Interest On Excess Reserves in a DSGE model with a Banking Sector
Incomplete and In progress

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Abstract

In 2008, the Federal Reserve implemented several new monetary policy tools. One of these tools included that it began to pay interest on a commercial bank's reserves, which created a channel system. A channel system describes a scenario where the central bank can establish an upper and a lower bound around an announced benchmark interest rate such as the federal funds rate. The penalty rate establishes the upper bound since a bank will not borrow from another commercial commercial bank above this rate. A benefit of paying interest on reserves is that IORs place a lower bound on the federal funds rate.

In order to analyze this new policy, this paper utilizes a DSGE model with a banking sector. The banking sector includes excess reserves in its balance sheet that receive interest that can be adjusted by the monetary authority. Exogenous shocks are applied to a deterministic model, where agents anticipate future shocks, and a stochastic model, where agents react to an unexpected shock, in order to analyze the impact on macroeconomic variables. I find that an expansionary IOR policy results in a lower price level compared to applying an expansionary OMO policy.

JEL Classification: E31, E32, E51, E52, E58.

1 Introduction

The purpose of this monograph is to examine the dynamics of a DSGE model when a monetary authority has the ability to adjust the interest rate that it pays on a bank's reserves. The experience of Canada, New Zealand, and Australia is that their respective central banks have been able to maintain tighter control over their target interest rates compared to the Federal Reserve by implementing a "channel system." This is where the target interest rate's ceiling is the penalty rate (commonly referred to as the discount rate) and the target rate's floor is the interest paid by the monetary authority on a banks reserves.

Until October of 2008, required reserves remained idle on a bank's balance sheet and therefore did not generate income for a bank. Thus, required reserves were seen as an implicit tax on all financial intermediaries that were subject to balance requirements. Taxes, explicit or implicit, are considered a distortion to markets. By allowing the monetary authority to pay interest on a financial intermediaries reserves, there is not always an inverse relationship between the target rate and the aggregate money supply.

Since October 2008, the Federal Reserve has had the authority to pay interest to commercial banks for funds stored at the district banks. Simultaneously, the Federal Reserve significantly increased the country's money supply, and thus its balance sheet, through an unprecedented process called quantitative easing (QE). Despite the significant increase in the money supply, inflation in the US has remained under 2 percent –contrary to what economic theory would predict.

This paper develops a general equilibrium model that includes a banking sector that earns interest on its total reserves. Simulations are then conducted to analyze the impact on the model's endogenous variables as a result of changing the interest paid on reserves. Specifically, I am interested to see how excess reserves can be manipulated through interest on reserves in order to influence the equilibrium price level and aggregate output. I compare expansionary OMO policy with IOR policy. The paper's model finds that an expansionary IOR policy results in a lower price level compared to an expansionary OMO policy.

2 Implementation of IOR Policy by the Federal Reserve

Throughout each business day, there are deposits and withdrawals among financial intermediaries (FIs). Banks strive to keep their excess reserves (ERs) at a minimum because there is an opportunity cost to maintaining them. That cost is the interest that could have been earned if the bank had lent out its excess reserves. Concurrently, some banks are subject to maintain a certain amount of required reserves (RR). Banks attempt to avoid insufficient reserves at the end of the business day since it is costly to make up the difference. Holding at least some excess reserves is necessary to protect against unexpected withdrawals. Hence, *maintaining ERs acts as a safety margin, or insurance, against a dearth of required reserves.*

Required reserves have been considered an implicit tax on financial intermediaries because of the opportunity cost of not being able to lend them out. An additional cost of reserves is the time and expense of rearranging the banks balance sheet for the purpose of avoiding or decreasing RRs. To stay competitive, a bank will expend resources moving funds in and out of customer's deposit accounts in order to decrease its required reserves. By implementing IOR policy, the opportunity cost of holding both required and excess reserves is either reduced or eliminated. In addition, the monetary authority has additional leverage over monetary policy when it pays interest on a bank's reserves.

In 2008 and afterward, several changes to monetary policy took place in order to provide liquidity to US financial markets. As a result, the Federal Reserves balance sheet increased to $2 trillion in that year from $800 billion just three months earlier (Hornstein 2010). One such change was that the Federal Reserve began paying interest to financial intermediaries for both required reserves and excess reserves that were held at a Federal Reserve district bank.

On October 13, 2006, Congress gave permission to the Federal Reserve to pay interest on reserves (IORs) starting in October, 2011. However, this date was moved back to October 2008 because of the Emergency Economic Stabilization Act in order to facilitate bank liquidity. The interest rate set on required reserves was initially set at 140 basis points. The original policy was to pay an interest rate 10 basis points below the federal funds rate (ffr). The policy regarding excess reserves was to pay an interest rate of 75 basis points below the target ffr. A few weeks later, the spread was adjusted to 35 basis points. By the end of 2008, the rates on both types of reserves were set to 25 basis points and were held constant until December, 2015. These IOR rates were increased again on December, 2016. Both increases were by 25 basis points.

Changing the interest on reserve rate is loosely analogous to changing the required reserve ratio. That is, by adjusting the IOR rate, banks will choose to also adjust the amount of reserves to hold rather than being forced to hold a required amount. Another implication is that the monetary authority can adjust the policy rate without having to also manipulate the money supply via open market operations.

The remainder of this section explains the purpose and theory of why a monetary authority would want to pay depository institutions to hold excess reserves.

2.1 Channel System Model

The textbook description of the federal funds market includes a demand curve R^D that has a downward sloping segment and a horizontal segment determined by the IOR rate. In addition, there is a supply of reserves curve, R^S, which consists of a perfectly vertical and a perfectly horizontal segment as in Figure **??**. The vertical line at NBR_1 implies that the monetary authority is a monopolist of nonborrowed reserves (NBRs) and therefore has perfect controls over NBRs. The horizontal segment is set at the primary credit discount rate. The horizontal segments of the R^S and R^D curves create an upper and lower bound, or channel, for the ffr. Banks have no incentive to borrow from other FIs at a rate above the discount rate. Concurrently, FI's have no incentive to lend below the IOR rate. When the vertical R^S curve intersects the R^D curve on the downward sloping segment, the equilibrium ffr is between the discount rate and IOR rate.

The pre-QE approach to adjusting the ffr was with open market operations (OMOs). An OMO purchase increases the NBRs and shifts the R^S curve to the right , which causes the ffr to decrease. The dynamic OMO purchase is shown in Figure **??** with the equilibrium moving from point 1 to point 2. An OMO sale will drain the market of reserves and shift the R^S curve from NBR_2 to NBR_1 and therefore raise the ffr.

Related but lesser known tools are repurchase agreements and reverse repurchase agreements. These are also known as repos and reverse repos respectively. A repo is a short-term OMO purchase with a stipulation that the bank will repurchase the security at a predetermined price and date. A reverse repo is therefore a OMO sale with a predetermined price and date of a resale of securities back to the FR. RRPs are essentially short term loans to the FR. Repos and reverse repos are useful for very short term defensive OMOs and can be used to maintain the ffr target rate during holidays when FIs tend to hold relatively larger amounts of ERs. Term RRPS are repurchased after a few days or weeks. Overnight repos (ON RRPS) are repos that are repurchased the next day. See Ihrig, Mease, and Weinbach (2015).

Since 2008, the demand for reserve curve includes a horizontal segment reflecting the IOR rate. Figure **??** amends the narrative from Figure 2 to show how IORs impact the federal funds market. After the OMO purchase, The R^S curve shifted from NBR_1 to NBR_2 and the ffr decreased. As a result, the equilibrium moved from point 1 to point 2. With an additional OMO purchase, NBRs increase from NBR_2 to NBR_3 and the ffr falls until it coincides with the IOR rate. That is, the IOR rate provides a nonzero floor. Finally, a third OMO purchase shifts the supply curve to the right from NBR_3 to NBR_4 while the ffr remains fixed. The equilibrium point moves from point 3 to point 4. Without IORs, the ffr would be zero.

By paying IORs, the Federal Reserve can now narrow the channel from below without effecting the market ffr as shown in Figure **??**. However, if the equilibrium takes place on the horizontal segment of the demand curve like points 3 and 4 in Figure 3, the monetary authority can lift the ffr without implementing OMOs by using the IOR tool. As shown in Figure **??**, an increase in the IOR will also increase the ffr from equilibrium point 1 to equilibrium point 2. In fact, the relativly new IOR tool is now necessary to increase interst rates in the aftermath of QE. As we will see below, IOR behavior in theory is not exactly IOR in practice. Though the two may eventually become the same when the reserve market resembles pre-QE conditions.

With IORs, monetary policy (MP) can now achieve various combinations of quantities of reserves and ffrs. The benefit of this independence is that the FR can target financial markets and the macroeconomy separately where this was not possible before. More specifically, the FR can now adjust aggregate bank reserves independently of the ffr. This would be particularly useful to counter any future shocks to the financial system without disrupting the overall economy (Goodfriend, 2002). On the perfectly elastic segment of the R^D curve, it is now possible to adjust reserves while holding the ffr constant. Alternatively, by adjusting IORs the central bank has the option of adjusting the ffr while holding reserves constant. See Irland, 2011; Goodfriend, 2002.

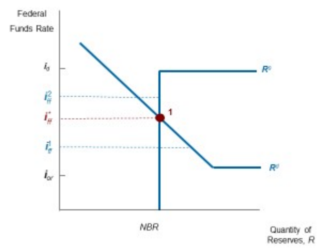

Figure 1: The demand for reserves is downward sloping except at the IOR rate, where the demand curve becomes infinitely elastic. The Supply for reserves is a vertical line at the amount of non-borrowed reserves, except at the penalty rate, where the supply curve becomes infinitely elastic

2.2 ERs and the Money Multiplier

In addition to providing a floor for the federal funds rate, IORs can be utilized for the purpose of affecting the money supply. In this section, we review a basic model that relates the monetary base and the money multiplier to the money supply (Mishkin, 2015). We then apply IORs to the money multiplier to show how IOR policy can be used to change the money supply.

The conventional tool for adjusting the monetary base is open market operations. The monetary base, along with the money multiplier, determines the money supply such that

$$MB \cdot m = M$$

where MB is the monetary base and m is the money multiplier. The three variables that characterize the money multiplier are the required reserve ratio, r; the currency ratio, c; and the excess reserve ratio, e. The money multiplier equation is described in the literature as

$$m = \frac{1+c}{r+c+e}.$$

The required reserve ratio is the percentage of a banks deposits that the bank must hold and therefore is not allowed to lend. Required reserves, RR, are determined by multiplying the required reserve ratio by deposits

$$D \cdot r = RR. \tag{1}$$

Equivalently, the required reserve ratio is the ratio of the required reserves to the bank's

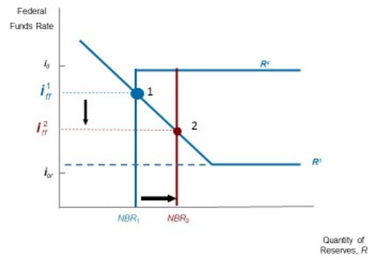

Figure 2: The traditional approach to adjusting the federal funds rate is to change the amount of reserves throughout the banking system.

