

Forward Guidance and Durable Goods Demand

Online Appendix

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June 1, 2021

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A Derivation of Equation (11)

Starting with the smooth-pasting condition

$$\mathbb{E}_t \frac{d}{dt} V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) = \mathbb{E}_t \frac{d}{dt} V_t(a, d, z)$$

and substituting the evolution of the value function conditional on not adjusting yields,

$$\mathbb{E}_t \frac{d}{dt} V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) = \rho V_t(a, d) - u(c_t, d)$$

Using Ito's Lemma, we determine the evolution of the left-hand-side,

$$\begin{aligned} & V_{a,t}(a_t^*, d_t^*, z) [\dot{a} + (1-f)p_t \dot{d} - \dot{p}_t(d_t^* - (1-f)d) - p_t(\dot{d}_t^* + d_{m,t}^* \dot{m} + d_{z,t}^* \mathbb{E}_t \dot{z} + d_{zz,t}^* \frac{\sigma_z^2}{2})] \\ & + V_{d,t}(a_t^*, d_t^*, z) (\dot{d}_t^* + d_{m,t}^* \dot{m} + d_{z,t}^* \mathbb{E}_t \dot{z} + d_{zz,t}^* \frac{\sigma_z^2}{2}) + V_{z,t}(a_t^*, d_t^*, z) \mathbb{E}_t \dot{z} + V_{zz,t}(a_t^*, d_t^*, z) \frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) \\ & = \rho V_t(a, d, z) - u(c_t, d). \end{aligned}$$

A.1 LTV constraint not binding

If the household is not borrowing constrained in making a durable adjustment, then the terms involving the optimal choice of d^* drop out (envelope condition),

$$\begin{aligned} & V_{a,t}(a_t^*, d_t^*, z) [\dot{a} + (1-f)p_t \dot{d} - \dot{p}_t(d_t^* - (1-f)d)] + V_{z,t}(a_t^*, d_t^*, z) \mathbb{E}_t \dot{z} + V_{zz,t}(a_t^*, d_t^*, z) \frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) \\ & = \rho V_t(a, d, z) - u(c_t, d). \end{aligned}$$

Next, we substitute the HJB equation post-adjusting,

$$\begin{aligned} & V_{a,t}(a_t^*, d_t^*, z) [\dot{a} + (1-f)p_t \dot{d} - \dot{p}_t(d_t^* - (1-f)d) - \dot{a}_t^*] - V_{d,t}(a_t^*, d_t^*, z) \dot{d}_t^* + \rho V_t(a_t^*, d_t^*, z) - u(c_t^*, d_t^*) \\ & = \rho V_t(a, d, z) - u(c_t, d) + \theta [V_t^{adj}(a_t^* + (1-f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]. \end{aligned}$$

Using the value-matching condition, first-order condition for adjustment, and dividing by $V_{a,t}$ yields,

$$\begin{aligned} & \dot{a} + (1-f)p_t \dot{d} - \dot{p}_t(d_t^* - (1-f)d) - \dot{a}_t^* - p_t \dot{d}_t^* = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)} [u(c_t^*, d_t^*) - u(c_t, d)] \\ & + \frac{\theta}{V_{a,t}(a_t^*, d_t^*, z)} [V_t^{adj}(a_t^* + (1-f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)] \end{aligned}$$

Substituting the evolution of liquid assets and the durable stock yields,

$$(r_t p_t + \nu p_t + \delta p_t - \dot{p}_t) (d_t^* - d) + f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)d + (c_t^* - c_t) \quad (1)$$

$$= \frac{1}{V_{a,t}(a_t^*, d_t^*, z)} [u(c_t^*, d_t^*) - u(c_t, d)] + \frac{\theta}{V_{a,t}(a_t^*, d_t^*, z)} [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]$$

Finally, we plug in the definition of the contemporaneous user cost, $r_t^d = r_t p_t + \nu p_t + \delta p_t - \dot{p}_t$. This yields equation (11) for the unconstrained case, $p_t V_{a,t} = V_{d,t}$ and assuming $\theta = 0$.

A.2 LTV constraint binding

If the household is LTV-constrained, then $d_t^* = \frac{1}{1 - \lambda(1 - f)} \frac{m_t}{p_t}$, and the smooth pasting condition is

$$\mathbb{E}_t \frac{d}{dt} V_t(a - p_t(d_t^* - (1 - f)d), d_t^*, z) = \rho V_t(a, d) - u(c_t, d)$$

$$\mathbb{E}_t \frac{d}{dt} V_t(m_t - p_t d_t^*, d_t^*, z) = \rho V_t(a, d) - u(c_t, d)$$

$$\mathbb{E}_t \frac{d}{dt} V_t\left(-\frac{\lambda(1 - f)}{1 - \lambda(1 - f)} m_t, \frac{1}{1 - \lambda(1 - f)} \frac{m_t}{p_t}, z\right) = \rho V_t(a, d, z) - u(c_t, d).$$

Using Ito's Lemma,

$$-V_{a,t}(a_t^*, d_t^*, z) \frac{\lambda(1 - f)}{1 - \lambda(1 - f)} \dot{m}_t + \frac{1}{1 - \lambda(1 - f)} V_{d,t}(a_t^*, d_t^*, z) \frac{1}{p_t} (\dot{m}_t - m_t \frac{\dot{p}_t}{p_t})$$

$$+ V_{z,t}(a_t^*, d_t^*, z) \mathbb{E}_t \dot{z} + V_{zz,t}(a_t^*, d_t^*, z) \frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) = \rho V_t(a, d, z) - u(c_t, d)$$

