# ON-LINE APPENDIX News Shocks under Financial Frictions* 

Christoph Görtz<br>University of Birmingham

John D. Tsoukalas<br>University of Glasgow

Francesco Zanetti<br>University of Oxford

March 2021

[^0]
## 1 Supporting details and results

### 1.1 Robustness to max FEV credit spread shock indicator

Figure 1 displays the variance shares explained by the max FEV EBP shock discussed in the main body, section 2.


Figure 1: Variance Decomposition FEV of variable ' $x$ ' of the max FEV EBP shock (median solid line). The shaded gray areas are the $16 \%$ and $84 \%$ posterior bands generated from the posterior distribution of VAR parameters. Vertical axes show percentages.

Figure 2 displays IRFs to a TFP news shock (as shown in Figure 1 in the main body) and the median responses to a single shock that maximizes the FEV of the GZ spread over forecast horizons six to thirty-two quarters (red dashed line). As shown in the main body for the EBP, also when we use the GZ spread as target variable to identify the shock that maximizes variation in the spread, the responses to this shock are qualitatively and quantitatively very similar to the responses to a TFP news shock.

### 1.2 Robustness to VAR methodology

The results in the main body of the paper are generated using the Francis et al. (2014) identification approach (referred to as Max share method). This section reports VAR findings using two alternative approaches. First, the identification scheme in Barsky and Sims (2011)


Figure 2: TFP news shock and max FEV GZ shock. Median IRFs to a TFP news shock (solid black line) and a max FEV GZ spread shock (dashed red line) from seven-variable VARs. The shaded gray areas are the $16 \%$ and $84 \%$ posterior bands of the TFP news shock generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.
that recovers the news shock by maximizing the variance of TFP over the horizons zero to 40 quarters, and the restriction that the news shock does not move TFP on impact. Second, the identification scheme in Kurmann and Sims (2016), that recovers the news shock by maximizing the FEV of TFP at a very long horizon (80 quarters) without however imposing the zero impact restriction on TFP conditional on the news shock. ${ }^{1,2}$

Figures 3 and 4 show the responses obtained from the two alternative identification methods in relation to the responses shown in the main body. The IRFs are qualitatively and quantitatively very similar to each other. Qualitatively, all methods suggest that TFP rises significantly above zero only with a significant delay, except that the Kurmann and Sims (2016) method allows TFP to jump on impact (though the response is not significant different from zero). Importantly, the results suggest that the identified news shocks from

[^1]the three alternative methods are qualitatively and in the majority of cases quantitatively very similar to each other.


## Excess Bond Premium S\&P 500






Inflation


Figure 3: TFP news shock. Impulse responses to a TFP news shock from a seven-variable VAR. the black solid line shows the median using the baseline news shock identification and the shaded gray areas are the corresponding $16 \%$ and $84 \%$ posterior bands generated from the posterior distribution of VAR parameters. The dashed blue lines show the median and posterior bands when using the Barsky and Sims (2011) identification. The units of the vertical axes are percentage deviations.

### 1.3 Robustness of VAR results to alternative samples

In addition to the results reported in the main body of the paper for the sample 1984Q12017Q1, we also report results for a sample without the Great Recession period (1984Q12007Q3). Independently, we identify responses to a TFP news shock and, using the agnostic approach in Uhlig (2003), we identify the single shock that maximizes the forecast error variance of the EBP at business cycle frequencies. Figure 5 shows responses to these shocks based on seven-variable VARs estimated with 3 lags (to account for the relatively short sample length). The results for the shorter sample without the Great Recession are consistent with the ones shown in the main body. Most notably, the EBP declines significantly on impact and both the max FEV EBP shock and the TFP news shock trigger very similar dynamic responses.


Figure 4: TFP news shock. Impulse responses to a TFP news shock from a seven-variable VAR. The black solid line shows the median using the baseline news shock identification and the shaded gray areas are the corresponding $16 \%$ and $84 \%$ posterior bands generated from the posterior distribution of VAR parameters. The dashed blue lines show the median and posterior bands when using the Kurmann and Sims (2016) identification. The units of the vertical axes are percentage deviations.


Figure 5: TFP news shock and max-EBP shock. Sample without Great Recession. Median IRFs to a TFP news shock (solid black line) and a max FEV EBP shock (dashed red line) from seven-variable VARs. The shaded gray areas are the $16 \%$ and $84 \%$ posterior bands of the TFP news shock generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.

### 1.4 Robustness to VAR results: TFP news and financial shocks

In this section, we identify TFP news and financial shocks within the same VAR framework. We identify the TFP news shock as desrcibed in section 2.2 and then identify the financial shock as the innovation to the excess bond premium (EBP), similar to the approach in Gilchrist and Zakrajsek (2012). The IRFs to the identified financial shock displayed in Figure 6 are qualitatively very similar to the ones generated from the max FEV EBP shock and the TFP news shock (Figure 3). One key difference that distinguishes between a financial shock and a news TFP shock is the behavior of inflation. The former is an inflationary shock, i.e. inflation rises with activity, while the latter is a dis-inflationary shock, i.e. inflation declines with activity.

Finally, we have undertaken an additional robustness exercise with respect to the identification of the news TFP shock. Specifically, the TFP news shock is identified as the shock that maximizes the variance of TFP at the 40 quarter horizon with a zero impact restriction but crucially where the latter is now a linear combination of the reduced form innovations from the remaining variables in the VAR, excluding the EBP. In other words, this identification does not assume a-priori any correlation between movements in EBP and future TFP caused by a TFP shock. Figure 7 below displays the IRFs from a TFP news shock identified as described above. Importantly, the IRFs from this alternative identification are qualitatively consistent with the ones based on the baseline identification used in the main body of the paper.



Figure 7: TFP news shock - alternative identification. Impulse responses to a TFP news shock from a seven-variable VAR. The shaded gray areas are the $16 \%$ and $84 \%$ posterior bands generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.

### 1.5 Robustness of DSGE model results

We scrutinise our baseline DSGE model results in four dimensions. First, we extend our baseline DSGE model by incorporating a wedge between the model implied sectoral spreads and the corresponding corporate spread concepts in the data. The wedge follows the process

$$
\text { wedge }_{t}=\rho_{\text {wedge }} \text { wedge }_{t-1}+\varepsilon_{\text {wedge }, t},
$$

where $\rho_{\text {wedge }} \in(0,1)$ and $\varepsilon_{\text {wedge, }}$ is $i . i . d$. $N\left(0, \sigma_{\text {wedge }}^{2}\right)$. The wedge is introduced as an reduced form way to account for variation in the spread that could reflect factors we do not model, such as agents' default risk (although our VAR findings do not suggest this is a major consideration) or other non-fundamental factors in the pricing of corporate bond as recently argued by Gilchrist and Zakrajsek (2012). We report the variance decomposition at business cycle frequencies for our baseline model and the extended model with measurement error in the corporate spread equations in Table 1. Results are consistent across the two model specifications in the way that they point towards a quantitatively important role of TFP news shocks.

Second, we estimate the baseline model using a sample that excludes the Great Recession (1984Q1-2007Q3), addressing concerns about misspecification of the monetary policy rule when the policy rate approaches the zero lower bound, as well as concerns that high volatility
in corporate bond spreads and disruptions in financial markets may, at least partly, drive the important role of TFP news shocks. It is evident from the variance decomposition provided in Table 2 that the DSGE model's prediction on the quantitative importance of TFP news shocks as drivers of aggregate fluctuations is robust to excluding the Great Recession from the sample.

Third, we estimate a one-sector model without financial frictions similar to the ones described in Fujiwara et al. (2011), Khan and Tsoukalas (2012), and Justiniano et al. (2010). ${ }^{3}$ Table 3 shows variance decomposition results for our baseline model and the one-sector model without financial frictions. Consistent with the comparison of the baseline model with a twosector model without financial sector in the main body, the absence of the financial sector also limits the importance of TFP news shocks in the one sector model, but a much more prominent role is assigned to the MEI shock. These results are consistent with the findings reported in the studies mentioned above.

[^2]Table 1: Variance Decomposition at Business Cycle Frequencies

|  | Baseline model |  |  |  |  | Baseline with measurement errors in spread eqs |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | all TFP unanticipated | all TFP <br> news | MEI | all Bank equity | all other shocks | all TFP unanticipated | all TFP <br> news | MEI | all Bank equity | Measurem. error | all other shocks |
| Output | 19.8 | 52.3 | 7.6 | 0.2 | 20.1 | 15.6 | 51.0 | 6.1 | 0.2 | 8.5 | 18.6 |
| Consumption | 15.0 | 50.8 | 0.0 | 0.0 | 34.2 | 19.9 | 66.0 | 0.0 | 0.0 | 4.2 | 9.9 |
| Investment | 13.8 | 42.6 | 6.4 | 0.2 | 37.1 | 13.2 | 36.9 | 5.5 | 0.1 | 5.2 | 39.1 |
| Total Hours | 10.0 | 50.1 | 4.9 | 0.1 | 34.8 | 11.8 | 46.9 | 3.7 | 0.1 | 5.6 | 31.8 |
| Real Wage | 10.2 | 49.0 | 0.0 | 0.0 | 40.8 | 9.2 | 45.7 | 0.0 | 0.0 | 0.5 | 44.6 |
| Nominal Interest Rate | 3.0 | 36.0 | 4.5 | 0.1 | 56.4 | 3.6 | 48.6 | 2.9 | 0.1 | 6.5 | 38.4 |
| C-Sector Inflation | 0.6 | 4.0 | 0.0 | 0.0 | 95.4 | 0.7 | 13.7 | 0.0 | 0.0 | 3.6 | 82.0 |
| GZ Spread | 4.5 | 37.3 | 12.0 | 7.8 | 38.4 | 9.1 | 28.0 | 10.6 | 7.9 | 15.5 | 28.7 |
| Bank Equity | 3.9 | 23.4 | 0.3 | 69.9 | 2.6 | 3.2 | 29.7 | 0.2 | 63.0 | 1.7 | 2.1 |
| Rel. Price of Investment | 3.3 | 10.7 | 8.0 | 11.9 | 66.0 | 7.4 | 21.6 | 5.7 | 9.6 | 3.0 | 52.7 |
| Corporate Equity | 13.3 | 21.1 | 0.0 | 0.0 | 65.7 | 14.4 | 19.6 | 0.0 | 0.0 | 0.1 | 65.9 |

[^3]Table 2: Variance Decomposition at Business Cycle Frequencies

|  | Baseline model |  |  |  |  | Baseline model without Great Recession |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | all TFP <br> unanticipated | all TFP <br> news | MEI | all Bank equity | all other shocks | all TFP <br> unanticipated | all TFP <br> news | MEI | all Bank equity | all other shocks |
| Output | 19.8 | 52.3 | 7.6 | 0.2 | 20.1 | 16.2 | 51.0 | 8.0 | 0.2 | 24.5 |
| Consumption | 15.0 | 50.8 | 0.0 | 0.0 | 34.2 | 12.1 | 40.7 | 0.0 | 0.0 | 47.1 |
| Investment | 13.8 | 42.6 | 6.4 | 0.2 | 37.1 | 15.8 | 33.3 | 6.7 | 0.2 | 44.0 |
| Total Hours | 10.0 | 50.1 | 4.9 | 0.1 | 34.8 | 12.6 | 44.3 | 4.8 | 0.1 | 38.1 |
| Real Wage | 10.2 | 49.0 | 0.0 | 0.0 | 40.8 | 11.8 | 49.3 | 0.0 | 0.0 | 38.9 |
| Nominal Interest Rate | 3.0 | 36.0 | 4.5 | 0.1 | 56.4 | 2.4 | 41.1 | 4.4 | 0.1 | 52.0 |
| C-Sector Inflation | 0.6 | 4.0 | 0.0 | 0.0 | 95.4 | 0.5 | 4.4 | 0.0 | 0.0 | 95.1 |
| GZ Spread | 4.5 | 37.3 | 12.0 | 7.8 | 38.4 | 6.5 | 29.6 | 13.3 | 7.7 | 42.9 |
| Bank Equity | 3.9 | 23.4 | 0.3 | 69.9 | 2.6 | 1.7 | 21.2 | 0.2 | 73.5 | 3.3 |
| Rel. Price of Investment | 3.3 | 10.7 | 8.0 | 11.9 | 66.0 | 5.0 | 9.3 | 6.9 | 10.9 | 67.9 |
| Corporate Equity | 13.3 | 21.1 | 0.0 | 0.0 | 65.7 | 16.8 | 14.4 | 0.0 | 0.0 | 68.8 |

[^4]Table 3: Variance decomposition - business cycle frequencies (6-32 quarters)

|  | Baseline model |  |  |  | One sector model without financial frictions |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | all TFP unanticipated | all TFP news | MEI | all other shocks | all TFP unanticipated | all TFP news | MEI | all other shocks |
| Output | 19.8 | 52.3 | 7.6 | 20.3 | 10.0 | 22.3 | 29.5 | 38.2 |
| Consumption | 15.0 | 50.8 | 0.0 | 34.2 | 5.0 | 12.6 | 33.1 | 49.3 |
| Investment | 13.8 | 42.6 | 6.4 | 37.3 | 7.6 | 25.5 | 36.7 | 30.2 |
| Total Hours | 10.0 | 50.1 | 4.9 | 35.0 | 2.2 | 24.0 | 32.0 | 41.9 |
| Real Wage | 10.2 | 49.0 | 0.0 | 40.8 | 18.3 | 31.9 | 3.9 | 45.9 |
| Nominal Interest Rate | 3.0 | 36.0 | 4.5 | 56.5 | 1.7 | 9.3 | 76.4 | 12.6 |
| C-Sector Inflation | 0.6 | 4.0 | 0.0 | 95.4 | 4.0 | 9.4 | 52.3 | 34.4 |
| GZ Spread | 4.5 | 37.3 | 12.0 | 46.2 | n.a. | n.a. | n.a. | n.a. |
| Bank Equity | 3.9 | 23.4 | 0.3 | 72.5 | n.a. | n.a. | n.a. | n.a. |
| Rel. Price of Investment | 3.3 | 10.7 | 8.0 | 77.9 | 55.9 | 43.9 | 0.0 | 0.0 |
| Corporate Equity | 13.3 | 21.1 | 0.0 | 65.7 | n.a. | n.a. | n.a. | n.a. |

Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.

### 1.6 Risk appetite shocks

Full decomposition of model described in section 4.3. The streamlined version of the model discussed in section 4.3 is obtained from the baseline version. All structural equations are identical and as described in the detailed model Appendix 3. The difference, compared to the baseline model, is the removal of equations that describe the exogenous processes for the investment sector mark-up, equation (3.79), preference, equation (3.83), MEI, equation (3.86), stationary TFP in the C sector, equation (3.87), stationary TFP in the I sector, equation (3.88), as these shocks are not considered in the estimation. We report the variance decomposition corresponding to the streamlined version with "risk appetite" shocks in Table 4. As discussed in section 4.3 this model version allows for a significant role of financial shocks in terms of real activity variables and the GZ spread. Moreover, financial shocks account for 58.9 percent of the variance in bank equity. TFP news shocks' quantitative importance is very similar to the baseline model, in fact slightly increased for activity variables.

Table 4: Variance decomposition: model with risk appetite shocks

|  | $z$ | $v$ | $\eta_{m p}$ | $\lambda_{p}^{C}$ | $\lambda_{w}$ | $\varsigma_{C}$ | $\Xi$ | $\Xi^{4}$ | $\Xi^{8}$ | $\Xi^{12}$ | $z^{4}$ | $z^{8}$ | $z^{12}$ | All TFP news cols. 11-13 | All <br> Financial cols. 7-10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output | 15.3 | 2.2 | 0.3 | 0.5 | 1.8 | 0.9 | 2.2 | 3.9 | 0.4 | 2.9 | 43.1 | 3.5 | 23.2 | 69.7 | 9.3 |
| Consumption | 16.6 | 3.2 | 0.5 | 2.3 | 1.4 | 0.4 | 2.0 | 3.9 | 0.4 | 3.3 | 38.3 | 1.6 | 25.9 | 65.9 | 9.7 |
| Investment | 2.8 | 3.5 | 0.0 | 0.9 | 1.0 | 1.2 | 3.6 | 6.6 | 0.7 | 5.2 | 36.2 | 3.9 | 34.5 | 74.6 | 16.0 |
| Total Hours | 1.5 | 2.6 | 0.3 | 0.4 | 2.6 | 1.1 | 2.7 | 4.7 | 0.5 | 3.6 | 29.4 | 4.8 | 46.0 | 80.2 | 11.4 |
| Real Wage | 14.5 | 5.4 | 0.1 | 9.2 | 5.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 34.7 | 2.1 | 28.2 | 64.9 | 0.0 |
| Nominal Int. Rate | 1.9 | 2.4 | 4.3 | 19.9 | 1.8 | 0.6 | 0.9 | 1.5 | 0.1 | 0.9 | 16.3 | 2.1 | 47.2 | 65.6 | 3.5 |
| C-Sector Inflation | 3.8 | 0.8 | 0.6 | 25.3 | 3.7 | 0.2 | 0.1 | 0.2 | 0.0 | 0.1 | 30.7 | 2.3 | 32.3 | 65.2 | 0.3 |
| GZ Spread | 0.0 | 3.1 | 0.2 | 0.9 | 0.1 | 15.3 | 1.6 | 12.5 | 4.1 | 48.5 | 4.5 | 0.6 | 8.6 | 13.7 | 66.6 |
| Bank Equity | 0.2 | 0.0 | 0.1 | 0.0 | 0.0 | 39.5 | 15.6 | 20.2 | 2.0 | 21.1 | 0.1 | 0.0 | 1.1 | 1.3 | 58.9 |
| Rel. Price of Inv. | 0.9 | 0.3 | 0.1 | 0.3 | 0.3 | 5.4 | 16.6 | 30.0 | 3.0 | 23.8 | 7.7 | 0.6 | 11.0 | 19.4 | 73.4 |
| Corporate Equity | 12.6 | 44.8 | 0.0 | 4.6 | 0.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 19.7 | 1.4 | 16.0 | 37.0 | 0.1 |

$z=$ TFP growth shock in consumption sector, $v=$ TFP growth shock in investment sector, $\eta_{m p}=$ monetary policy shock, $\lambda_{p}^{C}=$ C-sector price markup shock, $\lambda_{w}=$ wage markup shock, $\varsigma_{C}=$ consumption sector bank equity shock, $\Xi=$ risk appetite shock, $\Xi^{x}=x$ quarters ahead risk appetite news shock, $z^{x}=x$ quarters ahead consumption sector TFP growth news shock. Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.

