BKK the EZ Way

International Long-Run Growth News and Capital Flows

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ONLINE APPENDIX

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A: Data Sources

Country specific variables. The measure of productivity is obtained from the Penn World Table V.8 (Feenstra, Inklaar and Timmer 2013), and it accounts for variation in both the share of labor income and capital depreciation across countries and over time (series denoted as $rtfpna$). The Penn World table is available at `https://pwt.sas.upenn.edu/cic_main.html`.

Data for the construction of the price-dividend ratio for RoW countries are from the "International research returns" section of Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The US price-to-dividend ratio is obtained from the website of Robert Shiller (`http://www.econ.yale.edu/~shiller/data/ie_data.xls`). The price-to-dividend ratios for the RoW countries are calculated using cum- and ex-dividend country value-weighted dollar index returns (using "All 4 Data Items Not Reqd" series). French's data begin in 1977; for previous years we use price-dividend ratios from Campbell (2003).

Data on consumption and investment are from the Penn World Table and are expressed in constant national prices. To construct consumption from the Penn Word Tables dataset, we multiply the consumption share (denoted as csh_c) by GDP expressed in constant national prices (series denoted as $rgdpna$). We repeat the same procedure for investment using the investment share data series (denoted as csh_i).

The real risk-free rates are computed using data from the International Financial Statistics (IFS) dataset provided by the International Monetary Fund. The IFS dataset is available at `http://elibrary-data.imf.org/DataExplorer.aspx`. For each country, the real risk-free rate is computed as the difference between the nominal interest rate on government bills and realized inflation measured by the consumer price index for all items. For the United Kingdom, the retail index is used to calculate inflation. Germany's and Italy's risk-free rate series calculated by the IMF begin in 1975 and 1976, respectively. For earlier years we use data from Campbell (2003).

Net exports data and subcomponents. This section discusses how total net exports and net exports of capital goods are measured. For the bilateral Net Exports, we use two data sources. Table 1 in the main text is based on data collected from the IMF Direction of Trade Statistics (IMF-DOTS) for all pairwise combinations of the US and the rest of the G7 countries. Imports correspond to the series denoted as "Goods, Value of Imports, Cost, Insurance, Freight (CIF), US Dollars." Exports correspond to the series denoted as "Goods, Value of Exports, Free on board (FOB), US Dollars." The results in table B2 of appendix B.3 are instead based on the imports and exports series collected from Mitchell (2007*a*,*b*,*c*). These series correspond to the values reported in the books' sections called "External Trade with Main Trading Partners."

For the broader aggregate of net exports, whose results are reported in table 1 (column labeled "*NX* (US Total)"), we use annual data from the Bureau of Economic Analysis (BEA) table 4.2.5 "Exports and imports by type of products." For both exports and imports, data are aggregated in six main components: (C1) Foods, feeds, and beverages; (C2) Industrial supplies and materials; (C3) Capital goods, except automotive; (C4) Automotive vehicles, engines, and parts; (C5) Consumer goods, except automotive; and (C6) Services.

In this paper, we study a model that abstracts away from both consumer durable goods and government expenditure. For this reason, we exclude the following subcomponents from both imports and exports in our empirical investigation: " Transfers under U.S. military agency sales contracts" included under (C6) services, and "Consumer Durable goods" included under (C5).

The BEA provides a detailed list of the items that are considered industrial supplies. In the context of our model, the most relevant subcomponents of these supplies (for example, finished and unfinished metals, finished and unfinished building materials, and fabrics) are better interpreted as nonperishable investment goods. For this reason, our net exports of capital goods, NXI, comprise both industrial supplies (C2 above), and capital goods (C3 above). A somewhat more accurate allocation of these supplies across investment and consumption goods may be achieved using the BEA detailed goods trade data. Unfortunately, this would come at the cost of basing our inference on a significantly shorter sample, as data are available only from 1989. Data are available at `https://www.bea.gov/international/detailed\_trade\_data.htm`.

B: Additional Empirical Results

In this section, we report additional results relative to the empirical analysis discussed in section I of the main text.

1: Estimates of the Productivity Process

We document some properties of the system that we use to describe the joint evolution of productivity and p/d ratios in equations (1)-(2) of the main text. We begin by allowing all the parameters to differ across countries, thus estimating an unconstrained GMM. Specifically, we estimate the parameters

$$\{\beta_a^{US}, \beta_a^{RoW}, \varsigma^{US}, \varsigma^{RoW}, \varrho^{US}, \varrho^{RoW}, \varsigma_z^{US}, \varsigma_z^{RoW}, \varrho_a, \varrho_z\},$$

using the following orthogonality conditions:

1) $\frac{1}{T}\sum_{t=1}^{T}\left(\Delta a_t^{US} - \beta_a^{US}\cdot pd_{t-1}^{US}\right)\cdot pd_{t-1}^{US} = 0,$

2) $\frac{1}{T}\sum_{t=1}^{T}\left(\Delta a_t^{RoW} - \beta_a^{RoW}\cdot pd_{t-1}^{RoW}\right)\cdot pd_{t-1}^{RoW} = 0,$

3) $\frac{1}{T}\sum_{t=1}^{T}\left(\Delta a_t^{US} - \beta_a^{US}\cdot pd_{t-1}^{US}\right)^2 - \left(\varsigma^{US}\right)^2 = 0,$

4) $\frac{1}{T}\sum_{t=1}^{T}\left(\Delta a_t^{RoW} - \beta_a^{RoW}\cdot pd_{t-1}^{RoW}\right)^2 - \left(\varsigma^{RoW}\right)^2 = 0,$

Table B1—Productivity Dynamics

	β_a	ς	R^2	ϱ	ς_z	ϱ_a	ϱ_z
Panel A: GDP Weighted							
Estimate	0.009	1.145	0.060	0.938	0.101	0.442	0.789
	(0.001)	(0.074)		(0.036)	(0.011)	(0.056)	(0.062)
$H_0 : US = RoW$	[0.112]	[0.013]		[0.527]	[0.077]		
Panel B: Equally Weighted							
Estimate	0.009	1.130	0.059	0.939	0.103	0.500	0.848
	(0.001)	(0.074)		(0.035)	(0.012)	(0.055)	(0.050)
$H_0 : US = RoW$	[0.174]	[0.009]		[0.470]	[0.079]		
Panel C: Market Cap Weighted							
Estimate	0.009	1.136	0.061	0.939	0.103	0.495	0.818
	(0.001)	(0.081)		(0.036)	(0.009)	(0.055)	(0.057)
$H_0 : US = RoW$	[0.530]	[0.011]		[0.322]	[0.271]		

Note: In this table we report estimates of the parameters that govern the transition dynamics of productivity featured in the system of equations (1) and (2) in the main text, for the case in which $z^i_{t-1} = \beta_a \cdot pd^i_{t-1}$. The numbers in parentheses correspond to standard errors. The numbers in square brackets are the p-values associated with the null hypothesis that each coefficient is identical in the US and the RoW. The RoW quantities are obtained by aggregating the remaining G7 countries using GDP shares (panel A), equal weights (panel B), and market capitalization shares (panel C). Data sources are detailed in appendix A. Our sample starts in 1973 and ends in 2006.

