Perspectives on the Labor Share Online Appendix Loukas Karabarbounis

Appendix A provides details for the measurement of the labor share. Appendix B presents some additional empirical results for the labor share of the United States and other countries. Appendix C provides algebraic derivations for various models of the labor share and for tests of dynamic inefficiency. Finally, Appendix D discusses in more detail some work cited in the main text.

A Details on Measurement

Compensation of employees. The primary data source is the Quarterly Census of Employment and Wages (QCEW) of the Bureau of Labor Statistics (BLS), which covers roughly 97 percent of non-farm payroll. The BEA adjusts these data to account for misreporting and compensation not covered by unemployment insurance. The BEA aggregates industry-level estimates using industry payroll data from the U.S. Census Bureau.

Equity pay is included in the national accounts under compensation to employees when it is taxed as ordinary income (that is, reported under a W-2 form). Compensation to employees does not include incentive stock options, which are deferred and taxed as capital gains when exercised, and includes equity pay only when it is reported as taxable income and not when it is granted. Eisfeldt, Falato, and Xiaolan (2023) document that between one-third and one-half of stock options are taxed as capital gains. They argue that total compensation should equal the wage component plus the value of all newly granted equity compensation, which alters the dynamics of the labor share if newly granted equity compensation increases over time as a fraction of measured compensation.

Types of unambiguous capital income. In the NIPA accounts, corporate profits refer to the profits from the current production of U.S. corporations. This item includes profits reported by both C and S corporations. Corporate profits are inclusive of corporate income taxes. They exclude dividend income and capital gains and losses. The primary data source for the BEA, when available, is data collected on a tax-accounting basis from corporate income tax returns filed to the IRS. The NIPA measure of corporate profits includes adjustments for inventories and capital consumption. Guvenen, Mataloni, Rassier, and Ruhl (2022) argue that U.S. multinationals book profits in low tax foreign countries by transferring ownership of their intangible assets to foreign affiliates. If these intangibles were developed in the United States, then the income they generate should be attributed to the U.S. and not to foreign GDP. The

authors reattribute income generated abroad to U.S. corporate profits and find that the U.S. labor share declined by almost an additional percentage point between 1987 and 2008.

Rental income is net of depreciation and includes tenant- and owner-occupied housing, nonresidential properties, royalties, and an adjustment for capital consumption, which converts tax-based depreciation to economic depreciation. The source data for housing rentals is the American Community Survey of the Census Bureau, with an imputation by the BEA of the service flow to owner-occupied housing. Business current transfer payments are payments or transfers to persons, governments, or the rest of the world when no service is provided. Examples include insurance payouts, scholarships, and debt write-offs. The surplus of government enterprises is the revenues minus expenses of government agencies net of any subsidies. Examples include utilities, housing, and transportation agencies. The difference with the government sector is that these enterprises are charging prices so that revenues exceed 50 percent of costs. The System of National Accounts (SNA) calls these enterprises quasi-corporations.

Proprietors' labor input. My labor share measure "Total: Proprietors Same Wage" uses persons working in each sector and not hours to scale the labor share. This amounts to assuming that employees and proprietors have the same earnings instead of the same wage. I believe both are equally defensible choices, so I opt for the measure with the lower data requirement. I call this measure "Same Wage" for comparability with previous literature.

Mechanically, in Figure 1, the labor share measure "Total: Proprietors Same Wage" is declining before World War II because the share of proprietors' labor input in total labor input is declining without a corresponding decline of the compensation share. I investigated this more, and I am not convinced that proprietors' labor input is well measured in the period before World War II. The raw data for proprietors' labor input come from the NIPA, Table 6.7. I compared the NIPA estimates with those presented by Margo (1998) for the non-agricultural sector. The self-employment rate reported by the NIPA is 13.2 percent in 1930 for the nonagricultural sector, which matches closely with the 13.4 percent estimate reported by Margo. However, the self-employment rate in the NIPA increases to 15.7 percent in 1932. The change is so large because the NIPA report an increase in the number of self-employed workers relative to its level in 1929, whereas the number of employed fell by roughly 25 percent. I use the selfemployment rate reported by Margo to correct the NIPA measure until 1940. The adjustment is significant between 1931 and 1933 and becomes negligible by 1940 because the self-employment rates in the NIPA and in Margo converge. The labor share measure "Total: Proprietors Same Wage" in Figure 1 spikes even after this adjustment, and without this adjustment it would increase significantly more in the early 1930s. I suspect but cannot confirm that the NIPA also underestimate the decline of the number of proprietors during the Great Recession in agriculture, so an adjustment of agricultural proprietors would also be reasonable.

Labor share measure that excludes housing. For the labor share excluding housing, I subtract compensation of employees in the housing sector from aggregate compensation in the numerator of the labor share. In the denominator, I subtract housing value added from GDP, subtract proprietors' income in the housing sector from total proprietors' income, and net out taxes on production and imports less subsidies from the corresponding taxes recorded in the housing sector.

Labor share measure that excludes the government. For the labor share excluding government, I subtract compensation of employees in the government sector from aggregate compensation in the numerator of the labor share and government's gross value added from GDP in the denominator of the labor share. The government does not have proprietors' income and taxes on production, so no other adjustment is necessary.

Labor share measure that imputes capital income to the government. Government's gross value added equals compensation of employees and depreciation, because national accounts treat government as making zero net capital income from its operations. To impute net capital income produced by government's assets to GDP and to capital income, I multiply government fixed assets by the return net of depreciation that accrues to private non-residential fixed assets. The return is calculated as total capital income excluding the income from government and housing divided by private non-residential fixed assets plus inventories.

BLS labor share measure. There are three differences between the BLS measure and "Total Proprietors: Same Wage." First, my series is more comprehensive in terms of coverage, as it includes the entire economy. Second, I have used the assumption of equal earnings to impute proprietors' labor income, whereas the BLS uses the assumption of equal wages. Third, the BLS allocates a fraction of taxes on production to capital. In Figure 1, I scale the BLS series to have the same value as my series in 1947.

The BLS approach is to allocate a fraction of taxes on production, namely the taxes that concern property and motor vehicles, to capital income. Taxes on property and motor vehicles are subcategories of "other taxes on production," separate from "taxes on products," which mainly concern sales and excise taxes. I do not find this practice persuasive because changes in these taxes may also affect the cost of labor. As an example, consider a restaurant earning 100 dollars of income net of intermediate inputs. The restaurant pays 50 dollars in wages to its employees and 10 dollars in production taxes for its structure, which produces food, and its vehicles, which deliver food. The remaining 40 dollars accrue to its owner. Allocating one-half of property and vehicle taxes to labor income and one-half to capital income implies that the effective labor cost scales with these production taxes. This is a more appealing assumption than entirely allocating production taxes to capital income, because doing so does not take into account that the size of the structure and the number of vehicles likely scale with the size of the workforce.

Corporate sector. In the United States, both C and S corporations belong to the corporate sector, whereas partnerships and sole proprietorships do not. An important difference between C and S corporations is that the latter do not pay corporate income taxes but passthrough their income to the owners. Smith, Yagan, Zidar, and Zwick (2022) argue that, after the Tax Reform Act of 1986, the share of the corporate sector organized as S rose over time, and owners of S corporations started to prefer recording their income as profits instead of labor income for preferential tax treatment. Therefore, the tax reform caused some of the decline of the corporate labor share (around 1 percentage point).

The NIPA measure of corporate profits is closely related to the measures for corporations in the SNA. However, the strict exclusion of housing from the corporate sector applies only to the United States. For other countries, the SNA categorizes cooperatives, limited liability partnerships, and quasi-corporations in the corporate sector and these legal forms may own some housing. However, I argued that excluding the housing sector from capital income may not be desirable, and thus it is not obvious if the corporate labor share is a better proxy for the labor share in the United States relative to other countries. In addition, the corporate sector in the SNA includes quasi-corporations such as government enterprises and limited liability partnerships, which are excluded under the NIPA measures.

