# Online Appendix: <br> Invisible Geniuses: Could the Knowledge Frontier Advance Faster? by Ruchir Agarwal and Patrick Gaule 

March 1, 2020

## A Is there a causal effect of medals on long-term performance?

In the main text, we document a link between IMO scores and long-term performance. Could this be because earning a high score at the IMO boosts one's self-confidence in mathematics or facilitates access to better schools? Could IMO scores - by themselves - have a causal effect on long-term performance? If scoring well at the IMO generates a success-begets-success dynamic, IMO scores could affect long term performance even if talent is not relevant to the production of knowledge.

While we cannot directly test for a causal effect of scores, we can investigate whether IMO medals may have a causal effect on performance. At the IMO, medals are awarded based on explicit cutoffs in the IMO score. For example, in the year 2000, all participants who scored 30 points and above received a gold medal, those scoring between 21 and 29 received a silver medal, and those scoring between 11 and 20 received a bronze medal. To the extent that medals play a useful role in summarizing and communicating IMO performance to outsiders - as one might expect given that IMO medals are frequently mentioned on CVs and LinkedIn profiles, whereas raw IMO scores are not - the causal effect of IMO medals may be informative about the causal effect of IMO performance more generally.

The IMO medal allocation mechanism is a natural setting for a regression discontinuity (RD) design comparing those who just made the medal threshold (or threshold for a better medal) versus those who nearly missed it. The assignment variable here is the number of points scored which solely determines medal awards. Importantly, the number of points scored cannot be precisely manipulated by participants, and in any case the medal thresholds are not known when the participants solve the problems. ${ }^{1}$ Moreover, since the thresholds are different each year, these results are

[^0]likely to be robust to any sharp non-linearities in the function linking IMO score and performance.

We have three different time-varying thresholds corresponding to gold, silver and bronze medals. To maximize power, we pool observations across the three thresholds and analyze data at the individual IMO participant-threshold level. Specifically, we generate three copies of the data corresponding to each of the three medals thresholds, and express the IMO score as a distance to the respective threshold. The effect of being above the threshold is thus a weighted average of the effect of being above the gold threshold, being above the silver threshold and being above the bronze threshold. For each outcome variable, we select a set of individuals narrowly above and below the cutoff using the optimal bandwidth selector of Calonico, Cattaneo and Titiunik (2014). ${ }^{2}$

## (insert Table A4)

The results are presented in Table A4, while a graphical version is displayed in appendix Figure A2. We also report the effect of crossing the threshold for a bronze medal (table A5) or for a gold medal (table A6). The point estimates generally very imprecisely estimated but the evidence suggests that, controlling for score, being awarded a better medal appears to have no additional impact on becoming a professional mathematician or future knowledge production.

[^1]
## Appendix tables

Table A1: Summary statistics on IMO participants (1981-2000)

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| :--- | :--- | :--- | :--- | :--- | :--- |
| IMO Score | 4,710 | 16.0 | 11.3 | 0 | 42 |
| Gold Medal | 4,710 | 0.08 | 0.27 | 0 | 1 |
| Silver Medal | 4,710 | 0.16 | 0.37 | 0 | 1 |
| Bronze Medal | 4,710 | 0.24 | 0.43 | 0 | 1 |
| Honourable Mention | 4,710 | 0.10 | 0.30 | 0 | 1 |
| Olympiad Year | 4,710 | 1992.4 | 5.5 | 1981 | 2000 |
| Math PhD | 4,710 | 0.22 | 0.41 | 0 | 1 |
| Math PhD (top 10) | 4,710 | 0.07 | 0.25 | 0 | 1 |
| Pubs | 4,710 | 3.3 | 11.6 | 0 | 264 |
| Cites | 4,710 | 34.6 | 221.2 | 0 | 11,062 |
| ICM speaker | 4,710 | 0.01 | 0.09 | 0 | 1 |
| Fields medalist | 4,710 | 0.001 | 0.04 | 0 | 1 |
| High-income country | 4,710 | 0.48 | 0.50 | 0 | 1 |
| Upper middle-income country | 4,710 | 0.18 | 0.38 | 0 | 1 |
| Lower middle-income country | 4,710 | 0.23 | 0.42 | 0 | 1 |
| Low-income country | 4,710 | 0.11 | 0.31 | 0 | 1 |

Notes: The table displays descriptive statistics on the sample of all individuals who participated in any IMO from 1981 to 2000. IMO medals are based on the number of points scored (IMO score). Multiple gold, silver and bronze medals are awarded at every IMO. Math PhD is based on the Mathematics Genealogy Project. Math PhD (top 10) is based on the list of the 10 top schools listed in appendix table ??. Publication and cites are from MathSciNet. ICM speaker stands for speaker at the International Congress of Mathematicians. Country income groups are based on the 2000 World Bank classification.