5) $\frac{1}{T}\sum_{t=1}^{T}\left(\beta_a^{US}\cdot pd_t^{US} - \varrho^{US}\cdot\beta_a^{US}\cdot pd_{t-1}^{US}\right)\cdot\beta_a^{US}\cdot pd_{t-1}^{US} = 0,$

6) $\frac{1}{T}\sum_{t=1}^{T}\left(\beta_a^{RoW}\cdot pd_t^{RoW} - \varrho^{RoW}\cdot\beta_a^{RoW}\cdot pd_{t-1}^{RoW}\right)\cdot\beta_a^{RoW}\cdot pd_{t-1}^{RoW} = 0,$

7) $\frac{1}{T}\sum_{t=1}^{T}\left(\beta_a^{US}\cdot pd_t^{US} - \varrho^{US}\cdot\beta_a^{US}\cdot pd_{t-1}^{US}\right)^2 - \left(\varsigma_z^{US}\cdot\varsigma^{US}\right)^2 = 0,$

8) $\frac{1}{T}\sum_{t=1}^{T}\left(\beta_a^{RoW}\cdot pd_t^{RoW} - \varrho^{RoW}\cdot\beta_a^{RoW}\cdot pd_{t-1}^{RoW}\right)^2 - \left(\varsigma_z^{RoW}\cdot\varsigma^{RoW}\right)^2 = 0,$

9) $\frac{1}{T}\sum_{t=1}^{T}\left(\Delta a_t^{US} - \beta_a^{US}\cdot pd_{t-1}^{US}\right)\left(\Delta a_t^{RoW} - \beta_a^{RoW}\cdot pd_{t-1}^{RoW}\right) - \varrho_a\varsigma^{US}\cdot\varsigma^{RoW} = 0,$

10) $\frac{1}{T}\sum_{t=1}^{T}\left(\beta_a^{US}\cdot pd_t^{US}\right)\cdot\left(\beta_a^{RoW}\cdot pd_t^{RoW}\right) - \varrho_z\cdot\varsigma_z^{US}\cdot\varsigma_z^{RoW} = 0,$

where all the variables are de-meaned. Given that we typically cannot reject the null hypothesis that all the parameters are identical across countries, in what follows we focus on the pooled case in which $\beta_a^{US} = \beta_a^{RoW} = \beta_a$, $\varsigma_a^{US} = \varsigma_a^{RoW} = \varsigma_a$, $\varrho^{US} = \varrho^{RoW} = \varrho$, $\varsigma_z^{US} = \varsigma_z^{RoW} = \varsigma_z$.

In table B1 we report the pooled estimated parameters, the associated standard error, and the p-value for the null hypothesis that the estimated coefficients are identical in the US and the RoW for the system in (1) and (2) in the main text.

We document several results. First, all the parameters governing the transition dynamics of productivity in the auxiliary system are tightly identified. Second, the autocorrelation of the predictive component of productivity growth is extremely high and close to 1. This means that shock to the expected component have a lasting impact on future growth. Third, we typically cannot reject the null hypothesis that the US and the RoW feature the same parameters. The only parameter for which we cannot consistently reject our null hypothesis is the short-run volatility of productivity (σ). The point estimate for this parameter is 1.35 (with a standard error of 0.19) for the US and 0.93 (with a standard error of 0.11) for the RoW. We explain the lower volatility of the RoW aggregates as originating from the imperfect correlation of the growth rates of productivity in the remaining G7 countries. Mechanically, the aggregation of the remaining six countries smooths fluctuations. Last, but not least, the correlations of the expected components of productivity are always larger than their unanticipated counterparts, and very close to unity.

2: Estimates of Response to Shocks

In what follows, we shall denote

$$\begin{aligned}
\varepsilon_{a,t}^{US} &= \Delta a_t^{US} - \beta_a \cdot pd_{t-1}^{US} \\
\varepsilon_{a,t}^{RoW} &= \Delta a_t^{RoW} - \beta_a \cdot pd_{t-1}^{RoW} \\
\varepsilon_{z,t}^{US} &= \beta_a \cdot pd_t^{US} - \varrho \cdot \beta_a \cdot pd_{t-1}^{US} \\
\varepsilon_{z,t}^{RoW} &= \beta_a \cdot pd_t^{RoW} - \varrho \cdot \beta_a \cdot pd_{t-1}^{RoW},
\end{aligned}$$

where the parameters $\{\beta_a, \varrho\}$ are jointly estimated with each set of moment conditions reported below.

Net Exports regression. We estimate the parameters featured in equation (3) in the manuscript along with $\beta_a^{US} = \beta_a^{RoW} = \beta_a$, and $\varrho^{US} = \varrho^{RoW} = \varrho$, using moment conditions (1), (2), (5), and (6) in appendix B.1 together with the following moment conditions:

$$11)\ \tfrac{1}{T}\sum_{t=1}^{T}\left[\Delta\left(\tfrac{NX_t^{US}}{GDP_t}\right) - \beta_{NX,a}\left(\varepsilon_{a,t}^{US} - \varepsilon_{a,t}^{RoW}\right) - \beta_{NX,pd}\left(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW}\right)\right]\left(\varepsilon_{a,t}^{US} - \varepsilon_{a,t}^{RoW}\right) = 0$$

$$12)\ \tfrac{1}{T}\sum_{t=1}^{T}\left[\Delta\left(\tfrac{NX_t^{US}}{GDP_t}\right) - \beta_{NX,a}\left(\varepsilon_{a,t}^{US} - \varepsilon_{a,t}^{RoW}\right) - \beta_{NX,pd}\left(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW}\right)\right]\left(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW}\right) = 0.$$

Investments regressions. We estimate the parameters featured in equation (4) in the manuscript along with $\beta_a^{US} = \beta_a^{RoW} = \beta_a$, and $\varrho^{US} = \varrho^{RoW} = \varrho$, using moment conditions (1), (2), (5), and (6) in appendix B.1 together with the following moment conditions:

$$13)\ \tfrac{1}{T}\sum_{t=1}^{T}\left[\Delta I_t^{US} - \Delta I_t^{RoW} - \beta_{I,a}\cdot(\varepsilon_{a,t}^{US} - \epsilon_{a,t}^{RoW}) - \beta_{I,pd}\cdot(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW})\right]\cdot(\varepsilon_{a,t}^{US} - \epsilon_{a,t}^{RoW}) = 0$$

$$14)\ \tfrac{1}{T}\sum_{t=1}^{T}\left[\Delta I_t^{US} - \Delta I_t^{RoW} - \beta_{I,a}\cdot(\varepsilon_{a,t}^{US} - \epsilon_{a,t}^{RoW}) - \beta_{I,pd}\cdot(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW})\right]\cdot(\varepsilon_{z,t}^{US} - \epsilon_{z,t}^{RoW}) = 0.$$

TABLE B2—EMPIRICAL EVIDENCE

	NX (*Bilateral, Mitchell*)	***NXI*** (*Total US NX of Investments*)
Panel A: GDP Weighted		
a	-0.122	-0.275
	(0.017)	(0.034)
z	1.223	1.383
	(0.529)	(0.689)
Panel B: Equally Weighted		
a	-0.119	-0.274
	(0.232)	(0.046)
z	1.202	1.412
	(0.452)	(0.682)
Panel C: Market Cap Weighted		
a	-0.133	-0.275
	(0.022)	(0.046)
z	1.137	1.436
	(0.480)	(0.724)

Note: In this table we report estimates for the response of the bilateral net exports between the US and the RoW, and total US net exports of investments to relative shocks to the unanticipated (a) and expected (z) components of productivity. Bilateral net exports data are obtained from Mitchell (2007*a*,*b*,*c*). The RoW quantities are aggregated by weighting the remaining G7 countries by their share of GDP (panel A), equally (panel B), and market capitalization (panel C). Data sources are detailed in appendix A. Our sample starts in 1973 and ends in 2006.

