# Online Appendix to "Discounts and Deadlines in Consumer Search" 

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## A Survey of Deadlines in Consumer Search

From September 27th to November 1st, 2018, Qualtrics administered a survey on our behalf to a panel of consumers. Qualtrics is a survey administration company that recruits survey participants through a variety of means, including websites, member referrals, targeted email lists, gaming sites, social media, and other sources. Panelists are incentivized to complete the survey through some small monetary compensation or through points toward a particular product loyalty program. These panelists are thus likely to be comfortable with online activity.

Members of the Qualtrics panel were selected at random to receive an email offering them the opportunity to participate in our survey. Consumers who opted to start the survey were given the following screening question to identify participants who could recall an item for which they had searched:

Can you think of a recent purchase for which you considered searching at multiple locations (either online or offline) in order to find a good price? Note: Think back only on non-food items. Examples might include a phone/tablet/laptop (or other consumer electronic item), a toy, an item of clothing or accessory, a sporting good, a book, an appliance or other household item, or even a car.

- Yes
- No

Consumers who responded "No" were given no further questions. Consumers who responded "Yes" entered into our sample and were given the following survey. Respondents were required to make a response to all questions. Questions 1, 2, 3, 5, and 6 were freeresponse questions. Questions 4 and 7 were check-box questions, and the respondents were allowed to select as many of the options as desired, but were required to select at least one. Questions 8-12 were radio-button questions, and the respondents were required to select one and only one option.

1. What was the item you purchased? Describe it in just a few a words.
2. About how much money (in dollars) did you pay for it?
3. About how much money (in dollars) do you think you saved by searching around?
4. Where did you search? Select ALL that apply:

[^0](a) Amazon
(b) eBay
(c) Google
(d) Large retailer's physical store
(e) Small retailer's physical store
(f) Other
5. How many times did you visit a physical store in attempting to find the item?
6. How many times did you visit an online retail site in attempting to find the item?
7. Select ALL that apply to the item you purchased: [Respondents were allowed to select as many of the following as desired, but were required to select at least one.]
(a) The item was a gift for someone
(b) I wanted/needed this item for an upcoming event
(c) I wanted/needed this item more as time went by
(d) I knew where I could find this item for sure at a high price, but I searched around to find a low price
(e) None of the above
8. Which of the following best describes the urgency with which you wanted/needed the item? [Respondents were required to select one and only one of the following]
(a) I wanted/needed this item as soon as possible
(b) It wasn't urgent that I get the item as soon as possible, just as long as it came in time for a particular deadline or a particular use of the item I had in mind
(c) None of the above
9. If you hadn't found/purchased the item when you did, which of the following best describes what you would have done next in your attempt to get it? [Respondents were required to select one and only one of the following]
(a) Given up searching.
(b) Kept trying to find a good price, and eventually purchased it even if it had cost a little more than (respondent's answer to Q.2)
(c) Kept trying to find a good price, and eventually purchased it only if it had cost (respondent's answer to Q.2) or less
10. Which response best completes the following sentence? "If I hadn't purchased this item when I did, I would have been fine getting this item anytime within the next $\qquad$ ."
(a) one day
(b) one week
(c) two weeks
(d) month
(e) two months
(f) four months
(g) six months
(h) one year
(i) century (in other words, anytime would have been fine - I had no timeline for getting this item)
11. Which response best completes the following sentence? "I was aware that I wanted/needed to eventually buy this item about $\qquad$ before I purchased it."

- (Same options as prior question except the last)

12. Select the answer that best describes what you were trying to learn from your search:
(a) I was only trying to find the best price; I knew exactly what item I wanted
(b) I was mainly trying to find the best price, but I was also trying to find which product was the best fit for me
(c) Price and product fit were equally important to me in my search
(d) I was mainly trying to find which product was the best fit for me, but I was also trying to find the best price
(e) I was only trying to find which product was the best fit for me, independent of price
(f) None of the above

Qualtrics screens for non-serious responders in several ways. First, the company collects responses until 50 consumers have completed the survey. The company then computes a speed threshold (by computing the median time taken on the survey among those first 50 completers, and setting the threshold to half of that time); any respondent (or subsequent respondent) who completes the survey faster than that threshold (which in our case is 1.15 minutes) is not considered a serious respondent. Second, Qualtrics allowed us to examine responses to identify those in which the free response questions were non-serious (e.g. an answer of 0 for Q.2; answers such as "I don't know" or "none" for Q.1; or answers for Q. 1 that describe food, which violates the screening question.).

The survey responses are summarized by price range in Table A1 and by product categories in Table A2. Categories were determined from respondents' free-response item descriptions (Q.1) as follows: Automotive (vehicles and parts), Technology (computers, TVs, phones, game consoles), Entertainment (video games, books, sports equipment, toys), Household (appliances, furniture), Clothing (clothes, jewelry), and Other. The responses show remarkable consistency across the various products and prices. A notable exception is with automotive purchases, which are much more expensive, are rarely motivated by a special event, are less likely to be needed more over the search spell, and have more searches occur but at specialized websites rather than popular consumer websites.

Using the respondents' estimated savings, we consider whether those who completed their purchase relatively early in their search span saved more, consistent with our model's prediction. To account for the wide price range and differing potential search spans, we measure both variables in percentage rather than absolute terms. Table A3 reports the regression results. Despite heterogeneous goods and potentially imprecise guesses from respondents on savings and potential search span, we find a positive correlation between early purchases and greater savings. The estimate is quite noisy in the first column. The point estimate and its precision increase as we narrow the sample to those whose reported motives most closely fit the model assumptions, such as having several weeks or more to search, or being willing to pay more over time, or searching purely for the best price rather than across competing products.

Table A1: Survey Summary Statistics by Price Range

|  |  | > \$33 \& |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\leq \$ 33$ | $\leq \$ 150$ | > \$150 | Total |
|  |  | N | 416 | 397 | 397 | 1210 |
| Q2 | Purchase price | (mean) | 16 | 77 | 2600 | 884 |
|  |  | (sd) | 9 | 35 | 7213 | 4299 |
| Q3 | \% saved | (mean) | 39 | 29 | 22 | 30 |
| Q10\&11 | Potential search span | (mean) | 46 | 67 | 99 | 70 |
|  |  | (sd) | 66 | 85 | 135 | 101 |
|  | \% of search remaining | (mean) | 50 | 49 | 45 | 48 |
|  | Unlimited potential span | (\%) | 3.1 | 1.8 | 2.5 | 2.5 |
| Q5 | \# of physical searches | (mean) | 2.4 | 1.8 | 2. | 2.1 |
| Q6 | \# of online searches | (mean) | 3.1 | 3.8 | 5.5 | 4.1 |
| Q4 | a. Searched Amazon | (\%) | 74 | 73 | 59 | 69 |
|  | b. Searched eBay | (\%) | 31 | 28 | 25 | 28 |
|  | c. Searched Google | (\%) | 24 | 25 | 27 | 25 |
| Q7 | $\mathrm{a}-\mathrm{b}$ : For a special event |  | 36 | 38 | 23 | 32 |
|  | a-c: Needed more over time | (\%) | 65 | 66 | 64 | 65 |
|  | d. Knew high-price option | (\%) | 43 | 47 | 50 | 47 |
| Q8 | a. Needed ASAP | (\%) | 40 | 44 | 53 | 46 |
|  | b. Needed by deadline | (\%) | 45 | 42 | 38 | 42 |
| Q9 | b. Willing to pay more in future | (\%) | 66 | 63 | 64 | 64 |
| Q12a | a. Only searching on price | (\%) | 52 | 48 | 46 | 49 |

Notes: Table provides means and standard deviations for a participants' survey responses. The first column denotes the question number and, in some cases, the response letter corresponding to the survey questions described in the text of Technical Appendix A. The second column provides an abbreviated explanation of the survey question. The final column contains statistics for the full sample. The columns labeled with monetary amounts (e.g. " $\leq \$ 33$, ") report statistics for a particular subsample based on the participant's reported purchase price.

Table A2: Survey Summary Statistics by Category

|  |  |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & Z \\ & \hline \end{aligned}$ |  |  | $\begin{aligned} & \overrightarrow{0} \\ & 0 \\ & \stackrel{\rightharpoonup}{v} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 嵳 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | N | 52 | 329 | 110 | 210 | 183 | 326 | 1210 |
| Q2 | Purchase price | (mean) (sd) | $\begin{aligned} & 15,613 \\ & 14,213 \end{aligned}$ | $\begin{aligned} & 398 \\ & 476 \end{aligned}$ | $\begin{aligned} & 56 \\ & 98 \end{aligned}$ | $\begin{array}{r} 375 \\ 6194 \end{array}$ | $\begin{array}{r} 63 \\ 109 \end{array}$ | $\begin{array}{r} 92 \\ 455 \end{array}$ | $\begin{array}{r} 884 \\ 4,299 \end{array}$ |
| Q3 | \% saved | (mean) | 14 | 27 | 35 | 29 | 33 | 32 | 30 |
| Q10\&11 | Potential search span | (mean) (sd) | $\begin{aligned} & 122 \\ & 149 \end{aligned}$ | $\begin{array}{r} 78 \\ 117 \end{array}$ | 55 79 | 91 118 | 56 72 | 54 74 | 70 101 |
|  | \% of search remaining | (mean) | 41 | 47 | 50 | 50 | 49 | 48 | 48 |
|  | Unlimited potential span | (\%) | 3.8 | 2.1 | 3.6 | 1.9 | 2.2 | 2.8 | 2.5 |
|  | Span $>20$ days | (\%) | 85 | 74 | 72 | 77 | 78 | 65 | 73 |
| Q5 | \# of physical searches | (mean) | 2.7 | 1.8 | 1.3 | 1.7 | 4.1 | 1.6 | 2.1 |
| Q6 | \# of online searches | (mean) | 6.5 | 4.5 | 3.4 | 4.2 | 3.4 | 3.8 | 4.1 |
| Q4 | a. Searched eBay | (\%) | 13 | 29 | 37 | 21 | 25 | 33 | 28 |
|  | b. Searched Google | (\%) | 7.7 | 29 | 27 | 23 | 26 | 25 | 25 |
|  | c. Searched Amazon | (\%) | 17 | 73 | 84 | 66 | 65 | 71 | 69 |
| Q7 | a-b. For a special event | (\%) | 1.9 | 30 | 47 | 25 | 43 | 34 | 32 |
|  | a-c. Needed more over time | (\%) | 42 | 66 | 65 | 63 | 65 | 67 | 65 |
|  | d. Knew high-price option | (\%) | 56 | 46 | 45 | 45 | 51 | 45 | 47 |
| Q8 |  | (\%) | $54$ | $52$ | $34$ | $44$ | $38$ | $48$ | 46 |
|  | b. Needed by deadline | (\%) | 35 | $36$ | $50$ | $40$ | $50$ | 42 | 42 |
| Q9 | b. Willing to pay more in future | (\%) | 62 | 64 | 57 | 66 | 64 | 66 | 64 |
| Q12 | a. Only searching on price | (\%) | 46 | 46 | 65 | 40 | 50 | 50 | 49 |

Notes: Table provides descriptive statistics for the same survey responses as in Table A1, but broken down by product category (based on the participants' responses to survey Q1).

