# The Paradox of Global Thrift Online Appendix

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# A Additional lemmas

**Lemma 1** Suppose that the market for non-tradable goods clears competitively, so that the (AD) and (MP) equations hold, and that the world interest rate R and the inflation target  $\bar{\pi}$  satisfy  $R\bar{\pi} > 1$ . Then there cannot be a stationary equilibrium with  $R_{i,t}^n = 1$  for all t. Moreover if  $C_h^T \ge C_l^T$  then  $R_h^n > 1$ .

**Proof.** To prove the first part of the lemma, consider that in a stationary equilibrium the (AD) equation in the low and high state can be written as

$$Y_h^N = \frac{R\bar{\pi}}{R_h^n} \frac{C_h^T}{C_l^T} Y_l^N \tag{A.1}$$

$$Y_l^N = \frac{R\bar{\pi}}{R_l^n} \frac{C_l^T}{C_h^T} Y_h^N. \tag{A.2}$$

Combining these two expressions gives

$$R_h^n R_l^n = (R\bar{\pi})^2 > 1. \tag{A.3}$$

Since  $R_{i,t}^n \ge 1$  then max  $\left\{ R_h^N, R_l^N \right\} > 1$ .

We now prove that if  $C_h^T \ge C_l^T$  then  $R_h^n > 1$ . Suppose that this is not the case and  $R_h^n = 1$ . We have just proved that if  $R_h^h = 1$  then  $R_l^n > 1$ , and so  $Y_l^N = 1$ . We can thus write the (AD) equation in the high state as

$$Y_h^N = R\bar{\pi}C_h^T/C_l^T. \tag{A.4}$$

 $R\bar{\pi} > 1$  and  $C_h^T \ge C_l^T$  imply that the right-hand side is larger than one. Since  $Y_h^N \le 1$ , we have found a contradiction. So  $C_h^T \ge C_l^T$  implies  $R_h^n > 1$ .

## **B** Proofs

## B.1 Proof of Proposition 1

**Proposition 1** In a laissez-faire equilibrium with vanishing liquidity if  $R^{lf} \ge R^* \equiv (\bar{\pi}\beta)^{-1/2}$  then  $Y_h^N = Y_l^N = 1$ , otherwise  $Y_h^N = 1$  and  $Y_l^N = (R^{lf}/R^*)^2 < 1$ .

**Proof.** Since we are considering a stationary equilibrium satisfying  $R\bar{\pi} > 1$  and  $C_h^T > C_l^T$ , Lemma 1 applies and so  $R_h^n > 1$ . By the (MP) equation then  $Y_h^N = 1$ . The (AD) equation in the low state can then be written as

$$Y_l^N = \frac{R\bar{\pi}}{R_l^n} \frac{C_l^T}{C_h^T} = \frac{R^{lf}\bar{\pi}}{R_l^n} \frac{Y_l^T}{Y_h^T},$$

where the second equality makes use of the equilibrium relationships  $R = R^{lf}$ ,  $C_h^T = Y_h^T$  and  $C_l^T = Y_l^T$ . Define  $R^* \equiv (\bar{\pi}\beta)^{-1/2}$ . Combining the expression above with (??) and (MP), gives that if  $R^{lf} \ge R^*$ , then  $Y_l^N = 1$  and  $R_l^n \ge 1$ , otherwise  $R_l^n = 1$  and  $Y_l^N = (R^{lf}/R^*)^2 < 1$ .

#### **B.2** Proof of Proposition 2

**Proposition 2** Suppose that  $1/\bar{\pi} < R < 1/\beta$ . Define  $\bar{R}^* \equiv (\omega/(\bar{\pi}\beta))^{1/2}$ . A stationary solution to the national planning problem satisfies  $B_l = 0$  and  $B_h = \max\{\mathcal{B}_h^p(R), 0\}$ , where the function  $\mathcal{B}_h^p(R)$  is defined by

$$\mathcal{B}_{h}^{p}(R) = \begin{cases} \frac{\beta}{\omega+\beta} \left(Y_{h}^{T} - \frac{\omega Y_{l}^{T}}{\beta R}\right) & \text{if } R < \bar{R}^{*} \\ \frac{Y_{h}^{T} - R\bar{\pi}Y_{l}^{T}}{1 + R^{2}\bar{\pi}} & \text{if } \bar{R}^{*} \le R < R^{*} \\ \frac{\beta}{1+\beta} \left(Y_{h}^{T} - \frac{Y_{l}^{T}}{\beta R}\right) & \text{if } R^{*} \le R. \end{cases}$$
(B.1)

Moreover,  $\bar{\mu}_h > 0$  if  $\mathcal{B}_h^p(R) < 0$ , otherwise  $\bar{\mu}_h = 0$ . Finally,  $Y_h^N = 1$  and  $Y_l^N = \min\{1, R\bar{\pi}(Y_l^T + RB_h)/(Y_h^T - B_h)\}$ .

**Proof.** We break down the proof in several steps. We start by proving the the zero lower bound does not bind in the high state, and then we show that the borrowing constraint binds in the low state.

1. Zero lower bound does not bind in high state ( $\bar{v}_h = 0, Y_h^N = 1$ ). Suppose that  $\bar{v}_h > 0$ and  $R_h^n = 1$ . Since Lemma 1 applies then  $R_l^n > 1, Y_l^N = 1$  and  $C_l^T > C_h^T$ . But  $C_l^T > C_h^T$  only if  $B_h > 0$  and so if  $\bar{\mu}_h = 0$ . We can then write the Euler equation (30) in the high state as

$$\frac{\omega + \bar{v}_h Y_h^N}{C_h^T} = \beta R \frac{\omega}{C_l^T} - \bar{v}_h Y_h^N \frac{\partial C_l^T / \partial B_h}{C_l^T}.$$
(B.2)

Since  $\beta R < 1$  and  $\partial C_l^T / \partial B_h \ge 0$ , this expression implies  $C_l^T < C_h^T$ . We have thus reached a contradiction and proved that  $\bar{v}_h = 0$  and  $Y_h^N = 1$ .

2. Borrowing constraint binds in low state ( $\bar{\mu}_l > 0$ ). Suppose instead that  $\bar{\mu}_l = 0$ . Thus,

considering that  $Y_h^N = 1$  and  $\bar{v}_h = 0$ , the Euler equation (30) in the low state implies

$$\frac{\omega + \bar{v}_l Y_l^N}{C_l^T} = \beta R \frac{\omega}{C_h^T} - \bar{v}_l Y_l^N \frac{\partial C_h^T / \partial B_l}{C_h^T}.$$
(B.3)

Since  $\beta R < 1$  and  $\partial C_h^T / \partial B_l \ge 0$ , the following condition needs to hold  $C_h^T < C_l^T$ . Since  $Y_h^T > Y_l^T$  and  $B_l \ge 0$ , this is possible only if  $B_h > 0$  and so if  $\bar{\mu}_h = 0$ . Then the Euler equation (30) in the high state is

$$\frac{\omega}{C_h^T} = \beta R \frac{\omega + \bar{v}_l Y_l^N}{C_l^T}.$$
(B.4)

By combining (B.3) and (B.4), and using  $\partial C_h^T / \partial B_l \ge 0$ , we obtain  $\beta R \ge 1$ . This contradicts the condition  $\beta R < 1$ . Thus, it must be that  $\bar{\mu}_l > 0$ .

We now derive the function  $\mathcal{B}_h^p(R)$ . This function captures the planner's demand for bonds in the high state  $(B_h)$  when the borrowing constraint does not bind  $\bar{\mu}_h = 0$ .

**3.**  $\mathcal{B}_{h}^{p}(R)$  for  $R \geq R^{*}$ . We start by showing that if  $\bar{\mu}_{h} = 0$  then  $\bar{v}_{l} = 0$ . Suppose instead that  $\bar{v}_{l} > 0$ . Since  $\bar{v}_{h} = 0$ , we can write (30) as

$$\frac{\omega}{C_h^T} = \frac{\beta R}{C_l^T} \left( \omega + \bar{v}_l Y_l^N \right). \tag{B.5}$$

It must then be that  $C_l^T/C_h^T > \beta R$ . Using the (AD) in the low state we can then write

$$Y_l^N = \bar{\pi} R C_l^T / C_h^T > \bar{\pi} \beta R^2.$$
(B.6)

Since  $Y_l^N \leq 1$ , the expression above implies  $\bar{\pi}\beta R^2 < 1$ . Since we are focusing on the case  $R \geq R^*$  we have found a contradiction and proved that  $\bar{v}_l = 0$ .

Hence, (B.5) implies that  $\beta R C_h^T = C_l^T$ . Using the resource constraint it is then easy to show that if  $\bar{\mu}_h = 0$  then

$$B_h = \frac{\beta}{1+\beta} \left( Y_h^T - \frac{Y_l^T}{\beta R} \right)$$

4.  $\mathcal{B}_{h}^{p}(R)$  for  $R^{*} > R \geq \bar{R}^{*}$ . Using the same logic of step 3 above, it is easy to check that if  $R < R^{*}$  and  $\bar{\mu}_{h} = 0$  then  $\bar{\nu}_{l} > 0$ . We start by showing that for  $R^{*} > R \geq \bar{R}^{*}$  if  $\bar{\mu}_{h} = 0$  then the economy operates at full employment in the low state  $(Y_{l}^{N} = 1)$ . We can write (30) in the high state as

$$\frac{\omega}{C_h^T} = \frac{\beta R}{C_l^T} (\omega + \bar{\nu}_l Y_l^N) = \frac{\beta R}{C_l^T} (1 - \bar{\nu}_l), \tag{B.7}$$

where the second equality makes use of  $Y_l^N = 1$  and (25). Moreover, since  $\bar{v}_l > 0$  the (AD) equation in the low state implies

$$1 = R\bar{\pi}C_l^T/C_h^T. \tag{B.8}$$

Combining (B.7) and (B.8) gives

$$1 = \frac{\bar{\pi}\beta R^2 (1 - \bar{\nu}_l)}{\omega}.$$
(B.9)

Since we are free to set  $\bar{\nu}_l$  to any non-negative number, the expression above implies that a sufficient condition for  $Y_l^N = 1$  to be a solution is that  $R \ge (\omega/(\bar{\pi}\beta))^{1/2} \equiv \bar{R}^*$ . We have thus proved that if  $R^* > R \ge \bar{R}^*$  and  $\bar{\mu}_h = 0$  then  $Y_l^N = 1$ .

To solve for  $B_h$ , again assuming  $\bar{\mu}_h = 0$ , we can use (B.8),  $C_h^T = Y_h^T - B_h$  and  $C_l^T = Y_l^T + RB_h$  to write

$$B_h = \frac{Y_h^T - R\bar{\pi}Y_l^T}{1 + R^2\bar{\pi}}.$$
 (B.10)

5.  $\mathcal{B}_{h}^{p}(R)$  for  $R < \overline{R}^{*}$ . Suppose that the equilibrium is such that  $\overline{\mu}_{h} = 0$ . From the logic above we know that  $\overline{v}_{l} > 0$  and  $Y_{l}^{N} < 1$ . We set  $\overline{\nu}_{l} = 0$  and we use (25) to obtain  $\overline{v}_{l} = (1 - \omega)/Y_{l}^{N}$ . Plugging this condition in the Euler equation for the high state gives

$$\frac{C_l^T}{C_h^T} = \frac{\beta R}{\omega}.\tag{B.11}$$

By combining the expression above with (AD) in the low state we can write

$$Y_l^N = \bar{\pi} \frac{\beta R^2}{\omega} < 1. \tag{B.12}$$

To solve for  $B_h$ , again assuming that  $\bar{\mu}_h = 0$ , we use  $C_h^T = \beta R C_l^T / \omega$ ,  $C_h^T = Y_h^T - B_h$  and  $C_l^T = Y_l^T + R B_h$  to write

$$B_h = \frac{\beta}{\omega + \beta} \left( Y_h^T - \frac{\omega Y_l^T}{\beta R} \right). \tag{B.13}$$

6. Solution to the planning problem. We have showed that  $B_l = 0$  and  $B_h = \max \{\mathcal{B}_h^p(R), 0\}$ . Moreover, we have proved that  $Y_h^N = 1$ . Using  $C_h^T = Y_h^T - B_h$  and  $C_l^T = Y_l^T + RB_h$  we can then write output in the low state as  $Y_l^N = \min \{1, R\bar{\pi}(Y_l^T + RB_h)/(Y_h^T - B_h)\}$ .

#### B.3 Proof of Corollary 1

**Corollary 1** Consider a small open economy facing the world interest rate  $R^{lf}$ . If  $R^{lf} < R^*$  the national planner allocation features higher  $Y_l^N$ ,  $B_h$  and welfare compared to laissez faire, otherwise the two allocations coincide.

**Proof.** Since  $1/\bar{\pi} < R^{lf} < 1/\beta$  Proposition 2 applies. It is then straightforward to check that if  $R^{lf} \ge R^*$  the two allocations coincide (and feature  $Y_l^N = 1$  and  $B_h = 0$ ), while if  $R^{lf} < R^*$  then the planning allocation features higher  $Y_l^N$  and  $B_h$  compared to laissez faire.