In the instant after an adjustment takes place, the value function satisfies $u(c_t^*, d_t^*) + V_{a,t}(a_t^*, d_t^*, z) \dot{a}_t^* + V_{d,t}(a_t^*, d_t^*, z) \dot{d}_t^* + V_{z,t}(a_t^*, d_t^*, z) \mathbb{E}_t \dot{z} + V_{zz,t}(a_t^*, d_t^*, z) \frac{\sigma_z^2}{2} + \dot{V}_t(a_t^*, d_t^*, z) + \theta [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)] = \rho V_t(a_t^*, d_t^*, z)$. Substituting this into our previous equation yields,

$$V_{a,t}(a_t^*, d_t^*, z) \left[-\frac{\lambda(1 - f)}{1 - \lambda(1 - f)} \dot{m}_t - \dot{a}_t^* \right] + \frac{1}{1 - \lambda(1 - f)} V_{d,t}(a_t^*, d_t^*, z) \frac{1}{p_t} (\dot{m}_t - m_t \frac{\dot{p}_t}{p_t})$$

$$- V_{d,t}(a_t^*, d_t^*, z) \dot{d}_t^* - u(c_t^*, d_t^*) + \rho V_t(a_t^*, d_t^*, z) = \rho V_t(a, d, z) - u(c_t, d)$$

$$+ \theta [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]$$

Next we substitute the value-matching condition and $\dot{d}_t^* = -\delta(1-\chi)d_t^* = -\frac{\delta(1-\chi)}{1-\lambda(1-f)}\frac{m_t}{p_t}$ to further simplify,

$$\begin{aligned} V_{a,t}(a_t^*, d_t^*, z) & \left[-\frac{\lambda(1-f)}{1-\lambda(1-f)}\dot{m}_t - \dot{a}_t^* \right] + \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{m}_t + \delta(1-\chi)m_t - m_t\frac{\dot{p}_t}{p_t}) \\ & = u(c_t^*, d_t^*) - u(c_t, d) + \theta[V_t^{adj}(a_t^* + (1-f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)] \end{aligned}$$

The evolution of cash on hand conditional on not adjusting is given by,

$$\begin{aligned} \dot{m}_t & = \dot{a} + (1-f)p_t\dot{d} + (1-f)\dot{p}_t d_t \\ & = r_t a - (\nu + \chi\delta)p_t d - c_t + zy_t + (1-f)p_t\dot{d} + (1-f)\dot{p}_t d \\ & = r_t m_t - c_t + zy_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d, \end{aligned}$$

where we use y_t as compact notation for $(1-\tau_t)Y_t$. Since $a_t^* = -\frac{\lambda(1-f)}{1-\lambda(1-f)}m_t$ and $d_t^* = \frac{1}{1-\lambda(1-f)}\frac{m_t}{p_t}$, we then get

$$\begin{aligned} V_{a,t}(a_t^*, d_t^*, z) & \left[-\frac{\lambda(1-f)}{1-\lambda(1-f)}\{-c_t + zy_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d\} \right. \\ & \left. - \{-c_t^* + zy_t - (\nu + \chi\delta)p_t d_t^*\} \right] \\ & + \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{m}_t + \delta(1-\chi)m_t - m_t\frac{\dot{p}_t}{p_t}) \\ & = u(c_t^*, d_t^*) - u(c_t, d) + \theta[V_t^{adj}(a_t^* + (1-f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)] \end{aligned}$$

Next we distribute terms into distinct benefits and costs of adjusting,

$$\begin{aligned} V_{a,t}(a_t^*, d_t^*, z) & \left\{ -[r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d + (\nu + \chi\delta)p_t d_t^* \right\} \\ & + \frac{1}{1-\lambda(1-f)}V_{a,t}(a_t^*, d_t^*, z)\left\{ -zy_t + c_t + [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1-\chi)p_t - \dot{p}_t)]d \right\} \\ & + V_{a,t}(a_t^*, d_t^*, z)(c_t^* - c_t) \\ & + \frac{1}{1-\lambda(1-f)}V_{d,t}(a_t^*, d_t^*, z)\frac{1}{p_t}(\dot{m}_t + \delta(1-\chi)m_t - m_t\frac{\dot{p}_t}{p_t}) \\ & = u(c_t^*, d_t^*) - u(c_t, d) + \theta[V_t^{adj}(a_t^* + (1-f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)] \end{aligned}$$

Substituting the evolution of cash-on-hand,

$$\begin{aligned}
& V_{a,t}(a_t^*, d_t^*, z) \left\{ -[r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)]d + (\nu + \chi\delta)p_t d_t^* \right\} \\
& + \frac{1}{1 - \lambda(1 - f)} V_{a,t}(a_t^*, d_t^*, z) \left\{ -zy_t + c_t + [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)]d \right\} \\
& + V_{a,t}(a_t^*, d_t^*, z)(c_t^* - c_t) \\
& + \frac{1}{1 - \lambda(1 - f)} V_{d,t}(a_t^*, d_t^*, z) \frac{1}{p_t} \left([r_t + \delta(1 - \chi) - \frac{\dot{p}_t}{p_t}] m_t - c_t + zy_t - [r_t p_t + \nu + \delta - \dot{p}_t - f(r_t p_t + \delta(1 - \chi) - \dot{p}_t)]d \right) \\
& = u(c_t^*, d_t^*) - u(c_t, d) + \theta [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]
\end{aligned}$$