Table A3: Dependent Variable: Percentage Savings, Self-Reported

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |
| :--- | :---: | :---: | :---: | :---: |
| \% Remaining Search Time | 1.82 | 4.66 | 7.71 | 13.92 |
|  | $(3.15)$ | $(3.65)$ | $(4.78)$ | $(6.81)$ |
| Constant |  |  |  |  |
|  | $(1.61)$ | 28.5 | 27.1 | 23.8 |
|  |  |  |  |  |
| N | 764 | 534 | 347 | 162 |
| Willing to pay more in future | X | X | X | X |
| Span > 20 days |  | X | X | X |
| Exclude clothing and household |  |  | X | X |
| Only searching on price |  |  |  | X |

Notes: Table displays results of a regression of the percent saved by the consumer (computed as the response to Q3 divided by the response to Q2) regressed on the percent of search time remaining (computed as number of days corresponding to the response to Q10 divided by the sum of the days corresponding to Q10 and Q11), with progressively more restrictive samples used in Columns (1) through (4). Column (1) limits the sample to those respondents who indicated a willingness to pay more in the future (Q9b); column (2) adds a restriction that search span be greater than 20 days; column (3) excludes clothing and household items; and column (4) only includes those participants who were searching only for a good price (Q12a). Robust standard errors are displayed in parentheses.

## B Comparative Statics

## B. 1 Comparative Statics in the Buyer Equilibrium

In this section we discuss comparative statics results for the model parameters. Although our equilibrium has no closed-form solution, these comparative statics can be obtained by implicit differentiation of $\phi(k)$, which allows for analytic derivations reported below.

Table A4 reports the sign of the derivatives of four key statistics in the buyer equilibrium. The first and second are the average number of participants per auction, $\lambda^{*}$, which reflects how competitive the auction is among buyers, and the average mass of buyers in the market, $H^{*}$, which is always proportional to $\lambda^{*}$. Third is the measure of buyers who never win an auction and must use the posted-price listings; this crucially affects the profitability of the posted-price market in the market equilibrium. Fourth is the bid of new buyers in the market, indicating the effect on buyers' willingness to pay. This comparative static can be derived at any $s$ and has a consistent effect, but the simplest computation occurs at $s=T$. This comparative static also captures price dispersion, both within auctions and between auctions and posted prices. The posted price $z$ is fixed, so a lower $b^{*}(T)$ indicates greater dispersion.

Changes in $\alpha$ have an intuitive impact. With more frequent auctions (reduced search frictions) the value of continued search is greater as there are more opportunities to bid. The increase in auctions creates more winners, reducing the stock of bidders and the number of

Table A4: Comparative Statics on Key Statistics: Buyer Equilibrium

|  |  | $\partial / \partial \alpha$ | $\partial / \partial \tau$ | $\partial / \partial \rho$ | $\partial / \partial \beta$ | $\partial / \partial T$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Participants per Auction | $\lambda^{*}$ | - | + | 0 | 0 | + |
| Number of Buyers | $H^{*}$ | - | + | 0 | 0 | + |
| Measure of Buyers using Posted Price | $F^{\prime}(0)$ | - | - | 0 | 0 | - |
| Lowest Bid | $b^{*}(T)$ | - | $* *$ | - | + | - |

Notes: $* *$ indicates that the sign depends on parameter values. Sufficient conditions for a positive sign are $\delta \tau T>1$ and $\tau(\kappa-\alpha)>\rho>\tau(2 \kappa-\alpha) \sqrt{\tau \kappa T e^{-\lambda}}$. An exact condition is provided in the proof.
competitors per auction. Both of these effects lead bidders to lower reservation prices.
Changes in $\tau$ have nearly the reverse effect from that of $\alpha$, though there are opposing forces at work. A higher likelihood of participating also reduces the search friction of a given bidder, as she will participate in more of the existing auctions. However, all other bidders are more likely to participate as well. The net result is typically higher bids, because the greater number of competitors dominates the increased auction participation to reduce the value of search. However, this does depend on parameter values; in particular, when $\tau$ or $\rho$ are very close to zero, extra participation dominates extra competitors, leading to lower bids.

The rate of time preference has no impact on the number or distribution of bidders, as $\rho$ does not enter into equation (10) or (11). Intuitively, this is because the rate at which bidders exit is determined by how often auctions occur, which is exogenous here. Also, who exits depends on the ordinal ranking of their valuations, which does not change even if the cardinal values are altered. Their bids react as one would expect: buyers offer less when their utility from future consumption is valued less. By the same token, a decrease in $\beta$ has no effect on the distribution of bidders, but will reduce their bids because more utility from consumption is delayed until the deadline.

We can also consider the effect (not shown in Table A4) of the parameter change on the expected revenue generated in an auction. For the first four parameters, revenue moves in the same direction as bids because the number of participants per auction is either constant or moves in the same direction. For instance, more auctions will reduce the bids and reduce the number of bidders; thus expected revenue must be lower. The intriguing exception is when the deadline is farther away; there, the additional participants override the lower initial bid, driving up expected revenue.

## B. 2 Comparative Statics in the Market Equilibrium

For the market equilibrium, the computation of $\theta^{*}$ prevents analytic determination of the sign of the comparative statics, but numeric evaluation remains consistent over a large space of

Table A5: Comparative Statics on Key Statistics: Market Equilibrium

|  |  | $\partial / \partial \tau$ | $\partial / \partial \rho$ | $\partial / \partial T$ | $\partial / \partial c$ | $\partial / \partial \ell$ |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
| Auction Rate | $\alpha^{*}$ | + | - | - | - | - |
| Participants per Auction | $\lambda^{*}$ | + | + | + | + | + |
| $\%$ Buying via Posted Price | $\frac{F^{\prime}(0) H^{*}}{\delta}$ | - | + | + | + | + |
| Stock of Posted-Price Sellers | $P^{*}$ | + | - | - | - | - |
| Lowest Bid | $b^{*}(T)$ | - | - | - | + | + |
| Expected Revenue | $\theta^{*}$ | + | + | - | + | + |

Notes: Reported signs are numeric computations under estimated parameters.
parameter values. Table A5 summarizes these typical effects. We are particularly interested in how parameter changes affect the distribution of sellers across mechanisms. We find that more sellers join the discount market when buyers are more attentive $(\tau)$, less patient $(\rho)$, or have less time $(T)$. Higher seller costs (whether in listing fee, $\ell$, or production, $c$ ) also shift sellers from auctions to posted prices.

To examine the effects in greater depth, first consider an increase in $\tau$. In the buyer equilibrium, this leads to more participants per auction, who then are willing to bid more. In the market equilibrium, however, more attentive buyers also induce sellers to offer more auctions. This more than offsets the effect of more participants per auction, producing a net decline in bids. On net, however, expected revenue slightly increases.

Next, an increase in $\rho$ reduces bids but had no effect on the distribution of buyers in the buyer equilibrium. In a market equilibrium, bids will still fall, but sellers offer fewer auctions. Surprisingly, this leads to higher revenue per auction, as it concentrates more buyers per auction. Changes in $T$ behave similarly under either equilibrium definition.

For $c$, it is remarkable that even though increased production costs do not raise the retail price (by assumption), they still affect auctions in the distribution of buyers and their bids. Higher costs will shrink the margins in both markets, which the auction market responds to by reducing its flow of sellers. Fewer auctions necessarily mean that more buyers reach their deadline; and this increased demand for posted-price listings more than compensates for the smaller margin. That is, a larger stock of posted-price sellers is needed to return to normal profits. Also, with fewer available auctions, buyers have a lower continuation value from waiting for future discount buying opportunities. This drives up bidders' reservation prices, but not enough to prevent a smaller flow of auction sellers. The comparative statics for listing fees $\ell$ behave similarly, as discussed in the Section 6.4 of the text.

## B. 3 Derivation of Buyer Equilibrium Comparative Statics

Because we do not have a closed-form solution for the endogenous number of participants per auction, we use implicit differentiation of $\phi\left(H^{*}\right)=0$ from (10) to determine the effect of the exogenous parameters on $H^{*}$. In fact, we find it convenient to express this implicit differentiation in terms of the participants per auction, $\lambda^{*} \equiv \tau H^{*}$; so with slight abuse of notation, we refer to $\phi(\lambda)$ when literally it would be $\phi(\lambda / \tau)$. In preparation for implicit differentiation, we note that $\phi^{\prime}(\lambda)<0$ for all $\lambda$ :

$$
\begin{equation*}
\frac{\partial \phi}{\partial \lambda}=-\alpha e^{-\lambda}-\left(\tau T \alpha+e^{\lambda}\right) \delta e^{-\tau T \kappa}<0 \tag{28}
\end{equation*}
$$

where $\kappa \equiv \delta+\alpha e^{-\lambda}$ is used for notational convenience, though we treat $\kappa$ as a function of $\alpha$ and $\lambda$ when taking derivatives.

Also note that $H=\frac{\lambda^{*}}{\tau}$ and $F^{\prime}(0)=\kappa-\alpha$, while the lowest bid is:

$$
\begin{equation*}
b(T)=z e^{-\rho T} \cdot \frac{\kappa(\tau \kappa+\rho) e^{\lambda^{*}}}{\tau \kappa\left(\delta e^{\lambda^{*}}+\alpha e^{-\rho T}\right)+\rho\left(\delta e^{\lambda^{*}}+\alpha e^{\tau T \kappa}\right)} \tag{29}
\end{equation*}
$$

Because this is always evaluated at the equilibrium $\lambda^{*}$, we can substitute for $e^{\lambda^{*}}$ using $\phi\left(\lambda^{*}\right)=0$, which is $\delta e^{\lambda}=(\kappa-\alpha) e^{\tau T \kappa}$, thus obtaining:

$$
\begin{equation*}
b(T)=\frac{z e^{-\rho T}}{\delta} \cdot \frac{(\tau \kappa+\rho)(\kappa-\alpha)}{\tau\left(\kappa-\alpha+\alpha e^{-(\rho-\tau \kappa) T}\right)+\rho} \tag{30}
\end{equation*}
$$

## B.3.1 Auction Rate, $\alpha$

Using implicit differentiation, we compute the effect of $\alpha$ on $\lambda^{*}$.

$$
\begin{align*}
\frac{\partial \phi}{\partial \alpha} & =-1+e^{-\lambda}+\tau T \delta e^{-\tau T \kappa}  \tag{31}\\
& =-1+e^{-\lambda}\left(1+\left(\frac{\delta+\alpha e^{-\lambda}-\alpha}{\delta+\alpha e^{-\lambda}}\right) \ln \left(\frac{\delta e^{\lambda}}{\delta+\alpha e^{-\lambda}-\alpha}\right)\right) \tag{32}
\end{align*}
$$

The second equality comes from substituting for $T$ using a rearrangement of $\phi\left(\lambda^{*}\right)=0$, which is $T=\frac{1}{\tau \kappa} \ln \left(\frac{\delta e^{\lambda}}{\kappa-\alpha}\right)$.

