## Online Appendix

# Well-Being, Poverty and Labor Income Taxation: Theory and Application to Europe and the U.S.* 

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#### Abstract

This document provides six appendices (A-F) related to Maniquet and Neumann (2020): A) the proof of proposition 1, B) the proof that the theoretical results presented in the paper could have been obtained under the assumption that the poverty line depends on the distribution of consumption (after-tax income) levels, C) an extended description of the methodology used, the OECD tax-benefit calculator, including all data, specific settings and assumptions, D) a comparison of budget curves across household types, exploring whether having children makes it easier to get out of poverty, E) extensions to check the robustness of the results in Section 6 of the paper with respect to several assumptions made, and F) the full set of results of the application in Section 6 of the paper, namely simulated budget curves for all selected countries and six different household types and the values for the criterion developed in the paper to evaluate them.


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## A Proof of Proposition 1

Proof. We let the easy proof that $R^{l e x}$ satisfies Poverty Reduction, Pareto and Independence to the reader. We concentrate on the second statement. The proof is divided in three steps. In the first two steps, we prove that social preferences satisfying the three properties we have defined also satisfy other, stronger, properties. We begin by proving that if social preferences $R$ satisfy Poverty Reduction, Pareto and Independence, then they satisfy the following strengthening of Poverty Reduction.

Property A. 1 For all economies $\left(R_{1}, \ldots, R_{n}\right)$, for all pairs of allocations $z=\left(z_{1}, \ldots, z_{n}\right)$ and $z^{\prime}=\left(z_{1}^{\prime}, \ldots, z_{n}^{\prime}\right)$, if, for two agents $j$ and $k$

$$
\begin{aligned}
& \ell_{j}=\ell_{j}^{\prime}=\ell_{k}^{\prime}=\ell_{k}, \\
& c_{j}<c_{j}^{\prime}<p<c_{k}^{\prime}<c_{k},
\end{aligned}
$$

whereas $z_{i}=z_{i}^{\prime}$ for all other agents, then $z^{\prime} P z$.
We prove this claim in two steps, corresponding to two cases for the value of $\ell_{j}=\ell_{j}^{\prime}=\ell_{k}=\ell_{k}^{\prime}$. Case 1: $\ell_{j}=\ell_{j}^{\prime}=\ell_{k}=\ell_{k}^{\prime}=1$. Then there exist $z^{\prime \prime}=\left(z_{1}^{\prime \prime}, \ldots, z_{n}^{\prime \prime}\right)$ and $z^{\prime \prime \prime}=\left(z_{1}^{\prime \prime \prime}, \ldots, z_{n}^{\prime \prime \prime}\right)$ such that $z_{j}^{\prime \prime} I_{j} z_{j}, z_{j}^{\prime \prime \prime} I_{j} z_{j}^{\prime}, z_{k}^{\prime \prime} I_{k} z_{k}$ and $z_{k}^{\prime \prime \prime} I_{k} z_{k}^{\prime}$,

$$
\begin{aligned}
& \ell_{j}^{\prime \prime}=\ell_{j}^{\prime \prime \prime}=\ell_{k}^{\prime \prime}=\ell_{k}^{\prime \prime \prime}<1, \\
& c_{j}^{\prime \prime}<c_{j}^{\prime \prime \prime}<p<c_{k}^{\prime \prime \prime}<c_{k}^{\prime \prime}
\end{aligned}
$$

and $z_{i}^{\prime \prime}=z_{i}^{\prime \prime \prime}=z_{i}=z_{i}^{\prime}$ for all other agents. By Pareto, $z^{\prime \prime} I z$ and $z^{\prime \prime \prime} I z^{\prime}$. By transitivity, we now need to prove that $z^{\prime \prime \prime} P z^{\prime \prime}$, which corresponds to case 2.

Case 2: $\ell_{j}=\ell_{j}^{\prime}=\ell_{k}=\ell_{k}^{\prime}<1$. Let us assume, contrary to the claim, that

$$
\begin{equation*}
z R\left(R_{1}, \ldots, R_{n}\right) z^{\prime} \tag{1}
\end{equation*}
$$

where social preferences are written $R\left(R_{1}, \ldots, R_{n}\right)$ to denote their dependence on the profile of individual preferences. Let bundles $z_{j}^{1}, z_{j}^{2}$ and $z_{j}^{3}$ be defined by

$$
\begin{aligned}
& z_{j}^{\prime} P_{j} z_{j}^{3} I_{j} z_{j}^{2} P_{j} z_{j}^{1} I_{j} z_{j}, \\
& \ell_{j}^{1}=l_{j}^{2}=l_{j}^{\prime}, \\
& \ell_{j}^{3}=l_{j} .
\end{aligned}
$$

Now, we construct $z_{k}^{\prime \prime}, z_{k}^{1}, z_{k}^{2}, z_{k}^{3}$ and $R_{k}^{\prime \prime}$ in such a way that

$$
\begin{aligned}
& l_{k}^{\prime \prime}=l_{k}^{3}=l_{k}, l_{k}^{1}=l_{k}^{2}=l_{k}^{\prime}, \\
& c_{k}^{\prime \prime}>c_{k}, c_{k}^{3}-c_{k}^{\prime}=c_{j}^{\prime}-c_{j}^{3}, c_{k}^{2}-c_{k}^{1}=c_{j}^{1}-c_{j}^{2} \\
& z_{k}^{\prime \prime} I_{k}^{\prime \prime} z_{k}^{1} P_{k}^{\prime \prime} z_{k}^{2} I_{k}^{\prime \prime} z_{k}^{3} P_{k}^{\prime \prime} z_{k}^{\prime}, \text { and } \\
& I\left(z_{k}^{\prime}, R_{k}^{\prime \prime}\right)=I\left(z_{k}^{\prime}, R_{k}\right)
\end{aligned}
$$

The construction of those bundles and preferences are illustrated in Figure A. 1 .


Figure A.1: Illustration of the proof of the first claim

Let preferences $R_{k}^{\prime}$ be such that

$$
\begin{aligned}
I\left(z_{k}, R_{k}^{\prime}\right) & =I\left(z_{k}, R_{k}\right) \\
I\left(z_{k}^{\prime}, R_{k}^{\prime}\right) & =I\left(z_{k}^{\prime}, R_{k}\right) \\
I\left(z_{k}^{\prime \prime}, R_{k}^{\prime}\right) & =I\left(z_{k}^{\prime \prime}, R_{k}^{\prime \prime}\right)
\end{aligned}
$$

By Independence, Eq. 1 implies that

$$
\begin{equation*}
z R\left(R_{1}, \ldots, R_{k}^{\prime}, \ldots, R_{n}\right) z^{\prime} \tag{2}
\end{equation*}
$$

Let us define $z^{\prime \prime}=\left(z_{1}^{\prime \prime}, \ldots, z_{n}^{\prime \prime}\right)$ by fixing $z_{i}^{\prime \prime}=z_{i}$ for all $i \neq k\left(z_{k}^{\prime \prime}\right.$ is defined above). By Pareto,

$$
\begin{equation*}
z^{\prime \prime} P\left(R_{1}, \ldots, R_{k}^{\prime}, \ldots, R_{n}\right) z \tag{3}
\end{equation*}
$$

By transitivity, Eq. 2 and 3 imply

$$
\begin{equation*}
z^{\prime \prime} P\left(R_{1}, \ldots, R_{k}^{\prime}, \ldots, R_{n}\right) z^{\prime} \tag{4}
\end{equation*}
$$

By Independence,

$$
\begin{equation*}
z^{\prime \prime} P\left(R_{1}, \ldots, R_{k}^{\prime \prime}, \ldots, R_{n}\right) z^{\prime} \tag{5}
\end{equation*}
$$

We define $z^{1}, z^{2}$ and $z^{3}$ by: $z_{i}^{1}=z_{i}^{2}=z_{i}^{3}=z_{i}$ for all $i \neq j, k$ and the corresponding bundles of $j$ and $k$ are defined above. By Pareto, we can deduce that

$$
\begin{equation*}
z^{1} I\left(R_{1}, \ldots, R_{k}^{\prime \prime}, \ldots, R_{n}\right) z^{\prime \prime} \tag{6}
\end{equation*}
$$

By Poverty Reduction,

$$
\begin{equation*}
z^{2} P\left(R_{1}, \ldots, R_{k}^{\prime \prime}, \ldots, R_{n}\right) z^{1} \tag{7}
\end{equation*}
$$

By Pareto,

$$
\begin{equation*}
z^{3} I\left(R_{1}, \ldots, R_{k}^{\prime \prime}, \ldots, R_{n}\right) z^{2} \tag{8}
\end{equation*}
$$

By Poverty Reduction,

$$
\begin{equation*}
z^{\prime} P\left(R_{1}, \ldots, R_{k}^{\prime \prime}, \ldots, R_{n}\right) z^{3} \tag{9}
\end{equation*}
$$

Gathering Eq. 5, 6, 7, 8 and 10, we obtain

$$
\begin{equation*}
z^{\prime} P\left(R_{1}, \ldots, R_{k}^{\prime \prime}, \ldots, R_{n}\right) z^{\prime} \tag{10}
\end{equation*}
$$

the desired contradiction.
We proceed by proving that social preferences satisfying Poverty Reduction, Pareto and Independence, and, therefore, Property 4, also satisfy the following property.

Property A. 2 For all economies $\left(R_{1}, \ldots, R_{n}\right)$, for all pairs of allocations $z=\left(z_{1}, \ldots, z_{n}\right)$ and $z^{\prime}=\left(z_{1}^{\prime}, \ldots, z_{n}^{\prime}\right)$, if, for two agents $j$ and $k$

$$
\begin{aligned}
& \ell_{j}>\ell_{j}^{\prime}>\ell_{k}^{\prime}>\ell_{k}, \\
& c_{j}=c_{j}^{\prime}=c_{k}^{\prime}=c_{k}=p,
\end{aligned}
$$

whereas $z_{i}=z_{i}^{\prime}$ for all other agents, then $z^{\prime} P z$.
Let us assume, contrary to the claim, that

$$
\begin{equation*}
z R z^{\prime} \tag{11}
\end{equation*}
$$

The following construction is illustrated in Figure A.2. Let $\delta_{k}^{1}>0$ be such that there exists $c_{k}^{1}>p$ such that

$$
\left(\ell_{k}+\delta_{k}^{1}, c_{k}^{1}\right) I_{k} z_{k}
$$

The idea is to choose $\delta_{k}^{1}$ as close as possible to $\ell_{k}^{\prime}-\ell_{k}$, but the shape of the indifference curve through $z_{k}$ may be such that $\delta_{k}^{1}$ is bounded above. Let $c_{j}^{1}$ be defined by

$$
\left(\ell_{k}+\delta_{k}^{1}, c_{j}^{1}\right) I_{j} z_{j} .
$$

Such a $c_{j}^{1}$ always exists, thanks to our assumption that consumption is necessary. This is the only role played by this assumption. By Pareto,

$$
\begin{equation*}
\left(z_{1}, \ldots,\left(\ell_{k}+\delta_{k}^{1}, c_{j}^{1}\right), \ldots,\left(\ell_{k}+\delta_{k}^{1}, c_{k}^{1}\right), \ldots, z_{n}\right) I z . \tag{12}
\end{equation*}
$$