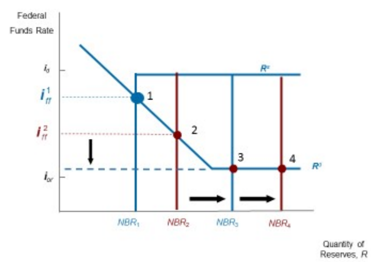

Figure 3: With IORs, the federal funds rate can be adjusted by changing the IOR rate

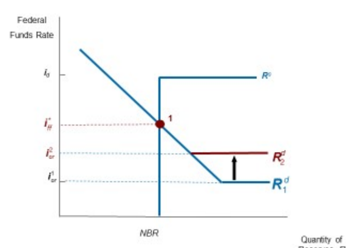

Figure 4: With IORs, the federal funds rate can be adjusted by changing the IOR rate

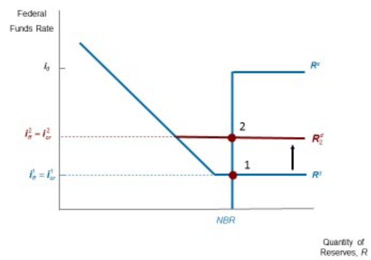

Figure 5: With IORs, the federal funds rate can be adjusted by changing the IOR rate without having to adjust the monetary base.

deposits. By rearranging equation (1) we get

$$r = \frac{RR}{D}.$$

The currency ratio is the amount of the currency, C, that the public holds relative to the amount of deposits within the banking system. The currency ratio is described by

$$c = \frac{C}{D}$$

and reflects the amount of currency that the public desires to hold. The excess reserve ratio reflects the amount of excess reserves, ER, the banking system possesses relative to the total amount of deposits. The excess reserve ratio is a function of how much of the bank's excess reserves that it wants to lend. The ER ratio is also determined by how much of the bank's ERs that the public wants to borrow. In addition, it also reflects how much banks want to borrow from each other in the federal funds market. The ER ratio is defined as

$$e = \frac{ER}{D}.$$

When a central bank pays IORs, the central bank can then influence the choice of how much ERs a bank will desire to hold. Hence, when the central bank increases IORs, we would expect the excess reserve ratio to become larger, the money multiplier to decrease, and the money supply to also decrease. In contrast, we would expect the money supply to increase when the central bank reduces the IOR rate.

The next section analyzes these variables along with other related monetary aggregate variables.

2.3 Empirical Monetary Aggregate Behavior Pre- and Post- Quntitiative Easing

This section analyzes the empirical relationship between IORs and ERs. With the option to influence ERs, we can then surmise the monetary authority's ability to manipulate other monetary aggregates. Figure **??** shows that there is a correlation between IORs and ERs. Unfortunately we cannot say if, or how much, of this relationship is causation. However, we can see what could happen to monetary aggregates by analyzing the behavior of ERs in the context of Quantitative Easing (QE).

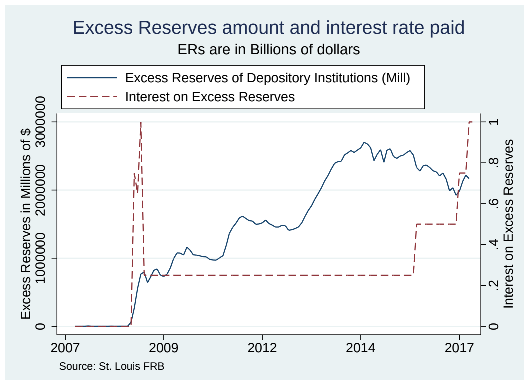

Figure 6: Quantitative Easing and IOR policy were both implemented in the last half of 2008. This figure shows the rapid increase in excess reserves After a few initial adjustments, the IOR rate was held at 25 basis points until December, 2015 and raised again December, 2016.

The first implementation of quantitative easing occurred in September, 2008, while IORs were introduced in October of the same year. At that time, excess reserves were \$1.8 billion dollars. Excess reserves reached a peak of \$2.7 trillion dollars in August, 2014, and have fluctuated around \$2.4 trillion ever since.

Other interesting changes in the behavior regarding excess reserve aggregates have also taken place since the implementation of QE. Until October, 2008, the nominal amount of excess reserves rarely increased above \$2 billion. The most notable exception was September, 2001, when it jumped to \$19 billion. However, excess reserves fell back to \$1.3 billion the next month and, for the most part, remained around or below \$2 billion. The two exceptions were August, 2003, and August, 2007, where excess reserves temporarily jumped to \$3.77 billion and \$4.8 billion, respectively. Since November, 2009, excess reserves have been around or above \$1 trillion. Figure **??** shows excess reserves in levels and Figure **??** shows the percent change. Figures **??** and **??** shows both ER and RR as a percentage of TR. With few exceptions, RRs made up between 97% and 99% of TRs. Again, the brief but notable exception was the year 2001 when RRs made up only 66% of TRs. We can

Figure 7: Excess Reserves increase from 1.9 million to 2.7 billion from August 2008 to August 2014.

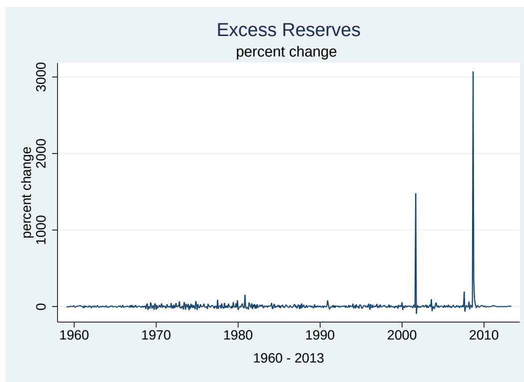

Figure 8: There are two significant changes in the percent change of excess reserves. The first occurred during the month of September in 2011. The most significant occurred with the implementation of QE 1 in September, 2008.

see that in the year 2008 is when ERs went from 4% of TRs to over 90% of TRs and have vacillated between 91% and 95% ever since.

Historically, the monetary base has been smaller than the M1 money supply. Before 2008, the amount of excess reserves within the monetary base was practically zero. Despite the significant increase in the monetary base by just under 380%, M1 has not increased at the same pace. The reason for this is because over half of the monetary base is composed of excess reserves. In fact, since 2008 the monetary base is now larger than M1. The base is not larger than M2, however. The behavior of the base and M1 are presented together in Figure **??**. The money multiplier can be calculated by dividing the money supply by the monetary base. Figure **??** shows how there was a significant decrease in the M1 money multiplier during the last half of the year 2008 and has remained below one

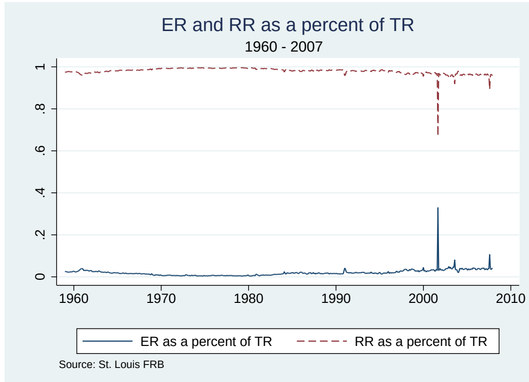

Figure 9: ER and TR as a percent of TRs, 1960-2007.

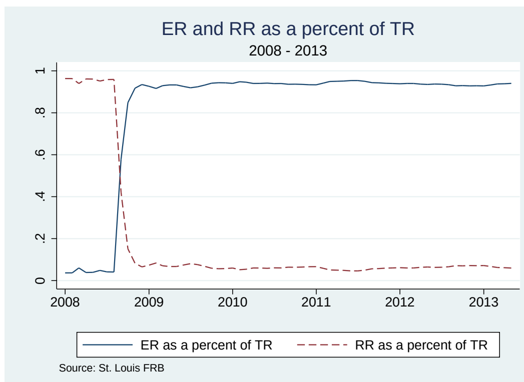

Figure 10: ER and TR as a percent of TRs, 2008-2013.

ever since. The large increase in the excess reserve ratio caused the money multiplier to decrease quickly. In contrast to the excess reserve ratio, a decrease in the currency reserve ratio causes the money multiplier to increase. Thus, the money multiplier would have been even lower if the currency ratio had not also decreased. Total Reserves also surpassed total checkable deposits as shown in Figure **??**. As the money supply increased while lending decreased, the excess reserve ratio increased, which is common during recessions as banks consider lending risky during recessions. Also, bank regulation contributed to the decrease in lending after 2008. Furthermore, low interest rates provided banks with a disincentive for lending –especially while the Federal Reserve was paying IORs. At the same time, the interest rate on riskless securities decreased to almost zero such that the opportunity cost of holding excess reserves became marginal. As a result of these factors, the excess reserve ratio increased dramatically in 2008 from essentially zero and has fluctuated between 1.5 and just under 3 ever since. The excess reserve ratio is shown in Figure **??**. The currency ratio is also included for comparison purposes. Even though we see abnormal behavior in

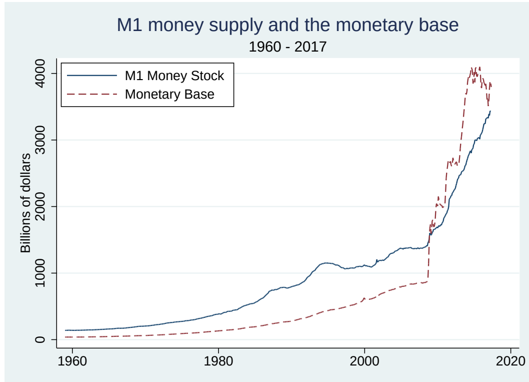

Figure 11: M1 money stock and the monetary base.

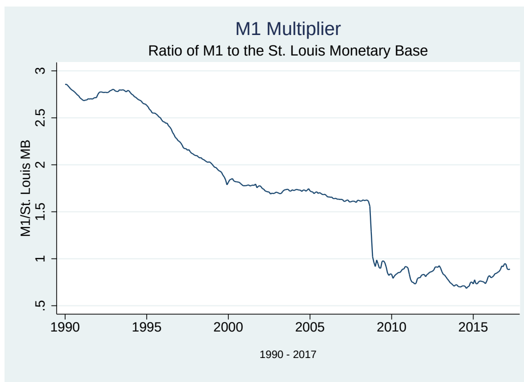

Figure 12: Theoretically, increasing the money supply is supposed to increase economic activity. When the money supply increases faster than the pace of lending, we can expect the money multiplier to decrease. That is exactly what we see in this graph

the currency ratio post-2008, the decrease in the currency ratio is explained by the use of debit cards rather than because of any type of monetary policy.

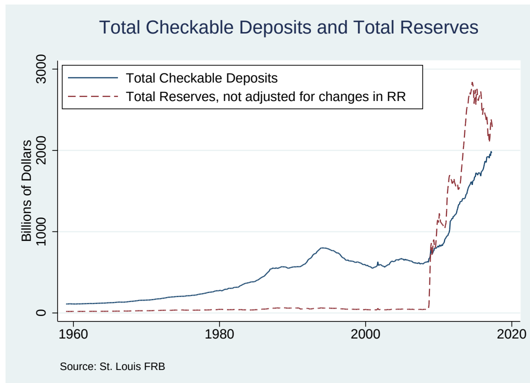

Figure 13: As a result of QE 1, total reserves actually surpassed the amount of checkable deposits.

