# The Social Value of Financial Expertise - Online Appendix 

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## A Equilibrium in the Baseline Model

## A. 1 Equilibrium Characterization

Taking $W(\theta)$ as given, let $p^{*}, \theta^{*}$ and $s^{*}$ be the highest- $p^{*}$ solution to the system of equations (15), (17) and (18). Furthermore, assume the following:

Assumption 1. $\frac{1}{p} \frac{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)}{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)+\lambda\left(1-\theta^{*}\right)} V<1$ for all $p>p^{*}$
Proposition 1. If Assumption 1 holds, there is a unique equilibrium in the trading stage, where:

1. Reservation prices are:

$$
p^{R}(i, s)=\left\{\begin{array}{cl}
\max \left\{p^{*}, \beta(s) V\right\} & \text { if } i \geq \lambda  \tag{A.1}\\
0 & \text { if } i<\lambda
\end{array}\right.
$$

2. The solution to the banks' problem is:

$$
\{\delta(\theta), p(\theta), \chi(\theta)\}=\left\{\begin{array}{cc}
\left\{\frac{1}{p^{*}}, p^{*}, \mathbb{I}(i \geq \lambda \theta)\right\} & \text { if } \theta \geq \theta^{*}  \tag{A.2}\\
\{0,0,0\} & \text { if } \theta<\theta^{*}
\end{array}\right.
$$

3. The densities over assets are:

$$
a(i ; \chi, p)=\left\{\begin{array}{cl}
\frac{\beta^{-1}\left(\frac{p}{V}\right) \chi(i)}{\frac{\int_{0}^{\lambda} \chi(i) d i+\int_{\chi}^{1} \chi(i) \beta^{-1}\left(\frac{p}{V}\right) d i}{\chi}} & \text { if } i \geq \lambda \text { and } p \geq p^{*}  \tag{A.3}\\
\frac{\chi(i)}{\int_{0}^{\lambda} \chi(i) d i+\int_{\lambda}^{1} \chi(i) \beta^{-1}\left(\frac{p}{V}\right) d i} & \text { if } i<\lambda \text { and } p \geq p^{*} \\
0 & \text { if } i \geq \lambda \text { and } p<p^{*} \\
\frac{\chi(i)}{\int_{0}^{\lambda} \chi(i) d i} & \text { if } i<\lambda \text { and } p<p^{*}
\end{array}\right.
$$

4. The rationing function is:

$$
\mu(p, i)=\left\{\begin{array}{cc}
1 & \text { if } i \geq \lambda, p \leq p^{*}  \tag{A.4}\\
\int_{\theta^{*}}^{\frac{i}{\lambda}} \frac{1}{\lambda(1-\theta)+s^{*}(1-\lambda)} \frac{1}{p^{*}} d W(\theta) & \text { if } i \in[\lambda \theta, \lambda), p \leq p^{*} \\
0 & \text { if } i<\lambda \theta, p \leq p^{*} \\
0 & \text { if } p>p^{*}
\end{array}\right.
$$

Proof.
(a) Equations (A.1)-(A.4) constitute an equilibrium.
i. Household optimization. (A.4) implies that:

$$
p^{L}(i)=\left\{\begin{array}{cl}
p^{*} & \text { if } i \geq \lambda \\
0 & \text { if } i<\lambda
\end{array}\right.
$$

This immediately implies that $p^{R}(i, s)$ from (A.1) solves the household's problem.
ii. Bank optimization.
A. $\chi(\theta)$ is the optimal acceptance rule because, given (A.3), any other rule that satisfies (8) includes a higher proportion of bad assets.
B. At any $p<p^{*}$, there are no good assets on sale so it is not optimal for
any bank to choose this. For any $p>p^{*}$ :

$$
\begin{align*}
\frac{1}{p} \frac{\beta^{-1}\left(\frac{p}{V}\right)}{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)+\lambda\left(1-\theta^{*}\right)} & <\frac{1}{p^{*}} \frac{s^{*}}{s^{*}(1-\lambda)+\lambda\left(1-\theta^{*}\right)} \\
\frac{p^{*}}{p} \frac{\beta^{-1}\left(\frac{p}{V}\right)}{s^{*}} & <\frac{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)+\lambda\left(1-\theta^{*}\right)}{s^{*}(1-\lambda)+\lambda\left(1-\theta^{*}\right)} \\
\frac{p^{*}}{p} \frac{\beta^{-1}\left(\frac{p}{V}\right)}{s^{*}} & <\frac{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)+\lambda(1-\theta)}{s^{*}(1-\lambda)+\lambda(1-\theta)} \quad \text { for all } \theta \geq \theta^{*} \\
\frac{1}{p} \frac{\beta^{-1}\left(\frac{p}{V}\right)}{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)+\lambda(1-\theta)} & <\frac{1}{p^{*}} \frac{s^{*}}{s^{*}(1-\lambda)+\lambda(1-\theta)} \quad \text { for all } \theta \geq \theta^{*} \tag{A.5}
\end{align*}
$$