Some researchers also look at the corporate non-financial labor share, which excludes financial firms' contributions to corporate compensation of employees, gross value added, and taxes on production. I find that this measure behaves quite similarly to the one for the entire corporate sector, and thus my analysis focuses on the measure that represents the larger share of the economy.

Unmeasured investments. Expenses accruing on brand equity, customer capital, firm-specific worker training, advertising, and marketing are not counted in GDP, because statistical agencies treat them as intermediate inputs (Corrado, Hulten, and Sichel, 2009). However, to the extent that these outlays are expected to generate returns in the future, economically they should be treated as investment flows. If statistical agencies decide to recognize these expenses as investments, then measured GDP will increase. Whether this income accrues to labor or capital depends on the ownership of the accumulated factors. If the ownership stays with the firm, or spillovers to other firms because of the non-exclusive nature of intangibles, then the income generated by the stock of intangibles is capital income. It is appropriate to allocate part of the intangibles' income to labor if this income is paid to skilled workers in the form of equity-based compensation, which is not recorded in compensation of employees.

The effect of recognizing these expenses as investments on the trend of the labor share

depends on the trend of these expenses relative to measured output and on the fraction of expenses that accrues to labor compensation. Expenses on unmeasured intangibles are increasing over time as a share of output and tangible investments. It also seems plausible that these expenses do not affect much labor income. Thus, if statistical agencies recognize these expenses as investments, the decline of the labor share is going to be even more pronounced.

Treatment of non-market output. Services provided by durable goods are not included in GDP and capital income. This contrasts with services provided by owner-occupied housing, which are included in GDP and capital income through imputed rent. The logic for this difference is that the durables are part of non-market production. In a broader measure of output that includes non-market production, the service flow of durables should be added to GDP and capital income. But if one adopts such a measure, consistency requires imputing the labor compensation generated in the home sector in the form of non-market time spent on activities such as cooking, cleaning, child care, and home and personal maintenance.

Differences between value added and compensation concepts in BLS-BEA integrated accounts and BEA industry accounts. Both labor share measures that I construct at the industry level use compensation from the integrated BEA-BLS accounts, which adds proprietors' labor income. The "BLS Industry" measure divides compensation to employees by value added for each industry from the BEA-BLS integrated accounts. The "BEA Industry" measure scales value added of each industry from the BEA-BLS integrated accounts by a constant ratio, calculated as the industry's average valued added in BEA industry accounts to its value added in BLS-BEA integrated accounts, and then subtracts taxes on production available only from the BEA industry accounts. For years before 1998, I impute taxes on production using the mean ratio of taxes on production from the BEA industry accounts to value added in the BLS-BEA integrated accounts. The fourth column of Table 3 shows industry shares of valued added less taxes on production, because the aggregate labor share in the BEA measure has value added less taxes on production in the denominator.

The two projects have different scopes. The BEA industry accounts are designed so that value added of industries adds up to GDP in the NIPA. Value added is organized similarly to the aggregate accounts and equals the sum of compensation to employees, gross operating surplus, and taxes on production less subsidies. Depreciation expenses and proprietors' income are included in the gross operating surplus of each industry. By contrast, the integrated BEA-BLS accounts are designed to estimate productivity and costs by industry. Value added equals the sum of compensation to workers, adjusted for proprietors' income, and various types of fixed assets. Different from the BEA industry accounts, compensation to capital in the integrated accounts follows the BLS practice of including imputed capital income to the government and part of taxes on production.

To confirm that proprietors' income is included in the integrated accounts, I compared compensation in the BLS-BEA integrated accounts to compensation produced by the BEA in its industry accounts which does not include proprietors' income. I find that compensation in the integrated accounts significantly exceeds compensation in the industry accounts in industries for which we expect a large share of proprietors, such as agriculture, construction, real estate, professional and business services, arts and entertainment, and other services. I also find that compensation is roughly equal between the two data sources in industries without a significant share of proprietors, such as oil and mining, utilities, manufacturing, wholesale, and government.

Differences between industry labor shares and those in Elsby, Hobijn, and Sahin (2013). My estimated declines of the labor share at the industry level are generally larger than those reported by Elsby, Hobijn, and Sahin (2013). An important difference with their measurement is that they use the BEA industry accounts, which do not allocate any of proprietors' income into labor compensation. Instead, I use the BEA-BLS integrated product accounts, which allocate part of proprietors' income to labor compensation. For example, the average labor share of agriculture in the BEA industry accounts is below 30 percent, compared with more than 60 percent in my measures.

Labor shares from the Penn World Table. Similar to my U.S. measures, all measures subtract taxes on products from the denominator of the labor share. The labor share resembling the "Total: Proprietors Same Share" for the United States is called "lab_sh2" in the PWT. The measures are not, however, strictly comparable, because some private businesses may be paying wages to their owners and some proprietors may be legally incorporated. The labor share which adjusts for the ratio of labor inputs between self-employed and employees is called "lab_sh3" in the PWT (similar to "Total: Proprietors Same Wage" measure for the United States). The measure "lab_sh4" adds all value added of agriculture to compensation of employees, and the measure "lab_sh1" allocates all mixed income to compensation of employees.

I restrict the sample to countries with at least 25 years of data. My preferred labor share series is "lab_sh2." I impute missing values for "lab_sh2" using the slope of the regression of "lab_sh2" on "lab_sh3," which is the most comparable measure to "lab_sh2." This allows me to expand the sample for a few countries (for example, Germany and the United Kingdom) by a few more years. When the "lab_sh2" series augmented with the imputation is unavailable for at least 25 years, I use, in order, "lab_sh3," "lab_sh4," and "lab_sh1."

B Additional Results

Additional labor share measures in the United States. Figure A.1 presents three additional measures of the labor share, in addition to "Total: Proprietors Same Share," "Total: Proprietors

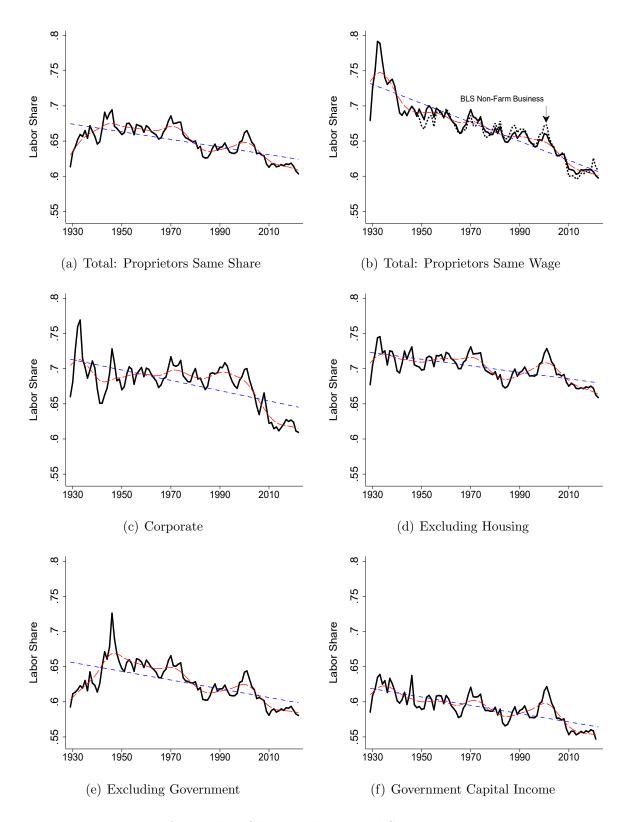


Figure A.1: Labor Share in the United States, 1929–2022

Notes: Solid black line is the labor share measure, dotted blue line is the linear trend of each measure, and red long-dashed line is the Hodrick-Prescott trend of each measure with a smoothing parameter of 100. The shorted-dashed black line is the BLS measure of the labor share for the non-farm business sector, scaled to equal "Total: Proprietors Same Wage" in the first year of its observation.

