# Online Appendix to “Exit Dilemma”

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## OA.1 Discussion of Equilibrium Outcomes

In this section we argue that any Nash equilibrium is outcome-equivalent to a perfect Bayesian equilibrium. Given a candidate equilibrium strategy profile, there are two possible observable deviations.

First, a firm may exit unexpectedly when it is supposed to remain in the market with probability one. Because exit is irreversible and the payoff upon exit is zero regardless of the behavior of the opponent in the continuation game, the specification of the off-path belief and behavior of the remaining firm after such a deviation play no role in sustaining on-path behavior. As a result, there is no discrepancy between perfect Bayesian equilibrium and Nash equilibrium outcome as far as this type of deviations are concerned.

Second, in principle, a firm may expect the opponent to exit with probability one and be surprised if this does not happen. However, there always exists a type of the opponent that finds it dominant to remain in business. That is, because the duopoly profits are strictly positive when the state is good, there always exists a private history of the opponent that has positive probability and makes the opponent sufficiently optimistic, so to render exiting a dominated action. Hence, starting from a Nash equilibrium, one can complete the description of an outcome-equivalent perfect Bayesian equilibrium along these histories using Bayes rule.

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## OA.2 Iterated Deletion of Conditionally Dominated Strategies and Nash Equilibrium

In this section, we argue that any Nash equilibrium strategy profile survives iterated deletion of conditionally dominated strategies.

Reasoning by contradiction, assume that a strategy profile $(\sigma_1^*, \sigma_2^*)$ is a Nash equilibrium but does not survive iterated deletion of conditionally dominated strategies. Hence, there exists another strategy for player $i$, $\hat{\sigma}^i$ which prescribes a different behavior at some history and potentially as some of its successors, and agrees with $\sigma^i$ at any other history, and yields a strictly higher expected continuation payoff starting at that history. If that history is on the path induced by the strategy profile $(\sigma_1^*, \sigma_2^*)$, this would contradict the best-reply property of the candidate Nash equilibrium $(\sigma_1^*, \sigma_2^*)$.

Suppose instead that the history is not on the path induced by the strategy profile $(\sigma_1^*, \sigma_2^*)$. By the argument in the previous section, the only such histories are the ones following an unexpected exit. But in light of our definition (see Section A.1), conditional dominance has no bite off the path because it requires the dominating strategy to yield a strictly higher expected continuation payoff for any consistent system of beliefs. As a result, the strategy profile must survive iterated deletion of conditionally dominated strategies, contradicting the hypothesis.

## OA.3 Capacity

In many industries, firms are able to reduce capacity and close individual plants. In this section, we show that our model could be extended to allow for capacity choices. That is, suppose that instead of choosing whether to exit, firms choose whether to close one of their plants, where the profit flow is given by

| | 2 plants | 1 plant |
|---|---|---|
| 2 plants | $2(\lambda^i_{\omega_t} R - c), 2(\lambda^i_{\omega_t} R - c)$ | $\alpha\lambda^i_{\omega_t} R - 2c, \lambda^i_{\omega_t} R - c/2$ |
| 1 plant | $\lambda^i_{\omega_t} R - c/2, \alpha\lambda^i_{\omega_t} R - 2c$ | $\lambda^i_{\omega_t} R - c/2, \lambda^i_{\omega_t} R - c/2$ |

where $\alpha > 2$. When best-replying to the other firm $j$ never downsizing first, firm $i$ chooses when to downsize to maximize

$$\max_{\tau} \mathbf{E}\left[\int_0^{\tau} 2e^{-rt}(\lambda^i_{\omega_t} R - c)\,\mathrm{d}t\right] + \mathbf{E}\left[\int_{\tau}^{\infty} e^{-rt}(\lambda^i_{\omega_t} R - c/2)\,\mathrm{d}t\right].$$

Subtracting $\mathbf{E}\left[\int_0^\infty e^{-rt}(\lambda^i_{\omega_t} R - c/2)\,\mathrm{d}t\right]$, the objective function becomes

$$\max_{\tau} \mathbf{E}\left[\int_0^\infty e^{-rt}\left(\lambda^i_{\omega_t} R - \frac{3}{2}c\right)\mathrm{d}t\right].$$

Hence, the model is equivalent to the one analyzed in the main body of the paper.