Collecting terms again,

$$\begin{aligned}
& \left(\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t} d_t^* - V_{a,t}(a_t^*, d_t^*, z)d \right) (r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) \\
& + V_{a,t}(a_t^*, d_t^*, z) \left\{ f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)d + (\nu + \chi\delta)p_t(d_t^* - d) \right\} \\
& + \frac{1}{1 - \lambda(1 - f)} \left[V_{a,t}(a_t^*, d_t^*, z) - \frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t} \right] \left\{ -zy_t + c_t + [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)]d \right\} \\
& + V_{a,t}(a_t^*, d_t^*, z)(c_t^* - c_t) \\
& = u(c_t^*, d_t^*) - u(c_t, d) + \theta [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]
\end{aligned}$$

Divide by the post-adjustment marginal utility of wealth $V_{a,t}(a_t^*, d_t^*, z)$

$$\begin{aligned}
& \left(\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} d_t^* - d \right) (r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) \\
& + f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)d + (\nu + \chi\delta)p_t(d_t^* - d) \\
& + \frac{1}{1 - \lambda(1 - f)} \left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1 \right] \left\{ zy_t - c_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1 - \chi) - \dot{p}_t)]d \right\} \\
& + (c_t^* - c_t) \\
& = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)} [u(c_t^*, d_t^*) - u(c_t, d)] + \frac{\theta}{V_{a,t}(a_t^*, d_t^*, z)} [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)].
\end{aligned}$$

Next we separate the first term into a component that is present for all household and

one that is only present for constrained households,

$$\begin{aligned}
& (r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) (d_t^* - d) + \left(\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1 \right) (r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) d_t^* \\
& + f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) d + (\nu + \chi\delta)p_t (d_t^* - d) \\
& + \frac{1}{1 - \lambda(1 - f)} \left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1 \right] \{ z y_t - c_t - [r_t p_t + \nu p_t + \delta p_t - \dot{p}_t - f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t)] d \} \\
& + (c_t^* - c_t) \\
& = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)} [u(c_t^*, d_t^*) - u(c_t, d)] + \frac{\theta}{V_{a,t}(a_t^*, d_t^*, z)} [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)],
\end{aligned}$$

which we can then combine with the other term affecting constrained households only,

$$\begin{aligned}
& (r_t p_t + \nu p_t + \delta p_t - \dot{p}_t) (d_t^* - d) + f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) d + (c_t^* - c_t) \\
& + \frac{1}{1 - \lambda(1 - f)} \left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1 \right] \left\{ \frac{m_t - (1 - f)p_t d_t}{p_t} (r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) \right. \\
& \left. + z y_t - c_t - (\nu + \delta\chi)p_t d \right\} \\
& = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)} [u(c_t^*, d_t^*) - u(c_t, d)] + \frac{\theta}{V_{a,t}(a_t^*, d_t^*, z)} [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]
\end{aligned}$$

Last, we substitute out cash on hand for liquid assets,

$$\begin{aligned}
& (r_t p_t + \nu p_t + \delta p_t - \dot{p}_t) (d_t^* - d) + f(r_t p_t + \delta(1 - \chi)p_t - \dot{p}_t) d + (c_t^* - c_t) \\
& + \frac{1}{1 - \lambda(1 - f)} \left[\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1 \right] \left\{ \dot{a}_t + a_t \left(\delta(1 - \chi) - \frac{\dot{p}_t}{p_t} \right) \right\} \\
& = \frac{1}{V_{a,t}(a_t^*, d_t^*, z)} [u(c_t^*, d_t^*) - u(c_t, d)] + \frac{\theta}{V_{a,t}(a_t^*, d_t^*, z)} [V_t^{adj}(a_t^* + (1 - f)p_t d_t^*, z) - V_t(a_t^*, d_t^*, z)]
\end{aligned}$$

If the household is not borrowing constrained, then $\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} = 1$ and this first order condition coincides with our earlier derivation (1). Thus our derivation given the borrowing constrained nests the unconstrained optimality condition as a special case.

To obtain equation (11), we plug in the definition of the contemporaneous user cost (9) $r_t^d = r_t p_t + \nu p_t + \delta p_t - \dot{p}_t$, the evolution of liquid assets (4), and set $\theta = 0$.

B Derivation of Equation (13)

Assume that an adjustment is optimal today. Then the integrated HJB equation (10) is

$$V_t^{adj}(x, z) = \max_{\{c_{t+s}\}, \tau, d} E \left\{ \int_0^\tau e^{-\rho s} [u(c_{t+s}, e^{-\delta(1-\chi)s} d)] ds + e^{-\rho\tau} V_{t+\tau}^{adj}(a_{t+\tau} + p_{t+\tau}(1-f)e^{-\delta(1-\chi)\tau} d, z_{t+\tau}) \right\}$$

subject to the borrowing constraint (3), where τ is the optimal stopping time. If between t and $t+s$ no further adjustment takes place, then liquid assets accumulate as

$$a_{t+s} = (x - p_t d) e^{\int_0^s r_{t+u} du} + \int_0^s e^{\int_k^s r_{t+u} du} [y_{t+k} - c_{t+k} - (\nu + \delta\chi) p_{t+k} e^{-\delta(1-\chi)k} d] dk.$$

