Fair social ordering, egalitarianism, and animal welfare: Online Appendix

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1 Existence of a most speciesist preference among the preferences that are at most as speciesist as R_i for all $i \in H$.

Proposition 1. *For any given profile $\boldsymbol{R} \in \mathcal{R}$, let $\mathcal{A}(R)$ denote the collection of preferences over $\mathbb{R}_+^2$ that are at most as speciesist as R_i for all $i \in H$. This collection has a well-defined most speciesist member, which is convex if every R_i is convex.*

Proof. If there is $i \in H$ such that $R_i = R^{AL}$, this individual has preferences that are at most as speciesist as everyone else, and then R^{AL} itself if the most speciesist among the preferences that are at most as speciesist as every R_i. In the rest of the proof, we restrict attention to the case in which all individual preferences are continuous and monotonic.

The first part of the proof consists in showing that the relation "at most as speciesist as", which we will denote by $\trianglerighteq$, generates a lattice structure on the set of continuous and strictly monotonic preferences. The proposition states that every collection of h preferences has a meet in the domain. This is indeed true if we have a lattice.

It suffices to show that every pair of preferences has a meet and a join for $\trianglerighteq$. We focus on the join, since the proof for the meet is essentially identical.

Continuous and strictly monotonic preferences have strictly decreasing indifference curves, which are therefore differentiable a.e. (almost everywhere). Let R, R' be two preferences and let $I(x, R)$ denote the indifference curve of R that contains $x = (x_o, x_i) \in \mathbb{R}_+^2$. Let $s(x, R)$ denote the derivative of the graph of the $I(x, R)$ (i.e., the slope of the curve, which must be negative a.e. by strict monotonicity of preferences).

The construction of the indifference map of the join $R \vee R'$ goes as follows:

1. At a point x where $s(x, R) > s(x, R')$, let the curve of $R \vee R'$ follow $I(x, R)$ for $y_i > x_i$ until the condition $s(y, R) > s(y, R')$ is no longer satisfied.

2. Symmetrically, when $s(x, R) < s(x, R')$, let the curve follow $I(x, R')$ for $y_i > x_i$ until the condition $s(y, R) > s(y, R')$ is no longer satisfied.

3. At a point x where $s(x, R) = s(x, R')$, let the curve follow $I(x, R)$ for $y_i > x_i$ until the condition $s(x, R) = s(x, R')$ is no longer satisfied.

This construction defines the indifference map of $R \vee R'$ a.e., and by continuity the construction is extended to the whole space.

This construction is unique because it is unique in the vicinity of a.e. point, and every point that is not included in steps 1-3 has a single curve that tends toward it (indeed, it has only one curve for each preference R, R' that tends toward it), so that the continuity extension is also unique.

A curve of this map is continuous because it is made of continuous bits of indifference curves and the continuity extension imposes continuity at the points where steps 1-3 do not apply.

Curves of $R \vee R'$ cannot cross because this would mean that at a particular point x, the construction is not unique.

A curve of $R \vee R'$ is decreasing a.e. by steps 1-3 because $s(x, R) < 0$ for all x and all R in the domain. It is therefore decreasing after the continuity extension.

In conclusion, $R \vee R'$ is in the domain of continuous, strictly monotonic preferences.

It is straightforward to check that $R'' \trianglerighteq R \vee R'$ for all R'' such that $R'' \trianglerighteq R$ and $R'' \trianglerighteq R'$. Indeed, the fact that $R'' \trianglerighteq R$ and $R'' \trianglerighteq R'$ implies that $s(x, R'') \geq \max\{s(x, R), s(x, R')\}$ for almost every x (i.e., every x where this is defined), and it is never the case that $s(x, R \vee R') > \max\{s(x, R), s(x, R')\}$.

This concludes the proof of existence of a join.

When both R and R' are convex, their indifference curves have non-decreasing slopes, and therefore this holds for the curves of $R \vee R'$ too, which are made of parts of these curves. When the join is taken for a finite set of more than two preferences, the same reasoning applies. $\square$