Online Appendix for "Segregation and the Initial Provision of Water in the United States"

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Derivation of the Interior Solution

The city planner wishes to maximize $U(N_W, N_B, z)$ subject to budget constraint, which for an interior solution is $B = z + c \times m + F$. This implies that:

$$\begin{aligned}
U(N_W, N_B, z) &= \alpha N_W + (1-\alpha)N_B + \beta z \\
&= \alpha \int_0^m (1-g(x))dx + (1-\alpha)\int_0^m g(x)dx + \beta z \\
&= \int_0^m \left[\alpha(1-g(x)) + (1-\alpha)g(x)\right] dx + \beta z \\
&= \int_0^m \left[\alpha + (1-2\alpha)g(x)\right] dx + \beta z \\
&= \int_0^m \left[\alpha + (1-2\alpha)g(x)\right] dx + \beta(B - c \times m - F)
\end{aligned}$$

Then $\frac{\partial U}{\partial m} = \alpha + (1-2\alpha)g(m) - \beta c$, which implies that the optimal main is when:

$$\begin{aligned}
(1-2\alpha)g(m^*) &= \beta c - \alpha \Rightarrow \\
g(m^*) &= \frac{\beta c - \alpha}{1-2\alpha} \Rightarrow \\
m^* &= g^{-1}\left(\frac{\beta c - \alpha}{1-2\alpha}\right) \\
&= g^{-1}\left(\frac{\alpha - \beta c}{2\alpha - 1}\right).
\end{aligned}$$

Proof of Propositions

Proposition 1 *For an interior solution, the size of the system decreases in the cost of mains and the preference for non-water public goods.*

Proof Since g is an increasing function, it follows that g^{-1} is an increasing function. Let $\lambda = \frac{\alpha - \beta c}{2\alpha - 1}$. Then $\frac{\partial \lambda}{\partial c} = \frac{-\beta}{2\alpha - 1} < 0$. Thus, an increase in the cost of a water main decreases optimal main mileage. Similarly, $\frac{\partial \lambda}{\partial \beta} = \frac{-c}{2\alpha - 1} < 0$ and an increase in the preferences for non-water public goods decreases optimal main mileage. ■

Proposition 2 *The size of the system m^* increases as γ increases.*

Proof This result follows immediately from the fact that

$$\begin{aligned} m^* &= g^{-1}\left(\frac{\alpha - \beta c}{2\alpha - 1}\right) \\ &= \gamma - \frac{1}{k} \ln\left(\frac{2\alpha - 1}{\alpha - \beta c} - 1\right). \blacksquare \end{aligned}$$

Proposition 3 *If the optimal main stops in a neighborhood that is less than one-half Black (i.e., $m^* < \gamma$) then a marginal increase in either segregation (k) or the preference for Whites (α) increases the size of the optimal water system. Conversely, if $m^* > \gamma$, then a marginal increase in either k or α decreases the size of the optimal water system.*

Proof Note that $\frac{\partial m^*}{\partial k} = \frac{1}{k^2} \ln\left(\frac{2\alpha - 1}{\alpha - \beta c} - 1\right)$, which implies that $\frac{\partial m^*}{\partial k} > 0$ if and only if

$$\begin{aligned} \frac{2\alpha - 1}{\alpha - \beta c} - 1 &> 1 \Leftrightarrow \\ \frac{2\alpha - 1}{\alpha - \beta c} &> 2 \Leftrightarrow \\ 2\alpha - 1 &> 2\alpha - 2\beta c \Leftrightarrow \\ \beta c > \frac{1}{2}. \end{aligned}$$

Now suppose that the optimal main stops in a majority White neighborhood. This implies that

$$\begin{aligned}
m^* &< \gamma \Leftrightarrow \\
g^{-1}\left(\frac{\alpha - \beta c}{2\alpha - 1}\right) &< \gamma \Leftrightarrow \\
\gamma - \frac{1}{k}\ln\left(\frac{2\alpha - 1}{\alpha - \beta c} - 1\right) &< \gamma \Leftrightarrow \\
0 &< \frac{1}{k}\ln\left(\frac{2\alpha - 1}{\alpha - \beta c} - 1\right) \Leftrightarrow \\
1 &< \frac{2\alpha - 1}{\alpha - \beta c} - 1 \Leftrightarrow \\
\frac{1}{2} &< \beta c,
\end{aligned}$$

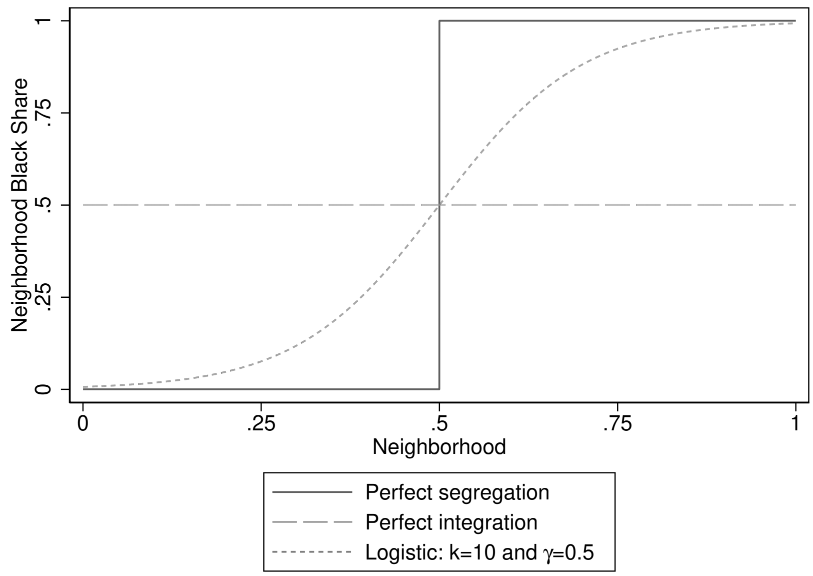
which implies that $\frac{\partial m^*}{\partial k} > 0$.

A symmetric argument will show that if the optimal main stops in a majority Black neighborhood, then $\beta c < \frac{1}{2}$ and $\frac{\partial m^*}{\partial k} < 0$.

As for the preferences for Whites (α), continue to let $\lambda = \frac{\alpha - \beta c}{2\alpha - 1}$. Then $\frac{\partial \lambda}{\partial \alpha} = \frac{(2\beta c - 1)}{(1 - 2\alpha)^2}$. This derivative is positive if $2\beta c - 1 > 0 \Rightarrow \frac{1}{2} < \beta c$ (when the optimal main stops in a majority White neighborhood) and negative when $\beta c < \frac{1}{2}$ (when the optimal main stops in a majority Black neighborhood). ■

Appendix Figures

Figure A.1: Examples of $g(.)$



Notes: The lines corresponds to hypothetical cities with the same Black share (50% in this case), but differ in their level of segregation. Neighborhoods are ordered based on their Black share with 0 being the neighborhood with the smallest Black share and 1 being the neighborhood with the largest Black share. All neighborhoods are assumed to be of the same size.