Same Wage," and the corporate labor share discussed previously in the text. The first measure is based on "Total: Proprietors Same Share," but excludes the housing sector by removing its contribution to compensation of employees, GDP, proprietors' income, and taxes on production. The second measure is also based on "Total: Proprietors Same Share," but excludes the government sector by removing its contribution to compensation of employees and GDP. Government's gross value added equals compensation of employees and depreciation, because national accounts treat government as making zero net capital income from its operations. The last measure imputes to capital income and to GDP the net capital income produced by government's assets.

Excluding the housing and the government sectors matters quite a bit for the estimated trends of the labor share in the early years of the sample. Estimates become more robust after World War II and are robust in the last subperiods, which include the decline of the labor share in the 2000s. Excluding government generates a dramatic increase in the labor share around the end of World War II, because government's labor share decreased substantially during this period and the government accounted for roughly 20 percent of GDP. The measure of the labor share that imputes government's capital income to output exhibits behavior similar to the other measures, except again in the early years of the sample. Notably, this measure does not display the spike of "Total: Proprietors Same Share" in the aftermath of World War II, because wartime fixed assets substantially increased imputed capital income.

Table A.1 presents the estimated trends in the additional measures of the labor share and for additional subsamples. Relative to the results reported in Table 2 in the main text, the labor share decreases by more in the first subperiod, remains roughly constant until the end of the 20th century, and declines by less in the last subperiod. Table A.2 presents estimates of the trend of each labor share measure using a linear trend, a Hodrick-Prescott trend, and a 5-year moving average, separately for the whole sample between 1929 and 2022 and the sample after World War II between 1946 and 2022.

Labor shares and collective bargaining. Figure A.2 shows the evolution of the labor share for countries in which the share of employees with the right to bargain exceeds 50 percent but remains relatively stable between the early 2000s and the late 2010s. The labor share data is from the PWT and the data on the share of employees with the right to bargain is from the OECD. As the figure shows, most countries experienced declines of their labor shares, including three of the four Scandinavian countries.

Tests of dynamic inefficiency. An economy is dynamically inefficient if it can increase its consumption per capita forever by reducing its capital per capita in a balanced growth path. The economy is dynamically inefficient if, in a balanced growth path, the marginal product of capital is lower than the growth rate of aggregate output. In competitive economies, this is