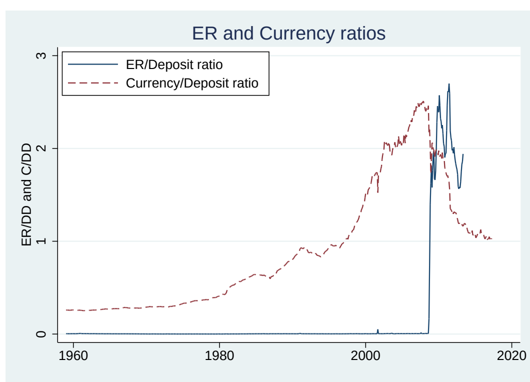

Figure 14: The excess reserve ratio increased by a factor of 1,000 because of the implementation of QE and has fluctuated around two ever since.

2.4 FR policy in practice since 2008

Before QE, traditional OMOs were effective at manipulating interest rates because of a scarcity of reserves in the federal funds market. However, this has not been the case since the implementation of QE. As we saw above, by December 2014 there were \$2.6 trillion of reserves in the banking system. The bulk of which were excess reserves. Other tools will therefore be necessary until the federal funds market can be drained of the plethora of Long Term Assets (LTAs). The process of returning to a pre QE state is referred to as "normalization," which the FOMC said it will do gradually. This is because the FOMC wants to focus on the ffr rather than the quantity of reserves. An aggressive sell-off of LTAs would distort money markets by creating unintended and unpredictable consequences. See Frost, Logan, Martin, McCabe, Natalucci, and Remache, 2015.

Beginning in December 2008, the FOMC declared a target range of 0-25 basis points instead of a specific target rate. As the FR began to raise the ffr in December 2015, the FOMC continued to target a range with a 25 point basis point spread. The process of increasing interest rates after QE has been named "liftoff."

A few months after IOR policy was implemented, the IOR rate has been set to the upper bound of the target range. Since "liftoff" began in December 2015, the ON RRP rate has been set to the lower bound of the target range. The FOMC intends to temporarily offer ON RRPs as a complementary tool to IORs until ON RRPs are no longer needed to support the ffr. Eventually IORs will become the sole tool for maintaining a floor for the ffr (Frost et al., 2015).

The 25 basis point spread between the two rates intentionally encourages arbitrage in order to increase the ffr. As can be seen in Figure **??**, the ffr consistently stays within the target range. Unlike what theory described above predicts, the IOR rate has not provided a floor. This is because arbitrage has not been complete. There are a few reasons for this. One reason is that participants who can earn IORs are only a subset of those who can lend in the federal funds market i.e. government sponsored enterprises and money market funds. Moreover, banks already have a plethora of excess reserves and there are costs associated with arbitrage. Adding additional reserves increases the required amount of FDIC insurance and requires banks to hold more capital.

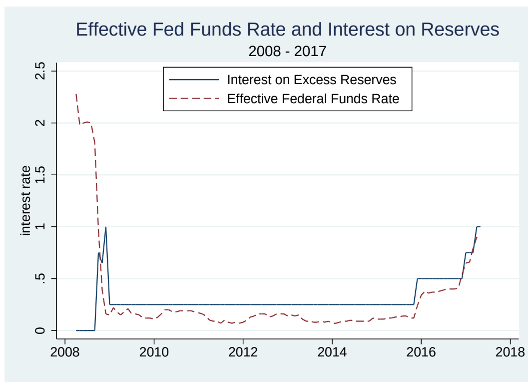

Figure 15: IORs are considered a interest rate floor for the federal funds market. To date, this has empirically not been the case. We can see that the ffr rate has consistently remained below the IOR rate since its implementation

3 DSGE Model

Heretofore, our discussion has described the experience of IOR policy since its inception in 2008. Therefore, we have analyzed IORs in the context of QE. The next step is to create a model that can describe IORs as an alternative or complementary monetary policy tool to OMOs. We would like to study the effectiveness of IORs in a state without QE. This section develops a DSGE cash-in-advance (CIA) model. The model in this paper follows closely Nason and Cogley (1994) and Schorfheide (2000).

There are three sectors in this hypothetical economy: the household, the firm, and the banking sectors. The monetary authority is a minor fourth agent in this scenario. Firms and banks are owned by the households and therefore pay dividends to the households. Households choose to hold money and how much money to deposit in their interest-bearing deposit accounts. Firms are perfectly competitive. The parameter $\beta \leq 1$ is the time discount factor. A relatively low value of β implies a low present value for the respective sectors.

The modification I make to the original model is that I include excess reserves in the bank's balance sheet. In addition, I also include a default rate for the loans that are made to firms. An additional assumption I make is that the central bank follows a standard form of the Taylor rule. Also, firms borrow money from the bank in order to rent capital in each time period. This is in contrast to Nason and Cogley (1994), where firms borrow money in order to pay workers' wages.

In section 4, there is a deterministic shock to the interest rate on reserves in order to see the impact throughout the macro-economy. Section 5 examines a stochastic model where the Taylor rule simultaneously determine the IOR rate, the deposit rate and the benchmark policy rate. The shocks are applied to technology, the money supply, and the Taylor rule.

Households

An infinitely lived, representative household maximizes its expected utility by choosing the optimal path of consumption spending, c_t; the amount it holds as bank deposits, d_t; and how much labor to supply; h_t. At the beginning of each time period, the household receives the money stock from the previous time period and choose how much to keep as deposits. The household solves the expected utility function described by

$$\max_{\{c_t\}_{t=0}^{\infty},\{H_t\}_{t=0}^{\infty},\{M_{t+1}\}_{t=0}^{\infty},\{D_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \ [(1-\psi)\ ln\ c_t + \psi\ ln\ (1-h_t) \right\}, \quad 0 < \beta,\ \psi < 1$$

subject to two constraints. The first is the CIA constraint:

$$P_t c_t \leq W_t h_t + M_t - d_t, \quad 0 \leq d_t.$$

The price level of consumption is P_t, and W_t is the wage rate in nominal terms. This constraint is specified so that cash minus deposits from the end of the previous time period plus labor wages can be used for consumption spending in the current time period. Since deposits can never be negative, we included the qualifier that $0 \leq d_t$. The second constraint is the households resources constraint. Money carried into the next time period is a function of current period dividend income from firms and banks, interest earned on deposits, income from supplying labor, and current money holdings net of current period deposits and consumption spending. The intertemporal budget constraint is thus

$$M_{t+1} \leq f_t + b_t + RH_t d_t + W_t h_t + M_t - d_t - P_t c_t,$$

where f_t and b_t are dividend income from firms and banks, respectively. The gross nominal interest rate that households earn from holding deposits is RH_t.

Banks

The objective of the representative bank is to maximize the dividends, b_t, that it pays to the households over time. Dividends are discounted by $t+1$ to reflect that the marginal utility of consumption by households take place in the time period after the dividend payments are made. The problem that banks solve is:

$$\max_{\{b_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{d_t\}_{t=0}^{\infty}, \{ER_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^{t+1} \frac{b_t}{c_{t+1} P_{t+1}} \right\},$$

subject to three constraints. The first constraint is the banks budget constraint

$$b_t \leq RF_t l_t (1 - \eta(l_t^{\upsilon})) - l_t(1 - \eta(l_t^{\upsilon})) + d_t - RH_t d_t + R_{ior} ER_t,$$

where l_t are the loans that banks make to firms, and RF_t is the interest rate that firms must pay on those loans. A fraction of the loans are never paid back. The default rate of loans is η, where $\eta = l^{\upsilon}, \upsilon > 1$. This implies that as the amount of lending increases, banks lend to riskier borrowers at an increasing rate and the wedge between the interest rates becomes larger. I assume that $RF_t > R_{ior}$ since commercial banks would not have an incentive to lend otherwise. In addition, ER_t are the banks' excess reserves, and R_{ior} is the interest rate that the central bank pays the bank for holding reserves.

Because a bank's liabilities must be less than or equal to its assets, the banks balance sheet is its second constraint:

$$d_t \leq l_t(1 - \eta(l_t^{\upsilon})) + ER_t.$$

Since we assume that banks do not hold capital, the inequality becomes an equality. The zero profit condition for the bank is determined by the equilibrium path

$$RH_t d_t = RF_t \cdot l_t + R_{ior} ER_t.$$

The amount of reserves that banks want to hold is determined by solving optimization problem subject to the bank's balance sheet:

$$\begin{aligned} E(R) &= ER \cdot R_{ior} + l \cdot (1 - \eta(l)) \cdot RF_t \\ s.t.\ d &= ER + l. \end{aligned}$$

The default rate on loans is η, which is a function of the shock parameter to the default risk, ϕ. Thus, it is the banks that internalize the default risk shock parameter. The first derivative of η with respect to loans implies the probability of default, η' .

$$\eta = \phi \cdot l^{\upsilon}, \quad \upsilon > 1 \tag{2}$$

$$\eta' = \upsilon \cdot \phi \cdot l^{\upsilon - 1} \tag{3}$$

The quadratic term ν implies defaults increase at an increasing rate as banks lend to the least risky borrowers first. As the amount of lending increases, banks lend to riskier borrowers at an increasing rate. Also, $RF_t > R_{ior}ER_t$ since commercial banks would not have an incentive to lend otherwise.