By rearrangement, $\frac{\partial \phi}{\partial \alpha} \leq 0$ if and only if:

$$
\begin{equation*}
\ln \left(\frac{\delta e^{\lambda}}{\delta+\alpha e^{-\lambda}-\alpha}\right)-\left(e^{\lambda}-1\right) \frac{\delta+\alpha e^{-\lambda}}{\delta+\alpha e^{-\lambda}-\alpha} \leq 0 \tag{33}
\end{equation*}
$$

As $\lambda \longrightarrow 0$, the left-hand side approaches 0 . If we take the derivative of the left-hand side w.r.t. $\lambda$, we obtain:

$$
\begin{equation*}
-\frac{\left(e^{\lambda}-1\right)\left(\alpha+\delta e^{\lambda}\right)\left(2 \alpha+e^{\lambda}(\delta-\alpha)\right)}{\left(\alpha+(\delta-\alpha) e^{\lambda}\right)^{2}} \tag{34}
\end{equation*}
$$

Each parenthetical term is strictly positive for all $\lambda>0$, so the left-hand side of (33) is strictly decreasing in $\lambda$. Thus, (33) strictly holds for any $\lambda>0$, including the equilibrium $\lambda^{*}$. Therefore, $\frac{\partial \phi}{\partial \alpha}<0$, and $\frac{\partial \lambda}{\partial \alpha}=-\left(\frac{\partial \phi}{\partial \alpha}\right) /\left(\frac{\partial \phi}{\partial \lambda}\right)<0$. Specifically,

$$
\begin{equation*}
\frac{\partial \lambda}{\partial \alpha}=-\frac{1-(1+\tau T(\kappa-\alpha)) e^{-\lambda}}{\kappa-\alpha+(1+\tau T(\kappa-\alpha)) \alpha e^{-\lambda}} . \tag{35}
\end{equation*}
$$

Next, consider the impact on the fraction purchasing from posted-price listings, which is affected both directly by $\alpha$ and indirectly through $\lambda$ :

$$
\begin{equation*}
\frac{\partial F^{\prime}(0)}{\partial \alpha}=e^{-\lambda}-1+\alpha \cdot \frac{\partial \lambda}{\partial \alpha} \tag{36}
\end{equation*}
$$

This is strictly negative because $e^{-\lambda}<1$ and $\frac{\partial \lambda}{\partial \alpha}<0$.
To demonstrate the effect to $\alpha$ on the bidding function, we use the alternate depiction in terms of the function $g(t)$ :

$$
b(T)=\frac{g(T)}{g(T)+\rho \int_{0}^{T} g(t) d t},
$$

recalling that

$$
g(t) \equiv \tau e^{-\rho t}\left(\kappa-\alpha\left(1-e^{-t \tau \kappa}\right)\right)
$$

Of course, $g(t)$ is a function of $\alpha$ (including its effect on $\kappa$ ), so let $g_{\alpha}(t)$ denote its derivative with respect to $\alpha$. Thus,

$$
g_{\alpha}(t)=\tau e^{-\rho t}\left(e^{-\tau \kappa t}+\frac{\kappa\left(1-\alpha \tau t e^{-\tau \kappa t}\right)}{\alpha+(\kappa-\alpha)\left(e^{\lambda}+\alpha \tau T\right)}-1\right)
$$

When we take the derivative of $b(T)$ w.r.t. $\alpha$, we obtain:

$$
\frac{\partial b(T)}{\partial \alpha}=z \rho \frac{\int_{0}^{T}\left(g(t) g_{\alpha}(T)-g(T) g_{\alpha}(t)\right) d t}{\left(g(T)+\rho \int_{0}^{T} g(t) d t\right)^{2}}
$$

The denominator is clearly positive. The numerator is always negative; in particular, at each $t \in[0, T]$, the integrand is negative. This integrand simplifies to:

$$
-\frac{\kappa \tau^{2} e^{-(t+T)(\kappa \tau+\rho)}\left(\alpha^{2} \tau(T-t)+(\kappa-\alpha)\left(\alpha \tau(T-t) e^{\kappa \tau T}+e^{\lambda}\left(e^{\kappa \tau T}-e^{\kappa t \tau}\right)\right)\right)}{\alpha+(\kappa-\alpha)\left(e^{\lambda}+\alpha \tau T\right)}<0
$$

The inequality holds that because $T \geq t$ and $\kappa>\alpha$, making each parenthetical term in the expression positive.

## B.3.2 Attention, $\tau$

Using implicit differentiation, we compute the effect of $\tau$ on $\lambda^{*}$.

$$
\begin{equation*}
\frac{\partial \phi}{\partial \tau}=\delta \kappa T e^{-\tau T \kappa}>0 \tag{37}
\end{equation*}
$$

All of these terms are strictly positive. Because $\frac{\partial \phi}{\partial \lambda}<0$, by implicit differentiation, $\frac{\partial \lambda}{\partial \tau}=$ $-\left(\frac{\partial \phi}{\partial \tau}\right) /\left(\frac{\partial \phi}{\partial \lambda}\right)>0$. Specifically,

$$
\begin{equation*}
\frac{\partial \lambda}{\partial \tau}=\frac{\delta \kappa T e^{\lambda}}{\alpha e^{\tau T \kappa}+\delta e^{\lambda}\left(e^{\lambda}+\alpha \tau T\right)} . \tag{38}
\end{equation*}
$$

Next, consider the impact on the fraction purchasing from posted-price listings. The probability of participation $\tau$ has no direct effect on $F^{\prime}(0)$, but affects it only through $\lambda$ :

$$
\begin{equation*}
\frac{\partial F^{\prime}(0)}{\partial \tau}=\frac{\partial F^{\prime}(0)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \tau}=-\alpha e^{-\lambda} \cdot \frac{\partial \lambda}{\partial \tau} \tag{39}
\end{equation*}
$$

which is always negative.
Finally, consider the effect on the lowest bid. Here, the sign of the derivative will depend on parameter values, so it is more convenient to take comparatives on (30) rather than examining it in terms of $g(t)$. Because $\kappa^{\prime}(\tau)=\alpha e^{-\lambda} \lambda^{\prime}(\tau)$, the comparative static on $b(T)$ works out to:

$$
\begin{equation*}
\frac{\partial b(T)}{\partial \tau}=\frac{z \alpha e^{\lambda} \psi}{(\kappa-\alpha)\left(\tau \alpha+(\tau(\kappa-\alpha)+\rho) e^{T(\rho+\tau \kappa)}\right)^{2}\left(\alpha+(\kappa-\alpha)\left(\tau \alpha T+e^{\lambda}\right)\right)} . \tag{40}
\end{equation*}
$$

where

$$
\begin{aligned}
\psi \equiv & e^{\lambda}(\kappa-\alpha)^{2}\left(\rho(\tau \delta T-1)+\delta \kappa \tau^{2} T-\frac{\alpha e^{-\lambda} \rho}{\kappa-\alpha}\right) \\
& +\delta e^{\lambda+\rho T}\left(\rho\left(e^{\lambda}(\kappa-\alpha)+\alpha\right)-T(\kappa \tau+\rho)\left(\tau(\kappa-\alpha)^{2}+\kappa \rho\right)\right)
\end{aligned}
$$

The lowest bid is increasing in $\tau$ if and only if $\psi>0$ because the remaining terms in $\frac{\partial b(T)}{\partial \tau}$ are always positive.

To verify the sufficient conditions listed under Table A4 in the paper, note that $\tau \delta T>1$ ensures that the first term in the first line is positive. For the remaining terms of the first line, note that $\delta \kappa \tau^{2} T>\kappa \tau$ by the same assumption. Moreover, because $\kappa>\alpha$ and $1>e^{-\lambda}$, then $\delta \kappa \tau^{2} T>\alpha \tau e^{-\lambda}$. Thus, the sufficient condition $\tau(\kappa-\alpha)>\rho$ ensures that $\delta \kappa \tau^{2} T>\frac{\alpha e^{-\lambda} \rho}{\kappa-\alpha}$.

For the second line, we note that by omitting the first and last $\alpha$ in the first step, then applying the second sufficient condition twice in the second, we get:

$$
\rho\left(e^{\lambda}(\kappa-\alpha)+\alpha\right)-T(\kappa \tau+\rho)\left(\tau(\kappa-\alpha)^{2}+\kappa \rho\right)>\rho e^{\lambda}(\kappa-\alpha)-T(\tau \kappa+\rho)^{2} \kappa
$$

$$
>\frac{\rho^{2} e^{\lambda}}{\tau}-T(\tau(2 \kappa-\alpha))^{2} \kappa .
$$

The third sufficient condition, $\rho>\tau(2 \kappa-\alpha) \sqrt{\tau \kappa T e^{-\lambda}}$, ensures that this last term is positive.

## B.3.3 Impatience, $\rho$

The rate of time preference $\rho$ does not enter into $\phi$, so therefore $\frac{\partial \phi}{\partial \rho}=0$ and $\frac{\partial \lambda}{\partial \rho}=0$. Similarly, $\rho$ has no direct effect on $F^{\prime}(0)$ or indirect effect through $\lambda$.

To demonstrate the effect to $\rho$ on the bidding function, we use the alternate depiction in terms of the function $g(t)$ :

$$
b(T)=\frac{g(T)}{g(T)+\rho \int_{0}^{T} g(t) d t},
$$

recalling that

$$
g(t) \equiv \tau e^{-\rho t}\left(\delta+\alpha\left(e^{-\lambda}+e^{-t \tau\left(\delta+\alpha e^{-\lambda}\right)}-1\right)\right) .
$$

Of course, $g(t)$ is a function of $\rho$, so let $g_{\rho}(t)$ denote its derivative with respect to $\rho$. Thus,

$$
g_{\rho}(t)=-t \tau e^{-\rho t}\left(\delta+\alpha\left(e^{-\lambda}+e^{-t \tau\left(\delta+\alpha e^{-\lambda}\right)}-1\right)\right) .
$$

Therefore, when we take the derivative of $b(T)$ w.r.t. $\rho$, we obtain:

$$
\frac{\partial b(T)}{\partial \rho}=z \frac{\int_{0}^{T}\left(\rho g(t) g_{\rho}(T)-\rho g(T) g_{\rho}(t)-g(t) g(T)\right) d t}{\left(g(T)+\rho \int_{0}^{T} g(t) d t\right)^{2}} .
$$

The denominator is necessarily positive. We will show that the integrand is negative for all $t$, implying that $\frac{\partial b(T)}{\partial \rho}<0$. The integrand simplifies to:
$\frac{\tau^{2}(\rho(t-T)-1)}{e^{(t+T)\left(\tau\left(\alpha e^{-\lambda}+\delta\right)+\rho\right)}} \cdot\left(\left(\alpha\left(1-e^{-\lambda}\right)-\delta\right) e^{\tau t\left(\alpha e^{-\lambda}+\delta\right)}-\alpha\right) \cdot\left(\left(\alpha\left(1-e^{-\lambda}\right)-\delta\right) e^{\tau T\left(\alpha e^{-\lambda}+\delta\right)}-\alpha\right)$.
Because $t \leq T$, the numerator is always negative, and the exponential term in the denominator is always positive. Finally, we note that $\alpha\left(1-e^{-\lambda}\right)-\delta<0$ because $\delta-\alpha\left(1-e^{-\lambda}\right)-$ $\delta e^{\lambda-\tau T\left(\delta+\alpha e^{-\lambda}\right)}=0$ in equilibrium. This ensures that second and third parenthetical terms are negative.