Baseline model with risk appetite shocks. We also report results from an extended baseline model with risk appetite shocks. This extended model considers all shocks featured in the baseline model and incorporates the risk appetite shocks as described in the streamlined version above. Table 5 below reports the variance decomposition. The key finding from this exercise is that TFP news shocks' quantitative importance for real activity variables is broadly similar to the baseline model. The variance shares of output and consumption are somewhat smaller, compared to the baseline in the main text, but still substantial at 41.5\% and $32.8 \%$ respectively. The variance shares for investment and hours worked estimated at $40.1 \%$ and $51.1 \%$ respectively are very similar to the shares reported for the baseline model. Hence the role of TFP news shocks is very robust. In the baseline model, risk appetite shocks remain important for the variance in the GZ spread and bank equity, with FEV shares estimated at $30.8 \%$ of the former and $48.5 \%$ of the latter. However, the quantitative importance of risk appetite shocks for real activity variables is at best very limited when the full menu of shocks is present in the estimation. To gain some insight into this finding we isolate MEI shocks, that soak up a non-negligible share of variation in real activity variables. As emphasized by Justiniano et al. (2011), MEI shocks can be thought of as ad-hoc proxies for
financial market frictions and are thus, similar in flavour to risk appetite shocks. Thus, they compete directly with risk appetite shocks for accounting in the FEV of the observables. In the extended model, MEI shocks account for $8.8 \%, 8.9 \%, 6.1 \%$ of the FEV in output, investment, and hours worked. These shares are comparable in magnitude to the FEV explained by the risk appetite shocks in the streamlined model as reported in Table 5 in the main text, where MEI shocks are not present in the menu of shocks. Both type of shocks generate similar dynamics in terms of co-movements of real activity variables. However, MEI shocks are investment supply shifters, whereas risk appetite shocks are investment demand shifters and imply different covariances of the relative price of investment and other observables. Although many data moments inform the estimation and consequently determine the relative significance of shocks, we note in particular the strong negative correlation between inflation and the (change in) the relative price of investment and the negative (near zero) correlations between inflation and growth of output (investment) in the data. Risk appetite shocks predict counterfactual correlations compared to MEI shocks and for this reason appear to be displaced when MEI shocks compete with them in the estimation.

Table 5: Variance decomposition: baseline model with risk appetite shocks

|  | $z$ | all $z$ news | $v$ | all $v$ news | $q$ | $\beta$ | $\Xi$ | all $\Xi$ news | all other shocks | all TFP surprise cols. $1+3$ | all TFP <br> news 4 <br> cols. $2+4$ | MEI and preference cols. $5+6$ | all risk appetite cols. $7+8$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output | 21.1 | 37.3 | 4.9 | 4.2 | 8.8 | 3.6 | 1.5 | 0.4 | 18.2 | 26.0 | 41.5 | 12.4 | 1.9 |
| Consumption | 14.0 | 27.6 | 3.3 | 5.2 | 0.0 | 48.4 | 0.0 | 0.0 | 1.5 | 17.3 | 32.8 | 48.4 | 0.0 |
| Investment | 6.9 | 33.9 | 5.9 | 6.2 | 8.9 | 3.6 | 1.5 | 0.4 | 32.7 | 12.8 | 40.1 | 12.5 | 1.9 |
| Total Hours | 6.5 | 47.9 | 3.7 | 3.2 | 6.1 | 2.3 | 1.1 | 0.3 | 28.9 | 10.2 | 51.1 | 8.4 | 1.4 |
| Real Wage | 7.9 | 28.9 | 3.3 | 9.6 | 0.0 | 6.9 | 0.0 | 0.0 | 43.3 | 11.2 | 38.5 | 7.0 | 0.0 |
| Nominal Interest Rate | 2.4 | 26.1 | 0.2 | 0.7 | 2.8 | 5.8 | 0.5 | 0.2 | 61.3 | 2.7 | 26.8 | 8.6 | 0.7 |
| C-Sector Inflation | 0.0 | 2.7 | 0.0 | 0.4 | 0.0 | 9.0 | 0.0 | 0.0 | 87.8 | 0.0 | 3.1 | 9.0 | 0.0 |
| GZ Spread | 7.0 | 25.9 | 5.2 | 5.2 | 9.4 | 0.1 | 1.7 | 29.2 | 16.3 | 12.3 | 31.1 | 9.5 | 30.8 |
| Bank Equity | 9.0 | 36.7 | 0.9 | 1.5 | 0.4 | 0.2 | 40.9 | 7.6 | 2.7 | 9.9 | 38.2 | 0.7 | 48.5 |
| Rel. Price of Investment | 0.3 | 5.6 | 2.3 | 3.6 | 10.7 | 2.1 | 16.3 | 5.5 | 53.5 | 2.6 | 9.3 | 12.8 | 21.9 |
| Corporate Equity | 2.6 | 10.8 | 7.9 | 15.7 | 0.0 | 0.0 | 0.0 | 0.0 | 63.0 | 10.5 | 26.5 | 0.0 | 0.0 |

### 1.7 DSGE based forecasting results

We use our DSGE model as the data generating process and perform the forecasting regressions as in Gilchrist and Zakrajsek (2012) to assess the predictive ability of credit spreads for economic activity.

In particular, we first draw parameters from the posterior distributions, simulate the model and reconstruct the time series in levels. We then use these time series to estimate the following forecasting specification,

$$
\begin{equation*}
\nabla^{h} Y_{t+h}=\alpha+\sum_{i=1}^{p} \beta_{i} \nabla Y_{t-i}+\gamma_{1} R F F_{t}+\gamma_{2} \text { Spread }_{t}+\epsilon_{t+h} \tag{1.1}
\end{equation*}
$$

where $\nabla^{h} Y_{t+h} \equiv \frac{c}{1+h} \ln \left(\frac{Y_{t+h}}{Y_{t-1}}\right)$ denotes output growth. The forecasting horizon is denoted by $h$ and $c=400$ is a scaling constant calibrated consistent with the quarterly frequency of our data. $R F F_{t}$ denotes the real interest rate defined as the difference of the nominal interest rate and expected (consumer's) inflation for the next quarter. Spread ${ }_{t}$ denotes the credit spread, and $\epsilon_{t+h}$ is the forecast error. The lag length $p$ is determined by the Akaike Information Criterion (AIC). We estimate the equation using ordinary least squares so that our procedure and the forecasting specification resemble exactly the setup in Gilchrist and Zakrajsek (2012). The only exception is that we omit the term spread as a right-hand variable since the model does not include bonds with different maturity structure that would allow us to generate time series for this variable.

We draw 200 times from the posterior distributions and estimate equation (1.1). ${ }^{4}$ Table 6 summarizes the results of this exercise where we focus on one and four quarter forecasting horizons. Consistent with the findings in Gilchrist and Zakrajsek (2012) the spread is a statistically highly significant predictor of economic activity at either the one or four quarter horizon, while the real interest rate is at best marginally significant. At the one-quarter horizon, the median for the real interest rate is insignificant for both model specifications.

[^5]At the four-quarter horizon, the median is significant at the $10 \%$ level, but only as long as the spread is not included as explanatory variable. Comparing results based on models with and without the spread, shows that including the spread in the forecasting equation leads to an increase in the adjusted R -squared.

Table 6: DSGE-model based forecasting results.

| Financial Indicator | Forecast horizon: 1 quarter |  |  |  |  |  | Forecast horizon: 4 quarters |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | percentiles |  |  | percentiles |  |  | percentiles |  |  | percentiles |  |  |
|  | 16 | 50 | 84 | 16 | 50 | 84 | 16 | 50 | 84 | 16 | 50 | 84 |
| Real Interest <br> Rate | $\begin{array}{r} 0.003 \\ {[0.520]} \end{array}$ | $\begin{array}{r} 0.028 \\ {[0.746]} \end{array}$ | $\begin{array}{r} 0.057 \\ {[0.926]} \end{array}$ | $\begin{gathered} -0.187 \\ {[0.038]} \end{gathered}$ | $\begin{gathered} -0.141 \\ {[0.119]} \end{gathered}$ | $\begin{gathered} -0.086 \\ {[0.348]} \end{gathered}$ | $\begin{array}{r} 0.076 \\ {[0.029]} \end{array}$ | $\begin{array}{r} 0.120 \\ {[0.084]} \end{array}$ | $\begin{array}{r} 0.150 \\ {[0.264]} \end{array}$ | $\begin{gathered} -0.138 \\ {[0.027]} \end{gathered}$ | $\begin{array}{r} -0.102 \\ {[0.108]} \end{array}$ | $\begin{gathered} -0.055 \\ {[0.395]} \end{gathered}$ |
| Spread |  |  |  | $\begin{gathered} -0.316 \\ {[0.000]} \end{gathered}$ | $\begin{gathered} -0.291 \\ {[0.000]} \end{gathered}$ | $\begin{gathered} -0.267 \\ {[0.000]} \end{gathered}$ |  |  |  | $\begin{gathered} -0.401 \\ {[0.000]} \end{gathered}$ | $\begin{gathered} -0.380 \\ {[0.000]} \end{gathered}$ | $\begin{gathered} -0.364 \\ {[0.000]} \end{gathered}$ |
| Adjusted $R^{2}$ | 0.224 | 0.248 | 0.268 | 0.296 | 0.310 | 0.329 | 0.277 | 0.323 | 0.362 | 0.479 | 0.493 | 0.509 |

Notes: Dependent variable is $\nabla^{h} Y_{t+h}$, where $Y_{t}$ denotes real GDP in quarter $t$ and $h$ is the forecast horizon. In addition to the specified financial indicator in quarter $t$, each specification also includes a constant and $p$ lags of $\nabla^{h} Y_{t-1}$ (not reported), where $p$ is determined by the AIC. Entries in the OLS coefficients associated with each financial indicator. Entries in brackets correspond to p-values. Each estimate is based on data simulated from the DSGE model with corresponding trends added. Time series length is 133 quarters (after 100 quarters are discarded) which corresponds to the length of our baseline horizon (1984Q1-2017Q1). We generate 200 sets of time series by drawing from the posterior distributions of the DSGE model parameters.

### 1.8 Specification for the Minnesota prior in the VAR

The prior for the VAR coefficients $A$ is of the form

$$
\operatorname{vec}(A) \sim N(\underline{\beta}, \underline{V}),
$$

where $\beta$ is one for variables which are in log-levels, and zero for the corporate bond spread as well as inflation. The prior variance $\underline{V}$ is diagonal with elements,

$$
\underline{V}_{i, j j}=\left\{\begin{array}{l}
\frac{a_{1}}{p^{2}} \text { for coefficients on own lags }  \tag{1.2}\\
\frac{a_{2} \sigma_{j i}}{p^{2} \sigma_{j j}} \text { for coefficients on lags of variable } j \neq i \\
\underline{a}_{3} \sigma_{i i} \text { for intercepts }
\end{array}\right.
$$

where, $p$ denotes the number of lags. Here $\sigma_{i i}$ is the residual variance from the unrestricted $p$-lag univariate autoregression for variable $i$. The degree of shrinkage depends on the hyperparameters $\underline{a}_{1}, \underline{a}_{2}, \underline{a}_{3}$. We set $\underline{a}_{3}=1$ and we select $\underline{a}_{1}, \underline{a}_{2}$ by searching on a grid and selecting the prior that maximizes the in-sample fit of the VAR, as measured by the Bayesian Information Criterion. ${ }^{5}$

### 1.9 Calibration and estimation

Calibration. Table 7 describes the calibrated parameters referred to in section 3.2. We set the quarterly depreciation rate to be equal across sectors, $\delta_{C}=\delta_{I}=0.025$. From the steady state restriction $\beta=\pi_{C} / R$, we set $\beta=0.997$. The shares of capital in the production functions, $a_{C}$ and $a_{I}$, are assumed equal across sectors and fixed at 0.3 . These are standard values used in the extant literature. The steady state value for the ratios of nominal investment to consumption is calibrated to be consistent with the average value in the data.

The steady state sectoral inflation rates are set to the sample averages and the sectoral steady state mark-ups are assumed to be equal to $15 \%$. We also calibrate the steady state (deterministic) growth of TFP in the consumption/investment sectors in line with the sample average growth rates of output in the two sectors. This yields $g_{a}=0.15 \%$ and $g_{v}=0.48 \%$ per

[^6]Table 7: Calibrated Parameters

Parameter Value Description

|  |  |  |
| :--- | :--- | :--- |
| $\delta_{C}$ | 0.025 | Consumption sector captial depreciation |
| $\delta_{I}$ | 0.025 | Investment sector captial depreciation |
| $a_{c}$ | 0.3 | Consumption sector share of captial |
| $a_{I}$ | 0.3 | Investment sector share of captial |
| $\beta$ | 0.997 | Discount factor |
| $\pi_{C}-1$ | 0.66 | Steady state consumption sector net inflation rate (percent quarterly) |
| $\pi_{I}-1$ | 0.14 | Steady state investment sector net inflation rate (percent quarterly) |
| $\lambda_{p}$ | 0.15 | Steady state price markup |
| $\lambda_{w}$ | 0.15 | Steady state wage markup |
| $g_{a}$ | 0.15 | Steady state C-sector TFP growth (percent quarterly) |
| $g_{v}$ | 0.48 | Steady state I-sector TFP growth (percent quarterly) |
| $p_{i} \frac{i}{c}$ | 0.445 | Steady state investment / consumption |
| $\theta_{B}$ | 0.965 | Fraction of bankers that survive |
| $R^{B}-R$ | 0.5 | Steady state spread (percent quarterly) |
| $\frac{Q S}{Q^{h} S^{h}}$ | 0.25 | corporate bonds over equity market capitalization |

Notes. $\beta, \pi_{C}, \pi_{I}, g_{a}, g_{v}, p_{i} \frac{i}{c}, R^{B}-R$ and $\frac{Q S}{Q^{h} S^{h}}$ are based on sample averages.
quarter. There are three parameters specific to financial intermediation. The parameter $\theta_{B}$, which determines the banker's average life span does not have a direct empirical counterpart and is fixed at 0.965 , similar to the value used by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). This value implies an average survival time of bankers of slightly over six years. The parameters $\varpi$ and the steady state leverage ratio are implied by steady state values and the estimate for $\lambda_{B}$. Our value for $\varpi=0.0051$ is very close to the calibration in Gertler and Kiyotaki (2010) and the steady state leverage ratio implied by these values (3.3005) is within the range of values reported in the literature and the average leverage ratio we compute from the data. Also the parameters for governing fixed equity adjustment costs, $\gamma^{h}=0.0286$ and $\gamma=0.0299$, are pinned down by steady state ratios as shown in Appendix 3.6.

## 2 Sample, Data Sources and Time Series Construction

There are several considerations for focusing attention on a Great Moderation period. Adrian and Shin (2010) and Jermann and Quadrini (2012) argue that the importance of the financial sector for the determination of credit and asset prices, which is the main focus of our study, has risen significantly during this period. Further, Jermann and Quadrini (2009) discuss a variety of financial innovations that were taking place or intensified in the 1980s, including banking liberalization, and flexibility in debt issuance through the introduction of the Asset Backed Securities market. The corporate bond market-relative to equity markets-has grown tremendously as a source of finance, suggesting that developments in the corporate bond market may more accurately reflect future economic conditions. According to the Securities Industry and Financial Markets Association (SIFMA) over the period 1990 to 2013 the volume of US corporate bonds outstanding more than quantipled from $\$ 1.35$ trillion to $\$ 7.46$ trillion. The same body reports that in 2010, total corporate debt was 5.1 times common stock issuance. Philippon (2009) argues that corporate bond spreads may contain news about future corporate fundamentals and provides evidence that information extracted from corporate bond markets, in contrast to the stock market, is informative for U.S. business fixed investment.

Table 8 provides an overview of the data used to construct the observables. All the data transformations we have made in order to construct the dataset used for the estimation of the model are described in detail below. As described in the main body, a subset of variables are used for estimating the various VAR specifications and they enter in levels. The majority of the raw data described below were retrieved from the Federal Reserve of St.Luis FRED database. The exceptions are, the Market equity for banks which is from CRSP, TFP data series which is from Fernald (2014) at the Federal reserve bank of San Francisco, and the GZ spread and excess bond premium which are from the Federal reserve board. ${ }^{6}$

Sectoral definition. To allocate a sector to the consumption or investment category,

[^7]Table 8: Time Series used to construct the observables and steady state relationships

|  |  |  |  |
| :--- | :--- | :--- | :--- |
| Time Series Description | Units | Code | Source |
|  |  |  |  |
| Gross domestic product |  |  |  |
| Gross Private Domestic Investment | CP, SA, billion $\$$ | GDP | St. Louis FED FRED |
| Real Gross Private Domestic Investment | CP, SA, billion $\$$ | GPDI | St. Louis FED FRED |
| Personal Consumption Exp: Durable Goods | CP, SA, billion $\$$ | GPDIC1 | Pt. Louis FED FRED |
| Real Personal Consumption Exp: Durable Goods | CVM, SA, billion $\$$ | PCDGCC96 | St. Louis FED FRED |
| Personal Consumption Expenditures: Services | CP, SA, billion $\$$ | PCESV | St. Louis FED FRED |
| Real Personal Consumption Expenditures: Services | CVM, SA, billion $\$$ | PCESVC96 | St. Louis FED FRED |
| Personal Consumption Exp: Nondurable Goods | CP, SA, billion $\$$ | PCND | St. Louis FED FRED |
| Real Personal Consumption Exp: Nondurable Goods | CVM, SA, billion $\$$ | PCNDGC96 | St. Louis FED FRED |
| Civilian Noninstitutional Population | NSA, 1000s | St. Louis FED FRED |  |
| Non-farm Business Sector: Compensation Per Hour | SA, Index 2005=100 | COMPNFB | St. Louis FED FRED |
| Non-farm Business Sector: Hours of All Persons | SA, Index 2005=100 | HOANBS | St. Louis FED FRED |
| Effective Federal Funds Rate | NSA, percent | FEDFUNDS | St. Louis FED FRED |
| Implicit GDP deflator | SA, percent | St. Louis FED FRED |  |
| S\&P 500 Index |  | St. Louis FED FRED |  |
| BAA corporate spread |  | Robert Shiller |  |
| GZ Spread |  | St. Louis FED FRED |  |
| Excess bond premium |  | FRB |  |
| Market Equity |  | FRB |  |
| Corporate equity (non financial corporate sector) | Flow of Funds | (Z.1 Financial Accounts) | St. Louis FED FRED |
| SLOOS |  | St. Louis FED FRED |  |
| TFP utilization adjusted |  | John Fernald |  |

$\mathrm{CP}=$ current prices, $\mathrm{CVM}=$ chained volume measures, $\mathrm{SA}=$ seasonally adjusted, $\mathrm{NSA}=$ not seasonally adjusted, $\mathrm{FRB}=$ Federal Reserve Board.
we used the 2005 Input-Output tables. The Input-Output tables track the flows of goods and services across industries and record the final use of each industry's output into three broad categories: consumption, investment and intermediate uses (as well as net exports and government). First, we determine how much of a 2-digit industry's final output goes to consumption as opposed to investment or intermediate uses.