which we substitute into the integrated HJB above equation and the borrowing constraint below,

$$a_{t+s} \geq -\lambda(1-f)e^{-\delta(1-\chi)s} p_{t+s} d$$

Letting the Lagrange multiplier on the borrowing constraint be Ψ_{t+s} , then we can rewrite value function as

$$\begin{aligned} V_t^{adj}(x, z) = & \max_{\{c_{t+s}\}, \tau, d} \mathbb{E}_t \left\{ \int_0^\tau e^{-\rho s} [u(c_{t+s}, e^{-\delta(1-\chi)s} d)] ds + e^{-\rho\tau} V_{t+\tau}^{adj} \left((x - p_t d) e^{\int_0^\tau r_{t+u} du} + \right. \right. \\ & + \left. \int_0^\tau e^{\int_s^\tau r_{t+u} du} [y_{t+s} - c_{t+s} - (\nu + \delta\chi) p_{t+s} e^{-\delta(1-\chi)s} d] ds + p_{t+\tau}(1-f)e^{-\delta(1-\chi)\tau} d, z_{t+\tau} \right) + \\ & + \mathbb{E}_t \int_0^\tau e^{-\rho s} \Psi_{t+s} \left[(x - p_t d) e^{\int_0^s r_{t+u} du} + \int_0^s e^{\int_k^s r_{t+u} du} [y_{t+k} - c_{t+k} - (\nu + \delta\chi) p_{t+k} e^{-\delta(1-\chi)k} d] dk \right. \\ & \left. + \lambda(1-f)e^{-\delta(1-\chi)k} p_{t+s} d \right] ds \end{aligned}$$

The first order condition for the durable stock is,

$$\begin{aligned} \mathbb{E}_t \int_0^\tau e^{-(\rho+\delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s} d) ds = & \\ & + \mathbb{E}_t e^{-\rho\tau} V_{x,t+\tau}^{adj} \left[p_t e^{\int_0^\tau r_{t+u} du} + (\nu + \delta\chi) \int_0^\tau e^{\int_k^\tau r_{t+u} du - \delta(1-\chi)k} p_{t+k} dk - (1-f)e^{-\delta(1-\chi)\tau} p_{t+\tau} \right] \\ & + \mathbb{E}_t \int_0^\tau e^{-\rho s} \Psi_{t+s} \left[p_t e^{\int_0^s r_{t+u} du} + (\nu + \delta\chi) \int_0^s e^{\int_k^s r_{t+u} du - \delta(1-\chi)k} p_{t+k} dk - \lambda(1-f)e^{-\delta(1-\chi)s} p_{t+s} \right] ds \end{aligned}$$

Substituting the definition of the cumulative user cost $r_{t,t+s}^d$ yields the equation (13) in the text.

C Net Benefit of Adjusting

Appendix Figures A.1-A.3 show that the patterns in Figure 3 also hold for different levels of liquid assets and income. In each case we plot a different slice of the durable stock near the lower adjustment threshold, which corresponds to an upward adjustment of the durable stock.

D Forward Guidance and Long-term Debt

Here we describe an extension of the model with long-term debt. After describing the environment, we show that the model with long-term debt yields identical decision rules to the model with only short-term debt conditional on the initial state variables (a_{i0}, d_{i0}, z_{i0}) . Any valuation effects from long-term debt are captured by the distribution over the initial states (a_{i0}, d_{i0}, z_{i0}) . In Section D.2 we quantify the importance of valuation effects of long-term debt relative to the model with only short-term debt.

The long-term bond trades at a price q_t . We can allow for an arbitrary sequence of coupon payments so long as the coupon payments on each bond are known and not idiosyncratic. In contrast to a mortgage, our setup does not allow for the option to prepay the loan at face value. Omitting the prepayment option allows us to focus on the role of financing duration.

No-arbitrage implies that all assets must pay the same return on a perfect foresight path. Therefore, the return on the long-term bond r_t^b is equal to the short-term interest rate, $r_{t+k}^b = r_{t+k}$ for $k \geq 0$.

D.1 Equivalence Result with Short-term Debt Model

We redefine a_{it} as the total value of liquid assets including holdings of short- and long-term bonds. As all assets pay the same return along a perfect foresight path, the return on total liquid assets is equal to the short-rate r_t irrespective of the portfolio weights on short-term and long-term bonds.

To prove the equivalence with the short-term debt model, we show that the household in the long-term debt model faces exactly the same constraints as the household in short-term