Rearrange the balance sheet so that $ER = d + l$ and then substitute into the objective function.

$$E[R] = (d - l) \cdot R_{ior} + l \cdot (1 - \eta(l)) \cdot RF$$

Take the FOC with respect to loans and solve for the optimal level of loans:

$$\begin{aligned} 0 &= -R_{ior} + (1 - \eta(l)) \cdot RF - l \cdot \eta'(l) \cdot RF \\ 0 &= -R_{ior} + (1 - \phi \cdot l^{\upsilon}) \cdot RF - l \cdot \upsilon \cdot \phi \cdot l^{\upsilon - 1} \cdot RF \\ R_{ior} &= RF - RF \cdot \phi \cdot l^{\upsilon} - RF \cdot \upsilon \cdot \phi \cdot l^{\upsilon} \\ RF - R_{ior} &= RF \cdot \phi \cdot l^{\upsilon} + RF \cdot \upsilon \cdot \phi \cdot l^{\upsilon} \\ &= RF \cdot l^{\upsilon} \cdot \phi(1 + \upsilon) \end{aligned}$$

Solving for the optimal level of loans:

$$\begin{aligned} l^{\upsilon} = \frac{RF - R_{ior}}{RF \cdot \phi(1+\upsilon)} \rightarrow l &= \left(\frac{RF - R_{ior}}{RF \cdot \phi(1+\upsilon)} \right)^{\frac{1}{\upsilon}} \\ &= \left(\frac{1}{\phi(1+\upsilon)} - \frac{R_{ior}}{RF \cdot \phi(1+\upsilon)} \right)^{\frac{1}{\upsilon}} \\ &= \left(\frac{1}{\phi(1+\upsilon)} \left\{ 1 - \frac{R_{ior}}{RF} \right\} \right)^{\frac{1}{\upsilon}} \\ &= \left(\frac{1}{\phi(1+\upsilon)} \right)^{\frac{1}{\upsilon}} \left\{ 1 - \frac{R_{ior}}{RF} \right\}^{\frac{1}{\upsilon}} \end{aligned}$$

$$\begin{aligned} &\equiv \quad \Xi \cdot \left\{ 1 - \frac{R_{ior}}{RF} \right\}^{\frac{1}{\upsilon}} \\ &\equiv \quad \chi. \end{aligned}$$

Substitute the optimal level of loans back into the balance sheet in order to determine the optimal level of excess reserves:

$$ER = d - \chi$$

Firms

Firms attempt to maximize the dividends they pay to households over time analogous to the bank. In addition, a firm chooses how much dividends to pay and how much capital to accumulate during each time period. The firm's choice variables are dividends, next period's capital stock, how much labor to hire, and the amount of loans. Furthermore, the firm faces a trade-off between increasing dividend payoffs and its capital accumulation. Just like in the case for banks, dividends are discounted by $t+1$. The other choice variables are loans, deposits, and excess reserves. The firm's objective function is

$$\max_{\{f_t\}_{t=0}^{\infty}, \{k_{t+1}\}_{t=0}^{\infty}, \{n_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^{t+1} \frac{f_t}{c_{t+1} P_{t+1}} \right\},$$

subject to three constraints. The budget constraint of the firm is

$$f_t + RF_t \, l_t(1 - \eta(l_t^{\upsilon})) + W_t n_t - l_t(1 - \eta(l_t^{\upsilon})) \leq P_t[y_t - i_t],$$

where gross investment is described by the law of motion of capital:

$$i_t = k_{t+1} - (1 - \delta)k_t, \quad 0 < \delta < 1,$$

and the firms output is produced with a CRS production function:

$$y_t = k_t^{\alpha}[A_t n_t]^{1-\alpha}, \quad 0 < \alpha < 1.$$

The second constraint that the firm faces is that it must finance its current capital costs by borrowing from the bank, such that

$$R_t k_t \leq l_t \ (1 - \eta(l_t^{\upsilon})). \tag{4}$$

This constraint says that capital costs, the rental rate times the amount of capital, must be less than or equal to the loan from the bank minus the loans that were defaulted.

Central Bank

The central bank follows the Taylor rule when choosing the interbank lending rate, i.e., the federal funds rate, (ffr). The monetary authority responds to a convex combination of a GDP gap and inflation gap. The ffr is represented as

$$RH_t = ffr_t = \Phi \cdot RH^{*}_{t-1} + (1-\Phi) \cdot [\phi_y \cdot (y_t - y^{\star}) + \phi_\pi \cdot (\pi_t - \pi^{\star})], \tag{5}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate, $\pi^{\star}$ is the inflation target, $(y_t - y^{\star})$ is the GDP gap, $(\pi_t - \pi^{\star})$ is an inflation gap, and R^{*} is the equilibrium rate of interest. The persistence of the inflation target is reflected in Φ. In this model we assume that the monetary authority sets the interest on reserve rate equal to the federal funds rate, and commercial banks pay interest on deposits, RH, equal to the federal funds rate.

3.1 Equilibrium

Because not all loans to firms are paid back to the bank, there is a default wedge of $1-\eta(l^v)$ between the interest paid on deposits and the interest charged for loans. As a result, the interest rate that firms pay for loans is higher than the interest rate that households receive from holding deposits to account for the difference in risk. In equilibrium,

$$RF_t(1-\eta(l^v_t)) = RH_t. \tag{6}$$

Equation (2.4) tells us that a bank must charge a higher rate on loans than it pays deposits in order to make a normal profit. The interest rate that households receive on their deposits equals interest paid on reserves:

$$RH_t = R_{ior}. \tag{7}$$

From the last two optimality conditions, we can write

$$RF_t(1-\eta(l^v_t)) = RH_t = R_{ior} \equiv R. \tag{8}$$

We restrict $R \leq RF$ in order to reflect a financial friction that results from default loans. Labor supply equals labor demand in the labor market

$$h_t = n_t.$$

Equilibrium in the money market is described by

$$P_t c_t = M_t.$$

which is the equation of exchange where velocity is equal to one. Equilibrium in the goods market implies that consumption and investment spending is equal to aggregate output

$$c_t + k_{t+1} - (1-\delta)k_t = k^{\alpha}_t [A_t n_t]^{1-\alpha}.$$

3.2 Optimality

The first Euler equation in this model describes optimality in the goods market

$$E_t \left\{ \frac{P_t}{c_{t+1}P_{t+1}} = \beta \frac{P_{t+1}[\alpha k_{t+1}^{\alpha-1} A_{t+1}\, n_{t+1}^{1-\alpha} + (1-\delta)]}{c_{t+2}P_{t+2}} \right\}.$$

Solving for the loan rate R in equation (3), we get the borrowing constraint of the firm

$$R_t = \frac{l_t\ (1-\eta(l_t^v))}{k_t}.$$

Equating the supply of labor, the demand for labor, and the marginal rate of substitution between consumption and leisure, the optimality condition for the intratemporal labor market becomes

$$\left(\frac{-\psi}{1-\psi} \right) \frac{c_t P_t}{1-n_t} + W = 0.$$

The second intertemporal Euler equation describes optimality in the credit market:

$$\frac{1}{c_t P_t} - \beta R_t E_t \left\{ \frac{1}{c_{t+1}P_{t+1}} \right\}.$$

The credit market is in equilibrium when the nominal interest rate equals the marginal product of capital. Thus,

$$RF_t = P_t \alpha k_t^{\alpha-1} A_t^{1-\alpha} n_t^{1-\alpha}.$$

3.3 Stochastic Detrending

There are two sources of nonstationarity in this model. The first source is that real variables, with the exception of labor, increase along with technology, A_t. The second source of nonstationarity results from the growth of the nominal variables: the money supply, M_t, and prices, P_t. In this section, we stochastically detrend the variables to make them stationary so that we are able to solve for various steady states in the model.

The notation for a detrended, stationary variable is to include a hat over the variable. The operation for the stochastic detrending of real variables is $\hat{q}_t = \frac{q_t}{A_t}$, where q_t is equal to $[y_t\ c_t\ i_t\ k_{t+1}]$. We can apply this stationarizing process to the equation for output. Let $\hat{y}_t = \frac{y_t}{A_t}$ and $\hat{K}_t = \frac{k_t}{A_t}$ be the detrended variables of output and capital, respectively. It therefore follows that $\hat{k}_t = \frac{k_{t-1}}{A_{t-1}}$. The detrended, stationarized output equation is then derived by

$$\frac{y_t}{A_t} = \left(\frac{k_{t-1}}{A_{t-1}} \right)^{\alpha} A_t^{1-\alpha} n_t^{1-\alpha} A_t^{-1} A_{t-1}^{\alpha},$$

which can also be written as

$$\hat{y}_t = \hat{k}_{t-1}^{\alpha} n_t^{1-\alpha} \left(\frac{A_t}{A_{t-1}} \right)^{-\alpha}$$

The last step is to take the exponential of the motion of technology so that we can rewrite the aggregate production function as:

$$\hat{y}_t = \hat{k}_{t-1}^{\alpha} {n_t}^{1-\alpha}\ exp(-\alpha(\gamma + \epsilon_{A,t})).$$

When we apply this stochastically detrending process to the aggregate resource constraint, the transformed equation is

$$\hat{c}_t + \hat{k}_{t+1} = exp\{-\alpha[\gamma + \epsilon_t]\}\hat{k}_t^{\alpha} n_t^{1-\alpha} + (1-\delta)exp\{-[\gamma + \epsilon_t]\}\hat{k}_t.$$

Similarly, prices are detrended by $\hat{P}_t = \frac{P_t A_t}{M_t}$, where Q_t is equal to $[y_t\ c_t\ i_t\ k_{t+1}]$, and the nominal aggregate variables are detrended, such that $\hat{Q}_t = \frac{Q_t}{M_t}$, where Q_t is equal to $[d_t\ l_t\ m_t\ ER_t]$. Thus, the money market equilibrium with the stochastically detrended variables becomes

$$\hat{P}_t \hat{c}_t = m_t,$$

and the detrended equilibrium credit market equation after transformation is now

$$\hat{d}_t = \hat{l}_t(1 - \hat{\eta}(\hat{l}_t^{\,\upsilon})) + \widehat{ER}_t.$$

3.4 Equilibrium and First-Order Conditions

Once we maximize the functions discussed above, we get the first-order conditions and equilibrium conditions. All of the stationarized equations are listed here:

$$E_t\left\{\frac{\hat{P}_t}{\hat{c}_{t+1}\hat{P}_{t+1}} = \beta\frac{\hat{P}_{t+1}[\alpha\hat{k}_{t+1}^{\alpha-1}A_{t+1}\,n_{t+1}^{1-\alpha} + (1-\delta)]}{\hat{c}_{t+2}\hat{P}_{t+2}}\right\} \tag{9}$$

$$R_t = \frac{\hat{l}_t\ (1 - \eta(\hat{l}_t^{\upsilon}))}{\hat{k}_t} \tag{10}$$

$$\frac{\psi}{1-\psi}\left[\frac{\hat{c}_t\hat{P}_t}{1-n_t}\right] = \hat{W}_t \tag{11}$$

$$R_t = \hat{P}_t \cdot \alpha \cdot \hat{k}_t^{\alpha-1} A_t^{1-\alpha} n_t^{1-\alpha} \tag{12}$$

$$\frac{1}{\hat{c}_t\hat{P}_t} = \beta\frac{(1-\alpha)\hat{P}_t e^{-\alpha(\gamma+\epsilon_{A,t+1})}\hat{k}_{t-1}^{\alpha} n_t^{1-\alpha}}{E[\hat{l}_t m_t \hat{c}_t \hat{P}_{t+1}]} \tag{13}$$

$$\hat{c}_t + \hat{k}_{t+1} = exp\{-\alpha[\gamma + \epsilon_t]\}\hat{k}_t^{\alpha} n_t^{1-\alpha} + (1-\delta)exp\{-[\gamma + \epsilon_t]\}\hat{k}_{t-1} \tag{14}$$

$$\hat{P}_t\hat{c}_t = m_t \tag{15}$$

$$\hat{d}_t = \hat{l}_t(1 - \eta(\hat{l}_t^{\upsilon})) + \widehat{ER}_t \tag{16}$$

$$RF_t(1 - \eta(l_t^{\upsilon})) = RH_t = R_{ior} \equiv R \tag{17}$$

$$\hat{y}_t = \hat{k}_{t-1}^{\alpha} {n_t}^{1-\alpha}\ exp(-\alpha(\gamma + \epsilon_{A,t})) \tag{18}$$

$$\widehat{ER}_t = \Theta \cdot (RF_t - R_{ior}) \tag{19}$$

Here we describe the above FOCs, equations (8) - (13), and the equilibrium conditions, equations (14) - (18). The FOCs are derived in detail in the appendix. Note that the hats above certain variables are the variables that have been transformed into stationary ones. Equation (8) is the Euler equation for the goods market. This equation shows the intertemporal consumption over time. Equation (9) shows the firm's borrowing constraint. As described above, the firm must borrow from the bank in order to pay its current capital costs. Equation (10) combines the supply and demand for labor, as well as the optimal allocation of labor over time.