Figure A.2: Illustration of the proof of the second claim

Then, we can choose $\delta_{j}^{1}$ such that

$$
\begin{equation*}
\frac{\delta_{j}^{1}}{\delta_{k}^{1}}<\frac{\ell_{j}-\ell_{j}^{\prime}}{\ell_{k}^{\prime}-\ell_{k}} \tag{13}
\end{equation*}
$$

and define $c_{j}^{1 *}$ by

$$
\left(\ell_{k}+\delta_{k}^{1}, c_{j}^{1 *}\right) I_{j}\left(\ell_{j}-\delta_{j}^{1}, p\right)
$$

By Property 4,

$$
\begin{align*}
& \left(z_{1}, \ldots,\left(\ell_{k}+\delta_{k}^{1}, c_{j}^{1 *}\right), \ldots,\left(\ell_{k}+\delta_{k}^{1}, p\right), \ldots, z_{n}\right) \\
& P\left(z_{1}, \ldots,\left(\ell_{k}+\delta_{k}^{1}, c_{j}^{1}\right), \ldots,\left(\ell_{k}+\delta_{k}^{1}, c_{k}^{1}\right), \ldots, z_{n}\right) . \tag{14}
\end{align*}
$$

By Pareto,

$$
\begin{gather*}
\left(z_{1}, \ldots,\left(\ell_{j}-\delta_{j}^{1}, p\right), \ldots,\left(\ell_{k}+\delta_{k}^{1}, p\right), \ldots, z_{n}\right) \\
P\left(z_{1}, \ldots,\left(\ell_{k}+\delta_{k}^{1}, c_{j}^{1 *}\right), \ldots,\left(\ell_{k}+\delta_{k}^{1}, p\right), \ldots, z_{n}\right) . \tag{15}
\end{gather*}
$$

By transitivity, Eq. 1214 and 15 imply

$$
\begin{equation*}
\left(z_{1}, \ldots,\left(\ell_{j}-\delta_{j}^{1}, p\right), \ldots,\left(\ell_{k}+\delta_{k}^{1}, p\right), \ldots, z_{n}\right) P z^{\prime} \tag{16}
\end{equation*}
$$

Iterating this chain of social indifference and strict preference a finite number of times and constructing $\delta_{k}^{2}, \delta_{j}^{2}, \ldots, \delta_{k}^{M}, \delta_{j}^{M}$ that satisfy Eq. 13 at each iteration, we arrive at labor time

$$
\begin{aligned}
& \ell_{j}^{\prime \prime}=\ell_{j}-\sum_{m=1}^{M} \delta_{j}^{m}>\ell_{j}^{\prime}, \\
& \ell_{k}^{\prime \prime}=\ell_{k}+\sum_{m=1}^{M} \delta_{k}^{m}>\ell_{k}^{\prime},
\end{aligned}
$$

and allocation

$$
z^{\prime \prime}=\left(z_{1}, \ldots,\left(\ell_{j}^{\prime \prime}, p\right), \ldots,\left(\ell_{k}^{\prime \prime}, p\right), \ldots, z_{n}\right)
$$

such that, by iteration of the sequence leading to Eq. 16 ,

$$
z^{\prime \prime} P z^{\prime},
$$

violating Pareto, because $\ell_{i}^{\prime \prime} \geq \ell_{i}^{\prime}$ for all $i, \ell_{j}^{\prime \prime}>\ell_{j}^{\prime}$ and $\ell_{k}^{\prime \prime}>\ell_{k}^{\prime}$.
We now complete the proof of the proposition. In this third step, all allocations will be composed of bundles containing a consumption level equal to $p$. Only labor times will vary, so that the objects we define are essentially unidimensional. The resulting proof boils down to the adaptation to this setting of Hammond (1982)'s characterization of the leximin in utility. Let economy $\left(R_{1}, \ldots, R_{n}\right)$ and allocations $z=\left(z_{1}, \ldots, z_{n}\right)$ and $z^{\prime}=\left(z_{1}^{\prime}, \ldots, z_{n}^{\prime}\right)$ be such that

$$
\min _{i} W^{p}\left(z_{i}\right)>\min _{i} W^{p}\left(z_{i}^{\prime}\right) .
$$

We need to prove that $z P z^{\prime}$. For all $i \in\{1, \ldots, n\}$, let us define $z_{i}^{\prime \prime}=$ $\left(W^{p}\left(z_{i}, R-i\right), p\right)$ and $z_{i}^{\prime \prime \prime}=\left(W^{p}\left(z_{i}^{\prime}, R-i\right), p\right)$. By Pareto, $z^{\prime \prime} I z$ and $z^{\prime \prime \prime} I z^{\prime}$, so that, by transitivity, $z^{\prime \prime \prime} R z^{\prime \prime}$. Without loss of generality, let us fix

$$
\begin{aligned}
& z_{1}=\min _{i} W^{p}\left(z_{i}, R_{i}\right) \\
& z_{2}=\min _{i} W^{p}\left(z_{i}^{\prime}, R_{i}\right) .
\end{aligned}
$$

Let us choose $\ell_{i}^{*}$ for each $i \in\{1, \ldots, n\}$ such that $\ell_{i}^{*} \geq \ell_{1}^{*}$ and $\ell_{1}<\ell_{1}^{*} \leq$ $\ell_{i}<\ell_{2}^{\prime}$. Let $z^{*}=\left(\left(\ell_{i}^{*}\right), \ldots,\left(\ell_{n}^{*}\right)\right)$. Using Property 5 iteratively for each $i \in\{2, \ldots, n\}$, we get the conclusion that $z^{*} P z^{\prime \prime}$. By transitivity, $z^{*} P z^{\prime \prime}$, violating Pareto.

## B Endogenous poverty line

The Poverty Reduction axiom in the paper is written with an exogenous poverty line. A mistaken interpretation of this assumption would be that it enters in conflict with the common practice of having it depend on the entire distribution of incomes. In this section, we prove that the assumption of an exogenous poverty line made in the paper can be replaced with the (much weaker) assumption that the poverty line depends on the distribution of incomes. More precisely, we derive a new property based on the latter assumption, and show that, combined with Pareto, it entails the Poverty Reduction property from the paper.

As a poverty line typically depends on after-tax incomes, we make it a function of the distribution of consumption levels $c_{i}$ only. To capture the fact that only the distribution of incomes matters, we further require that the poverty line function be anonymous, that is, the poverty line remains constant if incomes are permuted across agents. Finally, we also impose it to be independent of the preference profiles. Note that this last assumptions forces us to change the model into one in which the preference profile may vary. The following property formally summarizes these requirements.

## Property B. 1 Endogenous Poverty Line Poverty Reduction

There exists an anonymous poverty line function $P: \mathbb{R}_{+}^{n} \rightarrow \mathbb{R}_{++}$such that for all economies $\left(R_{1}, \ldots, R_{n}\right)$, all pairs of allocations $z=\left(z_{1}=\left(\ell_{1}, c_{1}\right), \ldots, z_{n}=\right.$ $\left.\left(\ell_{n}, c_{n}\right)\right)$ and $z^{\prime}=\left(z_{1}^{\prime}=\left(\ell_{1}^{\prime}, c_{1}^{\prime}\right), \ldots, z_{n}^{\prime}=\left(\ell_{n}^{\prime}, c_{n}^{\prime}\right)\right)$, if, for two agents $j$ and $k$ and a positive quantity $\Delta$

$$
\begin{align*}
\ell_{j} & =\ell_{j}^{\prime}=\ell_{k}=\ell_{k}^{\prime},  \tag{17}\\
c_{j}^{\prime} & =c_{j}+\Delta \leq P\left(c_{1}, \ldots, c_{n}\right) \leq c_{k}^{\prime}=c_{k}-\Delta, \tag{18}
\end{align*}
$$

whereas $z_{i}=z_{i}^{\prime}$ for all other agents, then $z^{\prime}$ is socially strictly preferred to $z$.
The main difficulty associated with this common practice of making the poverty line income depend on the income distribution is, again, that it does not take labor time into account. Therefore, in line with the general project of this paper, we study which constraint on the poverty line function $P$ can be deduced from combining this property with Pareto. As proven in the following proposition, the result is that the poverty line should be constant. That is, allowing the poverty line to depend on the distribution of incomes necessarily conflicts with individual well-being.

Proposition B. 1 If social preferences $R$ satisfy Endogenous Poverty Line Poverty Reduction and Pareto, then it satisfies Poverty Reduction.

Proof. We need to prove that there exists $p \in \mathbb{R}_{++}$such that $P\left(c_{1}, \ldots, c_{n}\right)=$ $p$ for all $\left(c_{1}, \ldots, c_{n}\right) \in \mathbb{R}_{+}^{n}$. As $P$ is anonymous, we can rename agents $j$ and $k$ as 1 and 2 . We prove the proposition in three steps.

Step 1: Let $\left(R_{1}, \ldots, R_{n}\right)$ and $z=\left(z_{1}=\left(\ell_{1}, c_{1}\right), \ldots, z_{n}=\left(\ell_{n}, c_{n}\right)\right)$ be given such that $\ell_{1}=\ell_{2}$. Let $p=P\left(c_{1}, \ldots, c_{n}\right)$. In particular, this means that $c_{1}<p<c_{2}$. In this step, we prove that $p$ can only depend on $c_{1}$ and $c_{2}$. That is, for all $z^{\prime}=\left(z_{1}^{\prime}=\left(\ell_{1}^{\prime}, c_{1}^{\prime}\right), \ldots, z_{n}^{\prime}=\left(\ell_{n}^{\prime}, c_{n}^{\prime}\right)\right)$ such that $\ell_{1}^{\prime}=\ell_{2}^{\prime}$, if $c_{1}^{\prime}=c_{1}$ and $c_{2}^{\prime}=c_{2}$, then $P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)=p$. The simple proof builds on the fact that the poverty line does not depend on labor times whereas we need to satisfy Pareto. It is illustrated in Figure B.1. The core of the proof lies in the fact that for all $i \in\{3, \ldots, n\}$, whatever $\ell_{i}, c_{i}$ and $c_{i}^{\prime}$, it is always possible to define a labor time $\ell_{i}^{\prime \prime}$ such that agent $i$ is indifferent between the bundles $z_{i}=\left(\ell_{i}, c_{i}\right)$ and $\left(\ell_{i}^{\prime \prime}, c_{i}^{\prime}\right)$, so that, by Pareto, if transferring $\Delta$ from agent 2 to agent 1 is a social improvement at $z$ it is also a social improvement at $z^{\prime}$. More formally, let $z^{\prime \prime}=\left(z_{1}^{\prime \prime}=\left(\ell_{i}^{\prime \prime}, c_{i}^{\prime \prime}\right), \ldots, z_{n}^{\prime \prime}=\left(\ell_{n}^{\prime \prime}, c_{n}^{\prime \prime}\right)\right)$ be such that $z_{1}^{\prime \prime}=z_{1}$ and $z_{2}^{\prime \prime}=z_{2}$, and both $c_{i}^{\prime \prime}=c_{i}^{\prime}$ and $z_{i}^{\prime \prime} I_{i} z_{i}$ for all $i \in\{3, \ldots, n\}$. By Pareto, $z$ and $z^{\prime \prime}$ are socially equivalent, and $\left(\left(\ell_{1}, c_{1}+\Delta\right),\left(\ell_{2}, c_{2}-\Delta\right), z_{3}, \ldots, z_{n}\right)$ is also equally good as $\left(\left(\ell_{1}^{\prime \prime}, c_{1}^{\prime \prime}+\Delta\right),\left(\ell_{2}^{\prime \prime}, c_{2}^{\prime \prime}-\Delta\right), z_{3}^{\prime \prime}, \ldots, z_{n}^{\prime \prime}\right)$. That means that the property of Endogenous Poverty Line Poverty Reduction works exactly the same way between agents 1 and 2 at $z$ and $z^{\prime \prime}$, or, to put it differently, $P\left(c_{1}^{\prime \prime}, \ldots, c_{n}^{\prime \prime}\right)=P\left(c_{1}, \ldots, c_{n}\right)=p$. Now, because $P$ only depends on the distribution of incomes, $P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)=P\left(c_{1}^{\prime \prime}, \ldots, c_{n}^{\prime \prime}\right)$. That completes the first step of the proof. Now, the definition of Endogenous Poverty Line Poverty Reduction does not allow $P$ to depend on the preferences of the agents. That means that, despite the fact that $z^{\prime \prime}$ above was constructed as a function of the preferences, the conclusion that $P\left(c_{1}, \ldots, c_{n}\right)=P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)$ is valid for all population profiles.