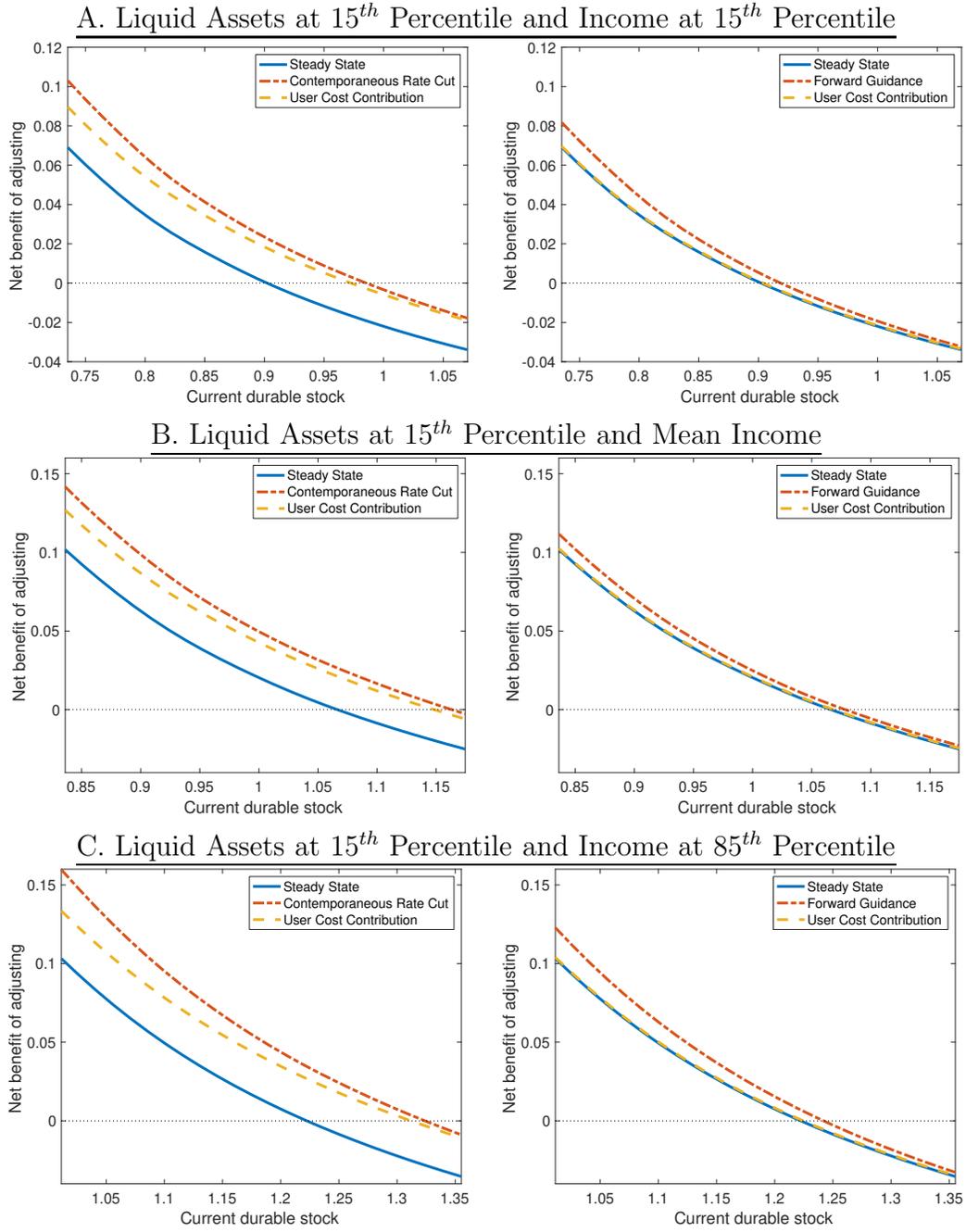
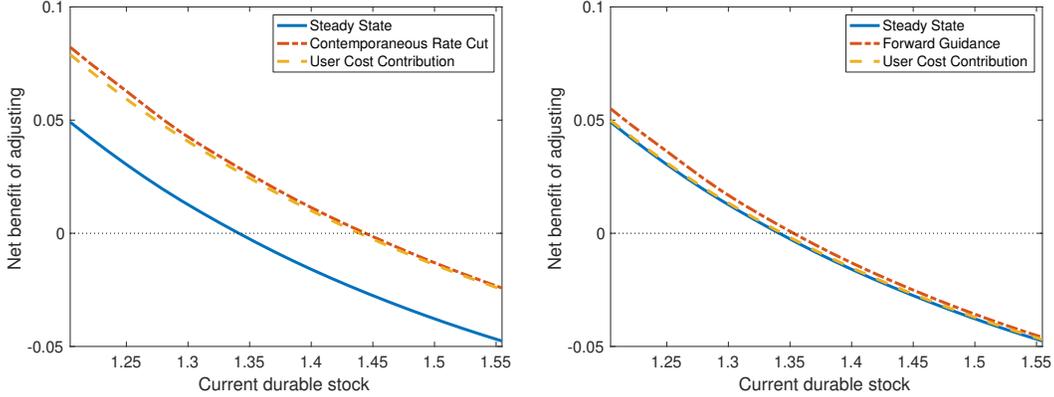
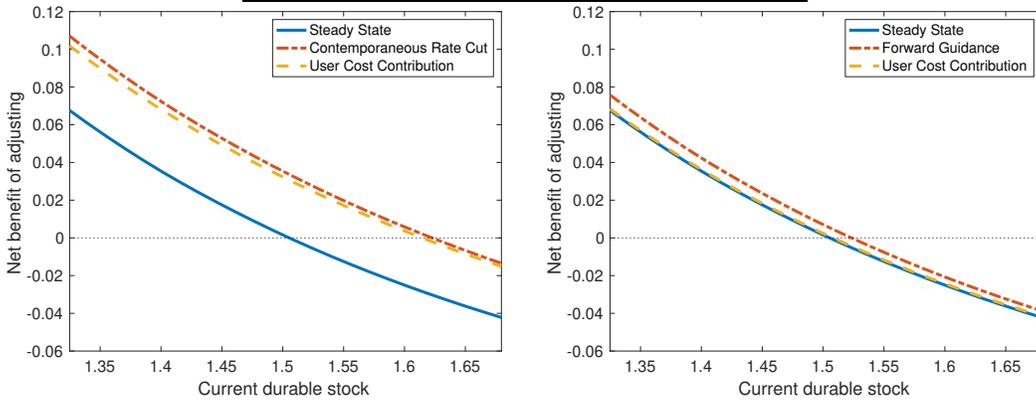