The equilibrium interest rate is expressed by equation (11). The interest rate is determined by the marginal revenue product of capital. Equation (12) describes the Euler equation in the credit market. This equation tells us the net present value of future consumption that results from current savings. Equation (13) is the resource constraint for the economy, where aggregate production is equal to consumption and net capital spending.

The remaining equations are the equilibrium conditions. The money market equilibrium condition is equation (14). Aggregate demand is represented by nominal consumption, $P_t c_t$, and money banks want to hold as excess reserves, ER, which are equal to the nominal money supply, m. Equation (15) is the bank's balance sheet. The bank's liabilities, which are its deposits, are equal to the bank's assets, loans plus excess reserves. Equation (16) shows the wedge between the interest rate that firms pay and the interest rate that deposits receive. Moreover, interest on deposits equals the interest on reserves. The firm's production function produces aggregate output, Y, in equation (17). The demand for excess reserves rule for banks is reflected in equation (18).

4 Deterministic Model

In the deterministic model, there are no unpredictable stochastic shocks. This is analogous to the Federal Reserve's forward guidance policy, where the FOMC commits to long term policy actions. The calibrated values are standard quarterly values from the literature intended to reflect US data. Empirically, the labor share of income is approximately 2/3 and the capital share of income is the remaining 1/3. Thus, α is set to 0.33. The discount factor is commonly set to 0.99, which implies that the interest rate averages 4 percent annually. The leisure weight on utility, ψ, is set at 0.65. This parameter reflects the leisure-labor ratio and implies that about 1/3 of each day is allocated to work. Depreciation averages 4 percent per annum, which is equal to 20 percent each quarter. Thus, δ is calibrated to 0.01. Technology growth is calibrated to 0.003 per quarter. I set the weights of the excess reserve demand coefficints to 0.5 each so that $\Theta = 0.05$. The parameters for the money growth process are $\bar{m}$ and ρ. These values are calculated by applying an AR(1) process to the monetary base (Schorfheide, 2000). An annual money growth rate of 4.1 percent implies a quarterly growth rate of $\bar{m} = 1.011$.

The remaining parameters are the smoothing parameters included in the Taylor rule. The Federal Reserve commonly sets it inflation target rate at 2 percent annually. So the quarterly target rate is set to $\phi_\pi = 0.005$. The parameters ϕ_π and ϕ_y are set to 0.5, which implies that the FR places equal importance on eliminating the inflation gap and the GDP gap. The parameter Φ is a smoothing coefficient for the policy benchmark rate and is calibrated to 0.80. The values of the parameters are summarized in Table **??**.

Table 1: Parameter Values

Parameter	Value	Description
α	0.330	capital share
β	0.990	discount factor
γ	0.003	Average technology growth
$\bar{m}$	1.011	Average money growth rate
ρ	0.02	Autocorrelation money process
ψ	0.787	Leisure weight in utility
δ	0.020	depreciation
Φ	0.800	Inflation Target Persistance
ϕ_{pi}	0.500	inflation gap weight
ϕ_y	0.500	GDP gap weight
$\pi^\star$	0.005	inflation target

4.1 Shock to Interest on Reserve Rate

The monetary authority increases the IOR rate, and the rate change is expected by the households, firms and banks. The IOR rate is equal to the deposit rate, and the loan rate is a fixed rate above the deposit rate. The difference between the loan rate and the other two interest rates is equal to the default rate. In this scenario, the default rate initially is .007 percent. Thus, since the initial loan rate is 6.3 percent, the deposit rate and IOR rate are both 4.6 percent. The exogenous shock is implemented in t=20 when the monetary authority increases the IOR rate by 1%. In equilibrium, the deposit rate increases to the same amount and the loan rate also increases to the deposit rate plus the amount of the default wedge. These three interest rates, before and after the deterministic-exogenous shock, are shown in Figure 16 and the pre and post shock steady state values for all of the variables are shown in Table **??**.

Table 2: These are the pre and post steady states of the *deterministic* model.

variable	$Initial^*$	$NewSteadyState^\dagger$
$ior^\star$	1.046	1.056
m	1.011	1.011
P	2.19121	2.20836
c	0.461388	0.457806
ER	0.314696	0.317726
W	4.56493	4.55707
Rf	1.05377	1.06385
k	6.47879	6.4285
d	1.16729	1.17032
n	0.1817	0.18029
l	0.858926	0.858926
y	0.590964	0.586376
π	1	1
ffr	1.5335	1.5435
η	0.007377	0.007377

*Shock takes place at t = 20.
†after 2000 time periods.
$\star$ Exogenous Variable

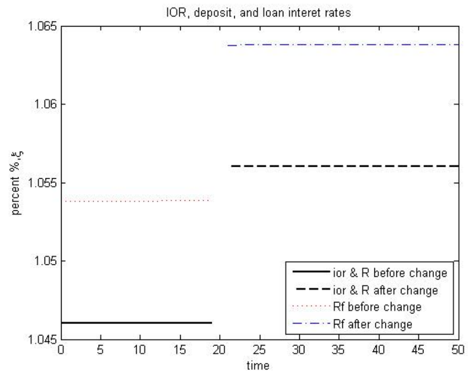

Figure 16: Interest Rates: deposit, loan, and IORs

The variables describing the equation of the bank's balance sheet, $d_t = l_t \cdot (1 - \eta(l_t^{\upsilon})) + ER_t$, is shown in Figure **??**. As can be expected, excess reserves experience a once time increase immediately after the IOR rate is increased. Banks adjust their excess reserves according to $ER_t = \Theta \cdot (RF_t - R_{ior})$, where $\eta = l_t^{\upsilon}, \upsilon > 1$ since defaults are an increasing function of loans as was explained above. That is, banks initially lend to the less risky borrowers. As lending increases, the relatively more risky borrowers receive bank loans.

As a result of the increase in the IOR rate, excess reserves increase at the same time from .31469 to .317726. Before the shock, the amount of loans gradually increases from the time of the original steady state of .5949 as firms anticipate the interest rate increase. At the time of the shock, the amount of loans significantly decreases from .8633 to .8542. Next, the amount of loans gradually increases again until it reaches a lower steady state of .5589 at time period t=105. The behavior of deposits over time is shown in panal 3. Deposits fall at the time of the shock despite the higher deposit rate. This is counter-intuitive.

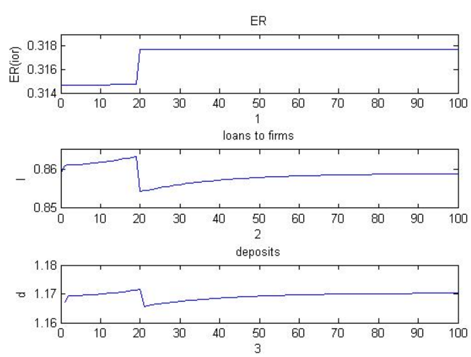

Figure 17: Bank's dynamic balance sheet

The variables that compose the demand for loans, $R_t k_t = l_t \cdot (1 - \eta(l_t^v))$, are shown in Figure 18. Capital and loans for capital initially increase as firms anticipate the higher, future borrowing costs. When the interest rate on loans increases, the higher borrowing costs cause the capital stock to begin to fall. The amount of loans jumps down immediately. However, capital cannot jump like the other aggregate variables that we are analyzing since capital follows a law of motion. That is, current capital is a function of last periods capital stock, and the depreciation rate. The default rate is an increasing function of loans. The time path of capital, loans to firms, and the default rate are presented in Figure **??**.

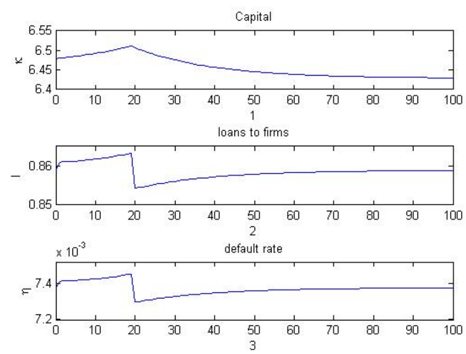

Figure 18: Equilibrium in the loans for capital market

As the demand for labor decreases, the wage rate falls and thus the labor supply decreases. Figure **??** shows labor and wages initially increasing until the time of the shock. Once interest rates increase, labor decreases from .1825 to .1795, and wages fall from 4.5691 to 4.5526. Both variables then gradually increase to their new, lower steady states.

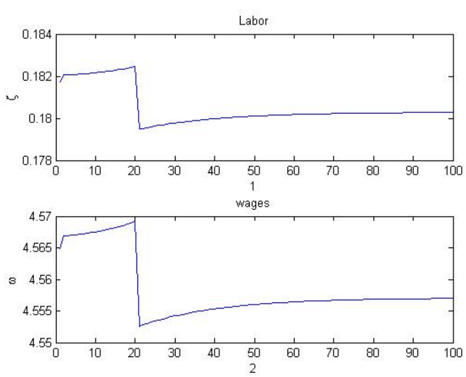

Figure 19: Labor Market

The behavior of the variables in the money market equilibrium equation, $P_t c_t = M_t$, are shown in Figure **??**. In this model, there are no monetary injection or any other type of monetary policy that affects the money supply. Thus, the money supply remains constant for all time periods as shown in panel 1. Panels 2 and 3 show the price level and consumption respectively. At the time of the shock, the price level gradually increases to its new steady state. As the price level increases, consumption gradually decreases to it new, lower steady state.

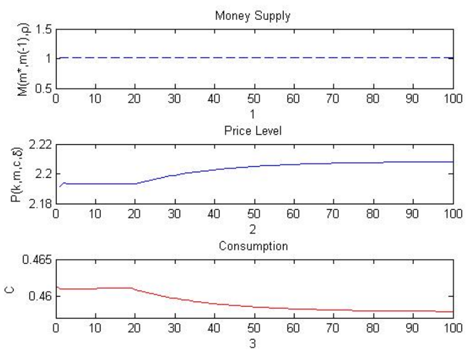

Figure 20: Dynamic behavior of the money market market equilibrium.

Figure **??** represents the resource constraint (13), and shows the relationships among output, consumption, and capital. After the shock, output jumps down and then slowly decreases even more to its new steady state. Consumption and Capital slowly decrease over time to their new steady state.