The first step is Assumption (1); the second is just rearranging; the third follows because the right hand side is increasing in $\theta$ and the last is just rearranging. Inequality (A.5) implies that all banks with $\theta \geq \theta^{*}$ prefer to buy at price $p^{*}$ than at higher prices. Therefore if they buy at all they buy at price $p^{*}$.
C. For $\theta>\theta^{*}, \tau(\theta)>0$ so the budget constraint (7) binds; for $\theta<\theta^{*}$ there is no $\chi(\theta)$ that satisfies (8) and leads to a positive value for the objective (6). Therefore $\delta(\theta)$ is optimal .
iii. Consistency of $A$ and $\mu$. Replacing reservation prices (A.1) into (10) and using this to replace $S(i ; p)$ into (5) leads to (A.3). Adding up demand using (A.2) and (12) and replacing in (13) implies (A.4).

## (b) The equilibrium is unique

Note first that since no feasible acceptance rule has $\chi(i) \neq \chi\left(i^{\prime}\right)$ for $i, i^{\prime} \geq \lambda$, this implies that $p^{L}(i)=p^{L}(\lambda)$ and $S(i, p)=S(\lambda, p)$ for all $i \geq \lambda$. Now proceed by contradiction.

Suppose there is another equilibrium with $p^{L}(\lambda)<p^{*}$. Households' optimization condition (4) and formula (10) for supply imply that for $p \in\left[p^{L}(\lambda), p^{*}\right]$ :

$$
S(i, p)=\left\{\begin{array}{cl}
\beta^{-1}\left(\frac{p}{V}\right) & \text { if } i \geq \lambda  \tag{A.6}\\
1 & \text { if } i<\lambda
\end{array}\right.
$$

(A.6) implies that all banks with $\theta>\theta^{*}$ can attain $\tau(\theta)>0$ by choosing $p^{*}$. By (A.5), they prefer $p^{*}$ to any $p^{\prime}>p^{*}$ and therefore in equilibrium they all chose
some $p(\theta) \in\left[p^{L}(\theta), p^{*}\right]$ and $\delta(\theta)=\frac{1}{p(\theta)}$. Using (5):

$$
a(i, \chi(\theta), p(\theta))=\frac{\beta^{-1}\left(\frac{p(\theta)}{V}\right)}{\beta^{-1}\left(\frac{p(\theta)}{V}\right)+\lambda(1-\theta)} \quad \text { for all } i \geq \lambda
$$

Using (13), this implies that

$$
\mu(p, \lambda)=\int_{\{\theta: p(\theta) \geq p\}} \frac{1}{\beta^{-1}\left(\frac{p(\theta)}{V}\right)+\lambda(1-\theta)} \frac{1}{p(\theta)} d W(\theta)
$$

and therefore

$$
\begin{align*}
\mu\left(p^{L}(\lambda), \lambda\right) & \geq \int_{\theta^{*}}^{1} \frac{1}{\beta^{-1}\left(\frac{p(\theta)}{V}\right)+\lambda(1-\theta)} \frac{1}{p(\theta)} d W(\theta) \\
& \geq \int_{\theta^{*}}^{1} \frac{1}{s^{*}+\lambda(1-\theta)} \frac{1}{p^{*}} d W(\theta) \\
& =1 \tag{A.7}
\end{align*}
$$

The first inequality follows because the set $\{\theta: p(\theta) \geq p(\lambda)\}$ includes $\left[\theta^{*}, 1\right]$; the second follows because $\beta^{-1}\left(\frac{p^{*}}{V}\right)=s^{*}, \beta^{-1}$ is increasing and $p^{*} \geq p(\theta)$; the last equality is just the market clearing condition (18). Furthermore, if $p(\theta)<p^{*}$ for a positive measure of banks, then (A.7) is a strict inequality, which leads to a contradiction. Instead, if $p(\theta)=p^{*}$ for almost all banks, then $p^{L}(\lambda)=p^{*}$, which contradicts the premise.
Suppose instead that there is an equilibrium such that $p^{L}(\lambda)>p^{*}$. This implies that there is no supply of good assets at any price $p<p^{L}(\lambda)$ and therefore no bank with $\theta<\theta^{*}$ chooses $\delta(\theta)>0$ and banks $\theta \in\left[\theta^{*}, 1\right]$ choose some price
$p(\theta) \geq p^{L}(\lambda)$ and $\delta(\theta) \leq \frac{1}{p(\theta)}$. Therefore, using (5) and (13), we have

$$
\begin{aligned}
\mu\left(p^{L}(\lambda), \lambda\right) & \leq \int_{\theta^{*}}^{1} \frac{1}{\beta^{-1}\left(\frac{p(\theta)}{V}\right)+\lambda(1-\theta)} \frac{1}{p(\theta)} d W(\theta) \\
& <\int_{\theta^{*}}^{1} \frac{1}{s^{*}+\lambda(1-\theta)} \frac{1}{p^{*}} d W(\theta) \\
& =1
\end{aligned}
$$