## B.3.4 Immediate Consumption, $\beta$

The fraction of immediate consumption has no impact on (10), so $\lambda^{*}$ will not change even if consumers obtain more utility at the time of purchase. Thus the number and distribution of buyers in the market are unaffected. The bid function is thus directly impacted as

$$
\begin{equation*}
\frac{\partial b(T)}{\partial \beta}=x \cdot \frac{\left(1-e^{-\rho T}\right) \delta e^{\lambda^{*}}(\tau \kappa+\rho)+\rho \alpha\left(e^{\tau \kappa T}-e^{-\rho T}\right)}{\tau \kappa\left(\delta e^{\lambda^{*}}+\alpha e^{-\rho T}\right)+\rho\left(\delta e^{\lambda^{*}}+\alpha e^{\tau \kappa T}\right)}>0 . \tag{41}
\end{equation*}
$$

The inequality holds because $e^{\tau \kappa T}>1>e^{-\rho T}$.

## B.3.5 Deadline, $T$

Using implicit differentiation, we compute the effect of $T$ on $\lambda^{*}$.

$$
\begin{equation*}
\frac{\partial \phi}{\partial T}=\delta \kappa \tau e^{\lambda^{*}} e^{-\tau T \kappa}, \tag{42}
\end{equation*}
$$

which is clearly positive. Then by implicit differentiation, $\frac{\partial \lambda}{\partial T}=-\left(\frac{\partial \phi}{\partial T}\right) /\left(\frac{\partial \phi}{\partial \lambda}\right)>0$. Specifically,

$$
\begin{equation*}
\frac{\partial \lambda}{\partial T}=\frac{\delta \tau \kappa}{\delta\left(1+\tau T \alpha e^{-\lambda^{*}}\right)+\alpha e^{\tau T \kappa-2 \lambda^{*}}} \tag{43}
\end{equation*}
$$

Moreover, the number of buyers $H^{*}$ is not directly affected by $T$, so it increases only because $\lambda^{*}$ increases.

Next, consider the impact on the fraction purchasing from posted-price listings. The deadline $T$ has no direct effect on $F^{\prime}(0)$, but affects it only through $\lambda$ :

$$
\begin{equation*}
\frac{\partial F^{\prime}(0)}{\partial T}=\frac{\partial F^{\prime}(0)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial T}=-\alpha e^{-\lambda} \cdot \frac{\partial \lambda}{\partial T} \tag{44}
\end{equation*}
$$

which is always negative.
To demonstrate the effect of $T$ on the bidding function, we again use the definition of $b(T)$ in terms of $g(t)$, but to distinguish between an intermediate time $t$ and the initial time $T$, we write it as:

$$
b(T)=\frac{g(T, T)}{g(T, T)+\rho \int_{0}^{T} g(t, T) d t},
$$

where

$$
g(t, T) \equiv \tau e^{-\rho t}\left(\kappa-\alpha\left(1-e^{-t \tau \kappa}\right)\right),
$$

where $T$ only affects the expression by changing $\lambda$ and hence changing $\kappa$.
The derivative of $b(T)$ w.r.t. $T$ is thus:

$$
\frac{\partial b(T)}{\partial T}=-\frac{z \rho\left(\int_{0}^{T}\left(\frac{g(T, T)^{2}}{T}-g(t, T) g_{t}(T, T)\right) d t+\int_{0}^{T}\left(g(T, T) g_{T}(t, T)-g(t, T) g_{T}(T, T)\right) d t\right)}{\left(g(T)+\rho \int_{0}^{T} g(t) d t\right)^{2}}
$$

where $g_{t}$ and $g_{T}$ are derivatives with respect to the first and second terms, respectively. Specifically, these evaluate to:

$$
g_{t}(T, T)=\left(\rho \tau(\alpha-\kappa)-\alpha \tau(\kappa \tau+\rho) e^{-\tau \kappa T}\right) e^{-T \rho}
$$

and

$$
g_{T}(t, T)=\alpha \tau\left(\alpha t \tau-e^{\kappa t \tau}\right) e^{-\lambda-t(\kappa \tau+\rho)} \lambda^{\prime}(T) .
$$

Because $\kappa>\alpha$, we know that $g_{t}(T, T)<0$ and $g(t, T)>0$ for all $t$. Thus, the first integral in the numerator is always positive.

The integrand of the second integral simplifies to $\mu(t) \alpha^{2} \tau^{2} \lambda^{\prime}(T) e^{-\lambda-(t+T)(\tau \kappa+\rho)}$, where:

$$
\mu(t) \equiv e^{\tau \kappa t}(\tau T(\alpha-\kappa)-1)+e^{\tau \kappa T}(t \tau(\kappa-\alpha)+1)+\alpha \tau(t-T) .
$$

We have already shown that $\lambda^{\prime}(T)>0$; thus, to show that the integral is positive, we only need to show that $\mu(t) \geq 0$ for all $t$. First note that $\mu(T)=0$ and $\mu(0)=e^{\tau \kappa T}-\tau \kappa T-1>0$. To see the latter inequality, note that this has the form $e^{x}-x-1$, which is equal to 0 at $x=0$ and has a positive derivative $e^{x}-1 \geq 0$ for all $x$.

Next, note that $\mu^{\prime \prime}(t)=-(1+\tau T(\kappa-\alpha)) \tau^{2} \kappa^{2} e^{\tau \kappa t}<0$ for all $t \in[0, T]$. Because $\mu(0)>$ $\mu(T)=0$ and $\mu^{\prime \prime}(t)<0, \mu(t)>0$ for all $t \in[0, T)$.

Thus, the integrand of the second integral is always positive. Thus $\frac{\partial b(T)}{\partial T}<0$.

## C Market Equilibrium Model

The equilibrium solution to the market model is presented here, along with the propositions for the existence of degenerate and dispersed equilibria and their proofs.

## C. 1 Equilibrium Solution

While the market equilibrium conditions simplify considerably, they do not admit an analytic solution and we must numerically solve for both $\alpha^{*}$ and $H^{*}$. Equilibrium is attained when both (10) and (20) simultaneously hold. To compute $\theta^{*}$, (14) must be evaluated using $b(s)$ and $F(s)$ from the buyer equilibrium; the resulting equation is cumbersome and is reported in the proof of Proposition 3. Once $\alpha^{*}$ and $H^{*}$ are found, the remaining equilibrium objects are easily solved as follows:

$$
\begin{align*}
\Pi_{a}^{*} & =c  \tag{45}\\
\Pi_{p}^{*} & =c  \tag{46}\\
A^{*} & =\frac{\alpha^{*}}{\eta}  \tag{47}\\
P^{*} & =\frac{(z(1-\ell)-c)\left(\delta-\alpha^{*}\left(1-e^{-\tau H^{*}}\right)\right)}{\rho c}  \tag{48}\\
\zeta^{*} & =\frac{\rho c}{z(1-\ell)-c}  \tag{49}\\
\sigma^{*} & =\frac{\alpha^{*}\left(1-e^{-\tau H^{*}}\right)}{\delta} \tag{50}
\end{align*}
$$

The following proposition demonstrates that these solutions are necessary for any equilibrium in which auctions actually take place.

Proposition 3. A market equilibrium with an active discount channel ( $\alpha^{*}>0$ ) must satisfy $\phi\left(H^{*}\right)=0$, equation (20), equations (11) through (14), and equations (45) through (50).

The solution described in Proposition 3 can be called a dispersed equilibrium, to use the language of equilibrium search theory, as we observe the homogeneous good being sold at a variety of prices and by multiple sales mechanisms. By contrast, in a degenerate equilibrium, the good is always sold at the same price. This only occurs if all goods are purchased via posted-price listings and no auctions are offered ( $\alpha^{*}=\sigma^{*}=0$ ). We can analytically solve for this degenerate equilibrium and for the conditions under which it exists, as described in the following proposition. ${ }^{47}$

Proposition 4. The degenerate market equilibrium, described by equations (11) and (12) and equations (45) through (50) with $\alpha^{*}=0$ and $H^{*}=\delta T$, exists if and only if

$$
\begin{equation*}
\beta x+\frac{(z-\beta x) \tau \delta}{1-e^{-\tau \delta T}} \cdot \frac{\tau \delta+(\rho T(\rho+\tau \delta)-\tau \delta) e^{-(\rho+\tau \delta) T}}{(\rho+\tau \delta)^{2}} \leq \frac{c}{1-\ell} \cdot\left(1+\frac{\rho}{\eta\left(1-e^{\tau \delta T}\right)}\right) . \tag{51}
\end{equation*}
$$

Moreover, if this condition fails, a dispersed market equilibrium will exist. Thus, an equilibrium always exists.

The left side of (51) calculates the expected revenue $\theta$ that a seller would earn by offering an auction when no one else does $(\alpha=0)$. For this equilibrium to exist, the expected revenue must be lower than the expected cost of entering the market (the right side of (51)). We can consider such a deviation because buyers still wait until their deadline before purchasing via the posted-price listing, and are willing to bid their reservation price $b(s)=\beta x+(z-\beta x) e^{-\rho s}$ if given the chance.

Equation (51) indicates that auctions are not viable when expected costs are high, such as high production costs or listing fees, or long delays before closing (small $\eta$ ). In contrast, the posted-price market can compensate for these costs by keeping a low stock of sellers so that the item is sold very quickly. Auctions can also be undermined by weak competition among bidders producing low expected revenue, which occurs with a small flow of buyers ( $\delta$ ) or few of them paying attention $(\tau)$.

Proposition 4 proves that an equilibrium always exists; we further conjecture that the equilibrium is always unique. This claim would require that at most one dispersed equilibrium

[^1]can occur, and that a dispersed equilibrium cannot occur when (51) holds - both of which are true if $\theta$ is a decreasing function of $\alpha$ (i.e., more auctions always lead to lower revenue). The complicated expression for $\theta$ in the dispersed equilibrium precludes an analytic proof, but we have consistently observed this relationship between $\alpha$ and $\theta$ in numerous calculations across a wide variety of parameters.

## C. 2 Proofs

Proof of Proposition 3. By Proposition 1, equations (11) and (12) and $\phi\left(H^{*}\right)=0$ must be satisfied in order to be a buyer equilibrium, as required in the first condition.

The solutions to $A^{*}$ and $\sigma^{*}$ are simply restatements of (16) and (18), respectively. It is apparent that $\sigma^{*} \geq 0$. To see that $\sigma^{*}<1$, note that the equilibrium condition $\phi\left(H^{*}\right)=0$ requires that $\alpha\left(1-e^{-\tau H}\right)<\delta$. This also ensures that $P^{*}>0$.

The profits stated in (45) and (46) are required by the third and second equilibrium conditions, respectively. From (13), profit solves as: $\Pi_{p}=\frac{\zeta z(1-\ell)}{\rho+\zeta}$, so for this to equal $c$, we require $\zeta^{*}=\frac{\rho c}{z(1-\ell)-c}$ as in equation (49). With this, (19) readily yields $P^{*}$ as listed in (48).

The only remaining element regards expected auction profit. Equation (15) solves as: $\Pi_{a}=\frac{\eta\left(1-e^{-\tau H}\right)(1-\ell) \theta}{\eta\left(1-e^{-\tau H}\right)+\rho}$. By setting this equal to $c$ and solving for $\theta$, we obtain (20).