We are left to prove that if  $R^{lf} < R^*$  the planning allocation features higher welfare compared to laissez faire. For households living in a country in the high-endowment state, the expected lifetime utility associated to  $(C_h^T, C_l^T, Y_h^N, Y_l^N)$  is

$$\mathcal{W} = \frac{1}{1-\beta^2} \left( \omega \log C_h^T + (1-\omega) \log Y_h^N + \beta \left( \omega \log C_l^T + (1-\omega) \log Y_l^N \right) \right)$$
(B.14)

Let us start from the laissez-faire case. Since  $R = R^{lf}$ , the Euler equation for high-state countries holds with equality, meaning that  $C_l^T/C_h^T = \beta R^{lf}$ . Moreover, the resource constraint for tradable goods (13) and  $B_l = 0$  imply

$$C_{h}^{T} + \frac{C_{l}^{T}}{R^{lf}} = Y_{h}^{T} + \frac{Y_{l}^{T}}{R^{lf}}.$$
(B.15)

We thus have that

$$C_h^T = \frac{1}{1+\beta} \left( Y_h^T + \frac{Y_l^T}{R^{lf}} \right) \tag{B.16}$$

$$C_l^T = \frac{\beta R}{1+\beta} \left( Y_h^T + \frac{Y_l^T}{R^{lf}} \right). \tag{B.17}$$

Finally,  $Y_h^N = 1$  and, since  $R^{lf} < R^*$ ,  $Y_l^N = R^{lf} \bar{\pi} C_l^T / C_h^T = (R^{lf})^2 \bar{\pi} \beta < 1$ . We can then write the expected lifetime utility under laissez faire as

$$\mathcal{W}^{lf} = \frac{1}{1-\beta^2} \left( \omega \log\left(\frac{1}{1+\beta}\right) + \beta \left( \omega \log\left(\frac{\beta R^{lf}}{1+\beta}\right) + (1-\omega) \log\left((R^{lf})^2 \bar{\pi}\beta\right) \right) + (1+\beta) \omega \log\left(Y_h^T + \frac{Y_l^T}{R^{lf}}\right) \right)$$
(B.18)

Turning to the planning allocation, start by considering the case  $R^{lf} < \bar{R}^*$ . Then  $C_l^T/C_h^T = \beta R^{lf}/\omega$  and so

$$C_h^T = \frac{\omega}{\omega + \beta} \left( Y_h^T + \frac{Y_l^T}{R^{lf}} \right)$$
(B.19)

$$C_l^T = \frac{\beta R^{lf}}{\omega + \beta} \left( Y_h^T + \frac{Y_l^T}{R^{lf}} \right).$$
(B.20)

Moreover  $Y_h^N = 1$  and, since  $R^{lf} < \bar{R}^*$  then  $Y_l^N = R^{lf} \bar{\pi} C_l^T / C_h^T = (R^{lf})^2 \bar{\pi} \beta / \omega$ . We can then write the expected lifetime utility under the planning allocation as

$$\mathcal{W}^{p} = \frac{1}{1-\beta^{2}} \left( \omega \log\left(\frac{\omega}{\omega+\beta}\right) + \beta \left( \omega \log\left(\frac{\beta R^{lf}}{\omega+\beta}\right) + (1-\omega) \log\left((R^{lf})^{2} \bar{\pi}\beta/\omega\right) \right) + (1+\beta)\omega \log\left(Y_{h}^{T} + \frac{Y_{l}^{T}}{R^{lf}}\right) \right)$$
(B.21)

After some algebra, the difference in welfare between the planning and the laissez-faire allocations can be written as

$$\mathcal{W}^p - \mathcal{W}^{lf} = \frac{1}{1 - \beta^2} \left( \omega (1 + \beta) \left( \log(1 + \beta) - \log(\omega + \beta) \right) + \left( \omega - \beta(1 - \omega) \right) \log \omega \right) \equiv \mathcal{F}(\omega).$$
(B.22)

We now show that the function  $\mathcal{F}(\omega)$  satisfies  $\mathcal{F}(\omega) > 0$  for  $0 < \omega < 1$ . First consider that  $\mathcal{F}(1) = 0$ . Moreover, differentiating (*B*.22) with respect to  $\omega$  and rearranging the resulting expression, we have that

$$\mathcal{F}'(\omega) = \frac{1}{1-\beta^2} \left( (1+\beta) \left( \log \left( \frac{\omega(1+\beta)}{\omega+\beta} \right) \right) - \frac{\beta^2(1-\omega)}{\omega(\omega+\beta)} \right).$$
(B.23)

This expression implies that  $\mathcal{F}'(\omega) < 0$  for  $0 < \omega < 1$ . It must then be that  $\mathcal{F}(\omega) > 0$  for  $0 < \omega < 1$ . We have thus proved that the planning allocation attains higher welfare compared to the laissez faire one.

To conclude the proof, we turn to the case  $\bar{R}^* \leq R^{lf} < R^*$ . In this case the planning allocation

features  $Y_h^N = 1$  and  $Y_l^N = R^{lf} \bar{\pi} C_l^T / C_h^T = 1$ . Since  $C_l^T / C_h^T = 1 / (R^{lf} \bar{\pi})$  we have

$$C_{h}^{T} = \frac{\bar{\pi} (R^{lf})^{2}}{1 + \bar{\pi} (R^{lf})^{2}} \left( Y_{h}^{T} + \frac{Y_{l}^{T}}{R^{lf}} \right)$$
(B.24)

$$C_{l}^{T} = \frac{R^{lf}}{1 + \bar{\pi} (R^{lf})^{2}} \left( Y_{h}^{T} + \frac{Y_{l}^{T}}{R^{lf}} \right).$$
(B.25)

After a few steps of algebra, and defining  $x \equiv (R^{lf})^2 \bar{\pi}$ , we can then write

$$\mathcal{W}^p - \mathcal{W}^{lf} = \frac{1}{1 - \beta^2} \left( \omega(1 + \beta) \log\left(\frac{x(1 + \beta)}{1 + x}\right) - \beta \log\left(\beta x\right) \right) \equiv \mathcal{G}(x).$$
(B.26)

Now notice that  $\omega/\beta < x < 1/\beta$  and  $\mathcal{G}(1/\beta) = 0$ . Now differentiating the function  $\mathcal{G}(x)$  gives

$$\mathcal{G}'(x) = \omega(1+\beta) \left(\frac{1}{x} - \frac{1}{1+x}\right) - \frac{\beta}{x}.$$
(B.27)

Using the fact that  $x \ge \omega/\beta$  and  $\omega < 1$  one can then check that  $\mathcal{G}'(x) < 0$  for  $\omega/\beta < x < 1/\beta$ . It must then be that  $\mathcal{G}(x) > 0$  for  $\omega/\beta < x < 1/\beta$ . We have thus completed the proof by showing that the planning allocation attains higher welfare compared to laissez faire when  $\bar{R}^* \le R^{lf} < R^*$ .<sup>1</sup>

#### B.4 Proof of Proposition 3

**Proposition 3** Global equilibrium with current account policies. Suppose that  $R^{lf} < R^*$ and  $\omega R^{lf} \bar{\pi} > 1$ . Then in a vanishing-liquidity equilibrium with current account policies  $R = R^p \equiv \omega R^{lf}$ . Moreover, for every country output and welfare are lower in the equilibrium with current account policies compared to the laissez-faire one.

**Proof.** In a vanishing-liquidity equilibrium with current account policies it must be that  $\mathcal{B}_h^p(R) = 0$ (so that  $B_h = 0$  and  $\bar{\mu}_h = 0$ ). We now show that if  $R^{lf} < R^*$  then there exist a unique equilibrium world interest rate  $R = R^p = \omega R^{lf}$ .

We will consider ranges of R for which  $R\bar{\pi} > 1$ , so that Proposition 2 applies.<sup>2</sup> Clearly  $R \ge R^*$ can't be a solution. In fact, for  $R \ge R^*$  the demand for bonds by national planners coincide with the one under laissez faire, and so  $\mathcal{B}_h^p(R) > 0$ . Moreover,  $\bar{R}^* \le R < R^*$  can't be a solution either. Consider that  $\mathcal{B}_h^p(R^*) > 0$ , and that over the range  $\bar{R}^* \le R^p < R^*$  we have  $\mathcal{B}_h^{p'}(R) > 0$ . This implies that there can't be a  $\bar{R}^* \le R < R^*$  such that  $\mathcal{B}_h^p(R) = 0$ . The equilibrium interest rate must then satisfy  $R < \bar{R}^*$ . But, from Proposition 2, in this range  $\mathcal{B}_h^p(R) = 0$  only if  $R = R^p \equiv \omega R^{lf}$ . We then have that the equilibrium world interest rate is  $R^p < R^{lf}$ .

We now show that  $Y_l^N$  and welfare are lower with current account interventions compared to laissez faire. Independently of whether governments intervene on the credit markets  $C_l^T =$ 

<sup>&</sup>lt;sup>1</sup>Following the same steps, it is easy to show that the same welfare result applies to countries in the low state.

<sup>&</sup>lt;sup>2</sup>By assumption, this condition holds for  $R \ge R^p$ . For completeness, if  $R < R^p$  it might be that  $R\bar{\pi} < 1$ . But in this case, as we discuss in Appendix F, an equilibrium does not exist.

 $Y_l^T, C_h^T = Y_h^T$  and  $Y_h^N = 1$ . Moreover, we can write non-tradable output in the low state as

$$Y_l^N = \min\left(R\bar{\pi}Y_l^T/Y_h^T, 1\right).$$

Since  $R^p < R^{lf}$  it immediately follows that  $Y_l^N$  is lower in the equilibrium with current account policy than in the laissez-faire equilibrium. Since the impact on welfare of credit market interventions is fully determined by  $Y_l^N$ , it follows that also welfare is lower in the equilibrium with current account policy than in the laissez-faire equilibrium

#### **B.5** Proof of Proposition 4

**Proposition 4** Multiple equilibria with current account policies. Suppose that  $R^{lf} \ge R^*$ . Then there exists a vanishing-liquidity equilibrium with current account policies with  $R = R^{lf}$ . This equilibrium is isomorphic to the laissez-faire one. However, if  $\omega R^{lf} < R^*$  and  $\omega R^{lf} \overline{\pi} > 1$ , there exists at least another equilibrium with current account policies associated with a world interest rate  $R = R^p \equiv \omega R^{lf}$ . This equilibrium features lower output and welfare than the laissez-faire one.

**Proof.** In an equilibrium with vanishing liquidity it must be that  $\mathcal{B}_h^p(R) = 0$  (so that  $B_h = 0$  and  $\bar{\mu}_h = 0$ ). Notice that  $R = R^{lf}$  is an equilibrium. This is the case because for  $R^{lf} \ge R^*$  Proposition 2 implies that the demand for bonds with current account interventions and under laissez faire coincide. If  $R^p \equiv \omega R^{lf} \ge R^*$ , this is the unique solution, because the demand for bonds are independent of current account interventions for any value of R. Now assume that  $R^p < R^*$ . Since by assumption  $R^p \bar{\pi} > 1$ , the results in Proposition 2*apply.Thenthereexistsasecondsolution* $R = R^p$ , because  $\mathcal{B}_h^p(R^p) = 0$ . Moreover, since  $R^p < R^*$  this second solution corresponds to a global liquidity trap. The welfare statement can be proved following the steps in the proof to Proposition 3.

#### **B.6** Proof of Proposition 5

Proposition 5 Global equilibrium with current account policies and positive liquidity. Suppose that  $Y_l^T = 0$ ,  $((\omega/\beta + 1)\overline{B}/Y_h^T)^{1/\phi}\overline{R}\overline{\pi} > 1$  and that under laissez faire the world is stuck in global liquidity trap. Then if  $\phi < \phi^*$ , for every country output and welfare are lower in the equilibrium with current account policies compared to the laissez-faire one.  $\phi^*$  is such that  $\omega^{\phi^*/2} = (\omega + \beta)/(1 + \beta)$ .

**Proof.** We start by showing that if  $\phi < \phi^*$  then  $Y_l^{Nlf} > Y_l^{Np}$ . To solve for  $Y_l^{Nlf}$ , consider that in a laissez faire equilibrium  $\beta RC_h^T = C_l^T$ . Using  $C_h^T = Y_h^T - 2B^{row}$  and  $C_l^T = 2RB^{row}$  gives

$$R^{lf} = \left(\frac{1+\beta}{\beta}\frac{\bar{B}}{Y_h^T}\right)^{\frac{1}{\phi}}\bar{R}.$$

Since  $\bar{\pi}R^{lf} > 1$  and  $C_h^T > C_l^T$  Lemma 1 implies  $Y_h^{Nlf} = 1$ . Output in the low state is then given by  $Y_l^{Nlf} = \bar{\pi}R^{lf}C_l^T/C_h^T = \bar{\pi}\beta(R^{lf})^2$ .

Turning to the equilibrium with current account policies, let us guess and verify that if  $\phi < \phi^*$ then  $Y_l^{Np} < 1$ . Following the steps outlined in the proof to Proposition 3, one finds that in the equilibrium with current account policies  $\beta RC_h^T = \omega C_l^T$ , while the equilibrium world rate is

$$R^p = \left(\frac{\omega + \beta}{\beta} \frac{\bar{B}}{Y_h^T}\right)^{\frac{1}{\phi}} \bar{R}.$$

Since  $\bar{\pi}R^p > 1$  and  $C_h^T > C_l^T$  then  $Y_h^{Np} = 1$ . Output in the low state is then given by  $Y_l^{Np} = \bar{\pi}R^pC_l^T/C_h^T = \bar{\pi}\beta(R^p)^2/\omega$ .