Consumption regressions. We estimate the parameters featured in equation (5) in the manuscript along with $\beta_a^{US} = \beta_a^{RoW} = \beta_a$, and $\varrho^{US} = \varrho^{RoW} = \varrho$, using moment conditions (1), (2), (5), and (6) in appendix B.1 together with the following moment conditions:

15) $\frac{1}{T}\sum_{t=1}^{T}\left[\Delta c_t^{US} - \Delta c_t^{RoW} - \beta_{c,a}\cdot(\varepsilon_{a,t}^{US} - \epsilon_{a,t}^{RoW}) - \beta_{c,pd}\cdot(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW})\right]\cdot(\varepsilon_{a,t}^{US} - \epsilon_{a,t}^{RoW}) = 0$

16) $\frac{1}{T}\sum_{t=1}^{T}\left[\Delta c_t^{US} - \Delta c_t^{RoW} - \beta_{c,a}\cdot(\varepsilon_{a,t}^{US} - \epsilon_{a,t}^{RoW}) - \beta_{c,pd}\cdot(\varepsilon_{z,t}^{US} - \varepsilon_{z,t}^{RoW})\right]\cdot(\varepsilon_{z,t}^{US} - \epsilon_{z,t}^{RoW}) = 0.$

3: Additional Series of Net Exports

In table B2 we report the results of the regression specification in equation (3) in the main text obtained by replacing the series on the left-hand side with the bilateral *NX* between the US and the RoW obtained by aggregating the data from Mitchell (2007*a*,*b*,*c*) (first column) and with total net exports of investments (*NXI*) of the US versus the remainder of the world (second column). The table documents the same behavior of net exports that we have highlighted in the main text, namely that net exports increase in response to a positive relative long-run shock and fall in response to a positive relative short-run shock. The results are robust to all the aggregation methods for the RoW variables that we have employed throughout the rest of the paper.

4: Decomposition of R^2 for NX Regressions

In Table B3, we report additional results for the regressions of bilateral net exports between the US and the RoW onto the spread of shocks to the expected and unanticipated component of productivity,

$$\Delta\left(\frac{NX_t^{US}}{GDP_t}\right) = \beta_{NX,a}\cdot\left(\varepsilon_{a,t}^{US}-\varepsilon_{a,t}^{RoW}\right)+\beta_{NX,z}\cdot\left(\varepsilon_{z,t}^{US}-\varepsilon_{z,t}^{RoW}\right)+\xi_t,$$

as in equation (3). The total R^2s in these regressions are usually in the range of 20%. A large fraction of these R^2s, ranging from about 60% to almost 75%, is accounted by the shocks to the predictive components of productivity, as documented in the last column of table B3.

Table B3—Relative $\mathbf{R^2}$

		$\mathbf{R^2}$	
a	**z**	**Total**	**LR share**
Panel A: GDP Weighted			
−0.096	1.072	0.218	0.596
(0.020)	(0.446)	(0.103)	(0.237)
Panel B: Equally Weighted			
−0.083	1.028	0.190	0.731
(0.025)	(0.402)	(0.103)	(0.218)
Panel C: Market Cap Weighted			
−0.100	0.917	0.183	0.642
(0.028)	(0.395)	(0.102)	(0.235)

Note: In this table we report the estimates for the regressions of bilateral net exports between the US and the RoW (columns 1 and 2) onto relative shocks to the unanticipated (a) and expected (z) components of productivity. Bilateral net exports data are obtained from IMF DOTS. Columns 3 and 4 report the total R^2 of each regression and the share of the R^2 that is represented by the two relative shocks. The RoW quantities are obtained by aggregating the remaining G7 countries using GDP shares (panel A), equal weights (panel B), and market capitalization shares (panel C). Data sources are detailed in appendix A. Our sample starts in 1973 and ends in 2006.

C: Additional Results for the Model

In this section, we derive the approximated solution of our two-period model. We then report our derivations for the infinite horizon model, along with key results for our aggregation with heterogeneous capital vintages.

1: Derivations for the Two-Period Model

Shock and information structure. In this section, we present a simplified two-period version of the model in order to provide intuition on the capital reallocation motives induced by recursive preferences. Specifically, at time $t = 1$ agents receive news θ about the productivity that capital will have at time $t = 2$. Since θ does not alter productivity at time $t = 1$, it represents a pure news shock. For simplicity, no other shock materializes at $t = 1, 2$.

At time $t = 0$, that is, before the arrival of the news, agents have the same wealth and consumption level, and exchange a complete set of θ-contingent securities to maximize their time-0 utility. As a result, the time-1 reallocation can be interpreted as a deviation from the symmetric steady state.

Utility and technology. In what follows, we take advantage of lognormality wherever possible. Up to a log linearization of the allocation shares, this modeling strategy enables us to get a simple closed-form solution. In this spirit, we start by assuming that agents have an IES equal to 1, that is, their preferences can be expressed as follows:

$$\text{(C1)} \qquad u_0^i = \begin{cases} (1-\beta)\log C_0^i + \frac{\beta}{1-\gamma}\log E_0[exp\{u_1^i(1-\gamma)\}] & \gamma \neq 1 \\ (1-\beta)\log C_0^i + \beta E_0[u_1^i] & \gamma = 1 \end{cases}$$

where

$$\text{(C2)} \qquad C_t = X_t^{\lambda} Y_t^{(1-\lambda)}, \quad C_t^* = X_t^{*(1-\lambda)} Y_t^{*\lambda}, \quad \text{for } t = 1, 2$$

and

$$u_1^i = (1-\beta)\log C_1^i + \beta(1-\beta)\log C_2^i.$$

Notice that in our setting uncertainty is fully resolved at time 1, and hence time-2 quantities are known at time 1. We set $C_0^h = C_0^f = \overline{C} > 0$ for symmetry and without loss of generality, as these variables play no role in the future allocation.

The resource constraints are specified as follows:

$$\text{(C3)} \qquad \begin{array}{ccc} 1 = X_1 + X_1^* + I_{x,1} + I_{y,t}, & 1 = Y_1 + Y_1^* + I_{x,1}^* + I_{y,1}^* & t = 1 \\ e^{\theta} G(I_{x,1}, I_{x,1}^*) = X_2 + X_2^*, & e^{-\theta} G^*(I_{y,1}, I_{y,1}^*) = Y_2 + Y_2^* & t = 2 \\ G = I_{x,1}^{\lambda_i} I_{x,1}^{*1-\lambda_i}, & G^* = I_{y,1}^{1-\lambda_i} I_{y,1}^{*\lambda_i} & \\ & \theta \sim iidN(0,\sigma) & \end{array},$$

and are consistent with the assumption of full capital depreciation in our benchmark setting. This assumption helps with log linearity.

Total production at time $t = 1$ is predetermined, like in a production economy in which labor does not adjust upon the arrival of news. Since the relative reallocation induced by θ does not depend on the time-1 size of the economy, we normalize total production to be 1.

Finally, we assume that θ affects both domestic and foreign productivity to preserve symmetry in our equations. In what follows, we show that the results are driven only by the relative cross-country productivity, 2θ.

Pareto problem. Under complete markets, the allocation can be recovered by solving the following Pareto problem:

$$\max_{\left\{\{X_t, X_t^*, Y_t, Y_t^*\}_{t=1,2}, I_{x,1}, I_{y,1}, I_{x,1}^*, I_{y,1}^*\right\}} \mu_0 u_0 + (1-\mu_0) u_0^*,$$

subject to the constraints specified in (C3). For the sake of symmetry, we assume $S_0 \equiv \frac{\mu_0}{1-\mu_0} = 1$.