(percentage points change per decade) 1929-1945 1946-1970 1971-1995 1996-2022 Total: Proprietors Same Share 3.0 -0.2 -1.6 -1.8 (0.6) (0.4) (0.3) (0.3) Total: Proprietors Same Wage -3.9 -0.9 -1.1 -2.3 Corporate -3.2 -0.1 -0.1 -3.3 Corporate -3.2 -0.1 -0.1 -3.3 Excluding Housing -0.1 0.3 -0.20 (0.3) Excluding Government 3.8 -1.3 -1.6 -2.0 (0.6) (0.7) (0.3) (0.3) (0.3) Government Capital Income -0.8 -0.3 -0.9 -2.3 (percentage points change per decade) 1929-1951 1952-1975 1976-1999 2000-2022 Total: Proprietors Same Wage -3.4 -0.8 -0.7 -2.6 (percentage points change per decade) 1929-1951 1952-1975 1976-1999 200-2022 Total: Proprietors Same Wage -3.4 -					
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$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		(1.8)	(0.5)	(0.3)	(0.3)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Excluding Housing	-0.1	0.3	-1.0	-2.0
0 (0.6) (0.7) (0.3) (0.3) Government Capital Income -0.8 -0.3 -0.9 -2.3 (0.9) (0.5) (0.2) (0.4) Median -0.5 -0.3 -1.1 -2.2 (percentage points change per decade) $1929 \cdot 1951$ $1952 \cdot 1975$ $1976 \cdot 1999$ $2000 \cdot 2022$ Total: Proprietors Same Share 1.7 0.1 -0.3 -2.3 (0.5) (0.4) (0.3) (0.3) Total: Proprietors Same Wage -3.4 -0.8 -0.7 -2.6 (1.1) (0.2) (0.2) (0.2) Corporate -1.6 0.4 0.1 -3.6 (1.0) (0.3) (0.3) (0.5) Excluding Housing -0.6 0.1 0.4 -2.6 (0.6) (0.2) (0.3) (0.3) Government Capital Income 3.3 -0.4 -0.3 -2.3 (0.6) (0.3) (0.3) (0.4)		(1.0)	(0.3)	(0.3)	(0.3)
Government Capital Income -0.8 -0.3 -0.9 -2.3 (0.9)(0.5)(0.2)(0.4)Median -0.5 -0.3 -1.1 -2.2 (percentage points change per decade)1929-19511952-19751976-19992000-2022Total: Proprietors Same Share1.70.1 -0.3 -2.3 (0.5)(0.4)(0.3)(0.3)Total: Proprietors Same Wage -3.4 -0.8 -0.7 -2.6 (1.1)(0.2)(0.2)(0.2)Corporate -1.6 0.40.1 -3.6 (1.0)(0.3)(0.3)(0.5)Excluding Housing -0.6 0.10.4 -2.6 (0.6)(0.2)(0.3)(0.3)Excluding Government 3.3 -0.4 -0.3 -2.3 Government Capital Income -1.2 0.00.2 -2.9 (0.6)(0.3)(0.3)(0.4) -1.2	Excluding Government	3.8	-1.3	-1.6	-2.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		(0.6)	(0.7)	(0.3)	(0.3)
Median -0.5 -0.3 -1.1 -2.2 (percentage points change per decade)1929-19511952-19751976-19992000-2022Total: Proprietors Same Share1.70.1 -0.3 -2.3 (0.5)(0.4)(0.3)(0.3)Total: Proprietors Same Wage -3.4 -0.8 -0.7 -2.6 (1.1)(0.2)(0.2)(0.2)Corporate -1.6 0.40.1 -3.6 (1.0)(0.3)(0.3)(0.5)Excluding Housing -0.6 0.10.4 -2.6 (0.6)(0.2)(0.3)(0.3)Excluding Government 3.3 -0.4 -0.3 -2.3 Government Capital Income -1.2 0.00.2 -2.9 (0.6)(0.3)(0.3)(0.4) -2.9	Government Capital Income	-0.8	-0.3	-0.9	-2.3
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		(0.9)	(0.5)	(0.2)	(0.4)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Median	-0.5	-0.3	-1.1	-2.2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$					
Total: Proprietors Same Wage -3.4 -0.8 -0.7 -2.6 (1.1)(0.2)(0.2)(0.2)Corporate -1.6 0.40.1 -3.6 (1.0)(0.3)(0.3)(0.5)Excluding Housing -0.6 0.10.4 -2.6 (0.6)(0.2)(0.3)(0.3)Excluding Government 3.3 -0.4 -0.3 -2.3 (0.7)(0.2)(0.3)(0.3)Government Capital Income -1.2 0.00.2 -2.9 (0.6)(0.3)(0.3)(0.4)	(percentage points change per decade)	1929-1951	1952-1975	1976-1999	2000-2022
$\begin{array}{cccccccccccccccccccccccccccccccccccc$					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		1.7	0.1	-0.3	-2.3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Total: Proprietors Same Share	1.7 (0.5)	0.1 (0.4)	-0.3 (0.3)	-2.3 (0.3)
Excluding Housing -0.6 0.1 0.4 -2.6 (0.6) (0.2) (0.3) (0.3) Excluding Government 3.3 -0.4 -0.3 -2.3 (0.7) (0.2) (0.3) (0.3) Government Capital Income -1.2 0.0 0.2 -2.9 (0.6) (0.3) (0.3) (0.4)	Total: Proprietors Same Share	1.7 (0.5) -3.4	0.1 (0.4) -0.8	-0.3 (0.3) -0.7	-2.3 (0.3) -2.6
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Total: Proprietors Same Share Total: Proprietors Same Wage	$ \begin{array}{c} 1.7 \\ (0.5) \\ -3.4 \\ (1.1) \end{array} $	$0.1 \\ (0.4) \\ -0.8 \\ (0.2)$	$-0.3 \\ (0.3) \\ -0.7 \\ (0.2)$	$ \begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \end{array} $
Excluding Government 3.3 -0.4 -0.3 -2.3 (0.7) (0.2) (0.3) (0.3) Government Capital Income -1.2 0.0 0.2 -2.9 (0.6) (0.3) (0.3) (0.4)	Total: Proprietors Same Share Total: Proprietors Same Wage	$ \begin{array}{r} 1.7 \\ (0.5) \\ -3.4 \\ (1.1) \\ -1.6 \end{array} $	$0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4$	$-0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1$	$ \begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ \end{array} $
Government Capital Income (0.7) (0.2) (0.3) (0.3) (0.6) (0.3) (0.3) (0.4)	Total: Proprietors Same Share Total: Proprietors Same Wage Corporate	$ \begin{array}{r} 1.7 \\ (0.5) \\ -3.4 \\ (1.1) \\ -1.6 \\ (1.0) \end{array} $	$0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4 \\ (0.3)$	$-0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1 \\ (0.3)$	$ \begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ (0.5) \end{array} $
Government Capital Income -1.2 0.0 0.2 -2.9 (0.6) (0.3) (0.3) (0.4)	Total: Proprietors Same Share Total: Proprietors Same Wage Corporate	$ \begin{array}{r} 1.7\\(0.5)\\-3.4\\(1.1)\\-1.6\\(1.0)\\-0.6\end{array} $	$\begin{array}{c} 0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4 \\ (0.3) \\ 0.1 \end{array}$	$-0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1 \\ (0.3) \\ 0.4$	$ \begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ (0.5) \\ -2.6 \\ \end{array} $
(0.6) (0.3) (0.3) (0.4)	Total: Proprietors Same Share Total: Proprietors Same Wage Corporate Excluding Housing	$ \begin{array}{r} 1.7\\(0.5)\\-3.4\\(1.1)\\-1.6\\(1.0)\\-0.6\\(0.6)\end{array} $	$\begin{array}{c} 0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4 \\ (0.3) \\ 0.1 \\ (0.2) \end{array}$	$\begin{array}{c} -0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1 \\ (0.3) \\ 0.4 \\ (0.3) \end{array}$	$ \begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ (0.5) \\ -2.6 \\ (0.3) \end{array} $
	Total: Proprietors Same Share Total: Proprietors Same Wage Corporate Excluding Housing	$ \begin{array}{r} 1.7\\(0.5)\\-3.4\\(1.1)\\-1.6\\(1.0)\\-0.6\\(0.6)\\3.3\end{array} $	$\begin{array}{c} 0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4 \\ (0.3) \\ 0.1 \\ (0.2) \\ -0.4 \end{array}$	$\begin{array}{c} -0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1 \\ (0.3) \\ 0.4 \\ (0.3) \\ -0.3 \end{array}$	$\begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ (0.5) \\ -2.6 \\ (0.3) \\ -2.3 \end{array}$
Median -0.9 0.1 -0.1 -2.6	Total: Proprietors Same Share Total: Proprietors Same Wage Corporate Excluding Housing Excluding Government	$ \begin{array}{r} 1.7\\(0.5)\\-3.4\\(1.1)\\-1.6\\(1.0)\\-0.6\\(0.6)\\3.3\\(0.7)\end{array} $	$\begin{array}{c} 0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4 \\ (0.3) \\ 0.1 \\ (0.2) \\ -0.4 \\ (0.2) \end{array}$	$\begin{array}{c} -0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1 \\ (0.3) \\ 0.4 \\ (0.3) \\ -0.3 \\ (0.3) \end{array}$	$\begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ (0.5) \\ -2.6 \\ (0.3) \\ -2.3 \\ (0.3) \end{array}$
	Total: Proprietors Same Share Total: Proprietors Same Wage Corporate Excluding Housing Excluding Government	$\begin{array}{c} 1.7\\(0.5)\\-3.4\\(1.1)\\-1.6\\(1.0)\\-0.6\\(0.6)\\3.3\\(0.7)\\-1.2\end{array}$	$\begin{array}{c} 0.1 \\ (0.4) \\ -0.8 \\ (0.2) \\ 0.4 \\ (0.3) \\ 0.1 \\ (0.2) \\ -0.4 \\ (0.2) \\ 0.0 \end{array}$	$\begin{array}{c} -0.3 \\ (0.3) \\ -0.7 \\ (0.2) \\ 0.1 \\ (0.3) \\ 0.4 \\ (0.3) \\ -0.3 \\ (0.3) \\ 0.2 \end{array}$	$\begin{array}{r} -2.3 \\ (0.3) \\ -2.6 \\ (0.2) \\ -3.6 \\ (0.5) \\ -2.6 \\ (0.3) \\ -2.3 \\ (0.3) \\ -2.9 \end{array}$

Table A.1: Changes of the U.S. Labor Share by Subperiod

Notes: Entries are percentage points changes per decade in different measures of the labor share. The estimates are from separate regressions of labor share measures on a linear trend within each subperiod. Robust standard errors are in parentheses.

equivalent to the criterion that the (net) rate of return on capital is lower than the growth rate. Such a case can arise in overlapping generations economies in which the conditions of the first welfare theorem do not apply because there is an infinite number of generations.

The problem with the criterion that compares the rate of return to the growth rate is that it is not obvious which rate of return to use. Abel, Mankiw, Summers, and Zeckhauser (1989) suggest an alternative criterion, which says that the economy is dynamically inefficient if its investment rate always exceeds its capital share of income. Intuitively, the economy is producing a dividend when capital income exceeds investment, whereas the economy is losing resources if it invests more than it is earning from its capital. The logic of why this criterion

Period: 1929-2022	Raw	Linear Trend	HP(100) Trend	MA(5) Trend
Total: Proprietors Same Share	-1.0	-5.1	-2.1	-2.1
Total: Proprietors Same Wage	-8.2	-12.5	-13.4	-11.7
Corporate	-5.1	-6.8	-9.0	-7.4
Excluding Housing	-1.8	-4.3	-4.4	-3.7
Excluding Government	-1.2	-5.8	-1.8	-2.2
Government Capital Income	-3.8	-5.4	-5.7	-4.9
Period: 1946-2022	Raw	Linear Trend	HP(100) Trend	MA(5) Trend
Total: Proprietors Same Share	-9.1	-6.3	-6.6	-7.3
Total: Proprietors Same Wage	-10.2	-9.4	-9.1	-9.0
100000 1 10pilotoito Stanio (1080	10.2	0.1	0.1	0.0
Corporate	-11.9	-7.4	-7.0	-8.2
			-	
Corporate	-11.9	-7.4	-7.0	-8.2

Table A.2: Trends of the U.S. Labor Share

Notes: "Raw" is the change between the value in 2022 and the value in either 1929 or 1946. The other entries are estimates of the trend of the labor share using a linear trend, a Hodrick-Prescott trend with a smoothing parameter of 100, a 5-year moving average, and a 20-year moving average. Changes in the column Median exclude the Raw estimates.

does not explicitly require measurement of the return to capital is that the capital share of income implicitly embeds the appropriate rate of return earned by an economy's capital.