Table A2: IMO scores and subsequent achievements

|  | $(1)$ <br> Math PhD | $(2)$ <br> Math PhD <br> (top 10) | $(3)$ <br> Pubs <br> $(\log )$ | $(4)$ <br> Cites <br> $(\log )$ | $(5)$ <br> ICM <br> speaker | $(6)$ <br> Field <br> medalist |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| IMO Score | 0.0101 | 0.0054 | 0.0261 | 0.0434 | 0.0012 | 0.0003 |
|  | $(0.0008)$ | $(0.0006)$ | $(0.0021)$ |  |  |  |
| Cohort FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Country FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 4,710 | 4,710 | 4,710 | 4,710 | 4,710 | 4,710 |
| Adjusted R2 | 0.1463 | 0.1094 | 0.1400 | 0.1465 | 0.0202 | 0.0012 |
| Mean of D.V. | 0.219 | 0.068 | 0.431 | 0.713 | 0.009 | 0.002 |

Notes: These regressions are run on the sample of all IMO participants who competed at any point between 1981 and 2000. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the $\log$ of mathematics publications plus one (column 3), the $\log$ of mathematics cites plus one (column 4), becoming an ICM speaker at the ICM Congress (column 5), becoming a Fields medalist (column 6). The variable of interest is the number of points scored controlling for cohort (olympiad year) fixed effects and country fixed effects. All regressions are estimated by OLS. Robust standard errors in parentheses.

Table A3: IMO scores on less and more difficult problems and subsequent achievements

|  | $(1)$ <br> Math PhD | $(2)$ <br> Math PhD <br> (top 10) | $(3)$ <br> Pubs <br> $(\log )$ | $(4)$ <br> Cites <br> $(\log )$ | $(5)$ <br> ICM <br> speaker | $(6)$ <br> Field <br> medalist |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Score on less | 0.0087 | 0.0044 | 0.0346 | 0.0208 | 0.0010 | 0.0002 |
| difficult problems | $(0.0011)$ | $(0.0007)$ | $(0.0044)$ | $(0.0027)$ | $(0.0003)$ | $(0.0001)$ |
| Score on more | 0.0129 | 0.0077 | 0.0616 | 0.0375 | 0.0019 | 0.0007 |
| difficult problems | $(0.0022)$ | $(0.0015)$ | $(0.0095)$ | $(0.0059)$ | $(0.0006)$ | $(0.0003)$ |
| Olympiad Year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Country FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 4,491 | 4,491 | 4,491 | 4,491 | 4,491 | 4,491 |
| Adjusted R2 | 0.1456 | 0.1112 | 0.1495 | 0.1428 | 0.0250 | 0.0041 |

Notes: These regressions are similar to those of table A2 but distinguish between the number of points scored on the less difficult problems ( $1,2,4$ and 5 ) and those scored on the more difficult problems ( 3 and 6 ); we do not have the score breakdown for the 1981 and 1983, hence the number of observations is slightly lower than in table A2. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log of mathematics publications plus one (column 3), the log of mathematics cites plus one (column 4), becoming an ICM speaker at the ICM Congress (column 5), becoming a Fields medalist (column 6). All regression are estimated by OLS. Robust standard errors in parentheses.

Table A4: Regression discontinuity estimates of the effect of (better) medals

|  | $(1)$ <br> Math PhD | $(2)$ <br> Math PhD <br> (top 10) | $(3)$ <br> Pubs <br> $(\mathrm{log})$ | $(4)$ <br> Cites <br> $(\log )$ |
| :--- | :---: | :---: | :---: | :---: |
| Above (better) | 0.0138 | 0.0147 | 0.0117 | 0.0114 |
| medal threshold | $(0.0250)$ | $(0.0142)$ | $(0.0719)$ | $(0.1043)$ |
| Distance from | 0.0098 | 0.0038 | 0.0202 | 0.0385 |
| threshold | $(0.0032)$ | $(0.0015)$ | $(0.0104)$ | $(0.0133)$ |
| Distance from | -0.0031 | 0.0011 | 0.0085 | 0.0039 |
| threshold X above threshold | $(0.0035)$ | $(0.0018)$ | $(0.0129)$ | $(0.0146)$ |
| Country FE | Yes | Yes | Yes | Yes |
| Cohort (Olympiad Year) FE | Yes | Yes | Yes | Yes |
| Threshold FE | Yes | Yes | Yes | Yes |
| Bandwidth | $[-9 ; 9]$ | $[-10 ; 10]$ | $[-8 ; 8]$ | $[-9 ; 9]$ |
| Observations | 5,208 | 5,775 | 4,564 | 5,208 |
| Mean of D.V. | 0.2757 | 0.0868 | 0.5680 | 0.9169 |