We thus have that  $Y_l^{Nlf} > Y_l^{Np}$  if and only if

$$\omega^{\frac{\phi}{2}} > \frac{\omega+\beta}{1+\beta},$$

which holds if  $\phi < \phi^*$ . Since by assumption  $Y_l^{Nlf} < 1$  then we have verified our guess  $Y_l^{Np} < 1$ . Concerning welfare, since if  $\phi < \phi^*$  then  $Y_l^{Np} < Y_l^{Nlf}$ , it is sufficient to show that the utility

Concerning welfare, since if  $\phi < \phi^*$  then  $Y_l^{NP} < Y_l^{NIJ}$ , it is sufficient to show that the utility associated with tradable consumption is lower in the equilibrium with current account policies compared to laissez faire. Following the steps in the proof to Corollary 1 one finds that this is the case if

$$\log \frac{\omega}{\omega+\beta} + \beta \log \frac{R^p}{\omega+\beta} - \log \frac{1}{1+\beta} + \beta \log \frac{R^{lf}}{1+\beta} < 0.$$

Since  $R^p < R^{lf}$ , the inequality above holds if

$$(1+\beta)\log\left(\frac{1+\beta}{\omega+\beta}\right) + \log\omega < 0.$$

The left-hand side of this inequality is equal to zero for  $\omega = 1$ , and it is easy to check that it is increasing in  $\omega$  for  $0 < \omega < 1$ . Hence, the inequality above holds for  $0 < \omega < 1$ . We have thus proved that welfare is lower when current account policies are implemented.

# C Microfoundations for the zero lower bound constraint

In this appendix we provide some possible microfoundations for the zero lower bound constraint assumed in the main text. First, let us introduce an asset, called money, that pays a private return equal to zero in nominal terms.<sup>3</sup> Money is issued exclusively by the government, so that the stock of money held by any private agent cannot be negative. Moreover, we assume that the money issued by the domestic government can be held only by domestic agents.

We modify the borrowing limit (3) to

$$B_{i,t+1} + \frac{B_{i,t+1}^n}{P_{i,t}^T} + \frac{M_{i,t+1}}{P_{i,t}^T} \ge -\kappa_{i,t}$$

 $<sup>^{3}</sup>$ Here we focus on the role of money as a saving vehicle, and abstract from other possible uses. More formally, we place ourselves in the cashless limit, in which the holdings of money for purposes other that saving are infinitesimally small.

where  $M_{i,t+1}$  is the stock of money held by the representative household in country *i* at the end of period *t*. The optimality condition for money holdings can be written as

$$\frac{\omega}{C_{i,t}^{T}} = \frac{P_{i,t}^{T}}{P_{i,t+1}^{T}} \frac{\beta\omega}{C_{i,t+1}^{T}} + \mu_{i,t} + \mu_{i,t}^{M},$$

where  $\mu_{i,t}^M \geq 0$  is the Lagrange multiplier on the non-negativity constraint for private money holdings, divided by  $P_{i,t}^T$ . Combining this equation with (5) gives

$$(R_{i,t}^n - 1) \frac{\beta \omega}{C_{i,t+1}^T} = \mu_{i,t}^M \frac{P_{i,t+1}^T}{P_{i,t}^T}.$$

Since  $\mu_{i,t}^M \ge 0$ , this expression implies that  $R_{i,t}^n \ge 1$ . Moreover, if  $R_{i,t}^n > 1$ , then agents choose to hold no money. If instead  $R_{i,t}^n = 1$ , agents are indifferent between holding money and bonds. We resolve this indeterminacy by assuming that the aggregate stock of money is infinitesimally small for any country and period.

# D Optimal discretionary monetary policy

We derive the constrained efficient allocation by taking the perspective of a benevolent central bank that operates in a generic country *i*, and solves its maximization problem in period  $\tau$ . For given initial net foreign assets  $B_{i,\tau}$  and paths  $\{Y_{i,t}^T, \kappa_{i,t}, R_t\}_{t \geq \tau}$ , the central bank maximizes equation (1) subject to equations (4), (6), (11), (13) and<sup>4</sup>

$$C_{i,t}^{N} = \min\left(\frac{R_{t}\pi_{i,t+1}C_{i,t}^{T}C_{i,t+1}^{N}}{R_{i,t}^{n}C_{i,t+1}^{T}}, 1\right)$$
(D.1)

 $C_{i,t}^N \le 1, \pi_{i,t} \ge \gamma$  with complementary slackness (D.2)

$$R_{i,t}^n \ge 1,\tag{D.3}$$

for any  $t \ge \tau$ . Start by considering that from equations (4), (6), (11) and (13) it is possible to solve for the paths  $\{C_{i,t}^T, B_{i,t+1}\}_{t\ge\tau}$  independently of monetary policy. Hence, monetary policy can affect utility only through its impact on  $\{C_{i,t}^N\}_{t\ge\tau}$ . Moreover, notice that  $B_{i,t+1}$  represents the only endogenous state variable of the economy.

We now restrict attention to a central bank that operates under discretion, that is by taking future policies as given. Since monetary policy cannot affect the state variables of the economy, it follows that a central bank operating under discretion cannot influence future variables at all. The problem of the central bank can be thus written as

$$\max_{R_{i,\tau}, C_{i,\tau}^N, \pi_{i,\tau}} \log(C_{i,\tau}^N), \tag{D.4}$$

<sup>&</sup>lt;sup>4</sup>Constraint (D.1) is obtained by combining (AD) and (12) with the restriction  $Y_{i,t}^N \leq 1$ . Constraint (D.2) is obtained by combining (10) with  $L_{i,t} = Y_{i,t}^N = C_{i,t}^N$  and  $P_{i,t}^N = W_{i,t}$ .

$$C_{i,\tau}^N = \min\left(\nu_{i,\tau}/R_{i,\tau}^n, 1\right) \tag{D.5}$$

 $C_{i,\tau}^N \leq 1, \pi_{i,\tau} \geq \gamma$  with complementary slackness (D.6)

$$R_{i,\tau}^n \ge 1,\tag{D.7}$$

where  $\nu_{i,\tau} \equiv R_{\tau} \pi_{i,\tau+1} C_{i,\tau}^T C_{i,\tau+1}^N / C_{i,\tau+1}^T$ . The central bank takes  $\nu_{i,\tau}$  as given because it is a function of present and future variables that monetary policy cannot affect.

The solution to this problem can be expressed as

$$R_{i,\tau}^n \ge 1, C_{i,\tau}^N \le 1$$
 with complementary slackness. (D.8)

Intuitively, it is optimal for the central bank to lower the policy rate until the economy reaches full employment or the zero lower bound constraint binds. Moreover, it follows from constraint (D.6) that any  $\pi_{i,\tau} \geq \gamma$  is consistent with constrained efficiency. In fact, as long as the central bank faces an infinitesimally small cost from deviating from its inflation target  $\bar{\pi}$ , then the constrained efficient allocation features  $\pi_{i,\tau} = \bar{\pi}$ .<sup>5</sup> This is exactly the policy implied by the rule (MP).

# E Transitional dynamics in the baseline model

In this appendix we briefly describe the transition toward the stationary equilibrium in our baseline model. Because of the zero liquidity assumption, transitional dynamics are extremely simple and take place in a single period.

Consider a case in which the world starts from an arbitrary bond distribution. At the end of period 0, by the zero liquidity assumption, every country holds zero bonds. It follows that

$$C_{i,0}^T = Y_{i,0}^T + R_{-1}B_{i,0}.$$
 (E.1)

The period 0 world interest rate is then given by

$$R_0 = \frac{1}{\beta} \max_{i} \left\{ \frac{Y_{i,0}^T + R_{-1}B_{i,0}}{Y_{i,1}^T} \right\}.$$
 (E.2)

Moreover, output in a generic country i is given by

$$C_{i,0}^{N} = \min\left\{1, R_{0}\bar{\pi}\frac{Y_{i,0}^{T} + R_{-1}B_{i,0}}{Y_{i,1}^{T}}C_{i,1}^{N}\right\}.$$
(E.3)

From period 1 on the economy converges to the stationary equilibrium described in the main text.

<sup>&</sup>lt;sup>5</sup>Recall that we are assuming  $\bar{\pi} > \gamma$ .

# **F** The case $R\pi \leq 1$

Throughout the paper we have focused on stationary equilibria in which the condition  $R\bar{\pi} > 1$  holds. In this appendix we describe what happens when  $R\bar{\pi} \leq 1$ , in the context of stationary two-period equilibria satisfying the assumptions stated in Section I.

The key observation here is that in a two-period stationary equilibria the following condition must hold

$$R_h^n R_l^n = (R\bar{\pi})^2. \tag{F.1}$$

This condition, which can be derived using the aggregate demand equation, ensures that agents are indifferent between investing in real and nominal bonds. To see this point, consider that the left-hand side captures the domestic-currency return from holding a domestic nominal bond for two periods. Instead, the right-hand side captures the return, again in domestic currency, from holding for two periods an international bond denominated in terms of the tradable good. In a two-period stationary equilibrium, indeed, on average tradable price inflation must be equal to the inflation target, and so  $\pi_h^T \pi_l^T = \bar{\pi}^2$ .

Let us consider now the case  $R\bar{\pi} < 1$ . Since  $R_{i,t}^n \ge 1$  the arbitrage condition (F.1) breaks down. Intuitively, households would make pure profits from borrowing in terms of the real international bond and investing in the domestic nominal bond. This investment strategy would not violate the borrowing constraint, since the two bonds enter symmetrically in the borrowing limit. But then, obviously, equilibrium on the credit market could not be reached.

One way to interpret this result is that any inflation target such that  $\bar{\pi} < 1/R$  is not sustainable. There is a parallel here with the standard New Keynesian model. In the standard New Keynesian model, in fact, the steady state real interest rate is equal to the inverse of the households' discount factor. This steady state condition, coupled with the zero lower bound on the nominal interest rate, implies that there exists a lower bound on the steady state inflation target that the central bank can implement. Following standard practice in the New Keynesian literature, we then focus on values of the inflation target such that condition (F.1) holds.

We now turn to the case  $R\bar{\pi} = 1$ . In this case the arbitrage condition (F.1) holds with  $R_h^n = R_l^n = 1$ . Hence, the economy is stuck in a permanent liquidity trap. But then it is easy to check that equilibrium output is not uniquely pinned down. In fact, there are an infinite number of pairs  $Y_l^N < Y_h^N \leq 1$  that satisfy the equilibrium conditions on the non-tradable good market. Intuitively, if monetary policy is permanently constrained by the zero lower bound it cannot pin down equilibrium output, which will then depend on agents' expectations.<sup>6</sup> While this case is interesting in principle, it arises only when the parameters satisfy the knife-edge condition  $R\bar{\pi} = 1$ . For this reason, we abstracted from this special case throughout the paper.

<sup>&</sup>lt;sup>6</sup>Notice that this is a different source of indeterminacy compared to the one described in Section IV.D. Here, in fact, output is not determined for a given value of the world interest rate R.

# G Planning problem under commitment

Under commitment, the planner chooses a sequence  $\{C_{i,t}^T, Y_{i,t}^N, B_{i,t+1}\}_t$  to maximize domestic households' utility

$$\sum_{t=0}^{\infty} \beta^t \left( \omega \log(C_{i,t}^T) + (1-\omega) \log(Y_{i,t}^N) \right), \tag{G.1}$$

subject to

$$C_{i,t}^{T} = Y_{i,t}^{T} - B_{i,t+1} + RB_{i,t}$$
(G.2)

$$B_{i,t+1} \ge -\kappa_{i,t} \tag{G.3}$$

$$Y_{i,t}^N \le 1 \tag{G.4}$$

$$Y_{i,t}^{N} \le C_{i,t}^{T} R \bar{\pi} \frac{Y_{t+1}^{N}}{C_{t+1}^{T}}.$$
(G.5)

The resource constraints are captured by (G.2) and (G.4). (G.3) implies that the government is subject to the same borrowing constraint imposed by the markets on individual households.<sup>7</sup> Instead, constraint (G.5), which is obtained by combining the (AD) and (MP) equations, encapsulates the requirement that production of non-tradable goods is constrained by private sector's demand.

Notice that, as in the case of discretion, since each country is infinitesimally small, the domestic planner takes the world interest rate R as given. This feature of the planning problem synthesizes the lack of international coordination in the design of current account policies.

The first order conditions of the planning problem can be written as

$$\bar{\lambda}_{i,t} = \frac{\omega}{C_{i,t}^T} + \bar{v}_{i,t} \frac{Y_{i,t}^N}{C_{i,t}^T} - \bar{v}_{i,t-1} \frac{Y_{i,t-1}^N}{\beta C_{i,t}^T}$$
(G.6)

$$\frac{1-\omega}{Y_{i,t}^N} + \bar{v}_{i,t-1} \frac{Y_{i,t-1}^N}{\beta Y_{i,t}^N} = \bar{\nu}_{i,t} + \bar{v}_{i,t}$$
(G.7)

$$\bar{\lambda}_{i,t} = \beta R \bar{\lambda}_{i,t+1} + \bar{\mu}_{i,t} \tag{G.8}$$

$$B_{i,t+1} \ge -\kappa_{i,t}$$
 with equality if  $\bar{\mu}_{i,t} > 0$  (G.9)

$$Y_{i,t}^N \le 1$$
 with equality if  $\bar{\nu}_{i,t} > 0$  (G.10)

$$Y_{i,t}^{N} \le C_{i,t}^{T} R \bar{\pi} \frac{Y_{t+1}^{N}}{C_{t+1}^{T}}$$
 with equality if  $\bar{v}_{i,t} > 0.$  (G.11)

 $\bar{\lambda}_{i,t}, \bar{\mu}_{i,t}, \bar{\nu}_{i,t}, \bar{\nu}_{i,t}$  denote respectively the nonnegative Lagrange multipliers on constraints (G.2), (G.3), (G.4) and (G.5).

<sup>&</sup>lt;sup>7</sup>To write this constraint we have used the equilibrium condition  $B_{i,t+1}^n = 0$ . It is straightforward to show that allowing the government to set  $B_{i,t+1}^n$  optimally would not change any of the results.

It is useful to combine (G.6) and (G.8) to obtain

$$\frac{1}{C_{i,t}^{T}} \left( \omega + \bar{v}_{i,t} Y_{i,t}^{N} - \bar{v}_{i,t-1} \frac{Y_{i,t-1}^{N}}{\beta} \right) = \frac{\beta R}{C_{i,t+1}^{T}} \left( \omega + \bar{v}_{i,t+1} Y_{i,t+1}^{N} - \bar{v}_{i,t} \frac{Y_{i,t}^{N}}{\beta} \right) + \bar{\mu}_{i,t}.$$
 (G.12)

We are now ready to define an equilibrium with current account policies under commitment.

