prompting him to require a share of the firm $\alpha = \frac{K-w}{e^*\theta W}$. In such a case, it is optimal for the manager to choose

$$\max_{\widehat{e}}\left(1 - \frac{K-w}{e^*\theta W}\right)\widehat{e}\theta W - w - \frac{\widehat{e}^2}{2\nu}$$

implying that $\frac{(e^*\theta W - K + w)}{e^*} = \frac{\widehat{e}}{\nu}$. Since in equilibrium $e^* = \widehat{e}$, we have $e^* = \frac{1}{2}\theta W\nu + \frac{1}{2}\sqrt{\theta^2 W^2\nu^2 - 4\nu(K-w)}$ and we see immediately that $e^*$ is increasing in $w$.

Plugging in for $e^*$ into the manager's expected payoff given in (14) we obtain

$$U^{incentives} := \frac{1}{2}\left(\left(\frac{1}{2}\theta W\nu + \frac{1}{2}\sqrt{\theta^2 W^2\nu^2 - 4\nu(K-w)}\right)\theta W - K - w\right)\left(\frac{w}{w^*_\theta}\right)^\beta.$$

Thus, $w_\theta^*$, is the solution to the first-order-condition

$$\frac{\nu\theta W}{\sqrt{\theta^2W^2\nu^2-4\nu\left(K-w\right)}}-1=\frac{\beta}{w_\theta^*}\left(\left(\frac{1}{2}\theta W\nu+\frac{1}{2}\sqrt{\theta^2W^2\nu^2-4\nu\left(K-w\right)}\right)\theta W-K-w\right).$$

It is straightforward to show that the second-order-condition is satisfied. Furthermore, we also have $\frac{\partial^2U^{incentives}|_{w=w_\theta^*}}{\partial w_\theta^*\partial\theta}<0$, implying by standard arguments that $\frac{dw_\theta^*}{d\theta}<0$ as claimed in the main text. **Q.E.D.**

**Proof: Time-varying disagreement and profitability (Section 5.7).** We briefly formalize our claims from Section 5.7.

(i) *Time varying disagreement*: Let $\theta\widetilde{W}^F$ be the financier's valuation of the firm given that he knows that, after investing, disagreement could disappear with probability $\lambda_\rho$ at any given point in time.[36] Applying the modified Ito's lemma for jump processes and following similar steps to Section 3, it is straightforward to show that the manager optimally hoards

$$w_\rho^*=\frac{\gamma}{\gamma-1}\frac{\left(\left(K-\theta\widetilde{W}^F\right)W^M+\frac{\lambda_\rho}{\lambda_\rho+r}\left(\theta W^M-K\right)\widetilde{W}^F\right)}{\left(W^M-\widetilde{W}^F\right)}$$

where $\gamma$ is the positive root to $\frac{1}{2}\sigma^2y\left(y-1\right)+\mu y-r-\lambda_\rho=0$. It is straightforward to verify that this leads to the same qualitative insights as in Section 3.[37]

(ii) *Time-varying profitability*: We illustrate the argument by making a simplifying assumption that allows us to solve the resulting problem analytically. Specifically, suppose that the NPV of the project from the financiers' point of view follows

$$\frac{d\left(K-\theta W^F\right)}{\left(K-\theta W^F\right)}=\mu_K dt+\sigma_K dZ_K$$

where $Z_K$ is a standard Brownian motion and $\sigma_K>0$ with a correlation $\psi$ to $Z$. We assume that $\mu_K<0$ implying that the NPV increases on average over time. To simplify the analysis, we further assume that the change in NPV is entirely due to a change in the investment cost $K$. Following similar steps to Proposition 1, the manager's expected payoff is the solution to the following partial differential equation

[36] If he disagreement is about the starting value of the flow process in analogy to Proposition 7, then $\theta\widetilde{W}^F=\theta W^F$.