Step 2: We can now focus on $c_{1}$ and $c_{2}$ and prove that $P$ cannot vary if $c_{1}$ and $c_{2}$ are replaced with $c_{1}^{\prime}$ and $c_{2}^{\prime}$ such that $c_{1}^{\prime}<c_{1}<c_{2}^{\prime}<c_{2}$. Let $z=\left(z_{1}=\left(\ell_{1}, c_{1}\right), \ldots, z_{n}=\left(\ell_{n}, c_{n}\right)\right)$ and $z^{\prime}=\left(z_{1}^{\prime}=\left(\ell_{1}^{\prime}, c_{1}^{\prime}\right), \ldots, z_{n}^{\prime}=\left(\ell_{n}^{\prime}, c_{n}^{\prime}\right)\right)$ be two allocations. By step 1, we can assume, without loss of generality, that $z_{i}=z_{i}^{\prime}$ for all $i \in\{3, \ldots, n\}$. Indeed, as soon as $c_{i} \neq c_{i}^{\prime}$ for some $i$, we know that we can choose $\ell_{i}$ and $\ell_{i}^{\prime}$ so as to satisfy $z_{i} I_{i} z_{i}^{\prime}$ without affecting $P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)$, and, by Pareto, replacing $z_{i}^{\prime}$ with $z_{i}$ does not affect the social ranking. The goal of this step is to show how the social desirability of a transfer from $c_{2}$ to $c_{1}$ yields social desirability of transfers between $c_{2}^{\prime}$ to $c_{1}^{\prime}$, as long as $c_{1}^{\prime}+\Delta \leq P\left(c_{1}, \ldots, c_{n}\right) \leq c_{2}^{\prime}-\Delta$. This step is illustrated in Figure B.2. The assumption is that $c_{1}^{\prime}<c_{1}<c_{2}^{\prime}<c_{2}$. A similar proof holds for the case $c_{1}<c_{1}^{\prime}<c_{2}<c_{2}^{\prime}$. We know that for all $\Delta$, a transfer of $\Delta$ from $c_{2}$ to $c_{1}$ is a social improvement. We need to prove the same for a transfer of $\Delta^{\prime}$ from $c_{2}^{\prime}$ to $c_{1}^{\prime}$. This is proven by assuming that $\left(R_{1}, \ldots, R_{n}\right), z_{1}, z_{1}^{\prime}, z_{2}$ and $z_{2}^{\prime}$


Figure B.1: Step 1.
are such that

$$
\begin{array}{rll}
z_{1}^{\prime} & I_{1} & z_{1} \\
z_{2}^{\prime} & I_{2} & z_{2} \\
\left(\ell_{1}^{\prime}, c_{1}^{\prime}+\Delta^{\prime}\right) & I_{1} & \left(\ell_{1}, c_{1}+\Delta\right) \\
\left(\ell_{2}^{\prime}, c_{2}^{\prime}+\Delta^{\prime}\right) & I_{2} & \left(\ell_{2}, c_{2}+\Delta\right) . \tag{22}
\end{array}
$$

By Pareto, the fact that $z$ and $z^{\prime}$ are equally good for society implies that $\left(\left(\ell_{1}, c_{1}+\Delta\right),\left(\ell_{2}, c_{2}-\Delta\right), z_{3}, \ldots, z_{n}\right)$ and $\left(\left(\ell_{1}^{\prime}, c_{1}^{\prime}+\Delta^{\prime}\right),\left(\ell_{2}^{\prime}, c_{2}^{\prime}-\Delta^{\prime}\right), z_{3}^{\prime}, \ldots, z_{n}^{\prime}\right)$ are also equally good. That proves that for all $\Delta^{\prime}$ such that $c_{1}^{\prime}+\Delta^{\prime} \leq$ $P\left(c_{1}, \ldots, c_{n}\right) \leq c_{2}^{\prime}-\Delta^{\prime}$, this transfer of $\Delta^{\prime}$ is a social improvement, that is $P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)=P\left(c_{1}, \ldots, c_{n}\right)$.

Step 3: We now assume that $c_{1}^{\prime}<c_{1}<c_{2}<c_{2}^{\prime}$. A similar proof holds for the case $c_{1}<c_{1}^{\prime}<c_{2}^{\prime}<c_{2}$. This step is illustrated in Figure B.3. The construction used in step 2 is impossible. Indeed, having (19), (20), (21) and (22) satisfied together with $c_{1}^{\prime}<c_{1}<c_{2}<c_{2}^{\prime}$ would conflict with the monotonicity of preferences. The proof goes by replicating the argument of step 2 twice and using a third allocation $z^{\prime \prime}=\left(z_{1}^{\prime \prime}=\left(\ell_{1}^{\prime \prime}, c_{1}^{\prime \prime}\right), \ldots, z_{n}^{\prime \prime}=\left(\ell_{n}^{\prime \prime}, c_{n}^{\prime \prime}\right)\right)$ such that $\ell_{1}^{\prime \prime}=\ell_{2}^{\prime \prime}, z_{i}=z_{i}^{\prime}$ for all $i \in\{3, \ldots, n\}$ and

$$
\begin{aligned}
c_{1}^{\prime \prime}<c_{1} & \leq P\left(c_{1}, \ldots, c_{n}\right) \leq c_{2}^{\prime \prime}<c_{2} \\
c_{1}^{\prime \prime}<c_{1}^{\prime} & \leq P\left(c_{1}, \ldots, c_{n}\right) \leq c_{2}^{\prime \prime}<c_{2}^{\prime} .
\end{aligned}
$$



Figure B.2: Step 2: the case in which $c_{1}<c_{1}^{\prime}<c_{2}<c_{2}^{\prime}$.

First, the argument of step 2 is used to prove that $P\left(c_{1}^{\prime \prime}, \ldots, c_{n}^{\prime \prime}\right)=P\left(c_{1}, \ldots, c_{n}\right)$. Second, it is used to prove that $P\left(c_{1}^{\prime \prime}, \ldots, c_{n}^{\prime \prime}\right)=P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)$. Obviously, though, the two arguments can only hold under different assumptions on preferences. To have a notation consistent with the figure, let's assume that the first part of the argument holds for profile $\left(R_{1}, \ldots, R_{n}\right)$ and the second part for $\left(R_{1}^{\prime}, \ldots, R_{n}^{\prime}\right)$. The key point is that the $P$ function does not depend on preferences. This is why the relationship $P\left(c_{1}^{\prime \prime}, \ldots, c_{n}^{\prime \prime}\right)=P\left(c_{1}^{\prime}, \ldots, c_{n}^{\prime}\right)=$ $P\left(c_{1}, \ldots, c_{n}\right)$ must hold for all preference profiles, what needed to be proven.

The proof of Proposition B. 1 crucially used the fact that social preferences are required to rank all allocations. This is why for a given income distribution, we had the freedom to fix labor time so as to be able to use Pareto. Another approach would be to look at the possibility of endogenizing the poverty line while restricting our attention to incentive compatible allocations. Of course, a proof like the one above would not be possible, because amounts of money can only be transferred between two agents through changes in the tax scheme, and that affects more than two agents.

We cannot, therefore, prove that making the poverty line depend on the income distribution would conflict with Pareto. We can prove, though, that it would not be in the interest of the poor. This is illustrated in Figure B.4, in which we apply the criterion that we obtained in Section 4 of the paper and the poverty line is fixed at $60 \%$ of the median income. We assume a three-agent economy. Allocations $\left(z_{1}, z_{2}, z_{3}\right)$ and $\left(z_{1}^{\prime}, z_{2}^{\prime}, z_{3}^{\prime}\right)$ are generated by


Figure B.3: Step 2: the case in which $c_{1}^{\prime}<c_{1}<c_{2}<c_{2}^{\prime}$.


Figure B.4: An endogenous poverty line could hurt the poor.
tax schemes $\tau$ and $\tau^{\prime}$ respectively. Median incomes are $c_{2}$ and $c_{2}^{\prime}$ respectively, so that the poverty lines are $p$ and $p^{\prime}$. Our criterion suggests to look at $y_{1}$ and $y_{1}^{\prime}$, and $y_{1}<y_{1}^{\prime}$ implies that we should prefer $\tau$ over $\tau^{\prime}$, according to our criterion, whereas $z_{1}^{\prime} P_{1} z_{1}$ (preferences $R_{2}^{*}$ and $R_{3}^{*}$ are not represented on the graph for the sake of clarity). Again, poverty is reduced at the expense of the well-being of the poor. Avoiding this paradox requires to fix $p$ independently of the generated income distribution.

## C Methodology: Constructing budget curves with the OECD tax-benefit calculator

This appendix provides a detailed description of the methodology used in Section 6 of the paper. In order to apply the criterion developed in the paper, we need to draw the actual budgets that agents face. Given that the tax function typically depends on the composition of the households, we partition the population into household types. Budgets are drawn using the OECD tax-benefit calculator which takes account of all relevant regulatory aspects that transform pre-tax incomes into after-tax incomes in OECD countries ${ }^{\text {P }}$

Selection of countries and systems. In our baseline application, we evaluate the ability of tax-transfer policies to alleviate poverty given official poverty lines, the official notion of disposable income and the legal minimum wage. As a result, we restrict our sample to the EU15 countries that have a legal minimum wag $母^{2}$ and the United States (US) $]^{3}$ and calculate budgets for tax-transfer rules of 2013.