**Definition 2** Equilibrium with current account policies under commitment. An equilibrium with current account policies under commitment is a path of real allocations  $\{C_{i,t}^T, Y_{i,t}^N, B_{i,t+1}, \bar{\mu}_{i,t}, \bar{\nu}_{i,t}, \bar{\nu}_{i,t}, \bar{\nu}_{i,t}\}_{i,t}$  and world interest rate  $\{R_t\}_t$ , satisfying (14), (20), (G.7), (G.9), (G.10), (G.11) and (G.12) given a path of endowments  $\{Y_{i,t}^T\}_{i,t}$ , a path for the borrowing limits  $\{\kappa_{i,t}\}_{i,t}$ , and initial conditions  $\{R_{-1}B_{i,0}\}_i$  and  $\bar{\nu}_{i,-1}$ .

#### G.1 Stationary equilibrium

Under the simplifying assumptions stated in Section I it is possible to solve analytically for the equilibrium with current account policies under commitment. The following proposition characterizes the allocation for a small open economy as a function of the world interest rate.

**Proposition 6** National planner allocation under commitment. Suppose that  $1/\bar{\pi} < R < 1/\beta$ . Define  $\bar{R}^{*c} \equiv ((\omega(1 + \beta) - 1)/(\bar{\pi}\beta^2))^{1/2}$ . A stationary solution to the national planning problem under commitment satisfies  $B_l = 0$  and  $B_h = \max\{\mathcal{B}_h^{pc}(R), 0\}$ , where the function  $\mathcal{B}_h^{pc}(R)$  is defined by

$$\mathcal{B}_{h}^{pc}(R) = \begin{cases} \frac{\beta^{2}}{\omega(1+\beta)-1+\beta^{2}} \left(Y_{h}^{T} - \frac{(\omega(1+\beta)-1)Y_{l}^{T}}{\beta^{2}R}\right) & \text{if } R < \bar{R}^{*c} \\ \frac{Y_{h}^{T} - R\bar{\pi}Y_{l}^{T}}{1+R^{2}\bar{\pi}} & \text{if } \bar{R}^{*c} \le R < R^{*} \\ \frac{\beta}{1+\beta} \left(Y_{h}^{T} - \frac{Y_{l}^{T}}{\beta R}\right) & \text{if } R^{*} \le R. \end{cases}$$
(G.13)

Moreover,  $\bar{\mu}_h > 0$  if  $\mathcal{B}_h^{pc}(R) < 0$ , otherwise  $\bar{\mu}_h = 0$ . Finally,  $Y_h^N = 1$  and  $Y_l^N = \min\{1, R\bar{\pi}(Y_l^T + RB_h)/(Y_h^T - B_h)\}$ .

**Proof.** The proof follows the steps of the proof to Proposition 2.

Using Proposition 6, it is easy to derive results similar to the ones in Corollary 1. That is, holding constant the world interest rate at  $R = R^{lf} < R^*$ , an economy with current account policies will feature higher  $B_h$ ,  $Y_l^N$  and welfare compared to laissez faire. Indeed, even compared to current account interventions under discretion, a planner endowed with the ability to commit will save more during booms, attain higher output during busts and increase overall welfare. Hence, governments endowed with the ability to commit have an incentive to exploit the forward guidance channel of current account policies.

Let us now trace the general equilibrium impact of current account policies under commitment. For concreteness, we consider a scenario in which  $R^{lf} < R^*$ , that is in which the laissez-faire equilibrium corresponds to a global liquidity trap. The first consideration is that, just as in the case of discretion, in a zero liquidity economy current account policies cannot alter the equilibrium path of tradable consumption. Moreover, following the steps outlined in the proof to Proposition 3, one can see that the equilibrium interest rate with current account policies satisfies

$$R = R^{pc} \equiv \frac{(\omega(1+\beta) - 1)Y_l^T}{\beta^2 Y_h^T} < R^{lf}.$$
 (G.14)

It is then easy to check that equilibrium output satisfies  $Y_l^N = R^{pc} \bar{\pi} Y_l^T / Y_h^T < Y_l^{Nlf}$ .<sup>8</sup> Hence, also in the case of commitment current account policies have a negative impact on output and, by extension, welfare.

# H Extended model and numerical analysis

In this appendix we report the results of our numerical analysis.

#### H.1 Setup and competitive equilibrium

As in the baseline model, we consider a world composed of a continuum of measure one of small open economies indexed by  $i \in [0, 1]$ . Time is discrete and indexed by  $t \in \{0, 1, ...\}$ . There is no uncertainty at the world level, but our small open economies are subject to idiosyncratic risk.

Each country is populated by a continuum of measure one of identical infinitely-lived households. The lifetime utility of the representative household in a generic country i is

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{i,t}^{1-\sigma} - 1}{1-\sigma} - \chi \frac{L_{i,t}^{1+\eta}}{1+\eta} \right) \right], \tag{H.1}$$

where  $E_t[\cdot]$  is the expectation operator conditional on information available at time  $t, 0 < \beta < 1$ ,  $\sigma > 0, \chi > 0$  and  $\eta \ge 0$ .  $L_{i,t}$  denotes labor effort. Consumption  $C_{i,t}$  is defined as

$$C_{i,t} = \left(\omega \left(C_{i,t}^{T}\right)^{1-\frac{1}{\xi}} + (1-\omega) \left(C_{i,t}^{N}\right)^{1-\frac{1}{\xi}}\right)^{\frac{\xi}{\xi-1}},$$
(H.2)

where  $0 < \omega < 1$  and  $\xi > 0$ .  $C_{i,t}^T$  and  $C_{i,t}^N$  denote consumption of respectively a tradable and a non-tradable good.

**Households.** Households can trade in one-period real and nominal bonds. Real bonds are denominated in units of the tradable consumption good and pay the gross interest rate  $R_t$ . The interest rate on real bonds is common across countries, and  $R_t$  can be interpreted as the *world* interest rate. Nominal bonds are denominated in units of the domestic currency and pay the gross

<sup>&</sup>lt;sup>8</sup>The attentive reader will have noticed that, if  $R^{lf} < R^*$ , for an equilibrium with current account policies to exist it must be that  $\omega(1+\beta) > 1$ . Intuitively, if this condition fails to hold planners' desire to save is so strong that international credit markets will fail to clear for any value of the world interest rate. This stark result is due to the fact that our simple model abstracts from many factors, such as disutility from working or uncertainty about the occurrence of future liquidity traps, that affect governments' interventions on the current account.

nominal interest rate  $R_{i,t}^n$ . To simplify the analysis, we assume that households cannot purchase foreign currency denominated bonds.<sup>9</sup>

The household budget constraint in terms of the domestic currency is

$$P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N + P_{i,t}^T B_{i,t+1} + B_{i,t+1}^n = W_{i,t} L_{i,t} + P_{i,t}^T Y_{i,t}^T + P_{i,t}^T R_{t-1} B_{i,t} + R_{i,t-1}^n B_{i,t}^n.$$
(H.3)

The left-hand side of this expression represents the household's expenditure.  $P_{i,t}^T$  and  $P_{i,t}^N$  denote respectively the price of a unit of tradable and non-tradable good in terms of country *i* currency. Hence,  $P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N$  is the total nominal expenditure in consumption.  $B_{i,t+1}$  and  $B_{i,t+1}^n$  denote respectively the purchase of real and nominal bonds made by the household at time *t*. If  $B_{i,t+1} < 0$ or  $B_{i,t+1}^n < 0$  the household is holding a debt.

The right-hand side captures the household's income.  $W_{i,t}$  denotes the nominal wage, and hence  $W_{i,t}L_{i,t}$  is the household's labor income. Labor is immobile across countries and so wages are country-specific.  $Y_{i,t}^T$  is an endowment of tradable goods received by the household. Changes in  $Y_{i,t}^T$  can be interpreted as movements in the quantity of tradable goods available in the economy, or as shocks to the country's terms of trade.  $P_{i,t}^T R_{t-1} B_{i,t}$  and  $R_{i,t-1}^n B_{i,t}^n$  represent the gross returns on investment in bonds made at time t - 1.

We model idiosyncratic fluctuations in the tradable good endowment by assuming that  $Y_{i,t}^T$  follows the log-normal AR(1) process

$$\log\left(Y_{i,t}^{T}\right) = \rho \log\left(Y_{i,t-1}^{T}\right) + \epsilon_{i,t}$$

where  $\epsilon_{i,t}$  is normally distributed with zero mean and standard deviation  $\sigma_{\epsilon}$ . The shock  $\epsilon_{i,t}$  is uncorrelated across countries, and hence the world endowment of tradable goods is constant over time.

There is a limit to the amount of debt that a household can take. In particular, the end-ofperiod bond position has to satisfy

$$B_{i,t+1} + \frac{B_{i,t+1}^n}{P_{i,t}^T} \ge -\kappa_{i,t} + \theta \left( R_{t-1}B_{i,t} + R_{i,t-1}^n \frac{B_{i,t}^n}{P_{i,t}^T} \right),$$
(H.4)

where  $\kappa_{i,t} \geq 0$  and  $\theta \geq 0$ . In our numerical simulations we will consider the case  $\kappa_{i,t} > 0$ , so that countries will be able to accumulate positive amounts of debt. We will also, following Justiniano et al. (2015) and Guerrieri and Iacoviello (2017), introduce inertia in the borrowing limit by setting  $\theta > 0$ . One reason to consider an inertial adjustment of the borrowing limit is the fact that the model features only debt contracts that last one period, which in our numerical simulations corresponds to one year. In reality, however, debt typically takes longer maturities. This formalization of the borrowing constraint captures in a tractable way the fact that long-term

<sup>&</sup>lt;sup>9</sup>Due to the presence of uncertainty, here the assumption that households cannot trade foreign nominal bonds is no longer innocuous. In fact, if they were allowed to, households would diversify their portfolio of bonds to insure against the shocks hitting their country. The resulting model, however, would be extremely complicated to solve. For this reason we have chosen to prevent households from holding foreign-currency denominated bonds.

debt allows agents to adjust gradually to episodes of tight access to credit.

Countries are subject to financial shocks, modeled as idiosyncratic fluctuations in the borrowing limit  $\kappa_{i,t}$ . Our aim is to capture economies that alternate between tranquil times and financial crises. The simplest way to formalize this notion is to assume that  $\kappa_{i,t}$  transitions between two values,  $\kappa_h$  and  $\kappa_l$  with  $\kappa_h > \kappa_l$ , according to a first-order Markov process. As we will see periods of tight access to credit, i.e. periods in which  $\kappa_{i,t} = \kappa_l$ , will trigger dynamics similar to a financial crisis event in countries featuring a significant stock of external debt.

Each household chooses its desired amount of hours worked, denoted by  $L_{i,t}^s$ . However, due to the presence of nominal wage rigidities to be described below, the household might end up working less than its desired amount of hours, i.e.

$$L_{i,t} \le L_{i,t}^s,\tag{H.5}$$

where  $L_{i,t}$  is taken as given by the household.

The household's optimization problem consists in choosing a sequence  $\{C_{i,t}^T, C_{i,t}^N, B_{i,t+1}, B_{i,t+1}^n, L_{i,t}^s\}_t$ to maximize lifetime utility (H.1), subject to the budget constraint (H.3), the borrowing limit (H.4) and the constraint on hours worked (H.5), taking initial wealth  $P_0^T R_{-1} B_{i,0} + R_{i,-1}^n B_{i,0}^n$ , a sequence for income  $\{W_{i,t}L_{i,t} + P_{i,t}^TY_{i,t}^T\}_t$ , and prices  $\{R_t, R_{i,t}^n, P_{i,t}^T, P_{i,t}^N\}_t$  as given. The household's first-order conditions can be written as

$$\frac{\omega C_{i,t}^{\frac{1}{\xi}-\sigma}}{\left(C_{i,t}^{T}\right)^{\frac{1}{\xi}}} = \beta R_{t} E_{t} \left[ \frac{\omega C_{i,t+1}^{\frac{1}{\xi}-\sigma}}{\left(C_{i,t+1}^{T}\right)^{\frac{1}{\xi}}} - \theta \mu_{i,t+1} \right] + \mu_{i,t}$$
(H.6)

$$\frac{\omega C_{i,t}^{\frac{1}{\xi}-\sigma}}{\left(C_{i,t}^{T}\right)^{\frac{1}{\xi}}} = \beta R_{i,t}^{n} E_{t} \left[ \frac{P_{i,t}^{T}}{P_{i,t+1}^{T}} \left( \frac{\omega C_{i,t+1}^{\frac{1}{\xi}-\sigma}}{\left(C_{i,t+1}^{T}\right)^{\frac{1}{\xi}}} - \theta \mu_{i,t+1} \right) \right] + \mu_{i,t}$$
(H.7)

$$B_{i,t+1} + \frac{B_{i,t+1}^n}{P_{i,t}^T} \ge -\kappa_{i,t} + \theta \left( R_{t-1}B_{i,t} + R_{i,t-1}^n \frac{B_{i,t}^n}{P_{i,t}^T} \right) \quad \text{with equality if} \quad \mu_{i,t} > 0$$
(H.8)

$$C_{i,t}^{N} = \left(\frac{1-\omega}{\omega} \frac{P_{i,t}^{T}}{P_{i,t}^{N}}\right)^{\xi} C_{i,t}^{T},\tag{H.9}$$

$$L_{i,t}^{s} = \left(\frac{1-\omega}{\chi} \frac{W_{i,t}}{P_{i,t}^{N}} \frac{C_{i,t}^{\frac{1}{\xi}-\sigma}}{\left(C_{i,t}^{N}\right)^{\frac{1}{\xi}}}\right)^{\frac{1}{\eta}},\tag{H.10}$$

where  $\mu_{i,t}$  is the nonnegative Lagrange multiplier associated with the borrowing constraint. Equations (H.6) and (H.7) are the Euler equations for, respectively, real and nominal bonds. Equation (H.8) is the complementary slackness condition associated with the borrowing constraint. Equation (H.9) determines the optimal allocation of consumption expenditure between tradable and non-tradable goods. Equation (H.10) gives the household's labor supply.