After simplifying common coefficients, the optimality condition for the allocation of good X_1 is

$$S_1(\theta) \frac{\partial \log C_1}{\partial X_1} = \frac{\partial \log C_1^*}{\partial X_1} \tag{C4}$$

with

$$S_1(\theta) = \begin{cases} S_0, & \gamma = 1 \\ S_0 \frac{\frac{e^{u_1(1-\gamma)}}{E_0[e^{u_1(1-\gamma)}]}}{\frac{e^{u_1^*(1-\gamma)}}{E_0[e^{u_1^*(1-\gamma)}]}} = S_0 e^{(u_1(\theta) - u_1^*(\theta))(1-\gamma)} & \gamma \neq 1, \end{cases} \tag{C5}$$

where the second equality in the case of $\gamma \neq 1$ holds because of the symmetry of our problem (at the equilibrium $E_0[e^{u_1(1-\gamma)}] = E_0[e^{u_1^*(1-\gamma)}]$).

Equation (C4) establishes that the optimal allocation can be found as in a regular static problem, for a *given* value of S_1. Equation (C5) pins down S_1 and states two important results. First, in the time-additive case, the share of resources is time invariant, that is, it is not affected by the actual realization of θ. This is consistent with the special log case considered by Cole and Obstfeld (1991).

Second, with recursive preferences, agents have a preference for the variance of their future utility and hence their time-1 marginal utility depends on the time-1 level of their utility. If agents prefer early resolution of uncertainty ($\gamma > 1$), a higher relative future utility implies a lower relative marginal utility and hence a lower share of allocated resources ($u_1(\theta) > u_1^*(\theta) \to S_1(\theta) < S_0$). The opposite is true when $\gamma < 1$. Because of the dependence of S_1 on future utility levels, $u_1(\theta) - u_1^*(\theta)$, θ prompts a reallocation at time 1.

Similarly to the results derived for time 1, the optimality condition for the allocation of good X_2 is

$$S_2(\theta)\frac{\partial \log C_2}{\partial X_2} = \frac{\partial \log C_2^*}{\partial X_2} \tag{C6}$$

where

$$S_2(\theta) = S_1(\theta)$$

because uncertainty is fully resolved at time 1 and hence agents face no further news going forward, that is, $\frac{\exp\{u_2(\theta)(1-\gamma)\}}{E_1[\exp\{u_2(\theta)(1-\gamma)\}]} = \frac{\exp\{u_2^*(\theta)(1-\gamma)\}}{E_1[\exp\{u_2^*(\theta)(1-\gamma)\}]} = 1 \quad \forall\theta$. As a result, no additional variation of the pseudo-Pareto weights takes place at time 2.

CRRA case. In this case, the allocation at time 1 can be computed exactly as $S_1 = S_0 = 1$. As a result, from the home-country perspective we have the following:

$$I_{x,1} = \lambda_i \frac{\beta}{1+\beta}, \quad I_{y,1} = (1-\lambda_i)\frac{\beta}{1+\beta},$$
$$X_1 = \lambda\frac{1}{1+\beta}, \quad X_1^* = (1-\lambda)\frac{1}{1+\beta},$$

that is, a fraction $1/(1+\beta)$ of time-1 output is devoted to consumption, whereas $\beta/(1+\beta)$ is devoted to investment. The fraction of good X used for domestic consumption equals the consumption home-bias parameter λ. The fraction of good X used for investment in the home country equals the investment home-bias parameter λ_i. By symmetry, similar results apply to the foreign country. It is then possible to establish that

$$NX_1^C \equiv X_1^* - p_1 Y_1 = 0 \quad \forall\theta$$

and

$$NX_1^I \equiv I_{y,1} - p_1 I_{x,1}^* = 0 \quad \forall\theta,$$

that is, news promotes no current account adjustment.

EZ case. In this case, the allocations at $t = 1, 2$ are a nonlinear function of $S_1(\theta)$. We log linearize them with respect to $s_1 \equiv \log S_1$ around $\bar{s}_1 = s_0 = 0$ and verify that at the equilibrium the following holds:

$$u_1 = const + \lambda_u^s s_1 + \lambda_u^\theta \theta, \qquad u_1^* = const - \lambda_u^s s_1 - \lambda_u^\theta \theta, \tag{C7}$$
$$s_1 = \lambda_s^\theta \theta, \tag{C8}$$

where λ_i^j denotes the elasticity of the variable i with respect to variable j. The derivations reported in what follows prove that

$$\lambda_s^\theta = 2\frac{(2\lambda-1)(1-\gamma)}{1-2\lambda_u^s(1-\gamma)}, \tag{C9}$$

where $\lambda_u^s \geq 0$ if $\lambda \geq 1/2$, as detailed in equation (C18).

Time 2. In the final period, the allocation of $\{X_2, X_2^*, Y_2, Y_2^*\}$ satisfies what follows:

$$\begin{aligned}
S_2(\theta)\frac{\partial \log C_2}{\partial X_2} &= \frac{\partial \log C_2^*}{\partial X_2} \\
S_2(\theta)\frac{\partial \log C_2}{\partial Y_2} &= \frac{\partial \log C_2^*}{\partial Y_2^*} \\
e^{\theta} G(\theta) &= X_2 + X_2^* \\
e^{-\theta} G^*(\theta) &= Y_2 + Y_2^*.
\end{aligned}$$

We report the solution only for the home country allocation using the condition $S_2(\theta) = S_1(\theta)$:

$$\begin{aligned}
X_2(\theta) &= SH^X(\theta) e^{\theta} G(\theta) = \frac{\kappa S_1(\theta)}{1 + \kappa S_1(\theta)} e^{\theta} G(\theta) \\
Y_2(\theta) &= SH^Y(\theta) e^{-\theta} G^*(\theta) = \frac{1/\kappa S_1(\theta)}{1 + 1/\kappa S_1(\theta)} e^{-\theta} G^*(\theta),
\end{aligned}$$

where $\kappa = \lambda/(1-\lambda)$, and SH^z is the share of good $z = X, Y$ allocated to the home (H) country. Using the resource constraints, $X_2^* = (1 - SH^X)e^{\theta}G(\theta)$, and $Y_2^* = (1 - SH^Y)e^{-\theta}G(\theta)$. After log linearizing the share processes with respect to $s_1(\theta)$ around $\overline{s} = 0$,

$$\begin{aligned}
\log SH^X(\theta) &\approx \log(\lambda) + (1-\lambda)s_1(\theta) \\
\log SH^Y(\theta) &\approx \log(1-\lambda) + \lambda s_1(\theta),
\end{aligned}$$

we get

$$\begin{aligned}
\text{(C10)} \quad \log C_2(\theta) &\approx \overbrace{\lambda \log(\lambda) + (1-\lambda)\log(1-\lambda)}^{constant} + \overbrace{(2\lambda - 1)}^{\lambda_{C_2}^{\theta}}\theta + \overbrace{2\lambda(1-\lambda)}^{\lambda_{C_2}^{s}} s_1(\theta) \\
&+ \lambda \log G(\theta) + (1-\lambda)\log G^*(\theta) \\
\text{(C11)} \quad \log C_2^*(\theta) &\approx constant - \lambda_{C_2}^{\theta}\theta - \lambda_{C_2}^{s} s_1(\theta) + (1-\lambda)\log G(\theta) \\
&+ \lambda \log G^*(\theta).
\end{aligned}$$

Time 1. At time 1, the planner needs to allocate both consumption goods $\{X_1, X_1^*, Y_1, Y_1^*\}$ and capital $\{I_x, I_x^*, I_y, I_y^*\}$ to solve the following problem:

$$\max(1-\delta)[S_1 \cdot (\log C_1 + \delta \log C_2) + (\log C_1^* + \delta \log C_2^*)],$$

subject to

$$\begin{aligned} 1 &= X_1 + X_1^* + I_{x,1} + I_{y,t} \\ 1 &= Y_1 + Y_1^* + I_{x,1}^* + I_{y,1}^*. \end{aligned}$$

The rescaling factor $(1-\delta)$ is reported just for consistency with the specification of our preferences, and it does not play any relevant role. This optimization is implemented taking θ and hence $s_1(\theta)$ as given. After solving this allocation problem, we can characterize $u_1 - u_1^*$ in equation (16) and solve a fixed point for the joint dynamics of $u_1 - u_1^*$ and $s_1(\theta)$, i.e., our main computational goal.