The first inequality follows from $\delta(\theta) \leq \frac{1}{p(\theta)}$; the second follows because $\beta^{-1}\left(\frac{p^{*}}{V}\right)=$ $s^{*}, \beta^{-1}$ is increasing and $p^{*}<p(\theta)$; the last equality is just the market clearing condition (18). Again, this is a contradiction.
Therefore any equilibrium must have $p^{L}(\lambda)=p^{*}$. The rest of the equilibrium objects follow immediately.

## A. 2 The Role of Assumption 1

The equilibrium concept gives banks the option to buy assets at prices other than $p^{*}$. Buying at lower prices is clearly worse than buying at $p^{*}$ because the reservation price for good assets is at least $p^{*}$ so no good assets are on sale at lower prices. Assumption 1 ensures that buying at higher price is not preferred either. Given the reservation prices (A.1), the surplus per unit of wealth for bank $\theta^{*}$ if it buys at price $p>p^{*}$ is:

$$
\frac{1}{p}\left[\frac{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda) V}{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)+\lambda\left(1-\theta^{*}\right)}-p\right]
$$

In principle, the bank faces a tradeoff: better selection (because $\beta^{-1}$ is an increasing function) but a higher price. Assumption 1 ensures that the direct higher-price effect dominates and a bank with expertise $\theta^{*}$ has no incentive to pay higher prices to ensure better selection. It is then possible to show that if this is true for the marginal bank $\theta^{*}$, it is true for all banks: higher- $\theta$ banks care even less about selection because they can filter assets themselves and lower- $\theta$ banks can never earn surplus in a market where $\theta^{*}$ would not. One can still solve for equilibria where Assumption 1 does not hold, but they are somewhat more
complicated. Wilson (1980), Stiglitz and Weiss (1981) and Arnold and Riley (2009) analyze the implications of models where an analogue of Assumption 1 doesn't hold.

## B Continuous $q(i)$

## B. 1 Computing $r$

Define:

$$
\begin{aligned}
S_{L}(p) & \equiv \int_{0}^{\lambda} s^{*}(i, p) d i \\
S_{H}(p) & \equiv \int_{\lambda}^{1} s^{*}(i, p) d i
\end{aligned}
$$

These represent, respectively, the quantity of bad and good assets offered on sale at price $p$. Further define:

$$
\begin{aligned}
& Q_{L}(p) \equiv \int_{0}^{\lambda} s^{*}(i, p) q(i) d i \\
& Q_{H}(p) \equiv \int_{\lambda}^{1} s^{*}(i, p) q(i) d i
\end{aligned}
$$

These represent, respectively, total dividends of bad and good assets offered on sale. Their derivatives are given by:

$$
\begin{aligned}
S_{L}^{\prime}(p) & =\int_{0}^{\lambda} \frac{\partial s^{*}(i, p)}{\partial p} d i \\
S_{H}^{\prime}(p) & =\int_{\lambda}^{1} \frac{\partial s^{*}(i, p)}{\partial p} d i \\
Q_{L}^{\prime}(p) & =\int_{0}^{\lambda} \frac{\partial s^{*}(i, p)}{\partial p} q(i) d i \\
Q_{H}^{\prime}(p) & =\int_{\lambda}^{1} \frac{\partial s^{*}(i, p)}{\partial p} q(i) d i
\end{aligned}
$$

The equilibrium conditions (58) and (59) can be rewritten as:

$$
\begin{align*}
& p^{*}=\frac{\left(1-\theta^{*}\right) Q_{L}\left(p^{*}\right)+Q_{H}\left(p^{*}\right)}{\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)}  \tag{B.1}\\
& p^{*}=\int_{\theta^{*}}^{1} \frac{1}{(1-\theta) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)} d W(\theta) \tag{B.2}
\end{align*}
$$

and in matrix form:

$$
K\left(p^{*}, \theta^{*}\right)=\binom{p^{*}-\frac{\left(1-\theta^{*}\right) Q_{L}\left(p^{*}\right)+Q_{H}\left(p^{*}\right)}{\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)}}{p^{*}-\int_{\theta^{*}}^{1} \frac{w(\theta)}{(1-\theta) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)} d \theta}=\binom{0}{0}
$$

