Online Appendix

Optimal Trend Inflation

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Appendix A - Derivation of the Sticky-Price Economy

A1. Firms' Optimization Problem

Let W_t denote the nominal wage and r_t the real rental rate of capital, firm j chooses the factor input mix so as to minimize production costs $K_{jt}P_t r_t + L_{jt}W_t$ subject to the constraints imposed by the production function (2). Let

$$I_{jt} \equiv F_t + Y_{jt}/(A_t Q_{t-s_{jt}} G_{jt})$$

denote the units of factor inputs $(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}})$ required to produce Y_{jt} units of output. We show below that cost minimization implies that the marginal costs of I_{jt} are given by

$$\text{(A1)} \qquad MC_t = \left(\frac{W_t}{1/\phi}\right)^{\frac{1}{\phi}} \left(\frac{P_t r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}}.$$

The previous expression allows for a simpler representation of firms' optimization problem further below.

The cost minimization problem of firm j,

$$\min_{K_{jt},L_{jt}} K_{jt}r_t + L_{jt}W_t/P_t \quad s.t. \quad Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t\right),$$

yields the first-order conditions

$$0 = r_t + \left(1 - \frac{1}{\phi}\right) \lambda_t A_t Q_{t-s_{jt}} G_{jt} \left(\frac{L_{jt}}{K_{jt}}\right)^{\frac{1}{\phi}}$$
$$0 = W_t/P_t + \frac{1}{\phi}\lambda_t A_t Q_{t-s_{jt}} G_{jt} \left(\frac{L_{jt}}{K_{jt}}\right)^{\frac{1}{\phi}-1},$$

where λ_t denotes the Lagrange multiplier. The first-order conditions imply that the optimal capital labor ratio is the same for all $j \in [0, 1]$, i.e.,

$$\frac{K_{jt}}{L_{jt}} = \frac{W_t}{P_t r_t}(\phi - 1).$$

Plugging the optimal capital labor ratio into the technology of firm j and solving for the factor inputs yields the factor demand functions

$$L_{jt} = \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jt} \tag{A2}$$

$$K_{jt} = \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}} I_{jt}. \tag{A3}$$

Firm j demands these amounts of labor and capital, respectively, to combine them to I_{jt}, which yields Y_{jt} units of output. Accordingly, the firm's cost function to produce I_{jt} is

$$MC_t I_{jt} = W_t \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jt} + P_t r_t \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}} I_{jt}, \tag{A4}$$

where MC_t denotes nominal marginal (or average) costs. This equation can be rearranged to obtain equation (A1).

Now consider a firm that either experienced a δ-shock or a Calvo shock in period t and that can freely choose its price. Let α denote the Calvo probability that the firm has to keep its previous price ($0 \leq \alpha < 1$), the firm will not be able to reoptimize its price with probability $\alpha(1-\delta)$ at any future date, i.e., whenever it undergoes neither a δ- shock nor a Calvo shock. The price-setting problem of a firm that can optimize its price in period t is thus given by

$$\begin{aligned} \max_{P_{jt}} E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \frac{\Omega_{t,t+i}}{P_{t+i}} \left[(1+\tau) P_{jt+i} Y_{jt+i} - MC_{t+i} I_{jt+i}\right] \\ s.t. \quad I_{jt+i} = F_{t+i} + Y_{jt+i}/A_{t+i} Q_{t-s_{jt}} G_{jt+i}, \\ Y_{jt+i} = \left(P_{jt+i}/P_{t+i}\right)^{-\theta} Y_{t+i}, \\ P_{jt+i+1} = \Xi_{t+i,t+i+1} P_{jt+i}. \end{aligned} \tag{A5}$$

where τ denotes a sales tax/subsidy and $\Omega_{t,t+i}$ denotes the representative household's discount factor between periods t and $t+i$. The first constraint captures the firm's technology, the second constraint captures the demand function faced by the firm, as implied by equation (1), and the last constraint captures how the firm's price is indexed over time (if at all) in periods in which prices are not reset optimally.

A2. Price-Setting Problem of Firms

The price-setting problem of the firm j, see equation (A5), implies that the optimal product price is given by

$$P^{\star}_{jt} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}Y_{t+i}\left(\Xi_{t,t+i}/P_{t+i}\right)^{-\theta}\frac{MC_{t+i}/P_{t+i}}{A_{t+i}Q_{t-s_{jt}}G_{jt+i}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}Y_{t+i}\left(\Xi_{t,t+i}/P_{t+i}\right)^{1-\theta}}.$$

Rewriting this equation yields

$$\frac{P^{\star}_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)$$

(A6)

$$= \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{1-\theta}}.$$

The multi-period growth rate of the cohort effect relative to the experience effect corresponds to

$$\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}} = \frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}},$$

for $i>0$, and equals unity for $i=0$. Hence, this growth rate is independent of the index j, because when going forward in time, firms are subject to the same experience effect. Thus, we can rewrite the equation (A6) according to

$$\frac{P^{\star}_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t},$$

where the numerator N_t and denominator D_t are are given by

$$N_t = E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \Omega_{t,t+i} \frac{Y_{t+i}}{Y_t} \left(\frac{\Xi_{t,t+i} P_t}{P_{t+i}} \right)^{-\theta} \frac{MC_{t+i}}{P_{t+i} A_{t+i} Q_{t+i}} \left(\frac{q_{t+i} \times \cdots \times q_{t+1}}{g_{t+i} \times \cdots \times g_{t+1}} \right)$$
$$D_t = E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \Omega_{t,t+i} \frac{Y_{t+i}}{Y_t} \left(\frac{\Xi_{t,t+i} P_t}{P_{t+i}} \right)^{1-\theta}$$

The numerator and denominator can furthermore be expressed recursively as

$$\text{(A7)} \quad N_t = \frac{MC_t}{P_t A_t Q_t} + \alpha(1-\delta) E_t \left[\Omega_{t,t+1} \frac{Y_{t+1}}{Y_t} \left(\Xi_{t,t+1} \right)^{-\theta} \left(\frac{P_{t+1}}{P_t} \right)^{\theta} \left(\frac{q_{t+1}}{g_{t+1}} \right) N_{t+1} \right]$$
$$\text{(A8)} \quad D_t = 1 + \alpha(1-\delta) E_t \left[\Omega_{t,t+1} \frac{Y_{t+1}}{Y_t} \left(\Xi_{t,t+1} \right)^{1-\theta} \left(\frac{P_{t+1}}{P_t} \right)^{\theta-1} D_{t+1} \right].$$

A3. First-Order Conditions to the Household Problem

The first-order conditions that belong to the household problem comprise the household's budget constraint, a no-Ponzi scheme condition, the transversality condition, and the following equations:

$$\frac{W_t}{P_t} = -\frac{U_{Lt}}{U_{Ct}}$$
$$\Omega_{t,t+1} = \beta \frac{\xi_{t+1}}{\xi_t} \frac{U_{Ct+1}}{U_{Ct}}$$
$$1 = E_t \left[\Omega_{t,t+1} \left(\frac{1+i_t}{\Pi_{t+1}} \right) \right]$$
$$1 = E_t \left[\Omega_{t,t+1} (r_{t+1} + 1 - d) \right].$$

Here, we denote by $U(.)$ the period utility function. Our assumption that $U(C_t, L_t) = ([C_t V(L_t)]^{1-\sigma} - 1)/(1-\sigma)$ implies

$$U_{Ct} = C_t^{-\sigma} V(L_t)^{1-\sigma}$$
$$U_{Lt} = C_t^{1-\sigma} V(L_t)^{-\sigma} V_{Lt},$$

where $U_{Ct} = \partial U(C_t, L_t)/\partial C_t$ and $V_{Lt} = \partial V(L_t)/\partial L_t$.

A4. Recursive Evolution of the Price Level

Let $P^\star_{t-s,t-k}$ denote the optimal price of a firm that last experienced a δ-shock in $t-s$ and that has last reset its price in $t-k$ ($s \geq k \geq 0$). In period t, this firm's price is equal to $\Xi_{t-k,t}P^\star_{t-s,t-k}$, where $\Xi_{t-k,t} = \prod_{j=1}^{k}\Xi_{t-k+j-1,t-k+j}$ captures the cumulative effect of price indexation (with $\Xi_{t-k,t} \equiv 1$ in the absence of price indexation). Let $\Lambda_t(s)$ denote the weighted average price in period t of the cohort of firms that last experienced a δ-shock in period $t-s$, where all prices are raised to the power of $1-\theta$, i.e.,

$$\text{(A9)} \qquad \Lambda_t(s) = (1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P^\star_{t-s,t-k})^{1-\theta} + \alpha^s(\Xi_{t-s,t}P^\star_{t-s,t-s})^{1-\theta}.$$

There are α^s firms that have not had a chance to optimally reset prices since receiving the δ-shock and $(1-\alpha)\alpha^k$ firms that have last adjusted $k < s$ periods ago. From equation (7) it follows that one can use the cohort average prices $\Lambda_t(s)$ to express the aggregate price level as

$$\text{(A10)} \qquad P_t^{1-\theta} = \sum_{s=0}^{\infty}(1-\delta)^s\delta\Lambda_t(s),$$

where δ is the mass of firms that experience a δ-shock each period and $(1-\delta)^s$ is the share of those firms that have not undergone another δ-shock for s periods.