## OA.4 Complements to Section 5

First, as in Section 5, we reformulate each firm's objective. At any time $t$, along the history with no exit, firm $i$ maximizes

$$\mathbf{E}^{(\sigma^1,\sigma^2)}\left[\int_t^{\sigma^i} e^{-r(s-t)}\left(\left(D^i - \pi^i_{\omega_s}\right) + \mathbf{1}_{\{\sigma^j < s\}}(M^i - D^i)\right)\mathrm{d}s\right],$$

where the expectation is taken with respect to the information available to the firm at time $t$.

Recall that we assume that

$$\begin{aligned}
\pi^i_{\mathcal{G}} &= D^i + \frac{\kappa}{\lambda^i_{\mathcal{B}}},\\
\pi^i_{\mathcal{B}} &= \frac{\kappa}{\lambda^i_{\mathcal{B}}},\\
M^i &= D^i + \frac{\eta}{\lambda^i_{\mathcal{B}}},
\end{aligned}$$

where $\eta > \kappa > 0$.

In this case, we define a stationary cutoff strategy for firm $i$, $\sigma^i_p$ as the strategy that prescribes exiting with probability one as soon as the posterior belief that the new market is unprofitable falls below the cutoff $p$. The proof of the following lemma is immediate and omitted.

**Lemma OA.1.** *Firm $i$'s best reply to $\sigma^2_0$ (i.e., firm 2's strategy prescribing never entering the new market) is $\sigma^1_{\pi^*(\lambda^1_G)}$, where $\pi^*(\lambda^1_{\mathcal{B}})$ is the cutoff belief in the conclusive news exit game when $c = \kappa$, and $R^i = D^i$.*

We now formally state how Theorem 2 generalizes to this setup. Notice that whenever we consider a game in which firms learn at rates $(\lambda^1_{\mathcal{B}}, \lambda^2_{\mathcal{B}})$ respectively, we also adjust the payoffs accordingly, so to guarantee that (OA.4) always holds. Also, in order to guarantee $\pi^i_{\mathcal{B}} < D^i$, we shall restrict attention to $\lambda^i_{\mathcal{B}} > \kappa/D$.

**Theorem OA.1.A.** *If $r > \gamma$, for any $\lambda^1_{\mathcal{B}} > \kappa/D$, there exists a $\overline{\lambda}^2_{\mathcal{B}} > \lambda^1_{\mathcal{B}}$ such that for $(\lambda^1_{\mathcal{B}}, \lambda^2_{\mathcal{B}})$, $\lambda^2_{\mathcal{B}} > \overline{\lambda}^2_{\mathcal{B}}$, $(\sigma^1_0, \sigma^2_{\pi^*(\lambda^2_{\mathcal{B}})})$ is the unique strategy profile that survives iterated deletion of conditionally dominated strategies.*

*Proof.* To adapt the proof of Theorem OA.1 to this setup, we need to identify an upper bound on firm's continuation payoff at time $2\tau^*(\lambda^2_{\mathcal{B}})$ along the history in which firm 2 does not observe any signal in the interval $[0, 2\tau^*(\lambda^2_{\mathcal{B}}))$. In fact, in light of Lemma OA.1, the first rounds of deletion can be performed with no change. At time $2\tau^*(\lambda^2_{\mathcal{B}})$, firm 2's expected gain from remaining in the established market is bounded above by the following

$$\begin{aligned}
&\int_{2\tau^*(\lambda^2_{\mathcal{B}})}^{\tau^*(\lambda^1_{\mathcal{B}})} e^{-r(t-2\tau^*(\lambda^2_{\mathcal{B}}))} \Big(\Big(\pi^*(\lambda^2_{\mathcal{B}}) e^{-\gamma(t-2\tau^*(\lambda^2_{\mathcal{B}}))}(D^2 - \pi^2_{\mathcal{B}}) \\
&\qquad\qquad + \left(1 - \pi^*(\lambda^2_G) e^{-\gamma(t-2\tau^*(\lambda^2_{\mathcal{B}}))}\right)(D^2 - \pi^2_{\mathcal{G}})\Big)\Big)\, \mathrm{d}t \qquad \text{(OA.1)} \\
&+ e^{-r\left(\tau^*(\lambda^1_{\mathcal{B}}) - 2\tau^*(\lambda^2_{\mathcal{B}})\right)} \Big(\pi^*(\lambda^2_{\mathcal{B}}) e^{-\gamma(\tau^*(\lambda^1_{\mathcal{B}}) - 2\tau^*(\lambda^2_{\mathcal{B}}))}(M^2 - \pi^2_{\mathcal{B}})/r \\
&\qquad\qquad + \left(1 - \pi^*(\lambda^2_{\mathcal{B}}) e^{-\gamma(\tau^*(\lambda^1_{\mathcal{B}}) - 2\tau^*(\lambda^2_{\mathcal{B}}))}\right)(M^2 - \pi^2_{\mathcal{G}})/r\Big).
\end{aligned}$$