Then we adopt the following criterion: if the majority of an industry's final output is allocated to final consumption demand it is classified as a consumption sector; otherwise, if the majority of an industry's output is allocated to investment or intermediate demand, it is classified as an investment sector. Using this criterion, mining, utilities, transportation and warehousing, information, manufacturing, construction and wholesale trade industries are classified as the investment sector and retail trade, real estate, rental and leasing, professional and business services, educational services, health care and social assistance, arts, entertainment, recreation, accommodation and food services and other services except government are classified as the consumption sector. ${ }^{7}$

[^8]Real and nominal variables. We describe the exact source of each data series below as provided by FRED.

Gross domestic product, current prices: U.S. Bureau of Economic Analysis, Gross Domestic Product [GDP], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/GDP, Sep 15, 2017.

Gross Private Domestic Investment, current prices: U.S. Bureau of Economic Analysis, Gross Private Domestic Investment [GPDI], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/GPDI, Sep 15, 2017.

Real Gross Private Domestic Investment: U.S. Bureau of Economic Analysis, Real Gross Private Domestic Investment [GPDIC1], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/GPDIC1, Sep 15, 2017.

Personal Consumption Exp.: Durable Goods, current prices: U.S. Bureau of Economic Analysis, Personal Consumption Expenditures: Durable Goods [PCEDG], retrieved from FRED, Federal Reserve Bank of St. Louis; https ://fred.stlouisfed.org/series/PCEDG, Sep 16, 2017.

Real Personal Consumption Exp.: Durable Goods: U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures: Durable Goods [PCEDGC96], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/PCEDGC96, Sep 15, 2017.

Personal Consumption Expenditures: Services, current prices: U.S. Bureau of Economic Analysis, Personal Consumption Expenditures: Services [PCES], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/PCES, Sep 16, 2017. Real Personal Consumption Expenditures: Services: U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures: Services [PCESC96], retrieved from FRED, Fed-
eral Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/PCESC96, Sep 16, 2017.

Personal Consumption Exp.: Nondurable Goods, current prices: U.S. Bureau of Economic Analysis, Personal Consumption Expenditures: Nondurable Goods [PCEND], retrieved from FRED, Federal Reserve Bank of St. Louis; https ://fred.stlouisfed.org/series/PCEND, Sep 16, 2017.

Real Personal Consumption Exp.: Nondurable Goods: U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures: Nondurable Goods [PCENDC96], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/PCENDC96, Sep 16, 2017.

Civilian Noninstitutional Population: U.S. Bureau of Labor Statistics, Population Level [CNP16OV], retrieved from FRED, Federal Reserve Bank of St. Louis; https : // fred.stlouisfed.org/se Sep 15, 2017.

Non-farm Business Sector: Compensation Per Hour: U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Compensation Per Hour [COMPNFB], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/COMPNFB, Sep 15, 2017.

Non-farm Business Sector: Hours of All Persons: U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Hours of All Persons [PRS85006031], retrieved from FRED, Federal Reserve Bank of St. Louis; https : //fred.stlouisfed.org/series/PRS85006031, Sep 15, 2017. Effective Federal Funds Rate: Board of Governors of the Federal Reserve System (US), Effective Federal Funds Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis; https : // fred.stlouisfed.org/series/FEDFUNDS, Sep 15, 2017.

Implicit GDP deflator: U.S. Bureau of Economic Analysis, Gross Domestic Product: Implicit Price Deflator [A191RI1Q225SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis; https : // fred.stlouisfed.org/series/A191RI1Q225SBEA, Sep 16, 2017.

The raw data are transformed as follows for the analysis. Consumption (in current
prices) is defined as the sum of personal consumption expenditures on services and personal consumption expenditures on non-durable goods. The times series for real consumption is constructed as follows. First, we compute the shares of services and non-durable goods in total (current price) consumption. Then, total real consumption growth is obtained as the chained weighted (using the nominal shares above) growth rate of real services and growth rate of real non-durable goods. Using the growth rate of real consumption we construct a series for real consumption using 2005 as the base year. The consumption deflator is calculated as the ratio of nominal over real consumption. In the DSGE model inflation of consumer prices is the growth rate of the consumption deflator. In the VAR model we use the log change in the GDP deflator as our inflation measure, however results are nearly identical when we use the consumption deflator or CPI inflation. Analogously, we construct a time series for the investment deflator using series for (current price) personal consumption expenditures on durable goods and gross private domestic investment and chain weight to arrive at the real aggregate. The relative price of investment is the ratio of the investment deflator and the consumption deflator. Real output is GDP expressed in consumption units by dividing current price GDP with the consumption deflator.

The hourly wage is defined as total compensation per hour. Dividing this series by the consumption deflator yields the real wage rate. Hours worked is given by hours of all persons in the non-farm business sector. All series described above as well as the equity capital series (described below) are expressed in per capita terms using the series of noninstitutional population, ages 16 and over. The nominal interest rate is the effective federal funds rate. We use the monthly average per quarter of this series and divide it by four to account for the quarterly frequency of the model. The time series for hours is in logs. Moreover, all series used in estimation (including the financial time series described below) are expressed in deviations from their sample average.

## Financial variables.

The GZ spread and excess bond premium. The GZ spread and excess bond premium series is directly obtained from the FED reserve board ('Updating the recession risk
and the excess bond premium'); The Excel file with the GZ spread can be accessed at: https : //www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebpcsv.csv. The methodology is described in Gilchrist and Zakrajsek (2012).

The BAA spread. eries is downloaded directly from FRED; the monthly rates are converted to quarterly using the three month average for the quarter; ederal Reserve Bank of St . Louis, Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity [BAA10YM], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/BAA10YM, Sep 16, 2017.

The S\&P 500 index is obtained from Robert Shiller's website (http : //www.econ.yale.edu/ shiller /do and has been converted to a real per capita index by dividing with the consumption deflator and non-institutional population, ages 16 and over.

Market equity. The market value of commercial bank's equity is constructed using monthly data from the Centre for Research in Security Prices (CRSP) and have been accessed from the Wharton research data services (WRDS): https : //wrds - www.wharton.upenn.edu/. From the raw data we retain companies with the following SIC codes to cover the commercial banking sector: 6021 (National Commercial Banks), 6022 (State Commercial Banks), 6029 (Commercial Banks, not elsewhere classified), 6081 (Branches and Agencies of Foreign Banks), 6153 (Short-Term Business Credit Institutions, except Agricultural), 6159 (Miscellaneous Business Credit Institutions) and 6111 (Federal and Federally-Sponsored Credit Agencies). Market value is calculated as the product of Price (PRC) and Shares Outstanding (SHROUT). We transform the data to quarterly frequency by considering the market value on the last trading day per quarter. The final series for total equity is generated by taking the log after dividing by Civilian Noninstitutional Population and the consumption deflator. Senior officer opinion survey of bank lending practices (SLOOS). The SLOOS is downloaded directly from FRED (Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms). Board of Governors of the Federal Reserve System (US), Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms
[DRTSCILM], retrieved from FRED, Federal Reserve Bank of St. Louis; The survey panel contains domestic banks headquartered in all 12 Federal Reserve Districts, with a minimum of 2 and a maximum of 12 domestic banks in the panel from each district. In general, up to 60 domestically chartered U.S. commercial banks participated in each survey from 1990 through mid-2012; beginning with the July 2012 survey, the size of the domestic panel was increased to include as many as 80 institutions. As described in the Federal Register Notice authorizing the SLOOS, the panel of domestic respondents as of September 30, 2011 contained 55 banks, 34 of which had assets of $\$ 20$ billion or more. The combined assets of the respondent banks totaled $\$ 7.5$ trillion and accounted for 69 percent of the $\$ 10.9$ trillion in total assets at domestically chartered institutions. The respondent banks also held between 40 percent and 80 percent of total commercial bank loans outstanding in each major loan category regularly queried in the survey, with most categories falling in the upper end of that range. The particular survey question we consider is the net percentage of domestic respondents reporting tightening lending standards for commercial and industry loans for large and medium-sized firms.

## 3 Model Details and Derivations

We provide the model details and derivations required for solution and estimation of the model. We begin with the pricing and wage decisions of firms and households, the financial sector followed by the normalization of the model to render it stationary, the description of the steady state and the log-linearized model equations.

### 3.1 Intermediate and Final Goods Producers

Intermediate producers pricing decision. A constant fraction $\xi_{p, x}$ of intermediate firms in sector $x=C, I$ cannot choose their price optimally in period $t$ but reset their price - as
in Calvo (1983) - according to the indexation rule,

$$
\begin{aligned}
P_{C, t}(i) & =P_{C, t-1}(i) \pi_{C, t-1}^{\iota_{p_{C}}} \pi_{C}^{1-\iota_{p_{C}}} \\
P_{I, t}(i) & =P_{I, t-1}(i) \pi_{I, t-1}^{\iota_{p_{I}}} \pi_{I}^{1-\iota_{p_{I}}}\left[\left(\frac{A_{t}}{A_{t-1}}\right)^{-1}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1-a_{c}}{1-a_{i}}}\right]^{\iota_{p_{I}}},
\end{aligned}
$$

where $\pi_{C, t} \equiv \frac{P_{C, t}}{P_{C, t-1}}$ and $\pi_{I, t} \equiv \frac{P_{I, t}}{P_{I, t-1}}\left(\frac{A_{t}}{A_{t-1}}\right)^{-1}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1-a_{c}}{1-a_{i}}}$ is gross inflation in the two sectors and $\pi_{C}, \pi_{I}$ denote steady state values. The factor that appears in the investment sector expression adjusts for investment specific progress.

The remaining fraction of firms, $\left(1-\xi_{p, x}\right)$, in sector $x=C, I$ can adjust the price in period $t$. These firms choose their price optimally by maximizing the present discounted value of future profits.

The resulting aggregate price index in the consumption sector is,

$$
P_{C, t}=\left[\left(1-\xi_{p, C}\right) \tilde{P}_{C, t}^{\frac{1}{\lambda_{p, t}}}+\xi_{p, C}\left(\left(\frac{\pi_{C, t-1}}{\pi}\right)^{\iota_{p_{C}}} \pi_{C}^{1-\iota_{p}} P_{C, t-1}\right)^{\frac{1}{\lambda_{p, t}}}\right]^{\lambda_{p, t}^{C}}
$$

The aggregate price index in the investment sector is,

$$
P_{I, t}=\left[\left(1-\xi_{p, I}\right) \tilde{P}_{I, t}^{\frac{1}{p, t}}+\xi_{p, I}\left(P_{I, t-1}\left(\frac{\pi_{I, t-1}}{\pi}\right)^{\iota_{p_{I}}} \pi_{I}^{1-\iota_{p_{I}}}\left[\left(\frac{A_{t}}{A_{t-1}}\right)^{-1}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1-a c}{1-a_{i}}}\right]^{\iota_{p_{I}}}\right)^{\frac{1}{\lambda_{p, t}}}\right]^{\lambda_{p, t}^{I}}
$$

Final goods producers. Profit maximization and the zero profit condition for final good firms imply that sectoral prices of the final goods, $P_{C, t}$ and $P_{I, t}$, are CES aggregates of the prices of intermediate goods in the respective sector, $P_{C, t}(i)$ and $P_{I, t}(i)$,

$$
P_{C, t}=\left[\int_{0}^{1} P_{C, t}(i)^{\frac{1}{\lambda_{p, t}}} d i\right]^{\lambda_{p, t}^{C}}, \quad P_{I, t}=\left[\int_{0}^{1} P_{I, t}(i)^{\frac{1}{\lambda_{p, t}}} d i\right]^{\lambda_{p, t}^{I}} .
$$

The elasticity $\lambda_{p, t}^{x}$ is the time varying price markup over marginal cost for intermediate firms. It is assumed to follow the exogenous stochastic process,

$$
\log \left(1+\lambda_{p, t}^{x}\right)=\left(1-\rho_{\lambda_{p}^{x}}\right) \log \left(1+\lambda_{p}^{x}\right)+\rho_{\lambda_{p}^{x}} \log \left(1+\lambda_{p, t-1}^{x}\right)+\varepsilon_{p, t}^{x},
$$

where $\rho_{\lambda_{p}^{x}} \in(0,1)$ and $\varepsilon_{p, t}^{x}$ is i.i.d. $N\left(0, \sigma_{\lambda_{p}^{x}}^{2}\right)$, with $x=C, I$.

### 3.1.1 Household's wage setting

Each household $j \in[0,1]$ supplies specialized labor, $L_{t}(j)$, monopolistically as in Erceg et al. (2000). A large number of competitive "employment agencies" aggregate this specialized labor into a homogenous labor input which is sold to intermediate goods producers in a competitive market. Aggregation is done according to the following function,

$$
L_{t}=\left[\int_{0}^{1} L_{t}(j)^{\frac{1}{1+\lambda_{w, t}}} d j\right]^{1+\lambda_{w, t}}
$$

The desired markup of wages over the household's marginal rate of substitution (or wage mark-up), $\lambda_{w, t}$, follows the exogenous stochastic process,

$$
\log \left(1+\lambda_{w, t}\right)=\left(1-\rho_{w}\right) \log \left(1+\lambda_{w}\right)+\rho_{w} \log \left(1+\lambda_{w, t-1}\right)+\varepsilon_{w, t},
$$

where $\rho_{w} \in(0,1)$ and $\varepsilon_{w, t}$ is i.i.d. $N\left(0, \sigma_{\lambda_{w}}^{2}\right)$.
Profit maximization by the perfectly competitive employment agencies implies the labor demand function,

$$
\begin{equation*}
L_{t}(j)=\left(\frac{W_{t}(j)}{W_{t}}\right)^{-\frac{1+\lambda_{w, t}}{\lambda_{w, t}}} L_{t} \tag{3.1}
\end{equation*}
$$

where $W_{t}(j)$ is the wage received from employment agencies by the supplier of labor of type $j$, while the wage paid by intermediate firms for the homogenous labor input is,

$$
W_{t}=\left[\int_{0}^{1} W_{t}(j)^{\frac{1}{\lambda_{w, t}}} d j\right]^{\lambda_{w, t}}
$$

Following Erceg et al. (2000), in each period, a fraction $\xi_{w}$ of the households cannot freely adjust its wage but follows the indexation rule,

$$
W_{t+1}(j)=W_{t}(j)\left(\pi_{c, t} e^{z_{t}+\frac{a_{c}}{1-a_{i}} v_{t}}\right)^{\iota_{w}}\left(\pi_{c} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}}\right)^{1-\iota_{w}}
$$

The remaining fraction of households, $\left(1-\xi_{w}\right)$, chooses an optimal wage, $W_{t}(j)$, by maximizing,

$$
E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[-b_{t+s} \varphi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu}+\Lambda_{t+s} W_{t}(j) L_{t+s}(j)\right]\right\}
$$

subject to the labor demand function (3.1). The aggregate wage evolves according to,

$$
W_{t}=\left\{\left(1-\xi_{w}\right)\left(\tilde{W}_{t}\right)^{\frac{1}{\lambda_{w}}}+\xi_{w}\left[\left(\pi_{c} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}}\right)^{1-\iota_{w}}\left(\pi_{c, t-1} e^{z_{t-1}+\frac{a_{c}}{1-a_{i}} v_{t-1}}\right)^{\iota_{w}} W_{t-1}\right]^{\frac{1}{\lambda_{w}}}\right\}^{\lambda_{w}},
$$

where $\tilde{W}_{t}$ is the optimally chosen wage.

### 3.2 Physical capital producers

Capital producers in sector $x=C, I$ use a fraction of investment goods from final goods producers and undepreciated capital stock from capital services producers (as described above) to produce new capital goods, subject to investment adjustment costs as proposed by Christiano et al. (2005). These new capital goods are then sold in perfectly competitive capital goods markets to capital services producers. The technology available for physical capital production is given as,

$$
O_{x, t}^{\prime}=O_{x, t}+\mu_{t}\left(1-S\left(\frac{I_{x, t}}{I_{x, t-1}}\right)\right) I_{x, t},
$$

where $O_{x, t}$ denotes the amount of used capital at the end of period $t, O_{x, t}^{\prime}$ the new capital available for use at the beginning of period $t+1$. The investment adjustment cost function $S(\cdot)$ satisfies the following: $S(1)=S^{\prime}(1)=0$ and $S^{\prime \prime}(1)=\kappa>0$, where " $/$ "s denote differentiation. The optimization problem of capital producers in sector $x=C, I$ is given as,

$$
\max _{I_{x, t, O_{x, t}}} E_{t} \sum_{t=0}^{\infty} \beta^{t} \Lambda_{t}\left\{Q_{x, t}\left[O_{x, t}+\mu_{t}\left(1-S\left(\frac{I_{x, t}}{I_{x, t-1}}\right)\right) I_{x, t}\right]-Q_{x, t} O_{x, t}-\frac{P_{I, t}}{P_{C, t}} I_{x, t}\right\}
$$

where $Q_{x, t}$ denotes the price of capital (i.e. the value of installed capital in consumption units). The first order condition for investment goods is,
$\frac{P_{I, t}}{P_{C, t}}=Q_{x, t} \mu_{t}\left[1-S\left(\frac{I_{x, t}}{I_{x, t-1}}\right)-S^{\prime}\left(\frac{I_{x, t}}{I_{x, t-1}}\right) \frac{I_{x, t}}{I_{x, t-1}}\right]+\beta E_{t} Q_{x, t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_{t}}\left[S^{\prime}\left(\frac{I_{x, t+1}}{I_{x, t}}\right)\left(\frac{I_{x, t+1}}{I_{x, t}}\right)^{2}\right]$.
From the capital producer's problem it is evident that any value of $O_{x, t}$ is profit maximizing. Let $\delta_{x} \in(0,1)$ denote the depreciation rate of capital and $\bar{K}_{x, t-1}$ the capital stock available at the beginning of period $t$ in sector $x=C, I$. Then setting $O_{x, t}=(1-\delta) \xi_{x, t}^{K} \bar{K}_{x, t-1}$ implies
the available (sector-specific) capital stock in sector $x$, evolves according to,

$$
\begin{equation*}
\bar{K}_{x, t}=\left(1-\delta_{x}\right) \xi_{x, t}^{K} \bar{K}_{x, t-1}+\mu_{t}\left(1-S\left(\frac{I_{x, t}}{I_{x, t-1}}\right)\right) I_{x, t}, \quad x=C, I \tag{3.2}
\end{equation*}
$$

as described in the main text.

### 3.3 Financial Intermediaries

This section describes in detail how the setup of Gertler and Karadi (2011) is adapted for the two sector model and describes in detail how the equations for financial intermediaries in the main text are derived.