To evaluate the integrals in (14), we first note that by interchanging the sum and integral and evaluating the sum, expected revenue simplifies to:

$$
\begin{equation*}
\theta=\frac{\lambda}{1-e^{-\lambda}}\left(e^{-\lambda} b(T)+\lambda \int_{0}^{T} b(s) F(s) F^{\prime}(s) e^{-\lambda F(s)} d s\right) . \tag{52}
\end{equation*}
$$

After substituting for $b(s)$ and $F(s)$ from the buyer equilibrium, this evaluates to:

$$
\begin{aligned}
\theta=\beta x+\frac{z-\beta x}{1-e^{-\tau H}} \cdot( & 1+\frac{1}{(\rho+\kappa \tau)\left(\rho \delta+\tau(\kappa-\alpha)\left(\delta+\alpha e^{-\tau H-\rho T}\right)\right)} \\
& \left(\tau(\alpha-\kappa) e^{-\tau H-\rho T}\left(\kappa \tau(\kappa-H \rho)-H \rho^{2}\right)-\delta \rho(2 \kappa \tau+\rho)\right. \\
& \left.\left.+\kappa \rho \tau\left(\delta \Psi\left(1-\frac{\kappa}{\alpha}\right)+(\alpha-\kappa) e^{-\tau H-\rho T} \Psi\left(1-\frac{\kappa e^{\tau H}}{\alpha}\right)\right)\right)\right),
\end{aligned}
$$

where $\kappa \equiv \delta+\alpha e^{-\tau H}$ and $\Psi(q)$ is Gauss's hypergeometric function with parameters $a=1$, $b=-1-(\rho / \tau \kappa), c=-\rho / \tau \kappa$, evaluated at $q$. Under these parameters, the hypergeometric function is equivalent to the integral:

$$
\Psi(q) \equiv-\left(1+\frac{\rho}{\tau \kappa}\right) \int_{0}^{1} \frac{t^{-2-\frac{\rho}{\tau \kappa}}}{1-q t} d t .
$$

While not analytically solvable for these parameters, $\Psi$ is readily computed numerically.

Proof of Proposition 4. The proposed Buyer and Market Equilibria still apply when $\alpha^{*}=$ 0 , bearing in mind that as $\alpha \rightarrow 0$, the solution to $\phi\left(H^{*}\right)=0$ approaches $H^{*}=\delta T$. In the absence of auctions, the distribution of bidders is uniformly distributed across $[0, T]$ because none of them exit early; so $F^{*}(s)=s / T$ and $H^{*}=\delta T$. Moreover, the buyer's willingness to bid (if an auction unexpectedly occurred) reduces to: $b(s)=\beta x+(z-\beta x) e^{-\rho s}$.

For $\alpha^{*}=0$ to be a market equilibrium, we need $\Pi_{a}^{*} \leq \Pi_{p}^{*}$. To prevent further entry, $\Pi_{p}^{*}=c$ is still required. From (15), a seller would earn $\Pi_{a}^{*}=\frac{\eta\left(1-e^{\tau \delta T}\right)(1-\ell) \theta}{\rho+\eta\left(1-e^{\tau \delta T}\right)}$ by offering an auction unexpectedly. Thus, the expected profit comparison simplifies to: $\theta \leq \frac{c}{1-\ell} \cdot\left(1+\frac{\rho}{\eta\left(1-e^{\tau \delta T}\right)}\right)$. This is equivalent to (51), where the left-hand side is evaluated from (52):

$$
\begin{aligned}
\theta & =\frac{\tau \delta T}{1-e^{-\tau \delta T}}\left(e^{-\tau \delta T} b(T)+\int_{0}^{T} b(s) F(s) F^{\prime}(s) e^{-\tau \delta T F(s)} d s\right) \\
& =\beta x+\frac{\tau \delta T}{1-e^{-\tau \delta T}}\left(e^{-\tau \delta T}(z-\beta x) e^{-\rho T}+\int_{0}^{T}(z-\beta x) e^{-\rho s} \frac{s}{T^{2}} e^{-\tau \delta s} d s\right) \\
& =\beta x+\frac{(z-\beta x) \tau \delta}{1-e^{-\tau \delta T}} \cdot \frac{\tau \delta+(\rho T(\rho+\tau \delta)-\tau \delta) e^{-(\rho+\tau \delta) T}}{(\rho+\tau \delta)^{2}}
\end{aligned}
$$

Thus, if (51) holds, then the profit from offering an auction is never greater than continuing to offer a posted-price listing, making $\alpha^{*}=0$ an equilibrium. If (51) fails to hold, then $\alpha^{*}=0$ cannot be an equilibrium because some firms will earn greater profit by deviating and offering an auction.

To prove the last claim, first note that in a buyer equilibrium, $H \rightarrow 0$ as $\alpha \rightarrow \infty$. In addition, $b(s) \rightarrow 0$ for all $s>0$, because auctions occur every instant, in which the buyer faces no competition. Thus, expected revenue is 0 in the limit, yielding profit $\Pi_{a}<0$ for $\alpha \rightarrow \infty$. At the same time, the violation of (51) is equivalent to $\Pi_{a}>0$ for $\alpha=0$. Because expected revenue is continuous in $\alpha$, by the intermediate value theorem there must exist an $\alpha^{*}>0$ such that $\Pi_{a}\left(\alpha^{*}\right)=0$, which will constitute a dispersed equilibrium.

## D Extensions

## D. 1 Alternative Mechanisms: Physical Search, Bargaining, or Lotteries

Our model of non-stationary search for discounts can be readily adapted for settings beyond auctions. Here, we briefly outline several examples of how the search problems could be formulated, changing the discount mechanism in (3) while maintaining the deadlines embedded in the $-V^{\prime}(s)$ term and the full price option $z$.

To our knowledge, these non-stationary bargaining and lottery problems have not been studied before. We believe they present interesting settings for future work.

## D.1.1 Physical Search

First, consider physical search for a homogeneous good where sellers post a price, but discovering these sellers is time consuming. At each encounter, the buyer learns a specific seller's price but has to purchase immediately or lose the opportunity. The buyer in state $s$ formulates a reservation price $b(s)$, purchasing if and only if the quoted price is at or below $b(s)$. Let $G(s)$ depict the cumulative distribution of sellers offering a price at or above $b(s)$. One could say that a firm charging $b(s)$ is targeting buyers of type $s$, and will only sell to those who have $s$ or less time remaining. In this case, the probability that a buyer "wins" the discount is:

$$
\begin{equation*}
W(s)=1-G(s), \tag{53}
\end{equation*}
$$

because the buyer will reject any discount targeted at buyers more desperate than herself. The expected payment would be:

$$
\begin{equation*}
M(s)=\int_{s}^{T} b(t) d G(t) \tag{54}
\end{equation*}
$$

When offered, the buyer accepts any price between $b(T)$ and $b(s)$, but pays nothing if a higher price is offered (which occurs with probability $G(s)$ ).

We now consider physical search from the seller's perspective. A deeper discount results in lower revenue but a higher likelihood of sale because it will be acceptable to more buyers. A seller who targets buyers with $s$ time remaining will only complete the sale to fraction $F(s)$ of buyers but will be paid $b(s)$ when the sale is completed. Thus, the discount mechanism generates an expected profit of:

$$
\begin{equation*}
\rho \Pi_{a}=\eta F(s)\left((1-\ell) b(s)-\Pi_{a}\right) . \tag{55}
\end{equation*}
$$

To obtain price dispersion, each targeted price $b(s)$ must yield the same expected profit $\Pi_{a}$. The equilibrium in this environment is closely related to the labor market model of Akın and Platt (2012).

## D.1.2 Bargaining

Alternatively, consider an environment in which buyers are randomly paired with sellers and enter Nash bargaining. Again, let $G\left(s^{\prime}\right)$ denote the distribution of seller states, where a seller in state $s^{\prime}$ is willing to accept any price at or above $b\left(s^{\prime}\right)$. Upon meeting, their private states are revealed. Matches with negative surplus are dissolved, while matches with positive surplus lead to a sale with a price $\omega b(s)+(1-\omega) b\left(s^{\prime}\right)$, where $\omega$ is the Nash bargaining power of the seller. Here, a buyer in state $s$ will only make a purchase if the seller is willing to accept a
lower price than $b(s)$, which occurs if $s^{\prime}>s$; so the buyer "wins" the discount with probability:

$$
\begin{equation*}
W(s)=1-G(s) . \tag{56}
\end{equation*}
$$

The expected payment would be:

$$
\begin{equation*}
M(s)=\int_{s}^{T}\left(\omega b(s)+(1-\omega) b\left(s^{\prime}\right)\right) d G\left(s^{\prime}\right) \tag{57}
\end{equation*}
$$

Now consider Nash bargaining from the seller's perspective. A seller of type $s^{\prime}$ would only find a mutually agreeable price with buyers of type $s<s^{\prime}$, which occurs in a random match with probability $F\left(s^{\prime}\right)$. The exact price $\omega b(s)+(1-\omega) b\left(s^{\prime}\right)$ depends on the type of the buyer, so we integrate over all possibilities.

$$
\begin{equation*}
\rho \Pi_{a}\left(s^{\prime}\right)=\eta\left((1-\ell) \int_{0}^{s^{\prime}}\left(\omega b(s)+(1-\omega) b\left(s^{\prime}\right)\right) d F(s)-F\left(s^{\prime}\right) \Pi_{a}\left(s^{\prime}\right)\right) . \tag{58}
\end{equation*}
$$

## D.1.3 Lottery

Finally, consider a lottery setting. Here, buyers are occasionally presented with a lottery as the discount option, with the freedom to buy as many tickets $k(s)$ as desired, with one being selected at random to win. If the number of lottery tickets purchased by other buyers collectively are distributed according to $G\left(k^{\prime}\right)$, then the probability of winning would be:

$$
\begin{equation*}
W(s)=\int_{0}^{T} \frac{k(s)}{k(s)+k^{\prime}} d G\left(k^{\prime}\right) \tag{59}
\end{equation*}
$$

If $p$ denotes the price of one lottery ticket, then the expected payment would be:

$$
\begin{equation*}
M(s)=p k(s) . \tag{60}
\end{equation*}
$$

A seller's revenue in a lottery setting is simply the number of tickets sold, while the lottery will result in a winner for sure at its close. The expected profit would then be:

$$
\begin{equation*}
\rho \Pi_{a}=\eta\left((1-\ell) \int_{0}^{T} p k^{\prime} d G\left(k^{\prime}\right)-\Pi_{a}\right) . \tag{61}
\end{equation*}
$$

## D. 2 Endogenous Posted Price and Reserve Price

The model assumes that all posted-price sellers charge the same exogenous price $z$. If the model were to be expanded to allow each seller to endogenously choose her own posted price, there would still exist an equilibrium in which all sellers would choose the same $z$. Specifically, if buyers anticipate that all sellers charge the same posted price $z$, they will expend no effort in searching among available sellers, but will choose one at random. Thus, a seller who deviates
by posting a lower price does not sell any faster but sacrifices some profit. Moreover, a seller who deviates by posting a higher price will always be rejected because the buyer anticipates that another seller can immediately be found who charges price $z$. Of course, other equilibria are certainly possible, posing an interesting avenue for future research.