Figure A.1: The net benefit of adjusting now rather than waiting for an instant for different levels of the durable stock. Liquid assets equal to the 15th percentile of the steady state distribution. Income equal to the 15th percentile, mean, and 85th percentile of the steady state distribution. The net benefit is the left-hand side minus the right-hand side of equation (11). The left column shows the change in net benefit after a contemporaneous 10% real interest rate cut that lasts for one quarter, and the contribution of the contemporaneous user cost. The right column shows the change in net benefit after an announced 10% real interest rate cut in one year, and the contribution of the contemporaneous user cost.

A. Mean Liquid Assets and Income at 15th Percentile



B. Mean Liquid Assets and Mean Income



C. Mean Liquid Assets and Income at 85th Percentile

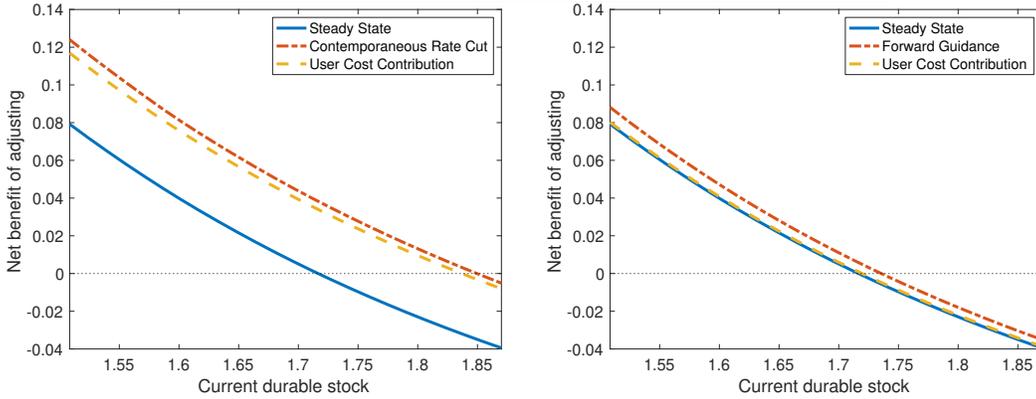


Figure A.2: The net benefit of adjusting now rather than waiting for an instant for different levels of the durable stock. Liquid assets equal to the mean of the steady state distribution. Income equal to the 15th percentile, the mean, and the 85th percentile of the steady state distribution. The net benefit is the left-hand side minus the right-hand side of equation (11). The left column shows the change in net benefit after a contemporaneous 10% real interest rate cut that lasts for one quarter, and the contribution of the contemporaneous user cost. The right column shows the change in net benefit after an announced 10% real interest rate cut in one year, and the contribution of the contemporaneous user cost.