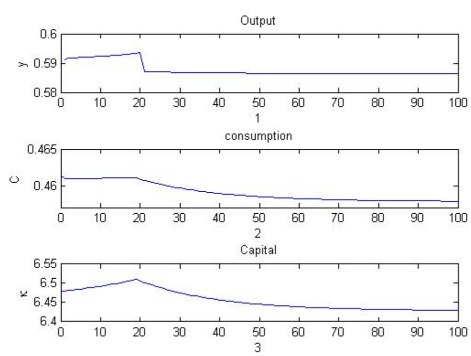

Figure 21: The resource constraint is $\hat{c}_t + \hat{k}_{t+1} = exp\{-\theta[\gamma + \epsilon_t]\}\hat{k}_t^{\theta} n_t^{1-\theta} + (1-\delta)exp\{-[\gamma + \epsilon_t]\}\hat{k}_{t-1}$

Next, we describe the relationship between the price level, the inflation rate, and the federal funds rate. At the time the agents learn about the future interest rate increase

, there is a one period price level jump. Thus, the inflation rate also jumps for just one period. At the time of the interest rate shock, the price level increases over time at a decreasing rate. Hence, the inflation rate jumps up and slowly falls as the rate of the price level increase decreases.

The monetary authority follows the Taylor Rule. The federal funds rate mimics the time path of the price level by also increasing at a decreasing rate. These three variables are show in Figure **??**.

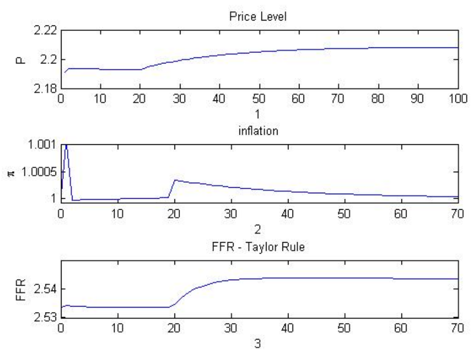

Figure 22: The Price Level, Inflation Rate, and the Federal Funds Rate. The monetary authority follows the Taylor Rule.

5 Stochastic Model

5.1 Exogenous Disturbances

In this section, we explore the reaction of agents to unanticipated shocks to the exogenous variables by utilizing a stochastic model. There are three exogenous processes: the technology shock, which is a real shock to the economy, a monetary shock, which is a nominal shock, and a shock to the monetary rule.

The technology shock evolves according to a random walk with drift:

$$lnA_t = \gamma + lnA_{t-1} + \epsilon_{A,t}, \quad \epsilon_{A,t} \sim \mathrm{N}(0, \sigma_A^2),$$

which can be rewritten as

$$lnA_t - lnA_{t-1} = \gamma + \epsilon_{A,t}.$$

Take the exponential of each side of the equation to get

$$dA_t \equiv \frac{A_t}{A_{t-1}} = exp(\gamma + \epsilon_{A,t}).$$

The growth rate of a monetary injection follows the exogenous stochastic process

$$ln\ m_t = (1-\rho)\ ln\ m^* + \rho\ ln\ m_{t-1} + \epsilon_{M,t}, \quad \epsilon_{M,t} \sim \mathrm{N}(0, \sigma_M^2).$$

This equation is interpreted as a simple monetary rule, where the growth rate of the money stock is $m_t = \frac{M_{t+1}}{M_t}$. The parameters m* and ρ imply a significant shift in the conduct of monetary policy. Schorfheide (2000) describes changes in these two parameters as reflecting "rare regime shifts." However, the shock variable implies that the monetary authority's conventional decision was unexpected.

The modification I make to the Taylor Rule in this paper's stochastic model is that the central bank sets the ior rate equal to the ffr rate. Thus, the IOR rate is now determined endogenously unlike in the previous section. The central bank follows the Taylor Rule when choosing the interbank lending rate just like it did in Section 4. However, now we add a shock variable that implies that the monetary authorities decision was unexpected. Like before, the monetary authority responds to a convex combination of a GDP gap and inflation gap. The ffr is again described by the Taylor rule from (2.3) with an additive structural shock term:

$$RH = ffr_t = \Phi \cdot RH_{t-1}^* + (1-\Phi) \cdot [\phi_y \cdot (y_t - y^\star) + \phi_\pi \cdot (\pi_t - \pi^\star)] + \varepsilon_{ffr,\ t}.$$

In this stochastic model setting, the Taylor rule now includes the serially uncorrelated innovation ε_{ffr}, which has mean zero and variance σ_{ffr}^2:

$$\varepsilon_{ffr} \sim \mathrm{N}(0, \sigma_{ffr}^2).$$

5.2 Impulse Response Functions

In this section, we demonstrate how an unanticipated structural shock passes through the model with impulse response functions. The model is perturbed by a one standard deviation impulse to the structural shocks ϵ_A, ϵ_M and, ϵ_{ffr} in the first time period. We examine the responses from the technology shock, money supply shock, and the interest rate shock.

The impulse responses function from a permanent technology shock on the endogenous variables are presented in Figure **??**. The horizontal zero line is the new steady state. A permanent technology shock results in a increase in total factor productivity (TFP) and therefore in aggregate output. The higher level of output is divided between the consumption good and capital production. A higher level of aggregate output lowers the price level and thus the inflation rate. The lower price level also stimulates consumption spending. Because of the increase in consumption spending, there is less deposits and therefore less excess reserves.

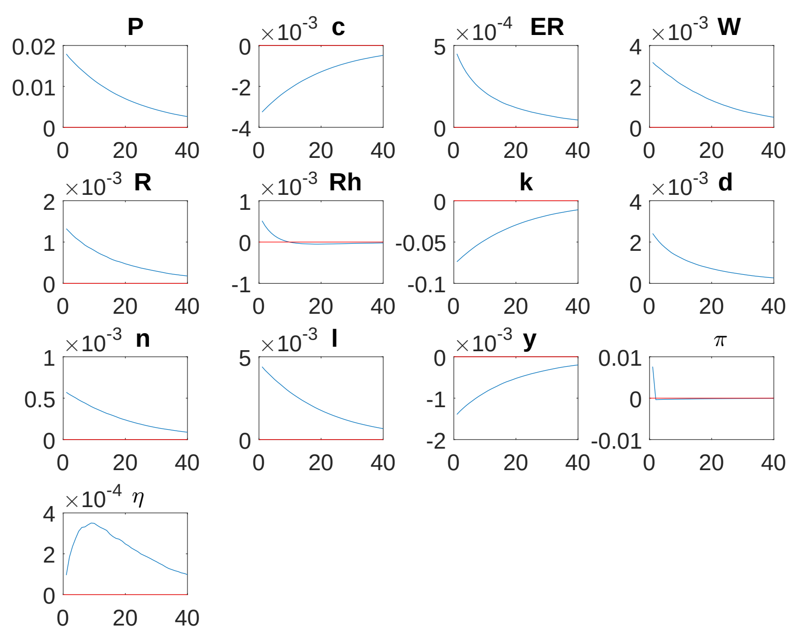

Figure 23: Orthogonalized shock to ϵ_A. The horizontal line at zero is the new steady state that results from a permanent shock to technology.

Moreover, a decrease in the price level is associated with relatively lower interest rates.

Despite the lower lending rate, firms borrow less since the increased TFP increases capital accumulation. In addition, TFP decreases the demand for labor so the wage rate falls. The decrease in the wage rate then prompts workers to provide less labor hours. Another observation is that the default initially increases, but than returns to its steady state as the number of loans decreases.

Figure **??** shows the impulse responses from a temporary money supply shock on the endogenous variables. The money supply, price level, and thus inflation all jump up from their steady states on impact. The money supply returns to its steady state in about 10 quarters while the price level and inflation rate fall slightly below their respective steady states. Because of the higher prices, consumption initially decreases but recovers in 10 quarters as the price level falls. Consumption remains above the steady state as long as the price level stays below its own steady state. The higher price level and inflation rate prompt the monetary authority to increase its benchmark rate. Thus, the lending, deposit, and IOR rates also increase. The higher interest rates result in an increase in the demand for excess reserves and also in an increase in deposits.

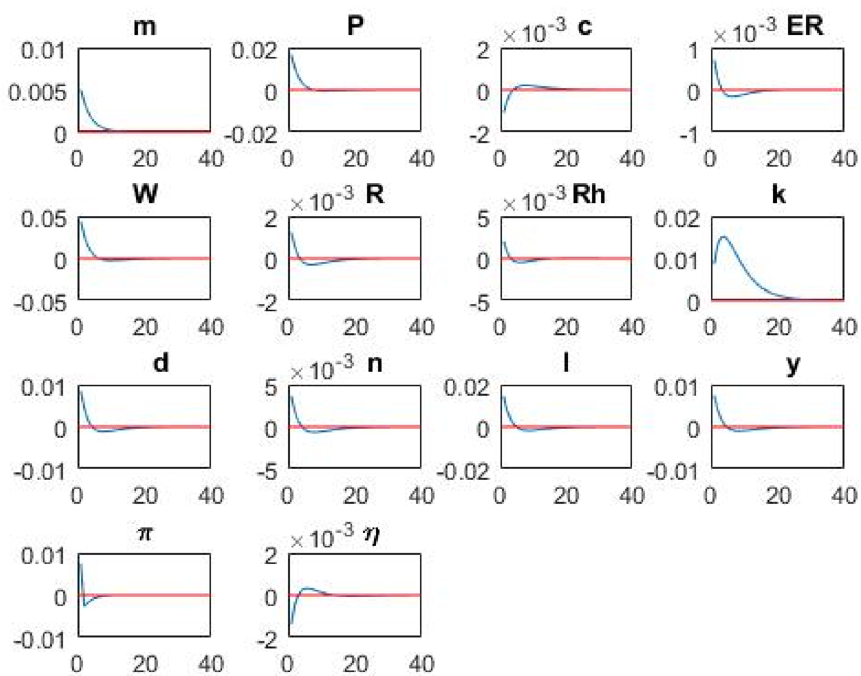

Figure 24: Orthogonalized shock to ϵ_M. The y-axis is the deviations from the steady state.

The unanticipated money shock stimulates output by the firm. The firm raises wages to attract more worker hours. Despite the higher borrowing rate, firms rent more capital so the amount of loans increases. Capital has a smooth transition because it follows the law of motion. At first, defaults on loans fall, but over time as banks lend to relatively more risky borrowers, the default rate increases.

The higher interest on reserve rate also causes an increase an excess reserves. That is, the increases in deposits are allocated between both more loans and more excess reserves. Since capital and labor are complements, labor hours increase, and aggregate output also increases.