The gradient of $K$ is:

$$
D=\left(\begin{array}{ll}
d_{1 p} & d_{1 \theta} \\
d_{2 p} & d_{2 \theta}
\end{array}\right)
$$

with

$$
\begin{aligned}
& {\left[\left(1-\theta^{*}\right) Q_{L}^{\prime}\left(p^{*}\right)+Q_{H}^{\prime}\left(p^{*}\right)\right]\left[\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right] } \\
& d_{1 p}=1-\frac{-\left[\left(1-\theta^{*}\right) S_{L}^{\prime}\left(p^{*}\right)+S_{H}^{\prime}\left(p^{*}\right)\right]\left[\left(1-\theta^{*}\right) Q_{L}\left(p^{*}\right)+Q_{H}\left(p^{*}\right)\right]}{\left[\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right]^{2}} \\
& d_{1 \theta}=\frac{Q_{L}\left(p^{*}\right) S_{H}\left(p^{*}\right)-S_{L}\left(p^{*}\right) Q_{H}\left(p^{*}\right)}{\left[\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right]^{2}} \\
& d_{2 p}=1+\int_{\theta^{*}}^{1} w(\theta)\left[(1-\theta) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right]^{-2}\left((1-\theta) S_{L}^{\prime}\left(p^{*}\right)+S_{H}^{\prime}\left(p^{*}\right)\right) d \theta \\
& d_{2 \theta}=\frac{w\left(\theta^{*}\right)}{\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)}
\end{aligned}
$$

In equilibrium, bad assets will be rationed and, in order to compute how much surplus is created (or destroyed) from trades of bad assets, I need to compute the fraction $\mu$ of bad assets put on sale that will actually be sold. Buyer $\theta$ will buy a total of

$$
\frac{1}{p} \frac{(1-\theta) S_{L}(p)}{(1-\theta) S_{L}(p)+S_{H}(p)}
$$

bad assets per unit of wealth, so total demand of bad assets will be

$$
\frac{1}{p} \int_{\theta^{*}}^{1} \frac{(1-\theta) S_{L}(p)}{(1-\theta) S_{L}(p)+S_{H}(p)} d W(\theta)
$$

and therefore

$$
\begin{equation*}
\mu=\frac{1}{p^{*}} \int_{\theta^{*}}^{1} \frac{(1-\theta)}{(1-\theta) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)} d W(\theta) \tag{B.3}
\end{equation*}
$$

The total surplus will be

$$
S=\underbrace{\mu\left(p^{*}, \theta^{*}\right)}_{\text {rationing }} \int_{0}^{\lambda}[\underbrace{q(i) \int_{0}^{s\left(i, p^{*}\right)}(1-\beta(s)) d s}_{\text {gains from trade in asset } i<\lambda}] d i+\int_{\lambda}^{1}[\underbrace{q(i) \int_{0}^{s\left(i, p^{*}\right)}(1-\beta(s)) d s}_{\text {gains from trade in asset } i \geq \lambda}] d i
$$

Taking the derivative, the marginal social value of an increase in expertise is given by:

$$
\begin{equation*}
S^{\prime}\left(\theta_{j}\right)=\mu\left(p^{*}, \theta^{*}\right) \int_{0}^{1}\left(q(i)\left(1-\beta\left(s^{*}\left(i, p^{*}\right)\right)\right) \frac{d s^{*}\left(i, p^{*}\right)}{d \theta_{j}}+q(i) \frac{d \mu\left(p^{*}, \theta^{*}\right)}{d \theta_{j}} \int_{0}^{s\left(i, p^{*}\right)}(1-\beta(s)) d s\right) d i \tag{B.4}
\end{equation*}
$$

I need to compute $\frac{d s^{*}\left(i, p^{*}\right)}{d \theta_{j}}$. and $\frac{d \mu\left(p^{*}, \theta^{*}\right)}{d \theta_{j}}$ to replace in (B.4). Rewrite $\frac{d s^{*}\left(i, p^{*}\right)}{d \theta_{j}}$ as

$$
\begin{equation*}
\frac{d s^{*}\left(i, p^{*}\right)}{d \theta_{j}}=\frac{\partial s^{*}\left(i, p^{*}\right)}{\partial p^{*}} \frac{\partial p^{*}}{\partial \theta_{j}} \tag{B.5}
\end{equation*}
$$

The term $\frac{\partial s^{*}\left(i, p^{*}\right)}{\partial p^{*}}$ can be computed directly from (57). Using the implicit function theorem:

$$
\begin{equation*}
\binom{\frac{\partial p^{*}}{\partial \epsilon}}{\frac{\partial \theta^{*}}{\partial \epsilon}}=-D^{-1}\binom{0}{\frac{\partial K_{2}^{*}}{\partial \theta_{j}}} \tag{B.6}
\end{equation*}
$$

where, using (59):

$$
\begin{equation*}
\frac{\partial K_{2}}{\partial \theta_{j}}=-w_{j}\left(\left(1-\theta_{j}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right)^{-2} S_{L}\left(p^{*}\right) \tag{B.7}
\end{equation*}
$$