The balance sheet for the consumption or investment sector branch can be expressed as,

$$
P_{C, t} Q_{x, t} S_{x, t}=P_{C, t} N_{x, t}+B_{x, t}, \quad x=C, I
$$

where $S_{x, t}$ denotes the quantity of financial claims held by the intermediary branch and $Q_{x, t}$ denotes the sector-specific price of a claim. The variable $N_{x, t}$ represents the bank's wealth (or equity) at the end of period $t$ and $B_{x, t}$ are the deposits the intermediary branch obtains from households. The sector-specific assets held by the financial intermediary pay the stochastic return $R_{x, t+1}^{B}$ in the next period. Intermediaries pay at $t+1$ the non-contingent real gross return $R_{t}$ to households for their deposits made at time $t$. Then, the intermediary branch equity evolves over time as,

$$
\begin{aligned}
N_{x, t+1} P_{C, t+1} & =R_{x, t+1}^{B} \pi_{C, t+1} P_{C, t} Q_{x, t} S_{x, t}-R_{t} B_{x, t} \\
N_{x, t+1} \frac{P_{C, t+1}}{P_{C, t}} & =R_{x, t+1}^{B} \pi_{C, t+1} Q_{x, t} S_{x, t}-R_{t}\left(Q_{x, t} S_{x, t}-N_{x, t}\right) \\
N_{x, t+1} & =\left[\left(R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}\right) Q_{x, t} S_{x, t}+R_{t} N_{x, t}\right] \frac{1}{\pi_{C, t+1}} .
\end{aligned}
$$

The premium, $R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}$, as well as the quantity of assets, $Q_{x, t} S_{x, t}$, determines the growth in bank's equity above the riskless return. The bank will not fund any assets with a negative discounted premium. It follows that for the bank to operate in period $i$ the following inequality must hold,

$$
E_{t} \beta^{i} \Lambda_{t+1+i}^{B}\left(R_{x, t+1+i}^{B} \pi_{C, t+1+i}-R_{t+i}\right) \geq 0, \quad i \geq 0
$$

where $\beta^{i} \Lambda_{t+1+i}^{B}$ is the bank's stochastic discount factor, with,

$$
\Lambda_{t+1}^{B} \equiv \frac{\Lambda_{t+1}}{\Lambda_{t}}
$$

where $\Lambda_{t}$ is the Lagrange multiplier on the household's budget equation. Under perfect capital markets, arbitrage guarantees that the risk premium collapses to zero and the relation always holds with equality. However, under imperfect capital markets, credit constraints rooted in the bank's inability to obtain enough funds may lead to positive risk premia. As long as the above inequality holds, banks will keep building assets by borrowing additional funds from households. Accordingly, the intermediary branch objective is to maximize expected terminal wealth,

$$
\begin{align*}
V_{x, t} & =\max E_{t} \sum_{i=0}\left(1-\theta_{B}\right) \theta_{B}^{i} \beta^{i} \Lambda_{t+1+i}^{B} N_{x, t+1+i} \\
& =\max E_{t} \sum_{i=0}\left(1-\theta_{B}\right) \theta_{B}^{i} \beta^{i} \Lambda_{t+1+i}^{B}\left[\left(R_{x, t+1+i}^{B} \pi_{C, t+1+i}-R_{t+i} \frac{Q_{x, t+i} S_{x, t+i}}{\pi_{C, t+1+i}}+\frac{R_{t+i} N_{x, t+i}}{\pi_{C, t+1+i}}\right],\right. \tag{3.3}
\end{align*}
$$

where $\theta_{B} \in(0,1)$ is the fraction of bankers at $t$ that survive until period $t+1$.
Following the setup in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) the banks are limited from infinitely borrowing additional funds from households by a moral hazard/costly enforcement problem. On the one hand, the agent who works in the bank can choose, at the beginning of each period, to divert the fraction $\lambda_{B}$ of available funds and transfer it back to the household. On the other hand, depositors can force the bank into bankruptcy and recover a fraction $1-\lambda_{B}$ of assets. Note that the fraction, $\lambda_{B}$, which intermediaries can divert is the same across sectors to guarantee that the household is indifferent between lending funds between different branches.

Given this tradeoff, depositors will only lend funds to the intermediary when the latter's maximized expected terminal wealth is larger or equal to the gain from diverting the fraction $\lambda_{B}$ of available funds. This incentive constraint can be formalized as,

$$
\begin{equation*}
V_{x, t} \geq \lambda_{B} Q_{x, t} S_{x, t}, \quad 0<\lambda_{B}<1 \tag{3.4}
\end{equation*}
$$

Using equation (3.3), the expression for $V_{x, t}$ can be written as the following first-order
difference equation,

$$
V_{x, t}=\nu_{x, t} Q_{x, t} S_{x, t}+\eta_{x, t} N_{x, t}
$$

with,

$$
\begin{aligned}
\nu_{x, t} & =E_{t}\left\{\left(1-\theta_{B}\right) \Lambda_{t+1}^{B}\left(R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}\right)+\theta_{B} \beta Z_{1, t+1}^{x} \nu_{x, t+1}\right\}, \\
\eta_{x, t} & =E_{t}\left\{\left(1-\theta_{B}\right) \Lambda_{t+1}^{B} R_{t}+\theta_{B} \beta Z_{2, t+1}^{x} \eta_{x, t+1}\right\},
\end{aligned}
$$

and,

$$
Z_{1, t+1+i}^{x} \equiv \frac{Q_{x, t+1+i} S_{x, t+1+i}}{Q_{x, t+i} S_{x, t+i}}, \quad Z_{2, t+1+i}^{x} \equiv \frac{N_{x, t+1+i}}{N_{x, t+i}}
$$

The variable $\nu_{x, t}$ can be interpreted as the expected discounted marginal gain of expanding assets $Q_{x, t} S_{x, t}$ by one unit while holding wealth $N_{x, t}$ constant. The interpretation of $\eta_{x, t}$ is analogous: it is the expected discounted value of having an additional unit of wealth, $N_{x, t}$, holding the quantity of financial claims, $S_{x, t}$, constant. The gross growth rate in assets is denoted by $Z_{1, t+i}^{x}$ and the gross growth rate of net worth is denoted by $Z_{2, t+i}^{x}$.

Then, using the expression for $V_{x, t}$, we can express the intermediary's incentive constraint (3.4) as,

$$
\nu_{x, t} Q_{x, t} S_{x, t}+\eta_{x, t} N_{x, t} \geq \lambda_{B} Q_{x, t} S_{x, t} .
$$

As indicated above, under perfect capital markets banks will expand borrowing until the risk premium collapses to zero which implies that in this case $\nu_{x, t}$ equals zero as well. Imperfect capital markets however, limit the possibilities for this kind of arbitrage because the intermediaries are constrained by their equity capital. If the incentive constraint binds it follows that,

$$
\begin{align*}
Q_{x, t} S_{x, t} & =\frac{\eta_{x, t}}{\lambda_{B}-\nu_{x, t}} N_{x, t} \\
& =\varrho_{x, t} N_{x, t} . \tag{3.5}
\end{align*}
$$

In this case, the quantity of assets which the intermediary can acquire depends on the equity capital, $N_{x, t}$, as well as the intermediary's leverage ratio, $\varrho_{x, t}$, limiting the bank's ability to
acquire assets. This leverage ratio is the ratio of the bank's intermediated assets to equity. The bank's leverage ratio is limited to the point where its maximized expected terminal wealth equals the gains from diverting the fraction $\lambda_{B}$ from available funds. However, the constraint (3.5) binds only if $0<\nu_{x, t}<\lambda_{B}$ (given $N_{x, t}>0$ ). This inequality is always satisfied with our estimates.

Using the leverage ratio (3.5) we can express the evolution of the intermediary's wealth as,

$$
N_{x, t+1}=\left[\left(R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}\right) \varrho_{x, t}+R_{t}\right] \frac{N_{x, t}}{\pi_{C, t+1}} .
$$

From this equation it also follows that,

$$
Z_{2, t+1}^{x}=\frac{N_{x, t+1}}{N_{x, t}}=\left[\left(R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}\right) \varrho_{x, t}+R_{t}\right] \frac{1}{\pi_{C, t+1}},
$$

and,

$$
Z_{1, t+1}^{x}=\frac{Q_{x, t+1} S_{x, t+1}}{Q_{x, t} S_{x, t}}=\frac{\varrho_{x, t+1} N_{x, t+1}}{\varrho_{x, t} N_{x, t}}=\frac{\varrho_{x, t+1}}{\varrho_{x, t}} Z_{2, t+1}^{x} .
$$

Financial intermediaries which are forced into bankruptcy are replaced by new entrants. Therefore, total wealth of financial intermediaries is the sum of the net worth of existing, $N_{x, t}^{e}$, and new ones, $N_{x, t}^{n}$,

$$
N_{x, t}=N_{x, t}^{e}+N_{x, t}^{n} .
$$

The fraction $\theta_{B}$ of bankers at $t-1$ which survive until $t$ is equal across branches. Then, the law of motion for existing bankers is given by,

$$
\begin{equation*}
N_{x, t}^{e}=\theta_{B}\left[\left(R_{x, t}^{B} \pi_{C, t}-R_{t-1}\right) \varrho_{x, t-1}+R_{t-1}\right] \frac{N_{x, t-1}}{\pi_{C, t}}, \quad 0<\theta_{B}<1 . \tag{3.6}
\end{equation*}
$$

where a main source of variation is the ex-post excess return on assets, $R_{x, t}^{B} \pi_{C, t}-R_{t-1}$.
New banks receive startup funds from their respective household, equal to a small fraction of the value of assets held by the existing bankers in their final operating period. Given that the exit probability is i.i.d., the value of assets held by the existing bankers in their final
operating period is given by $\left(1-\theta_{B}\right) Q_{x, t} S_{x, t}$. The transfer to new intermediaries is a fraction, $\varpi$, of this value, leading to the following formulation for new banker's wealth,

$$
\begin{equation*}
N_{x, t}^{n}=\varpi Q_{x, t} S_{x, t}, \quad 0<\varpi<1 \tag{3.7}
\end{equation*}
$$

Existing banker's net worth (3.6) and entering banker's net worth (3.7) lead to the law of motion for total net worth,

$$
N_{x, t}=\left(\theta_{B}\left[\left(R_{x, t}^{B} \pi_{C, t}-R_{t-1}\right) \varrho_{x, t-1}+R_{t-1}\right] \frac{N_{x, t-1}}{\pi_{C, t}}+\varpi Q_{x, t} S_{x, t}\right)_{\varsigma_{x, t}}
$$

where $\varsigma_{x, t}$ is s shock to bank's equity capital. The excess return, $x=C, I$ can be defined as,

$$
R_{x, t}^{S}=R_{x, t+1}^{B} \pi_{C, t+1}-R_{t} .
$$

Since $R_{t}, \lambda_{B}, \varpi$ and $\theta_{B}$ are equal across sectors, the institutional setup of the two representative banks in the two sectors is symmetric. Both branches hold deposits from households and buy assets from firms in the sector they provide specialized lending. Their performance differs because the demand for capital differs across sectors resulting in sectorspecific prices of capital, $Q_{x, t}$, and nominal rental rates for capital, $R_{x, t}^{K}$. Note that the institutional setup of banks does not depend on firm-specific factors. Gertler and Karadi (2011) show that this implies a setup with a continuum of banks is equivalent to a formulation with a representative bank. Owing to the symmetry of the banks this also holds for our formulation of financial intermediaries in the two-sector setup.

### 3.4 Resource Constraints

The resource constraint in the consumption sector is,

$$
C_{t}+\left(a\left(u_{C, t}\right) \xi_{C, t}^{K} \bar{K}_{C, t-1}+a\left(u_{I, t}\right) \xi_{I, t}^{K} \bar{K}_{I, t-1}\right) \frac{A_{t} V_{t}^{\frac{a_{c}}{1-a_{i}}}}{V_{t}^{\frac{1}{1-a_{i}}}}=a_{l t} A_{t} L_{c, t}^{1-a_{c}} K_{c, t}^{a_{c}}-A_{t} V_{t}^{\frac{a_{c}}{1-a_{i}}} F_{C}
$$

The resource constraint in the investment sector is,

$$
I_{I, t}+I_{C, t}=v_{l t} V_{t} L_{I, t}^{1-a_{i}} K_{I, t}^{a_{i}}-V_{t}^{\frac{1}{1-a_{i}}} F_{I}
$$

Hours worked are aggregated as,

$$
L_{t}=L_{I, t}+L_{C, t} .
$$

Bank equity is aggregated as,

$$
N_{t}=N_{I, t}+N_{C, t} .
$$

### 3.5 Stationary Economy

The model includes two non-stationary TFP shocks, $A_{t}$ and $V_{t}$. This section shows how we normalize the model to render it stationary. Lower case variables denote normalized stationary variables.

The model variables can be stationarized as follows:

$$
\begin{align*}
& k_{x, t}=\frac{K_{x, t}}{V_{t}^{\frac{1}{1-a_{i}}}}, \quad \bar{k}_{x, t}=\frac{\bar{K}_{x, t}}{V_{t}^{\frac{1}{1-a_{i}}}}, \quad k_{t}=\frac{K_{t}}{V_{t}^{\frac{1}{1-a_{i}}}},  \tag{3.8}\\
& i_{x, t}=\frac{I_{x, t}}{V_{t}^{\frac{1}{1-a_{i}}}}, \quad i_{t}=\frac{I_{t}}{V_{t}^{\frac{1}{1-a_{i}}}}, \quad c_{t}=\frac{C_{t}}{A_{t} V_{t}^{\frac{a_{c}}{1-a_{i}}}},  \tag{3.9}\\
& r_{C, t}^{K}=\frac{R_{C, t}^{K}}{P_{C, t}} A_{t}^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{i}}}, \quad r_{I, t}^{K}=\frac{R_{I, t}^{K}}{P_{C, t}} A_{t}^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{i}}}, \quad w_{t}=\frac{W_{t}}{P_{C, t} A_{t} V_{t}^{\frac{a_{c}}{1-a_{i}}}} . \tag{3.10}
\end{align*}
$$

From

$$
\begin{aligned}
\frac{P_{I, t}}{P_{C, t}} & =\frac{m c_{C, t}}{m c_{I, t}} \frac{1-a_{c}}{1-a_{i}} \frac{A_{t}}{V_{t}}\left(\frac{K_{I, t}}{L_{I, t}}\right)^{-a_{i}}\left(\frac{K_{C, t}}{L_{C, t}}\right)^{a_{c}} \\
& =\frac{m c_{C, t}}{m c_{I, t}} \frac{1-a_{c}}{1-a_{i}} A_{t} V_{t}^{\frac{a_{c}-1}{1-a_{i}}}\left(\frac{k_{I, t}}{L_{I, t}}\right)^{-a_{i}}\left(\frac{k_{C, t}}{L_{C, t}}\right)^{a_{c}},
\end{aligned}
$$

follows that,

$$
\begin{equation*}
p_{i, t}=\frac{P_{I, t}}{P_{C, t}} A_{t}^{-1} V_{t}^{\frac{1-a_{C}}{1-a_{i}}} . \tag{3.11}
\end{equation*}
$$

and the multipliers are normalized as,

$$
\begin{equation*}
\lambda_{t}=\Lambda_{t} A_{t} V_{t}^{\frac{a_{c}}{1-a_{i}}}, \quad \phi_{x, t}=\Phi_{x, t} V_{t}^{\frac{1}{1-a_{i}}} . \tag{3.12}
\end{equation*}
$$

where $\Phi_{x, t}$ denotes the multiplier on the respective capital accumulation equation. Using
the growth of investment, it follows that the prices of capital can be normalized as,

$$
q_{x, t}^{T}=Q_{x, t}^{T} A_{t}^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{i}}}, \quad q_{x, t}^{h}=Q_{x, t}^{h} A_{t}^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{i}}}, \quad q_{x, t}=Q_{x, t} A_{t}^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{i}}}
$$

with the price of capital in sector $x$, defined as,

$$
\begin{gathered}
q_{x, t}^{T}=\phi_{x, t} / \lambda_{t}, \quad x=C, I . \\
s_{x, t}=\frac{S_{x, t}}{V_{t}^{\frac{1}{1-a_{i}}}}, \quad s_{x, t}^{h}=\frac{S_{x, t}^{h}}{V_{t}^{\frac{1}{1-a_{i}}}} .
\end{gathered}
$$

Then, it follows from entering bankers wealth equation (3.7) that,

$$
n_{x, t}^{n}=N_{x, t}^{n} A_{t}^{-1} V_{t}^{\frac{-a_{c}}{1-a_{i}}}
$$

Total wealth, wealth of existing and entering bankers has to grow at the same rate,

$$
n_{x, t}^{e}=N_{x, t}^{e} A_{t}^{-1} V_{t}^{\frac{-a_{c}}{1-a_{i}}}, \quad n_{x, t}=N_{x, t} A_{t}^{-1} V_{t}^{\frac{-a_{c}}{1-a_{i}}}
$$

### 3.5.1 Intermediate goods producers

Firm's production function in the consumption sector:

$$
\begin{equation*}
c_{t}=a_{l t} L_{C, t}^{1-a_{c}} k_{C, t}^{a_{c}}-F_{C} \tag{3.13}
\end{equation*}
$$

Firm's production function in the investment sector:

$$
\begin{equation*}
i_{t}=v_{l t} L_{I, t}^{1-a_{i}} k_{I, t}^{a_{i}}-F_{I} . \tag{3.14}
\end{equation*}
$$

Marginal costs in the consumption sector:

$$
\begin{equation*}
m c_{C, t}=\left(1-a_{c}\right)^{a_{c}-1} a_{c}^{-a_{c}}\left(r_{C, t}^{K}\right)^{a_{c}} w_{t}^{1-a_{c}} a_{l t}^{-1} . \tag{3.15}
\end{equation*}
$$

Marginal costs in the investment sector:

$$
\begin{equation*}
m c_{I, t}=\left(1-a_{i}\right)^{a_{i}-1} a_{i}^{-a_{i}} w_{t}^{1-a_{i}}\left(r_{I, t}^{K}\right)^{a_{i}} v_{l t}^{-1} p_{i, t}^{-1}, \quad \text { with } \quad p_{i, t}=\frac{P_{I, t}}{P_{C, t}} . \tag{3.16}
\end{equation*}
$$

Capital labour ratios in the two sectors:

$$
\begin{equation*}
\frac{k_{C, t}}{L_{C, t}}=\frac{w_{t}}{r_{C, t}^{K}} \frac{a_{c}}{1-a_{c}}, \quad \frac{k_{I, t}}{L_{I, t}}=\frac{w_{t}}{r_{I, t}^{K}} \frac{a_{i}}{1-a_{i}} \tag{3.17}
\end{equation*}
$$

### 3.5.2 Firms' pricing decisions

Price setting equation for firms that change their price in sector $x=C, I$ :

$$
\begin{equation*}
0=E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p, x}^{s} \beta^{s} \lambda_{t+s} \tilde{x}_{t+s}\left[\tilde{p}_{x, t} \tilde{\Pi}_{t, t+s}-\left(1+\lambda_{p, t+s}^{x}\right) m c_{x, t+s}\right]\right\} \tag{3.18}
\end{equation*}
$$

with

$$
\begin{aligned}
& \tilde{\Pi}_{t, t+s}=\prod_{k=1}^{s}\left[\left(\frac{\pi_{x, t+k-1}}{\pi_{x}}\right)^{\iota_{p_{x}}}\left(\frac{\pi_{x, t+k}}{\pi_{x}}\right)^{-1}\right] \text { and } \tilde{x}_{t+s}=\left(\frac{\tilde{P}_{x, t}}{P_{x, t}} \tilde{\Pi}_{t, t+s}\right)^{-\frac{1+\lambda_{p, t+s}^{x}}{\lambda_{p, t+s}}} x_{t+s} \\
& \text { and } \frac{\tilde{P}_{x, t}}{P_{x, t}}=\tilde{p}_{x, t} .
\end{aligned}
$$