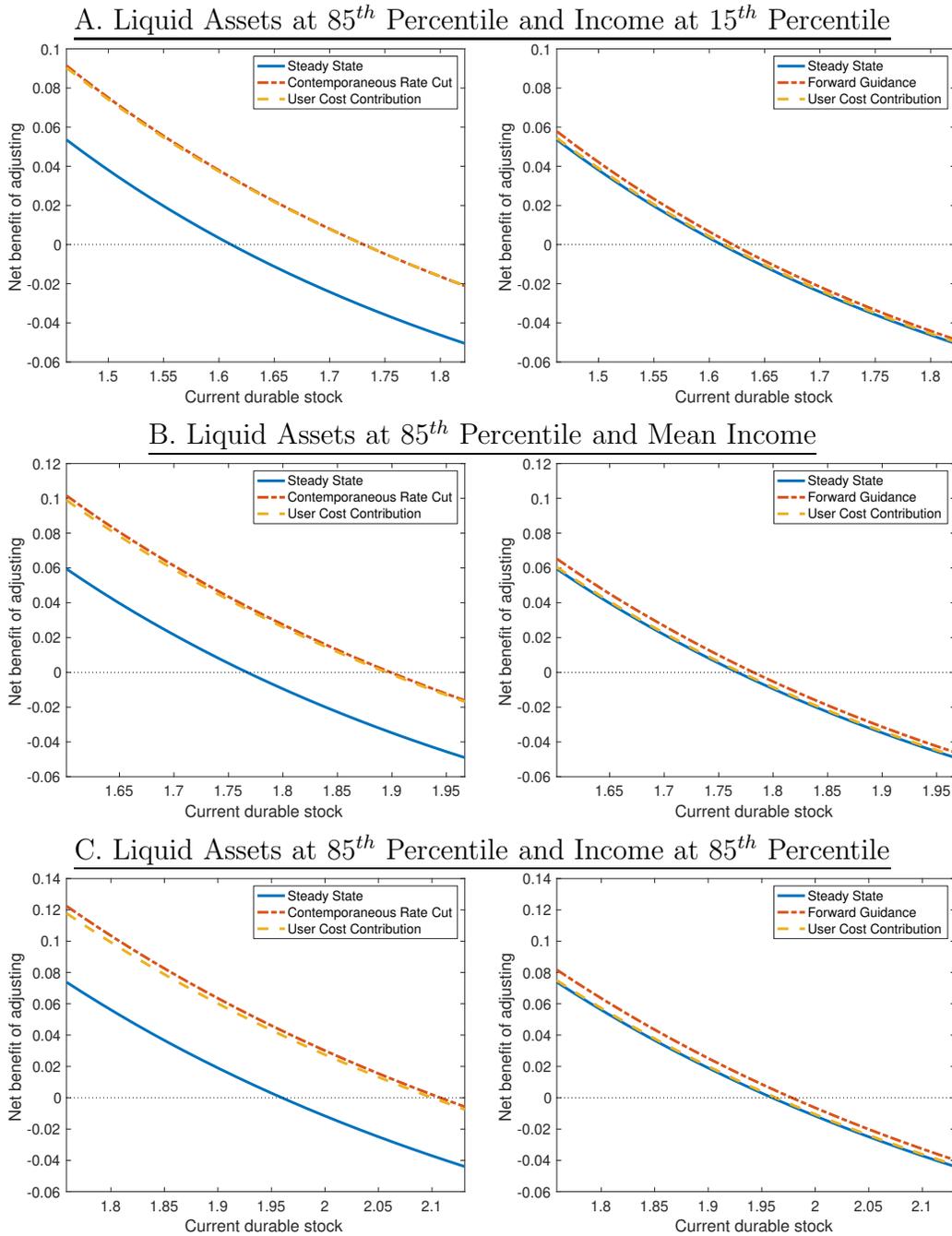


Figure A.3: The net benefit of adjusting now rather than waiting for an instant for different levels of the durable stock. Liquid assets equal to the 85th percentile of the steady state distribution. Income equal to the 15th percentile, the mean, and the 85th percentile of the steady state distribution. The net benefit is the left-hand side minus the right-hand side of equation (11). The left column shows the change in net benefit after a contemporaneous 10% real interest rate cut that lasts for one quarter, and the contribution of the contemporaneous user cost. The right column shows the change in net benefit after an announced 10% real interest rate cut in one year, and the contribution of the contemporaneous user cost.

debt model of section I conditional on the stock of liquid assets a_{i0} , the current durable stock d_{i0} , and the current level of productivity z_{i0} . These constraints are the budget constraint conditional on adjusting (1), the durable accumulation equation (2), the borrowing constraint (3), the budget constraint conditional on not adjusting (4), and the evolution of productivity (5). As the objective function is unchanged, once we have shown that the constraints are equivalent in the two models it follows that households will make the same optimal decisions given the same initial state variables.

The durable accumulation equation (2) and the evolution of productivity (5) are independent of the specification of assets. We next prove that the budget constraint conditional on adjusting (1) and the borrowing constraint (3) are identical. Let b_{it} be the holdings of the long-term bond and \tilde{a}_{it} be holdings of short-term assets. The definition of liquid assets a_{it} is then $a_{it} = \tilde{a}_{it} + q_t b_{it}$. When a household adjusts its durables it choose a new portfolio $(\tilde{a}'_{it}, b'_{it}, d'_{it})$ subject to

$$\tilde{a}'_{it} + q_t b'_{it} + p_t d_{it} = \tilde{a}_{it} + q_t b_{it} + (1 - f)p_t d_{it}.$$

Substituting the definition of a_{it} on both sides yields the same constraint as (1). Turning to the LTV constraint, we assume the borrowing limit applies to the total financial position

$$\tilde{a}_{it} + q_t b_{it} \geq -\lambda(1 - f)p_t d_{it}.$$

Substituting the definition of a_{it} yields (3).

It remains to show that the budget constraint conditional on not adjusting (4) is the same. Due to no-arbitrage, the total return on the household's financial assets does not depend on the composition of their portfolio between short- and long-term bonds.

Absent a durable adjustment, the evolution of total liquid assets is then

$$\begin{aligned} \dot{a}_{it} &= r_t \tilde{a}_{it} + r_t^b q_t b_{it} - (\nu + \chi\delta)p_t d_{it} - c_{it} + zy_{it} \\ &= r_t a_{it} - (\nu + \chi\delta)p_t d_{it} - c_{it} + zy_{it}, \end{aligned} \tag{2}$$

where the second line uses $r_t^b = r_t$ and the definition of a_{it} . Equation (2) is identical to (4) without a borrowing spread, $r^s = 0$, but the argument extends to positive spreads as well (see below). To sum up, the constraints (1)-(5) are the same in the models with and without long-term debt leading to identical policy rules.

D.2 Quantitative Model

While the partial equilibrium decision problem is unaffected by long-term debt, the equilibrium of the economy will reflect a valuation effect on a_{i0} as the asset price q_0 jumps upon news of the real interest rate path. Moreover, the government budget constraint is similarly affected by valuation effects yielding a different path for taxes. We now quantify the importance of these valuation effects and show that they slightly reduce the power of contemporaneous interest rates but overall our results are little changed.

We assume that household portfolios consist entirely of long-term debt. The total value of assets for each household is then $a_{it} = q_t b_{it}$. Like Farhi and Werning (2019) we then introduce short-term debt at the margin and make sure that households are not better off by including it in their portfolio. This implies that the return on both assets must be equalized, $r_t = r_t^b$. We assume that borrowing through the long-term bond incurs an intermediation fee r^s proportional to value of the debt so the cost of borrowing through the long-term asset is $r_t^b + r^s$. The budget constraint conditional on not adjusting then evolves as in the baseline model (equation (4)).