Now rewrite $\frac{d \mu\left(p^{*}, \theta^{*}\right)}{d \theta_{j}}$ as

$$
\begin{equation*}
\frac{d \mu}{d \theta_{j}}=\underbrace{\frac{\partial \mu}{\partial \theta_{j}}}_{\text {direct efffect }}+\underbrace{\frac{\partial \mu\left(i ; p^{*}, \theta^{*}\right)}{\partial p^{*}} \frac{\partial p^{*}}{\partial \theta_{j}}}_{\text {price effect }}+\underbrace{\frac{\partial \mu\left(i ; p^{*}, \theta^{*}\right)}{\partial \theta^{*}} \frac{\partial \theta^{*}}{\partial \theta_{j}}}_{\text {marginal seller effect }} \tag{B.8}
\end{equation*}
$$

Using (B.3), the direct effect is:

$$
\frac{\partial \mu}{\partial \theta_{j}}=-\frac{1}{p^{*}} \frac{S_{H}\left(p^{*}\right)}{\left[\left(1-\theta_{j}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right]^{2}}
$$

and the indirect effects are:

$$
\begin{aligned}
& \frac{\partial \mu\left(i ; p^{*}, \theta^{*}\right)}{\partial p^{*}}=-\left[\begin{array}{c}
\frac{1}{\left(p^{*}\right)^{2}} \int_{\theta^{*}}^{1} \frac{(1-\theta)}{(1-\theta) S_{L}\left(p^{*} *\right)+S_{H}\left(p^{*}\right)} d W(\theta) \\
+\frac{1}{p^{*}} \int_{\theta^{*}}^{1} \frac{\left((1-\theta) S_{L}^{\prime}\left(p^{*}\right)+S_{H}^{\prime}\left(p^{*}\right)\right)(1-\theta)}{\left((1-\theta) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right)^{-2}} d W(\theta)
\end{array}\right] \\
& \frac{\partial \mu\left(i ; p^{*}, \theta^{*}\right)}{\partial \theta^{*}}=-\frac{1}{p^{*}} \frac{\left(1-\theta^{*}\right) w\left(\theta^{*}\right)}{\left(1-\theta^{*}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)}
\end{aligned}
$$

with $\frac{\partial p^{*}}{\partial \theta_{j}}$ and $\frac{\partial \theta^{*}}{\partial \theta_{j}}$ given by (B.6) and (B.7). Replacing (B.5) and (B.8) into (B.4) gives the marginal social surplus.

Profits for bank $j$ are:

$$
w_{j} \tau\left(\theta_{j}\right)=\frac{w_{j}}{p^{*}}\left[\frac{(1-\theta) Q_{L}\left(p^{*}\right)+Q_{H}\left(p^{*}\right)}{(1-\theta) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)}-p^{*}\right]
$$

so the marginal private gain from increasing expertise is:

$$
\begin{equation*}
w_{j} \tau^{\prime}\left(\theta_{j}\right)=\frac{1}{p^{*}} \frac{S_{L}\left(p^{*}\right) Q_{H}\left(p^{*}\right)-Q_{L}\left(p^{*}\right) S_{H}\left(p^{*}\right)}{\left[\left(1-\theta_{j}\right) S_{L}\left(p^{*}\right)+S_{H}\left(p^{*}\right)\right]^{2}} \tag{B.9}
\end{equation*}
$$

Taking the ratio of (B.4) and (B.9) and simplifying:
$r=\frac{\binom{D_{12}^{-1} S_{L}\left(p^{*}\right) \mu\left[\int_{0}^{\lambda} q(i)\left(\left(1-\beta\left(s^{*}\left(i, p^{*}\right)\right)\right) \frac{\partial s^{*}\left(i, p^{*}\right)}{\partial p^{*}}\right) d i+\int_{\lambda}^{1} q(i)\left(1-\beta\left(s^{*}\left(i, p^{*}\right)\right)\right) \frac{\partial s^{*}\left(i, p^{*}\right)}{\partial p^{*}} d i\right]}{+\left(-\frac{1}{p^{*}} S_{H}\left(p^{*}\right)+\left(\frac{\partial \mu\left(i ; p^{*}, \theta^{*}\right)}{\partial p^{*}} D_{12}^{-1}+\frac{\partial \mu\left(i ; p^{*}, \theta^{*}\right)}{\partial \theta^{*}} D_{22}^{-1}\right) S_{L}\left(p^{*}\right)\right) \int_{0}^{\lambda} q(i)\left(\int_{0}^{s\left(i, p^{*}\right)}(1-\beta(s)) d s\right) d i}}{\frac{1}{p^{*}}\left(S_{L}\left(p^{*}\right) Q_{H}\left(p^{*}\right)-Q_{L}\left(p^{*}\right) S_{H}\left(p^{*}\right)\right)}$
which does not depend on $\theta_{j}$ or $w_{j}$, so Proposition 1 holds.