Policies modeled. Some of the policies we are interested in are conditional. Our aim is to try to draw the budgets of the poorest people. As a result, when a benefit is means-tested, we assume that the conditions are satisfied. When the conditions do not bear on means, we assume they are not satisfied. Precisely, the tax-transfer policies that are taken into account are income support and social assistance (SA).$^{4}$ family and child benefits (FB).$^{5}$

[^1]housing benefits (HB), in-work benefits (IW), labor income taxes (IT) and social insurance contributions (SC). ${ }^{6}$ Unemployment insurance benefits are not taken into account as they are typically conditional on past labor force participation and social contributions. As a result, young or long-term unemployed people typically do not benefit from it. Unemployment assistance benefits, which are not based on previous contributions, are considered to be part of SA. Finally, we assume that the policies implemented to fight against non take-up and fiscal evasion are distinct from the definition of the tax-transfer system itself and thus abstract from the latter phenomena here.

Regional heterogeneity. When taxes or benefits vary across region, the OECD calculator chooses a region that it considers typical. For instance, the whole tax-transfer system of Michigan, a typical manufacturing region, is used to represent the US (see footnote 3).

Household types, wages, labor time. We consider six household types: singles and couples without children, with one child (aged 10), with two children (aged 10 and 12). $7^{7}$ All adults belong to the working-age population and we assume away any specific needs for adults or children due to, e.g., disability, sickness or invalidity. The wage earned is the legal monthly minimum wage in each country in 2013 (applying to the situation of January 1st), as reported by EUROSTAT (2015a). Germany introduced a legal minimum wage in 2015. The wage for Germany is this minimum wage deflated to 2013. In the OECD calculator, average wages (AW) are the key input data. Minimum wages therefore need to be defined as a percentage of AW and AWs will also be needed for the specification of various policies, e.g. to define certain thresholds. Table C.1 shows the minimum wages by country together with AWs calculated by the OECD (2019). For single earners, disposable income is derived on the basis of the labor income that is earned when increasing hours worked from zero to full-time. Given that the OECD calculator can only assume a fixed number of hours worked for the second earner, we construct the budget curves for couples combining two different situations. First, we assume that the second earner does not work at all and the principal earner varies her hours from zero to full-time, such that the

[^2]household earns once the minimum wage. Second, at this point the curve is continued under the assumption that the second earner works full-time and the principal earner again varies her hours from zero to full-time, such that at this latter point the household earns twice the minimum wage and works twice the full-time of a single earner.

|  | $A W$ | $w_{m}$ | $\frac{w_{m}}{A W}$ |
| :--- | :---: | :---: | :---: |
| US | $4,064.5$ | $1,265.0$ | 31.1 |
| BE | $3,849.8$ | $1,994.6$ | 39.0 |
| FR | $3,067.3$ | $1,899.5$ | 46.6 |
| GE | $3,725.0$ | $1,938.8$ | 39.2 |
| GR | $1,730.5$ | 908.1 | 39.5 |
| IR | $2,762.8$ | $1,941.5$ | 52.9 |
| LU | $4,457.1$ | $2,489.1$ | 42.0 |
| NL | $4,035.1$ | $1,951.5$ | 36.4 |
| PT | $1,470.0$ | 751.5 | 38.5 |
| SP | $2,168.9$ | 999.9 | 34.7 |
| UK | $2,923.4$ | $1,659.9$ | 36.3 |

Note: $A W$ is the average wage. $w_{m}$ is the legal minimum wage. Germany introduced a legal minimum wage in 2015. $w_{m}$ for Germany is this minimum wage deflated to 2013. $A W$ and $w_{m}$ in monthly USD. $\frac{w_{m}}{A W}$ in \%. Sources: OECD and EUROSTAT.

Table C.1: Average wage and legal minimum wage by country
Further assumptions. Further assumptions are made within the OECD tax-benefit calculator to implement the different policies. We report a few main assumptions in the following.
a) Time aspects. All tax-transfer amounts relate to the year of 2013 and are generally computed using the rules and regulations that were in force on July 1, 2013. All taxes, benefits and net incomes are thus determined for a particular month. Benefits and income taxes, which depend on annual incomes, are determined in relation to the annualized amounts (i.e. multiplied by 12). Any timing issues in the assessment of claimants' entitlement or the payment of benefits are disregarded.
b) Specific assumptions on in-work benefits. By construction of the policy, in-work benefits are paid only to those with earnings or those who have worked more than a certain number of hours per week. Thus, our assumptions made above about hours worked and wages earned determine the level of employment-conditional benefits. Delays in payment of benefits (which can be long for IW) are ignored.
c) Specific assumptions on housing benefits. From the countries under analysis, FR, GE, IR, LU, NL and the UK grant housing benefits (HB). Cash HB are particularly complex and might depend on various characteristics such as the rent, the size of dwelling and the region, amongst several others. Given this complexity, it is not possible to derive typical or average HB for low-income households with the OECD calculator. Due to our income definition above, in our baseline scenario we nevertheless include HB and make three main simplifying assumptions: i) families live in privately rented accommodation; ii) where size is relevant and where housing benefits vary across region, we stick to the default choice of the OECD calculator; iii) for the level of rent, the OECD calculator's default assumption is $20 \%$ of AW for the housing cost (the OECD average), regardless of country, household type or income position. This is too coarse for our purpose. To arrive at rent levels that are somewhat more specific to our context, we use information from EUROSTAT (2016) on what share of their disposable income lowincome households (below $60 \%$ of median equivalized disposable income) use to pay privately rented accommodation. We proxy household disposable income needed to implement this information with the household specific mean income (for minimum-wage households as defined above) that is derived with the OECD calculator when housing benefits are assumed away. The derived housing costs need to be implemented as percentage of AW in the OECD calculator. Tables C. 2 and C.3 show the share of disposable income used for rents and the accordant share of AW. The former information is available for single and couple households with or without children. Therefore, the level of $\frac{r e n t}{n e t}$ is the same for singles with one child and two children (couples with one child and two children) in a given country. We show the information for all countries under analysis (and not only for those that grant housing benefits) because the level of rent sometimes determines other policies in some countries, e.g. the calculation of social assistance in the US. The EUROSTAT data only covers the European countries under analysis and we do not have similar information for the US. We use the household-type specific average $\frac{r e n t}{A W}$ over all other countries for the US (not shown in Tables C.2 and C.3).

|  | $\frac{\text { rent }}{n e t}$ | $\frac{\text { rent }}{\text { AW }}$ |
| :---: | :---: | :---: |
| BE s0ch | 46.90 | 15.21 |
| BE s1ch | 30.50 | 11.72 |
| BE s2ch | 30.50 | 13.81 |
| BE c0ch | 38.30 | 26.09 |
| BE c1ch | 32.35 | 23.67 |
| BE c2ch | 32.35 | 26.19 |
| FR s0ch | 47.80 | 18.20 |
| FR sch | 32.00 | 14.64 |
| FR s2ch | 32.00 | 16.16 |
| FR c0ch | 37.80 | 30.38 |
| FR c1ch | 31.05 | 26.17 |
| FR c2ch | 31.05 | 27.40 |
| GE s0ch | 40.30 | 11.59 |
| GE s1ch | 31.50 | 12.12 |
| GE s2ch | 31.50 | 15.99 |
| GE c0ch | 35.10 | 20.76 |
| GE c1ch | 28.50 | 19.55 |
| GE c2ch | 28.50 | 22.14 |
| GR s0ch | 71.50 | 23.58 |
| GR s1ch | 68.40 | 23.69 |
| GR s2ch | 68.40 | 24.82 |
| GR c0ch | 63.30 | 45.93 |
| GR c1ch | 52.30 | 39.67 |
| GR c2ch | 52.30 | 41.40 |
| IR s0ch | 30.80 | 15.59 |
| IR s1ch | 21.80 | 17.54 |
| IR s2ch | 21.80 | 20.91 |
| IR c0ch | 39.70 | 40.36 |
| IR c1ch | 28.00 | 33.37 |
| IR c2ch | 28.00 | 36.83 |

Note: $\frac{r e n t}{n e t}$ is the share of disposable household income paid for the rent in \%. $\frac{r e n t}{A W}$ is the rent as share of the average wage in \%. s0ch denotes a low-income single household without children, ..., c2ch a low-income couple household with two children. Sources: OECD and EUROSTAT.

Table C.2: Rents paid as percentage of disposable income and average wage, by country and household type - Part 1

|  | $\frac{\text { rent }}{\text { net }}$ | $\frac{\text { rent }}{\text { AW }}$ |
| :--- | :---: | :---: |
| LU s0ch | 43.60 | 16.47 |
| LU s1ch | 39.20 | 18.19 |
| LU s2ch | 39.20 | 22.67 |
| LU c0ch | 38.10 | 32.36 |
| LU c1ch | 29.25 | 28.06 |
| LU c2ch | 29.25 | 31.73 |
| NL s0ch | 49.70 | 14.80 |
| NL s1ch | 35.40 | 15.21 |
| NL s2ch | 35.40 | 16.17 |
| NL c0ch | 44.30 | 26.23 |
| NL c1ch | 30.50 | 20.70 |
| NL c2ch | 30.50 | 21.79 |
| PT s0ch | 31.30 | 10.72 |
| PT s1ch | 34.30 | 12.64 |
| PT s2ch | 34.30 | 13.81 |
| PT c0ch | 29.10 | 19.56 |
| PT c1ch | 48.75 | 34.49 |
| PT c2ch | 48.75 | 36.37 |
| SP s0ch | 53.10 | 17.26 |
| SP s1ch | 57.00 | 19.16 |
| SP s2ch | 57.00 | 19.80 |
| SP c0ch | 54.40 | 35.35 |
| SP c1ch | 50.55 | 33.23 |
| SP c2ch | 50.55 | 33.61 |
| UK s0ch | 58.10 | 19.21 |
| UK s1ch | 38.10 | 19.43 |
| UK s2ch | 38.10 | 23.14 |
| UK c0ch | 47.00 | 32.62 |
| UK c1ch | 44.95 | 35.91 |
| UK c2ch | 44.95 | 41.75 |
|  |  |  |

Note: $\frac{r e n t}{n e t}$ is the share of disposable household income paid for the rent in \%. rent $\frac{\text { is }}{A W}$ the rent as share of the average wage in \%.s0ch denotes a low-income single household without children, ..., c2ch a low-income couple household with two children. Sources: OECD and EUROSTAT.