Figure A.3 reports the share of capital income and the share of investment in GDP for the 16 largest economies of the world by GDP in 2015. The data come from the PWT. The capital share is defined as one minus the labor share. We observe that in most countries the capital share exceeds the investment rate, and thus most countries are not dynamically inefficient.

In assessing dynamic inefficiency, the comparison of one minus the labor share to the investment rate is appropriate when there are no economic profits. However, if some of the residual payments accrue to profits instead of capital, this comparison is not anymore appropriate. I adjust the criterion in Section C to take into account markups and fixed costs. The adjustment shows that economic profits need to increase significantly for economies to become dynamically inefficient. To be clear, by economic profits I refer to the gap between revenues and costs, irrespective of whether the gap derives from markups or returns to scale that are not constant.

C Models of the Labor Share

Labor share solution. Consider a firm that operates in partial equilibrium and chooses prices and quantities of output and inputs. Output y is produced with capital k and labor ℓ according

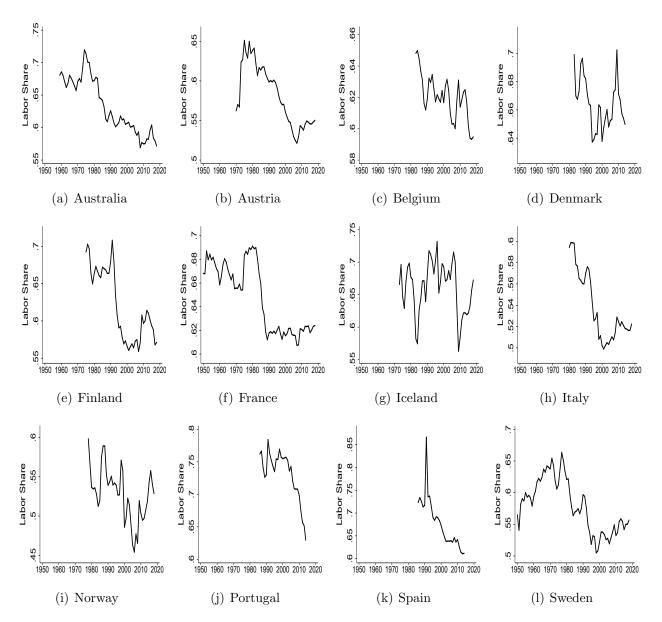


Figure A.2: Labor Share with High and Stable Share of Right-to-Bargain Employees

to the production function

$$y = F(k,\ell) - f = \left(\alpha(A_kk)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_\ell\ell)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - f,\tag{A.1}$$

where $\sigma \geq 0$ is the constant elasticity of substitution between capital k and labor ℓ , $A_k \geq 0$ is capital-augmenting technology, and $A_\ell \geq 0$ is labor-augmenting technology. The distribution factor α is between 0 and 1. Parameter f subtracts output for given capital and labor. The firm faces increasing returns to scale when f > 0, constant returns to scale when f = 0, and decreasing returns to scale when f < 0. Denote by λ the solution of the following equation: $\lambda y = F = F_k k + F_\ell \ell \Longrightarrow \lambda = 1 + \frac{f}{F(k,\ell)-f}$. I call λ returns to scale, and note that returns to scale generally vary with the optimal choices of inputs.

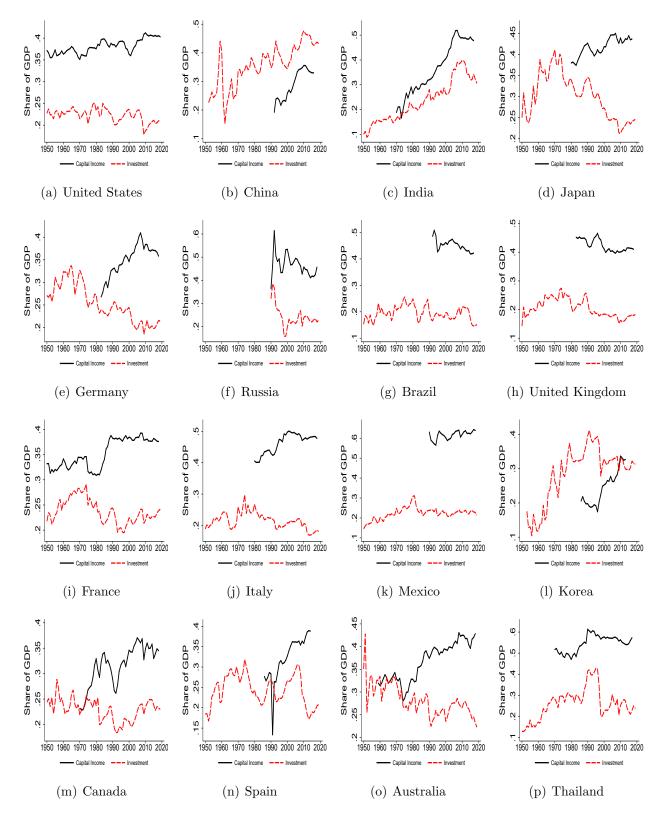


Figure A.3: Tests of Dynamic Inefficiency

The firm faces product demand p(y) and labor supply $\ell(w)$. The firm takes as given the user cost of capital R. The profit maximization problem of the firm is

$$\max_{k,\ell} \pi = p(y) \cdot y(k,\ell) - w(\ell) \cdot \ell - R \cdot k, \tag{A.2}$$

subject to the production technology $y = F(k, \ell) - f$, the product demand p(y) and the labor supply $\ell(w)$. In the profit maximization problem, the firm internalizes that its price p varies with output y and that its wage w varies with labor ℓ . The first-order conditions with respect to capital and labor are

$$pF_k = p\alpha A_k^{\frac{\sigma-1}{\sigma}} \left(\frac{\lambda y}{k}\right)^{\frac{1}{\sigma}} = \mu R,$$
(A.3)

$$pF_{\ell} = p(1-\alpha)A_{\ell}^{\frac{\sigma-1}{\sigma}} \left(\frac{\lambda y}{\ell}\right)^{\frac{1}{\sigma}} = \mu\theta w, \tag{A.4}$$

where the expressions for the two markups are given by $\mu = (-y'(p)p/y)/(-y'(p)p/y-1) \ge 1$ and $\theta = (\ell'(w)w/\ell + 1)/(\ell'(w)w/\ell) \ge 1$, where y'(p) is the slope of product demand and $\ell'(w)$ is the slope of labor supply. Dividing the two first-order conditions, I get that the optimal capital-to-labor ratio is

$$\frac{k}{\ell} = \left(\frac{\alpha}{1-\alpha}\frac{\theta w}{R}\right)^{\sigma} \left(\frac{A_k}{A_\ell}\right)^{\sigma-1}.$$
(A.5)

The capital-to-labor ratio does not depend on product market markups μ , because μ distorts capital and labor proportionally, and on fixed costs f, because fixed costs do not affect marginal products.