## B. 2 Computing $\alpha, f$ and $\eta$

Banks' average profitability is given by:

$$
\alpha=\frac{\mu Q_{L}+Q_{H}}{\int_{\theta^{*}}^{1} w(\theta) d \theta}
$$

The numerator is the total dividends from assets that are actually sold; the denominator is the total funds that buyers spend on assets.

The fraction of bad assets among traded assets is

$$
f=1-\frac{p^{*} S_{H}}{\int_{\theta^{*}}^{1} d W(\theta)}
$$

The total number of good assets traded is

$$
G=S_{H}\left(p^{*}\right)
$$

so its elasticity with respect to a capital inflow is given by:

$$
\eta=\frac{S_{H}^{\prime}\left(p^{*}\right) \frac{d p^{*}}{d \Delta}}{S_{H}\left(p^{*}\right)}
$$

where

$$
\begin{aligned}
\frac{d p^{*}}{d \Delta} & =-D_{12}^{-1} \frac{\partial K_{2}^{*}}{\partial \Delta} \\
& =D_{12}^{-1} p^{*}
\end{aligned}
$$

so

$$
\eta=\frac{S_{H}^{\prime}\left(p^{*}\right)}{S_{H}\left(p^{*}\right)} D_{12}^{-1} p^{*}
$$

## C "False Negative" Information

Suppose a bank with expertise $\theta$ observes a signal that, instead of following equation (3), is given by:

$$
x(i, \theta)=\mathbb{I}(i \geq 1-\theta(1-\lambda))
$$

This means banks make "false negative" as opposed to "false positive" mistakes. A bank that buys an asset with $x(i, \theta)=1$ is sure that the asset is good, but for some good assets it observes $x(i, \theta)=0$. Among good assets, those with higher $i$ are more transparent, since more banks notice that they are good. For each good asset $i \in[\lambda, 1]$, define

$$
\hat{\theta}(i) \equiv \frac{1-i}{1-\lambda}
$$

Expertise $\hat{\theta}(i)$ is the lowest $\theta$ that is sufficient to realize that asset $i$ is good. Assume the following conditions hold:

## Assumption 2.

1. $w(\theta)$ is strictly decreasing
2. 

$$
\begin{equation*}
p^{C}(i) \geq p^{N S}(i) \tag{C.1}
\end{equation*}
$$

where $p^{C}(i)$ is the highest solution to

$$
\begin{equation*}
p=\frac{1}{\beta^{-1}\left(\frac{p}{V}\right)(1-\lambda)} w(\hat{\theta}(i)) \tag{C.2}
\end{equation*}
$$

and $p^{N S}(i)$ is the highest solution to

$$
p=\frac{(i-\lambda) \beta^{-1}\left(\frac{p}{V}\right)}{(i-\lambda) \beta^{-1}\left(\frac{p}{V}\right)+\lambda} V
$$

Part 1 of Assumption 2 says that there is less wealth at higher levels of expertise. While stated directly in terms of the wealth/expertise distribution, which is endogenous, it's straightforward to find sufficient conditions on endowments and costs of expertise such that this assumption holds. Part 2 ensures that no bank wants to buy assets for which it observes $x(i, \theta)=0$. Kurlat (2016) shows that the equilibrium takes the following form (and also characterizes equilibrium for the case where Assumption 2 does not hold).

Let $p(i)=\min \left\{p^{C}(i), V\right\}$. In equilibrium, households are able to sell asset $i$ at price $p(i)$; formally $\mu(p, i)=\mathbb{I}(p \leq p(i))$. Households' indifference condition (15) defines a cutoff seller for asset $i$ given by:

$$
\begin{equation*}
s^{*}(i)=\beta^{-1}\left(\frac{p(i)}{V}\right) \tag{C.3}
\end{equation*}
$$

Buyer $\hat{\theta}(i)$ buys all the units of asset $i$ that are sold, and if $p(i)<1$, he exhausts his wealth doing so, which implies the price of asset $i$ must satisfy the cash-in-the-market condition:

$$
\begin{equation*}
p(i)=\frac{w(\hat{\theta}(i))}{s^{*}(i)(1-\lambda)} \tag{C.4}
\end{equation*}
$$