To express the evolution of P_t in a recursive form, consider the optimal price $P^\star_{t-s,t}$ of a firm that sustained a δ-shock $s > 0$ periods ago, but can adjust the price in t due to the occurrence of a Calvo shock. Also, consider the price $P^\star_{t,t}$ of a firm where a δ-shock occurs in period t. The optimal price setting equation (10) then implies

$$\text{(A11)} \qquad P^\star_{t,t} = P^\star_{t-s,t}\left(\frac{g_t \times \cdots \times g_{t-s+1}}{q_t \times \cdots \times q_{t-s+1}}\right).$$

The previous equation shows that a stronger cohort productivity trend (higher values for q) causes the firm that experiences a δ-shock in period t to choose lower prices relative to firms that experienced δ-shocks further in the past, as a stronger cohort trend makes this firm relatively more productive. Conversely, a stronger experience effect (higher values for g) increases the optimal relative price of the firm that underwent a δ-shock in t. The net effect depends on the relative strength of the cohort versus the experience effect.

Plugging the weighted average price of a cohort, equation (A9), into the price level, equation (A10), yields

$$P_t^{1-\theta} = \delta(\Xi_{t,t} P_{t,t}^{\star})^{1-\theta} + \sum_{s=1}^{\infty}(1-\delta)^s \delta \left[(1-\alpha) \sum_{k=0}^{s-1} \alpha^k (\Xi_{t-k,t} P_{t-s,t-k}^{\star})^{1-\theta} + \alpha^s (\Xi_{t-s,t} P_{t-s,t-s}^{\star})^{1-\theta} \right].$$

Telescoping the sums yields:

$$\begin{aligned} P_t^{1-\theta} &= \delta(\Xi_{t,t} P_{t,t}^{\star})^{1-\theta} \\ &\quad + \delta(1-\delta)^1 \big[(1-\alpha)(\Xi_{t,t} P_{t-1,t}^{\star})^{1-\theta} + \alpha(\Xi_{t-1,t} P_{t-1,t-1}^{\star})^{1-\theta}\big] \\ &\quad + \delta(1-\delta)^2 \big[(1-\alpha)(\Xi_{t,t} P_{t-2,t}^{\star})^{1-\theta} + (1-\alpha)\alpha(\Xi_{t-1,t} P_{t-2,t-1}^{\star})^{1-\theta} + \alpha^2(\Xi_{t-2,t} P_{t-2,t-2}^{\star})^{1-\theta}\big] \\ &\quad + \ldots . \end{aligned}$$

Collecting optimal prices that were set at the same date in square brackets yields:

$$\begin{aligned} &P_t^{1-\theta} = \\ &\delta \Xi_{t,t}^{1-\theta} \left[(P_{t,t}^{\star})^{1-\theta} + (1-\alpha)(1-\delta) \left\{ (P_{t-1,t}^{\star})^{1-\theta} + (1-\delta)(P_{t-2,t}^{\star})^{1-\theta} + (1-\delta)^2 (P_{t-3,t}^{\star})^{1-\theta} + \ldots \right\} \right] \\ &+ [\alpha(1-\delta)]\delta \Xi_{t-1,t}^{1-\theta} \left[(P_{t-1,t-1}^{\star})^{1-\theta} + (1-\alpha)(1-\delta) \left\{ (P_{t-2,t-1}^{\star})^{1-\theta} + (1-\delta)(P_{t-3,t-1}^{\star})^{1-\theta} + \ldots \right\} \right] \\ &+ \ldots . \end{aligned}$$

Using equation (A11) and the definition of p_t^e in equation (16), we can replace the terms in curly brackets in the previous equation by p_t^e. This yields

$$
\begin{aligned}
P_t^{1-\theta} &= \delta(\Xi_{t,t} P^{\star}_{t,t})^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p^e_t)^{\theta-1}}{\delta} - 1\right\}\right] \\
&\quad + [\alpha(1-\delta)]^1 \delta(\Xi_{t-1,t} P^{\star}_{t-1,t-1})^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p^e_{t-1})^{\theta-1}}{\delta} - 1\right\}\right] \\
&\quad + [\alpha(1-\delta)]^2 \delta(\Xi_{t-2,t} P^{\star}_{t-2,t-2})^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p^e_{t-2})^{\theta-1}}{\delta} - 1\right\}\right] + \dots .
\end{aligned}
$$

Rearranging the previous equation yields

$$
\begin{aligned}
P_t^{1-\theta} &= (\Xi_{t,t} P^{\star}_{t,t})^{1-\theta}\left[\alpha\delta + (1-\alpha)(p^e_t)^{\theta-1}\right] \\
&\quad + \alpha(1-\delta)(\Xi_{t-1,t})^{1-\theta}\Big\{(\Xi_{t-1,t-1} P^{\star}_{t-1,t-1})^{1-\theta}\left[\alpha\delta + (1-\alpha)(p^e_{t-1})^{\theta-1}\right] \\
&\quad + \alpha(1-\delta)(\Xi_{t-2,t-1} P^{\star}_{t-2,t-2})^{1-\theta}\left[\alpha\delta + (1-\alpha)(p^e_{t-2})^{\theta-1}\right] + \dots\Big\} .
\end{aligned}
$$

The term in curly brackets in the previous equation corresponds to $P_{t-1}^{1-\theta}$, which yields the price level equation (15) in the main text.

A5. Equilibrium Definition

We are now in a position to define the market equilibrium:

DEFINITION 1: *An equilibrium is a state-contingent path for* $\{(P_{jt}, L_{jt}, K_{jt})$ *for* $j \in [0, 1]$, $W_t, r_t, i_t, C_t, K_{t+1}, L_t, B_t, T_t\}_{t=0}^{\infty}$ *such that*

1) *the firms' choices* $\{P_{jt}, L_{jt}, K_{jt}\}_{t=0}^{\infty}$ *maximize profits for all* $j \in [0, 1]$, *given the price adjustment frictions,*

2) *the household's choices* $\{C_t, K_{t+1}, L_t, B_t\}_{t=0}^{\infty}$ *maximize expected household utility,*

3) *the government flow budget constraint holds each period, and*

4) *the markets for capital, labor, final and intermediate goods and government bonds clear,*

given the initial values $B_{-1}(1+i_{-1})$, K_0, $P_{j,-1}$, *and* $A_{-1}Q_{-1-s_{j,-1}}G_{j,-1}$, *with* $j \in [0,1]$.

A6. Aggregate Technology and Aggregate Productivity

To derive the aggregate technology, we combine firms' technology to produce the differentiated product in equation (2) with product demand $Y_{jt}/Y_t = \left(P_{jt}/P_t\right)^{-\theta}$ to obtain

$$\frac{Y_t}{A_t Q_t}\left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} = \left(\frac{K_{jt}}{L_{jt}}\right)^{1-\frac{1}{\phi}} L_{jt} - F_t.$$

Integrating over all firms with $j \in [0,1]$, using labor market clearing, $L_t = \int_0^1 L_{jt}\,\mathrm{dj}$, and the fact that optimizing firms maintain the same (and hence the aggregate) capital labor ratio yields

$$\frac{Y_t}{A_t Q_t}\int_0^1 \left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} \mathrm{dj} = K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t.$$

Rearranging this equation and defining the (inverse) endogenous component of aggregate productivity as in equation (18) in the main text yields the aggregate technology (17).