Income shares are defined as

$$s_{\ell} = \frac{w\ell}{py}, \quad s_k = \frac{Rk}{py}, \quad s_{\pi} = \frac{\pi}{py}.$$
(A.6)

I note that fixed costs f affect output directly and do not reflect demand for labor or capital inputs. Thus, compensation to labor and compensation to capital do not include f. To express income shares in terms of markups, returns to scale, and payments to factors, we use the definition of returns to scale λ (evaluated at the optimal choices) and the first-order conditions to derive profits as

$$\pi = py - w\ell - Rk = \left(\frac{pF_k}{\lambda} - R\right)k + \left(\frac{pF_\ell}{\lambda} - w\right)\ell = \left(\frac{\mu}{\lambda} - 1\right)Rk + \left(\frac{\mu\theta}{\lambda} - 1\right)w\ell.$$
(A.7)

Therefore, income shares are

$$s_{\ell} = \frac{\lambda}{\mu} \left(\frac{w\ell}{\theta w\ell + Rk} \right), \quad s_{k} = \frac{\lambda}{\mu} \left(\frac{Rk}{\theta w\ell + Rk} \right), \quad s_{\pi} = 1 - \frac{\lambda}{\mu} \left(\frac{w\ell + Rk}{\theta w\ell + Rk} \right). \tag{A.8}$$

One expression for the labor share is obtained by applying the income shares in equation (A.8) to the capital-to-labor ratio in equation (A.5),

$$s_{\ell} = \frac{\lambda}{\mu\theta} \left(\frac{1}{1 + \left(\frac{\alpha}{1 - \alpha}\right)^{\sigma} \left(\frac{A_k \theta w}{A_\ell R}\right)^{\sigma - 1}} \right).$$
(A.9)

The expression that appears in the main text derives from applying the income shares in equation (A.8) to the first-order condition with respect to capital in equation (A.3),

$$s_{\ell} = \frac{\lambda}{\mu\theta} \left(1 - \alpha^{\sigma} \left(\frac{A_k}{\mu R} \right)^{\sigma - 1} \right), \tag{A.10}$$

where I normalize the product price to one at the optimal choices, so R represents the cost of capital relative to the product price. The solution for the labor share shown in the main text is equation (A.10) for the case of constant returns to scale, $\lambda = 1$.

The misleading equation (A.9). A difference between equations (A.9) and (A.10) is that labor-augmenting technology A_{ℓ} and the wage w appear only in the former. Here, I want to draw a distinction between partial and general equilibrium. If all firms experience a change in labor-augmenting technology, we expect the equilibrium wage to grow proportionally to technology. Thus, keeping the wage constant in equation (A.9) is misleading for thinking about changes in labor-augmenting technology at the aggregate level. It is also confusing to think how the wage affects the aggregate labor share, because the wage is typically determined in general equilibrium as a function of labor-augmenting technology and potentially other primitives. So, it makes little sense to do comparative statics with respect to the wage.

The more useful equation (A.10). Using equation (A.10) avoids some of the pitfalls from changing one determinant of the labor share in partial equilibrium without thinking about how this determinant is determined in general equilibrium. Mechanically, A_{ℓ} and w do not appear in equation (A.10), because I used the capital's first-order condition for profit maximization to derive the labor share. Equation (A.10) may not be misleading for thinking about the effects of capital-augmenting technology A_k on the labor share, to the extent that the cost of capital Rand markups μ, θ do not respond to A_k . Similarly, it may not be misleading to think about the effects of the cost of capital on the labor share, to the extent that the cost of capital is pinned down in equilibrium independently of A_k , μ , and θ .¹ Several economic models share these properties under infinitely elastic supply of capital in the long run, isoelastic preferences, and monopolistic or monopsonistic markets. However, the same caveats about partial equilibrium versus general equilibrium reasoning apply in models that relax these assumptions.

Industry equilibrium with two types of firms. Consider an industry populated by two types of firms. The labor-intensive firm produces with technology $y_{\ell} = \ell$. The capital-intensive firm

¹With fixed costs, f > 0, returns to scale λ decline with a higher A_k or a lower R because production expands.

produces with technology $y_k = A_k k$. The output prices of the firms are given by p_ℓ and p_k . With linear production we obtain $p_\ell = w$ and $p_k = R/A_k$, where w is the wage and R is the cost of capital.

Consumers have CES preferences over the output of the two firms, with an elasticity ε . Demand functions are given by $y_{\ell} = (p_{\ell}/p)^{-\epsilon} E$ and $y_k = (p_k/p)^{-\epsilon} E$, where E is the aggregate expenditure of consumers across all industries and p is the industry price index.

The labor share of income for the industry equals the share of output produced by the labor-intensive firm,

$$s_{\ell} = \frac{w\ell}{w\ell + Rk} = \frac{p_{\ell}y_{\ell}}{p_{\ell}y_{\ell} + p_{k}y_{k}} = \frac{p_{\ell}^{1-\varepsilon}}{p_{\ell}^{1-\varepsilon} + p_{k}^{1-\varepsilon}} = \frac{1}{1 + \left(\frac{w}{A_{k}R}\right)^{\varepsilon-1}}.$$
(A.11)

Consider an increase in A_k , holding constant w and R. Equation (A.11) shows that the labor share of the industry declines if consumers perceive the two goods as substitutes, $\varepsilon > 1$. The HHI of sales concentration for the industry is given by $s_{\ell}^2 + (1 - s_{\ell})^2$, which declines with s_{ℓ} if $s_{\ell} < 1/2$. The 1/2 threshold can be relaxed by introducing more parameters or in a richer environment in which firms are not as striking.

The point is that we have constructed an economy in which, if $\varepsilon > 1$ and $s_{\ell} < 1/2$: larger firms have a lower labor share; no firm substitutes between labor and capital; capitalaugmenting technical change leads to a decline of the labor share; and capital-augmenting technical change leads to an increase in sales concentration without changing employment or capital concentration.

Here, again, we need to be cautious about general equilibrium effects as we have not allowed w/R to respond to A_k . The relative input price w/R is unresponsive to A_k if the change in A_k is at the industry level and the industry is small enough relative to the rest of the economy. Alternatively, if all industries experience the same change in A_k , the assumption is satisfied in a small open economy with perfectly elastic capital supply and labor mobility. Relaxing the labor mobility assumption and assuming that labor is completely inelastic in the aggregate, in the symmetric-industry equilibrium the wage satisfies $w^{1-\varepsilon} = \mathbb{C} - (R/A_k)^{1-\varepsilon}$ for a constant \mathbb{C} . In this case, the solution for the labor share in general equilibrium becomes $s_\ell = 1 - 1/(\mathbb{C}(R/A_k)^{\varepsilon-1})$, which again is decreasing in R/A_k if $\varepsilon > 1$.

Labor share with imported intermediate inputs: Model I. I consider the problem of a firm in partial equilibrium. The firm produces gross output with technology

$$g = \left((1-\omega)\ell^{\frac{\sigma-1}{\sigma}} + \omega y(k,m)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},\tag{A.12}$$

$$y = \left(\alpha (A_k k)^{\frac{\phi-1}{\phi}} + (1-\alpha)(A_m m)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}.$$
(A.13)

Labor ℓ and the composite good y nest in gross output g with an elasticity of substitution σ . Capital k and imported intermediate inputs m nest in the composite good y with an elasticity of substitution ϕ . The price of g is p, the price of ℓ is w, the price of k is R, and the price of m is q. Domestic product equals pg - qm.

We can split the problem of the firm in two steps. In the first step, we obtain the price of the composite good y from minimizing the cost v = qm + Rk of producing one unit of the composite good,

$$v = \left(\alpha^{\phi} A_k^{\phi-1} R^{1-\phi} + (1-\alpha)^{\phi} A_m^{\phi-1} q^{1-\phi}\right)^{\frac{1}{1-\phi}}.$$
(A.14)

Given v, the firm maximizes its profits $\pi = p(g)g - vy - w\ell$, internalizing that it faces an isoelastic demand for its product p(g).