Replacing (C.3) into (C.4) gives equation (C.1), which defines $p^{C}(i)$.
Using the implicit function theorem:

$$
p^{\prime}(i)=-\frac{\beta^{\prime}\left(s^{*}(i)\right) w^{\prime}(\hat{\theta}(i))}{(1-\lambda)^{2}\left[\beta^{\prime}\left(s^{*}(i)\right) s^{*}(i)+\beta\left(s^{*}(i)\right)\right]}
$$

so Assumption 2.1 ensures that $p(i)$ is increasing, creating transparency premium: more transparent assets trade at a higher price. Therefore $p(i)$ is the cheapest price at which bank $\hat{\theta}(i)$ can detect good assets on sale, so it's optimal for it to buy at that price. Furthermore, Assumption 2.2 ensures that it is unprofitable for any bank to buy assets without being
selective. If a bank decided to buy at price $p(i)$ and accept any asset offered on sale, it would face a pool containing $\lambda$ bad assets plus $s^{*}(i)$ of each asset in $[\lambda, i]$. Hence it would obtain an average quality of

$$
\frac{(i-\lambda) s^{*}(i)}{(i-\lambda) s^{*}(i)+\lambda} V
$$

Assumption 2.2 ensures that this is below $p(i)$, so buying non-selectively is unprofitable.
Bank $j$ 's profits are given by:

$$
w_{j} \tau\left(\theta_{j}\right)=w_{j} \frac{V}{p\left(1-\theta_{j}(1-\lambda)\right)}
$$

so the marginal private value of expertise is:

$$
\begin{equation*}
w_{j} \tau^{\prime}\left(\theta_{j}\right)=w_{j} V \frac{(1-\lambda) p^{\prime}\left(1-\theta_{j}(1-\lambda)\right)}{\left[p\left(1-\theta_{j}(1-\lambda)\right)\right]^{2}} \tag{C.5}
\end{equation*}
$$

The social surplus generated by selling asset $i$ is

$$
\int_{0}^{s^{*}(i)}(1-\beta(s)) d s
$$

Therefore the marginal social value of having a unit of wealth at expertise level $\hat{\theta}(i)$ is

$$
\begin{equation*}
\Sigma(i)=\frac{d s^{*}(i)}{d w(\hat{\theta}(i))}\left(1-\beta\left(s^{*}(i)\right)\right) \tag{C.6}
\end{equation*}
$$

and the marginal social value of expertise for a bank with expertise $\theta_{j}$ and wealth $w_{j}$ is

$$
\begin{align*}
S^{\prime}\left(\theta_{j}\right) & =\left.w_{j} \frac{d \Sigma(i)}{d i}\right|_{i=1-\theta_{j}(1-\lambda)} \frac{1}{d \theta / d i} \\
& =-\left.w_{j}(1-\lambda) \frac{d \Sigma(i)}{d i}\right|_{i=1-\theta_{j}(1-\lambda)} \tag{C.7}
\end{align*}
$$

Using the implicit function theorem:

$$
\begin{equation*}
\frac{d s^{*}}{d w}=\frac{1}{V(1-\lambda)\left[\beta^{\prime}\left(s^{*}(i)\right) s^{*}(i)+\beta\left(s^{*}(i)\right)\right]} \tag{C.8}
\end{equation*}
$$

so replacing (C.8) in (C.6) and taking the derivative:

$$
\frac{d \Sigma(i)}{d i}=-\frac{\frac{d s^{*}}{d i}}{V(1-\lambda)} \frac{\beta^{\prime}\left(s^{*}(i)\right)\left[\beta^{\prime}\left(s^{*}(i)\right) s^{*}(i)+\beta\left(s^{*}(i)\right)\right]}{+\left[\beta^{\prime \prime}\left(s^{*}(i)\right) s^{*}(i)+2 \beta^{\prime}\left(s^{*}(i)\right)\right]\left(1-\beta\left(s^{*}(i)\right)\right)}\left[\begin{array}{l} 
\\
{\left[\beta^{\prime}\left(s^{*}(i)\right) s^{*}(i)+\beta\left(s^{*}(i)\right)\right]^{2}} \tag{C.9}
\end{array}\right.
$$

where, using the implicit function theorem:

$$
\begin{equation*}
\frac{d s^{*}}{d i}=-\frac{w^{\prime}(\hat{\theta}(i))}{V(1-\lambda)^{2}\left[\beta^{\prime}\left(s^{*}(i)\right) s^{*}(i)+\beta\left(s^{*}(i)\right)\right]} \tag{C.10}
\end{equation*}
$$

The ratio of marginal social to private value of expertise $r_{j}=\frac{S^{\prime}\left(\theta_{j}\right)}{w_{j} \tau^{\prime}\left(\theta_{j}\right)}$ can be found using equations (C.5)-(C.10).

Proposition 1 does not apply, so $r_{j}$ could be different for different banks. It's easy to find examples with $r$ above or below 1 , or even with $r$ above 1 for some banks and below 1 for others. Figure 1 shows a numerical example.


Figure 1: Equilibrium and welfare with "false negative" information. The example uses $\lambda=0.1, \beta(s)=0.1+0.7 s, w(\theta)=0.5-0.3 \theta$ and $V=1$.