It is useful to combine (H.6) and (H.7) to obtain a no arbitrage condition between real and nominal bonds

$$R_{i,t}^{n} = R_{t} \frac{E_{t} \left[ \frac{\omega C_{i,t+1}^{\frac{1}{\xi} - \sigma}}{(C_{i,t+1}^{T})^{\frac{1}{\xi}}} - \theta \mu_{i,t+1} \right]}{E_{t} \left[ \frac{P_{i,t}^{T}}{P_{i,t+1}^{T}} \left( \frac{\omega C_{i,t+1}^{\frac{1}{\xi} - \sigma}}{(C_{i,t+1}^{T})^{\frac{1}{\xi}}} - \theta \mu_{i,t+1} \right) \right]}.$$
 (H.11)

We can then use (H.9) and (H.11) to get the analogue of the baseline model's AD equation

$$C_{i,t}^{N} = C_{i,t}^{T} \left( \frac{R_{t}}{R_{i,t}^{n}} \frac{E_{t} \left[ \frac{\omega C_{i,t+1}^{\frac{1}{\xi} - \sigma}}{(C_{i,t+1}^{T})^{\frac{1}{\xi}}} - \theta \mu_{i,t+1} \right]}{E_{t} \left[ \pi_{i,t+1} \left( \frac{C_{i,t+1}^{T}}{C_{i,t+1}^{N}} \right)^{\frac{1}{\xi}} \left( \frac{\omega C_{i,t+1}^{\frac{1}{\xi} - \sigma}}{(C_{i,t+1}^{T})^{\frac{1}{\xi}}} - \theta \mu_{i,t+1} \right) \right]} \right)^{\xi}.$$
 (H.12)

where  $\pi_{i,t} \equiv P_{i,t}^N / P_{i,t-1}^N$ .

Firms and nominal rigidities. Non-traded output  $Y_{i,t}^N$  is produced by a large number of competitive firms. Labor is the only factor of production, and the production function is

$$Y_{i,t}^N = L_{i,t}.\tag{H.13}$$

Profits are given by  $P_{i,t}^N Y_{i,t}^N - W_{i,t}L_{i,t}$ , and the zero profit condition implies that in equilibrium  $P_{i,t}^N = W_{i,t}$ . Using this condition we can simplify the labor supply equation (H.10) to

$$L_{i,t}^{s} = \left(\frac{1-\omega}{\chi} \frac{C_{i,t}^{\frac{1}{\xi}-\sigma}}{\left(C_{i,t}^{N}\right)^{\frac{1}{\xi}}}\right)^{\frac{1}{\eta}}.$$
(H.14)

Nominal wages are subject to the downward rigidity constraint

$$W_{i,t} \ge \gamma W_{i,t-1}$$

where  $\gamma > 0$ . Equilibrium on the labor market is captured by the condition

$$L_{i,t} \leq L_{i,t}^s, \quad W_{i,t} \geq \gamma W_{i,t-1}$$
 with complementary slackness. (H.15)

This condition implies that unemployment, defined as a downward deviation of hours worked from the household's desired amount, arises only if the constraint on wage adjustment binds.

Monetary policy and inflation. The objective of the central bank is to set  $\pi_{i,t} = \bar{\pi}$ . As in the baseline model, we focus on the case  $\bar{\pi} > \gamma$ , so that  $\pi_{i,t} = \bar{\pi} \to L_{i,t} = L_{i,t}^s$ . The central bank runs monetary policy by setting the nominal interest rate  $R_{i,t}^n$ , subject to the zero lower bound constraint  $R_{i,t}^n \ge 1$ . We also, as in the baseline model, restrict attention to the constant-inflation limit  $\bar{\pi} \to \gamma$ . Hence monetary policy can be described by the rule

$$R_{i,t}^{n} = \begin{cases} \geq 1 & \text{if } Y_{i,t}^{N} = L_{i,t}^{s} \\ = 1 & \text{if } Y_{i,t}^{N} < L_{i,t}^{s}, \end{cases}$$
(H.16)

where we have used (H.15) and the equilibrium relationships  $W_{i,t} = P_{i,t}^N$  and  $L_{i,t} = Y_{i,t}^N$ .

Market clearing and definition of competitive equilibrium Since households inside a country are identical, we can interpret equilibrium quantities as either household or country specific. For instance, the end-of-period net foreign asset position of country i is equal to the end-of-period holdings of bonds of the representative household,  $NFA_{i,t} = B_{i,t+1} + B_{i,t+1}^n / P_{i,t}^T$ . Throughout, we focus on equilibria in which nominal bonds are in zero net supply, so that

$$B_{i,t}^n = 0, \tag{H.17}$$

for all *i* and *t*. This implies that the net foreign asset position of a country is exactly equal to its investment in real bonds, i.e.  $NFA_{i,t} = B_{i,t+1}$ .

Market clearing for the non-tradable consumption good requires that in every country consumption is equal to production

$$C_{i,t}^N = Y_{i,t}^N.$$
 (H.18)

Instead, market clearing for the tradable consumption good requires

$$C_{i,t}^{T} = Y_{i,t}^{T} + R_{t-1}B_{i,t} - B_{i,t+1}.$$
(H.19)

Finally, we generalize slightly, compared to the baseline economy, the world bond market clearing condition. In fact, we allow our model economy to run imbalances with respect to the rest of the world. More specifically, the bond market clearing condition is now

$$\int_0^1 B_{i,t+1} \,\mathrm{d}i = B^{rw},\tag{H.20}$$

where  $B^{rw}$  is a constant, corresponding to bond supply by the rest of the world. This formulation allows us to capture, in our numerical simulations, the negative net foreign asset position toward the rest of the world characterizing our sample of advanced economy.

We are now ready to define a competitive equilibrium.

**Definition 3** Competitive equilibrium. A competitive equilibrium is a path of real allocations  $\{C_{i,t}, L_{i,t}, L_{i,t}^s, C_{i,t}^T, C_{i,t}^N, Y_{i,t}^N, B_{i,t+1}, B_{i,t+1}^n, \mu_{i,t}\}_{i,t}$ , policy rates  $\{R_{i,t}^n\}_{i,t}$  and world interest rate  $\{R_t\}_t$ , satisfying (H.2), (H.6), (H.8), (H.12), (H.13), (H.14), (H.16), (H.17), (H.18), (H.19) and (H.20) given a path of endowments  $\{Y_{i,t}^T\}_{i,t}$ , a path for the borrowing limits  $\{\kappa_{i,t}\}_{i,t}$ , and initial conditions  $\{B_{i,0}\}_i$ .

#### H.2 National planning problem and equilibrium with current account policies

To streamline the exposition of the planning problem, we impose, as in the numerical analysis, the parametric restriction  $\sigma = 1/\xi$ . This assumption simplifies the derivation of the planning problem. In particular, it implies that the labor supply equation (H.10) reduces to

$$L_{i,t}^{s} = \left(\frac{1-\omega}{\chi}\right)^{\frac{1}{\eta+\frac{1}{\xi}}} \equiv L^{s}, \tag{H.21}$$

where we have also used the fact that, when households work their desired amount of hours,  $L_{i,t}^{s} = C_{i,t}^{N}$ .

Define  $z_{i,t} \equiv \{Y_{i,t}^T, \kappa_{i,t}\}$ . The problem of the national planner in a generic country *i* can be represented as

$$V(B_{i,t}, z_{i,t}) = \max_{C_{i,t}^T, Y_{i,t}^N, B_{i,t+1}} \frac{\omega(C_{i,t}^T)^{1-\frac{1}{\xi}} + (1-\omega)(Y_{i,t}^N)^{1-\frac{1}{\xi}} - 1}{1-\frac{1}{\xi}} - \frac{\chi(Y_{i,t}^N)^{1+\eta}}{1+\eta} + \beta E_t \left[ V(B_{i,t+1}, z_{i,t+1}) \right]$$
(H.22)

subject to

$$C_{i,t}^{T} = Y_{i,t}^{T} - B_{i,t+1} + R_{t-1}B_{i,t}$$
(H.23)

$$B_{i,t+1} \ge -\kappa_{i,t} + \theta R_{t-1} B_{i,t} \tag{H.24}$$

$$Y_{i,t}^N \le L^s \tag{H.25}$$

$$Y_{i,t}^{N} \le C_{i,t}^{T} \left(\frac{R_{t}}{\bar{\pi}}\right)^{\xi} \Psi(B_{i,t+1}, z_{i,t+1}).$$
(H.26)

The resource constraints are captured by (H.23) and (H.25). (H.24) implies that the government is subject to the same borrowing constraint imposed by the markets on individual households.<sup>10</sup> Instead, constraint (H.26), which is obtained by combining the (H.12) and (H.16) equations, encapsulates the requirement that production of non-tradable goods is constrained by private sector's demand. The function  $\Psi(B_{i,t+1}, z_{i,t+1})$  captures how the future planners' decisions affect constraint (H.26) in the present.<sup>11</sup> Since the current planner cannot make credible commitments about its

$$\Psi(B_{i,t+1}, z_{i,t+1}) \equiv \left(\frac{E_t \left[\frac{\omega}{\mathcal{C}^T(B_{i,t+1}, z_{i,t+1})^{\frac{1}{\xi}}} - \theta\mu(B_{i,t+1}, z_{i,t+1})\right]}{E_t \left[\left(\frac{\mathcal{C}^T(B_{i,t+1}, z_{i,t+1})}{\mathcal{Y}^N(B_{i,t+1}, z_{i,t+1})}\right)^{\frac{1}{\xi}} \left(\frac{\omega}{\mathcal{C}^T(B_{i,t+1}, z_{i,t})^{\frac{1}{\xi}}} - \theta\mu(B_{i,t+1}, z_{i,t+1})\right)\right]}\right)^{\frac{\xi}{\xi}}$$

where  $\mathcal{C}^{T}(B_{i,t+1}, Y_{i,t+1}^{T})$  and  $\mathcal{Y}^{N}(B_{i,t+1}, Y_{i,t+1}^{T})$  determine respectively consumption of tradable goods and production of non-tradable goods in period t + 1 as a function of the state variables at the beginning of next period. In turn,  $\mu(B_{i,t+1}, z_{i,t+1})$ , households' Lagrange multiplier on the borrowing constraint, is defined as

$$\mu(B_{i,t+1}, z_{i,t+1}) = \frac{\omega}{\mathcal{C}^T(B_{i,t+1}, z_{i,t+1})^{\frac{1}{\xi}}} - \beta R_{t+1} E_t \left[ \frac{\omega}{\mathcal{C}^T(B_{i,t+2}, z_{i,t+2})^{\frac{1}{\xi}}} - \theta \mu(B_{i,t+2}, z_{i,t+2}) \right].$$

<sup>&</sup>lt;sup>10</sup>To write this constraint we have used the equilibrium condition  $B_{i,t+1}^n = 0$ .

<sup>&</sup>lt;sup>11</sup>Formally, the function  $\Psi(B_{i,t+1},z_{i,t+1})$  is defined as

future actions, these variables are not into its direct control. However, the current planner can still influence these quantities through its choice of net foreign assets. In what follows, we focus on equilibria in which  $\Psi(B_{i,t}, z_{i,t})$  is differentiable. This is the case in the numerical simulations considered in the paper.

To solve this problem, we start by guessing that constraint (H.25) does not bind. The planner's first order conditions can then be written as

$$\bar{\lambda}_{i,t} = \frac{\omega}{(C_{i,t}^T)^{-\frac{1}{\xi}}} + \bar{v}_{i,t} \frac{Y_{i,t}^N}{C_{i,t}^T}$$
(H.27)

$$\bar{v}_{i,t} = \frac{1-\omega}{(Y_{i,t}^N)^{\frac{1}{\xi}}} - \chi(Y_{i,t}^N)^{\eta}$$
(H.28)

$$\bar{\lambda}_{i,t} = \beta R_t E_t \left[ \bar{\lambda}_{i,t+1} - \theta \bar{\mu}_{i,t+1} \right] + \bar{\mu}_{i,t} + \bar{v}_{i,t} Y_{i,t}^N \frac{\Psi_B(B_{i,t+1}, z_{i,t+1})}{\Psi(B_{i,t+1}, z_{i,t+1})}$$
(H.29)

$$B_{i,t+1} \ge -\kappa_{i,t} + \theta R_{t-1} B_{i,t} \quad \text{with equality if} \quad \bar{\mu}_{i,t} > 0 \tag{H.30}$$

$$Y_{i,t}^{N} \le C_{i,t}^{T} \left(\frac{R_{t}}{\bar{\pi}}\right)^{\xi} \Psi(B_{i,t+1}, z_{i,t+1}) \quad \text{with equality if} \quad \bar{\upsilon}_{i,t} > 0, \tag{H.31}$$

where  $\lambda_{i,t}, \bar{\mu}_{i,t}, \bar{v}_{i,t}$  denote respectively the nonnegative Lagrange multipliers on constraints (H.23), (H.24) and (H.26), while  $\Psi_B(B_{i,t+1}, z_{i,t+1})$  is the partial derivative of  $\Psi(B_{i,t+1}, z_{i,t+1})$  with respect to  $B_{i,t+1}$ .