As $\lambda^2_{\mathcal{B}} \to \infty$, $\tau^*(\lambda^2_{\mathcal{B}}) \to 0$, and $\pi^*(\lambda^2_{\mathcal{B}})\lambda^2_{\mathcal{B}} \to 0$. Hence, in the limit, as $\lambda^2_{\mathcal{B}} \to \infty$, the bound converges to

$$\begin{aligned}
&\frac{1 - e^{-r\tau^*(\lambda^1_{\mathcal{B}})}}{r}(D^2 - \pi^2_{\mathcal{G}}) + \frac{e^{-r\tau^*(\lambda^1_{\mathcal{B}})}}{r}(M^2 - \pi^2_{\mathcal{G}}) \\
&= -\frac{1 - e^{-r\tau^*(\lambda^1_{\mathcal{B}})}}{r}\frac{\kappa}{\lambda^2_{\mathcal{B}}} + \frac{e^{-r\tau^*(\lambda^1_{\mathcal{B}})}}{r}\frac{\eta - \kappa}{\lambda^2_{\mathcal{B}}}. \qquad \text{(OA.2)}
\end{aligned}$$

From (5), replacing the parameters with those in Lemma OA.1,

$$\tau^*(\lambda^1_{\mathcal{B}}) \geq \frac{1}{\gamma^1 + \lambda^1_G - \lambda_B} \ln\left(\frac{\lambda^1_{\mathcal{B}}((\lambda^1_{\mathcal{B}} + \gamma)D^1 - \kappa)}{\gamma\kappa}\right). \qquad \text{(OA.3)}$$

Replacing $\tau^*(\lambda^1_G)$ with this bound in (OA.2), we obtain

$$-\frac{\kappa}{\lambda^2_{\mathcal{B}}} + \frac{\eta}{\lambda^2_{\mathcal{B}}}\left(\frac{\lambda^1_{\mathcal{B}}((\lambda^1_{\mathcal{B}} + \gamma)D^1 - \kappa)}{\gamma\kappa}\right)^{-\frac{r}{\gamma + \lambda^1_{\mathcal{B}}}}. \qquad \text{(OA.4)}$$

If $\lambda^1_{\mathcal{G}}$ is larger than the positive root of the following quadratic equation,

$$D^1 x^2 - (\kappa - \gamma D^1)x - \eta\gamma = 0,$$

and $r > \gamma + \lambda^1_{\mathcal{G}}$, then (OA.4) is strictly negative. Hence, investing in the new market is dominant for firm 2. The remainder of the proof follows the same steps as the proof of Theorem 2.A. $\square$

**Theorem OA.1.B.** *There exists an open set of pairs $(\lambda^1_{\mathcal{B}}, \lambda^2_{\mathcal{B}})$, $\lambda^2_{\mathcal{B}} < \lambda^1_{\mathcal{B}}$, under which $(\sigma^1_0, \sigma^2_{\pi^*(\lambda^2_G)})$ is the unique strategy profile that survives iterated deletion of conditionally dominated strategies, provided that $D^1$ is high enough and that $r$ and $\gamma$ are high enough.*

*Proof.* Proceeding as in the proof of Theorem 2.B, we want to show that (OA.1) is negative for $(\lambda^1_{\mathcal{B}}, \lambda^2_{\mathcal{B}})$ appropriately chosen, provided that $D^1$ is high enough. First, we can choose $\lambda^2_{\mathcal{B}}$ arbitrarily close to $\kappa/D^2$, so that that $\tau^*(\lambda^2_{\mathcal{B}})$ is arbitrarily close to 0. Second, using again the bound in (OA.3), it can be shown $\arg\max_{\lambda_{\mathcal{B}^1}} \tau^*(\lambda_{\mathcal{B}^1})$ can be taken to be arbitrarily large, by increasing $D^1$, so that the second and third line in (OA.1) converge to zero. By definition the first and second line are negative and bounded away from zero, and the result follows.

$\square$