The FOCs with respect to X_1, X_1^*, I_x, and I_y are

$$X_1^* = 1/(\kappa S_1) X_1 \tag{C12}$$

$$X_1 = \frac{1}{\lambda_i \beta [1 + 1/(\kappa S_1])} I_x \tag{C13}$$

$$I_y = \frac{1-\lambda_i}{\lambda_i} \frac{\lambda + (1-\lambda) S_1}{1 - \lambda + \lambda S_1} I_x \tag{C14}$$

Equations (C12)–(C14) together with the resource constraint imply the following:

$$I_x = \lambda_i \delta \frac{1}{1 + \delta \lambda_i + \delta(1-\lambda_i)\frac{\lambda + (1-\lambda)S_1}{1-\lambda+\lambda S_1}}. \tag{C15}$$

A log-linearization of equations (C12)–(C15) with respect to $s_1(\theta)$ around $\overline{s} = 0$ produces

$$\begin{aligned} \log I_x &\approx \log\left(\frac{\lambda_i \delta}{1+\delta}\right) + \underbrace{\frac{\delta}{1+\delta}(1-\lambda_i)(2\lambda - 1)}_{\lambda_i^s} s_1 \\ \log I_y &\approx \log\left(\frac{(1-\lambda_i)\delta}{1+\delta}\right) + \underbrace{\frac{1}{1+\delta}(1+\lambda_i\delta)(1-2\lambda)}_{\lambda_{i_y}^s} s_1 \\ \log X_1 &\approx \log\left(\frac{\lambda}{1+\delta}\right) + \underbrace{\left[\frac{\delta}{1+\delta}(1-\lambda_i)(2\lambda-1) + 1 - \lambda\right]}_{\lambda_x^s} s_1 \\ \log X_1^* &\approx \log\left(\frac{1-\lambda}{1+\delta}\right) + \underbrace{\left[\frac{\delta}{1+\delta}(1-\lambda_i)(2\lambda-1) - \lambda\right]}_{\lambda_{x^*}^s} s_1. \end{aligned} \tag{C16}$$

By symmetry:

$$\begin{aligned}\log I_x^* &\approx \log\left(\frac{(1-\lambda_i)\delta}{1+\delta}\right) - \lambda_{i_y}^s s_1, \qquad \log I_y^* \approx \log\left(\frac{\lambda_i\delta}{1+\delta}\right) - \lambda_i^s s_1,\\ \log Y_1^* &\approx \log\left(\frac{\lambda}{1+\delta}\right) - \lambda_x^s s_1, \qquad \log Y_1 \approx \log\left(\frac{1-\lambda}{1+\delta}\right) - \lambda_{x^*}^s s_1.\end{aligned}$$

We are now ready to characterize the utility functions at time 1:

$$\begin{aligned}u_1 &= (1-\delta)(\log C_1 + \delta \log C_2) \approx const + \lambda_u^\theta \theta + \lambda_u^s s_1 = const + \overbrace{(\lambda_u^\theta + \lambda_u^s \cdot \lambda_s^\theta)}^{\lambda_u}\theta\\ u_1^* &= (1-\delta)(\log C_1^* + \delta \log C_2^*) \approx const - \lambda_u^\theta \theta - \lambda_u^s s_1 = const - \lambda_u \theta,\end{aligned}$$

where

$$\begin{aligned}&\text{(C17)} & \lambda_u^\theta &= \lambda_{C_2}^\theta (1-\delta)\delta = (2\lambda-1)(1-\delta)\delta\\ &\text{(C18)} & \lambda_u^s &= (1-\delta)[\lambda \cdot \lambda_x^s + (1-\lambda)(-\lambda_{x^*}^s)]\\ & & &+ \delta(1-\delta)(2\lambda-1)(\lambda_i \cdot \lambda_i^s + (1-\lambda_i)(-\lambda_{i_y}^s))\\ &\text{(C19)} & \lambda_s^\theta &: s_1(\theta) = \lambda_s^\theta \theta.\end{aligned}$$

Given the equilibrium condition (16),

$$\lambda_s^\theta = -2(\gamma-1)\lambda_u \rightarrow \lambda_s^\theta - 2\lambda_u^s(\gamma-1)\lambda_u = \lambda_u,$$

and hence

$$\text{(C20)} \qquad \lambda_s^\theta = \frac{2(1-\gamma)(1-\delta)\delta(2\lambda-1)}{1+2\lambda_u^s(\gamma-1)}.$$

LEMMA 1: *If $\lambda > 1/2$, $\lambda_i \in (0,1)$, and $\gamma > 1$, then $\lambda_s^\theta < 0$.*

PROOF:

If $\lambda > 1/2$ and $\gamma > 1$, the numerator of equation (C20) is negative. Since $\lambda_i \in (0,1)$, in the system of equations (C16) home bias implies that (i) $\lambda_i^s > 0$ and $-\lambda_{i_y}^s > 0$, and (ii) $\lambda_x^s > 0$ and $-\lambda_{x^*}^s > 0$. As a result, according to equation (C18) we have $\lambda_u^s > 0$. Given these conditions, the denominator of (C20) is positive.

Time-1 net exports. According to the definition of net exports of consumption goods, from the home-country perspective we have

$$\frac{NX_1^C}{X_1} = \frac{X_1^*}{X_1} - \underbrace{\left(\frac{1-\lambda}{\lambda}\frac{X_1}{Y_1}\right)}_{p_1}\frac{Y_1}{X_1} = -\frac{1-\lambda}{\lambda}\left(1 - \frac{1}{S_1}\right),$$

where p_1 is the terms of trade. Similarly, for the net exports of investment we obtain

$$\frac{NX_1^I}{I_x} = \frac{I_y}{I_x} - \underbrace{\left(\frac{1-\lambda_i}{\lambda_i}\frac{I_x}{I_x^*}\right)}_{p_1}\frac{I_x^*}{I_x} = -\frac{1-\lambda_i}{\lambda_i}\left(1-\frac{S_1+\kappa}{\kappa S_1+1}\right).$$