Aggregate price index in the consumption sector:

$$
1=\left[\left(1-\xi_{x, p}\right)\left(\tilde{p}_{x, t}\right)^{\frac{1}{\lambda_{p, t}^{x}}}+\xi_{x, p}\left[\left(\frac{\pi_{x, t-1}}{\pi_{x}}\right)^{\iota_{p x}}\left(\frac{\pi_{x, t}}{\pi_{x}}\right)^{-1}\right]^{\frac{1}{\lambda_{p, t}^{x}}}\right]^{\lambda_{p, t}^{x}} .
$$

It further holds that

$$
\begin{equation*}
\frac{\pi_{I, t}}{\pi_{C, t}}=\frac{p_{i, t}}{p_{i, t-1}} \tag{3.19}
\end{equation*}
$$

### 3.5.3 Household's optimality conditions and wage setting

Marginal utility of income:

$$
\begin{equation*}
\lambda_{t}=\frac{b_{t}}{c_{t}-h c_{t-1}\left(\frac{A_{t-1}}{A_{t}}\right)\left(\frac{V_{t-1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}}}-\beta h \frac{b_{t+1}}{c_{t+1}\left(\frac{A_{t+1}}{A_{t}}\right)\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}}-h c_{t}} . \tag{3.20}
\end{equation*}
$$

Euler equation:

$$
\lambda_{t}=\beta E_{t} \lambda_{t+1}\left(\frac{A_{t}}{A_{t+1}}\right)\left(\frac{V_{t}}{V_{t+1}}\right)^{\frac{a_{c}}{1-a_{i}}} R_{t} \frac{1}{\pi_{c, t+1}} .
$$

Labor supply

$$
\lambda_{t} w_{t}=b_{t} \varphi\left(L_{C, t}+L_{I, t}\right)^{\nu}
$$

Purchase of financial claims

$$
\begin{equation*}
1=E_{t} \beta \frac{\lambda_{t+1}}{\lambda_{t}} z_{t+1}^{-1} v_{t+1}^{-\frac{a_{c}}{1-a_{i}}} \frac{R_{x, t}^{h}}{\pi_{c, t+1}} \tag{3.21}
\end{equation*}
$$

### 3.5.4 Capital services

Optimal capital utilization:

$$
r_{C, t}^{K}=a_{C}^{\prime}\left(u_{C, t}\right), \quad r_{I, t}^{K}=a_{I}^{\prime}\left(u_{I, t}\right) .
$$

Definition of capital services:

$$
\begin{equation*}
k_{C, t}=u_{C, t} \xi_{C, t}^{K} \bar{k}_{C, t-1}\left(\frac{V_{t-1}}{V_{t}}\right)^{\frac{1}{1-a_{i}}}, \quad k_{I, t}=u_{I, t} \xi_{I, t}^{K} \bar{k}_{I, t-1}\left(\frac{V_{t-1}}{V_{t}}\right)^{\frac{1}{1-a_{i}}} . \tag{3.22}
\end{equation*}
$$

Optimal choice of available capital in sector $x=C, I$ :

$$
\begin{equation*}
\phi_{x, t}=\beta E_{t} \xi_{x, t+1}^{K}\left\{\lambda_{t+1}\left(\frac{V_{t}}{V_{t+1}}\right)^{\frac{1}{1-a_{i}}}\left(r_{x, t+1}^{K} u_{x, t+1}-a\left(u_{x, t+1}\right)\right)+(1-\delta) E_{t} \phi_{x, t+1}\left(\frac{V_{t}}{V_{t+1}}\right)^{\frac{1}{1-a_{i}}}\right\}, \tag{3.23}
\end{equation*}
$$

Optimal financing from households

$$
\begin{array}{r}
0=\beta E_{t} \frac{\lambda_{t+1}}{\lambda_{t}}\left[r_{x, t+1}^{K} u_{x, t+1} \frac{q_{x, t}^{h}}{q_{x, t}^{T}}-a_{x}\left(u_{x, t+1}\right) \frac{q_{x, t}^{h}}{q_{x, t}^{T}}\right]-\gamma^{h} v_{t+1}^{\frac{1}{1-a_{i}}} \\
-\Gamma_{s_{x}^{h}}\left[\left(\frac{s_{x, t}^{h}}{s_{x}^{h}} v_{t}^{\frac{1}{1-a_{i}}}-e^{\left(\frac{1}{1-a_{i}}\right) g_{v}}\right),\left(\frac{s_{x, t}}{s_{x}} v_{t}^{\frac{1}{11-a_{i}}}-e^{\left(\frac{1}{1-a_{i}}\right) g_{v}}\right)\right] v_{t}^{\frac{1}{1-a_{i}}} \frac{v_{t+1}^{\frac{1}{1-a_{i}}}}{s_{x}^{h}} \tag{3.24}
\end{array}
$$

Optimal financing from financial intermediaries

$$
\begin{array}{r}
0=\beta E_{t} \frac{\lambda_{t+1}}{\lambda_{t}}\left[r_{x, t+1}^{K} u_{x, t+1} \frac{q_{x, t}}{q_{x, t}^{T}}-a_{x}\left(u_{x, t+1}\right) \frac{q_{x, t}}{q_{x, t}^{T}}\right]-\gamma v_{t+1}^{\frac{1}{1-a_{i}}} \\
-\Gamma_{s_{x}}\left[\left(\frac{s_{x, t}}{s_{x}} v_{t}^{\frac{1}{1-a_{i}}}-e^{\left(\frac{1}{1-a_{i}}\right) g_{v}}\right),\left(\frac{s_{x, t}}{s_{x}} v_{t}^{\frac{1}{1-a_{i}}}-e^{\left(\frac{1}{1-a_{i}}\right) g_{v}}\right)\right] v_{t}^{\frac{1}{11-a_{i}}} \frac{v_{t+1}^{\frac{1}{1-a_{i}}}}{s_{x}} \tag{3.25}
\end{array}
$$

Household's return on claims

$$
\begin{equation*}
R_{x, t}^{h}=\left[r_{x, t+1}^{K} u_{x, t+1}+q_{x, t+1}^{h}\left(1-\delta_{x}\right)-a_{x}\left(u_{x, t+1}\right)\right] \frac{z_{t+1}}{\frac{1-a_{c}}{\frac{1-a_{i}}{1}} q_{x, t}^{h}} v_{t+1} . \tag{3.26}
\end{equation*}
$$

Total value of acquired capital

$$
\begin{equation*}
q_{x, t}^{T} \bar{k}_{x, t}=q_{x, t}^{h} s_{x, t}^{h}+q_{x, t} s_{x, t} . \tag{3.27}
\end{equation*}
$$

### 3.5.5 Physical capital producers

Optimal choice of investment in sector $x=C, I$ :

$$
\begin{align*}
\lambda_{t} p_{i, t}= & \phi_{x, t} \mu_{t}\left[1-S\left(\frac{i_{x, t}}{i_{x, t-1}}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1}{1-a_{i}}}\right)-S^{\prime}\left(\frac{i_{x, t}}{i_{x, t-1}}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1}{1-a_{i}}}\right) \frac{i_{x, t}}{i_{x, t-1}}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1}{1-a_{i}}}\right] \\
& +\beta E_{t} \phi_{x, t+1} \mu_{t+1}\left(\frac{V_{t}}{V_{t+1}}\right)^{\frac{1}{1-a_{i}}}\left[S^{\prime}\left(\frac{i_{x, t+1}}{i_{x, t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{1}{1-a_{i}}}\right)\left(\frac{i_{x, t+1}}{i_{x, t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{1}{1-a_{i}}}\right)^{2}\right] . \tag{3.28}
\end{align*}
$$

Accumulation of capital in sector $x=C, I$ :

$$
\begin{equation*}
\bar{k}_{x, t}=\left(1-\delta_{x}\right) \xi_{x, t}^{K} \bar{k}_{x, t-1}\left(\frac{V_{t-1}}{V_{t}}\right)^{\frac{1}{1-a_{i}}}+\mu_{t}\left(1-S\left(\frac{i_{x, t}}{i_{x, t-1}}\left(\frac{V_{t}}{V_{t-1}}\right)^{\frac{1}{1-a_{i}}}\right)\right) i_{x, t} \tag{3.29}
\end{equation*}
$$

### 3.5.6 Household's wage setting

Household's wage setting:

$$
\begin{equation*}
E_{t} \sum_{s=0}^{\infty} \beta^{s} \xi_{w}^{s} \lambda_{t+s} \tilde{L}_{t+s}\left[\tilde{w}_{t} \tilde{\Pi}_{t, t+s}^{w}-\left(1+\lambda_{w, t+s}\right) b_{t+s} \varphi \frac{\tilde{L}_{t+s}^{\nu}}{\lambda_{t+s}}\right]=0 \tag{3.30}
\end{equation*}
$$

with

$$
\begin{aligned}
& \tilde{\Pi}_{t, t+s}^{w}=\prod_{k=1}^{s}\left[\left(\frac{\pi_{C, t+k-1} e^{a_{t+k-1}+\frac{a_{c}}{1-a_{i}} v_{t+k-1}}}{\pi_{c} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}}}\right)^{\iota_{w}}\left(\frac{\pi_{C, t+k} e^{a_{t+k}+\frac{a_{c}}{1-a_{i}} v_{t+k}}}{\pi_{C} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}}}\right)^{-1}\right] \\
& \quad \text { and } \\
& \tilde{L}_{t+s}=\left(\frac{\tilde{w}_{t} \tilde{\Pi}_{t, t+s}^{w}}{w_{t+s}}\right)^{-\frac{1+\lambda_{w, t+s}}{\lambda_{w, t+s}}} L_{t+s} .
\end{aligned}
$$

Wages evolve according to

$$
w_{t}=\left\{\left(1-\xi_{w}\right) \tilde{w}_{t}^{\frac{1}{\lambda_{w, t}}}+\xi_{w}\left[\left(\frac{\pi_{c, t-1} e^{a_{t-1}+\frac{a_{c}}{1-a_{i}} v_{t-1}}}{\pi_{c} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}}}\right)^{l_{w}}\left(\frac{\pi_{c, t} e^{a_{t}+\frac{a_{c}}{1-a_{i}} v_{t}}}{\pi_{c} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}}}\right)^{-1} w_{t-1}\right]^{\frac{1}{\lambda_{w, t}}}\right\}^{\lambda_{w, t}} .
$$

### 3.5.7 Financial Intermediation

The stationary stochastic discount factor can be expressed as,

$$
\lambda_{t+1}^{B}=\frac{\lambda_{t+1}}{\lambda_{t}} .
$$

Then, one can derive expressions for $\nu_{x, t}$ and $\eta_{x, t}$,

$$
\begin{aligned}
\nu_{x, t} & =E_{t}\left\{\left(1-\theta_{B}\right) \lambda_{t+1}^{B} \frac{A_{t}}{A_{t+1}}\left(\frac{V_{t}}{V_{t+1}}\right)^{\frac{a_{c}}{1-a_{i}}}\left(R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}\right)+\theta_{B} \beta z_{1, t+1}^{x} \nu_{x, t+1}\right\}, \\
\eta_{x, t} & =E_{t}\left\{\left(1-\theta_{B}\right) \lambda_{t+1}^{B} \frac{A_{t}}{A_{t+1}}\left(\frac{V_{t}}{V_{t+1}}\right)^{\frac{a_{c}}{1-a_{i}}} R_{t}+\theta_{B} \beta z_{2, t+1}^{x} \eta_{x, t+1}\right\},
\end{aligned}
$$

with

$$
z_{1, t+1+i}^{x} \equiv \frac{q_{x, t+1+i} s_{x, t+1+i}}{q_{x, t+i} s_{x, t+i}} \frac{A_{t+1}}{A_{t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}}, \quad z_{2, t+1+i}^{x} \equiv \frac{n_{x, t+1+i}}{n_{x, t+i}} \frac{A_{t+1}}{A_{t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}} .
$$

It follows that if the bank's incentive constraint binds it can be expressed as,

$$
\begin{aligned}
& \nu_{x, t} q_{x, t} s_{x, t}+\eta_{x, t} n_{x, t}=\lambda_{B} q_{x, t} s_{x, t} \\
\Leftrightarrow & q_{x, t} s_{x, t}=\varrho_{x, t} n_{x, t},
\end{aligned}
$$

with the leverage ratio given as,

$$
\varrho_{x, t}=\frac{\eta_{x, t}}{\lambda_{B}-\nu_{x, t}} .
$$

It further follows that:

$$
z_{2, t+1}^{x}=\frac{n_{x, t+1}}{n_{x, t}} \frac{A_{t+1}}{A_{t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}}=\left[\left(R_{x, t+1}^{B} \pi_{C, t+1}-R_{t}\right) \varrho_{x, t}+R_{t}\right] \frac{1}{\pi_{C, t+1}},
$$

and

$$
z_{1, t+1}^{x}=\frac{q_{x, t+1} s_{x, t+1}}{q_{x, t} s_{x, t}} \frac{A_{t+1}}{A_{t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}}=\frac{\varrho_{x, t+1} n_{x, t+1}}{\varrho_{x, t} n_{x, t}} \frac{A_{t+1}}{A_{t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}}=\frac{\varrho_{x, t+1}}{\varrho_{x, t}} z_{2, t+1}^{x} .
$$

The normalized equation for bank's wealth accumulation is,

$$
n_{x, t}=\left(\theta_{B}\left[\left(R_{x, t}^{B} \pi_{C, t}-R_{t-1}\right) \varrho_{x, t-1}+R_{t-1}\right] \frac{A_{t-1}}{A_{t}}\left(\frac{V_{t-1}}{V_{t}}\right)^{\frac{a_{c}}{1-a_{i}}} \frac{n_{x, t-1}}{\pi_{C, t}}+\varpi q_{x, t} s_{x, t}\right) \varsigma_{x, t} .
$$

The leverage equation:

$$
q_{x, t} s_{x, t}=\varrho_{x, t} n_{x, t} .
$$

Bank's stochastic return on assets can be described in normalized variables as:

$$
R_{x, t+1}^{B}=\frac{r_{x, t+1}^{K} u_{x, t+1}+q_{x, t+1}\left(1-\delta_{x}\right)-a\left(u_{x, t+1}\right)}{q_{x, t}} \xi_{x, t+1}^{K} \frac{A_{t+1}}{A_{t}}\left(\frac{V_{t+1}}{V_{t}}\right)^{-\frac{1-a_{c}}{1-a_{i}}},
$$

knowing from the main model that

$$
r_{x, t}^{K}=\frac{R_{x, t}^{K}}{P_{x, t}} A_{t}^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{i}}} .
$$

### 3.5.8 Monetary policy and market clearing

Monetary policy rule:

$$
\frac{R_{t}}{R}=\left(\frac{R_{t-1}}{R}\right)^{\rho_{R}}\left[\left(\frac{\pi_{C, t}}{\pi_{C}}\right)^{\phi_{\pi}}\left(\frac{y_{t}}{y_{t-1}}\right)^{\phi_{\Delta Y}}\right]^{1-\rho_{R}} \eta_{m p, t},
$$

Resource constraint in the consumption sector:

$$
c_{t}+\left(a\left(u_{C, t}\right) \xi_{C, t}^{K} \bar{k}_{C, t-1}+a\left(u_{I, t}\right) \xi_{I, t}^{K} \bar{k}_{I, t-1}\right)\left(\frac{V_{t-1}}{V_{t}}\right)^{\frac{1}{1-a_{i}}}=a_{l t} L_{C, t}^{1-a_{c}} k_{C, t}^{a_{c}}-F_{C} .
$$

Resource constraint in the investment sector:

$$
i_{t}=v_{l t} L_{I, t}^{1-a_{i}} k_{I, t}^{a_{i}}-F_{I} .
$$

Definition of GDP:

$$
\begin{equation*}
y_{t}=c_{t}+p_{i, t} i_{t}+\left(1-\frac{1}{g_{t}}\right) y_{t} . \tag{3.31}
\end{equation*}
$$

Moreover

$$
L_{t}=L_{I, t}+L_{C, t}, \quad i_{t}=i_{C, t}+i_{I, t}, \quad n_{t}=n_{C, t}+n_{I, t} .
$$

### 3.6 Steady State

This section describes the model's steady state.

From the optimal choice of available capital (3.23) and the optimal choice of investment (3.28) in both sectors:

$$
\begin{align*}
& r_{C}^{K}=\left(\frac{e^{\frac{1}{1-a_{i}} g_{v}}}{\beta}-\left(1-\delta_{C}\right)\right) p_{i},  \tag{3.32}\\
& r_{I}^{K}=\left(\frac{e^{\frac{1}{1-a_{i}} g_{v}}}{\beta}-\left(1-\delta_{I}\right)\right) p_{i} . \tag{3.33}
\end{align*}
$$

From firm's price setting in both sectors (3.18),

$$
\begin{equation*}
m c_{C}=\frac{1}{1+\lambda_{p}^{C}}, \quad m c_{I}=\frac{1}{1+\lambda_{p}^{I}} . \tag{3.34}
\end{equation*}
$$

Using equations (3.34) and imposing knowledge of the steady state expression for $r_{C}^{K}$ and $r_{I}^{K}$, one can derive expressions for the steady state wage from the equations that define marginal costs in the two sectors ((3.15) and (3.16)).