To derive the recursive representation of Δ_t shown in equation (19), we rewrite equation (18) according to

$$\frac{\Delta_t}{P_t^{\theta}} = \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}}\right)\left(P_{jt}\right)^{-\theta} \mathrm{dj},$$

using the processes describing the evolution of Q_t and G_{jt}. As for the price level, we proceed with the aggregation in two steps. First, we aggregate the optimal prices of all firms operating within a particular cohort. Second, we aggregate all cohorts in the economy. To this end, we rewrite Δ_t/P_t^{θ} in the previous equation according to

$$\frac{\Delta_t}{P_t^{\theta}} = \sum_{s=0}^{\infty}(1-\delta)^s \delta \widehat{\Lambda}_t(s), \tag{A12}$$

using

$$\widehat{\Lambda}_t(s) = \begin{cases} \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right) \left[(1-\alpha) \sum_{k=0}^{s-1} \alpha^k (\Xi_{t-k,t} P^\star_{t-s,t-k})^{-\theta} + \alpha^s (\Xi_{t-s,t} P^\star_{t-s,t-s})^{-\theta}\right] & \text{if} \quad s \geq 1 \, , \\ (\Xi_{t,t} P^\star_{t,t})^{-\theta} & \text{if} \quad s = 0 \, . \end{cases}$$

Substituting out for $\widehat{\Lambda}_t(s)$ in equation (A12) yields

$$\begin{aligned} \frac{\Delta_t}{P_t^\theta} &= \delta(\Xi_{t,t} P^\star_{t,t})^{-\theta} \\ &+ \delta \sum_{s=1}^{\infty} (1-\delta)^s \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right) \left[(1-\alpha) \sum_{k=0}^{s-1} \alpha^k (\Xi_{t-k,t} P^\star_{t-s,t-k})^{-\theta} + \alpha^s (\Xi_{t-s,t} P^\star_{t-s,t-s})^{-\theta}\right]. \end{aligned}$$

We rearrange the previous equation following corresponding steps to those in appendix A.A4. This yields

$$\begin{aligned} \frac{\Delta_t}{P_t^\theta} &= (\Xi_{t,t} P^\star_{t,t})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] \\ &+ \alpha(1-\delta) \left(\frac{q_t}{g_t}\right) (\Xi_{t-1,t} P^\star_{t-1,t-1})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right] \\ &+ [\alpha(1-\delta)]^2 \left(\frac{q_t q_{t-1}}{g_t g_{t-1}}\right) (\Xi_{t-2,t} P^\star_{t-2,t-2})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots \, . \end{aligned}$$

We rearrange the previous equation further to obtain that

$$\begin{aligned} \frac{\Delta_t}{P_t^\theta} &= (\Xi_{t,t} P^\star_{t,t})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] \\ &+ \alpha(1-\delta) \left(\frac{q_t}{g_t}\right) (\Xi_{t-1,t})^{-\theta} \Bigg\{ (P^\star_{t-1,t-1})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right] \\ &+ \alpha(1-\delta) \left(\frac{q_{t-1}}{g_{t-1}}\right) (\Xi_{t-2,t-1} P^\star_{t-2,t-2})^{-\theta} \left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots \Bigg\}. \end{aligned}$$

The term in curly brackets in the previous equation is equal to $\Delta_{t-1}/P_{t-1}^\theta$, which yields

$$\frac{\Delta_t}{P_t^\theta} = \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] (\Xi_{t,t} P^\star_{t,t})^{-\theta} + \alpha(1-\delta) \left(\frac{q_t}{g_t}\right) (\Xi_{t-1,t})^{-\theta} \frac{\Delta_{t-1}}{P_{t-1}^\theta}.$$

Multiplying the previous equation by P_t^θ yields equation (19) in the main text.

A7. Consolidated Budget Constraint

Consolidating the household's and the government's budget constraints shown in the main text yields

$$C_t + K_{t+1} = (1-d)K_t + r_t K_t + \frac{W_t}{P_t} L_t + \frac{\int_0^1 \Theta_{jt}\,\mathrm{dj}}{P_t} - \tau \left(\frac{\int_0^1 P_{jt} Y_{jt} \mathrm{dj}}{P_t} \right). \tag{A13}$$

To compute aggregate firm profits denoted by $\int_0^1 \Theta_{jt}$ dj, we use marginal costs in equation (A4) and combine them with the factor demands for L_{jt} and K_{jt}, equations (A2) and (A3), which yields that $MC_t I_{jt} = W_t L_{jt} + P_t r_t K_{jt}$. We use this equation and product demand $Y_{jt}/Y_t = \left(P_{jt}/P_t\right)^{-\theta}$ to rewrite aggregate firm profits according to

$$\begin{aligned}
\int_0^1 \Theta_{jt}\,\mathrm{dj} &= (1+\tau) \int_0^1 P_{jt} Y_{jt}\,\mathrm{dj} - \int_0^1 MC_t I_{jt}\,\mathrm{dj} \\
&= (1+\tau) \int_0^1 P_{jt} Y_{jt}\,\mathrm{dj} - \int_0^1 (W_t L_{jt} + P_t r_t K_{jt})\,\mathrm{dj} \\
&= (1+\tau) P_t Y_t - W_t L_t - P_t r_t K_t,
\end{aligned}$$

with $L_t = \int_0^1 L_{jt}$ dj and $K_t = \int_0^1 K_{jt}$ dj. Thus, the consolidated budget constraint (A13) reduces to

$$K_{t+1} = (1-d)K_t + Y_t - C_t.$$

Dividing the previous equation by trend growth Γ_t^e yields

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t,$$

where $\gamma_t^e = \Gamma_t^e / \Gamma_{t-1}^e$ denotes the gross trend growth rate.

A8. Transformed Sticky-Price Economy

We define $p_t^\star = P_{t,t}^\star / P_t$ and $mc_t = MC_t / (P_t (\Gamma_t^e)^{1/\phi})$ and $\omega_t = W_t / (P_t \Gamma_t^e)$ and $c_t = C_t / \Gamma_t^e$. We also use that $p_t^e = 1/\Delta_t^e$, which follows from the equations (16) and (25). This yields the following equations that describe the transformed sticky-price economy.

$$\text{(A14)} \qquad 1 = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{1-\theta} + \alpha(1-\delta)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta-1}$$

$$\text{(A15)} \qquad \Delta_t = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right]\left(p_t^\star\right)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1}$$

$$\text{(A16)} \qquad p_t^\star = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t}$$

$$\text{(A17)} \qquad N_t = \frac{mc_t}{\Delta_t^e} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right]$$

$$\text{(A18)} \qquad D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right]$$

$$\text{(A19)} \qquad mc_t = \left(\frac{w_t}{1/\phi}\right)^{\frac{1}{\phi}}\left(\frac{r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}}$$

$$\text{(A20)} \qquad r_t k_t = (\phi-1)w_t L_t$$

$$\text{(A21)} \qquad y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left(k_t^{1-\frac{1}{\phi}}L_t^{\frac{1}{\phi}} - f\right)$$

$$\text{(A22)} \qquad \gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t$$

$$\text{(A23)} \qquad \gamma_t^e = (a_t q_t \Delta_{t-1}^e/\Delta_t^e)^{\phi}$$

$$\text{(A24)} \qquad \left(\Delta_t^e\right)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta}$$

$$\text{(A25)} \qquad w_t = -c_t\left(\frac{V_{Lt}}{V(L_t)}\right)$$

$$\text{(A26)} \qquad 1 = E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right]$$

$$\text{(A27)} \qquad 1 = E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right]$$

$$\text{(A28)} \qquad \Omega_{t,t+1} = \beta\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\gamma_{t+1}^e c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{V(L_{t+1})}{V(L_t)}\right)^{1-\sigma}$$

After adding a description of monetary policy and a price indexation rule, these seventeen equations determine the paths of the seventeen variables i_t, Π_t, y_t, c_t, k_t, L_t, r_t, w_t, mc_t, γ_t^e, Δ_t, Δ_t^e, $p_t^\star$, $\Xi_{t-1,t}$, N_t, D_t, $\Omega_{t-1,t}$ given the four exogenous shocks q_t, g_t, a_t, ξ_t.

A9. Steady State in the Transformed Sticky-Price Economy

We consider a steady state in the transformed sticky-price economy, in which g and q are constant and the government maintains a constant inflation rate Π, which also implies a constant rate of price indexation Ξ.

To solve for the model variables in this steady state, we first solve for the ratio Δ/Δ^e as a function of model parameters and the inflation rate Π only. To this end, we derive an expression for $p^\star$ as a function of Δ using the equations (A14) and (A15). Both equations can be rearranged to obtain, respectively,

$$(1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{1-\theta} \tag{A29}$$

$$\Delta\left(1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}\right) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{-\theta}. \tag{A30}$$

Dividing the equation (A29) by the equation (A30) yields

$$p^\star = \Delta^{-1}\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\right). \tag{A31}$$

We substitute this expression for $p^\star$ into the equation (A30), which yields

$$\left(\frac{\Delta}{\Delta^e}\right)^{1-\theta} = \frac{\alpha\delta(\Delta^e)^{\theta-1}+1-\alpha}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\right)^{-\theta}.$$

We use equation (A24) to substitute for $(\Delta^e)^{\theta-1}$ on the right hand side of the previous equation and rearrange the result to obtain

$$\frac{\Delta(\Pi)}{\Delta^e} = \left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}\left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)\,(\Pi/\Xi)^{\theta}\,(g/q)^{-1}}\right), \tag{A32}$$

where we have indicated that $\Delta(\Pi)$ depends on the steady-state inflation rate Π. For later use, we define the relative price distortion as

$$\rho(\Pi) = \frac{\Delta^e}{\Delta(\Pi)}. \tag{A33}$$

Combining the pricing equations (A16) to (A18) yields

$$\frac{1}{mc} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1}{p^{\star}\Delta^{e}}\right)\left(\frac{1-\alpha(1-\delta)[\beta(\gamma^{e})^{1-\sigma}]\,(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)[\beta(\gamma^{e})^{1-\sigma}]\,(\Pi/\Xi)^{\theta}\,(g/q)^{-1}}\right).$$

Using the expression for $p^{\star}$ in equation (A31) to substitute for $p^{\star}$ in the previous equation and the solution for $\Delta(\Pi)/\Delta^{e}$ in equation (A32), we thus obtain a solution for $1/mc$. Again for later use, we denote the average markup by $\mu = 1/mc$ and thus obtain the solution

(A34)

$$\mu(\Pi) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{1}{\theta-1}}\left(\frac{1-\alpha(1-\delta)[\beta(\gamma^{e})^{1-\sigma}]\,(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)[\beta(\gamma^{e})^{1-\sigma}]\,(\Pi/\Xi)^{\theta}\,(g/q)^{-1}}\right).$$

Again, we indicate here that $\mu(\Pi)$ depends on the steady-state inflation rate.