[37] Furthermore, single crossing holds for $\lambda$ sufficiently small, which helps to extend also the results from Section 4.

$$\begin{aligned} rU &= \mu w U_w + \frac{1}{2}\sigma^2 w^2 U_{ww} + \mu_K \left(K - \theta W^F\right) U_K \\ &\quad + \frac{1}{2}\sigma_K^2 \left(K - \theta W^F\right)^2 U_{KK} + \psi\sigma\sigma_K w \left(K - \theta W^F\right) U_{wK} \end{aligned} \tag{26}$$

where the subscripts $w$ and $K$ denote the partials with respect to $w$ and $\left(K - \theta W^F\right)$, respectively. Define $\chi = \frac{w}{(K - \theta W^F)}$ so that $U(w, K) = \left(K - \theta W^F\right) U(\chi)$, where we use that $U$ is homogenous of degree one in $(w, \left(K - \theta W^F\right))$ (doubling $w$ and $\left(K - \theta W^F\right)$ would merely double the value of the growth opportunity). We have

$$\begin{aligned} U_w &= U_\chi;\ U_{ww} = \frac{1}{(K - \theta W^F)} U_{\chi\chi};\ U_{wK} = -\frac{w}{(K - \theta W^F)^2} U_{\chi\chi} \\ U_K &= U - \frac{w}{(K - \theta W^F)} U_\chi;\ U_{KK} = \frac{w^2}{(K - \theta W^F)^3} U_{\chi\chi}. \end{aligned}$$

Plugging into (26), we obtain the simple ordinary differential equation

$$\underbrace{(r - \mu_K)}_{r'} U = \underbrace{(\mu - \mu_K)}_{\mu'} \chi U_\chi + \underbrace{\left(\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_K^2 - \psi\sigma\sigma_K\right)}_{\sigma'} \chi^2 U_{\chi\chi} \tag{27}$$

with a value matching condition $U(\chi^*) = \left(\frac{1-\chi^*}{\theta W^F}\right) \theta W^M - \chi^*$. Defining $\phi$ as the positive root to $\frac{1}{2}\sigma'^2 y(y-1) + \mu' y = r'$ (where $r'$, $\mu'$ and $\sigma'$ are defined in (27)), and following the same steps as in Section 3, we obtain

$$\chi^* = \frac{w^*}{(K^* - \theta W^F)} = \frac{\phi}{\phi - 1}\left(\frac{\theta W^M}{\theta W^M - \theta W^F}\right).$$

We see, thus, that the optimal co-investment $w$ and the NPV from the financier's point of view are in a constant proportion at the optimal investment barrier. Along this barrier, the optimal cash level $w^*$ increases with the investment cost $K^*$, and this level is lower when the investment opportunities are better (high $\theta$). **Q.E.D.**

# Appendix B

**Verifying Mailath's Conditions for a Separating Equilibrium** In what follows, we verify that the regularity conditions required by Mailath (1987) are indeed satisfied and that $\frac{\partial^2}{\partial\widehat{w}\partial\theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta) < 0$. Mailath's conditions are:

1) Smoothness: $U(\cdot)$ is twice continuously differentiable.

2) Belief monotonicity: $\frac{\partial}{\partial \widehat{\theta}} U(\cdot)$ is either strictly positive or strictly negative.

3) Type monotonicity: $\frac{\partial^2}{\partial \theta \partial w^*} U(\cdot)$ is either strictly positive or strictly negative.

4) Strict quasiconcavity: $\frac{\partial}{\partial w^*} U(\cdot)\,|_{\widehat{\theta}=\theta} = 0$ has a unique solution in $w$ that maximizes $U(\cdot)\,|_{\widehat{\theta}=\theta}$, and $\frac{\partial^2}{\partial (w^*)^2} U(\cdot)\,|_{\widehat{\theta}=\theta} < 0$ at this solution.

5) Boundedness: There is $k > 0$ such that for all $(\theta,\ w) \in \left[\underline{\theta}, \overline{\theta}\right] \times \mathbb{R}_+$, $\frac{\partial^2}{\partial (w^*)^2} U(\cdot)\,|_{\widehat{\theta}=\theta} \geq 0$ implies $\left|\frac{\partial}{\partial w^*} U(\cdot)_{\widehat{\theta}=\theta}\right| > k$.