Table C.3: Rents paid as percentage of disposable income and average wage, by country and household type - Part 2

Budget curve decomposition. Figures C. 1 and C. 2 show exemplarily for the US, how budget curves for the different household types evolve as the sum of the different tax-transfer components. SA mainly includes the Supplemental Nutrition Assistance Program (SNAP), which is designed to increase the food purchasing power of eligible low-income households (the former Food Stamps). It is dependent on household size and there is an income threshold for receiving SNAP. IW refers to the Earned Income Tax Credit (EITC) which is a refundable tax credit. Eligible for EITC are working families with children under 24 years of age and childless working persons aged between 25 and 65 that meet certain income thresholds. There are no cash HB. FB refer to the Temporary Assistance for Needy Families (TANF). Each state may establish its own benefit levels and determine its own benefit calculation. Michigan TANF are increasing in the number of family members and are somewhat above the average for all states. TANF is income dependent and Michigan applies certain income disregards in the calculation of the benefit. Concerning IT, there are several tax reliefs applicable, especially related to children. In 2013, the rate for employee SC was $7.65 \%$.


All values in USD per month.
Figure C.1: Budget curve decomposition for US single households

US (2013)
Couple without children

Couple with one child

Couple with two children


All values in USD per month.
Figure C.2: Budget curve decomposition for US couple households

Poverty lines and equivalence scales. Finally, in order to evaluate tax schemes with respect to poverty alleviation, we have to introduce poverty thresholds. In our baseline scenario, we stick to official poverty thresholds. 8 In the Europe 2020 strategy (European Commission, 2010), the European member states have agreed to use $60 \%$ of national median equivalized disposable income as the at-risk-of-poverty indicator. We thus use this poverty threshold for the European countries in our analysis, as reported by EUROSTAT (2015b), As the needs of a household increase with the number of its household members in a non-proportional way due to economies of scale in consumption, we also have to adjust poverty thresholds in an appropriate way to households of different size when comparing them in Appendix D. The equivalence scale applied for the European countries is the modified OECD equivalence scale which assigns a value of 1 to the household head, of 0.5 to each additional adult member and of 0.3 to each child. Thus, the poverty threshold for a single-parent household with two children, for instance, will equal the poverty threshold of a pure single household multiplied by a factor of 1.6. For the US, we rely on the Supplemental Poverty Measure reported by the U.S. Census Bureau and an accordant specific and more complex threeparameter equivalence scale to account for family size ${ }_{-}^{9}$ In Appendix E.1, we compare our baseline results to the assumption of a poverty line that is $50 \%$ of the mean income in each country.

[^3]
## D Does it help to have children?

In this appendix we compare whether it is easier for households with or without children to get out of poverty. We do that by rescaling budget curves so as to measure equivalized disposable income on the vertical axis as described in Appendix C. We find that Germany, the UK and the US are the only countries in which social welfare is unambiguously higher among households with children ${ }^{10}$

Figure D. 1 compares budget curves of US singles on the left and couples on the right, with zero, one and two children. In both cases, the labor time that is required for those households to reach a disposable income equal to the poverty line is lower when there are children in the household. Maybe surprisingly, that does not come from differences in the shape of the budget curve, but from differences in the basic income. That is, the in-work benefits (the EITC) simply adjust the shape of the budget curves to the equivalence scale, so that they are almost parallel to each other. Maybe contrary to the received wisdom, the difference rather comes from the family benefits on the one hand and the social assistance that is more generous for households with children on the other hand (see decomposition above, Figures C.1 and C.2).

Greece, Portugal and Spain are found to be the countries in our sample in which social welfare is unambiguously higher among households without children. In all cases, this is due to the simplicity and lack of generosity of the tax-transfer system. Figure D. 2 shows the budget curves for all household types in Spain. The system is characterized by limited social assistance, which fades out at a tax rate of $100 \%$, above which no benefit exists anymore. The difference in slopes that we can see in the graphs reflect the rescaling of disposable incomes according to the equivalence scale.

The other countries do not offer any clear pattern. In France, for instance, family benefits are more generous for the second child of a single-parent household than for the first child, but social assistance is lower for single parents with two children than with one. All in all, the budget curve of a single-parent with one child makes it easier for that household to get out of poverty than for singles without or with two children, as can be seen in Figure D.3. For couples, on the contrary, the labor time it takes to get out of poverty increases with the number of children.

[^4]US (2013)




$$
\text { couple: - without children - with one child } \quad \text { - with two children }
$$

Wage $=31.1 \%$ of AW (= legal minimum wage $)$.
Figure D.1: Budget curves for single and couple households with zero, one or two children: US

SP (2013)


Wage $=34.7 \%$ of AW (= legal minimum wage) .
Figure D.2: Budget curves for single and couple households with zero, one or two children: SP

FR (2013)


Wage $=46.6 \%$ of AW (= legal minimum wage $)$.
Figure D.3: Budget curves for single and couple households with zero, one or two children: FR

## E Extensions

The application provided in the paper was developed under the assumptions that i) the relevant poverty line was the official one, ii) housing benefits are granted if existent in a country, iii) individual labor supplies were not rationed. We remove these three assumptions in this appendix.

## E. 1 Mean income poverty lines

All official poverty lines in the European countries under analysis are defined as $60 \%$ of the equivalized median income. Median income is determined both by the general living standards of a country and by the income inequality in that country. To remove the impact of income inequality onto the poverty line and make it depend only on the ability of the economy to produce income, we change the poverty line from the official ones to $50 \%$ of the mean income, taken from EUROSTAT (2015b), As a consequence, countries with a larger income inequality are expected to do worse than previously according to our measure of social welfare.

Figure E. 1 is the variant of Figure 9 in the paper when the vertical axis is rescaled to take country mean income into account. Differences across countries become much larger compared to the baseline scenario. Most striking is the fact that the budget set of single parents with two children in the US is now the lowest one, dominated even by Greece (except in a small interval of incomes), Portugal and Spain. In those countries, welfare systems have suffered a lot from the sovereign debt crisis or are known for their quasi-absence of important benefits, e.g., the lack of a basic-income scheme in Greece. In Appendix F.5, we show that budget curves for other types of households display slightly different patterns, with the US always belonging to the set of the two or three worst performing countries.

## E. 2 Removing housing benefits

As outlined in Appendix C] among the tax-transfer policies that we simulate, housing benefits (HB) require most assumptions (e.g. related to housing cost, housing size, reference region). It is also the policy with the most demanding eligibility conditions. In addition to that, it may induce a bias in our country comparisons, because countries that do not offer HB in cash may have a large and generously subsidized social housing (such as Belgium) directed towards the poorest. This subsidy is not added to the disposable income in those countries within our simulations. It may be interesting, therefore, to apply our criterion to a notion of disposable income that does not include HB.

Budget schemes - single with two children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure E.1: Cross-country comparison of entire budget curves with $50 \%$ of mean income poverty line for single households with two children

Table E. 1 reproduces Table 1 in the paper for the six countries under analysis that provide HB , when the eligibility to HB is assumed away ${ }^{11}$ Removing HB from the computation of disposable incomes has two effects. First, it decreases $b$, the disposable income of those who do not work. That effect varies between modest 157.69 USD in Luxemburg and the huge amount of 1016.49 USD in the UK, which equals the amount of HB granted at zero earnings for singles with two children.

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 119.10 | $2,262.25$ | $1,185.59$ | 53.18 | $1,899.48$ | $2,225.79$ | 85.34 | 53.27 |
| GE | 67.09 | $1,300.80$ | $1,337.20$ | 45.10 | $1,938.84$ | $2,076.62$ | 93.37 | 64.39 |
| IR | 13.99 | 271.59 | $1,770.27$ | 45.89 | $1,941.48$ | $2,025.62$ | 95.85 | 87.39 |
| LU | 24.05 | 598.57 | $3,038.07$ | 84.76 | $2,489.11$ | $3,538.16$ | 70.35 | 85.87 |
| NL | 87.91 | $1,715.61$ | $1,941.74$ | 74.69 | $1,951.51$ | $2,213.57$ | 88.16 | 87.72 |
| UK | 39.94 | 663.06 | $1,494.22$ | 58.52 | $1,659.93$ | $1,986.20$ | 83.57 | 75.23 |

Note: $W_{\min }^{p}$ in \% of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in \%. All other values in monthly USD.
Table E.1: Social welfare and its decomposition when eligibility for housing benefits is removed, for single households with two children

Second, it decreases $t$, because HB typically fade out as income increases. This effect is the weakest (namely zero) in the Netherlands and Luxemburg, where HB do not depend on income as long as income is below a threshold much larger than the minimum wage. It is the largest in Ireland, where HB decrease with earned income at a one-to-one dollar rate, decreasing the effective tax rates on low incomes by 29.48 percentage points.

As a result, the effect on social welfare (first column) is the largest for the Netherlands. Because this country has a $100 \%$ marginal tax rate on low incomes, its performance on our measure of social welfare is extremely sensitive to whether or not the basic income is lower or larger than the poverty line. When we subtract HB from the definition of disposable income, it decreases below the poverty line, making it much harder for low-skill lone parents with two children to earn sufficient income to reach the poverty line.

[^5]At the other extreme, because HB fade out quickly even for low incomes in Ireland, the labor time it takes to reach the poverty line does not increase by much when HB are not taken into account. This illustrates that social welfare as we measure it is much more sensitive to changes in the definition of disposable income when tax and transfers are designed in such a way that $t$ is low. It also means that partial take-up or variations in eligibility conditions are more likely to have limited impact on social welfare when $t$ is low.

## E. 3 Taking unemployment rates into account

So far we have made the assumption that the earners in a household are free to choose their labor time. There are many reasons why this could not be the case, in particular in countries experiencing a high unemployment rate. The figures that we provided up to now should then be read as evaluating the ability of tax-transfer systems to help people get out of poverty conditional on them having found a job. This is consistent with the idea that tax-transfer systems should not depend on short-term fluctuations on the labor market. Consequently, as mentioned above, unemployment insurance benefits are not taken into account in the budget curves presented here.

One may argue, though, that the possibility to find jobs should be part of the evaluations of the opportunities that are given to households to get out of poverty. We apply that idea in this appendix. The expected gross income of such an individual willing to work full-time at the minimum wage is simply the minimum wage weighted by the probability to find a job, approximated in our application by one minus the unemployment rate of low-skill individuals.

Unemployment rates of low-skill individuals vary between $9 \%$ in Luxemburg and the Netherlands and $33 \%$ in Spain in $2013{ }^{12}$ Low skill means not having completed high school. Figure E. 2 is the variant of Figure 9 in the paper when the horizontal axis is rescaled to take country-specific unemployment rates of low-skill individuals into account. The main lesson is that the picture does not change much. On the one hand, the countries with the tax-transfer system offering the least opportunities to get out of poverty, Greece, Portugal and Spain, are also the countries experiencing the highest unemployment rates. On the other hand, the countries providing the highest social welfare are also those offering high basic incomes, and they are not affected when unemployment rates are taken into account. Countries with an intermediary level of social welfare do not differ much in their unemployment rate of low-skill individuals and its effect on social welfare.

[^6]Budget schemes - single with two children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure E.2: Cross-country comparison of entire budget curves when minimum wages are adjusted for unemployment rates, for single households with two children

## F Full set of results

This appendix presents the full set of results of the applications presented in the paper and in Appendices Cto Elabove. Those applications focus on the analysis of tax schemes for single-parent households with two children and in Appendices $C$ and $D$ on selected countries. This appendix presents results for all countries and all household types as specified in Appendix C.