2: Pareto Problem, Infinite Horizon

For the sake of brevity, in this appendix we suppress notation to denoting state and histories and retain only subscripts for time. We represent the Epstein and Zin (1989) utility preference in the following compact way:

$$U_t = W(\widetilde{C}_t, U_{t+1}),$$

so that the dependence of current utility on j-step-ahead consumption can easily be denoted as follows:

$$\text{(C21)} \qquad \frac{\partial U_t}{\partial \widetilde{C}_{t+j}} = W_{2,t+1} \cdot W_{2,t+2} \cdot \dots \cdot W_{2,t+j} W_{1,t+j},$$

where $W_{2,t+j} \equiv \frac{\partial U_{t+j-1}}{\partial U_{t+j}}$ and $W_{1,t+j} \equiv \frac{\partial U_{t+j}}{\partial \widetilde{C}_{t+j}}$. Given this notation, the intertemporal marginal rate of substitution between $\widetilde{C}_t$ and $\widetilde{C}_{t+1}$ is

$$\text{(C22)} \qquad IMRS_{\widetilde{C},t+1} = \frac{W_{2,t+1}W_{1,t+1}}{W_{1,t}} = M_{t+1}\pi_{t+1},$$

where π_{t+1} is the probability of a specific state, and M_{t+1} is the stochastic discount factor in $\widetilde{C}$ units with the following form:

$$\text{(C23)} \qquad M_{t+1} = \beta\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}}{E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

The consumption bundle, $\widetilde{C}$, depends on both the consumption aggregate, C, and labor, N:

$$\widetilde{C}_t = \widetilde{C}(C_t, N_t).$$

The consumption aggregate combines two goods, x and y:

$$C_t = C(X_t, Y_t).$$

The planner faces the following constraints:

$$F(A_t, K_t, N_t) \geq X_t + X_t^* + I_{x,t} + I_{y,t} \tag{C24}$$
$$F(A_t^*, K_t^*, N_t^*) \geq Y_t + Y_t^* + I_{x,t}^* + I_{y,t}^* \tag{C25}$$
$$K_t \leq (1-\delta)K_{t-1} + e^{\omega_t} G(I_{x,t-1}, I_{x,t-1}^*) \tag{C26}$$
$$K_t^* \leq (1-\delta)K_{t-1}^* + e^{\omega_t^*} G^*(I_{y,t-1}, I_{y,t-1}^*), \tag{C27}$$

where A_t and A_t^* are the exogenous stochastic productivity processes in equation (19). The processes $w_t = -\frac{1-\alpha}{\alpha}(\Delta a_t - \mu)$ and $w_t^* = -\frac{1-\alpha}{\alpha}(\Delta a_t^* - \mu)$ result from the vintage capital structure assumed in Ai, Croce and Li (2013).

The social planner chooses $\left\{X_t, X_t^*, Y_t, Y_t^*, N_t, N_t^*, K_t, K_t^*, I_{x,t}, I_{y,t}, I_{x,t}^*, I_{y,t}^*\right\}_t$ to maximize

$$\mu_0 W_0 + (1-\mu_0) W_0^*,$$

subject to sequences of constraints (C24)–(C27). Specifically, let $\lambda_{i,t}$ be the Lagrangian multiplier for the time t constraint (Bi); then the Lagrangian is

$$\begin{aligned}
\Omega =& \mu_0 W_0 + (1-\mu_0) W_0^* \\
&+ ... \\
&+ \lambda_{1,t}(F(A_t, K_t, N_t) - X_t - X_t^* - I_{x,t} - I_{y,t}) \\
&+ \lambda_{2,t}(F(A_t^*, K_t^*, N_t^*) - Y_t - Y_t^* - I_{x,t}^* - I_{y,t}^*) \\
&+ \lambda_{3,t}((1-\delta)K_{t-1} + e^{\omega_t} G(I_{x,t-1}, I_{x,t-1}^*) - K_t) \\
&+ \lambda_{4,t}((1-\delta)K_{t-1}^* + e^{\omega_t^*} G^*(I_{y,t-1}, I_{y,t-1}^*) - K_t^*) \\
&+ ...
\end{aligned}$$

The optimality condition for the allocation of good X_t for $t = 1, 2, ...$ in each possible state is

$$\mu_0 \cdot \left(\prod_{j=1}^{t} W_{2,j}\right) \cdot W_{1,t} \widetilde{C}_{C,t} C_{x,t} = \lambda_{1,t} = C_{x^*,t}^* \widetilde{C}_{C^*,t}^* W_{1,t}^* \cdot \left(\prod_{j=1}^{t} W_{2,j}^*\right) \cdot \mu_0^*, \tag{C28}$$

where $\mu_0^* = (1-\mu_0)$, $\widetilde{C}_{C,t} = \partial\widetilde{C_t}/\partial C_t$, $C_{x,t} = \partial C_t/\partial x_t$, and the analogous partial derivatives for the foreign country are denoted by an asterisk.

Define μ_t as the date t Pareto weight for the home country. Using equation

(C22), we obtain

$$\begin{aligned}\mu_t &= \mu_0 \cdot \left(\prod_{j=1}^{t} W_{2,j}\right) \cdot W_{1,t} C_t \\ &= \mu_{t-1} \cdot W_{2,t}^{i} \cdot \frac{W_{1,t}}{W_{1,t-1}} \cdot \frac{C_t}{C_{t-1}} = \mu_{t-1} \cdot M_t \cdot \frac{C_t}{C_{t-1}}.\end{aligned}$$

It follows that equation (C28) can be rewritten as

$$\mu_t \cdot \widetilde{C}_{C,t} C_{x,t} \frac{1}{C_t} = \frac{1}{C_t^*} C^*_{x^*,t} \widetilde{C}^*_{C^*,t} \cdot \mu_t^*. \tag{C29}$$

Let $S_t \equiv \mu_t / \mu_t^*$, and note that with GHH preferences, $\widetilde{C}_{C,t} = 1$; that is, equation (C23) holds also for the discount factor in C units. Then the optimality condition in equation (C29) can be represented by the following system of recursive equations:

$$\begin{aligned}S_t \cdot C_{x,t} \cdot \frac{1}{C_t} &= C^*_{x^*,t} \cdot \frac{1}{C_t^*} \\ S_t &= S_{t-1} \frac{M_t e^{\Delta c_t}}{M_t^* e^{\Delta c_t^*}}.\end{aligned} \tag{C30}$$

In a similar fashion, the optimal allocation of good Y is determined by

$$S_t \cdot C_{y,t} \cdot \frac{1}{C_t} = C^*_{y^*,t} \cdot \frac{1}{C_t^*}.$$

Given our GHH preferences, the optimal allocation of labor implies the following standard intratemporal conditions:

$$\begin{aligned}\widetilde{C}_{N,t} &= -F_{N,t} C_{X,t} \\ \widetilde{C}^*_{N^*,t} &= -F^*_{N^*,t} C^*_{Y^*,t},\end{aligned}$$

where $C_{X,t} = \partial C_t / \partial X_t$, $C^*_{Y^*,t} = \partial C_t^* / \partial Y_t^*$, $\widetilde{C}_{N,t} = \partial \widetilde{C}_t / \partial N_t$, and $F_{N,t} = \partial F_t / \partial N_t$.