Now, we rewrite marginal costs in equation (A19) as

$$mc = \left(\frac{w}{r}(\phi-1)\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right),$$

and use equation (A20) to obtain $mc = \left(\frac{k}{L}\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right)$ or

$$r = \mu(\Pi)^{-1}\left(1-\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{-\frac{1}{\phi}}, \tag{A35}$$

after also using $\mu = 1/mc$. Analogous steps for the wage rate also imply

$$w = \mu(\Pi)^{-1}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}. \tag{A36}$$

Furthermore, the aggregate technology (A21), the aggregate resource constraint (A22) and the household's optimality conditions (A25) to (A28) imply the following four equations:

$$
\begin{aligned}
y &= \rho(\Pi)\left(\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} L - f\right) \\
w &= c\left(-\frac{V_L}{V(L)}\right) \\
r &= \frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d \\
y &= c + (\gamma^e - 1 + d)k,
\end{aligned}
$$

where we have used $\rho(\Pi) = \Delta^e/\Delta(\Pi)$. To simplify these four equations further, we use the equations (A35) and (A36) to substitute out for w and r. Then, we express all the remaining variables relative to hours worked, which yields the following four equations:

$$\text{(A37)} \qquad \frac{y}{L} = \rho(\Pi)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1}$$

$$\text{(A38)} \qquad \frac{c}{L} = \mu(\Pi)^{-1}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(-\frac{V(L)}{LV_L}\right)$$

$$\text{(A39)} \qquad \frac{k}{L} = \mu(\Pi)^{-1}\left(1 - \frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-1}$$

$$\text{(A40)} \qquad \frac{y}{L} = \frac{c}{L} + (\gamma^e - 1 + d)\frac{k}{L}.$$

We now show that these four equations determine the four variables y, c, L, k, given a steady-state inflation rate Π and assuming that the ratio of fixed costs over output, f/y, is a calibrated parameter.

First, we solve for hours worked as a function of Π by substituting the equations (A37) to (A39) into equation (A40). This yields

$$\mu(\Pi)\rho(\Pi)\left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} = \left(\frac{1}{\phi}\right)\left(-\frac{V(L)}{LV_L}\right) + \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right)\left(1 - \frac{1}{\phi}\right),$$

or

$$\left(-\frac{V(L)}{LV_L}\right) = \phi\mu(\Pi)\rho(\Pi)\left(1+\rho(\Pi)\frac{f}{y}\right)^{-1} - (\phi-1)\left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right)$$
$$= \mathcal{L}(\Pi),$$

where $\mathcal{L}(\Pi)$ abbreviates the right-hand-side term, which is a function of the steady-state inflation rate. The previous equation provides an implicit solution for L. We obtain an explicit solution for L, if we assume a functional form for $V(L)$. Using that $V(L) = 1 - \psi L^\nu$, with $\nu > 1$ and $\psi > 0$ yields

$$-\frac{V(L)}{LV_L} = \frac{1-\psi L^\nu}{\psi\nu L^\nu}$$

and hence

$$L(\Pi) = \left(\frac{1}{\psi + \psi\nu\mathcal{L}(\Pi)}\right)^{1/\nu}, \tag{A41}$$

where we have indicated that in general, steady-state hours worked L depend on the steady-state inflation rate Π through $\mathcal{L}(\Pi)$. Recall that in order to compute $\mathcal{L}(\Pi)$, the equations (A32), (A33) and (A34) are required. The solutions for k, c, and y can be recursively computed from the equations (A37) to (A39). These solutions are

$$k(\Pi) = \mu(\Pi)^{-\phi}\left(1-\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-\phi} L \tag{A42}$$

$$c(\Pi) = \mu(\Pi)^{-1}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(-\frac{V(L)}{V_L}\right) \tag{A43}$$

$$y(\Pi) = c + (\gamma^e - 1 + d)k. \tag{A44}$$

Again, we indicate that these solutions depend on the steady-state inflation rate.

APPENDIX B - PLANNER PROBLEM AND ITS SOLUTION

The planner allocates resources across firms and time by maximizing expected discounted household utility subject to firms' technologies and feasibility constraints. The planner problem can be solved in two steps. The first step determines the allocation of given amounts of capital and labor between heterogenous firms at date t. The second step determines the allocation of aggregate capital, consumption and labor over time. Endogenous variables in the planner solution are indicated by superscript e.

B1. Intratemporal Planner Problem

The intratemporal problem corresponds to

$$\max_{L^e_{jt},K^e_{jt}} \left(\int_0^1 (Y^e_{jt})^{\frac{\theta-1}{\theta}} \mathrm{dj}\right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad Y^e_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left((K^e_{jt})^{1-\frac{1}{\phi}} (L^e_{jt})^{\frac{1}{\phi}} - F_t\right),$$

and given L^e_t and K^e_t, with $L^e_t = \int_0^1 L^e_{jt}$ dj and $K^e_t = \int_0^1 K^e_{jt}$ dj. Optimality conditions yield $K^e_{jt}/L^e_{jt} = K^e_t/L^e_t$ and hence that all firms maintain the same capital labor ratio. Thus, the problem can be recast in terms of the optimal mix of input factors, $I^e_{jt} = (K^e_{jt})^{1-1/\phi}(L^e_{jt})^{1/\phi}$:

$$\max_{I^e_{jt}} \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(I^e_{jt} - F_t\right)\right]^{\frac{\theta-1}{\theta}} \mathrm{dj}\right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad I^e_t = \int_0^1 I^e_{jt} \, \mathrm{dj},$$

with $I^e_t = (K^e_t)^{1-1/\phi}(L^e_t)^{1/\phi}$ being given. Equating the first-order conditions to this problem for two different firms j and k to each other yields the condition

$$Z_{jt} \left[Z_{jt} \left(I^e_{jt} - F_t\right)\right]^{-\frac{1}{\theta}} = Z_{kt} \left[Z_{kt} \left(I^e_{kt} - F_t\right)\right]^{-\frac{1}{\theta}},$$

where $Z_{jt} = Q_{t-s_{jt}} G_{jt}$ denotes productivity of the firm j at date t. Rearranging this condition yields $I^e_{jt} - F_t = (Z_{jt}/Z_{kt})^{\theta-1} \left(I^e_{kt} - F_t\right)$, and aggregating this equation over all j's yields

$$I^e_{kt} - F_t = \frac{(G_{kt} Q_{t-s_{kt}}/Q_t)^{\theta-1}}{\int_0^1 (G_{jt} Q_{t-s_{jt}}/Q_t)^{\theta-1} \, \mathrm{dj}} (I^e_t - F_t). \tag{B1}$$

Thus, the optimal input mix of the firm k net of fixed costs is proportional to the optimal aggregate input mix net of fixed costs, and the factor of proportionality corresponds to the (weighed) productivity of the firm k relative to the (weighed) aggregate productivity in the economy. Thus, equation (B1) shows that the productivity distribution determines the efficient allocation of the optimal input mix across firms.

To obtain the aggregate technology in the planner economy, we combine equation (B1) with equation (2) and the Dixit-Stiglitz aggregator (1). This yields

$$Y_t^e = \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(\frac{(Q_{t-s_{jt}} G_{jt})^{\theta-1}}{\int_0^1 (Q_{t-s_{jt}} G_{jt})^{\theta-1}\,\mathrm{dj}} (I_t^e - F_t)\right)\right]^{\frac{\theta-1}{\theta}} \mathrm{dj}\right)^{\frac{\theta}{\theta-1}} .$$

Simplifying this equation yields the aggregate technology in the planner economy,

(B2) $$Y_t^e = \frac{A_t Q_t}{\Delta_t^e}\left((K_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}} - F_t\right),$$

where the efficient productivity adjustment factor is defined as

(B3) $$1/\Delta_t^e = \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}}/Q_t\right)^{\theta-1} \mathrm{dj}\right)^{\frac{1}{\theta-1}}$$

and evolves recursively. To see this, rewrite equation (B3) as

Assuming that the initial productivity distribution at $t = -1$ is consistent with the assumed productivity process we have

$$\begin{aligned}
(1/\Delta_t^e)^{\theta-1} &= \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}}\right)^{1-\theta} \mathrm{dj} \\
&= \delta\left\{1 + \sum_{s=1}^{\infty} (1-\delta)^s \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right)^{1-\theta}\right\} \\
&= \delta\left\{1 + (1-\delta)\left(\frac{q_t}{g_t}\right)^{1-\theta} + (1-\delta)^2\left(\frac{q_t q_{t-1}}{g_t g_{t-1}}\right)^{1-\theta} + \dots\right\} \\
&= (p_t^e)^{\theta-1} .
\end{aligned}$$

The last step follows from backward-iterating equation (16) and implies that the efficient productivity adjustment factor equals the relative price of firms hit by a δ-shock in period

t in the economy with flexible prices,

$$1/\Delta_t^e = p_t^e. \tag{B4}$$

It follows also from equation (16) that Δ_t^e evolves recursively as shown in equation (25). The intratemporal planner allocation then consists of equation (B1), which determines the efficient allocation of the optimal input mix across firms, and equations (B2) and (25), which describe the aggregate consequences of the efficient allocation at the firm level.