Precisely, Appendix F. 1 presents for all countries and household types simulated budget curves and their decomposition into the various tax-transfer components. Appendix F. 2 presents household-type specific tables with the main statistic for all countries, i.e. the social welfare measure and its components. Appendix F. 3 shows household-type specific figures that extend the cross-country comparison to entire budget curves. Appendix F. 4 presents country-specific figures that show whether single households on the one hand and couple households on the other hand are treated differently depending on the number of children in the household. In Appendix F. 5 the extended cross-country comparison is done with the assumption of a poverty threshold that is $50 \%$ of mean income. Appendix F. 6 shows the social welfare statistic when housing benefits are assumed away. Finally, Appendix F. 7 does the extended cross-country comparison when taking into account that household earners might need some time in order to find a job, before deciding about their hours of work.

## F. 1 Budget curve decomposition

Figures F. 1 to F. 22 show for all countries and household types how budget curves evolve as the sum of the different tax-transfer policies described above. They correspond to Figures C.1 and C. 2 in Appendix C. As mentioned there, unemployment insurance benefits (UB) could be modeled with the OECD tax-benefit calculator but are not taken into account here. That is why for all countries UB are still shown in the legend but no corresponding curve is drawn. The only exception, however, is the UK. There, the Jobseeker's Allowance has a part that is contribution-based (i.e. an insurance benefit) but also a part that is income-based (i.e. means-tested and thus an assistance benefit). Where lone parents are eligible to Income Support (SA), other family types claim this latter type of Jobseeker's Allowance. It is thus modeled but labeled under UB in the OECD calculator.
Single without children

Single with one child

Single with two children


All values in USD per month.
Figure F.1: Budget curve decomposition for single households: US

US (2013)
Couple without children

Couple with one child

Couple with two children


All values in USD per month.
Figure F.2: Budget curve decomposition for couple households: US

BE (2013)
Single without children

Single with one child

Single with two children


All values in USD per month.
Figure F.3: Budget curve decomposition for single households: BE

BE (2013)
Couple without children

Couple with one child


Couple with two children


All values in USD per month.
Figure F.4: Budget curve decomposition for couple households: BE

FR (2013)
Single without children

Single with one child


Single with two children


All values in USD per month.
Figure F.5: Budget curve decomposition for single households: FR

FR (2013)
Couple without children

Couple with one child


Couple with two children


All values in USD per month.
Figure F.6: Budget curve decomposition for couple households: FR

GE (2013)
Single without children

Single with one child


Single with two children


All values in USD per month.
Figure F.7: Budget curve decomposition for single households: GE

GE (2013)
Couple without children

Couple with one child


Couple with two children


All values in USD per month.
Figure F.8: Budget curve decomposition for couple households: GE

GR (2013)


All values in USD per month.
Figure F.9: Budget curve decomposition for single households: GR

GR (2013)


Couple with one child

Couple with two children


All values in USD per month.
Figure F.10: Budget curve decomposition for couple households: GR

## IR (2013)



All values in USD per month.
Figure F.11: Budget curve decomposition for single households: IR

## IR (2013)

Couple without children

Couple with one child

Couple with two children


All values in USD per month.
Figure F.12: Budget curve decomposition for couple households: IR

## LU (2013)

Single without children


Single with one child


Single with two children


All values in USD per month.
Figure F.13: Budget curve decomposition for single households: LU

## LU (2013)



All values in USD per month.
Figure F.14: Budget curve decomposition for couple households: LU

NL (2013)
Single without children

Single with one child


Single with two children


All values in USD per month.
Figure F.15: Budget curve decomposition for single households: NL

NL (2013)
Couple without children

Couple with one child


Couple with two children


All values in USD per month.
Figure F.16: Budget curve decomposition for couple households: NL


All values in USD per month.
Figure F.17: Budget curve decomposition for single households: PT
PT (2013)

Couple with one child


Couple with two children


All values in USD per month.
Figure F.18: Budget curve decomposition for couple households: PT

SP (2013)


Single with one child

Single with two children


All values in USD per month.
Figure F.19: Budget curve decomposition for single households: SP

SP (2013)


All values in USD per month.
Figure F.20: Budget curve decomposition for couple households: SP

UK (2013)
Single without children

Single with one child

Single with two children


All values in USD per month.
Figure F.21: Budget curve decomposition for single households: UK

UK (2013)


All values in USD per month.
Figure F.22: Budget curve decomposition for couple households: UK

## F. 2 Measuring and decomposing social welfare

Tables F. 1 to F. 6 show separately for each household type the measure of social welfare derived in the paper and its components. They correspond to Table 1 in the paper.

For the sake of completeness, we repeat the description of the statistics from the paper. The first column of each table shows $W_{\min }^{p}$, the measure of social welfare according to social preferences $R^{l e x}$, as percentage of a fulltime job. The second column shows the pre-tax income corresponding to this measure, $W_{\min }^{p} w_{m}$. The next columns of the tables decompose $W_{\min }^{p}$ into the three policy parameters that determine it. The first one is what can be called the "basic" income, $b$, that is, the disposable income of those who do not earn anything. It gives us the level of the opportunity set available to those who do not work. The second one is the rate $t$ at which low incomes are effectively taxed. It shows, given any additional dollar earned, how much of it is taken away by the tax-transfer system, on average, below the minimum wage. This tool gives us a summary of the shape of the opportunity set of low-skill individuals, that is, of how their labor is rewarded. The third one is the minimum wage itself, $w_{m}$, but its effect on $W_{\min }^{p}$ is best seen when it is expressed as a percentage of the poverty line, $\frac{w_{m}}{p}$, which is done in the last column. A level below $100 \%$ indicates that the level of the minimum wage is below that of the poverty line and that in this case, a household will not be able to reach the poverty line without additional benefits. The contrary is the case for a level above $100 \%$.

If the marginal tax rate were constant over low incomes at $t$, then our measure of social welfare would satisfy the equation $p=b+W_{\min }^{p} w_{m}(1-t)$, which gives us

$$
W_{\min }^{p}=\frac{p-b}{w_{m}(1-t)},
$$

illustrating how the combination of the three policy parameters, $b, t$ and $w_{m}$ determine social welfare and how they can be used to increase it. Social assistance, family benefits and housing benefits typically determine $b$. How these benefits fade out when gross income increases, in-work transfers, income tax and social security payments together determine $t$. Finally, $w_{m}$ is a direct policy instrument.

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| US | 70.10 | 886.69 | 200.05 | 27.92 | $1,264.96$ | 971.11 | 130.26 | 20.60 |
| BE | 80.00 | $1,595.65$ | $1,064.26$ | 70.38 | $1,994.57$ | $1,427.77$ | 139.70 | 74.54 |
| FR | 83.91 | $1,593.77$ | 948.37 | 67.62 | $1,899.48$ | $1,391.12$ | 136.54 | 68.17 |
| GE | 35.97 | 697.39 | $1,055.84$ | 79.45 | $1,938.84$ | $1,297.89$ | 149.38 | 81.35 |
| GR | 73.92 | 671.31 | 0.00 | 16.50 | 908.10 | 556.27 | 163.25 | 0.00 |
| IR | 0.00 | 0.00 | $1,469.93$ | 78.68 | $1,941.48$ | $1,266.01$ | 153.35 | 116.11 |
| LU | 13.10 | 325.96 | $1,932.25$ | 84.17 | $2,489.11$ | $2,211.35$ | 112.56 | 87.38 |
| NL | 0.00 | 0.00 | $1,609.45$ | 84.03 | $1,951.51$ | $1,383.48$ | 141.06 | 116.33 |
| PT | 82.08 | 616.80 | 236.60 | 42.48 | 751.48 | 542.99 | 138.40 | 43.57 |
| SP | 97.12 | 971.05 | 498.77 | 49.67 | 999.86 | 897.99 | 111.34 | 55.54 |
| UK | 0.00 | 0.00 | $1,306.87$ | 80.11 | $1,659.93$ | $1,241.38$ | 133.72 | 105.28 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.1: Social welfare and its decomposition for single households without children

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| US | 52.09 | 658.92 | 770.21 | 29.20 | $1,264.96$ | $1,465.41$ | 86.32 | 52.56 |
| BE | 90.00 | $1,795.11$ | $1,750.35$ | 86.05 | $1,994.57$ | $1,856.11$ | 107.46 | 94.30 |
| FR | 57.94 | $1,100.55$ | $1,336.09$ | 68.37 | $1,899.48$ | $1,808.45$ | 105.03 | 73.88 |
| GE | 3.06 | 59.35 | $1,639.35$ | 77.67 | $1,938.84$ | $1,687.25$ | 114.91 | 97.16 |
| GR | 90.89 | 825.34 | 102.58 | 16.59 | 908.10 | 723.16 | 125.57 | 14.19 |
| IR | 0.00 | 0.00 | $1,885.58$ | 68.73 | $1,941.48$ | $1,645.81$ | 117.96 | 114.57 |
| LU | 15.00 | 373.37 | $2,557.23$ | 85.74 | $2,489.11$ | $2,874.76$ | 86.59 | 88.95 |
| NL | 0.00 | 0.00 | $2,143.99$ | 74.69 | $1,951.51$ | $1,798.53$ | 108.51 | 119.21 |
| PT | 97.92 | 735.86 | 368.34 | 53.01 | 751.48 | 705.89 | 106.46 | 52.18 |
| SP | 121.90 | $1,218.85$ | 680.61 | 64.49 | 999.86 | $1,167.39$ | 85.65 | 58.30 |
| UK | 0.00 | 0.00 | $1,937.19$ | 77.68 | $1,659.93$ | $1,613.79$ | 102.86 | 120.04 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.2: Social welfare and its decomposition for single households with one child

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| US | 51.13 | 646.72 | $1,018.28$ | 7.02 | $1,264.96$ | $1,739.71$ | 72.71 | 58.53 |
| BE | 97.95 | $1,953.65$ | $2,128.09$ | 84.42 | $1,994.57$ | $2,284.44$ | 87.31 | 93.16 |
| FR | 97.00 | $1,842.41$ | $1,571.74$ | 62.58 | $1,899.48$ | $2,225.79$ | 85.34 | 70.61 |
| GE | 0.00 | 0.00 | $2,116.80$ | 68.25 | $1,938.84$ | $2,076.62$ | 93.37 | 101.93 |
| GR | 107.09 | 972.47 | 182.87 | 12.58 | 908.10 | 890.04 | 102.03 | 20.55 |
| IR | 0.00 | 0.00 | $2,353.30$ | 75.37 | $1,941.48$ | $2,025.62$ | 95.85 | 116.18 |
| LU | 15.95 | 397.07 | $3,195.76$ | 84.76 | $2,489.11$ | $3,538.16$ | 70.35 | 90.32 |
| NL | 0.00 | 0.00 | $2,321.35$ | 74.69 | $1,951.51$ | $2,213.57$ | 88.16 | 104.87 |
| PT | 112.99 | 849.07 | 495.40 | 61.36 | 751.48 | 868.79 | 86.50 | 57.02 |
| SP | 146.97 | $1,469.54$ | 771.64 | 70.32 | 999.86 | $1,436.79$ | 69.59 | 53.71 |
| UK | 0.00 | 0.00 | $2,510.71$ | 77.68 | $1,659.93$ | $1,986.20$ | 83.57 | 126.41 |
| Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD. |  |  |  |  |  |  |  |  |