We now relax the assumption that auction sellers always set their reserve price equal to $b(T)$, the lowest bid any buyer might make in equilibrium. There is clearly no incentive to reduce the reserve price below that point: doing so would not bring in any additional bidders, but would decrease revenue in those instances where only one bidder participates.

Now consider a seller who contemplates raising the reserve price to $R>b(T)$, taking the behavior of all others in the market as given. This will only affect the seller if a single bidder arrives or if the second-highest bid is less than $R$. With this higher reserve price, the seller would close the auction without sale in these situations and would re-list the good, a strategy that has a present expected profit of $\Pi_{a}$. Because $\Pi_{a}=c$ in equilibrium, deviating to the reserve price $R$ is certain to be unprofitable if $R<c$. In words, the optimal seller reserve price should equal the total cost of production. Thus, in our context, $b(T)$ is the optimal seller reserve price so long as $b^{*}(T) \geq c$.

If $b^{*}(T)<c$, then the seller would prefer to set a reserve price of $c$. One can still analyze this optimal reserve price in our model by endogenizing the buyer deadline, $T$. For instance, suppose that buyers who enter six months before their deadline are only willing to bid below the cost of production. By raising the reserve price, these bidders are effectively excluded from all auctions; it is as if they do not exist. They only begin to participate once they reach time $S$ such that $b^{*}(S)=c$. In other words, it is as if all buyers enter the market with $S$ units of time until their deadline. To express this in terms of our model, we would make $T$ endogenous, requiring $b^{*}\left(T^{*}\right)=c$ in equilibrium. All else would proceed as before.

Even with sellers using optimal reserve prices, the entry and exit of sellers will ensure that expected profits from entering the market are zero. Any gains from raising the reserve price are dissipated as more auctions are listed. To consider the absence of this competitive response, imagine one seller has monopoly control of both markets. The optimal market design for this monopolist would be to shut down the auction market, forcing all buyers to purchase at the highest price $z$. When there are numerous independent sellers, however, they cannot sustain this degenerate equilibrium (at least when the conditions for degeneracy from Proposition 4 are not satisfied). An individual seller always has an incentive to offer an auction if all other sellers offer posted-price listings: the product sells faster through auctions, even if at a slightly lower price.

## D. 3 Buyer and Seller Heterogeneity

The baseline model assumes ex-ante homogeneity of buyers and sellers. This focus is intentional in order to discipline the model and allow us to isolate the effect of consumer deadlines
on repeated bidding, price dispersion, and sales channel decisions rather than confounding these effects with differences among the market participants. However, the model can accommodate certain types of heterogeneity among buyers or sellers with minimal impact on the overall behavior. For example, some sellers might have stronger preferences than others for posted-prices over auctions; this would determine which sellers would participate in each mechanism, though the overall mix would be determined by the marginal seller, as in the baseline model. The same would occur if some buyers were to have a stronger distaste for auction participation.

Another potential extension would be to allow buyers to differ in their raw consumption utility, which is particularly straightforward when $\beta=0$ (all consumption utility is realized at the deadline). ${ }^{48}$ Suppose $x$ is a random variable drawn for each buyer, similar to the exogenously-given valuations in traditional auction models. If $x$ is bounded below by $z$, all of the model's results carry through without modification, as bids are chosen relative to the posted price (which all bidders have as their common outside option), rather than relative to their idiosyncratic consumption utility. ${ }^{49}$

## D. 4 Endogenous participation

A final group of extensions endogenizes when buyers start or conclude their participation in the discount mechanism. First, suppose that a buyer incurs some cost while searching for auctions. This would lead her to postpone her search until closer to her deadline in an effort to avoid the search cost while the chances of winning are exceptionally low. Relative to our baseline model, this would be a simple extension that would effectively endogenize $T$; buyers would be aware of their need earlier, but search would really begin only once the expected utility from search is equal to the cost of search.

Second, consider a case where buyers must also search to find a posted-price listing. This is in contrast to the baseline model, in which a posted-price option is always readily available. If such search were required, some buyers would abandon the discount market prior to their deadline to increase their chances of securing the good in the posted-price market (depending on the penalty for missing the deadline). This would effectively endogenize participation at the end of the search spell. This extension, and the costly search extension discussed in the previous paragraph, would affect when discount search would begin or end (and must be solved for numerically), but bids would still rise during the search spell and sellers would still find it profitable to utilize both mechanisms.

An alternative adjustment to participation would be to introduce exogenous heterogeneity

[^2]in the initial time-until-deadline $T$ or attention given to discount opportunities $\tau$. For the latter, a buyer might increase her attention $\tau(s)$ as her deadline approaches. Unlike the heterogeneity extensions in the preceding subsection, this type of heterogeneity would disrupt the analytic tractability of the solution; however, we have found that numerical solutions under this extension produce similar qualitative results to our baseline model.

At the same time, we note that observed participation already increases over the search duration in our baseline model, even though attention is assumed to be constant throughout the search. Song (2004) first noted that a buyer who arrives after the auction's current bid exceeds her reservation price will be precluded from submitting a bid and will remain unobserved. In our empirical application, we account for the feature of our model that buyers closer to their deadline have higher reservation prices, and increasing reservation prices also lead to a higher frequency of being observed. We use methods from Platt (2017) to explicitly account for unobserved participation in the structural estimation of the model, as described in Section 5.1 of the paper.

## E Shipping Speeds and Closing Times

We now present two empirical patterns that provide strong ancillary evidence that buyers grow more time-sensitive over the duration of their search. First, after repeated losses, buyers are increasingly likely to participate in auctions where expedited shipping is available, consistent with the time sensitivity we model. Here we define fast shipping as any shipping option with guaranteed delivery within 96 hours. The overall fraction of buyers bidding in auctions with fast shipping available is $44 \%$, and this fraction rises with the number of auctions a buyer has attempted. This can be seen in Figure A1.A. The horizontal axis indicates the total number of auctions a bidder participates in, and the vertical axis indicates the fraction of cases where the last auction the bidder participates in offers fast shipping. We find that those who have participated in more prior auctions gravitate toward fast shipping (roughly $2 \%$ more for each additional auction).

The choice of which auction to use is beyond the scope of the model, but we would expect that fast shipping would be most relevant to buyers within a week of their deadline. Of course, deadlines are not observed in the data, and so Figure A1.A proxies for closeness to the deadline by how many attempts a bidder has made. To give a sense of the magnitude of the effect, we use simulated data from the model to determine the relationship between bidder attempts and closeness to the deadline. This is reported in Figure A1.B, which shows the fraction of bidders who are in their last week at the time of their last bid.

Note that, in the data, participation in fast-shipping auctions is much more prevalent than would be suggested by the model if bidders only join such auctions in their last week. Yet fast-shipping participation still rises 10 percentage points from those who bid once to those who bid six times. In the model, the fraction who are in their last week grows 20 percentage

Figure A1: Shipping Speed


Notes: Panel (A) reports the fraction of bidders in the data participating in an auction with fast shipping during the last bid attempt on a product. Panel (B) reports the fraction of bidders in the simulated data who, at the time of their last bid attempt, are within a week of their deadline.
points. Thus, the participation in fast shipping rises about half as much as the rise in lastweek bidders. We see this as favorable evidence that buyers with longer auction sequences are feeling greater time pressures, though clearly this is not the only reason they participate in fast-shipping auctions.

Second, we find that as bidders move farther along in their search process they are increasingly likely to participate in auctions that are ending soon. Our main sample examines primarily bidders who participate just before the auction closes, so we broaden our analysis here to include non-serious bids. In this broader sample, a buyer's highest bid in a given auction is, on average, placed when there are 1.33 days remaining. Figure A2.A demonstrates that this number decreases steadily and significantly across auction attempts (with the average time until the auction closes falling by $2.43 \%$ per auction attempt), again consistent with growing time sensitivity during the search process. Hendricks and Sorensen (2018) report a similar fact in their data: high-value bidders tend to prefer auctions that end soon. While this preference toward soon-to-close auctions is not explicitly micro-founded in either model, deadlines provide one motivation: in the deadlines model, high-value bidders are precisely those who need the item sooner.

## F Consumer Surplus and Demand

Online retail markets are a rich source of data about consumer demand. However, demand data has wildly different interpretations depending on the model in which it is analyzed. For example, if consumers grow increasingly time sensitive over the duration of their search, ignor-

Figure A2: Days Left in Auction Regression (A) and Derived Demand Curve (B)


Notes: Panel (A) displays estimated coefficients for dummy variables for each auction number (i.e. where the auction appears in the sequence) from a regression of a dependent variable on these auction number dummies and on dummies for the length of auction sequence. The dependent variable is the number of days left in the auction when the bidder bid. Regressions are performed after removing outliers in the auction number variable (defined as the largest $1 \%$ of observations). $95 \%$ confidence intervals are displayed about each coefficient. Panel (B) reports the demand curve inferred from bids reported in Panel (C) of Figure 3 using our deadlines model (solid) vs. treating the data as though it came from a static model (dotted) or a stationary dynamic model (dashed). The dashed line is truncated, but would intersect the vertical axis at a price of 1.41.
ing this non-stationarity would lead to mis-measurement of demand and consumer surplus. ${ }^{50}$ To demonstrate this, we consider two alternatives to our non-stationary dynamic search model: a static model and a stationary dynamic model. Buyers in the static model only make one purchase attempt, while the stationary dynamic model allows multiple attempts; but in both, buyer valuations are exogenously given and constant.

For the static model, assume that the valuation of bidder type $s$ is denoted $x(s)$, which is a decreasing function of $s$. Types are independently drawn from an exogenous distribution $F(s)$. Each bidder has only one opportunity to bid. In such a model, the optimal bid will be $b(s)=x(s)$, so that bids precisely reveal the underlying utility of bidders.

For the stationary dynamic model, $x(s)$ still denotes the valuation of bidder type $s$, and these valuations are persistent throughout their search. Types in a given auction are distributed by $F(s)$, which could be endogenously determined. Bidders participate in auctions at rate $\tau \alpha$ with an average of $\lambda$ bidders per auction. In this dynamic environment, the con-

[^3]tinuation value of a bidder is:
$$
\rho V(s)=\tau \alpha\left(e^{-\lambda F(s)}(x(s)-V(s))-e^{-\lambda} b(T)-\int_{s}^{T} \lambda e^{-\lambda F(t)} b(t) F^{\prime}(t) d t\right)
$$

The optimal bid is $b(s)=x(s)-V(s)$; so after substituting this into the HJB equation, it simplifies to:

$$
\begin{equation*}
x(s) \equiv b(s)+\frac{\tau \alpha}{\rho}\left(e^{-\lambda F(s)} b(s)-e^{-\lambda}\left(b(T)+e^{\lambda} \int_{s}^{T} b(t) \lambda e^{-\lambda F(t)} F^{\prime}(t) d t\right)\right) \tag{62}
\end{equation*}
$$

In the static model, buyers reveal their valuations in their single truthful bid, so the econometrician can estimate demand by inverting the empirical CDF of bids. By way of comparison, if bidding data were generated by our model, but the data is then used to estimate demand under a static model, we obtain the dotted line in Figure A2.B, in a parametric plot of $(H \cdot F(s), b(s))$.