Conditions 1)-2) are satisfied. Proposition 1 shows that condition 4) is also satisfied. To check for condition 5), observe that if $\frac{\partial^2}{\partial (w^*)^2} U(\cdot)\,|_{\widehat{\theta}=\theta} \geq 0$, then since the first line on the RHS of

$$\begin{aligned}\frac{\partial^2}{\partial \left(w^*\right)^2} U(w_t, w^*, \widehat{\theta}, \theta) &= \frac{\beta}{\left(w^*\right)^2}\left(\frac{\widehat{\theta} W^F - K}{\widehat{\theta} W^F}\right) \theta W^M \left(\frac{w_t}{w^*}\right)^{\beta} \\ &\quad -\frac{\beta}{w^*}\left(-\frac{\beta}{w^*}\left(\left(1 - \frac{K - w^*}{\widehat{\theta} W^F}\right) \theta W^M - w^*\right) + \frac{\theta W^M}{\widehat{\theta} W^F} - 1\right)\left(\frac{w_t}{w^*}\right)^{\beta}\end{aligned}$$

is negative, $\frac{\partial}{\partial w^*} U(\cdot)\,|_{\widehat{\theta}=\theta} < 0$ for $\widehat{w} \in (0, k)$ where $k$ is bounded away from infinity. $w^*_\theta$ remains the unique equilibrium even if $k \to \infty$. To see this, observe that the single crossing property holds for $w^* \in (0, \infty)$. Hence, local incentive compatibility of $w^*_\theta$ guarantees also global incentive compatibility and $w^*_\theta$ is a separating equilibrium even if $k \to \infty$. Moreover, it is the unique equilibrium. Otherwise, there must be an alternative equilibrium with a type for whom $w^*_\theta(\theta) \to \infty$. However, for such a trigger, the option value component of this type's expected payoff is zero, whereas it is strictly positive for a positive trigger bounded away from infinity. This makes a deviation profitable, contradicting the existence of a different separating equilibrium than $w^*_\theta$.

Finally, we check when $\frac{\partial^2}{\partial w^* \partial \theta} U(\cdot) < 0$ holds–i.e., condition 3). $U$ is submodular in $w^*$ and $\theta$ if

$$\frac{\partial^2}{\partial w^* \partial \theta} U(w_t, w^*(\widehat{\theta}), \widehat{\theta}, \theta) = \left(-\frac{\beta}{\widehat{w}}\left(\widehat{\theta} W^F - K\right) + 1 - \beta\right) \frac{W^M}{\widehat{\theta} W^F}\left(\frac{w_t}{\widehat{w}}\right)^{\beta} < 0$$

and so it should hold

$$w^*(\widehat{\theta}) > \underline{w}_{\widehat{\theta}} := \frac{\beta}{\beta - 1}\left(K - \widehat{\theta} W^F\right). \tag{28}$$

Comparing this condition to (7), we see that submodularity holds as long as signaling does not require a too large distortion away from first best.

Clearly, condition (28) is satisfied for all types close enough to the lowest type, since $w^*_{\underline{\theta}} = w^{VM}_{\underline{\theta}} > \underline{w}_{\underline{\theta}}$. A sufficient condition that it is satisfied for all types is that $\theta W^F >$

$\theta W^M - \frac{1}{\beta}K$–i.e., that disagreement is not excessive.[38] Intuitively, if disagreement is excessively large, absent information asymmetry, all types find it optimal to accumulate large cash reserves and raise only little financing from outside financiers. Then, deviating too much from this strategy when there is information asymmetry, which may be needed for high types to separate from low types, could become too costly as the benefit from overcoming information asymmetry does not compensate for the cost of increasing exposure to outside financing and, thus, disagreement.

If condition (28) is violated, denote the lowest type for whom this is the case with $\theta'$. Then, we can construct a separating equilibrium for types $[\underline{\theta}, \theta']$ characterized by (22) and $w^*_{\underline{\theta}} = w^{VM}_{\underline{\theta}}$. The remaining types $(\theta', \overline{\theta}]$ then pool at the cash level $w^P < w^*_{\theta'}$ for which type $\theta'$ is indifferent between separating and pooling with the higher types. It is straightforward to find beliefs that support such an equilibrium.

[38] This condition can be derived from requiring that $\frac{dw^*_\theta}{d\theta} > \frac{d\underline{w}_\theta}{d\theta}$ for $w^*_\theta = \underline{w}_\theta$–i.e., that $w^*_\theta$ and $\underline{w}_\theta$ do not cross.