B2. Intertemporal Planner Problem

The intertemporal allocation maximizes expected discounted household utility subject to the intertemporal feasibility condition,

$$\max_{\{C_t^e, L_t^e, K_{t+1}^e\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t U(C_t^e, L_t^e) \quad s.t. \tag{B5}$$

$$C_t^e + K_{t+1}^e = (1-d)K_t^e + \frac{A_t Q_t}{\Delta_t^e} \left((K_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}} - F_t \right), \tag{B6}$$

with $U(.)$ denoting the period utility function and Δ_t^e given by equation (25). The first order conditions to this problem comprise the feasibility constraint and

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \tag{B7}$$

$$1 = \beta E_t \left[\frac{\xi_{t+1}}{\xi_t} \frac{U_{Ct+1}^e}{U_{Ct}^e} \left(Y_{Kt+1}^e + 1 - d \right) \right], \tag{B8}$$

denoting by Y_{Kt}^e the marginal product of capital and by Y_{Lt}^e the marginal product of labor. Thus, the planner allocation for aggregate variables is characterized by the aggregate technology, equation (B2), the efficient adjustment factor, equation (25), the feasibility condition, equation (B6), and the two first-order conditions (B7) and (B8).

Appendix C - Efficiency of the Flexible-Price Equilibrium

This appendix proves the following result:

PROPOSITION 6: *The flexible-price equilibrium ($\alpha = 0$) is efficient if condition 1 holds.*

To show that condition (23) holds under flexible prices, we divide equation (15) by $P_t^{1-\theta}$ and impose $\alpha = 0$ to find out that the optimal relative price $p_t^\star$ of firms experiencing a δ-shock in period t is equal to p_t^e. This and the equations (A16) to (A18) determining the optimal relative price of firms experiencing a δ-shock in t imply with $\alpha = 0$ that

$$p_t^e = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{mc_t}{\Delta_t^e}.$$

Combining the previous equation with the equation (B4) yields

$$1 = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)mc_t, \tag{C1}$$

which shows that real detrended marginal costs are constant in the economy with flexible prices. From equation (10) it follows that the optimal relative price of the firm j in the flexible-price model satisfies

$$\frac{P_{jt}^\star}{P_t}(G_{jt}Q_{t-s_{jt}}/Q_t) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{mc_t}{\Delta_t^e}.$$

Combining the previous equation with equation (C1), we obtain condition (23) in the main text. Plugging this condition into equation (18) shows that the flexible-price equilibrium implements $\Delta_t = \Delta_t^e$. Thus, the aggregate production function in equation (17) in the flexible-price equilibrium is given by

$$Y_t = \frac{A_t Q_t}{\Delta_t^e}\left((K_t)^{1-\frac{1}{\phi}}(L_t)^{\frac{1}{\phi}} - F_t\right), \tag{C2}$$

with $F_t = f \cdot (\Gamma_t^e)^{1-1/\phi}$ and $\Gamma_t^e = (A_t Q_t/\Delta_t^e)^\phi$, and the resource constraint (derived in Appendix A.A7) is given by

$$K_{t+1} = (1-d)K_t + Y_t - C_t. \tag{C3}$$

The two equations (C2) and (C3) are the same constraints faced by the planner under efficient allocation. Combined with the fact that the household decisions in the flexible price economy are undistorted in the presence of the corrective sales subsidy, it follows that the allocation of aggregate consumption, capital, labor, and output in the flexible-price equilibrium is identical to efficient allocation.

APPENDIX D - PROOF OF PROPOSITION 1

Establishing (1): First, we show that firms hit by a δ-shock in period t in the sticky-price economy choose the same optimal relative price as in the flexible-price economy. Let superscript e denote allocations and prices in the flexible-price economy, which we have shown reproduces the efficient allocation. Under flexible prices ($\alpha = 0$) and given condition 1, the optimal relative price implied by equation (10) for firms with a δ-shock in period t is given by

$$p_t^e = \frac{(P_{t,t}^\star)^e}{P_t^e} = \frac{MC_t^e}{P_t^e A_t Q_t}.$$

Under sticky prices ($\alpha > 0$) and the efficient allocation, combining this equation with equation (A7) implies

$$\text{(D1)} \qquad \frac{N_t}{p_t^e} = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}^e \frac{Y_{t+1}^e}{Y_t^e}\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^\theta \left(\frac{q_{t+1}}{g_{t+1}}\right)\left(\frac{p_{t+1}^e}{p_t^e}\right)\left(\frac{N_{t+1}}{p_{t+1}^e}\right)\right].$$

Furthermore, equation (A8) implies

$$\text{(D2)} \qquad D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}^e \frac{Y_{t+1}^e}{Y_t^e}\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1} D_{t+1}\right].$$

Firms hit by a δ-shock in period t in the sticky-price economy choose the same optimal relative price as firms receiving a δ-shock in period t in the flexible-price economy, i.e., $P_{t,t}^\star/P_t = N_t/D_t = p_t^e$ or equivalently $N_t/p_t^e = D_t$, if it holds that

$$\text{(D3)} \qquad \left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)\left(\frac{q_{t+1}}{g_{t+1}}\right)\left(\frac{p_{t+1}^e}{p_t^e}\right) = 1,$$

which follows from comparing the equations (D1) and (D2). To show that equation (D3) holds under the optimal inflation rate stated in proposition 1, we lag this equation by one period and rearrange it to obtain

$$\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right) p_t^e = p_{t-1}^e \frac{g_t}{q_t}.$$

Combining this equation with equation (16) implies that the optimal inflation rate as defined in equation (24) satisfies equation (D3).

Establishing (2): To show that, under the optimal inflation rate, firms that are subject to a Calvo shock in period t and hence can adjust their price do not find it optimal to change their price, we need to establish that

$$P^\star_{t-k,t} = \Xi^\star_{t-k,t} P^\star_{t-k,t-k}, \tag{D4}$$

for all $k > 0$. Dividing this equation by the (optimal) aggregate price level $P^\star_{t-k}$ and using the result from step (1), i.e., $P^\star_{t,t}/P^\star_t = p^e_t$, we obtain

$$\frac{P^\star_{t-k,t}}{P^\star_{t-k}} = \Xi^\star_{t-k,t}\left(\frac{P^\star_{t-k,t-k}}{P^\star_{t-k}}\right) = \Xi^\star_{t-k,t}\, p^e_{t-k}.$$

Using equation (A11), we can rewrite the previous equation as

$$\frac{P^\star_{t,t}}{P^\star_t}\left(\frac{q_t \times \cdots \times q_{t-k+1}}{g_t \times \cdots \times g_{t-k+1}}\right)\frac{P^\star_t}{P^\star_{t-k}} = \Xi^\star_{t-k,t}\, p^e_{t-k}.$$

Again using $P^\star_{t,t}/P^\star_t = p^e_t$ and that $\Xi_{t-k,t} = \prod_{j=1}^k \Xi_{t-k+j-1,t-k+j}$ further delivers

$$\left(\frac{p^e_t}{p^e_{t-k}}\right)\left(\frac{q_t \times \cdots \times q_{t-k+1}}{g_t \times \cdots \times g_{t-k+1}}\right)\left(\frac{\Pi^\star_t}{\Xi^\star_{t-1,t}} \times \cdots \times \frac{\Pi^\star_{t+1-k}}{\Xi^\star_{t-k,t+1-k}}\right) = 1.$$

Rewriting the previous equation as

$$\left(\frac{\Pi^\star_t}{\Xi^\star_{t-1,t}}\,\frac{q_t}{g_t}\,\frac{p^e_t}{p^e_{t-1}}\right)\times\left(\frac{\Pi^\star_{t-1}}{\Xi^\star_{t-2,t-1}}\,\frac{q_{t-1}}{g_{t-1}}\,\frac{p^e_{t-1}}{p^e_{t-2}}\right)\times\cdots\times\left(\frac{\Pi^\star_{t+1-k}}{\Xi^\star_{t-k,t+1-k}}\,\frac{q_{t+1-k}}{g_{t+1-k}}\,\frac{p^e_{t+1-k}}{p^e_{t-k}}\right) = 1$$

shows that each term in parenthesis is equal to unity under the optimal inflation rate, which follows from equation (D3). This establishes that firms that can adjust their price maintain the indexed price as given by equation (D4).