However, in our paper's environment, the buyer's value, $x e^{-\rho s}$, is no longer the same as willingness to pay, $b(s)=x e^{-\rho s}-V(s)$. Buyers are truthful about their willingness to pay, but they they do not bid their full value because tomorrow's discount opportunities provide positive expected surplus. When observed bids are adjusted to determine the valuations, ${ }^{51}$ it generates the true demand curve, depicted as the solid line in Figure A2.B. The static interpretation of data generated from a dynamic process will underestimate demand - on average by $1.4 \%$ of the retail price.

Of course, other dynamic models (Zeithammer, 2006; Said, 2011; Backus and Lewis, 2016; Bodoh-Creed et al., 2018; Hendricks and Sorensen, 2018) can make a similar critique because the option to participate in future discount opportunities reduces buyers' willingness to bid. However, these stationary dynamic models predict that the highest-valuation bidders have the greatest option value from search and thus shade their bids the most aggressively. This is not true in our model, where the highest-valuation bidders are about to abandon the discount mechanism and thus do not shade their bids. If bids were generated by deadline-motivated buyers but interpreted using a stationary dynamic model, it would overstate demand by $28.1 \%$ of the retail price (the dashed line in Figure A2.B). In the stationary model, low-valuation buyers are unlikely to win in current or future auctions and thus they are willing to pay nearly their full valuation. Meanwhile, high-valuations buyers are most likely to win in current and future auctions, so they shade their bids aggressively (by as much as $40 \%$ ). In our nonstationary model, however, high-valuation buyers are closer to their deadline and hence shade less than low-valuation bidders.

[^4]
## G Including Non-Serious Bids: Data and Model Results

Our main sample includes bids submitted in the last hour of the auction and the two highest bids prior to that time ("serious" bids). This screens out extremely low bids that have no chance of winning and yet are never raised later in that auction. ${ }^{52}$ Here we repeat the key analysis from the paper when these non-serious bids are included in the sample. ${ }^{53}$

In Figure A3, we replicate the data facts reported in Figures 1 and 2 of the paper. In Table A6, we report the parameter estimates obtained in this expanded sample compared to the main estimates from the paper. Figure A4 then replicates the comparison of model fit from the paper on key graphs where they are affected.

We note that including non-serious bids leads to more long sequences; thus we report sequences of length up to 10 . We observe the same pattern of increasing average bids in the data among all bidders (Figure A3.A and Figure A3.B), those bidding on expensive items (Figure A3.D), experienced bidders (Figure A3.E), and inexperienced bidders (Figure A3.F). We also observe line sequences that rarely cross. However, including non-serious bids pulls down the average bid amount by almost 20 percentage points, leading to a gap between average bids in the data (in Figure A3.A) and the model equivalent (Figure A4.A), illustrating the better fit of the serious bids sample. The rate of switching to posted prices (Figure A3.C) is essentially identical.

Some other comparisons to the data and model are similar even when non-serious bids are included, such as the distribution of sequence lengths (Figure A4.B) and the duration between bids (Figure A4.C). We find that, with non-serious bids included, the fitted model predicts many more auctions than are observed in the data (Figure A4.D).

Table A6 demonstrates that including non-serious bids has the largest impact in increasing the number of participants per auction $(\lambda)$ and the flow of participants entering the market $(\delta)$. Changes in these fitted parameters then lead to slightly shorter implied time frame for search $(T)$ than in the main model (decreasing from 4.3 to 2.52 months) and a (unrealistically large) estimate for the discount factor ( $\rho$ ), which increases from 0.056 in the paper to 0.381 when non-serious bids are included. The reason the model yields this large estimate for $\rho$ is as follows: the model rationalizes these extremely low-ball bids by treating these bidders as agents who will eventually be willing to pay the full price upon reaching their deadline, thus interrupting the large fraction of non-serious bids as though they (and all bids) must have been generated by agents who steeply discount the future.

[^5]Figure A3: Data Facts, Including Non-Serious Bids


Notes: Figure displays the equivalents of Figures 1 and 2 with non-serious bids included.

Table A6: Data Moments and Parameter Values, Including Non-Serious Bids

|  | Observed in Data |  | Theoretical Equivalent | Fitted Parameter |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Paper |  |  | Paper | With Nonserious |
| Bidders per completed auction | 2.55 | 5.30 | $\frac{\lambda \cdot P(\lambda)}{1-e^{-\lambda}}$ | $\begin{gathered} \hline \lambda=2.97 \\ (0.017) \end{gathered}$ | $\begin{gathered} 13.10 \\ (0.216) \end{gathered}$ |
| Completed auctions per month | 12.76 |  | $\alpha\left(1-e^{-\lambda}\right)$ | $\begin{gathered} \alpha=13.45 \\ (0.490) \end{gathered}$ | $\begin{gathered} 12.76 \\ (0.468) \end{gathered}$ |
| Auctions a bidder is observed in per month | 1.10 | 1.18 | $\frac{\tau \alpha P(\lambda)}{1-e^{-\tau \alpha P(\lambda)}}$ | $\begin{aligned} & \tau=0.019 \\ & (0.00060) \end{aligned}$ | $\begin{gathered} 0.064 \\ (0.0021) \end{gathered}$ |
| New bidders per month who never win | 16.18 | 39.11 | $\begin{gathered} (\delta-\alpha) . \\ \left(1-e^{-\tau \alpha T P(\lambda)}\right) \end{gathered}$ | $\begin{gathered} \delta=41.27 \\ (2.40) \end{gathered}$ | $\begin{aligned} & 81.64 \\ & (3.21) \end{aligned}$ |
| - |  |  | Eq. (10) | $\begin{gathered} T=4.30 \\ (0.139) \end{gathered}$ | $\begin{gathered} 2.54 \\ (0.050) \end{gathered}$ |
| Average revenue per completed auction | 0.853 |  | $\theta$ | $\begin{gathered} \rho=0.055 \\ (0.0023) \end{gathered}$ | $\begin{gathered} 0.379 \\ (0.011) \end{gathered}$ |
| Average listing fee paid | 0.116 |  | $\ell$ | $\begin{gathered} \ell=0.116 \\ (0.0028) \\ \eta=6.40 \end{gathered}$ |  |
| Average duration of an auction listing (months) | 0.156 |  | $1 / \eta$ | (0.027) |  |
| - | - |  | Eq. (20) | $\begin{gathered} c=0.747 \\ (0.0034) \end{gathered}$ | $\begin{gathered} 0.712 \\ (0.0037) \end{gathered}$ |

Notes: Table displays the equivalent of Table 2 with non-serious bids included in the data sample moments.

Figure A4: Model Fit, Including Non-Serious Bids


Notes: Figure displays the equivalents of Figures 3.A, Figure 4.A-B, and Figure 6.A with non-serious bids included in the data sample moments and with the model-fitting exercise performed using this expanded sample.

## H Differences Across Products in Behavior and Parameter Estimates

Not surprisingly, behavior in our data varies across products. We illustrate this here by examining the amount of repeat bidding and the rate of bid increase. For the average product, repeat bidders place $23 \%$ of bids, with an interquartile range of 13 percentage points across all products. The highest rates of repeat bidding occur with Computers/Tablets (26.5\%) and DVDs/Movies (24.9\%). We note that items with higher rates of repeat bidding typically have lower estimates for $T$. The rate of bid increase also varies across products. For the average product, bidders raise their bid on average by $1.9 \%$ per observed attempt, with an interquartile range of 2.4 percentage points. This rate is nearly the same across the major categories with the exception of Video Games (4.9\%).

We now explore estimation of the model's parameters separately for each product. The estimates presented in the paper (with the exception of Figure 6.B) uses aggregate data moments, aggregated across all products, yielding a fit that is representative of the average product in the market. Here, instead, we estimate the model's parameters product-by-product, matching the data moments for a given product to the theoretical moment to obtain productspecific parameter estimates.

Table A7 summarizes the mean, standard deviation, and median of these product-level parameter estimates. The first column in Table A7 displays our main estimates for comparison. For the parameters $\lambda, \alpha, \delta, \eta, \ell$ and $c$, the mean product-level estimates are nearly the same as the main estimates. In contrast, the average $\tau, \rho$, and $T$ are somewhat larger. For these three parameters, the median products estimates are smaller (and, for $\tau$ and $T$, are in closer agreement to our main estimates). A few factors contribute to the distinctions between our main estimates and these product-by-product estimates.

First, some targeted moments are not normally distributed across products. For instance, the monthly flow of new buyers is highly skewed, with $75 \%$ of the products below the mean of 15 , while the top $1 \%$ of products reach into triple digits. Attempted auctions per month are similarly skewed, though to a lesser degree. Second, the estimation procedure tends to add or exacerbate skewness in $\lambda$, due to the non-linearity of $P(\lambda)$.

Together, these factors lead to disproportionately skewed product-level estimated parameters. The aggregated targets are necessarily kept away from extremes, but any given product target could be an extreme, and such outliers have a large influence on the average of the product-level parameters. This skewness also explains why the median estimates are in closer agreement for some parameters.

We also note that $11 \%$ of individual products cannot fit the model under any parameters. Most of these misfits are due to cases where the average auction revenue is greater than the average posted price, which our model cannot rationalize. However, for about $2 \%$ of products, the lack of a solution is because bidders on those products are never observed bidding in more

Table A7: Comparison of Parameter Estimates

|  |  | Product-Level Parameter Estimates |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Main Parameter Estimates | Mean | Median | Std. Dev. | Coeff. Var. |
| $\lambda$ | 2.98 | 3.10 | 3.01 | 1.28 | 0.41 |
| $\alpha$ | 13.44 | 13.36 | 7.59 | 29.26 | 2.19 |
| $\tau$ | 0.019 | 0.033 | 0.020 | 0.039 | 1.19 |
| $\delta$ | 41.28 | 41.06 | 22.17 | 127.74 | 3.11 |
| $T$ | 4.3 | 7.75 | 5.43 | 7.37 | 0.95 |
| $\rho$ | 0.056 | 0.064 | 0.040 | 0.087 | 1.36 |
| $\eta$ | 6.39 | 6.91 | 6.24 | 2.46 | 0.36 |
| $\ell$ | 0.12 | 0.11 | 0.10 | 0.17 | 1.48 |
| $c$ | 0.75 | 0.73 | 0.76 | 0.14 | 0.19 |

Notes: Main parameter estimates come from Table 2 (which were estimated by fitting model moments to the average of product-level averages of data moments). The mean, median, and standard deviation columns display the mean, median, and standard deviation across products of parameters estimated separately for each product. Coefficient of variation displays the standard deviation over the mean.
than one auction (which is the data moment used to identify $\tau$ ).
While our data permits us to classify equivalent products together through an anonymized product id, it does not allow us to see what the product actually is. For example, we cannot tell whether a given product id corresponds to an X-box or a PlayStation. This limits our ability to consider whether particular parameters seem appropriate for a specific product. However, we can analyze heterogeneity using a broad category identifier and the average price level of the product.