Note that equation (H.28) implies that, as we guessed, constraint (H.25) does not bind. Intuitively, the labor supply decision of the planner coincides with the households' one.

It is useful to combine (H.27) and (H.29) to obtain

$$\frac{\omega}{(C_{i,t}^{T})^{-\frac{1}{\xi}}} + \bar{v}_{i,t}\frac{Y_{i,t}^{N}}{C_{i,t}^{T}} = \beta R_{t}E_{t} \left[\frac{\omega}{(C_{i,t+1}^{T})^{-\frac{1}{\xi}}} + \bar{v}_{i,t+1}\frac{Y_{i,t+1}^{N}}{C_{i,t+1}^{T}} - \theta\bar{\mu}_{i,t+1}\right] \\ + \bar{\mu}_{i,t} + \bar{v}_{i,t}Y_{i,t}^{N}\frac{\Psi_{B}(B_{i,t+1}, z_{i,t+1})}{\Psi(B_{i,t+1}, z_{i,t+1})} \quad (\text{H.32})$$

This is the planner's Euler equation. We are now ready to define an equilibrium with current account policy.

**Definition 4** Equilibrium with current account policy. An equilibrium with current account policy is a path of real allocations  $\{C_{i,t}^T, Y_{i,t}^N, B_{i,t+1}, \bar{\mu}_{i,t}, \bar{\nu}_{i,t}\}_{i,t}$  and world interest rate  $\{R_t\}_t$ , satisfying (H.20), (H.23), (H.28), (H.30), (H.31) and (H.32) given a path of endowments  $\{Y_{i,t}^T\}_{i,t}$ , a path for the borrowing limits  $\{\kappa_{i,t}\}_{i,t}$ , and initial conditions  $\{B_{i,0}\}_i$ . Moreover, the function  $\Psi(B_{i,t+1}, Y_{i,t+1}^T)$  has to be consistent with the national planners' decision rules.

#### H.3 Parameters

The extended model cannot be solved analytically, and we study its properties using numerical simulations. We employ a global solution method, described in Appendix H.7, in order to deal with the nonlinearities involved by the occasionally binding borrowing and zero lower bound constraints.

One period corresponds to one year. We set the coefficient of relative risk aversion to  $\sigma = 2$ , the elasticity of substitution between tradable and non-tradable goods to  $\xi = 0.5$ , and the share of tradable goods in consumption expenditure to  $\omega = 0.25$ , in line with the international macroeconomics literature. The inverse of the Frisch elasticity of labor supply  $\eta$  is set equal to 2.2, as in Galí and Monacelli (2016). We normalize  $\chi = 1 - \omega$ , which implies that equilibrium labor at full employment is equal to 1.<sup>12</sup>

The next set of parameters is selected to match some salient features characterizing advanced economies in the aftermath of the 2008 global financial crisis.<sup>13</sup> We set the discount factor to  $\beta = 0.988$ , so that under laissez faire the steady-state world interest rate  $R^{lf}$  is equal to 1.007. This target captures the low interest rate environment that has characterized advanced economies in the post-crisis years. In fact, 0.7% corresponds to the average real world interest rate over the period 2009-2015, estimated as in King and Low (2014). We calibrate  $B^{rw}$  and  $\bar{\pi}$  using data from a sample of advanced economies.<sup>14</sup> We set  $B^{rw}$ , the bond supply from the rest of the world, to reproduce the fact that advanced economies have been in the recent past net debtors toward the rest of the world.<sup>15</sup> In particular, we set  $B^{rw}$  so that under laissez-faire the net debt position of our model economies is equal to 9.4% of their aggregate GDP. This corresponds to the aggregate net debt-to-GDP ratio of our sample countries, averaged over the period 2009-2015.  $\bar{\pi}$  is chosen to match the average core inflation rate experienced by our sample countries between 2009 and 2015. This target implies  $\bar{\pi} = 1.0125$ .

We calibrate the tradable endowment process based on data on the cyclical component of tradable output in our sample countries. We identify tradable output in the data as per capita GDP in agriculture, forestry, fishing, mining, and manufacturing at constant prices. The sample period goes from 1970 to 2015. Since our model abstracts from aggregate shocks, we control for global movements in tradable output by subtracting, for each year, aggregate per-capita tradable output from the country-level series. We then extract the cyclical component from the resulting

<sup>&</sup>lt;sup>12</sup>As shown in Appendix H.1, in absence of nominal wage rigidities equilibrium labor in the extended model would be constant. This property arises due the fact that production takes place only in the non-tradable sector and the parametric assumption  $\sigma = 1/\xi$ , which implies that utility is separable in consumption of tradable and non-tradable goods.

<sup>&</sup>lt;sup>13</sup>Appendix I provides a detailed description of the data sources and the procedures we employed to calibrate the model.

<sup>&</sup>lt;sup>14</sup>Our sample of advanced economies is composed of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.

<sup>&</sup>lt;sup>15</sup>Indeed, in recent years advanced economies have been net recipients of capital inflows from emerging countries. As is well known, see for instance Bernanke (2005), a large driver of these capital flows has been the accumulation of reserves by central banks in emerging markets. It is not clear how to model the reaction of these flows to changes in the world interest rate. For this reason, in our baseline model we have opted for the simplest assumption of an inelastic supply of funds from the rest of the world. In Appendix H.8, however, we examine the robustness of our results to the presence of an elastic supply of funds from rest-of-the-world countries.

Table 1: F	Parameters
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	Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Elasticity consumption aggr.	$\xi = 0.5$	Standard value
Tradable share in expenditure	$\omega = 0.25$	Standard value
Frisch elasticity of labor supply	$1/\eta = 1/2.2$	Galí and Monacelli (2016)
Labor disutility coefficient	$\chi = 1 - \omega$	Normalization
Discount factor	$\beta = 0.988$	$R^{lf} = 0.7\%$
Bond supply r.o.w.	$B^{rw} = -0.376$	$B^{rw} / \int_0^1 GDP_{i,t} di = -9.4\%$
Inflation target	$\bar{\pi} = 1.0125$	Average core inflation
Tradable endowment process	$\rho = 0.87,  \sigma_{Y^T} = 0.056$	Estimate for advanced economies
Prob. negative financial shock	$p(\kappa_l   \kappa_h) = 0.125$	1% prob. financial crisis
Persistence negative financial shock	$p(\kappa_l   \kappa_l) = 0.2$	5 years crisis duration
Tight credit regime	$\kappa_l = 0$	Corr(CA/GDP, log(GDP)) = -0.21
	$\theta = 0.9$	mimics 10y debt maturity

series by subtracting a country-specific log-linear trend. The first order autocorrelation  $\rho$  and the standard deviation  $\sigma_{Y^T}$  of the tradable endowment process are set respectively to 0.87 and 0.056, to match their empirical counterparts. In the computations, we approximate the tradable endowment process with the quadrature procedure of Tauchen and Hussey (1991) using 7 nodes.

We are left to calibrate the parameters governing the borrowing limit and the financial shocks. We are interested in capturing economies that alternate between tranquil times, characterized by abundant access to credit, and financial crisis episodes triggered by sudden stops in capital inflows. We start by setting  $\kappa_h$  to a value high enough so that the borrowing constraint never binds when  $\kappa_{i,t} = \kappa_h$ . The parameters  $\kappa_l$  and  $\theta$ , joint with the transition probabilities  $p(\kappa_l | \kappa_h)$  and  $p(\kappa_l | \kappa_l)$ , thus determine how often the borrowing constraint binds, as well as agents' ability to smooth consumption in response to endowment shocks.

We set the probability of an adverse financial shock  $p(\kappa_l|\kappa_h)$  and its persistence  $p(\kappa_l|\kappa_l)$  to target the frequency and duration of financial crises in our sample countries. We follow Bianchi and Mendoza (2018) and define a financial crisis as a sharp improvement in the trade balance, capturing unusually large drops in foreign financing. Different from Bianchi and Mendoza (2018), since our model abstract from global financial shocks, to identify financial crisis episodes in the data we control for time fixed effects.<sup>16</sup> The resulting annual frequency of financial crises is 1% and their average duration is 5 years. We match these statistics by setting  $p(\kappa_l|\kappa_h) = 0.125$  and  $p(\kappa_l|\kappa_l) = 0.2$ .

To choose values for  $\theta$  and  $\kappa_l$  we employ the following strategy. To set  $\theta$  we exploit the fact that this parameter corresponds to the fraction of debt that can be rolled over every period, irrespective of whether the borrowing constraint binds or not. Hence, drawing a parallel with long-term debt,  $1 - \theta$  can be interpreted as the fraction of debt maturing in a given period. Following this logic we set  $\theta = 0.9$  to mimic an average debt maturity of 10 years, close to the average US households'

 $<sup>^{16}</sup>$ See Appendix I for a detailed description of the procedure that we use to identify financial crisis events in the data.

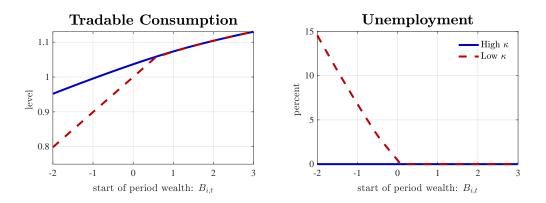


Figure 1: Policy functions in the laissez-faire steady state.

debt maturity reported by Jones et al. (2017).<sup>17</sup> To set  $\kappa_l$  we target the negative correlation between current account and GDP characterizing our sample countries. In fact, in absence of financial frictions our model would generate a counterfactual positive correlation between these two variables, since agents would smooth consumption by saving in good times and borrowing during downturns. As financial shocks become more severe, i.e. as  $\kappa_l$  falls, the correlation between current account and GDP implied by the model falls, until it eventually turns negative. Given  $\theta = 0.9$ , setting  $\kappa_l = 0$  generates a correlation between the current account-to-GDP ratio and the logarithm of GDP of -0.21, equal to its empirical counterpart.

#### H.4 Debt and liquidity traps under laissez faire

Before discussing the impact of current account policies, in this section we briefly describe the steady-state equilibrium under laissez faire. We will show that a country that has accumulated a high stock of debt is at risk of experiencing liquidity traps characterized by severe rises in unemployment.

Figure 1 displays the optimal choices for tradable consumption and unemployment as a function of  $B_{i,t}$ , i.e. the country's stock of wealth at the start of the period. The solid lines refer to countries with abundant access to credit ( $\kappa_{i,t} = \kappa_h$ ), while the dashed lines correspond to countries hit by negative financial shocks ( $\kappa_{i,t} = \kappa_l$ ).<sup>18</sup> The left panel of Figure 1 shows that, as it is natural, tradable consumption is increasing in wealth. Moreover, the figure shows that high-debt countries hit by negative financial shocks experience sharp falls in tradable consumption, triggered by the binding borrowing constraint. Taking stock, tradable consumption is low in high-debt countries, especially when these are hit by negative financial shocks.

The right panel of Figure 1 shows that high-debt countries with tight access to credit are exactly the ones experiencing high unemployment. To understand this result consider that, just as in the baseline model, demand for non-tradable consumption is increasing in consumption of tradable goods. Hence, the combination of high debt and tight access to credit depresses both consumption

<sup>&</sup>lt;sup>17</sup>Following Jones et al. (2017) and interpreting  $\theta$  as the fraction of debt that matures every period, average debt maturity D can be written as  $D = R/(\theta + R - 1)$ .

<sup>&</sup>lt;sup>18</sup>Both policy functions are conditional on  $Y_{i,t}^{T}$  being equal to its mean value.

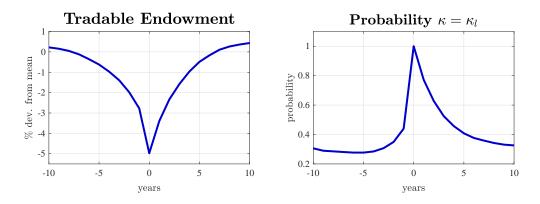


Figure 2: Liquidity trap events: tradable endowment and financial shocks.

of tradable goods and demand for non-tradables. Low demand for non-tradables, in turn, pushes the policy rate against the zero lower bound and the economy into a recessionary liquidity trap. This explains why high-debt countries are exposed to the risk of sharp rises in unemployment in the event of a negative financial shock.

Figures 2 and 3 provide a snapshot of the liquidity trap events generated by the model. To construct these figures, we simulated the behavior of a country under laissez faire for a large number of periods and collected all the liquidity trap events. We then took averages of several macroeconomic indicators across all these events, centering each episode around the period associated with the peak in unemployment.<sup>19</sup> Figure 2 displays the average path of the tradable endowment and financial shocks, while the solid lines in Figure 3 illustrate the dynamics of GDP, tradable consumption, current account and unemployment.

Large rises in unemployment are preceded by low realizations of the tradable endowment shock, to which households respond by accumulating debt in order to sustain tradable consumption. This explains the current account deficits characterizing the run up to the unemployment crisis. Debt accumulation, however, puts the economy at risk of a large drop in tradable consumption in the event of a tightening in the borrowing limit. This is exactly what happens in period 0, when a negative financial shock generates a current account reversal and a large drop in consumption of tradable goods. As tradable consumption falls also aggregate demand for non-tradables drops. Constrained by the zero lower bound, the central bank is unable to react to the decline in domestic demand. The result is a sharp recession lasting several years.<sup>20</sup>

Though negative financial shocks in our model are rare events, the fact that they trigger severe and persistent recessions imply that their impact on unemployment and output is significant. Indeed, in the laissez-faire equilibrium average unemployment is 1.26%.<sup>21</sup> Thus, the combination

<sup>&</sup>lt;sup>19</sup>More precisely, we say that a country is in a liquidity trap in a given period t if  $L_{i,t} < 1$ , that is if unemployment is positive. We then define the unemployment peak during a liquidity trap as the period in which unemployment is at its highest value compared to the 10 periods before and after. The period associated with the unemployment peak corresponds to period 0 in Figures 2 and 3.