Let s_{t+1} index the possible states at time $t+1$. The first-order condition with respect to $I_{x,t}$ is

$$\begin{aligned}& -\lambda_{1t} + \sum_{s_{t+1}} (\lambda_{3,t+1} e^{\omega_{t+1}} G_{I_x,t}) = 0 \\ \Leftrightarrow & \sum_{s_{t+1}} \left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{\lambda_{3,t+1}}{\lambda_{1,t+1}} \omega_{t+1}^{h} \right) = \frac{1}{G_{I_x,t}}.\end{aligned}$$

By definition, $IMRS^x_{t+1|t} = \frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\partial U_0/\partial x_{t+1}}{\partial U_0/\partial x_t} = M^x_{t+1}\pi_{t+1|t}$ for $i \in \{h, f\}$, where M^x_{t+1} is the stochastic discount factor in X-units. Substituting the stochastic discount factor into the above equation, we have

$$\text{(C31)} \qquad \frac{1}{G_{I_x,t}} = E_t[M^x_{t+1}P_{k,t+1}e^{\omega_{t+1}}],$$

where $G_{I_x,t} \equiv \frac{\partial G(I_{x,t}, I^*_{x,t})}{\partial I_{x,t}}$, and $P_{k,t+1} \equiv \frac{\lambda_{3,t+1}}{\lambda_{1,t+1}}$ is the cum-dividend price of capital in X-units. The optimal accumulation of K_t has to satisfy

$$\begin{aligned} -\lambda_{3,t} + \lambda_{1,t}F_{k,t} + \sum_{s_{t+1}}((1-\delta)\lambda_{3,t+1}) = 0 \\ \Leftrightarrow E_t[M^x_{t+1}(1-\delta)P_{k,t+1}] + F_{k,t} = P_{k,t}, \end{aligned}$$

where $F_{k,t} \equiv \frac{\partial F}{\partial k_t}$. Define $Q_{k,t} \equiv E_t[M^x_{t+1}P_{k,t+1}]$ as the ex-dividend price of capital. Then we have

$$\begin{aligned} P_{k,t} &= F_{k,t} + (1-\delta)Q_{k,t} \\ Q_{k,t} &= E_t[M^x_{t+1}P_{k,t+1}] \end{aligned}$$

and

$$R_{k,t+1} = \frac{P_{k,t+1}}{Q_{k,t}}.$$

The first-order condition with respect to $I_{y,t}$ states the following:

$$\begin{aligned} -\lambda_{1,t} + \sum_{s_{t+1}}\left(\lambda_{4,t+1}e^{\omega^*_{t+1}}G^*_{I_y,t}\right) = 0 \\ \Leftrightarrow \sum_{s_{t+1}}\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\frac{\lambda_{4,t+1}}{\lambda_{2,t+1}}\frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}e^{\omega^*_{t+1}}\right) = \frac{1}{G^*_{I_y,t}}, \end{aligned}$$

where $G^*_{I_y,t} \equiv \frac{\partial G^*_t}{\partial I_{y,t}}$. Similarly to what done for the home country, define $P^*_{k,t+1} \equiv \frac{\lambda_{4,t+1}}{\lambda_{2,t+1}}$ as the cum-dividend price of capital in Y-units and note that $P_{t+1} = \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}$ measures the terms of trade. It is then possible to obtain that

$$\text{(C32)} \qquad \frac{1}{G^*_{I_y,t}} = E_t[M^x_{t+1}P^*_{k,t+1}P_{t+1}e^{\omega^*_{t+1}}].$$

Define $M^y_{t+1} \equiv \frac{\lambda_{2,t+1}}{\lambda_{2,t}}$ as the SDF in Y-units. The remaining first-order conditions imply

$$\begin{aligned}\frac{1}{G^*_{I^*_y,t}} =& E_t[M^y_{t+1}P^*_{k,t+1}e^{\omega^*_{t+1}}] \\ P^*_{k,t} =& F^*_{k,t} + (1-\delta)Q^*_{k,t} \\ R^*_{k,t+1} =& \frac{P^*_{k,t+1}}{Q^*_{k,t}} \\ Q^*_{k,t} =& E_t[M^y_{t+1}P^*_{k,t+1}] \\ \frac{1}{G_{I^*_x,t}} =& E_t\left[M^y_{t+1}P_{k,t+1}\frac{1}{P_t}e^{\omega_{t+1}}\right].\end{aligned} \tag{C33}$$

We use perturbation methods to solve our system of equations. We compute our policy functions using the dynare++4.2.1 package. All variables included in our dynare++ code are expressed in log units.

3: Aggregation with Vintage Capital

In what follows, we confirm the aggregation results proved in Ai, Croce and Li (2013).

LEMMA 2: *Suppose there are m types of firms. For $i = 1, 2, 3, \cdots m$, the productivity of the type i firm is denoted by $A(i)$, and the total measure of the type i firm is denoted by $F(i)$. The production technology of the type i firm is given by*

$$y(i) = [A(i)\, n(i)]^{1-\alpha},$$

where $n(i)$ denotes the labor hired at firm i. The total labor supply in the economy is N. Then total output is given by

$$Y = \left[\sum_{i=1}^{m} F(i)\left[\frac{A(i)}{A(1)}\right]^{\frac{1-\alpha}{\alpha}}\right]^{\alpha}[A(1)\, N]^{1-\alpha}.$$

PROOF:

Without loss of generality, we assume that firms of the same type employ the same amount of labor. In this case, the total output in the economy is given by

$$\begin{aligned} Y = \max \sum_{i=1}^{m} F(i)\, A(i)^{1-\alpha}\, n(i)^{1-\alpha} \\ \textit{subject to } \sum_{i=1}^{m} F(i)\, n(i) = N. \end{aligned} \tag{C34}$$

The first-order condition of the above optimization problem implies that for all i,

$$\frac{n(i)}{n(1)} = \left(\frac{A(i)}{A(1)}\right)^{\frac{1-\alpha}{\alpha}}.$$

Using the resource constraint,

$$\sum_{i=1}^{m} F(i) \left(\frac{A(i)}{A(1)}\right)^{\frac{1-\alpha}{\alpha}} n(1) = N,$$

we determine the labor employed in firm 1*:*

$$n(1) = \left[\sum_{i=1}^{m} F(i) \left[\frac{A(i)}{A(1)}\right]^{\frac{1-\alpha}{\alpha}}\right]^{-1} N. \tag{C35}$$

Using equations (C34)–(C35), we have:

$$\begin{aligned} Y &= \sum_{i=1}^{m} F(i) A(i)^{1-\alpha} \left[\left(\frac{A(i)}{A(1)}\right)^{\frac{1-\alpha}{\alpha}} n(1)\right]^{1-\alpha} \\ &= [A(1) n(1)]^{1-\alpha} \left[\sum_{i=1}^{m} F(i) \left(\frac{A(i)}{A(1)}\right)^{\frac{1-\alpha}{\alpha}}\right] \\ &= [A(1) N]^{1-\alpha} \left[\sum_{i=1}^{m} F(i) A(i)^{\frac{1-\alpha}{\alpha}}\right]^{\alpha}, \end{aligned}$$

as needed.

At time t, there are $t+1$ types of operating production units in the economy, namely, production units of generation $-1, 0, 1, \cdots, t-1$. The measures of these production units are $(1-\delta_K)^t K_0$, $(1-\delta_K)^{t-1} G_0$, $(1-\delta)^{t-2} G_1, \cdots, G_{t-1}$. Using the above lemma, at date t, the total production in the economy is given by

$$Y_t = A_t \left[(1-\delta_K)^t K_0 + \sum_{\tau=0}^{t-1} (1-\delta_K)^{t-\tau-1} G_\tau \left(\frac{A_t^\tau}{A_t}\right)^{\frac{\alpha-1}{\alpha}}\right]^{\alpha} N_t^{1-\alpha}.$$

Clearly, if we define the $\{K_t\}_{t=0}^{\infty}$ process recursively according to equation (23), the aggregate production function is Cobb-Douglas, as in section III.