Notes: The IMO medals (gold, silver and bronze) are allocated solely based on the number of points scored at the IMO, with the medal thresholds varying from year to year. To maximize power, we pool observations across the three thresholds and analyze data at the individual IMO participant-threshold level. Specifically, we generate three copies of the data corresponding to each of the three medals thresholds, and express the IMO score as a distance to the respective threshold. The effect of being above the threshold is thus a weighted average of the effect of being above the gold threshold, being above the silver threshold and being above the bronze threshold. For each outcome variable, we select a set of individuals narrowly above and below the cutoff using the optimal bandwidth selector of Calonico, Cattaneo \& Titiunik (2014). Because the optimal bandwidth depends on the dependent variable, the number of observations varies across specifications. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log of mathematics publications plus (column 3), the log mathematics cites plus one (column 4). All regressions are estimated by OLS and include country fixed effect, cohort (olympiad year) fixed effects and threshold fixed effects. Standard errors clustered by olympiad participant in parentheses.

Table A5: Regression discontinuity estimates of the effect of getting a bronze medal

|  | $(1)$ <br> Math PhD | $(2)$ <br> Math PhD <br> (top 10) | $(3)$ <br> Pubs <br> $(\mathrm{log})$ | $(4)$ <br> Cites <br> $(\log )$ |
| :--- | :---: | :---: | :---: | :---: |
| Above bronze |  |  |  |  |
| medal threshold | -0.1112 | -0.0125 | 0.0074 | -0.0681 |
| Distance from | $(0.0498)$ | $(0.0200)$ | $(0.0774)$ | $(0.1327)$ |
| threshold | 0.0396 | 0.0037 | 0.0220 | 0.0476 |
|  | $(0.0142)$ | $(0.0035)$ | $(0.0100)$ | $(0.0197)$ |
| Distance from | -0.0174 | 0.0034 | -0.0015 | 0.0028 |
| threshold X above threshold | $(0.0189)$ | $(0.0055)$ | $(0.0166)$ | $(0.0326)$ |
| Country FE | Yes | Yes | Yes | Yes |
| Cohort (Olympiad Year) FE | Yes | Yes | Yes | Yes |
| Bandwidth | $[-4,4]$ | $[-6,6]$ | $[-7,7]$ | $[-7,7]$ |
| Observations | 1,328 | 1,868 | 2,464 | 2,163 |
| Mean of D.V. | 0.2304 | 0.0482 | 0.3779 | 0.6343 |

Notes: The IMO medals (gold, silver and bronze) are allocated solely based on the number of points scored at the IMO, with the medal thresholds varying from year to year. For each outcome variable, we select a set of individuals narrowly above and below the cutoff for a bronze medal using the optimal bandwidth selector of Calonico, Cattaneo \& Titiunik (2014). Because the optimal bandwidth depends on the dependent variable, the number of observations varies across specifications. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the $\log$ of mathematics publications plus one (column 3), the $\log$ mathematics cites plus one (column 4). All regressions are estimated by OLS and include country fixed effect, cohort (olympiad year) fixed effects and threshold fixed effects. Robust standard errors clustered by olympiad participant in parentheses.

Table A6: Regression discontinuity estimates of the effect of getting a gold medal

|  | $(1)$ <br> Math PhD | $(2)$ <br> Math PhD <br> (top 10) | $(3)$ <br> Pubs <br> $(\log )$ | $(4)$ <br> Cites <br> $(\log )$ |
| :--- | :---: | :---: | :---: | :---: |
| Above gold | 0.0097 | 0.0096 | 0.1831 | -0.1968 |
| medal threshold | $(0.1572)$ | $(0.1160)$ | $(0.4569)$ | $(0.5165)$ |
| Distance from | -0.0181 | -0.0171 | -0.1803 | 0.0214 |
| threshold | $(0.0857)$ | $(0.0621)$ | $(0.2588)$ | $(0.2011)$ |
| Distance from | 0.0309 | 0.0567 | 0.1771 | 0.0549 |
| threshold X above threshold | $(0.1003)$ | $(0.0707)$ | $(0.2944)$ | $(0.2495)$ |
| Country FE | Yes | Yes | Yes | Yes |
| Cohort (Olympiad Year) FE | Yes | Yes | Yes | Yes |
| Bandwidth | $[-2,2]$ | $[-2,2]$ | $[-2,2]$ | $[-3,3]$ |
| Observations | 352 | 352 | 352 | 482 |
| Mean of D.V. | 0.3977 | 0.1591 | 0.9353 | 1.5718 |