In Figure A5, we show how estimates for $T, \rho$, and $c$ vary across these classifications and within them. We focus on these three parameters because they are easily interpretable even beyond the eBay context (unlike, for example, $\lambda$ ). The box indicates the 25 th, 50 th, and 75 th percentiles for products within a given classification, while the whiskers extend to the 5th and 95th percentile. The categories we display include at least 100 products from our sample, while the price ranges split our products into roughly four quartiles.

While some groups of products have systematically higher $T$ (in the first row), such as toys or items under $\$ 12$, the variation within each group is very large. Similarly, estimates for $\rho$ (in the second row) can be very high for more expensive products, and for categories such as health or electronics. For the estimates for $c$ (in the third row), note that a lower estimated cost $c$ is equivalent to a higher percentage markup. We find that this markup appears to be higher among toys, movies, and health products, as well as lower-priced items.

Figure A5: Distribution of Product-Level Estimates


Notes: Panels (A), (C), and (E) display model parameters estimated separately product-by-product and then aggregated by product category for each category containing at least 100 products. In each panel, boxes indicate the 25 th, 50 th, and 75 th percentiles of the parameter estimate for products within a given category, while the whiskers extend to the 5th and 95th percentile. For categories with large values for the 95th percentile, the value of the 95 th percentile is shown in red type. Panels (B), (D), and (F) display similar results but aggregated by average posted price level of each product rather than by product category. Panels (A) and (B) display estimates of $T$, panels (C) and (D) display estimates of $\rho$, and panels (E) and (F) display estimates of $c$.

Table A8: eBay Fees Over Time

|  |  |  |  | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Auction | Insertion fee (by starting or reserve price) |  | 0.01-0.99 | \$0.30 | \$0.25 | \$0.20 | \$0.20 | \$0.15 | \$0.15 | \$0.10 | \$0.10 | \$0.10 | \$0.30 | \$0.30 | \$0.30 |
|  |  |  | 1.00-9.99 | \$0.35 | \$0.35 | \$0.35 | \$0.35 | \$0.35 | \$0.35 | \$0.25 | \$0.25 | \$0.25 | \$0.30 | \$0.30 | \$0.30 |
|  |  |  | 10.00-24.99 | \$0.60 | \$0.60 | \$0.60 | \$0.60 | \$0.55 | \$0.55 | \$0.50 | \$0.50 | \$0.50 | \$0.30 | \$0.30 | \$0.30 |
|  |  |  | 25.00-49.99 | \$1.20 | \$1.20 | \$1.20 | \$1.20 | \$1.00 | \$1.00 | \$0.75 | \$0.75 | \$0.75 | \$0.30 | \$0.30 | \$0.30 |
|  |  |  | 50.00-199.99 | \$2.40 | \$2.40 | \$2.40 | \$2.40 | \$2.00 | \$2.00 | \$1.00 | \$1.00 | \$1.00 | \$0.30 | \$0.30 | \$0.30 |
|  |  |  | 200.00-499.99 | \$3.60 | \$3.60 | \$3.60 | \$3.60 | \$3.00 | \$3.00 | \$2.00 | \$2.00 | \$2.00 | \$0.30 | \$0.30 | \$0.30 |
|  |  |  | 500.00+ | \$4.80 | \$4.80 | \$4.80 | \$4.80 | \$4.00 | \$4.00 | \$2.00 | \$2.00 | \$2.00 | \$0.30 | \$0.30 | \$0.30 |
|  | Final value fee (\% of closing price, cumulative) |  | 0.01-25.00 | 5.25 | 5.25 | 5.25 | 5.25 | 8.75 | 8.75 | 9.00 | 9.00 | 9.00 | 10.00 | 10.00 | 10.00 |
|  |  |  | 25.01-1000.00 | 2.75 | 2.75 | 3.00 | 3.25 | 3.50 | 3.50 | 9.00 | 9.00 | 9.00 | 10.00 | 10.00 | 10.00 |
|  |  |  | 1000+ | 1.50 | 1.50 | 1.50 | 1.50 | 1.50 | 1.50 | 9.00 | 9.00 | 9.00 | 10.00 | 10.00 | 10.00 |
|  |  |  | Maximum charge |  |  |  |  |  |  | \$50.00 | \$100.00 | \$250.00 | \$250.00 | \$250.00 | \$750.00 |
| Posted Price | Final value fee (\% of posted price, cumulative) | Consumer <br> Electronics | \$0.01-\$50.00 | same as Auction style | same as Auction style | same as <br> Auction style | same as Auction style | 8.00 | 8.00 | 8.00 | 7.00 | 7.00 |  | same as <br> Auction style | same as <br> Auction <br> style |
|  |  |  | \$50.01- \$1,000.00 |  |  |  |  | 4.50 | 4.50 | 5.00 | 5.00 | 5.00 |  |  |  |
|  |  |  | \$1,000.00+ |  |  |  |  | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 |  |  |  |
|  |  | Computers \& Networking | \$0.01-\$50.00 |  |  |  |  | 6.00 | 6.00 | 8.00 | 7.00 | 7.00 |  |  |  |
|  |  |  | \$50.01- \$1,000.00 |  |  |  |  | 3.75 | 3.75 | 5.00 | 5.00 | 5.00 |  |  |  |
|  |  |  | \$1,000.00+ |  |  |  |  | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 |  |  |  |
|  |  | Clothing, Shoes \& Accessories | \$0.01-\$50.00 |  |  |  |  | 12.00 | 12.00 | 12.00 | 10.00 | 10.00 |  |  |  |
|  |  |  | \$50.01- \$1,000.00 |  |  |  |  | 9.00 | 9.00 | 9.00 | 8.00 | 8.00 |  |  |  |
|  |  |  | \$1,000.00+ |  |  |  |  | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |  |  |  |
|  |  | Media | \$0.01-\$50.00 |  |  |  |  | 15.00 | 15.00 | 15.00 | 13.00 | 13.00 |  |  |  |
|  |  |  | \$50.01- \$1,000.00 |  |  |  |  | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |  |  |  |
|  |  |  | \$1,000.00+ |  |  |  |  | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |  |  |  |
|  |  | All Other Categories | \$0.01-\$50.00 |  |  |  |  | 12.00 | 12.00 | 12.00 | 11.00 | 11.00 |  |  |  |
|  |  |  | \$50.01- \$1,000.00 |  |  |  |  | 6.00 | 6.00 | 6.00 | 6.00 | 6.00 |  |  |  |
|  |  |  | \$1,000.00+ |  |  |  |  | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |  |  |  |
|  | Insertion fee (by posted price) | Media | \$1.00+ |  |  |  |  | \$0.15 | \$0.15 | \$0.50 | \$0.50 | \$0.50 | \$0.05 | \$0.05 |  |
|  |  | Other Categories | \$1.00+ |  |  |  |  | \$0.35 | \$0.35 | \$0.50 | \$0.50 | \$0.50 | \$0.30 | \$0.30 |  |
| (1) Final Value Fee (\%) at Average Price in Auction Sample (\$97) |  |  |  | 3.39 | 3.39 | 3.58 | 3.77 | 4.85 | 4.85 | 9.00 | 9.00 | 9.00 | 10.00 | 10.00 | 10.00 |
| (2) Final Value Fee (\%) at Median Price in Auction Sample (\$31) |  |  |  | 4.77 | 4.77 | 4.81 | 4.86 | 7.73 | 7.73 | 9.00 | 9.00 | 9.00 | 10.00 | 10.00 | 10.00 |
| (3) Einav et al. (2018) Fraction Revenue from Auctions |  |  |  | 0.83 | 0.79 | 0.75 | 0.71 | 0.62 | 0.49 | 0.49 | 0.49 | 0.45 | 0.40 | 0.35 | 0.30 |
| (4) Backus et al. (2018) Fraction Revenue from Auctions |  |  |  | -- | 0.82 | 0.76 | 0.73 | 0.66 | 0.56 | 0.52 | 0.48 | 0.42 | 0.36 | 0.32 | 0.27 |

Notes: Fees come from archived eBay.com pages on Wayback Machine (one snapshot in summer or fall of each year). No final value fee charged if item not sold. Starting in 2011, final value includes shipping fee. First 50 listings per month have no insertion fees starting in 2011 for auctions and 2013 for posted prices. "Media" category nests Books, Music, DVDs \& Movies, Video Games. "Consumer Electronics" category nests Consumer Electronics, Video Game Systems, Cameras \& Photo. Some additional category-specific exceptions are omitted from table, as are other optional promotion or listing add-on fees. Insertion and final value fees after 2015 are relatively constant and are omitted from table. Rows (1) and (2) at bottom of table show commission based only on auction final value fees evaluated at the same price in different years: $\$ 97$ for row (1) (the mean of auction price plus shipping in the paper sample) and $\$ 31$ for (2) (the median). Rows (3) comes from Figure 1 of Einav et al. (2018) and row (4) from Figure 1 of Backus et al. (2018).


[^0]:    ${ }^{46}$ Coey: Facebook, Core Data Science; coey@fb.com. Larsen: Stanford University, Department of Economics and NBER, bjlarsen@stanford.edu. Platt: Brigham Young University, Department of Economics, brennan_platt@byu.edu

[^1]:    ${ }^{47}$ In equilibrium search models, a degenerate equilibrium often exists regardless of parameter values, essentially as a self-fulfilling prophecy. Buyers won't search if there is only one price offered, and sellers won't compete with differing prices if buyers don't search. Yet in our auction environment, the degenerate equilibrium does not always exist. This is because our buyers do not incur any cost to watch for auctions; even if no auctions are expected, buyers are still passively available should one occur. In that sense, they are always searching, giving sellers motivation to offer auctions when (51) does not hold.

[^2]:    ${ }^{48}$ If $\beta>0$, some of the utility $x$ is immediately obtained on purchase, and becomes relevant in the bidding function. This disrupts analytic tractability of the equilibrium bidding function, but we have found that numeric solutions under this extension preserve the same qualitative features as the baseline model.
    ${ }^{49}$ The behavior is more nuanced if $x$ can be less than $z$; in such a setting, some bidders would be worse off purchasing at the posted price, and extending the model in this case would require specifying the consequences of missing the deadline.

[^3]:    ${ }^{50}$ Incorrect estimates of the demand curve could potentially distort calculations needed for profit maximization, price discrimination, regulation, and other applications. Moreover, individual-level estimates of willingness to pay are essential in providing individualized product recommendations, targeted advertising, and personalized pricing. One implication of consumer-specific deadlines is that firms engaged in personalized pricing based on consumer data (e.g. Kehoe et al. 2018) might benefit by including in their models a measure of a given consumer's observed search duration.

[^4]:    ${ }^{51}$ Here, we set $x=z$, which creates the smallest difference between the static model and ours.

[^5]:    ${ }^{52}$ Note that dropping these bids, for the most part, drops particular bidders who do not appear to be ever bidding seriously. Only $16 \%$ of bidders are observed in the data having a serious bid in one auction and a non-serious bid in another auction. All other bidders place only serious bids or only non-serious bids. This fact, along with the fact that non-serious bids do not affect final prices, suggests that non-serious bidding is unlikely to be an important strategic or outcome-driving feature of the marketplace.
    ${ }^{53}$ An alternative way to expand the sample would be to lengthen the window for a bid to qualify as serious; not surprisingly, such an approach yields results a mixture of the paper results and those presented here.