Unfortunately, the expression for $r_{j}$ does not neatly decompose into objects that have
empirical counterparts as it does in the false-positives case. Therefore it is not clear how to measure it empirically.

## D Data Sources and Variable Definitions

The Thomson Reuters/Securities Data Company contains data on all corporate bonds issued in the United States. For each bond, the database reports: date of issuance, dollar volume, maturity, coupon rate, yield and price at issuance. From this database I extract all bonds flagged as "high-yield", where high yield is defined as "having a Standard \& Poor's rating of BB+ and below or a Moody's rating of Ba1 and below". The Bloomberg database in principle contains the same universe of bonds, and reports the same variables on them. I select from it all the bonds rated below investment grade by either S\&P, Moody's or Fitch. Both databases contain bonds that are not included in the other, and in some instances they report inconsistent information about the same bond. I simply add the two databases together, eliminating duplicates and following SDC when there are discrepancies. The date ranges from 1977 to 2010 (by date of issuance). This leaves a total of 30,193 bonds in my main sample, of which I have price information for 17,872 .

For each bond, I also record the yield on a Treasury bond of the same maturity at the date of issuance. To construct the yield, I obtain from Bloomberg the Treasury rates of standard maturities ( $1,2,3,5,7,10,20$ and 30 -year) at the issuance date. Then, I interpolate them to build the yield of a Treasury bond that expires at the same time of the bond. For bonds with maturities larger than 30 years, I set the 30 -year Treasury bond yield as the corresponding Treasury yield.

For each bond, I calculate a price in two ways. The first is directly, by just taking the recorded price at issuance. The second is indirectly, by projecting all the coupon payments (assuming yearly interest-only coupons and a single principal payment at maturity) and discounting them at the recorded yield at issuance. By definition, the price at issuance, coupons and yield at issuance of a bond are linked by

$$
\begin{equation*}
p=\sum_{t=1}^{T} \frac{c_{t}}{(1+y)^{t}} \tag{D.1}
\end{equation*}
$$

where $c_{t}$ is the coupon payment at time $t$ (including principal and interest), $T$ is the bond's maturity, $y$ is the yield and $p$ is the price. This implies that in theory the indirect calculation should give the same answer as directly recording the price, up to some inaccuracy in the
exact timing of coupons, which the database does not detail. Indeed, for $90.3 \%$ of bonds, the two measures give answers within $1 \%$ of each other. However, there are some discrepancies in the database, often because the price-at-issuance is just recorded as equal to the face value. Because of this, the indirect calculation seems more reliable, and whenever I have information about the bond's yield at issuance I record the price using the indirect calculation; for bonds where the yield-at-issuance information is missing but I do have the issuance price I use the issuance price directly.

Since bonds differ across many dimensions, prices are not directly comparable across bonds. For instance, lower-coupon bonds will have a lower price than higher-coupon bonds of the same maturity and default probability. In order to have a measure of $p$ that is comparable across bonds, I first compute the promised present value for each bond by discounting the projected coupons at the maturity-matched Treasury rate. I then normalize the price of each bond by dividing it by the promised present value. From this I obtain a measure of price per unit of promised present value.

The NYU Salomon database contains a listing of all bonds issued in the same 1977-2010 sample period that subsequently defaulted, including those that were originally issued as junk bonds and those that were not. I add a binary default indicator to each bond in the main sample that is also found in the NYU Salomon database. I match a bond in the main sample to one in the NYU Salomon database whenever (a) the entry in main sample includes the CUSIP identifier and it matches an entry in the NYU Salomon database, (b) the entry in the main sample lacks a CUSIP identifier but the bond (i) is issued the same year, (ii) is issued be the same issuer and (iii) has either the same initial volume or the same coupon rate as an entry in the NYU Salomon database.

Investment bank profitability is measured as $\frac{\text { Net Income }}{\text { Net Worth }}$ for all firms in Compustat classified as investment banks. GDP growth is real GDP growth from NIPA. Stock market excess returns are the return on the S\&P500 index minus the return on 3-month T-Bills. The price-earnings ratio is the cyclically adjusted price-earnings ratio computed by Robert Shiller (http://www.econ.yale.edu/~shiller/data.htm).

## References

Arnold, L. G. and Riley, J. G.: 2009, On the possibility of credit rationing in the stiglitz-weiss model, American Economic Review 99(5), 2012-21.

Kurlat, P.: 2016, Asset markets with heterogeneous information, Econometrica 84, 33-85.

Stiglitz, J. E. and Weiss, A.: 1981, Credit rationing in markets with imperfect information, American Economic Review 71(3), 393-410.

Wilson, C.: 1980, The nature of equilibrium in markets with adverse selection, Bell Journal of Economics 11(1), 108-130.