Denoting the income shares of gross output by θ and the income shares of the composite good by χ , we obtain

$$\theta_{\ell} = \frac{w\ell}{pg} = \left(\frac{1}{\mu}\right) \left(\frac{1}{1 + \left(\frac{\omega}{1-\omega}\right)^{\sigma} \left(\frac{w}{v}\right)^{\sigma-1}}\right),\tag{A.15}$$

$$\chi_m = \frac{qm}{vy} = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\phi} \left(\frac{qA_k}{RA_m}\right)^{\phi-1}}.$$
(A.16)

The labor share of domestic income is

$$s_{\ell} = \frac{w\ell}{pg - qm} = \frac{w\ell}{pg} \frac{pg}{pg - qm} = \frac{\theta_{\ell}}{1 - \chi_m (1/\mu - \theta_{\ell})}.$$
(A.17)

The labor share of income is increasing in θ_{ℓ} because labor absorbs a larger part of gross output and some of gross output represents domestic income. The labor share of income is increasing in χ_m because χ_m increases the gap between gross output and domestic income and labor is paid from gross output.

For the comparative statics with respect to q, I impose the assumption that w and R do not respond in equilibrium. Thus, the comparative statics are subject to the same caveats I discussed above. The comparative statics of s_{ℓ} with respect to q depend on the relative values of σ and ϕ . If $\sigma > \phi$, then s_{ℓ} is an increasing function of q. The logic is that when imported intermediate inputs become cheaper, firms substitute more between labor and the composite good when σ is high, and firms substitute more between materials and capital when ϕ is high. The first effect tends to lower the labor share of gross output for high values of σ , whereas the second effect tends to close the gap between gross output and domestic income for high values of ϕ . When $\sigma > \phi$, the first effect dominates, and the labor share falls when the price of intermediate inputs decreases.

I do not find this to be a very intuitive way to think about the decline of the labor share. First, even if one accepts that globalization has caused a decline of q, commodity prices tend to be more volatile than domestic input prices, and so one would need to consider other forces that impact q relative to w and R. Second, for the theory that declines of q cause a decline of s_{ℓ} , we need labor to be more substitutable with imported intermediate inputs than capital is. But many imported goods are capital goods, so for these goods a better description of technology is that imported goods are more substitutable to domestic capital than to labor, $\phi > \sigma$.

Labor share with imported intermediate inputs: Model II. The same prediction arises if we change the way the three inputs are nested in the production function. Technology is

$$g = \left((1-\omega)k^{\frac{\sigma-1}{\sigma}} + \omega y(\ell, m)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},\tag{A.18}$$

$$y = \left(\alpha (A_{\ell}\ell)^{\frac{\phi-1}{\phi}} + (1-\alpha)(A_m m)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}.$$
(A.19)

Capital k and the composite good y nest in gross output g with an elasticity of substitution σ . Labor ℓ and imported intermediate inputs m nest in the composite good y with an elasticity of substitution ϕ . The price of g is p, the price of ℓ is w, the price of k is R, and the price of m is q. Domestic product equals pg - qm.

As was the case before, we can split the problem of the firm in two steps. In the first step, we obtain the price of the composite good y from minimizing the cost $v = qm + w\ell$ of producing one unit of the composite good,

$$v = \left(\alpha^{\phi} A_{\ell}^{\phi-1} w^{1-\phi} + (1-\alpha)^{\phi} A_{m}^{\phi-1} q^{1-\phi}\right)^{\frac{1}{1-\phi}}.$$
(A.20)

Given v, the firm maximizes its profits $\pi = p(g)g - vy - Rk$, internalizing that it faces an isoelastic demand for its product p(g).

Denoting the income shares of gross output by θ and the income shares of the composite good by χ , we obtain

$$\theta_y = \frac{vy}{pg} = \left(\frac{1}{\mu}\right) \left(\frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\sigma} \left(\frac{v}{R}\right)^{\sigma-1}}\right),\tag{A.21}$$

$$\chi_{\ell} = \frac{w\ell}{vy} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{\phi} \left(\frac{wA_m}{qA_{\ell}}\right)^{\phi-1}}.$$
(A.22)

The labor share of domestic income is

$$s_{\ell} = \frac{w\ell}{pg - qm} = \frac{w\ell}{vy} \frac{vy}{pg} \frac{pg}{pg - qm} = \frac{\chi_{\ell}\theta_y}{1 - \theta_y(1 - \chi_{\ell})}.$$
(A.23)

The labor share of income is increasing in χ_{ℓ} because labor absorbs a larger part of the composite good. The labor share of income is increasing in θ_y because θ_y increases the gap between the composite good and domestic income and labor is paid from the composite good.

For the comparative statics with respect to q, I again impose the assumption that w and R do not respond in equilibrium. The comparative statics of s_{ℓ} with respect to q depend on the

relative values of σ and ϕ . If $\phi > \sigma$, then s_{ℓ} is an increasing function of q. The logic is that when imported intermediate inputs become cheaper, firms substitute more between capital and the composite good when σ is high, and firms substitute more between materials and labor when ϕ is high. The first effect tends to close the gap between the composite good and domestic income for high values of σ , whereas the second effect tends to increase the labor share of the composite good for high values of ϕ . When $\phi > \sigma$, the second effect dominates, and the labor share falls when the price of intermediate inputs decreases. Thus, for the theory that declines of q cause a decline of s_{ℓ} , we again need labor to be more substitutable with imported intermediate inputs than capital is.

Dynamic inefficiency with economic profits. Denote the capital stock per capita that maximizes consumption per capita in balanced growth by k^* and the equilibrium capital stock per capita by \hat{k} . In the neoclassical models with market power considered by Ball and Mankiw (2023), the capital stock that maximizes consumption satisfies

$$F_k(k^*) = n + \delta + g, \tag{A.24}$$

where F_k is the marginal product of capital, n is population growth, δ is the depreciation rate, and g is the growth rate of output per capita. The equilibrium capital stock satisfies

$$F_k(\hat{k}) = \mu(r+\delta),\tag{A.25}$$

where μ is the markup of prices over marginal cost.

Dynamic inefficiency arises if $n + \delta + g > \mu(r + \delta)$. By multiplying both sides by k/y and using the fact that the balanced growth investment rate is $i/y = (n + g + \delta)k/y$, we obtain $i/y > \mu s_k$ where $s_k = (r + \delta)k/y$ is the capital share of income. For the case of no labor market power, this criterion simplifies to

$$\frac{i}{y} > \lambda - \mu s_{\ell},\tag{A.26}$$

where s_{ℓ} is the labor share of income and λ is returns to scale. The profit share in the case of no labor market power is $s_{\pi} = 1 - \lambda/\mu$. My analysis differs from Ball and Mankiw (2023) in that I express the criterion for dynamic inefficiency in terms of the labor share.

How does product market power change the calculus underlying the tests of dynamic inefficiency in Figure A.3? Table A.3 provides some illustrative calculations. Fixing the labor share at 0.6 in the first column, each row of the table varies the markup μ and returns to scale λ in the next two columns. The fourth column shows the profit share s_{π} , the fifth column shows the naive criterion used in Figure A.3 that equates the capital share with one minus the labor share, and the sixth column shows the right-hand-side of the criterion in equation (A.26), $\lambda - \mu s_{\ell}$, for the model with product market power. In the first panel, keeping returns to scale λ constant,

				Naive	Adjusted
s_ℓ	μ	λ	s_{π}	$1 - s_{\ell}$	$\lambda - \mu s_\ell$
0.60	1.00	1.00	0.00	0.40	0.40
0.60	1.10	1.00	0.09	0.40	0.34
0.60	1.20	1.00	0.17	0.40	0.28
0.60	1.30	1.00	0.23	0.40	0.22
0.60	1.40	1.00	0.29	0.40	0.16
0.60	1.20	1.08	0.10	0.40	0.36
0.60	1.30	1.17	0.10	0.40	0.39
0.60	1.40	1.26	0.10	0.40	0.42
0.60	1.50	1.35	0.10	0.40	0.45
0.60	1.20	1.10	0.08	0.40	0.38
0.60	1.30	1.15	0.12	0.40	0.37
0.60	1.40	1.20	0.14	0.40	0.36
0.60	1.50	1.25	0.17	0.40	0.35

Table A.3: Illustrations of Condition for Dynamic Inefficiency

we see that a significant increase in markups μ results in a significant decrease of the right-hand side of equation (A.26). In the second panel instead, returns to scale adjust to keep the profit rate constant as μ increases. In this case, the right-hand side of equation (A.26) increases with μ . Finally, the last panel adjusts λ less than in the second panel so that profits rise. But as the table shows the right-hand side of equation (A.26) does not decline significantly because the rise in λ tends to offset the rise in μ .