Establishing (3): We can establish the fact that the condition 2 causes initial prices to reflect initial relative productivities as follows. The pricing equations (10)-(A8) imply under flexible prices and no markup distortion that

$$\frac{P^{\star}_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)=\frac{MC_t}{P_tA_tQ_t}.$$

For a firm hit by a δ-shock in period t, this equation yields

$$p^e_t=\frac{MC_t}{P_tA_tQ_t}.$$

Combining both previous equations yields

$$\frac{P^{\star}_{jt}}{P_t}=\left(\frac{Q_t}{Q_{t-s_{jt}}G_{jt}}\right)p^e_t.$$

Plugging this equation into the aggregate price level, $P_t^{1-\theta}=\int_0^1 P_{jt}^{1-\theta}$ dj, yields

$$1=\int_0^1\left(\frac{Q_t}{Q_{t-s_{jt}}G_{jt}}\right)^{1-\theta}(p^e_t)^{1-\theta}\text{dj}.$$

Rewriting this equation and using $p^e_t=1/\Delta^e_t$ yields equation (22) for $t=-1$.

Appendix E - Discontinuity of the Optimal Inflation Rate

This appendix compares the optimal inflation rate in an economy with δ-shocks ($\delta>0$) to the economy in the absence of such shocks ($\delta=0$). We refer to the first economy as the δ-economy and to the latter as the 0-economy. Comparing these two economies is not as straightforward as it might initially appear: even if both economies are subject to the same fundamental shocks (a_t, q_t, g_t, ξ_t), the efficient allocation displays a discontinuity when considering the limit $\delta \to 0$. The discontinuity arises because aggregate productivity growth in the δ-economy is driven by $a_t q_t$, while it is driven by $a_t g_t$ in the 0-economy.

To properly deal with this issue, we construct a δ-economy whose efficient aggregate allocation (consumption, hours, capital) is identical to the efficient aggregate allocation in the 0-economy.[44] We then compare the optimal inflation rates in these two economies and show that the optimal inflation rate for the δ-economy differs from the optimal inflation rate for the 0-economy, even for the limit $\delta \to 0$.

Let $a_t^\delta, q_t^\delta, g_t^\delta$ denote the productivity disturbances in the δ-economy and let $A_{-1}^\delta G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta$ for $j \in [0,1]$ denote the initial distribution of firm productivities. This, together with the process $\{\delta_{jt}\}_{t=0}^\infty$ for all $j \in [0,1]$, determines the entire state-contingent values for A_t^δ, Q_t^δ, G_{jt}^δ, and $Q_{t-s_{jt}}^\delta$ for all $j \in [0,1]$ and all $t \geq 0$.

Next, consider the 0-economy and suppose it starts with the same initial capital stock as the δ-economy. For the 0-economy, we normalize $Q_{t-s_{jt}}^0 \equiv 1$ for all $j \in [0,1]$ and all t and then set the initial firm productivity distribution in the 0-economy equal to that in the δ-economy by choosing the initial conditions

$$
\begin{aligned}
A_{-1}^0 &= A_{-1}^\delta, \\
G_{j,-1}^0 &= G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta .
\end{aligned}
$$

Finally, let the process for common TFP in the 0-economy be given by

$$
A_t^0 = A_t^\delta \left(\int_0^1 \left(Q_{t-s_{jt}}^\delta G_{jt}^\delta \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}} \left(\int_0^1 \left(G_{jt}^0 \right)^{\theta-1} \mathrm{dj} \right)^{\frac{-1}{\theta-1}},
$$

where G_{jt}^0 is generated by an arbitrary process g_t^0, e.g., $g_t^0 = g_t^\delta$. In this setting, it is easily verified that aggregate productivity associated with the efficient allocation, defined as

$$
A_t Q_t / \Delta_t^e = A_t Q_t \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}} / Q_t \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}},
$$

[44]The two economies do of course differ in their underlying firm-level dynamics.

is the same in the δ-economy and the 0-economy.[45] We then have the following result:

PROPOSITION 7: *Under the assumptions stated in this section, the efficient allocations in the two economies, the δ-economy and the* 0*-economy, satisfy*

$$C_t^\delta = C_t^0, L_t^\delta = L_t^0, K_t^\delta = K_t^0$$

for all $t \geq 0$ *and all possible realizations of the disturbances.*

PROOF:

Since $A_t^\delta Q_t^\delta / \Delta_t^{e,\delta} = A_t^0 Q_t^0 / \Delta_t^{e,0}$ for all t, it follows from the planner's problem (B5)-(B6) and the fact that the initial capital stock is identical that both economies share the same efficient allocation.

The following proposition shows that (generically) the optimal inflation rate discontinuously jumps when moving from the 0-economy to the δ-economy, even if both economies are identical in terms of their efficient aggregate dynamics:[46]

LEMMA 2: *Under the assumptions stated in this section and provided conditions 1 and 2 hold, the optimal inflation rate in the* 0*-economy is* $\Pi_t^{\star,0} = 1$ *for all* t*. The optimal inflation rate in the δ-economy is given by equation (24); in particular, for* $g_t^\delta = g$ *and* $q_t^\delta = q$*, and in the absence of price indexation, the optimal rate of inflation in the δ-economy satisfies* $\lim_{t\to\infty} \Pi_t^{\star,\delta} = g/q$.

PROOF:

The results directly follow from proposition 1 and lemma 1.

The previous result illustrates the fragility of the optimality of strict price stability in standard sticky-price models, once non-trivial firm-level productivity trends are taken into account. Moreover, in combination with proposition 7, the result shows that two

[45] The fact that $A_t Q_t / \Delta_t^e$ is equal to aggregate productivity in the efficient allocation follows from equations (B6) and (22).

[46] Recall that the optimal inflation rates implement the efficient aggregate allocations in these economies.

economies that can be identical in terms of their aggregate efficient allocations may require different inflation rates for implementing these allocations.

APPENDIX F - PROOF OF PROPOSITION 2

Under the assumptions stated in the proposition, it is straightforward to show that the relative price distortion $\rho(\Pi)$ and the markup distortion $\mu(\Pi)$, which are defined in equations (A32), (A33) and (A34), are inversely proportional to each other,

$$\mu(\Pi) = 1/\rho(\Pi).$$

As a result, the solution of L determined in equation (A41) in appendix A.A9 simplifies to

$$L = \left(\frac{1}{\psi(1+\nu)}\right)^{1/\nu},$$

because $\mathcal{L}(\Pi) = 1$ and, therefore, L no longer depends on the steady-state inflation rate Π. This result implies that $L(1) = L(\Pi^\star)$, as stated in proposition 2.

In this case, the solutions for capital and consumption, equations (A42) and (A43), imply

$$k(\Pi) = \rho(\Pi)^\phi \left(1 - \frac{1}{\phi}\right)^\phi \left(\gamma^e - 1 + d\right)^{-\phi} L,$$
$$c(\Pi) = \rho(\Pi)^\phi \left(\frac{1}{\phi}\right) \left(1 - \frac{1}{\phi}\right)^{\phi-1} \left(\gamma^e - 1 + d\right)^{1-\phi} \left(-\frac{V(L)}{V_L}\right),$$

where we explicitly indicate that steady-state capital and consumption depend on Π.

Comparing steady-state consumption for the policy implementing the optimal inflation rate $\Pi^\star$ and the alternative policy implementing strict price stability in economies without price indexation yields

$$\frac{c(1)}{c(\Pi^\star)} = \left(\frac{\rho(1)}{\rho(\Pi^\star)}\right)^\phi.$$

Equations (A32) and (A33) imply that the relative price distortion $\rho(\Pi^\star) = 1$. This yields

$$\begin{aligned}\frac{c(1)}{c(\Pi^\star)} &= \rho(1)^\phi,\\ &= \left(\frac{\Delta^e}{\Delta(1)}\right)^\phi\\ &= \left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)}\right)^{\frac{\phi\theta}{\theta-1}}\left(\frac{1-\alpha(1-\delta)\,(g/q)^{-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^\phi,\end{aligned}$$

which is the expression in proposition 2.