<sup>&</sup>lt;sup>20</sup>Interestingly, the 6% peak drop in GDP during our typical crisis event is quantitatively in line with the Romer and Romer (2017) empirical estimates of the output response to financial crises in advanced economies.

<sup>&</sup>lt;sup>21</sup>Since we are focusing on a stationary equilibrium, here average unemployment refers both to the cross-sectional average, that is  $1 - \int_0^1 L_{i,t} di$ , as well as to the unconditional expected value for a given country.

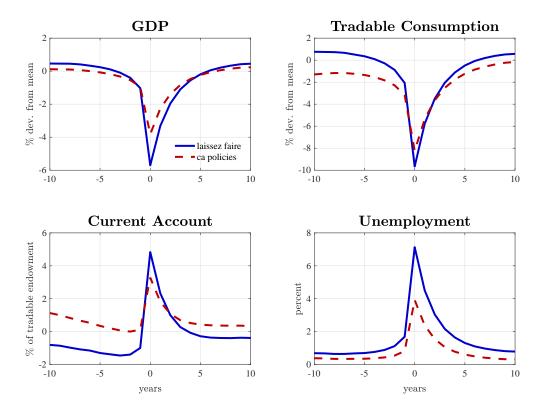


Figure 3: Liquidity trap events: macroeconomic indicators. Solid (dashed) lines refer to economies under laissez faire (current account policies). GDP is defined as  $GDP_{i,t} = Y_{i,t}^T + p^N Y_{i,t}^N$ , where  $p^N$  denotes the unconditional mean of  $P_{i,t}^N/P_{i,t}^T$  in the laissez-faire steady state.

of financial frictions and of the zero lower bound constraint on monetary policy implies that under laissez faire the world economy operates substantially below potential.

Summing up, the model is able to generate liquidity trap events characterized by severe and persistent rises in unemployment. Crucially, large recessions are triggered by negative financial shocks, and they are more likely to happen in high-debt countries. It is this feature of the model, as we will see in the next section, that creates space for current account policies.

#### H.5 Current account policies: a small open economy perspective

We now turn to government interventions on the international credit markets. As an intermediate step, it is useful to start by taking a partial equilibrium perspective, i.e. by abstracting from the impact of current account policies on the world interest rate. Hence, in this section we consider a single small open economy that implements the optimal current account policy, while the rest of the world sticks to laissez faire.

The dashed lines in Figure 3 show how public interventions on the current account affect the behavior of a country during the liquidity trap events described in the previous section.<sup>22</sup> The

<sup>&</sup>lt;sup>22</sup>To construct this figure, for each liquidity trap event identified under laissez faire we collected the value of net foreign assets in period t - 10, where period t corresponds to the unemployment peak during the event, as well as the path for the shocks in periods t - 10 to t + 10. We then, for each event, fed the corresponding sequence of shocks and initial value for the net foreign assets to the decision rules derived under current account policy. Finally, we took averages of our variables of interest across all the events.

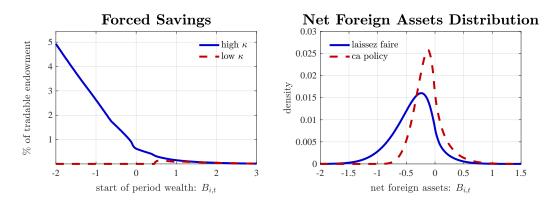


Figure 4: Forced savings and stationary net foreign asset distribution in a small open economy. Right panel: solid (dashed) lines refer to economies under laissez faire (current account policies).

key result is that the government intervenes in the run up to the crisis by reducing households' debt accumulation and improving the country's current account. Limiting debt accumulation, the reason is, reduces the exposure of the economy to negative financial shocks. As a result, both the current account reversal and the rise in unemployment occurring in period 0, when access to credit gets tight, are substantially milder under the optimal current account policy compared to laissez faire.

As in the baseline model, the government intervenes on the current account due to the presence of aggregate demand externalities. Private agents, in fact, do not internalize the impact of their borrowing decisions on aggregate demand and employment. It is then natural to think that a government will intervene more aggressively to improve the current account, when conditions are such that a negative financial shock will trigger a sharp rise in unemployment. This is precisely the result illustrated by Figure 4, which shows that the "forced savings" induced by current account interventions are larger in high-debt countries experiencing lax access to credit.<sup>23</sup>

Quantitatively, public interventions on the current account have a sizable impact on average savings. To illustrate this point, the right panel of Figure 4 compares the stationary net foreign asset distribution of a small open economy operating under laissez faire (solid line), against the one of a country with current account policies (dashed line). The implementation of current account policies induces a rightward shift of the net foreign asset distribution, corresponding to an increase in average savings. The counterpart of this rise in savings is a reduction in unemployment. In fact, the implementation of current account policies by a single country would reduce its average unemployment to 0.5%, down from the 1.26% average unemployment characterizing laissez-faire economies.

Of course, in our model it is perfectly possible for a single country to reduce its average unemployment by means of current account policies. In fact, since we are focusing on small open economies, a change in saving behavior by a single country will not affect the world interest rate. As we show next, matters are completely different when current account policies are adopted on a

<sup>&</sup>lt;sup>23</sup>Formally, forced savings are defined as  $C_{i,t}^T - \tilde{C}_{i,t}^T$ , where  $\tilde{C}_{i,t}^T$  is the notional consumption that would be chosen by households absent government intervention. In the figure,  $Y_{i,t}^T$  is kept equal to its mean value.

global scale.

#### H.6 Revisiting the paradox of global thrift

We have seen that, as in the baseline model, governments have a strong incentive to manipulate their country's current account when the zero lower bound is expected to bind in the future. It is then interesting to consider what happens when current account policies are implemented on a global scale. It turns out that, under our benchmark parametrization, the outcome is a large drop in the world interest rate, which ends up exacerbating the output and welfare losses due to the zero lower bound. This result shows that the logic of the paradox of global thrift goes beyond the simple baseline model presented in Section I.

Throughout this section we run the following experiment. Imagine that the world starts from the laissez-faire steady state. In period 0 all the countries in the world experience a previously unexpected change in the policy regime, so that governments start implementing the self-oriented optimal current account policy. We are interested in tracing the impact of this policy change on output and welfare.

Before moving on, a few words on multiplicity of equilibria under current account policies are in order. The logic of Proposition 3 applies also to the extended model, and thus the possibility that under some parametrizations multiple equilibria under current account policies exist cannot be discarded. That said, in all the numerical simulations that follow we could not find evidence of multiple equilibria.<sup>24</sup> We thus leave an analysis of equilibrium multiplicity in the extended model for future research.

#### H.6.1 Output response to current account policies

Figure 5 plots the path of the world interest rate and world GDP during the transition toward the steady state with current account policies. The change in policy regime induces a gradual drop in the world interest rate. Intuitively, public interventions on the current account increase the aggregate demand for bonds by our model economies. Given the fixed bond supply from the rest of the world the result is a large drop in the world rate, which falls by 170 basis points compared to its value under laissez-faire. The drop in the world interest rate, in turn, exacerbates the zero lower bound constraint on monetary policy and leads to a fall in world output. Indeed, world GDP in the steady state with public interventions on the current account is 1.2% lower than in the laissez-faire equilibrium.<sup>25</sup>

<sup>&</sup>lt;sup>24</sup>The analysis of the baseline model suggests that under current account policies multiple steady states are possible. Each steady state is characterized by a particular value of the world interest rate. In the extended model, however, it is not possible to derive analytically the conditions under which multiple steady states exist. To check for the existence of multiple steady states, we thus solved numerically the model for a grid of values of the world interest rate. In all our simulations we could find only a single value of the world interest rate that clears the global asset market. This indicates that, under the parametrizations that we considered, the model has a unique steady state.

<sup>&</sup>lt;sup>25</sup>The differences in terms of unemployment are even larger. In fact, steady state aggregate unemployment when governments' intervene on the current account is 2.9%, compared to the 1.3% aggregate unemployment in the laissez-faire steady state.

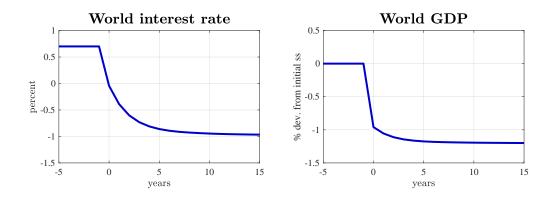


Figure 5: Transition toward steady state with current account policies. World GDP is defined as  $\int_0^1 GDP_{i,t} di = \int_0^1 Y_{i,t}^T + p^N Y_{i,t}^N di$ , where  $p^N$  denotes the unconditional mean of  $P_{i,t}^N/P_{i,t}^T$  in the laissez-faire steady state.

The first row of Table 2 shows the drop in the present value of expected output caused by the global implementation of current account policies, as a percent of expected output in the laissez-faire steady state.<sup>26</sup> On average, the cumulative output loss caused by current account interventions is equal to 1.22% of output in the laissez faire steady state. Moreover, the expected output losses are higher in countries starting the transition with a high stock of debt and tight access to credit. As it is intuitive, the countries that suffer the largest drops in expected output upon implementation of current account policies are those that start the transition inside a liquidity trap.

#### H.6.2 Welfare response to current account policies

We now turn to the impact that current account policies, and the associated drop in the world interest rate, have on welfare. As we discussed in the context of our baseline model, a lower world rate exacerbates the inefficiencies due to the zero lower bound and lead to an inefficiently low production of non-tradable goods. This effect is at the heart of the paradox of global thrift. In the extended model, however, there are two additional effects to consider. First, given that we have moved away from the zero liquidity limit, in the extended model a drop in the world rate redistributes wealth from creditor to debtor countries. Second, since the countries that form our economy are net debtors with respect to the rest of the world, a lower world interest rate redistributes wealth from rest-of-the-world countries toward our model economies.<sup>27</sup> In what follows, we start by discussing how current account policies affect total welfare. We then isolate the channel

$$E_0\left[\beta^t \sum_{t=0}^{\infty} \left(1 - \tau_i^y\right) GDP_{i,t}^{lf}\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t GDP_{i,t}^{tr}\right],$$

<sup>&</sup>lt;sup>26</sup>Formally, for any country i we computed the expected cumulative output loss  $\tau_i^y$  caused by current account policies as

where  $GDP_{i,t}^{lf}$  denotes GDP in the laissez-faire steady state, while  $GDP_{i,t}^{tr}$  refers to the path of GDP during the transition toward the steady state with current account policies. GDP is defined as  $GDP_{i,t} = Y_{i,t}^T + p^N Y_{i,t}^N$ , where  $p^N$  denotes the unconditional mean of  $P_{i,t}^N/P_{i,t}^T$  in the laissez-faire steady state.

<sup>&</sup>lt;sup>27</sup>As we explain in Section 5.2, rest-of-the-world agents are akin to noise traders. Hence, one must be careful when considering the welfare impact of a wealth redistribution between our model economies and the agents from the rest of the world.

	Average	Net fo	Net foreign assets $(B_{i,0}, \text{ perc.})$			Financia	Financial shock $(\kappa_{i,0})$	
		5th	25th	50th	75th	$\kappa_l$	$\kappa_h$	
Output losses	1.22	1.32	1.24	1.20	1.19	1.24	1.21	
Welfare losses	0.087	0.083	0.082	0.085	0.090	0.101	0.078	
Welfare losses (NT)	0.308	0.357	0.319	0.301	0.293	0.315	0.304	

Table 2. Impact of current account policies.

Notes: All numbers are in percent.

that is directly connected with the paradox of global thrift by focusing on the non-tradable sector.

The second row of Table 2 illustrates the impact of current account policies on total welfare, by reporting the proportional increase in consumption for all possible future histories that agents living in the laissez-faire equilibrium must receive, in order to be indifferent between the status quo or switching to the equilibrium with current account interventions.<sup>28</sup> These calculations explicitly consider the welfare effect of the whole transitional dynamics toward the steady state with current account policies. The table reports the results in terms of welfare losses, so a positive entry means that the implementation of current account policies lowers welfare compared to the laissez-faire equilibrium.

On average households experience a drop in welfare from governments' interventions on the current account. In fact, on average households are willing to give up permanently 0.087% of their consumption in the laissez-faire equilibrium to prevent the government from implementing the current account policies.<sup>29</sup> Interestingly, the welfare losses are evenly spread across debtor and creditor countries. This is the result of two opposing effects. On the one hand, high-debt countries experience larger output losses upon the implementation of current account policies. This effect points toward higher welfare losses in high debt countries. However, high-debt countries also experience a reduction in the cost of servicing their debt following the drop in the world rate. This effect points toward lower welfare losses in high-debt countries. The fact that the welfare losses are evenly distributed across the initial net foreign asset distribution means that these two effects essentially cancel out. Turning to the financial shock, the welfare losses tend to be higher in countries starting the transition during a period of tight access to credit. This is unsurprising, because these are the countries in which the output losses caused by the drop in the world rate are larger.

The third row of Table 2 illustrates the contribution of the non-tradable sector to the welfare

<sup>28</sup>More formally, for any country *i* we computed the welfare loss  $\tau_i^w$  as

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U\left((1-\tau_i^w)C_{i,t}^{lf}, L_{i,t}^{lf}\right)\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t U\left(C_{i,t}^{tr}, L_{i,t}^{tr}\right)\right],$$

where superscripts lf denote the value of the corresponding variable in the laissez faire steady state, while tr refers to the transition toward the steady state with current account interventions.