For this quantitative exercise, we model the long-duration bond as a perpetuity that pays exponentially declining coupons as in Hatchondo and Martinez (2009). Each unit of bonds pays a flow coupon ϕ with the quantity of bonds amortizing at rate Γ . The instantaneous return on the bond is

$$r_t^b \equiv \frac{\dot{q}_t + \phi}{q_t} - \Gamma. \quad (3)$$

We normalize dividend payments $\phi = r + \Gamma$ such that the steady state price of debt is $q = \frac{\phi}{r + \Gamma} = 1$. The valuation effect on assets at time 0 is then $a_{i0} = q_0 b_{i0}$ with b_{i0} given and the path for q_t determined by the no-arbitrage equation

$$r_t = \frac{\dot{q}_t + \nu}{q_t} - \Gamma \equiv r_t^b.$$

A technical issue with the quantitative model is that the valuation effects can cause households to immediately violate the borrowing constraint. To ensure this does not happen, we modify the constraint to apply to the number of long-term bonds outstanding rather than

their value,

$$b_{it} \geq -\lambda p_{it} d_{it}.$$

Thus, a household that is initially at the borrowing constraint with $b_{i0} = -\lambda p_{it} d_{it}$ will continue to satisfy it after the valuation effects take place.

The government maintains a constant quantity of debt \bar{B} . This implies that there are no discontinuous changes in tax policy from valuation effects. As in our baseline model, the government balances its budget. This requires raising taxes to finance dividend payments $\phi \bar{B}$ net of debt issuance $\Gamma q_t \bar{B}$ each instant. Thus, the aggregate tax rate is

$$\tau_t = \frac{(\phi - \Gamma q_t) \bar{B}}{Y_t}.$$

Relative to our baseline model, there is one additional parameter Γ governing the duration of the long-term asset (or debt). Setting the duration to $\Gamma^{-1} \rightarrow 0$ yields the baseline model as a special case. Doepke and Schneider (2006) calculate the maturity of assets held by the household sector to be approximately 4.5 years so we set $\Gamma^{-1} = 4.5$ (see their Figure 3).

Figure A.4 compares the effectiveness of forward guidance in the model with long-term debt with our baseline model. The output responses are very similar and contemporaneous interest rate reductions remain substantially more powerful at stimulating contemporaneous output than are expected future interest rate reductions. There are, however, some small difference in the results. First, contemporaneous interest rates are slightly less powerful in the long-term debt model. A lower real rate increases the asset price q_0 , which redistributes from debtors to creditors and partially offsets the redistribution from creditors to debtors from lower interest rate payments (Auclert, 2019). The asset price q_0 responds more strongly for more immediate interest rate reductions. Thus, contemporaneous interest rate changes lead to a larger redistribution from debtors to creditors than do future changes. This depresses the expansionary effects of contemporaneous interest rate changes relatively more than forward guidance.

Second, forward guidance is slightly more powerful with long-term debt. With long-term debt, τ_0 falls in response to future interest rate cuts because the revenue the government raises from issuing a unit of bond rises. In contrast, taxes react only to contemporaneous

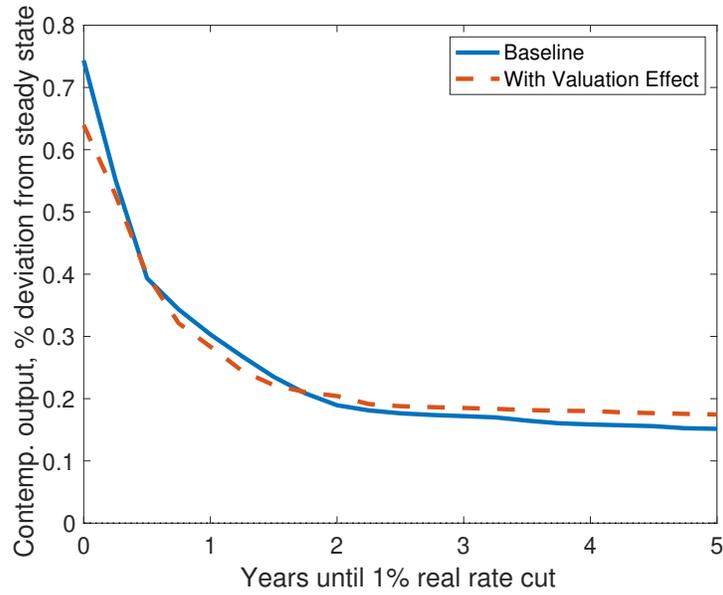


Figure A.4: Contemporaneous output response to promises of interest rate cuts at different horizons in the baseline model with short-term debt (solid blue line) and the model with long-term debt (dashed red line). At each horizon the real interest rate drops by 1 percentage point for one quarter.

interest rate changes with short-term debt. The reduction in τ_0 in response to future interest rate changes makes forward guidance slightly more powerful.

References

- AUCLERT, A. (2019): “Monetary policy and the redistribution channel,” *American Economic Review*, 109, 2333–67.
- DOEPKE, M. AND M. SCHNEIDER (2006): “Inflation and the redistribution of nominal wealth,” *Journal of Political Economy*, 114, 1069–1097.
- FARHI, E. AND I. WERNING (2019): “Monetary policy, bounded rationality, and incomplete markets,” *American Economic Review*, 109, 3887–3928.
- HATCHONDO, J. C. AND L. MARTINEZ (2009): “Long-duration bonds and sovereign defaults,” *Journal of International Economics*, 79, 117 – 125.