Table F.3: Social welfare and its decomposition for single households with two children

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| US | 46.46 | $1,175.48$ | 367.10 | 23.74 | $2,529.92$ | $1,373.36$ | 184.21 | 26.73 |
| BE | 56.54 | $2,255.40$ | $1,419.01$ | 53.56 | $3,989.13$ | $2,141.66$ | 186.26 | 66.26 |
| FR | 62.55 | $2,376.38$ | $1,274.58$ | 52.87 | $3,798.95$ | $2,086.68$ | 182.06 | 61.08 |
| GE | 34.95 | $1,355.21$ | $1,573.80$ | 67.20 | $3,877.68$ | $1,946.83$ | 199.18 | 80.84 |
| GR | 50.51 | 917.30 | 0.00 | 8.15 | $1,816.20$ | 834.41 | 217.66 | 0.00 |
| IR | 0.00 | 0.00 | $2,516.93$ | 41.68 | $3,882.97$ | $1,899.02$ | 204.47 | 132.54 |
| LU | 10.95 | 545.23 | $2,848.63$ | 68.20 | $4,978.22$ | $3,317.03$ | 150.08 | 85.88 |
| NL | 0.00 | 0.00 | $2,104.17$ | 74.78 | $3,903.02$ | $2,075.22$ | 188.08 | 101.39 |
| PT | 61.04 | 917.39 | 354.90 | 36.75 | $1,502.96$ | 814.49 | 184.53 | 43.57 |
| SP | 74.06 | $1,481.06$ | 648.40 | 33.00 | $1,999.72$ | $1,346.99$ | 148.46 | 48.14 |
| UK | 37.47 | $1,243.80$ | $1,583.69$ | 58.99 | $3,319.85$ | $1,862.06$ | 178.29 | 85.05 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.4: Social welfare and its decomposition for couple households without children

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| US | 31.51 | 797.21 | $1,018.28$ | 35.96 | $2,529.92$ | $1,844.28$ | 137.18 | 55.21 |
| BE | 64.49 | $2,572.48$ | $1,689.31$ | 54.05 | $3,989.13$ | $2,569.99$ | 155.22 | 65.73 |
| FR | 75.00 | $2,849.21$ | $1,495.78$ | 57.27 | $3,798.95$ | $2,504.01$ | 151.71 | 59.74 |
| GE | 23.98 | 929.85 | $2,045.72$ | 71.75 | $3,877.68$ | $2,336.19$ | 165.98 | 87.57 |
| GR | 57.47 | $1,043.74$ | 44.12 | 5.13 | $1,816.20$ | $1,001.29$ | 181.39 | 4.41 |
| IR | 0.00 | 0.00 | $3,093.50$ | 62.26 | $3,882.97$ | $2,278.82$ | 170.39 | 135.75 |
| LU | 14.05 | 699.32 | $3,406.62$ | 71.99 | $4,978.22$ | $3,980.43$ | 125.07 | 85.58 |
| NL | 54.95 | $2,144.52$ | $2,352.93$ | 73.45 | $3,903.02$ | $2,490.27$ | 156.73 | 94.49 |
| PT | 70.00 | $1,052.07$ | 476.51 | 41.15 | $1,502.96$ | 977.39 | 153.77 | 48.75 |
| SP | 87.46 | $1,749.03$ | 739.43 | 36.53 | $1,999.72$ | $1,616.39$ | 123.72 | 45.75 |
| UK | 0.00 | 0.00 | $2,342.26$ | 76.70 | $3,319.85$ | $2,234.48$ | 148.57 | 104.82 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.5: Social welfare and its decomposition for couple households with one child

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| US | 30.06 | 760.60 | $1,265.34$ | 33.28 | $2,529.92$ | $2,095.33$ | 120.74 | 60.39 |
| BE | 69.49 | $2,771.94$ | $2,006.01$ | 53.33 | $3,989.13$ | $2,998.33$ | 133.05 | 66.90 |
| FR | 87.45 | $3,322.04$ | $1,769.34$ | 62.99 | $3,798.95$ | $2,921.35$ | 130.04 | 60.57 |
| GE | 22.45 | 870.50 | $2,446.80$ | 73.08 | $3,877.68$ | $2,725.56$ | 142.27 | 89.77 |
| GR | 64.56 | $1,172.49$ | 65.94 | 1.00 | $1,816.20$ | $1,168.18$ | 155.47 | 5.64 |
| IR | 0.00 | 0.00 | $3,504.06$ | 70.04 | $3,882.97$ | $2,658.62$ | 146.05 | 131.80 |
| LU | 15.00 | 746.73 | $4,031.64$ | 75.99 | $4,978.22$ | $4,643.84$ | 107.20 | 86.82 |
| NL | 64.01 | $2,498.36$ | $2,497.10$ | 71.10 | $3,903.02$ | $2,905.31$ | 134.34 | 85.95 |
| PT | 79.48 | $1,194.56$ | 594.23 | 44.60 | $1,502.96$ | $1,140.29$ | 131.81 | 52.11 |
| SP | 101.01 | $2,019.89$ | 771.64 | 36.53 | $1,999.72$ | $1,885.79$ | 106.04 | 40.92 |
| UK | 0.00 | 0.00 | $2,787.53$ | 76.48 | $3,319.85$ | $2,606.89$ | 127.35 | 106.93 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.6: Social welfare and its decomposition for couple households with two children

## F. 3 Extended cross-country comparison

In Figures F. 23 to F.28, we draw the entire budget curves for all countries over the relevant income span, separately for each household type. They correspond to Figure 9 in the paper. Country-specific budget curves are made comparable by rescaling the axes such that all minimum wages (resp. poverty lines) correspond to coordinate 1 along the horizontal (resp. vertical) axis.

Budget schemes - single without children (2013)



Figure F.23: Cross-country comparison of entire budget sets for single households without children

Budget schemes - single with one child (2013)



Figure F.24: Cross-country comparison of entire budget sets for single households with one child

Budget schemes - single with two children (2013)



Figure F.25: Cross-country comparison of entire budget sets for single households with two children

Budget schemes - couple without children (2013)



Figure F.26: Cross-country comparison of entire budget sets for couple households without children

Budget schemes - couple with one child (2013)



Figure F.27: Cross-country comparison of entire budget sets for couple households with one child

Budget schemes - couple with two children (2013)



Figure F.28: Cross-country comparison of entire budget sets for couple households with two children

## F. 4 Does it help to have children?

Figures F. 29 to F. 39 compare separately for each country the budget curves for households of singles at the top and couples at the bottom, with zero, one and two children. The figures correspond to Figures D.1 to D.3 in Appendix D.

US (2013)


Wage $=31.1 \%$ of AW (= legal minimum wage).
Figure F.29: Budget curves for single and couple households with zero, one or two children: US

BE (2013)


Wage $=39 \%$ of AW (= legal minimum wage).
Figure F.30: Budget curves for single and couple households with zero, one or two children: BE

FR (2013)


Wage $=46.6 \%$ of AW (= legal minimum wage).
Figure F.31: Budget curves for single and couple households with zero, one or two children: FR

GE (2013)


Wage $=39.2 \%$ of AW (= legal min. wage of 2015 deflated to 2013).
Figure F.32: Budget curves for single and couple households with zero, one or two children: GE

GR (2013)


## Wage $=39.5 \%$ of AW (= legal minimum wage) .

Figure F.33: Budget curves for single and couple households with zero, one or two children: GR

IR (2013)


Wage $=52.9 \%$ of AW (= legal minimum wage).
Figure F.34: Budget curves for single and couple households with zero, one or two children: IR

LU (2013)


Wage $=42 \%$ of AW (= legal minimum wage).
Figure F.35: Budget curves for single and couple households with zero, one or two children: LU

NL (2013)


Wage $=36.4 \%$ of AW (= legal minimum wage).
Figure F.36: Budget curves for single and couple households with zero, one or two children: NL


Figure F.37: Budget curves for single and couple households with zero, one or two children: PT


couple: _ without children _ with one child _- with two children

Wage $=34.7 \%$ of AW (= legal minimum wage) .
Figure F.38: Budget curves for single and couple households with zero, one or two children: SP

UK (2013)


Wage $=36.3 \%$ of AW (= legal minimum wage).
Figure F.39: Budget curves for single and couple households with zero, one or two children: UK

## F. 5 Mean income poverty lines

Figures F. 40 to F. 45 correspond to Figure E. 1 in Appendix E. 1 and are variants of the figures in Appendix F. 3 when the vertical axis is rescaled to take country mean income into account.

Budget schemes - single without children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure F.40: Cross-country comparison of entire budget sets with $50 \%$ of mean income poverty line for single households without children

Budget schemes - single with one child (2013)




Figure F.41: Cross-country comparison of entire budget sets with $50 \%$ of mean income poverty line for single households with one child

Budget schemes - single with two children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure F.42: Cross-country comparison of entire budget sets with $50 \%$ of mean income poverty line for single households with two children

Budget schemes - couple without children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure F.43: Cross-country comparison of entire budget sets with $50 \%$ of mean income poverty line for couple households without children

Budget schemes - couple with one child (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure F.44: Cross-country comparison of entire budget sets with $50 \%$ of mean income poverty line for couple households with one child

Budget schemes - couple with two children (2013)



Figure F.45: Cross-country comparison of entire budget sets with $50 \%$ of mean income poverty line for couple households with two children

## F. 6 Removing housing benefits

Tables F. 7 to F. 12 correspond to Table E. 1 in Appendix E. 2 and reproduce the tables from Appendix F. 2 for the six countries under analysis that provide housing benefits (HB) when the eligibility to HB is assumed away.