Notes: The IMO medals (gold, silver and bronze) are allocated solely based on the number of points scored at the IMO, with the medal thresholds varying from year to year. For each outcome variable, we select a set of individuals narrowly above and below the cutoff for a gold medal using the optimal bandwidth selector of Calonico, Cattaneo \& Titiunik (2014). Because the optimal bandwidth depends on the dependent variable, the number of observations varies across specifications. The dependent variables are as follows: obtaining a math PhD (column 1), obtaining a math PhD from a top 10 school (column 2), the log of mathematics publications plus one (column 3), the log of mathematics cites plus one (column 4). All regressions are estimated by OLS and include country fixed effect, cohort (olympiad year) fixed effects and threshold fixed effects. Robust standard errors clustered by olympiad participant in parentheses.

Table A7: Link between IMO score, long-term performance, and occupation by country income group with standard error clustered by country

|  | (1) <br> Math PhD <br> Math PhD <br> (top 10) | (2) <br> Pubs | (4) <br> Cites | (5) <br> Academia <br> (math) | (6) <br> Academia <br> (non-math) | (7) <br> Industry |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income group <br> (country of origin): |  |  |  |  |  |  |  |
| Low-income | -0.152 | -0.031 | -0.337 | -0.560 | -0.111 | -0.011 | 0.039 |
|  | $(0.046)$ | $(0.029)$ | $(0.116)$ | $(0.190)$ | $(0.028)$ | $(0.018)$ | $(0.032)$ |
| Lower middle-income | -0.101 | -0.022 | -0.194 | -0.321 | -0.049 | -0.004 | -0.043 |
|  | $(0.029)$ | $(0.019)$ | $(0.074)$ | $(0.127)$ | $(0.041)$ | $(0.020)$ | $(0.036)$ |
|  |  |  |  |  |  |  |  |
| Upper middle-income | -0.040 | -0.025 | -0.083 | -0.171 | 0.002 | 0.082 | -0.064 |
|  | $(0.036)$ | $(0.019)$ | $(0.084)$ | $(0.137)$ | $(0.049)$ | $(0.021)$ | $(0.032)$ |
|  |  |  |  |  |  |  |  |
| IMO Score | 0.011 | 0.005 | 0.027 | 0.045 | 0.007 | 0.002 | -0.001 |
|  | $(0.001)$ | $(0.001)$ | $(0.003)$ | $(0.004)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ |
| Observations | 4,710 | 4,710 | 4,710 | 4,710 | 2,272 | 2,272 | 2,272 |
| Mean of D.V. | 0.219 | 0.068 | 0.431 | 0.713 | 0.241 | 0.125 | 0.177 |

Notes: This table is identical to table 2 in the main text but with standard errors clustered at the country level instead of robust standard errors.

## Appendix figures

Figure A1: Distribution of IMO scores expressed in terms of distance to the threshold for a (better) medal


Notes: For the regressions evaluating the causal effects of medals to be valid, there should be no discontinuity in the density of cases across the threshold for a (better) medal. To test this formally, we use the Frandsen (2017) test for manipulation in the regression discontinuity design when the running variable is discrete. The test does not reject the null of no manipulation ( p -value $=0.974, \mathrm{k}=0.02$ ). As in the main analysis, we pool observations across the three medal thresholds (gold, silver, bronze) with each individual appearing three times.

Figure A2: Distance to medal threshold and long-term performance


Notes: The IMO medals (gold, silver and bronze) are allocated solely based on the number of points scored at the IMO. The medal thresholds for a gold, silver, or bronze medal vary from year to year. For each medal threshold, we construct the sample of participants no more than 5 points from the threshold. We then stack these three samples and construct a unique distance (number of points) to the threshold for a (better) medal. The graph displays samples means by distance to the threshold for a (better) medal, with linear fits superimposed.


[^0]:    ${ }^{1}$ Appendix figure A1 plots the distribution of IMO scores expressed in terms of distance to medal threshold. The Frandsen (2017) test for manipulation in the regression discontinuity design when the running variable is discrete does

[^1]:    not reject the null of no manipulation ( p -value $=0.974$ for $\mathrm{k}=0.02$ ).
    ${ }^{2}$ The validity of the regression discontinuity design rests upon the assumption that there is no precise manipulation and that the density of cases is smooth around the cutoff. Figure A1 displays the distribution of scores by distance to the cutoff. We also use the Frandsen (2017) test for manipulation in the regression discontinuity design when the running variable is discrete. The test does not reject the null of no manipulation ( p -value $=0.974, \mathrm{k}=0.02$ ).