 $<sup>^{29}</sup>$ As we discuss in Appendix H.8, our model is likely to underestimate the welfare losses due to unemployment because it assumes that voluntary and involuntary leisure are perfect substitutes. There we show that reducing the Frisch elasticity of labor supply, which corresponds to an increase in the disutility from involuntary unemployment, from our benchmark value of 0.45 to 0.35 increases the welfare losses associated with current account policies by one order of magnitude.

losses. To this end, we computed a measure of welfare losses that takes into account only changes in non-tradable consumption and labor effort, thus neglecting the impact of changes in tradable consumption on welfare.<sup>30</sup> This statistic isolates the welfare costs directly linked to the paradox of global thrift, i.e. to the fact that the global implementation of current account policies exacerbates the inefficiencies due to the zero lower bound. In particular, this measure abstracts from the welfare gains driven by the transfer of wealth from the rest of the world to our model economies caused by the drop in the world interest rate.

The table shows that current account interventions substantially exacerbate the inefficiencies due to the zero lower bound. In fact, once we abstract from the wealth effect originating from changes in the world interest rate, on average households are willing to give up permanently 0.308% of their non-tradable consumption in the laissez-faire equilibrium to prevent the government from implementing the current account policies. Moreover, this welfare measure shows that high-debt countries are the ones who suffer the most from the inefficient drop in production caused by the global implementation of current account policies. Indeed, these are the countries in which monetary policy is most constrained by the zero lower bound.

Summing up, the results from the extended model largely confirm the analytic results that we derived using the simplified framework of Section I. Current account policies generate a large increase in global savings, giving rise to a sharp drop in the world interest rate. In turn, the lower world rate exacerbates the distortions due to the zero lower bound and leads to a drop in world output. The output drop is larger in countries with a high stock of debt and tight access to credit. Moreover, though governments design current account policies to increase their citizens' welfare, once implemented on a global scale these policy interventions can be welfare-reducing. Because our model is highly stylized, we interpret the quantitative results as being only suggestive. Still, the model points toward the possibility of significant output and welfare losses associated with the paradox of global thrift.<sup>31</sup>

$$U(C_{i,t}, L_{i,t}) = \frac{\left(\omega C_{i,t}^{T}\right)^{1-\sigma} - 1}{1-\sigma} + \frac{\left((1-\omega)C_{i,t}^{N}\right)^{1-\sigma} - 1}{1-\sigma} - \chi \frac{L_{i,t}^{1+\eta}}{1+\eta}.$$

Now define

$$U^{N}\left(C_{i,t}^{N}, L_{i,t}\right) \equiv \frac{\left((1-\omega)C_{i,t}^{N}\right)^{1-\sigma} - 1}{1-\sigma} - \chi \frac{L_{i,t}^{1+\eta}}{1+\eta}.$$

We computed the welfare losses pertaining to the non-tradable sector  $\tau_i^{wN}$  as

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U^N\left((1-\tau_i^{wN})C_{i,t}^{Nlf}, L_{i,t}^{lf}\right)\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t U^N\left(C_{i,t}^{Ntr}, L_{i,t}^{tr}\right)\right],$$

where superscripts lf denote the value of the corresponding variable in the laissez faire steady state, while tr refers to the transition toward the steady state with current account interventions.

<sup>31</sup>In Appendix H.8 we provide a sensitivity analysis and show how our quantitative results are affected by changes in some key model parameters. In particular, we consider changes in the disutility from involuntary unemployment and in inflation expectations. We also consider a version of the model in which the supply of bonds from the rest of the world responds to variations in the world interest rate.

<sup>&</sup>lt;sup>30</sup>Here we exploit the fact that under our parametrization the value function is separable in the consumption of tradable and non-tradable goods. To see this point, consider that throughout our numerical simulations we assumed  $\sigma = 1/\xi$ . Under this assumption it is easy to see that

#### H.7 Numerical solution method

To solve the model numerically we follow the method proposed by Guerrieri and Lorenzoni (2017).

We start by discussing the computations needed to solve for the steady state. Computing the steady state of the model involves finding the interest rate that clears the bond market at the world level. The first step consists in deriving the optimal policy functions  $C^T(B, z)$  and  $C^N(B, z)$ , where  $z = \{Y^T, \kappa\}$  for a given interest rate R. To compute the optimal policy functions we discretize the endogenous state variable B using a grid with 500 points, and then iterate on the Euler equation and on the intratemporal optimality conditions using the endogenous gridpoints method of Carroll (2006). The decision rule  $C^T(B, z)$ , coupled with the country-level market clearing condition for tradable goods, fully determines the transition for the country's bond holdings. Using the optimal policies, it is then possible to derive the inverse of the bond accumulation policy g(B, z). This is used to update the conditional bond distribution M(B, z) according to the formula  $M_{\tau}(B, z) = \sum_{z} M_{\tau-1} (g(B, \tilde{z}), \tilde{z}) P(z|\tilde{z})$ , where  $\tau$  is the  $\tau$ -th iteration and  $P(z|\tilde{z})$  is the probability that  $z_{t+1} = z$  if  $z_t = \tilde{z}$ . Once the bond distribution has converged to the stationary distribution, we check whether the market for bonds clears. If not, we update the guess for the interest rate.

To compute the transitional dynamics, we first derive the initial and final steady states. We then choose a T large enough so that the economy has approximately converged to the final steady state at t = T (we use T = 100, increasing T does not affect the results reported). The next step consists in guessing a path for the interest rate. We then set the policy functions for consumption in period T equal to the ones in the final steady state and iterate backward on the Euler equation and on the intratemporal optimality conditions to find the sequence of optimal policies  $\{C_t^T(B, z), C_t^N(B, z)\}$ . Next, we use the optimal policies to compute the sequence of bond distributions  $M_t(B, z)$  going forward from t = 0 to t = T, starting with the distribution in the initial steady state. Finally, we compute the world demand for bonds in every period and update the path for the interest rate until the market clears in every period.

#### H.8 Sensitivity analysis

In this appendix we discuss how the results are affected by changes in some key model parameters.

We start by considering changes in the Frisch elasticity of labor supply  $1/\eta$ . This is an important parameter, because it determines the impact on welfare of deviations of employment from its natural value. More precisely, the lower the Frisch elasticity the higher the welfare losses associated with involuntary unemployment. In our benchmark parametrization we considered a Frisch elasticity of 0.45, in line with the value used by the New Keynesian literature. However, in our setting this assumption is likely to underestimate the welfare costs of unemployment. This is due to the fact that in the benchmark New Keynesian model there is no involuntary unemployment. Instead, in our world characterized by wage rigidities all the fluctuation in employment are involuntary. It is then interesting to see how the results change when the welfare costs associated with fluctuations in unemployment increase.

	Output losses	Welfare losses	Welfare losses (NT)
Benchmark	1.22	0.09	0.30
Lower Frisch elasticity $(1/\eta = 0.35)$	1.80	0.24	0.53
Higher Frisch elasticity $(1/\eta = 0.55)$	0.54	-0.02	0.09
Lower inflation ( $\bar{\pi} = 1.01$ )	2.01	0.25	0.52
Higher inflation $(\bar{\pi} = 1.015)$	0.41	-0.03	0.06
Elastic $B^{rw}$ (low, $\zeta = 1$ )	0.73	0.00	0.16
Elastic $B^{rw}$ (high, $\zeta = 10$ )	0.25	-0.05	0.02

Table 3. Sensitivity analysis

Notes: All the numbers are in percent. For each variable the table shows its average cross-sectional value.

The second row of Table 3 shows that lowering the Frisch elasticity to 0.35 substantially increases both the output and welfare losses caused by current account policies. This result is due to the fact that higher welfare costs from unemployment induce governments to intervene more aggressively on the current account. Hence, the implementation of current account policies leads to a larger drop in the world interest rate, which exacerbates the inefficiencies due to the zero lower bound compared to our benchmark parametrization. As a result, lowering the Frisch elasticity to 0.35 more than doubles the welfare losses triggered by current account policies with respect to the benchmark parametrization. The third row of table 3 shows that, as it is natural, the opposite occurs for a higher value of the Frisch elasticity equal to 0.55.

In our second experiment we consider changes in inflation  $\bar{\pi}$ . As it is well known higher inflation expectations, in our model captured by a higher  $\bar{\pi}$ , reduce the constraint on monetary policy imposed by the zero lower bound on the policy rate. In our benchmark parametrization we have set  $\bar{\pi} = 1.0125$ . This is lower that the 2% inflation target characterizing countries such as the US or Euro area, but higher than the average inflation experienced by countries undergoing long-lasting liquidity traps such as Japan.

It turns out that in our model even relatively small variations in inflation expectations can have a substantial impact on the output and welfare losses triggered by current account interventions. For instance, lowering  $\bar{\pi}$  to 1.01 roughly doubles the output and welfare losses associated with current account policies. Instead, increasing  $\bar{\pi}$  to 1.015 substantially mitigates the drop in global output triggered by the implementation of current account policies. Moreover, in this case the average impact on welfare of current account policies is slightly positive. However, current account interventions still exacerbate the inefficiencies due to the zero lower bound. In fact, once the focus is restricted to the non-tradable sector, current account policies have a negative impact on welfare is negative. These results suggests that inflation expectations play a key role in shaping the impact of current account policies on the global economy.

To conclude, we relax the assumption of an inelastic bond supply from the rest of the world. In particular, we assume that the supply of bonds from the rest of the world is given by

$$B_t^{rw} = B^{rw} \left(\frac{R_t}{R^{lf}}\right)^{\zeta},$$

so that the supply of bonds by the rest of the world is increasing in the world interest rate. Notice that this specification implies that in the laissez-faire steady state the bond supply from rest-of-theworld countries takes the same value as in our benchmark calibration. The parameter  $\zeta$  captures the elasticity of  $B_t^{rw}$  with respect to  $R_t$ , and hence by how much the world interest rate falls as a consequence of the adoption of current account policies. Unfortunately, we could find reliable estimates for this elasticity.<sup>32</sup> Hence, we report the results for two benchmark value,  $\zeta = 1$  (low elasticity) and  $\zeta = 10$  (high elasticity).

The key difference with respect to the benchmark economy with inelastic  $B^{rw}$ , is that now the parameter  $\zeta$  is a key determinant of the response of R to the implementation of current account policies. More precisely, the higher  $\zeta$  the less R will drop after an increase in the supply of savings by our model economies. It is then natural to think that the negative impact that current account policies will have on world output will be milder the higher  $\zeta$ . This is precisely the result shown by the two last rows of Table 3. However, current account policy produce a substantial drop in world output even when  $\zeta$  takes the relatively high value of 10. A similar result applies to the welfare losses driven by the fact that current account policies exacerbate the inefficiencies due to the zero lower bound constraint. In fact, once the focus is restricted to the non-tradable sector, current account policies have a negative impact on welfare even for  $\zeta = 10$ . Summing up, while assuming an elastic supply of bonds from the rest of the world changes the quantitative predictions of the model, the results that current account policies depress global output and exacerbate the inefficiencies due to the zero lower bound hold for relatively high elasticities.

# I Data appendix

This appendix provides details on the construction of the series used in the calibration and to construct Figure !.

## I.1 Data used in the calibration

The countries in the sample are Australia, Austria, Canada, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.

- 1. World interest rate. The series for the world interest rate is constructed by Rachel and Smith (2015) following the methodology proposed by King and Low (2014).<sup>33</sup>
- 2. Net foreign assets. The data for the net foreign asset position and GDP come from External Wealth of Nations' dataset by Lane and Milesi-Ferretti (2007).

<sup>&</sup>lt;sup>32</sup>As we alluded to in the main text, the key challenge is that a significant fraction of lending from emerging to advanced countries is in the form of reserve accumulation by emerging countries' governments. These flows might be driven by different considerations than the standard trade-off between risk and return. Because of this, it is hard to pin down quantitatively how these flows react to changes in the world rate.

<sup>&</sup>lt;sup>33</sup>We thank Lukasz Rachel for providing us with the data.

- 3. *Core inflation rate.* Core inflation is computed as the percentage change with respect to the previous year of the CPI for all items excluding food and energy. The series are yearly and provided by the OECD.
- 4. Tradable endowment process. Tradable output is defined as the aggregate of value added in agriculture, hunting, forestry, fishing, mining, manufacturing and utilities. To extract the cyclical component from the actual series we used the following procedure. For each country we divided tradable output by total population and took logs. Since our model abstracts from aggregate shocks, for every year we subtracted from the country-level series the logarithm of the average cross-sectional tradable output per capita. For every country we then obtained the cyclical component of the resulting series by removing a country-specific log-linear trend. The first order autocorrelation and the standard deviation of the final series are respectively 0.87 and 0.056. We use yearly data for the period 1970-2015, coming from the United Nations' National account main aggregate database.
- 5. *Identifying financial crises.* We identify a financial crisis in the data as an episode in which the cyclical component of the trade balance is one standard deviation above its average and the cyclical component of tradable output, as defined above, is one standard deviation below its average. We define the start of a financial crisis as the first year in which the cyclical component of the trade balance is half standard deviation above its mean, while a financial crisis ends when the cyclical component of the trade balance of the trade balance falls below one standard deviation above its mean.

To compute the cyclical component of the trade balance we used the following procedure. We collected yearly series for the trade balance for the period 1970-2015 from the OECD. The data are in 2010 constant US dollars. For each country, we then divided by total population. Since our model abstracts from aggregate shocks, for every year we subtracted the cross-sectional average from the series. Finally, we obtained the cyclical component from the resulting series by subtracting a country-specific linear trend.

## I.2 Data used to construct Figure 1

- 1. *Policy rates.* Monthly series. US: Effective Federal Funds Rate from the Fed Board of Governors of the Federal Reserve System. Euro area and Japan: Central bank discount rate from the IMF International Financial Statistics database. UK: Bank of England policy rate.
- 2. GDP per capita. Constant prices, series from the World Bank.

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