Consumption sector:

$$
\begin{equation*}
w=\left(\frac{1}{1+\lambda_{p}^{C}}\left(1-a_{c}\right)^{1-a_{c}} a_{c}^{a_{c}}\left(r_{C}^{K}\right)^{-a_{c}}\right)^{\frac{1}{1-a_{c}}} \tag{3.35}
\end{equation*}
$$

Investment sector:

$$
\begin{equation*}
w=\left(\frac{1}{1+\lambda_{p}^{I}}\left(1-a_{i}\right)^{1-a_{i}} a_{i}^{a_{i}}\left(r_{I}^{K}\right)^{-a_{i}} p_{i}\right)^{\frac{1}{1-a_{i}}} \tag{3.36}
\end{equation*}
$$

Since labour can move across sectors the steady state wage has to be the same in the consumption and investment sector. The equality is verified by $p_{i}$. An expression for $p_{i}$ can be found by setting (3.35) equal to (3.36):

$$
\begin{align*}
&\left(\frac{1}{1+\lambda_{p}^{C}}\left(1-a_{c}\right)^{1-a_{c}} a_{c}^{a_{c}}\left(r_{C}^{K}\right)^{-a_{c}}\right)^{\frac{1}{1-a_{c}}}=\left(\frac{1}{1+\lambda_{p}^{I}}\left(1-a_{i}\right)^{1-a_{i}} a_{i}^{a_{i}}\left(r_{I}^{K}\right)^{-a_{i}} p_{i}\right)^{\frac{1}{1-a_{i}}} \\
& \Leftrightarrow\left(\frac{1}{1+\lambda_{p}^{C}}\left(1-a_{c}\right)^{1-a_{c}} a_{c}^{a_{c}}\left(\frac{e^{\frac{1}{1-a_{i}} g_{v}}}{\beta}-\left(1-\delta_{C}\right)\right)^{-a_{c}} p_{i}^{-a_{c}}\right)^{\frac{1}{1-a_{c}}} \\
& \quad=\left(\frac{1}{1+\lambda_{p}^{I}}\left(1-a_{i}\right)^{1-a_{i}} a_{i}^{a_{i}}\left(\frac{e^{\frac{1}{1-a_{i}} g_{v}}}{\beta}-\left(1-\delta_{I}\right)\right)^{-a_{i}} p_{i}^{-a_{i}} p_{i}\right)^{\frac{1}{1-a_{i}}} \\
& \Leftrightarrow p_{i}=\frac{\frac{1}{1+\lambda_{p}^{C}}\left(1-a_{c}\right)^{1-a_{c}} a_{c}^{a_{c}}\left(\frac{e^{\frac{1-1}{1-a_{i}} g_{v}}}{\beta}-\left(1-\delta_{C}\right)\right)^{-\alpha_{c}}}{\left[\frac{1}{1+\lambda_{p}^{I}}\left(1-a_{i}\right)^{1-a_{i}} a_{i}^{a_{i}}\left(\frac{e^{\frac{1}{1-a_{i}} g_{v}}}{\beta}-\left(1-\delta_{I}\right)\right)^{-\alpha_{i}}\right]^{\frac{1-a_{c}}{1-a_{i}}}} \tag{3.37}
\end{align*}
$$

Knowing $w, r_{C}^{K}$ and $r_{I}^{K}$, the expressions given in (3.17) can be used to find the steady state capital-to-labour ratios in the two sectors:

$$
\begin{align*}
& \frac{k_{C}}{L_{C}}=\frac{w}{r_{C}^{K}} \frac{a_{c}}{1-a_{c}},  \tag{3.38}\\
& \frac{k_{I}}{L_{I}}=\frac{w}{r_{I}^{K}} \frac{a_{i}}{1-a_{c}} . \tag{3.39}
\end{align*}
$$

The zero profit condition for intermediate goods producers in the consumption sector,
$c-r_{C}^{K} k_{C}-w L_{C}=0$, and (3.13) imply:

$$
\begin{aligned}
& L_{C}^{1-a_{c}} k_{C}^{a_{c}}-F_{C}-r_{C}^{K} k_{C}-w L_{C}=0 \\
\Leftrightarrow & \frac{F_{C}}{L_{C}}=\left(\frac{k_{C}}{L_{C}}\right)^{a_{c}}-r_{C}^{K} \frac{k_{C}}{L_{C}}-w .
\end{aligned}
$$

Analogously the zero profit condition for intermediate goods producers in the investment sector, $i-r_{I}^{K} k_{I}-w L_{I}=0$, and (3.14) imply:

$$
\frac{F_{I}}{L_{I}}=\left(\frac{k_{I}}{L_{I}}\right)^{a_{i}}-r_{I}^{K} \frac{k_{I}}{L_{I}}-w
$$

These expressions pin down the steady state consumption-to-labour and investment-tolabour ratios which follow from the intermediate firms' production functions ((3.13) and (3.14)):

$$
\begin{gathered}
\frac{c}{L_{C}}=\left(\frac{k_{C}}{L_{C}}\right)^{a_{c}}-\frac{F_{C}}{L_{C}}, \quad \frac{i}{L_{I}}=\left(\frac{k_{I}}{L_{I}}\right)^{a_{i}}-\frac{F_{I}}{L_{I}} . \\
1+\lambda_{p}^{C}=\frac{c+F_{C}}{c} \Leftrightarrow \lambda_{p}^{C} c=F_{C}, \quad \text { and } \quad 1+\lambda_{p}^{I}=\frac{i+F_{I}}{i} \Leftrightarrow \lambda_{p}^{I} i=F_{I} .
\end{gathered}
$$

This and the steady state consumption-to-labour ratio can be used to derive an expression for steady state consumption:

$$
\begin{aligned}
c & =\left(\frac{k_{C}}{L_{C}}\right)^{a_{c}} L_{C}-F_{C} \\
\Leftrightarrow c & =\left(\frac{k_{C}}{L_{C}}\right)^{a_{c}} L_{C}-\lambda_{p}^{C} c \\
\Leftrightarrow c & =\frac{1}{1+\lambda_{p}^{C}}\left(\frac{k_{C}}{L_{C}}\right)^{a_{c}} L_{C} .
\end{aligned}
$$

Analogously one can derive an expression for steady state investment:

$$
i=\frac{1}{1+\lambda_{p}^{I}}\left(\frac{k_{I}}{L_{I}}\right)^{a_{i}} L_{I} .
$$

Combining these two expressions leads to,

$$
\begin{aligned}
p_{i} \frac{i}{c} & =\frac{\frac{1}{1+\lambda_{p}^{I}}\left(\frac{k_{I}}{L_{I}}\right)^{a_{i}} L_{I}}{\left.\frac{1}{1+\lambda_{p}^{C}} \frac{k_{C}}{L_{C}}\right)^{a_{C}} L_{C}} p_{i} \\
\Leftrightarrow \frac{L_{I}}{L_{C}} & =p_{i} \frac{i \frac{1}{1+\lambda_{p}^{C}}\left(\frac{k_{C}}{L_{C}}\right)^{a_{C}}}{\frac{1}{1+\lambda_{p}}\left(\frac{k_{I}}{L_{I}}\right)^{a_{i}}} p_{i}^{-1} .
\end{aligned}
$$

Total labour $L$ is set to unity in the steady state. However, since $a_{i}$ and $a_{c}$ are not necessarily calibrated to be equal one needs to fix another quantity in addition to $L=1$. We fix the steady state investment-to-consumption ratio, $p_{i} \frac{i}{c}$, which equals 0.426 in the data. This allows us to derive steady state expressions for labour in the two sectors. Steady state labour in the investment sector is given by

$$
\begin{equation*}
L_{I}=1-L_{C}, \tag{3.40}
\end{equation*}
$$

and the two equations above imply that steady state labour in the consumption sector can be expressed as,

$$
\begin{equation*}
L_{C}=\left(1+p_{i} \frac{\left.i \frac{1}{c} \frac{\frac{1}{1+\lambda_{p}^{C}}\left(\frac{k_{C}}{L_{C}}\right)^{a_{c}}}{\frac{1}{1+\lambda_{p}^{I}}\left(\frac{k_{I}}{L_{I}}\right)^{a_{i}}} p_{i}^{-1}\right)^{-1} . . . . . . .}{}\right. \tag{3.41}
\end{equation*}
$$

The steady state values for labour in the two sectors imply:

$$
k_{C}=\frac{k_{C}}{L_{C}} L_{C}, \quad k_{I}=\frac{k_{I}}{L_{I}} L_{I}, \quad c=\frac{c}{L_{C}} L_{C}, \quad i=\frac{i}{L_{I}} L_{I}, \quad F_{C}=\frac{F_{C}}{L_{C}} L_{C}, \quad F_{I}=\frac{F_{I}}{L_{I}} L_{I} .
$$

It follows from (3.22) that,

$$
k_{C}=\bar{k}_{C} e^{-\frac{1}{1-a_{i}} g_{v}}, \quad \text { and } \quad k_{I}=\bar{k}_{I} e^{-\frac{1}{1-a_{i}} g_{v}}
$$

The accumulation equation of available capital (3.29) can be used to solve for investment in the two sectors:

$$
\begin{align*}
i_{C} & =k_{C}\left(e^{\frac{1}{1-a_{i}} g_{v}}-\left(1-\delta_{C}\right)\right),  \tag{3.42}\\
i_{I} & =k_{I}\left(e^{\frac{1}{1-a_{i}} g_{v}}-\left(1-\delta_{I}\right)\right) . \tag{3.43}
\end{align*}
$$

From the definition of GDP (3.31):

$$
y=c+p_{i} i+\left(1-\frac{1}{g}\right) y .
$$

From the marginal utility of income (3.20):

$$
\lambda=\frac{1}{c-h c e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}}}-\frac{\beta h}{c e^{g_{a}+\frac{a c}{1-a_{i}} g_{v}}-h c} .
$$

From the household's wage setting (3.30)

$$
\sum_{s=0}^{\infty} \beta^{s} \xi_{w}^{s} \lambda L\left[w-\left(1+\lambda_{w}\right) \varphi \frac{L^{\nu}}{\lambda}\right]=0
$$

follows the expression for $L$ :

$$
w-\left(1-\lambda_{w}\right) \varphi \frac{L^{\nu}}{\lambda}=0 \quad \Rightarrow \quad L=\left[\frac{w \lambda}{\left(1+\lambda_{w}\right) \varphi}\right]^{\frac{1}{\nu}}
$$

This expression can be solved for $\varphi$ to be consistent with $L=1$ :

$$
\begin{aligned}
1 & =\left[\frac{w \lambda}{\left(1+\lambda_{w}\right) \varphi}\right]^{\frac{1}{\nu}} \\
\Leftrightarrow \varphi & =\frac{\lambda w}{1+\lambda_{w}} .
\end{aligned}
$$

It further holds from equation (3.19) that,

$$
\frac{\pi_{I}}{\pi_{C}}=e^{g_{a}-\frac{1-a_{c}}{1-a_{i}} g_{v}}
$$

A system of 10 equations (3.32, 3.33, 3.35, 3.37, 3.38, 3.39, 3.40, 3.41, 3.42, 3.43) can be solved for the 10 steady state variables $k_{C}, k_{I}, w, i_{C}, i_{I}, r_{C}^{K}, r_{I}^{K}, L_{C}, L_{I}$ and $p_{i}$. The steady state values for the remaining variables follow from the expressions above.

Given these steady state variables, the remaining steady state values are mainly related to financial intermediaries and household's provision of funds for capital acquisition. These can be derived as follows.

The nominal interest rate is given from the Euler equation as,

$$
R=\frac{1}{\beta} e^{g_{a}+\frac{a_{c}}{1-a_{i}} g_{v}} \pi_{C}
$$

The bank's stationary stochastic discount factor can be expressed in the steady state as

$$
\lambda^{B}=1 .
$$

The steady state price of capital is given by

$$
q_{x, t}=p_{i, t} .
$$

The steady state leverage equation is given by

$$
\frac{q_{x} s_{x}}{n_{x}}=\varrho_{x}
$$

Given the non-linearity in the leverage ratio, we solve numerically for the steady state expressions for $\eta$ and $\nu$ using,

$$
\begin{aligned}
& \nu_{x}=\left(1-\theta_{B}\right) \lambda^{B} e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}}\left(R_{x}^{B} \pi_{C}-R\right)+\theta_{B} \beta z_{1}^{x} \nu_{x}, \\
& \eta_{x}=\left(1-\theta_{B}\right) \lambda^{B} e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}} R+\theta_{B} \beta z_{2}^{x} \eta_{x},
\end{aligned}
$$

with

$$
z_{2}^{x}=\left[\left(R_{x}^{B} \pi_{C}-R\right) \varrho_{x}+R\right] \frac{1}{\pi_{C}}, \quad \text { and } \quad z_{1}^{x}=z_{2}^{x}
$$

and the steady state leverage ratio,

$$
\varrho_{x}=\frac{\eta_{x}}{\lambda_{B}-\nu_{x}} .
$$

Recall that the parameter $\lambda_{B}$ is estimated. This information, the calibrated value for $\theta_{B}$ and the weighted quarterly average of the corporate spreads ( $R_{x}^{B}-R=0.5 \%$ ) allows us then to pin down $\varpi$ using the bank's wealth accumulation equation,

$$
\varpi=\left[1-\theta_{B}\left[\left(R_{x}^{B} \pi_{C}-R\right) \varrho_{x}+R\right] e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}} \frac{1}{\pi_{C}}\right]\left(\frac{q_{x} s_{x}}{n_{x}}\right)^{-1} .
$$

The steady state equations for bank's stochastic return can be solved to pin down

$$
q_{x}=\frac{r_{x}^{K}}{R_{x}^{B} e^{-g_{a}+\left(\frac{1-a_{c}}{1-a_{i}}\right) g_{v}}-\left(1-\delta_{x}\right)}
$$

Household's purchase of financial claims, equation (3.21), implies in steady state

$$
R_{x}^{h}=\frac{\pi_{c}}{\beta} e^{g_{a}+\left(\frac{a_{c}}{1-a_{i}}\right) g_{v}} .
$$

Household's sectoral returns on financial claims, equation (3.26) can be used to solve for $q_{C}^{h}$ and $q_{I}^{h}$

$$
R_{x}^{h}=\left[r_{x}^{K}+q_{x}^{h}\left(1-\delta_{x}\right)\right] \frac{e^{g_{a}}}{e^{\left(\frac{1-a_{c}}{1-a_{i}}\right) g_{v}} q_{x}^{h}}
$$

Since the price of capital $q_{C}^{T}=q_{I}^{T}=1$ and in the data the corporate bonds over equity
market capitalization, $\frac{q_{x} s_{x}}{q_{x}^{h} s_{x}^{h}}=0.25$, the additive identity for capital claims from equity and debt markets, equation (3.27), pins down $s_{x}^{h}$ as follows

$$
s_{x}^{h}=\frac{q_{x}^{T} \bar{k}_{x}}{\left(1+\frac{q_{x} s_{x}}{q_{x}^{h} s_{x}^{h}}\right) q_{x}^{h}} .
$$

This allows in turn to use equation (3.27) to pin down $s_{x}$. The conditions for bank's and firm's optimal financing, equations (3.24) and (3.25), pin down the fixed cost to portfolio adjustment

$$
\begin{aligned}
\gamma^{h} & =\beta r^{K} \frac{q^{h}}{q^{T}} e^{-\left(\frac{1}{1-a_{i}}\right) g_{v}} \\
\gamma & =\beta r^{K} \frac{q}{q^{T}} e^{-\left(\frac{1}{1-a_{i}}\right) g_{v}} .
\end{aligned}
$$

### 3.7 Log-linearized Economy

This section collects the log-linearized model equations. The log-linear deviations of all variables are defined as

$$
\hat{\varsigma}_{t} \equiv \log \varsigma_{t}-\log \varsigma,
$$

except for

$$
\begin{aligned}
\hat{z}_{t} & \equiv z_{t}-g_{a} \\
\hat{v}_{t} & \equiv v_{t}-g_{v} \\
\hat{\lambda}_{p, t}^{C} & \equiv \log \left(1+\lambda_{p, t}^{C}\right)-\log \left(1+\lambda_{p}^{C}\right), \\
\hat{\lambda}_{p, t}^{I} & \equiv \log \left(1+\lambda_{p, t}^{I}\right)-\log \left(1+\lambda_{p}^{I}\right), \\
\hat{\lambda}_{w, t} & \equiv \log \left(1+\lambda_{w, t}\right)-\log \left(1+\lambda_{w}\right) .
\end{aligned}
$$

### 3.7.1 Firm's production function and cost minimization

Production function for the intermediate good producing firm $(i)$ in the consumption sector:

$$
\hat{c}_{t}=\frac{c+F_{I}}{c}\left[\hat{a}_{l t}+a_{c} \hat{k}_{C, t}+\left(1-a_{c}\right) \hat{L}_{C, t}\right] .
$$

Production function for the intermediate good producing firm $(i)$ in the investment sector:

$$
\hat{i}_{t}=\frac{i+F_{I}}{i}\left[\hat{v}_{l t}+a_{i} \hat{k}_{I, t}+\left(1-a_{i}\right) \hat{L}_{I, t}\right] .
$$

Capital-to-labour ratios for the two sectors:

$$
\begin{equation*}
\hat{r}_{C, t}^{K}-\hat{w}_{t}=\hat{L}_{C, t}-\hat{k}_{C, t}, \quad \hat{r}_{I, t}^{K}-\hat{w}_{t}=\hat{L}_{I, t}-\hat{k}_{I, t} . \tag{3.44}
\end{equation*}
$$

Marginal cost in both sectors:

$$
\begin{equation*}
\hat{m} c_{C, t}=a_{c} \hat{r}_{C, t}^{K}+\left(1-a_{c}\right) \hat{w}_{t}-\hat{a}_{l t}, \quad \hat{m} c_{I, t}=a_{i} \hat{r}_{I, t}^{K}+\left(1-a_{i}\right) \hat{w}_{t}-\hat{v}_{l t}-\hat{p}_{i, t} . \tag{3.45}
\end{equation*}
$$

### 3.7.2 Firm's prices

Price setting equation for firms that change their price in sector $x=C, I$ :

$$
0=E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p, x}^{s} \beta^{s}\left[\hat{\tilde{p}}_{x, t} \hat{\tilde{\Pi}}_{t, t+s}-\hat{\lambda}_{p, t+s}^{x}-\hat{m} c_{x, t+s}\right]\right\}
$$

with

$$
\hat{\tilde{\Pi}}_{t, t+s}=\sum_{k=1}^{s}\left[\iota_{p_{x}} \hat{\pi}_{t+k-1}-\hat{\pi}_{t+k}\right]
$$

Solving for the summation

$$
\begin{aligned}
& \frac{1}{1-\xi_{p, x} \beta} \hat{\tilde{p}}_{x, t}= \\
&=E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p, x}^{s} \beta^{s}\left[-\hat{\Pi}_{t, t+s}+\hat{\lambda}_{p, t+s}^{x}+\hat{m} c_{x, t+s}\right]\right\} \\
&=-\hat{\Pi}_{t, t}+\hat{\lambda}_{p, t}^{x}+\hat{m} c_{x, t}-\frac{\xi_{p, x} \beta}{1-\xi_{p, x} \beta} \hat{\Pi}_{t, t+1} \\
&+\xi_{p, x} \beta E_{t}\left\{\sum_{s=1}^{\infty} \xi_{p, x}^{s-1} \beta^{s-1}\left[-\hat{\Pi}_{t+1, t+s}+\hat{\lambda}_{p, t+s}^{x}+\hat{m} c_{x, t+s}\right]\right\} \\
&= \hat{\lambda}_{p, t}^{x}+\hat{m} c_{x, t}+\frac{\xi_{p, x} \beta}{1-\xi_{p, x} \beta} E_{t}\left[\hat{\tilde{p}}_{x, t+1}-\hat{\Pi}_{t, t+1}\right]
\end{aligned}
$$

where we used $\hat{\Pi}_{t, t}=0$.