To show that $c(1)/c(\Pi^\star) \leq 1$, note that $c(1)/c(\Pi^\star) = 1$, if $g = q$ and hence $\Pi^\star = 1$. To show that the inequality holds strictly, $c(1)/c(\Pi^\star) < 1$, for $g \neq q$, we take the derivative of $c(1)/c(\Pi^\star)$ with respect to g/q. This yields

$$\frac{\partial}{\partial(g/q)}\left(\frac{c(1)}{c(\Pi^\star)}\right) = \left[\frac{c(1)}{c(\Pi^\star)}\right]\left[\frac{\alpha(1-\delta)\phi}{(g/q)^2}\right]\frac{1-(g/q)^\theta}{\left[1-\alpha(1-\delta)\,(g/q)^{-1}\right]\left[1-\alpha(1-\delta)(g/q)^{\theta-1}\right]}.$$

Terms in square brackets are positive, because we have assumed that $(1-\delta)(g/q)^{\theta-1} < 1$ (see equation (6)), $\alpha < 1$, and $g/q > \alpha(1-\delta)$. Therefore, the derivative is strictly positive if $1-(g/q)^\theta > 0$ and thus $g/q < 1$. The derivative is strictly negative if $1-(g/q)^\theta < 0$ and thus $g/q > 1$. The derivative is zero if $g/q = 1$.

Appendix G - Proof of Proposition 3

We start by deriving equation (30) in the proposition. Average employment per firm $\overline{L}_t$ can be written as

$$\overline{L}_t = \delta\overline{L}_t^\star + (1-\delta)\overline{L}_t^c, \tag{G1}$$

where $\overline{L}_t^\star$ denotes average employment of the firms that received a δ-shock in period t and $\overline{L}_t^c$ average employment of the remaining firms. Equation (2) and equation (17), respectively, imply

$$\frac{Y_{jt}}{A_t Q_{t-s_{jt}} G_{jt}} + F_t = (K_{jt}/L_{jt})^{1-\frac{1}{\phi}} L_{jt}$$
$$\frac{Y_t \Delta_t}{A_t Q_t} + F_t = (K_t/L_t)^{1-\frac{1}{\phi}} \overline{L}_t,$$

where we used the fact that due to there being a unit mass of firms, we have $L_t = \overline{L}_t$. Taking the ratio of the two previous equations and using the fact that each firm's capital-labor ratio is equal to the aggregate capital-labor ratio, we get

$$\frac{L_{jt}}{\overline{L}_t} = \left(\frac{1}{1 + F_t \frac{A_t Q_t}{Y_t \Delta_t}}\right)\left(\frac{Y_{jt}}{A_t Q_{t-s_{jt}} G_{jt}} \frac{A_t Q_t}{Y_t \Delta_t} + F_t \frac{A_t Q_t}{Y_t \Delta_t}\right).$$

Using $F_t = f \cdot (\Gamma_t^e)^{1-\frac{1}{\phi}}$ from equation (3), the definition of detrended output $y_t = Y_t / \Gamma_t^e$, and $\Gamma_t^e = (A_t Q_t / \Delta_t^e)^{\phi}$ from equation (20), the previous equation can be expressed as

$$\frac{L_{jt}}{\overline{L}_t} = \left(1 + \frac{f}{y_t \Delta_t / \Delta_t^e}\right)^{-1} \left(\frac{Y_{jt}}{Y_t \Delta_t}\left(\frac{Q_t}{Q_{t-s_{jt}} G_{jt}}\right) + \frac{f}{y_t \Delta_t / \Delta_t^e}\right).$$

Using the product demand function (8) to substitute Y_{jt}/Y_t, we get

$$\frac{L_{jt}}{\overline{L}_t} = \left(1 + \frac{f \Delta_t^e}{y_t \Delta_t}\right)^{-1} \left(\frac{f \Delta_t^e}{y_t \Delta_t} + \frac{1}{\Delta_t}\left(\frac{Q_t}{Q_{t-s_{jt}} G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta}\right).$$

Firms that receive a δ-shock at date t can charge the optimal price, i.e., $P_{jt}/P_t = P_{t,t}^{\star}/P_t = p_t^{\star}$. For these firms, the previous equation implies

$$\frac{\overline{L}_t^{\star}}{\overline{L}_t} = \left(1 + \frac{f \Delta_t^e}{y_t \Delta_t}\right)^{-1} \left[\frac{f \Delta_t^e}{y_t \Delta_t} + \frac{1}{\Delta_t} \left(p_t^{\star}\right)^{-\theta}\right],$$

where we used the fact that firms that receive a δ-shock are identical, so that on the left-hand side of the previous equation, we can write average employment of these firms in the numerator. Using equation (G1) to substitute for $\overline{L}_t^{\star}/\overline{L}_t$ in the previous equation yields

$$\left(1+\frac{f\,\Delta_t^e}{y_t\,\Delta_t}\right)\left(1-(1-\delta)\overline{L}_t^c/\overline{L}_t\right)-\delta\frac{f\,\Delta_t^e}{y_t\,\Delta_t}=\left(\frac{\Delta_t^e}{\Delta_t}\right)[\delta(\Delta_t^e)^{\theta-1}]\left(\Delta_t^e\,p_t^\star\right)^{-\theta}.$$

Equation (24) implies $\delta\left(\Delta_t^e\right)^{\theta-1}=1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}$. This allows us to rewrite the previous equation as

$$\text{(G2)}\qquad \left(\Delta_t^e\,p_t^\star\right)^{-\theta}=\left(\frac{\Delta_t}{\Delta_t^e}\right)\left(\frac{1-(1-\delta)\left[\frac{\overline{L}_t^c}{\overline{L}_t}+\frac{f\,\Delta_t^e}{y_t\,\Delta_t}\left(\frac{\overline{L}_t^c}{\overline{L}_t}-1\right)\right]}{1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}}\right).$$

From equation (A14) we obtain

$$1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}=\left[\alpha\delta(\Delta_t^e)^{\theta-1}+(1-\alpha)\right](\Delta_t^e\,p_t^\star)^{1-\theta}.$$

Using again $\delta\left(\Delta_t^e\right)^{\theta-1}=1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}$ allows us to rewrite the previous equation as

$$\text{(G3)}\qquad (p_t^\star\,\Delta_t^e)^{-\theta}=\left(\frac{1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}.$$

Equating the right-hand sides of equation (G2) and equation (G3) delivers equation (30) in the proposition for the special case with $f=0$.

We next derive equation (31) in the proposition. From equation (A15) we have

$$\Delta_t=\left[\alpha\delta(\Delta_t^e)^{\theta-1}+(1-\alpha)\right]\Delta_t^e\left(p_t^\star\,\Delta_t^e\right)^{-\theta}+\alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1}.$$

Equation (24) implies $\delta\left(\Delta_t^e\right)^{\theta-1}=1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}$. Substituting this into the previous equation and dividing by Δ_t^e delivers

$$\frac{\Delta_t}{\Delta_t^e}=\left[1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}\right]\left(p_t^\star\,\Delta_t^e\right)^{-\theta}+\alpha(1-\delta)\left(\frac{q_t}{g_t}\frac{\Delta_{t-1}^e}{\Delta_t^e}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\frac{\Delta_{t-1}}{\Delta_{t-1}^e}.$$

Using $\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star} = \frac{g_t}{\Delta_{t-1}^e}\frac{\Delta_t^e}{q_t}$ from equation (26) delivers

$$\frac{\Delta_t}{\Delta_t^e} = \left[1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}\right](p_t^\star\Delta_t^e)^{-\theta} + \alpha(1-\delta)\left(\frac{\left(\Pi_t/\Xi_{t-1,t}\right)^\theta}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)\frac{\Delta_{t-1}}{\Delta_{t-1}^e}.$$

Using equation (G3) to substitute $(p_t^\star\Delta_t^e)^{-\theta}$ in the previous equation delivers equation (31) in the proposition.

Appendix H - Robustness of Results to Positive Fixed Costs

From the proof of proposition 3 in appendix G, which covers the general case with non-negative fixed costs $f \geq 0$, it follows that equation (31) continues to hold for $f \geq 0$. From equations (G2) and (G3) it follows that equation (30) generalizes to

$$\left(\frac{\Delta_t}{\Delta_t^e}\right)^{-1}\left(\frac{1-\alpha(1-\delta)\left(\Pi_t/\Xi_{t-1,t}\right)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi_t^\star/\Xi_{t-1,t}^\star\right)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}} = \left(\frac{1-(1-\delta)\left[\frac{\overline{L}_t^c}{\overline{\overline{L}}_t} + \frac{f\Delta_t^e}{y_t\Delta_t}\left(\frac{\overline{L}_t^c}{\overline{\overline{L}}_t}-1\right)\right]}{1-(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}}\right). \tag{H1}$$

Using equations (31) and (H1), we then evaluate the sensitivity of the optimal inflation estimate in steady state ($y_t = y$) for different fixed cost, using the baseline parameters from table 1. We thereby set $\overline{L}_t^c/\overline{L}_t = 1.0703$, which is the sample mean of this ratio in the data and $\Pi_t/\Xi_{t-1,t} = 1.031$, which is equal to the sample mean of GDP deflator over the considered sample period, i.e., we assume no price indexation ($\Xi_{t-1,t} \equiv 1$). The steady state value of Δ_t^e/Δ_t then follows from (H1). We consider fixed costs in a range up to 10% of total (detrended) output, $f/y \in [0, 0.1]$, where $f/y = 0$ is the case considered in the main text. Figure H1 shows that the estimated optimal inflation rate is quite insensitive to assuming alternative fixed costs values: over the considered range of fixed costs, the optimal inflation rate increases, but the maximal effect on the optimal inflation rate is small and around 0.1%. This continues to be true for reasonably sized output fluctuations ($y_t \gtrless y$).