The impulse responses from an unanticipated shock to interest rates in Figure **??**. In this scenario, the monetary authority decreases its benchmark rate which simultaneously decreases the deposit rate and the banks' lending rate. When the benchmark interest rate decreases, firms expect a lower inflation rate so decrease the price level. As the price level decreases, consumption spending increases. A lower lending rate encourages firms to borrow more in order to rent more capital. There is a smooth increase in capital accumulation as it follows the law of motion before decreasing back to its initial steady state. Output increase on impact and then falls back to its steady state. Because of the initial

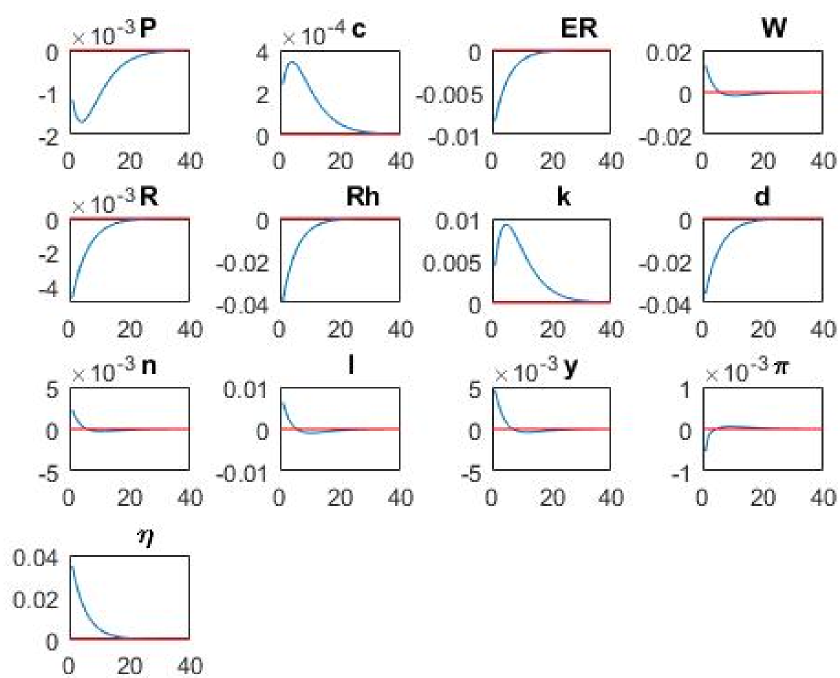

Figure 25: Orthogonalized shock to ϵ_{ffr}. The y-axis is the deviations from the steady state.

increase in production, labor demand increases since capital and labor are compliments. The increase in labor demand raises wages so that workers provide more labor hours. The increase in consumption spending in less deposits and, combines with a lower interest on reserve rate, less excess reserves.

We can now make some observation regarding these two stimulative monetary policy tools. As we saw above, an increase in the money supply raises the price level along with the deposit and lending rates. This was in contrast to the monetary authority reducing the IOR rate, which caused the lending and deposit rates to decrease along with the price level.

Moreover, the positive monetary shock also led to more deposits and an increase of excess reserves in contrast to lowering the IOR rate. Even though both tools increased output, the increase in output resulting from the money supply shock caused only an increase in capital accumulation but not an increase in consumption good production. Simultaneously, household income was deposited instead of spent on the consumption good because of the relatively higher price level. As a further result, excess reserves increased.

However, the output increase that resulted from the stimulative IOR policy increased capital accumulation as well as consumption good production. Another difference is that because of the increase in consumption spending, deposits decreased and therefore so did excess reserves. An unexpected difference is that loan defaults decreased with more lending in the money supply shock case.

In order to examine how well these IRFs coincide with empirical data, I compare correlation coefficient matrices with actual time series data with simulated model data.

6 Correlation Coefficients

Macroeconomic models can be judged based on how well they replicate correlations among time series variables. In this section, we compare the endogenous variables with each other to see how well the correlations among variables coincide with empirically correlated data. When there is a discrepancy, we will attempt to explain the difference and see where we can amend the model in order to match the data. There are assumptions about the model that are clearly at odds with reality, which is an inevitable process of model building.

I use time series data obtained from the St. Louis FRED. The variables used to compare with the model's variables are shown in Table **??**. The monthly data was transformed into quarterly data by taking a three month moving average. The H-P filter of the natural log was applied in order to remove the trend from the money supply, output, and employment. The empirical data for 1959 - December 2003 are shown in Table **??** panel A. December, 2003 was chosen as the end of this time series to avoid distortions to the variables resulting from the housing market bubble of the 2000s. The correlation coefficients for the empirical data from 2008 to the present is shown in panel B. The year 2008 is significant because of the Federal Reserves implementation of quantitative easing (QE) and its impact on

Table 3: Emprical Data Variables

variable	Description	Notes
ln_M2_hp	M2 money stock	HP log of 3-month moving average
CPIma	core consumer Price	3-month moving average
PCECC96	Real Personal Consumption Expenditures	Quarterly data
GDI	Gross Domestic Income	Quarterly data
TBillma	T-bill Treasury Bill	3-month moving average
ln_kfrm	Gross Fixed Capital Formation	log of quarterly data
DPOSma	Total Checkable Deposits	3-month moving average
EXRESma	Excess Reserves	3-month moving average
ln_n_hp	Employment Manufacturing	log of 3-month moving average
LOANSma	Loans and Leases of Commercial Banks	3-month moving average
IP_y	Industrial Production	3-month moving average
NPTLTL η	Nonperforming Loans	Quarterly; series begins in 1988.

Data source: St. Louis FRED. Original data modified by the author as explained under notes.

monetary aggregates. The correlations in panel B thus reflect the recession of 2007-2009 as well as its recovery thereafter. One caveat in regard to analyzing time series correlation coefficients is that they reflect q point in time where causation among variables experience lag effects.

The analysis begins by noting the long term relationships among the variables. We will primarily focus on the correlations in Table **??** A since these coefficients show long term trends. Since this is a monetary theory paper, we are interested in the role of money and its impact on other monetary aggregates. The money supply has an expected positive relationship with prices, consumption, income, excess reserves, loans, and output. Because of the positive relationship with loans, money also has a positive relationship with loan defaults. The money supply also has an expected negative relationship with interest rates. However, we also see an unexpected negative relationship with capital and labor. This may be because short term interest rates do not have much or any impact on these inputs.

Some other relationships we see is that wages and consumption are positively correlated, which is expected. However consumption and labor are negatively related, which is counter-intuitive. Loans and deposits are positively correlated as are excess reserves and deposits. Moreover, we see a positive relationship between capital and labor, which implies that the two inputs tend to be complements. Wages and labor are negatively related reflecting the law of demand for labor. Labor and deposits are positively correlated implying the marginal propensity to save. Also, wages and output are positively related reflecting the demand for labor. Another expected relationship is that there is a negative correlation between interest rates with both output and loans.

Output is positively related to capital and labor but inversely related to interest rates, which we expect. Though it is then difficult to explain why labor and capital are then positively related to interest rates. Similarly loans and output are positively correlated, which is intuitive. However, we unexpectedly also see a negative relationship of loans with capital and labor. Though we do see a positive relationship among loans, capital, and labor in panel B.

There are a few other observations to make regarding the differences in the empirical data correlation tables. The negative relation between the money supply and output in Table **??** B reflects QE during the 2007-2009 recession. Usually there is a positive relationship like what we see in Table **??** A. We also see a negative correlation between money and bank defaults in Table **??** B but not in Table **??** A. This is a result of the precipitous increase of mortgage defaults during QE. Another observation regarding the money supply is that there is a negative relationship with labor in **??** A. However, we except to see a positive relationship like what we see in Table **??** B.

Table 4: Correlation of Empirical Data

1960-Dec. 2003

	m	P	c	W	R	k	d	ER	n	l	y	η
	lnM2_hp	CPIma	PCECC96	GDI	TBillma	lnkfrmhp	DPOSma	EXRESma	ln_n_hp	LOANSma	IP_y	NPTLTL η
lnM2_hp M2	1.0000											
CPIma P	0.0977	1.0000										
PCECC96 c	0.1447	0.9824	1.0000									
GDI W	0.1306	0.9844	0.9936	1.0000								
TBillma R	-0.3494	-0.0880	-0.1220	-0.1655	1.0000							
lnkfrmhp k	-0.0277	-0.0555	-0.0221	-0.0361	0.1660	1.0000						
DPOSma d	0.0294	0.9471	0.8951	0.8917	-0.0879	-0.0286	1.0000					
EXRESma ER	0.2529	0.6825	0.6825	0.7042	-0.3099	-0.0701	0.6141	1.0000				
ln_n_hp n	-0.1783	-0.0262	-0.0042	-0.0039	0.3648	0.7546	-0.0327	-0.0490	1.0000			
LOANSma l	0.1666	0.9728	0.9899	0.9973	-0.1936	-0.0427	0.8668	0.7148	-0.0087	1.0000		
IP_y	0.0936	0.9635	0.9913	0.9793	-0.0578	0.0402	0.8695	0.6414	0.0683	0.9732	1.0000	
$NPTLTL^{\dagger}$ η	0.3813	-0.8190	-0.7608	-0.7706	0.3668	-0.2455	-0.2684	-0.2018	-0.0645	-0.7087	-0.8307	1.0000

Data source: St. Louis FRED
† Data series begins in 1988

(A)

2008-present

	m	P	c	W	R	k	d	ER	n	l	y	η
	lnM2_hp	CPIma	PCECC96	GDI	TBillma	lnkfrmhp	DPOSma	EXSRESma	ln_n_hp	LOANSma	IP_y	NPTLTL
lnM2_hp M2	1.0000											
CPIma P	0.1145	1.0000										
PCECC96 c	-0.0083	0.9407	1.0000									
GDI W	0.0861	0.9190	0.9797	1.0000								
TBillma R	-0.0782	-0.5776	-0.3268	-0.2496	1.0000							
lnkfrmhp k	-0.0142	0.4081	0.6389	0.6836	0.3939	1.0000						
DPOSma d	0.1524	0.9966	0.9472	0.9285	-0.5456	0.4454	1.0000					
EXSRESma ER	0.0522	0.9596	0.8750	0.8261	-0.6750	0.2342	0.9479	1.0000				
ln_n_hp n	0.1666	0.1297	0.3676	0.4663	0.5686	0.8761	0.1767	-0.0473	1.0000			
LOANSma l	0.4570	0.7312	0.7561	0.8244	-0.1968	0.4639	0.7454	0.6507	0.4444	1.0000		
IP_y	-0.1142	0.5973	0.8092	0.8322	0.2433	0.9362	0.6241	0.4588	0.7515	0.5839	1.0000	
NPTLTL η	-0.2609	-0.3285	-0.5478	-0.6451	-0.4745	-0.8207	-0.3675	-0.1931	-0.8916	-0.7026	-0.8099	1.0000

Data source: St. Louis FRED

(B)

6.1 Model Correlations

In this section, we will compare three hypothetical model scenarios to the empirical data. Correlation matrices generated by the model depicting various scenarios are presented in the next three tables. Table **??** shows correlations for when there is a positive technology and money supply shock. A positive shock to the benchmark rate is then included for the correlations in Table **??**. Table **??** presents correlations that reflect the IRFs from Figures 24 and 25 together. That is, the correlations reflect stimulative policy via a positive shock to the money supply and a simultaneous negative shock to the benchmark rate. The empirical coefficients from Table **??** A are also included for comparison purposes.

Macroeconomic theory predicts a positive relationship between the money supply and the price level. Not surprisingly, we see a positive correlation coefficient in all five tables. In addition, monetary theory predicts that there is a negative relationship between the money supply and interest rates. That is, when the monetary authority increases the money supply, interest rates will decrease. This is what the empirical tables confirm. However, in the model banks expect the monetary authority to increase then benchmark rate. Thus, banks immediately increase the deposit and lending rates. Hence we see a positive coefficient for the money supply and interest rate correlations in Tables **??** - **??**.