Prices evolve as

$$
0=\left(1-\xi_{p, x}\right) \hat{\tilde{p}}_{x, t}+\xi_{p, x}\left(\iota_{p_{x}} \hat{\pi}_{t-1}-\hat{\pi}\right)
$$

from which we obtain the Phillips curve in sector $x=C, I$ :

$$
\begin{align*}
& \hat{\pi}_{x, t}=\frac{\beta}{1+\iota_{p_{x}} \beta} E_{t} \hat{\pi}_{x, t+1}+\frac{\iota_{p_{x}}}{1+\iota_{p_{x}} \beta} \hat{\pi}_{x, t-1}+\kappa_{x} \hat{m} c_{x, t}+\kappa_{x} \hat{\lambda}_{p, t}^{x},  \tag{3.46}\\
& \text { with } \quad \kappa_{x}=\frac{\left(1-\xi_{p, x} \beta\right)\left(1-\xi_{p, x}\right)}{\xi_{p, x}\left(1+\iota_{p_{x}} \beta\right)} .
\end{align*}
$$

From equation (3.19) it follows that

$$
\hat{\pi}_{I, t}-\hat{\pi}_{C, t}=\hat{p}_{I, t}-\hat{p}_{I, t-1} .
$$

### 3.7.3 Households

Marginal utility:

$$
\begin{align*}
\hat{\lambda}_{t}= & \frac{e^{G}}{e^{G}-h \beta}\left[\hat{b}_{t}+\left(\hat{z}_{t}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t}\right)-\left(\frac{e^{G}}{e^{G}-h}\left(\hat{c}_{t}+\hat{z}_{t}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t}\right)-\frac{h}{e^{G}-h} \hat{c}_{t-1}\right)\right] \\
& -\frac{h \beta}{e^{G}-h \beta} E_{t}\left[\hat{b}_{t+1}-\left(\frac{e^{G}}{e^{G}-h}\left(\hat{c}_{t+1}+\hat{z}_{t+1}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t+1}\right)-\frac{h}{e^{G}-h} \hat{c}_{t}\right)\right] \\
\Leftrightarrow \hat{\lambda}_{t}= & \alpha_{1} E_{t} \hat{c}_{t+1}-\alpha_{2} \hat{c}_{t}+\alpha_{3} \hat{c}_{t-1}+\alpha_{4} \hat{z}_{t}+\alpha_{5} \hat{b}_{t}+\alpha_{6} \hat{v}_{t}, \tag{3.47}
\end{align*}
$$

with

$$
\begin{aligned}
\alpha_{1} & =\frac{h \beta e^{G}}{\left(e^{G}-h \beta\right)\left(e^{G}-h\right)}, \quad \alpha_{2}=\frac{e^{2 G}+h^{2} \beta}{\left(e^{G}-h \beta\right)\left(e^{G}-h\right)}, \quad \alpha_{3}=\frac{h e^{G}}{\left(e^{G}-h \beta\right)\left(e^{G}-h\right)}, \\
\alpha_{4} & =\frac{h \beta e^{G} \rho_{z}-h e^{G}}{\left(e^{G}-h \beta\right)\left(e^{G}-h\right)},
\end{aligned} \alpha_{5}=\frac{e^{G}-h \beta \rho_{b}}{e^{G}-h \beta}, \quad \alpha_{6}=\frac{\left(h \beta e^{G} \rho_{v}-h e^{G}\right) \frac{a_{c}}{1-a_{i}}}{\left(e^{G}-h \beta\right)\left(e^{G}-h\right)},
$$

This assumes the shock processes for $\hat{z}_{t}$ and $\hat{b}_{t}$.

Euler equation:

$$
\begin{equation*}
\hat{\lambda}_{t}=\hat{R}_{t}+E_{t}\left(\hat{\lambda}_{t+1}-\hat{z}_{t+1}-\hat{v}_{t+1} \frac{a_{c}}{1-a_{i}}-\hat{\pi}_{C, t+1}\right) . \tag{3.48}
\end{equation*}
$$

Purchase of financial claims

$$
\begin{equation*}
E_{t} \hat{\lambda}_{t+1}-\hat{\lambda}_{t}-\hat{z}_{t+1}-\left(\frac{a_{c}}{1-a_{i}}\right) \hat{v}_{t+1}+\hat{R}_{x, t}^{h}-\hat{\pi}_{c, t+1}=0 . \tag{3.49}
\end{equation*}
$$

### 3.7.4 Investment and Capital

Capital utilization in both sectors:

$$
\begin{equation*}
\hat{r}_{C, t}^{K}=\chi_{C} \hat{u}_{C, t}, \quad \hat{r}_{I, t}^{K}=\chi_{I} \hat{u}_{I, t}, \quad \text { where } \quad \chi_{x}=\frac{a_{x}^{\prime \prime}(1)}{a_{x}^{\prime}(1)} . \tag{3.50}
\end{equation*}
$$

Choice of investment for the consumption sector:

$$
\begin{align*}
\hat{q}_{C, t}= & e^{2\left(\frac{1}{1-a_{i}} g_{v}\right)} \kappa\left(\hat{i}_{C, t}-\hat{i}_{C, t-1}+\frac{1}{1-a_{i}} \hat{v}_{t}\right)-\beta e^{2\left(\frac{1}{1-a_{i}} g_{v}\right)} \kappa E_{t}\left(\hat{i}_{C, t+1}-\hat{i}_{C, t}+\frac{1}{1-a_{i}} \hat{v}_{t+1}\right) \\
& +\hat{p}_{i, t}-\hat{\mu}_{t}, \tag{3.51}
\end{align*}
$$

with $\hat{q}_{C, t}=\hat{\phi}_{C, t}-\hat{\lambda}_{t}$.
Choice of investment for the investment sector:

$$
\begin{align*}
\hat{q}_{I, t}= & e^{2\left(\frac{1}{1-a_{i}} g_{v}\right)} \kappa\left(\hat{i}_{I, t}-\hat{i}_{I, t-1}+\frac{1}{1-a_{i}} \hat{v}_{t}\right)-\beta e^{2\left(\frac{1}{1-a_{i}} g_{v}\right)} \kappa E_{t}\left(\hat{i}_{I, t+1}-\hat{i}_{I, t}+\frac{1}{1-a_{i}} \hat{v}_{t+1}\right) \\
& +\hat{p}_{i, t}-\hat{\mu}_{t}, \tag{3.52}
\end{align*}
$$

with $\hat{q}_{I, t}=\hat{\phi}_{I, t}-\hat{\lambda}_{t}$.
Capital services input in both sectors:

$$
\begin{equation*}
\hat{k}_{C, t}=\hat{u}_{C, t}+\xi_{C, t}^{K}+\hat{\bar{k}}_{C, t-1}-\frac{1}{1-a_{i}} \hat{v}_{t}, \quad \hat{k}_{I, t}=\hat{u}_{I, t}+\xi_{I, t}^{K}+\hat{\bar{k}}_{I, t-1}-\frac{1}{1-a_{i}} \hat{v}_{t} . \tag{3.53}
\end{equation*}
$$

Capital accumulation in the consumption and investment sector:

$$
\begin{align*}
& \hat{\bar{k}}_{C, t}=\left(1-\delta_{C}\right) e^{-\frac{1}{1-a_{i}} g_{v}}\left(\hat{\bar{k}}_{C, t-1}+\xi_{C, t}^{K}-\frac{1}{1-a_{i}} \hat{v}_{t}\right)+\left(1-\left(1-\delta_{C}\right) e^{-\frac{1}{1-a_{i}} g_{v}}\right) \hat{i}_{C, t},  \tag{3.54}\\
& \hat{\bar{k}}_{I, t}=\left(1-\delta_{I}\right) e^{-\frac{1}{1-a_{i}} g_{v}}\left(\hat{\bar{k}}_{I, t-1}+\xi_{I, t}^{K}-\frac{1}{1-a_{i}} \hat{v}_{t}\right)+\left(1-\left(1-\delta_{I}\right) e^{-\frac{1}{1-a_{i}} g_{v}}\right) \hat{i}_{I, t} . \tag{3.55}
\end{align*}
$$

Optimal financing from households

$$
\begin{align*}
& 0=\beta \frac{q_{x}^{h}}{q_{x}^{T}} r_{x}^{K}\left(\hat{\lambda}_{t+1}-\hat{\lambda}_{t}+\hat{q}_{x, t}^{h}-\hat{q}_{x, t}^{T}+\hat{r}_{x, t+1}^{K}+\hat{u}_{x, t+1}\right)-\beta \frac{q_{x}^{h}}{q_{x}^{T}} a^{\prime}\left(u_{x}\right) \hat{u}_{x, t+1} \\
&\left.-\gamma^{h} e^{\left(\frac{1}{1-a_{i}}\right) g_{v}}\left[\left(\frac{1}{1-a_{i}}\right) \hat{v}_{t+1}\right]-\kappa^{h} e^{\left(\frac{3}{1-a_{i}} g_{v}\right.}\right) \frac{1}{s_{x}^{h}}\left[\hat{s}_{x, t}^{h}+\left(\frac{1}{1-a_{i}}\right) \hat{v}_{t}\right] . \tag{3.56}
\end{align*}
$$

Optimal financing from financial intermediaries

$$
\begin{align*}
0= & \beta \frac{q_{x}}{q_{x}^{T}} r_{x}^{K}\left(\hat{\lambda}_{t+1}-\hat{\lambda}_{t}+\hat{q}_{x, t}-\hat{q}_{x, t}^{T}+\hat{r}_{x, t+1}^{K}+\hat{u}_{x, t+1}\right)-\beta \frac{q_{x}}{q_{x}^{T}} a^{\prime}\left(u_{x}\right) \hat{u}_{x, t+1} \\
& \left.-\gamma e^{\left(\frac{1}{1-a_{i}}\right) g_{v}}\left[\left(\frac{1}{1-a_{i}}\right) \hat{v}_{t+1}-z_{t+1}\right]-\kappa^{B} e^{\left(\frac{3}{1-a_{i}} g_{v}\right.}\right) \frac{1}{s_{x}}\left[\hat{s}_{x, t}+\left(\frac{1}{1-a_{i}}\right) \hat{v}_{t}\right], \tag{3.57}
\end{align*}
$$

Household's return on claims

$$
\left.\hat{R}_{x, t}^{h}=\frac{1}{r_{x}^{K}+q_{x}^{h}(1-\delta)}\left[r_{x}^{K}\left(\hat{r}_{x, t+1}^{K}+\hat{u}_{x, t+1}\right)+q_{x}^{h}(1-\delta) \hat{q}_{x, t+1}^{h}\right]-\hat{q}_{x, t}^{h}+z_{t+1}-\left(\frac{1-a_{c}}{1-a_{i}}\right) \hat{v}_{t+(\hat{l})} .58\right)
$$

Total value of acquired capital

$$
\begin{equation*}
\hat{q}_{x, t}^{T}+\hat{\bar{k}}_{x, t}=\frac{q_{x}^{h} s_{x}^{h}}{q_{x}^{T} \bar{k}_{x}}\left(\hat{q}_{x, t}^{h}+\hat{s}_{x, t}^{h}\right)+\frac{q_{x} s_{x}}{q_{x}^{T} \bar{k}_{x}}\left(\hat{q}_{x, t}+\hat{s}_{x, t}\right) \tag{3.59}
\end{equation*}
$$

### 3.7.5 Wages

The wage setting equation for workers renegotiating their salary:

$$
0=E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[\hat{\tilde{w}}_{t}+\hat{\tilde{\Pi}}_{t, t+s}^{w}-\hat{\lambda}_{w, t+s}-\hat{b}_{t+s}-\nu \hat{\tilde{L}}_{t+s}+\hat{\lambda}_{t+s}\right]\right\}
$$

with

$$
\begin{aligned}
\hat{\tilde{\Pi}}_{t, t+s}^{w}= & \sum_{k=1}^{s}\left[\iota_{w}\left(\hat{\pi}_{c, t+k-1}+\hat{z}_{t+k-1}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t+k-1}\right)-\left(\hat{\pi}_{c, t+k}+\hat{z}_{t+k}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t+k}\right)\right] \\
& \quad \text { and } \\
\hat{\tilde{L}}_{t+s}= & \hat{L}_{t+s}-\left(1+\frac{1}{\lambda_{w}}\right)\left(\hat{\tilde{w}}_{t}+\hat{\tilde{\Pi}}_{t, t+s}^{w}-\hat{w}_{t+s}\right) .
\end{aligned}
$$

Then using the labor demand function,

$$
\begin{aligned}
0= & E_{t}\left\{\sum _ { s = 0 } ^ { \infty } \xi _ { w } ^ { s } \beta ^ { s } \left[\hat{\tilde{w}}_{t}+\hat{\tilde{\Pi}}_{t, t+s}^{w}-\hat{\lambda}_{w, t+s}-\hat{b}_{t+s}\right.\right. \\
& \left.\left.-\nu\left(\hat{L}_{t+s}-\left(1+\frac{1}{\lambda_{w}}\right)\left(\hat{\tilde{w}}_{t}+\hat{\tilde{\Pi}}_{t, t+s}^{w}-\hat{w}_{t+s}\right)\right)+\hat{\lambda}_{t+s}\right]\right\} \\
\Leftrightarrow 0= & E_{t}\left\{\sum _ { s = 0 } ^ { \infty } \xi _ { w } ^ { s } \beta ^ { s } \left[\hat{\tilde{w}}_{t}\left(1+\nu\left(1+\frac{1}{\lambda_{w}}\right)\right)+\hat{\tilde{\Pi}}_{t, t+s}^{w}-\hat{\lambda}_{w, t+s}-\hat{b}_{t+s}\right.\right. \\
& \left.\left.-\nu\left(\hat{L}_{t+s}-\left(1+\frac{1}{\lambda_{w}}\right)\left(\hat{\tilde{\Pi}}_{t, t+s}^{w}-\hat{w}_{t+s}\right)\right)+\hat{\lambda}_{t+s}\right]\right\} .
\end{aligned}
$$

Solving for the summation,

$$
\begin{align*}
\frac{\nu_{w}}{1-\xi_{w} \beta} \hat{\tilde{\tilde{w}}}_{t} & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[-\left(1+\nu\left(1+\frac{1}{\lambda_{w}}\right)\right) \hat{\tilde{\Pi}}_{t, t+s}^{w}+\hat{\psi}_{t+s}\right]\right\} \\
& =-\nu_{w} \hat{\tilde{\Pi}}_{t, t}^{w}+\hat{\psi}_{t}+E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[-\nu_{w} \hat{\tilde{\Pi}}_{t, t+s}^{w}+\hat{\psi}_{t+s}\right]\right\} \\
& =\hat{\psi}_{t}-\frac{\xi_{w} \beta}{1-\xi_{w} \beta} \nu_{w} \hat{\Pi}_{t, t+1}^{w}+\xi_{w} \beta E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[-\nu_{w} \hat{\Pi}_{t+1, t+1+s}^{w}+\hat{\psi}_{t+1+s}\right]\right\} \\
& =\hat{\psi}_{t}+\frac{\xi_{w} \beta}{1-\xi_{w} \beta} \nu_{w} E_{t}\left[\hat{\tilde{w}}_{t+1}-\hat{\tilde{\Pi}}_{t, t+1}^{w}\right] . \tag{3.60}
\end{align*}
$$

where

$$
\begin{align*}
\hat{\psi}_{t} & \equiv \hat{\lambda}_{w, t}+\hat{b}_{t}+\nu \hat{L}_{t}+\nu\left(1+\frac{1}{\lambda_{w}}\right) \hat{w}_{t}-\hat{\lambda}_{t},  \tag{3.61}\\
\nu_{w} & \equiv 1+\nu\left(1+\frac{1}{\lambda_{w}}\right),
\end{align*}
$$

and recall that $\hat{\tilde{\Pi}}_{t, t}^{w}=0$.

Wages evolve as,

$$
\begin{align*}
\hat{w}_{t} & =\left(1-\xi_{w}\right) \hat{\tilde{w}}_{t}+\xi_{w}\left(\hat{w}_{t-1}+\iota_{w} \hat{\pi}_{c, t-1}+\iota_{w}\left(\hat{z}_{t-1}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t-1}\right)-\hat{\pi}_{c, t}-\hat{z}_{t}-\frac{a_{c}}{1-a_{i}} \hat{v}_{t}\right) \\
\Leftrightarrow \hat{w}_{t} & =\left(1-\xi_{w}\right) \hat{\tilde{w}}_{t}+\xi_{w}\left(\hat{w}_{t-1}+\hat{\tilde{\Pi}}_{t, t-1}^{w}\right) . \tag{3.62}
\end{align*}
$$

Equation (3.62) can be solved for $\hat{\tilde{w}}_{t}$. This expression, as well as the formulation for $\hat{\psi}_{t}$ given in (3.61) can be plugged into equation (3.60). After rearranging this yields the wage Phillips curve,

$$
\begin{align*}
\hat{w}_{t}= & \frac{1}{1+\beta} \hat{w}_{t-1}+\frac{\beta}{1+\beta} E_{t} \hat{w}_{t+1}-\kappa_{w} \hat{g}_{w, t}+\frac{\iota_{w}}{1+\beta} \hat{\pi}_{c, t-1}-\frac{1+\beta \iota_{w}}{1+\beta} \hat{\pi}_{c, t} \\
& +\frac{\beta}{1+\beta} E_{t} \hat{\pi}_{c, t+1}+\kappa_{w} \hat{\lambda}_{w, t}+\frac{\iota_{w}}{1+\beta}\left(\hat{z}_{t-1}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t-1}\right) \\
& -\frac{1+\beta \iota_{w}-\rho_{z} \beta}{1+\beta} \hat{z}_{t}-\frac{1+\beta \iota_{w}-\rho_{v} \beta}{1+\beta} \frac{a_{c}}{1-a_{i}} \hat{v}_{t} . \tag{3.63}
\end{align*}
$$

where

$$
\begin{aligned}
\kappa_{w} & \equiv \frac{\left(1-\xi_{w} \beta\right)\left(1-\xi_{w}\right)}{\xi_{w}(1+\beta)\left(1+\nu\left(1+\frac{1}{\lambda_{w}}\right)\right)}, \\
\hat{g}_{w, t} & \equiv \hat{w}_{t}-\left(\nu \hat{L}_{t}+\hat{b}_{t}-\hat{\lambda}_{t}\right)
\end{aligned}
$$

### 3.7.6 Financial sector

The part of the economy concerned with the banking sector is described by the following equations:

The stochastic discount factor:

$$
\begin{equation*}
\hat{\lambda}_{t}^{B}=\hat{\lambda}_{t}-\hat{\lambda}_{t-1} . \tag{3.64}
\end{equation*}
$$

Definition of $\nu$ for $x=C, I$ :

$$
\begin{align*}
\hat{\nu}_{x, t}= & \left(1-\theta_{B} \beta z_{1}^{x}\right)\left[\hat{\lambda}_{t+1}^{B}-\hat{z}_{t+1}-\frac{a_{c}}{1-a_{i}} \hat{v}_{t+1}\right] \\
& +\frac{1-\theta_{B} \beta z_{1}^{x}}{R_{x}^{B} \pi_{C}-R}\left[R_{x}^{B} \pi_{C} \hat{R}_{x, t+1}^{B}+R_{x}^{B} \pi_{C} \hat{\pi}_{C, t+1}-R \hat{R}_{t}\right]+\theta_{B} \beta z_{1}^{x}\left[\hat{z}_{1, t+1}^{x}+\hat{\nu}_{x, t+1}\right] . \tag{3.65}
\end{align*}
$$

Definition of $\eta$ :

$$
\begin{align*}
\hat{\eta}_{x, t}= & \left(1-\theta_{B} \beta z_{2}^{x}\right)\left[\hat{\lambda}_{t+1}^{B}-\hat{z}_{t+1}-\frac{a_{c}}{1-a_{i}} \hat{v}_{t+1}+\hat{R}_{t}\right] \\
& +\theta_{B} \beta z_{2}^{x}\left[\hat{z}_{2, t+1}^{x}+\hat{\eta}_{t+1}\right], \quad x=C, I . \tag{3.66}
\end{align*}
$$

Definition of $z_{1}$ :

$$
\begin{equation*}
\hat{z}_{1, t}^{x}=\hat{\varrho}_{x, t}-\hat{\varrho}_{x, t-1}+\hat{z}_{2, t}^{x}, \quad x=C, I . \tag{3.67}
\end{equation*}
$$

Definition of $z_{2}$ for $x=C, I$ :
$\hat{z}_{2, t}^{x}=\frac{\pi_{C}}{\left(R_{x}^{B}-R\right) \varrho_{x}+R}\left[R_{x}^{B} \varrho_{x}\left[\hat{R}_{x, t}^{B}+\hat{\pi}_{C, t}\right]+\frac{R}{\pi_{C}}\left(1-\varrho_{x}\right) \hat{R}_{t-1}+\left(R_{x}^{B} \pi_{C}-R\right) \frac{\varrho_{x}}{\pi_{C}} \hat{\varrho}_{x, t-1}\right]-\hat{\pi}_{C, t}$.

