

## Online Appendix: A Simple Model of the Credit Market

This model presents the intuition of section 2 in a simple mathematical form.

### 7.1 Model Set Up

Assume that there is a borrower with wealth  $w$  who wants to invest an amount  $k > w$  in a technology  $f(k)$  with a decreasing marginal product of capital. To do so he needs to borrow the rest,  $(k - w)$ . There is a competitive lending sector where the cost of capital is  $\rho$ . Assume that the loans are offered at an interest rate  $r$ .

We focus on the problem that the borrower can default. By expending a cost  $\eta k$ ,  $\eta < \rho$  he can avoid having to pay, in which case his payoff is  $f(k) - \eta k$ . If, instead, he chooses to respect the contract his payoff is  $f(k) - r(k - w)$ . The lender will not lend if the borrower has no incentive to repay. To avoid this possibility, lenders set  $k$  so that

$$f(k) - r(k - w) \geq f(k) - \eta k,$$

which<sup>1</sup> is equivalent to the condition that

$$k \leq \frac{r}{r - \eta} w.$$

The lender will not lend any more than this, because it knows that the borrower will default if he is given any more. The borrower, however, may very well want to borrow more. That will be the case when the borrower borrows the maximum allowed but his total investment  $k$  is such that the marginal product of capital is still greater than the interest rate:  $f'(k) > r$ . When this holds we say that the borrower is credit constrained. Obviously if the borrower happens to want less than  $r/(r - \eta) w$ , he is not going to be credit constrained. Notice that when  $r$  goes up, the credit limit  $k$  for a constrained borrower will go down. A higher interest rate creates greater incentive to default, which requires that the borrower be reined in even further.

<sup>1</sup> This equation makes it clear why we need that  $\eta < \rho$ . Otherwise the lender can always set  $r = \rho$  and then the borrower would never want to default.

To figure out what  $r$  will clear the market, we need to say something about the lender's costs. We already assumed that the cost of capital is  $\rho$ . In addition, assume that there is a fixed administrative cost associated with making a loan, which we will denote by  $\phi$ . If the lender does not pay this cost, the borrower will act as if  $\eta$  were zero and always default. Furthermore, assume that this cost does not vary with the size of the loan. Adding some costs that increase with loan size (the lender may want two references for someone who is borrowing a lot rather than one) will not change the spirit of our conclusions.

Given these assumptions, and the fact that competitive lenders are not meant to make profits,  $r$  will be determined by the requirement that the lender's revenues equal his cost,

$$r(k - w) = \rho(k - w) + \rho\phi.$$

We assume here that the lender spends the fixed cost at the time the loan is made, and so needs to cover the interest cost on that expense.

## 7.2 Model Implications

Now consider a credit constrained borrower who wants to borrow as much as he can. For him  $k = r/(r - \eta) w$ . Using this, a bit of algebra gives us,

$$r = \eta + \frac{\rho - \eta}{1 - \frac{\rho\phi}{\eta w}}$$

A number of observations follow from this equation.

First, when wealth  $w$  increases, the interest rate  $r$  clearly goes down. And the maximum size that can be invested ( $k$ ) increases, for two reasons: One is the effect of being able to post more collateral. The other is that the fall in the interest rate allows the borrower to borrow more. When wealth  $w$  is so low that  $\rho\phi/\eta w$  is close to 1, then the interest rate can be extremely large.

Second, for those who have so little of their own wealth that  $\rho\phi/\eta w$  is greater than 1, no lending is possible: some people with low wealth will not be able to borrow at all.

Third, when wealth is low, or the cost of capital or the cost of administering the loan are high, and  $\rho\phi/\eta w$  is close to 1, small changes in  $\eta$  or  $\phi$ , which measure how easy it is to monitor the borrower, will generate large shifts in the interest rate.

The key insight of this model is the *multiplier property*, which contracts the supply of credit: Borrowers with little wealth must get smaller loans, so the fixed administrative cost has to be covered by the interest payment on a small loan, which pushes the interest rate up. But high interest rates exacerbate the problem of getting borrowers to repay. Total lending therefore shrinks further, pushing

up interest rates even more until the loan is small enough and interest rate high enough to cover the fixed cost.

The predictions of this model fit the facts we saw above. The lending rate  $r$  can be much higher than deposit rate  $\rho$  when the borrowers are poor. As the borrower gets richer, he can borrow more and pays a lower rate, while the poorest borrower will face complete exclusion from the credit market. Small differences among poor borrowers, in terms of how easy it is to get information about them or to bully them, may turn into large differences in the interest rate that they face. In this sense interest rates are likely to be most variable when they are high, which is also consistent with what we see.