In the model, a determinant of excess reserves is the interest on reserve rate. Heretofore, there is not enough empirical data to confirm this relationship. The empirical tabls ostensibly show a negative relationship. Another difference between the model and the data is in regard to prices and consumption. Empirically, prices and consumption spending are positively correlated. However, in the model there is a single consumption good. So the consumption good follows the law of demand and therefore there is a inverse relationship.

There is a positive correlation between wages and consumption in the data. It is empirically true that higher wages lead to more consumption spending. But the model coefficients give conflicting results. Table 6 shows a positive relationship while Tables 5 and 7 show a negative relationship. Loans and interest rates are predictably inversely related in the data. In the model, this it true for Tables **??** and **??**, but positively related in Table **??**.

Interest rates and output are inversely related in the model and for the time series data in Table **??** A, which is expected. This is unexpectedly also true for deposits and capital. We expect a positive relationship since deposits are a proxy for for saving and savings provide funds for real investment spending such as capital.

This paper's model includes loan defaults. As the number of bank loans increases, the model predicts that the number of loan defaults will also increase. The empirical tables ostensibly imply that there is a negative relationship. This is because realistically there is a lag between when a loan is issued and the time it becomes nonperforming. The model assumes that defaults on loans take place in the same time period that they are issued.

Table 5: Correlation of Simulated Variables for the model

shocks to e_a and e_m, but not to e_{ffr}

	m	P	c	W	R	Rh	k	d	ER	n	l	y	η
m	1.0000												
lnM2_hp M2	1.0000												
P	0.2218	1.0000											
CPIma P	0.0977	1.0000											
c	0.1004	-0.9477	1.0000										
PCECC96 c	0.1447	0.9824	1.0000										
W	0.9678	0.4600	-0.1532	1.0000									
GDI W	0.1306	0.9844	0.9936	1.0000									
R	-0.0683	0.9571	-0.9987	0.1846	1.0000								
TBillma R	-0.3494	-0.0880	-0.1220	-0.1655	1.0000								
Rh	0.5944	0.6556	-0.4768	0.7079	0.5136	1.0000							
k	0.2903	-0.8684	0.9812	0.0402	-0.9735	-0.3433	1.0000						
lnkfrmhp k	-0.0277	-0.0555	-0.0221	-0.0361	0.1660		1.0000						
d	0.4557	0.9607	-0.8318	0.6624	0.8516	0.8178	-0.7117	1.0000					
DPOSma d	0.0294	0.9471	0.8951	0.8917	-0.0879		-0.0286	1.0000					
ER	0.0892	0.9759	-0.9669	0.3322	0.9772	0.6842	-0.9125	0.9264	1.0000				
EXRESma ER	0.2529	0.6825	0.6825	0.7042	-0.3099		-0.0701	0.6141	1.0000				
n	0.3285	0.9935	-0.9066	0.5556	0.9183	0.6937	-0.8083	0.9805	0.9523	1.0000			
ln_n_hp n	-0.1783	-0.0262	-0.0042	-0.0039	0.3648		0.7546	-0.0327	-0.0490	1.0000			
l	0.3688	0.9881	-0.8877	0.5909	0.9013	0.7153	-0.7822	0.9869	0.9432	0.9990	1.0000		
LOANSma l	0.1666	0.9728	0.9899	0.9973	-0.1936		-0.0427	0.8668	0.7148	-0.0087	1.0000		
y	0.6500	-0.5967	0.8210	0.4380	-0.8024	-0.0288	0.9155	-0.3750	-0.6892	-0.5036	-0.4663	1.0000	
IP_y	0.0936	0.9635	0.9913	0.9793	-0.0578		0.0402	0.8695	0.6414	0.0683	0.9732	1.0000	
η	-0.6541	0.3573	-0.5760	-0.5011	0.5413	-0.4435	-0.6806	0.0882	0.3503	0.2795	0.2405	-0.8101	1.0000
$NPTLTL^{\dagger}$ η	0.3813	-0.8190	-0.7608	-0.7706	0.3668		-0.2455	-0.2684	-0.2018	-0.0645	-0.7087	-0.8307	1.0000

Data source: St. Louis FRED

† Data series begins in 1988

Table 6: Correlation of Simulated Variables for the model

Positive shocks to e_a, e_m, and e_{ffr}

	m	P	c	W	R	Rh	k	d	ER	n	l	y	η
m	1.0000												
lnM2_hp M2	1.0000												
P	0.3944	1.0000											
CPIma P	0.0977	1.0000											
c	-0.2393	-0.9855	1.0000										
PCECC96 c	0.1447	0.9824	1.0000										
W	0.1882	-0.2015	0.2558	1.0000									
GDI W	0.1306	0.9844	0.9936	1.0000									
R	0.0863	0.3965	-0.4136	-0.9508	1.0000								
TBillma R	-0.3494	-0.0880	-0.1220	-0.1655	1.0000								
Rh	0.1017	0.3168	-0.3299	-0.9081	0.9752	1.0000							
k	-0.1313	-0.9491	0.9790	0.1198	-0.2489	-0.1578	1.0000						
lnkfrmhp k	-0.0277	-0.0555	-0.0221	-0.0361	0.1660		1.0000						
d	0.0887	0.3967	-0.4153	-0.9454	0.9980	0.9814	-0.2504	1.0000					
DPOSma d	0.0294	0.9471	0.8951	0.8917	-0.0879		-0.0286	1.0000					
ER	0.0898	0.3872	-0.4053	-0.9429	0.9971	0.9855	-0.2394	0.9997	1.0000				
EXRESma ER	0.2529	0.6825	0.6825	0.7042	-0.3099		-0.0701	0.6141	1.0000				
n	-0.0130	-0.2741	0.2974	0.9790	-0.9874	-0.9544	0.1350	-0.9834	-0.9819	1.0000			
ln_n_hp n	-0.1783	-0.0262	-0.0042	-0.0039	0.3648		0.7546	-0.0327	-0.0490	1.0000			
l	0.0788	-0.0734	0.1020	0.9735	-0.9420	-0.9285	-0.0583	-0.9378	-0.9387	0.9788	1.0000		
LOANSma l	0.1666	0.9728	0.9899	0.9973	-0.1936		-0.0427	0.8668	0.7148	-0.0087	1.0000		
y	-0.0449	-0.4604	0.4885	0.9522	-0.9925	-0.9501	0.3371	-0.9910	-0.9880	0.9775	0.9172	1.0000	
IP_y	0.0936	0.9635	0.9913	0.9793	-0.0578		0.0402	0.8695	0.6414	0.0683	0.9732	1.0000	
η	-0.0948	-0.3615	0.3780	0.9313	-0.9911	-0.9947	0.2093	-0.9958	-0.9976	0.9733	0.9366	0.9760	1.0000
$NPTLTL^{\dagger}$ η	0.3813	-0.8190	-0.7608	-0.7706	0.3668		-0.2455	-0.2684	-0.2018	-0.0645	-0.7087	-0.8307	1.0000

Data source: St. Louis FRED

† Data series begins in 1988

Table 7: Correlation of Simulated Variables for the model. Pos. ϵ_M and neg ϵ_{ffr}

Stimulative shocks to e_m, and e_{ffr}

	m	P	c	W	R	Rh	k	d	ER	n	l	y	η
m	1.0000												
lnM2_hp M2	1.0000												
P	0.9515	1.0000											
CPIma P	0.0977	1.0000											
c	-0.6033	-0.8195	1.0000										
PCECC96 c	0.1447	0.9824	1.0000										
W	0.9150	0.9071	-0.6465	1.0000									
GDI W	0.1306	0.9844	0.9936	1.0000									
R	0.1216	0.3396	-0.6534	-0.0630	1.0000								
TBillma R	-0.3494	-0.0880	-0.1220	-0.1655	1.0000								
Rh	0.0233	0.2176	-0.5203	-0.2013	0.9862	1.0000							
k	0.5809	0.3035	0.2953	0.4039	-0.4627	-0.4369	1.0000						
lnkfrmhp k	-0.0277	-0.0555	-0.0221	-0.0361	0.1660		1.0000						
d	0.1366	0.3409	-0.6288	-0.0698	0.9982	0.9907	-0.4165	1.0000					
DPOSma d	0.0294	0.9471	0.8951	0.8917	-0.0879		-0.0286	1.0000					
ER	0.0392	0.2376	-0.5427	-0.1795	0.9903	0.9996	-0.4419	0.9938	1.0000				
EXRESma ER	0.2529	0.6825	0.6825	0.7042	-0.3099		-0.0701	0.6141	1.0000				
n	0.6512	0.6881	-0.5705	0.9020	-0.2488	-0.4037	0.1381	-0.2775	-0.3796	1.0000			
ln_n_hp n	-0.1783	-0.0262	-0.0042	-0.0039	0.3648		0.7546	-0.0327	-0.0490	1.0000			
l	0.7880	0.8154	-0.6454	0.9690	-0.1392	-0.2924	0.2376	-0.1596	-0.2683	0.9796	1.0000		
LOANSma l	0.1666	0.9728	0.9899	0.9973	-0.1936		-0.0427	0.8668	0.7148	-0.0087	1.0000		
y	0.7137	0.7087	-0.5074	0.93207	-0.3118	-0.4581	0.2796	-0.3330	-0.4354	0.9895	0.9837	1.0000	
IP_y	0.0936	0.9635	0.9913	0.9793	-0.0578		0.0402	0.8695	0.6414	0.0683	0.9732	1.0000	
η	-0.0948	-0.3615	0.3780	0.9313	-0.9911	-0.9947	0.2093	-0.9958	-0.9976	0.9733	0.9366	0.9760	1.0000
$NPTLTL^{\dagger}$ η	0.3813	-0.8190	-0.7608	-0.7706	0.3668		-0.2455	-0.2684	-0.2018	-0.0645	-0.7087	-0.8307	1.0000

Data source: St. Louis FRED

† Data series begins in 1988

7 Conclusion

Dynamic stochastic general equilibrium models are the workhorse of macroeconomics. With the relatively new Federal Reserve policy of paying interest paid on a commercial bank's reserves, incorporating an interest rate on excess reserves within a DSGE model is a natural extension of the DSGE model literature. This paper has modeled interest on reserve policy into a deterministic model and stochastic model in order to analyze the effects on the macro-economy. Using this new tool of monetary policy, the Federal Reserve no longer has to face a trade-off between interest rates and the money supply.

This paper compares the effects of stimulative OMO policy with IOR policy. Even though both MP tools increased output, I find that IOR policy is deflationary while OMO policy is inflationary. While increasing the money supply will decrease nominal interest rates, the model shows that real interest rates will increase which leads to higher prices. As a result, consumption spending increases from IOR policy, but decreases from OMO policy. The increase in the money supply from OMOs get absorbed in excess reserves.

I then create correlation coefficient matrices for empirical time series data and from various shock scenarios in the model. This was done to judge the accuracy of the model at describing empirical aggregate relationships. A further extension is that we will know how well the model can be utilized for making predictions regarding various MP tools.

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