The leverage ratio:

$$
\begin{equation*}
\hat{\varrho}_{x, t}=\hat{\eta}_{x, t}+\frac{\nu}{\lambda_{B}-\nu} \hat{\nu}_{x, t}, \quad x=C, I . \tag{3.69}
\end{equation*}
$$

The leverage equation:

$$
\begin{equation*}
\hat{q}_{x, t}+\hat{s}_{x, t}=\hat{\varrho}_{x, t}+\hat{n}_{x, t} . \tag{3.70}
\end{equation*}
$$

The bank's wealth accumulation equation

$$
\begin{align*}
\hat{n}_{x, t}= & \theta_{B} \frac{\varrho_{x}}{\pi_{C}} e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}}\left[R_{x}^{B} \pi_{C}\left[\hat{R}_{x, t}^{B}+\hat{\pi}_{C, t}\right]+\left(\frac{1}{\varrho_{x}}-1\right) R \hat{R}_{t-1}+\left(R_{x}^{B} \pi_{C}-R\right) \hat{\varrho}_{x, t-1}\right] \\
& +\frac{\theta_{B}}{\pi_{C}} e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}}\left[\left(R_{x}^{B} \pi_{C}-R\right) \varrho_{x}+R\right]\left[-\hat{z}_{t}-\frac{a_{c}}{1-a_{i}} \hat{v}_{t}+\hat{n}_{x, t-1}-\hat{\pi}_{C, t}\right] \\
& +\left(1-\frac{\theta_{B}}{\pi_{C}} e^{-g_{a}-\frac{a_{c}}{1-a_{i}} g_{v}}\left[\left(R_{x}^{B} \pi_{C}-R\right) \varrho_{x}+R\right]\right)\left[\hat{q}_{x, t}+\hat{s}_{x, t}\right]+\hat{\varsigma}_{x, t}, \quad x=C, I . \tag{3.71}
\end{align*}
$$

The bank's stochastic return on assets in sector $x=C, I$ :

$$
\begin{equation*}
\hat{R}_{x, t}^{B}=\frac{1}{r_{x}^{K}+q_{x}\left(1-\delta_{x}\right)}\left[r_{x}^{K}\left(\hat{r}_{x, t}^{K}+\hat{u}_{x, t}\right)+q_{x}\left(1-\delta_{x}\right) \hat{q}_{x, t}\right]-\hat{q}_{x, t-1}+\xi_{x, t}^{K}+\hat{z}_{t}-\frac{1-a_{c}}{1-a_{i}} \hat{v}_{t} . \tag{3.72}
\end{equation*}
$$

Excess (nominal) return:

$$
\begin{equation*}
\hat{R}_{x, t}^{S}=\frac{R_{x}^{B} \pi_{C}}{R_{x}^{B} \pi_{C}-R}\left(\hat{R}_{x, t+1}^{B}+\hat{\pi}_{C, t+1}\right)-\frac{R}{R_{x}^{B} \pi_{C}-R} \hat{R}_{t}, \quad x=C, I \tag{3.73}
\end{equation*}
$$

### 3.7.7 Monetary policy and market clearing

Monetary policy rule:

$$
\begin{equation*}
\hat{R}_{t}=\rho_{R} \hat{R}_{t-1}+\left(1-\rho_{R}\right)\left[\phi_{\pi} \hat{\pi}_{c, t}+\phi_{\Delta Y}\left(\hat{y}_{t}-\hat{y}_{t-1}\right)\right]+\hat{\eta}_{m p, t} \tag{3.74}
\end{equation*}
$$

Resource constraint in the consumption sector:

$$
\begin{equation*}
\hat{c}_{t}+\left(r_{C}^{K} \frac{\bar{k}_{C}}{c} \hat{u}_{C, t}+r_{I}^{K} \frac{\bar{k}_{I}}{c} \hat{u}_{I, t}\right) e^{-\frac{1}{1-a_{i}} g_{v}}=\frac{c+F_{c}}{c}\left[\hat{a}_{l t}+a_{c} \hat{k}_{C, t}+\left(1-a_{c}\right) \hat{L}_{C, t}\right] \tag{3.75}
\end{equation*}
$$

Resource constraint in the investment sector:

$$
\begin{equation*}
\hat{i}_{t}=\frac{i+F_{I}}{i}\left[\hat{v}_{l t}+a_{i} \hat{k}_{I, t}+\left(1-a_{i}\right) \hat{L}_{I, t}\right] \tag{3.76}
\end{equation*}
$$

Definition of GDP:

$$
\begin{equation*}
\hat{y}_{t}=\frac{c}{c+p_{i} i} \hat{c}_{t}+\frac{p_{i} i}{c+p_{i} i}\left(\hat{i}_{t}+\hat{p}_{i, t}\right)+\hat{g}_{t} . \tag{3.77}
\end{equation*}
$$

Market clearing:

$$
\begin{equation*}
\frac{L_{C}}{L} \hat{L}_{C, t}+\frac{L_{I}}{L} \hat{L}_{I, t}=\hat{L}_{t}, \quad \frac{i_{C}}{i} \hat{i}_{C, t}+\frac{i_{I}}{i} \hat{i}_{I, t}=\hat{i}_{t}, \quad \frac{n_{C}}{n} \hat{n}_{C, t}+\frac{n_{I}}{n} \hat{n}_{I, t}=\hat{n}_{t} . \tag{3.78}
\end{equation*}
$$

### 3.7.8 Exogenous processes

The 11 exogenous processes of the model can be written in log-linearized form as follows:
Price markup in sector $x=C, I$ :

$$
\begin{equation*}
\hat{\lambda}_{p, t}^{x}=\rho_{\lambda_{p}^{x}} \hat{\lambda}_{p, t-1}^{x}+\varepsilon_{p, t}^{x} . \tag{3.79}
\end{equation*}
$$

The TFP growth (consumption sector):

$$
\begin{equation*}
\hat{z}_{t}=\rho_{z} \hat{z}_{t-1}+\varepsilon_{t}^{z} . \tag{3.80}
\end{equation*}
$$

The TFP growth (investment sector):

$$
\begin{equation*}
\hat{v}_{t}=\rho_{v} \hat{v}_{t-1}+\varepsilon_{t}^{v} . \tag{3.81}
\end{equation*}
$$

Wage markup:

$$
\begin{equation*}
\hat{\lambda}_{w, t}=\rho_{w} \hat{\lambda}_{w, t-1}+\varepsilon_{w, t} . \tag{3.82}
\end{equation*}
$$

Preference:

$$
\begin{equation*}
\hat{b}_{t}=\rho_{b} \hat{b}_{t-1}+\varepsilon_{t}^{b} . \tag{3.83}
\end{equation*}
$$

Monetary policy:

$$
\begin{equation*}
\hat{\eta}_{m p, t}=\varepsilon_{t}^{m p} . \tag{3.84}
\end{equation*}
$$

Government spending:

$$
\begin{equation*}
\hat{g}_{t}=\rho_{g} \hat{g}_{t-1}+\varepsilon_{t}^{g} . \tag{3.85}
\end{equation*}
$$

The Marginal Efficiency of Investment (MEI):

$$
\begin{equation*}
\hat{\mu}_{t}=\rho_{\mu} \hat{\mu}_{t-1}+\varepsilon_{t}^{\mu} \tag{3.86}
\end{equation*}
$$

The TFP stationary (consumption sector):

$$
\begin{equation*}
\hat{a}_{l t}=\rho_{a_{l}} \hat{a}_{l, t-1}+\varepsilon_{t}^{a_{l}} . \tag{3.87}
\end{equation*}
$$

The TFP stationary (investment sector):

$$
\begin{equation*}
\hat{v}_{l t}=\rho_{v_{l}} \hat{v}_{l, t-1}+\varepsilon_{t}^{v_{l}} . \tag{3.88}
\end{equation*}
$$

Bank equity capital:

$$
\begin{equation*}
\hat{\zeta}_{x, t}=\rho_{\varsigma_{x}} \hat{\varsigma}_{x, t-1}+\epsilon_{x, t}^{\varsigma} . \tag{3.89}
\end{equation*}
$$

The entire log-linear model is summarized by equations (3.44) - (3.59) and (3.63) - (3.78) as well as the shock processes (3.79) - (3.89).

### 3.8 Measurement equations

For estimation, model variables are linked with observables using measurement equations. Letting a superscript "d" denote observable series, then the model's measurement equations are as follows:

Real consumption growth,

$$
\Delta C_{t}^{d} \equiv \log \left(\frac{C_{t}}{C_{t-1}}\right)=\log \left(\frac{c_{t}}{c_{t-1}}\right)+\hat{z}_{t}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t}
$$

Real investment growth,

$$
\Delta I_{t}^{d} \equiv \log \left(\frac{I_{t}}{I_{t-1}}\right)=\log \left(\frac{i_{t}}{i_{t-1}}\right)+\frac{1}{1-a_{i}} \hat{v}_{t},
$$

Real wage growth,

$$
\Delta W_{t}^{d} \equiv \log \left(\frac{W_{t}}{W_{t-1}}\right)=\log \left(\frac{w_{t}}{w_{t-1}}\right)+\hat{z}_{t}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t},
$$

Real output growth,

$$
\Delta Y_{t}^{d} \equiv \log \left(\frac{Y_{t}}{Y_{t-1}}\right)=\log \left(\frac{y_{t}}{y_{t-1}}\right)+\hat{z}_{t}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t}
$$

Consumption sector inflation,

$$
\pi_{C, t}^{d} \equiv \pi_{C, t}=\hat{\pi}_{C, t} \quad \text { and } \quad \hat{\pi}_{C, t}=\log \left(\pi_{C, t}\right)-\log \left(\pi_{C}\right)
$$

Relative price of investment

$$
\Delta\left(\frac{P_{t}^{I}}{P_{t}^{C}}\right)^{d} \equiv \log \left(\frac{p_{t}^{i}}{p_{t-1}^{i}}\right)+\hat{z}_{t}+\frac{a_{c}-1}{1-a_{i}} \hat{v}_{t},
$$

Total hours worked,

$$
L_{t}^{d} \equiv \log L_{t}=\hat{L}_{t},
$$

Nominal interest rate (federal funds rate),

$$
R_{t}^{d} \equiv \log R_{t}=\log \hat{R}_{t},
$$

Corporate bond spread,

$$
R_{t}^{S, d} \equiv \log R_{t}^{S}=\left(e^{\log \hat{R}_{C, t+1}^{B}} e^{\hat{\pi}_{C, t+1}}-e^{\log \hat{R}_{t}}\right) * w_{C}+\left(e^{\log \hat{R}_{I, t+1}^{B}} e^{\hat{\pi}_{C, t+1}}-e^{\log \hat{R}_{t}}\right) * w_{I}
$$

where $w_{C}$ and $w_{I}$ are steady state shares of assets (as a fraction of bank equity) in banks portfolios in the consumption and investment sector respectively.

Real total bank equity capital growth,

$$
\begin{aligned}
\Delta N_{t}^{d} & \equiv \log \left(\frac{N_{t}}{N_{t-1}}\right) \\
& =e^{g_{a}+\frac{a_{c}}{1-g_{i}}}\left(\frac{n_{C}}{n_{C}+n_{I}}\left(\hat{n}_{C, t}-\hat{n}_{C, t-1}\right)+\frac{n_{I}}{n_{C}+n_{I}}\left(\hat{n}_{I, t}-\hat{n}_{I, t-1}\right)+\hat{z}_{t}+\frac{a_{c}}{1-a_{i}} \hat{v}_{t}\right) .
\end{aligned}
$$

Capital claims from equity markets

$$
\begin{aligned}
\Delta S_{t}^{h, d} \equiv & \log \left(\frac{S_{t}^{h}}{S_{t-1}^{h}}\right) \\
= & e^{\frac{1}{1-a_{i}} g_{v}}\left(\frac{s_{C}^{h}}{s_{C}^{h}+s_{I}^{h}}\left(\hat{s}_{C, t}^{h}-\hat{s}_{C, t-1}^{h}+\hat{q}_{C, t}^{h}-\hat{q}_{C, t-1}^{h}\right)\right. \\
& \left.+\left(1-\frac{s_{C}^{h}}{s_{C}^{h}+s_{I}^{h}}\right)\left(\hat{s}_{I, t}^{h}-\hat{s}_{I, t-1}^{h}+\hat{q}_{I, t}^{h}-\hat{q}_{I, t-1}^{h}\right)+\frac{1}{1-a_{i}} \hat{v}_{t}\right) .
\end{aligned}
$$

## References

Adrian, Tobias, M. E. and Shin, H. S. (2010). Financial intermediation asset prices and macroeconomic fundamentals. Federal Reserve Bank of New York Staff Report, (422).

Barsky, R. B. and Sims, E. R. (2011). News shocks and business cycles. Journal of Monetary Economics, 58(3):273-289.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12(3):383-398.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy, 113(1):1-45.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics, 46(2):281-313.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. Working Paper, (2012-19).

Forni, M., Gambetti, L., and Sala, L. (2014). No news in business cycles. Economic Journal, 124:1168-1191.

Francis, N., Owyang, M., Roush, J., and DiCecio, R. (2014). A flexible finite-horizon alternative to long-run restrictions with an application to technology shocks. Review of Economics and Statistics, 96:638-647.

Fujiwara, I., Hirose, Y., and Shintani, M. (2011). Can news be a major source of aggregate fluctuations? A Bayesian DSGE approach. Journal of Money, Credit and Banking, 43(1):1-29.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. Journal of Monetary Economics, 58(1):17-34.

Gertler, M. L. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In Friedman, B. M. and Woodford, M., editors, Handbook of Monetary Economics, volume 3, chapter 11, pages 547-599. Elsevier, 1 edition.

Gilchrist, S. and Zakrajsek, E. (2012). Credit spreads and business cycle fluctuations. American Economic Review, 102(4):1692-1720.

Jermann, U. and Quadrini, V. (2009). Financial innovations and macroeconomic volatility. mimeo.

Jermann, U. and Quadrini, V. (2012). Macroeconomic effects of financial shocks. American Economic Review, 102(1):238-71.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2010). Investment shocks and business cycles. Journal of Monetary Economics, 57(2):132-145.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2011). Investment shocks and the relative price of investment. Review of Economic Dynamics, 14(1):101-121.

Khan, H. and Tsoukalas, J. (2012). The Quantitative Importance of News Shocks in Estimated DSGE Models. Journal of Money, Credit and Banking, 44(8):1535-1561.

Kurmann, A. and Sims, E. (2016). Revisions in utilization-adjusted tfp and robust identification of news shocks. mimeo.

Philippon, T. (2009). The bond market's q. Quarterly Journal of Economics, 124:1011-1056.

Uhlig, H. (2003). What moves real gnp? Technical report, Humboldt University Mimeo.


[^0]:    *Görtz: University of Birmingham, Department of Economics, Email: c.g.gortz@bham.ac.uk. Tsoukalas: University of Glasgow, Adam Smith Business School/Economics, Email: john.tsoukalas@glasgow.ac.uk. Zanetti: University of Oxford, Department of Economics, Email: francesco.zanetti@economics.ox.ac.uk.

[^1]:    ${ }^{1}$ These authors argue that allowing TFP to jump freely on impact, conditional on a news shock, produces robust inference to cyclical measurement error in the construction of TFP.
    ${ }^{2}$ A third, alternative identification proposed in the literature is the Forni et al. (2014) long-run identification scheme. This method identifies the news shock by imposing the zero impact restriction on TFP, and seeks to maximise the impact of the news shock on TFP in the long run. As such it is very similar in spirit to the Max Share method we employ as a baseline identification. Responses are qualitatively and quantitatively very similar between these two identification schemes. We don not show these results for space considerations, but IRFs are available upon request.

[^2]:    ${ }^{3}$ In comparison to our baseline setup, this model version turns off the financial channel, i.e. the balance sheet identity (15), the leverage constraint (16), the evolution of equity capital (17), and the financial constraint (9) that describe the financial sector as well as equations (7), (10) and (11) that allow capital services producers to raise funds from households. The one-sector model can be written as a special case of the two-sector model. It imposes a perfectly competitive investment sector and perfect capital mobility.

[^3]:    Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE models and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities.

[^4]:    Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE models and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities.

[^5]:    ${ }^{4}$ The lag length $p$ is determined based on the AIC each time for a set of time series based on a particular draw. The maximum number of lags considered is eight which is never chosen in any specification.

[^6]:    ${ }^{5}$ The grid of values we use is: $\underline{a}_{1}=(1 \mathrm{e}-5: 1 \mathrm{e}-4: 9 \mathrm{e}-5,1 \mathrm{e}-4: 1 \mathrm{e}-4: 9 \mathrm{e}-4,0.001: 0.001: 0.009,0.01: 0.01: 0.5), \underline{a}_{2}=$ ( $0.01,0.05,0.1,0.2,0.3,0.4,0.5$ ). We consider all possible pairs of $\underline{a}_{1}$ and $\underline{a}_{2}$ in the above grids.

[^7]:    ${ }^{6}$ The TFP data can be accessed at www.frbsf.org/economic research/economists/jfernald/quarterly_tfp.xls.

[^8]:    ${ }^{7}$ The investment sectors' NAICS codes are: $\begin{array}{lllllllll}21 & 22 & 23 & 31 & 32 & 33 & 42 & 48 & 49 \\ 51\end{array}$ (except 491). The