Appendix I - Proof to Proposition 4

We start by deriving the optimal inflation rate (33) and the recursive equation (34). In the absence of price rigidities, firms choose at all times their price such that their

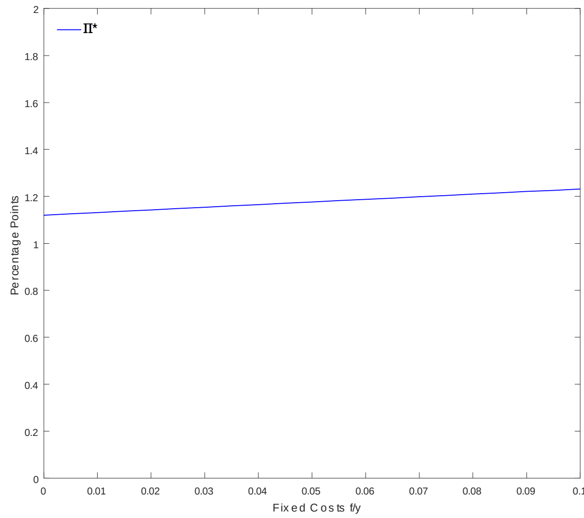

FIGURE H1. ROBUSTNESS OF OPTIMAL INFLATION ESTIMATES TOWARDS POSITIVE FIXED COSTS.

relative price is inversely proportional to their relative productivity. This follows from the equation (23), which determines the optimal relative price in the absence of price rigidities and is reproduced here for convenience:

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}. \tag{I1}$$

Condition 2 implies that the previous equation holds also for $t = -1$.

We now show that the optimal relative price (I1) can also be achieved by firm j in an economy *with* price setting frictions and non-constant δ-shock intensities under the optimal inflation rate stated in the proposition. This is so because absent δ-shocks, the optimal inflation rate insures that the firm's nominal price either remains constant (when there is no price indexation) or evolves over time in line with the price indexation rule, while equation (I1) continues to hold. Taking growth rates of equation (I1) and imposing

$P_{jt} = \Xi_{t-1,t} P_{j,t-1}$, which holds in the absence of δ-shocks, delivers[47]

$$\frac{\Pi_t^{\star}}{\Xi_{t-1,t}^{\star}} = \frac{\Delta_t^e}{\Delta_{t-1}^e}\,\frac{g_t}{q_t}.$$

The previous equation implies equation (33).

To derive equation (34), we can rewrite the definition of Δ_t^e in equation (22) according to[48]

$$\begin{aligned}
(\Delta_t^e)^{1-\theta} &= \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}}\right)^{1-\theta} \text{dj} \\
&= \delta_0 + \delta_0(1-\delta_1)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right)^{1-\theta} \\
&= \delta_0 + \delta_0(1-\delta_1)\,(q_t/g_t)^{1-\theta} \\
&+ (1-\delta)\,(q_t/g_t)^{1-\theta}\left\{\delta_0(1-\delta_1)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\left(\frac{q_{t-1} \times \cdots \times q_{t-s}}{g_{t-1} \times \cdots \times g_{t-s}}\right)^{1-\theta}\right\},
\end{aligned}$$

where the term in parenthesis is equal to $(\Delta_{t-1}^e)^{1-\theta} - \delta_0$. This delivers equation (34) in the proposition.

In the absence of economic disturbances, equation (34) implies that Δ_t^e converges to

$$\Delta^e = \left(\frac{\delta}{1-\delta_1+\delta}\right)^{\frac{1}{1-\theta}} \left(\frac{1-(\delta_1-\delta)(g/q)^{\theta-1}}{1-(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{1}{1-\theta}}.$$

The steady state result in the proposition then follows from equation (33) and the assumption of no price indexation $\left(\Xi_{t-1,t}^{\star} \equiv 1\right)$.

Appendix J - Proof to Proposition 5

For simplicity, we shall refer to P_t^N, which contains only products of age N or higher, as the measured price level and to $\Pi_t^N = P_t^N/P_{t-1}^N$ as the measured inflation rate. As before, we let P_t denote the ideal price level (using all products) and Π_t the ideal inflation

[47]In the presence of δ-shocks, prices are flexible so that equation (I1) can easily be achieved.

[48]The following derivations assume that the initial productivity distribution at $t = -1$ is consistent with the assumed productivity process.

rate. The proof proceeds in two steps. In a first step, we derive the measured inflation rate $\Pi_t^{N\star}$ in a setting where monetary policy implements $\Pi_t^\star$ from proposition 1 for the ideal inflation rate. In a second step, we show that if monetary policy implements $\Pi_t^{N\star}$ for the measured inflation rate, then this policy implements the same relative product prices as in the case where monetary policy implements $\Pi_t^\star$ for the ideal rate.

Step 1: In analogy to equation (A10), which defines the ideal price level, the measured price level is defined as

$$(P_t^N)^{1-\theta} = \delta \sum_{s=0}^{\infty} (1-\delta)^s \Lambda_t(s+N), \tag{J1}$$

where the weighted average cohort price $\Lambda_t(\cdot)$ is defined in equation (A9). From proposition 1 it follows that under the optimal inflation rate $\Pi_t^\star$, firms with a Calvo shock do not find it optimal to adjust their price, so that we have for $s \geq k \geq 0$

$$P_{t-s,t-k}^\star = \Xi_{t-s,t-k}^\star P_{t-s,t-s}^\star.$$

Using this result to rewrite equation (A9) shows that the weighted average cohort price under the optimal inflation rate $\Pi_t^\star$ is

$$\Lambda_t(s) = (\Xi_{t-s,t}^\star P_{t-s,t-s}^\star)^{1-\theta}. \tag{J2}$$

The previous equation implies

$$\begin{aligned} \Lambda_t(s+N) &= \left(\frac{\Xi_{t-(N+s),t}^\star}{\Xi_{t-(N+s),t-N}^\star} \right)^{1-\theta} \Lambda_{t-N}(s) \\ &= (\Xi_{t-N,t}^\star)^{1-\theta} \Lambda_{t-N}(s). \end{aligned}$$

Substituting this into equation (J1) yields

$$(P_t^{N\star})^{1-\theta} = (\Xi_{t-N,t}^\star)^{1-\theta} \left[\delta \sum_{s=0}^{\infty} (1-\delta)^s \Lambda_{t-N}(s) \right],$$

where the expression in brackets is the ideal price level defined in equation (A10) shifted

N periods into the past. For a policy that implements the optimal inflation rate from proposition 1 for the ideal inflation measure, we thus have

$$P_t^{N\star} = \Xi^\star_{t-N,t} P^\star_{t-N}. \tag{J3}$$

From the previous equation we get that measured inflation is then given by

$$\Pi_t^{N\star} = \frac{\Xi^\star_{t-1,t}}{\Xi^\star_{t-N-1,t-N}} \Pi^\star_{t-N},$$

which is the inflation rate stated in the proposition.

Step 2: Using equation (J2) to rearrange equation (J1) delivers

$$\begin{aligned}(P_t^{N\star})^{1-\theta} &= \delta \sum_{s=0}^{\infty} (1-\delta)^s (\Xi^\star_{t-(s+N),t} P^\star_{t-(s+N),t-(s+N)})^{1-\theta} \\ &= \delta(\Xi^\star_{t-N,t} P^\star_{t-N,t-N})^{1-\theta} + (1-\delta)(\Xi^\star_{t-1,t} P^{N\star}_{t-1})^{1-\theta}.\end{aligned}$$

Dividing the previous equation by $(P_t^{N\star})^{1-\theta}$ and using equation (J3) one obtains

$$\Pi_t^{N\star} / \Xi^\star_{t-1,t} = \left(\frac{1 - \delta \left(P^\star_{t-N,t-N} / P^\star_{t-N} \right)^{1-\theta}}{1-\delta} \right)^{\frac{1}{\theta-1}}. \tag{J4}$$

The previous equation shows how the relative price of firms with a δ-shock $(P^\star_{t-N,t-N}/P^\star_{t-N})$ is determined so as to be consistent with $\Pi_t^{N\star}$. When monetary policy targets $\Pi_t^{N\star} = \frac{\Xi^\star_{t-1,t}}{\Xi^\star_{t-N-1,t-N}} \Pi^\star_{t-N}$, as assumed, then equation (J4) coincides with equation (24) shifted back by N periods. Since equation (23) implies $1/\Delta_t^e = P^\star_{t,t}/P^\star_t$, this shows that monetary policy implements the same relative prices as a policy that implements $\Pi_t^\star$ from proposition 1 for the ideal inflation measure.