In sum, increasing market power can lead to dynamic inefficiency only if it is associated with a significant increase in the economic profit share. If the rise of market power is accompanied with rising fixed costs, so that economic profits do not change much, then the analysis in Figure A.3 does not need to be modified significantly.

D Further Discussions

Discussion of the elasticity of substitution between capital and labor. Usual strategies to estimating the elasticity of substitution σ involve variants of our labor share solution (A.9) to project relative inputs or factor shares on input costs such as the wage and the cost of capital. Earlier, I explained the conceptual problems of using this equation for comparative statics. The same problems arise from an econometric perspective.

Chirinko (2008) summarizes the earlier estimates, which suggest an elasticity of substitution around 0.5. Several of these estimates appeal to within-country or within-industry variation in factor shares, relative inputs, and relative costs. These estimation techniques tend to identify short-term elasticities, which are expected to be lower than long-run elasticities once adjustments in output and input markets take place. They are prone to measurement errors, which attenuate the elasticity of substitution, because it is difficult to measure the marginal cost of labor and capital at high frequencies. Additionally, these techniques are plagued by endogeneity concerns and require strong assumptions about the time series behavior of markups μ , θ and factor-augmenting technologies A_k, A_ℓ .

At the micro level, Oberfield and Raval (2021) examine how establishment's wage bill to capital ratio varies with local wages to estimate micro-level elasticities between 0.3 and 0.7. The authors estimate an equation similar to (A.9), by projecting $w\ell/k$ on w. This approach amounts to delegating A_k , A_ℓ , θ , μ , and R into the error term. Yet, another alternative is to use the capital-to-output ratio that I solved for in equation (A.5), which again requires assumptions about labor market markups and factor-augmenting technologies. The problem of using wages as the right-hand side variable is that we expect them to comove closely with labor-augmenting technology. Thus, using variation in wages to estimate the elasticity of substitution is not a credible strategy.

More recent estimates sidestep some of these problems by looking at long-term and crosssectional variation in factor shares and the cost of capital. Karabarbounis and Neiman (2014b) estimate an elasticity of substitution of around 1.3 using an equation similar to (A.10) to project a function of the long-run change in labor share on the change in the relative price of investment, which does not require assumptions or measurements of wages and labor market markups. Further, we expect the cost of capital to move one-to-one with the relative price of investment in the long run, which makes the use of the relative price of investment a good proxy of the cost of capital. Hubmer (2023) exploits differential exposure across U.S. goods to the secular decline of the relative price of investment and estimates an elasticity of substitution of around 1.4. These approaches, however, still require assumptions on capital-augmenting technology and product market markups.

The impossibility of separating σ from A_k , A_ℓ without further structure is stressed by Diamond, McFadden, and Rodriguez (1978). Leon-Ledesma, McAdam, and Willman (2010) show that joint estimation using both the production function and the first-order conditions provides a useful alternative.

Houthakker (1955) famously demonstrated that the aggregation of micro-level Leontief technologies with independently distributed Pareto productivities leads to an aggregate Cobb-Douglas production function with a unitary elasticity of substitution. See Growiec (2013) for a microfoundation of aggregate production functions with an elasticity of substitution higher than one and Kaymak and Schott (Forthcoming) for industry elasticities higher than one despite unitary elasticities at the establishment level. Kehrig and Vincent (2021) document that within-establishment labor shares do not change much over time, which is consistent with unitary elasticities at the establishment level.

Discussion of the rise of markups. De Loecker, Eeckhout, and Unger (2020) apply the production approach pioneered by Hall (1988) to Compustat financial data. They infer markups as the ratio of the revenue elasticity of cost of goods sold to the revenue share of cost of goods sold (COGS). There are several concerns about their methodology, which infers an extremely large increase in markups.

- 1. Compustat represents only 30 percent of economic activity, and it is a selected sample of firms. So, it is not obvious how to generalize their results to the rest of the economy.
- 2. In Compustat, it is impossible to separate payments to flexible inputs from payments to inputs which require costs to adjust. The accounting item COGS does not reflect only payments to flexible inputs such as materials, as it also includes production and overhead labor costs. Conversely, as pointed out by Traina (2018), labor payments to non-production workers and some variable inputs are also included in the accounting item selling, general, and administrative (SG&A). There has been a shift of reported expenses over time away from COGS toward SG&A, because the ratio of sales to the sum of COGS and SG&A has been relatively constant over time. It is not obvious if this shift represents changes in market structure, changing interpretations of what financial accountants mean by production, an economic substitution of production activities performed by labor toward production activities performed by capital, or a rise in outsourcing, which could cause a reclassification of otherwise economically similar expenses. The shift away from COGS toward SG&A is observed in many other countries (Karabarbounis and Neiman, 2019).
- 3. The authors do not aggregate firm-level markups appropriately. To be precise, if one is interested in the aggregate markup and the aggregate labor share, then the weights that need to be applied are cost weights and not revenue weights for the case in which cost elasticities are equalized across firms (Edmond, Midrigan, and Xu, 2023). If cost elasticities are not equalized, one needs to use harmonic revenue-weighted markups (Hasenzagl and Perez, 2023). Applying either of these alternative weighting schemes changes dramatically the magnitude of the increase of markups.
- 4. Compustat does not include prices and quantities of goods, only firm sales. In a static profit maximization problem, the ratio of revenue elasticities to revenue costs for a flexible input, which is what De Loecker, Eeckhout, and Unger (2020) calculate, is identically equal to one and thus uninformative about markups (Bond, Hashemi, Kaplan, and Zoch, 2021). If inputs are distorted, as is likely with inputs included in COGS, then the ratio identifies distortions between the input price and the cost of hiring the input. To see this point, consider the cost minimization problem of a firm that uses a flexible input m and a distorted input ℓ : $\min_{m,\ell} C = qm + (1 + \tau)w\ell$ subject to $y(m,\ell) < \bar{y}$. The notion of

distortion here is that τ drives a wedge between the cost of hiring ℓ and the observed price of ℓ , which is just w. From the first-order conditions for cost minimization and the revenue function py, we obtain that $1 + \tau = [(\partial(py)/\partial \ell)/(py/\ell)]/[(w\ell)/(py)]$, which is the ratio of revenue elasticity to revenue share. If instead we use the flexible input, the ratio is identically one: $1 = [(\partial(py)/\partial m)/(py/m)]/[(qm)/(py)]$.

5. Echoing the arguments of Basu (2019), with constant returns to scale, moving from 20 percent to 60 percent markups implies an increase in the profit share from 17 percent to 38 percent for a representative firm. This is because, in the absence of labor market distortions, the profit share is equal to $s_{\pi} = 1 - \lambda/\mu$, where λ are returns to scale. Since the labor share is around 60 percent, opportunity costs must absorb almost none of income. To partly avoid this absurd prediction, one must argue that fixed costs have increased over time. While rising fixed costs are consistent with a reallocation of economic activity from low fixed cost and high marginal cost industries to high fixed cost and low marginal cost industries, the measured decreases in the labor share and rise of markups are mostly within-industry phenomena. Arguments that rely on fixed costs being large are inconsistent with research which indicates that the typical industry has constant or decreasing returns to scale (Basu and Fernald, 1997), and it is unclear what economic forces have caused fixed costs to increase by so much over time within industry or within firm.

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