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 83.91 | $1,593.77$ | 658.66 | 52.75 | $1,899.48$ | $1,391.12$ | 136.54 | 47.35 |
| GE | 90.05 | $1,745.95$ | 507.33 | 52.79 | $1,938.84$ | $1,297.89$ | 149.38 | 39.09 |
| IR | 66.92 | $1,299.22$ | $1,070.45$ | 58.87 | $1,941.48$ | $1,266.01$ | 153.35 | 84.55 |
| LU | 20.95 | 521.53 | $1,793.90$ | 83.52 | $2,489.11$ | $2,211.35$ | 112.56 | 81.12 |
| NL | 84.07 | $1,640.56$ | $1,230.48$ | 80.81 | $1,951.51$ | $1,383.48$ | 141.06 | 88.94 |
| UK | 74.93 | $1,243.80$ | 485.88 | 49.14 | $1,659.93$ | $1,241.38$ | 133.72 | 39.14 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.7: Social welfare and its decomposition when eligibility for housing benefits is removed, for single households without children

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 93.99 | $1,785.34$ | 987.99 | 53.18 | $1,899.48$ | $1,808.45$ | 105.03 | 54.63 |
| GE | 82.91 | $1,607.46$ | $1,039.71$ | 55.45 | $1,938.84$ | $1,687.25$ | 114.91 | 61.62 |
| IR | 11.91 | 231.22 | $1,426.11$ | 45.55 | $1,941.48$ | $1,645.81$ | 117.96 | 86.65 |
| LU | 21.90 | 545.23 | $2,399.54$ | 85.29 | $2,489.11$ | $2,874.76$ | 86.59 | 83.47 |
| NL | 28.02 | 546.85 | $1,797.57$ | 74.69 | $1,951.51$ | $1,798.53$ | 108.51 | 99.95 |
| UK | 39.94 | 663.06 | $1,048.94$ | 58.52 | $1,659.93$ | $1,613.79$ | 102.86 | 65.00 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.8: Social welfare and its decomposition when eligibility for housing benefits is removed, for single households with one child

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 119.10 | $2,262.25$ | $1,185.59$ | 53.18 | $1,899.48$ | $2,225.79$ | 85.34 | 53.27 |
| GE | 67.09 | $1,300.80$ | $1,337.20$ | 45.10 | $1,938.84$ | $2,076.62$ | 93.37 | 64.39 |
| IR | 13.99 | 271.59 | $1,770.27$ | 45.89 | $1,941.48$ | $2,025.62$ | 95.85 | 87.39 |
| LU | 24.05 | 598.57 | $3,038.07$ | 84.76 | $2,489.11$ | $3,538.16$ | 70.35 | 85.87 |
| NL | 87.91 | $1,715.61$ | $1,941.74$ | 74.69 | $1,951.51$ | $2,213.57$ | 88.16 | 87.72 |
| UK | 39.94 | 663.06 | $1,494.22$ | 58.52 | $1,659.93$ | $1,986.20$ | 83.57 | 75.23 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.9: Social welfare and its decomposition when eligibility for housing benefits is removed, for single households with two children

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 62.55 | $2,376.38$ | 987.99 | 45.49 | $3,798.95$ | $2,086.68$ | 182.06 | 47.35 |
| GE | 60.97 | $2,364.20$ | 916.39 | 49.21 | $3,877.68$ | $1,946.83$ | 199.18 | 47.07 |
| IR | 50.47 | $1,959.83$ | $1,788.69$ | 41.68 | $3,882.97$ | $1,899.02$ | 204.47 | 94.19 |
| LU | 15.00 | 746.73 | $2,701.55$ | 67.85 | $4,978.22$ | $3,317.03$ | 150.08 | 81.44 |
| NL | 63.46 | $2,476.92$ | $1,757.74$ | 68.47 | $3,903.02$ | $2,075.22$ | 188.08 | 84.70 |
| UK | 54.96 | $1,824.55$ | 762.71 | 40.97 | $3,319.85$ | $1,862.06$ | 178.29 | 40.96 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.10: Social welfare and its decomposition when eligibility for housing benefits is removed, for couple households without children

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 75.00 | $2,849.21$ | $1,185.59$ | 49.26 | $3,798.95$ | $2,504.01$ | 151.71 | 47.35 |
| GE | 59.44 | $2,304.85$ | $1,266.12$ | 49.54 | $3,877.68$ | $2,336.19$ | 165.98 | 54.20 |
| IR | 24.01 | 932.21 | $2,132.84$ | 55.94 | $3,882.97$ | $2,278.82$ | 170.39 | 93.59 |
| LU | 77.50 | $3,858.12$ | $3,262.66$ | 71.46 | $4,978.22$ | $3,980.43$ | 125.07 | 81.97 |
| NL | 64.01 | $2,498.36$ | $1,973.32$ | 67.23 | $3,903.02$ | $2,490.27$ | 156.73 | 79.24 |
| UK | 39.53 | $1,312.39$ | $1,325.76$ | 56.16 | $3,319.85$ | $2,234.48$ | 148.57 | 59.33 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.11: Social welfare and its decomposition when eligibility for housing benefits is removed, for couple households with one child

|  | $W_{\min }^{p}$ | $W_{\min }^{p} w_{m}$ | $b$ | $t$ | $w_{m}$ | $p$ | $\frac{w_{m}}{p}$ | $\frac{b}{p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FR | 87.45 | $3,322.04$ | $1,383.19$ | 53.08 | $3,798.95$ | $2,921.35$ | 130.04 | 47.35 |
| GE | 58.55 | $2,270.23$ | $1,563.62$ | 48.62 | $3,877.68$ | $2,725.56$ | 142.27 | 57.37 |
| IR | 24.01 | 932.21 | $2,477.00$ | 64.42 | $3,882.97$ | $2,658.62$ | 146.05 | 93.17 |
| LU | 81.55 | $4,059.62$ | $3,887.93$ | 75.41 | $4,978.22$ | $4,643.84$ | 107.20 | 83.72 |
| NL | 72.94 | $2,846.85$ | $2,117.49$ | 67.23 | $3,903.02$ | $2,905.31$ | 134.34 | 72.88 |
| UK | 37.47 | $1,243.80$ | $1,771.04$ | 56.16 | $3,319.85$ | $2,606.89$ | 127.35 | 67.94 |

Note: $W_{\min }^{p}$ in $\%$ of full-time. $\frac{w_{m}}{p}$ and $\frac{b}{p}$ in $\%$. All other values in monthly USD.
Table F.12: Social welfare and its decomposition when eligibility for housing benefits is removed, for couple households with two children

## F. 7 Taking unemployment rates into account

Figures F. 46 to F. 51 correspond to Figure E. 2 in Appendix E. 3 and are variants of the figures in Appendix F.3 when the horizontal axis is rescaled to take country-specific unemployment rates of low-skill individuals into account.

Unemployment rates for low-skill individuals are given in Table F.13, taken from the OECD (2015), Low skill means not having completed high school.

|  | Unemp. rate |
| :--- | :---: |
| US | 0.13 |
| BE | 0.14 |
| FR | 0.14 |
| GE | 0.12 |
| GR | 0.29 |
| IR | 0.20 |
| LU | 0.09 |
| NL | 0.09 |
| PT | 0.17 |
| SP | 0.33 |
| UK | 0.10 |

Table F.13: Unemployment rates for low-skill individuals (below upper secondary education) aged 25-64

Budget schemes - single without children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure F.46: Cross-country comparison of entire budget sets when minimum wages are adjusted for unemployment rates, for single households without children

Budget schemes - single with one child (2013)


Figure F.47: Cross-country comparison of entire budget sets when minimum wages are adjusted for unemployment rates, for single households with one child

Budget schemes - single with two children (2013)




Figure F.48: Cross-country comparison of entire budget sets when minimum wages are adjusted for unemployment rates, for single households with two children

Budget schemes - couple without children (2013)


| US | BE | FR | GE | GR | IR |
| :---: | :---: | :---: | :---: | :---: | :---: |



Figure F.49: Cross-country comparison of entire budget sets when minimum wages are adjusted for unemployment rates, for couple households without children

Budget schemes - couple with one child (2013)



Figure F.50: Cross-country comparison of entire budget sets when minimum wages are adjusted for unemployment rates, for couple households with one child

Budget schemes - couple with two children (2013)


| -US | Z | BE | - | FR | - | GE | - |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



Figure F.51: Cross-country comparison of entire budget sets when minimum wages are adjusted for unemployment rates, for couple households with two children

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[^1]:    ${ }^{1}$ See the OECD website "Benefits and wages" (OECD, 2019), which gives access to the OECD tax-benefit calculator as well as country-specific information on the policies modeled.
    ${ }^{2}$ Belgium (BE), France (FR), Germany (GE), Greece (GR), Ireland (IR), Luxemburg (LU), the Netherlands (NL), Portugal (PT), Spain (SP), the United Kingdom (UK).
    ${ }^{3}$ Tax-transfer systems in the US are largely state specific. The OECD calculator uses the tax-transfer system of Michigan, a typical manufacturing region, to represent the US. Michigan's Temporary Assistance for Needy Families (TANF) and unemployment insurance benefits (not modeled in the present paper) are somewhat above the average for all States. In the following, for simplicity and consistency, we continue referring to the US to denote Michigan's tax-transfer system.
    ${ }^{4}$ Eligibility to SA might be conditional on behavior, especially whether the individual is actively searching for work. It is assumed that all individuals fulfill all requirements for full social assistance benefits to be received. In some countries, additional benefits can be paid conditional on participation in active labor market programs. Such benefits are not taken into account.
    ${ }^{5}$ Childcare benefits for parents with children in externally provided childcare and the costs of that care can partly be modeled with the OECD tax-benefit calculator. Though, this would involve further assumptions and complicate the analysis and comparability

[^2]:    across countries. We thus do not implement childcare benefits here.
    ${ }^{6}$ Only personal income tax and employees' social security contributions payable in respect of earnings and benefits are included. Central, state and local government income taxes are included, but council tax in the United Kingdom is excluded. In general, only standard tax reliefs are included when calculating tax payments.
    ${ }^{7}$ Child and family benefits often vary with the age of children. We found their average levels across child ages to be mostly around the intermediate ages of 10 and 12 . That is why we chose them.

[^3]:    ${ }^{8}$ Ideally, a poverty line should be defined in terms of the financial resources available to people for consumption. The official US poverty line was defined in pre-tax income and was criticized for that reason (and many others). See Citro and Michael (1995) and Iceland (2005) for accounts of those criticisms. The Supplemental Poverty Measure, published by the U.S. Census Bureau since 2011, is defined in a much closer way to disposable income. All EU member state's official poverty lines are defined in terms of disposable incomes. Of course, there are many ways of defining disposable income, e.g. as a function of whether in-kind or consumption subsidies are considered as income. See Meyer and Sullivan (2012) for a detailed account of the many issues raised by the definition of the poverty line.
    ${ }^{9}$ Detailed information about this measure and its derivation for 2013 can be found in Short (2014). A historical comparison of official poverty measures used in the US up to the Supplemental Poverty Measure can be found in Meyer and Sullivan (2012).

[^4]:    ${ }^{10}$ Cp. Appendix F. 4

[^5]:    ${ }^{11}$ Note that this is different from just subtracting HB from the simulated disposable income in the former applications as there might be interactions of HB with other policies. We therefore have to re-simulate accordant budget curves. As a matter of fact, removing HB in the OECD calculator is done by setting housing costs (rents) to zero. Besides the possible interaction of removed HB with other policies, there might be a second implication of this approach, namely when the level of rents also has an influence on other policies independently of HB, as already mentioned in Appendix C. Among the six countries of interest, this is the case for FR and GE. While the impact is limited, this is of course an unintended effect but we are not able to alter the OECD calculator in the preferred way at that time.

[^6]:    ${ }^{12}$ See Table F. 13 for an entire list of the unemployment rates.