4: Sensitivity Analysis

In this section we assess the sensitivity of our results with respect to the key elements of our study, i.e., (i) the preference parameters related to the recursive risk-sharing motive, and (ii) the degree of home bias. Starting from the EZ-BKK

Table C1—Model Sensitivity Analysis

Panel A: the role of the IES (ψ)					
Moments	Data		EZ-BKK	IES=1	IES=1.5
$E[r_f]$ (%)	1.32 (0.64)		2.31	2.96	1.03
corr($\Delta c, \Delta c^*$)	0.37 (0.11)		0.25	0.24	0.47
β_{LR}^{NX}	1.07 (0.45)		1.24	1.29	1.21
Panel B: the role of the RRA (γ)					
Moments	Data		EZ-BKK	RRA=5	RRA=15
StD(Δi)/Std(Δx^T)	2.36 (0.26)		2.24	2.24	2.41
$E[r_f]$ (%)	1.32 (0.64)		2.31	3.60	1.24
StD(Δe)	5.93 (0.77)		4.65	3.88	5.14
corr($\Delta c, \Delta c^*$)	0.37 (0.11)		0.25	0.16	0.32
β_{LR}^{NX}	1.07 (0.45)		1.24	1.10	1.33
Panel C: the role of home bias ($\lambda = \lambda_I$)					
Moments	Data	EZ-BKK	$\lambda = .85$	$\lambda = .95$	$\lambda = .95$ (RRA=3)
corr($\Delta \frac{NX}{X^T}, \Delta x^T$)	-0.36 (0.13)	-0.16	-0.53	0.27	-0.32
corr($\Delta i, \Delta i^*$)	0.33 (0.17)	0.39	0.07	0.61	0.34
corr($\Delta c, \Delta c^*$)	0.37 (0.11)	0.25	0.19	0.27	0.11
Quant. Anomaly	0.15 (0.09)	-0.14	-0.06	-0.17	-0.05
StD(Δe)	5.93 (0.77)	4.65	2.52	7.41	4.44
β_{SR}^{NX}	-0.10 (0.02)	-0.01	-0.08	0.02	-0.01
β_{LR}^{NX}	1.07 (0.45)	1.24	1.45	1.14	0.76

Note: Empirical moments are computed using annual data from 1973 to 2006. All data sources are discussed in section I. Numbers in parentheses are Newey-West adjusted standard errors. All the parameters are calibrated as in table 2, unless otherwise specified. The entries for the models are obtained by repetitions of small-sample simulations.

model calibrated as in table 2, we vary one parameter of interest at a time and report the moments that change significantly in table C1. Our results refer to the case in which $\phi_0 = 1$. We have conducted the same sensitivity analysis for our EZ-BKK model with vintage capital and found virtually identical results.

The role of the IES. As we increase the IES from 1 to 1.5, the average risk-free rate declines, as is common in any economy with EZ preferences. Most importantly, the contemporaneous correlation of the growth rates of consumption increases toward the upperbound of our confidence interval, whereas the sensitivity coefficients of NX to long-run shocks declines. These results implicitly impose a relevant upper bound on what the IES should be in order to match international trade data.

The role of the RRA. Similarly to the IES case, an increase in the risk aversion coefficient decreases the average risk-free rate (precautionary motive) and increases the international correlation of consumption growth. Since the reallocation effect depends on $\gamma - 1/\psi$, higher risk aversion tends to increase the exposure of the net exports to long-run shocks. Additionally, it enhances the incentives to trade investment goods, and hence it amplifies the volatility of both national investment growth and the exchange rate.

The role of home bias. As discussed in section II.A, consumption home bias is an important driver of the reallocation motives with respect to news shocks. In the original BKK calibration, the home bias is set to 0.85 to match total US trade. Focusing on the extent of trade of the US with the remaining G7 countries, a higher value of 0.95 is more appropriate. We consider both of these values in the last panel of table C1.

Reducing the extent of home bias makes domestic and foreign resources more substitutable. As a result, the productivity channel is more pronounced with respect to short-run shocks and the net exports become more countercyclical. Unfortunately, this channel makes the international trade of investment goods excessive and investment becomes less correlated across countries.

If we increase the consumption home bias, the risk-sharing channel becomes stronger and it dominates even with respect to short-run shocks. The net exports become positively correlated with output and their exposure to short-run shocks becomes positive as well, in contrast to the data. We note that this problem can be easily solved by simultaneously lowering the risk aversion parameter to a value as low as three. After considering this refinement, a more pronounced value of the consumption home bias enhances most of our quantitative results, including the quantity anomaly.

Heterogeneous home bias. Recent studies have documented that home bias is more pronounced for investment goods than consumption goods (see, among others, Boileau (1999), Erceg, Guerrieri and Gust (2008) and Engel and Wang (2011)). In order to capture this feature, we also consider the case $\lambda_I < \lambda$. As discussed in appendix C.3, our main results are robust to, and often enhanced by, this further extension.

Additional sensitivity analysis. In table C2, we report a comprehensive list of moments produced by our EZ-BKK model; our model augmented with vintage capital friction (EZ-BKK (II)); and our model augmented with both vintage capital and heterogeneous home bias (EZ-BKK (III)). We set the extent of home bias for investment goods, λ_I, to 0.85, a value consistent with both US data and prior literature. For comparability with the EZ-BKK setting, we retain a total imports share of 8% by setting $\lambda = 0.95$. Our main results continue to hold and are often enhanced.

D: Indirect inference details

In this section, we describe the details of our indirect inference estimation procedure, along with a robustness exercise.

1: Econometric Methodology

Moment conditions of the Auxiliary Model. Let $m_T(Y_T, \phi)$ be a vector consisting of moment conditions (1)-(16) defined in appendix B.1, where $\beta_a^{US} = \beta_a^{RoW} = \beta_a$, and $\varrho^{US} = \varrho^{RoW} = \varrho$.

Estimation. We adopt the following procedure to estimate the vector of model's parameters θ.

Table C2—Additional Sensitivity Analysis

Panel A: Domestic Moments	Vol. Relative to GDP			Asset Prices		Correlation($\Delta\cdot, \Delta\cdot$)				ACF(1)
	Δn	Δc	Δi	E$[r_f]$(%)	E$[r^{ex}]$(%)	(c,i)	(c,n)	$(\frac{NX}{X^T}, x^T)$	$(\frac{NXI}{X^T}, x^T)$	$\frac{NX}{X^T}$
Data:	0.74	0.65	2.36	1.32	4.58	0.69	0.82	-0.36	-0.52	0.88
	(0.08)	(0.06)	(0.26)	(0.64)	(2.15)	(0.07)	(0.03)	(0.13)	(0.13)	(0.08)
EZ-BKK	0.49	0.66	2.24	2.31	0.11	0.93	0.89	-0.16	-0.56	0.85
EZ-BKK (II)	0.49	0.61	2.53	1.99	3.26	0.83	0.84	-0.24	-0.58	0.83
EZ-BKK (III)	0.49	0.63	2.57	1.98	3.30	0.83	0.85	-0.35	-0.52	0.78

Panel B: International Moments	$\rho_h = \text{corr}(\Delta h, \Delta h^*)$				StD($\cdot$)(%)		Sensitivity to News			
	ρ_c	$\rho_{x^T} - \rho_c$	ρ_i	ρ_n	Δe	NX/X	β_{SR}^{NX}	β_{LR}^{NX}	R_{SR}^2/R^2	R_{LR}^2/R^2
Data:	0.37	0.15	0.33	0.53	5.93	0.56				
	(0.11)	(0.09)	(0.17)	(0.11)	(0.77)	(0.07)				
EZ-BKK	0.25	-0.14	0.39	0.25	4.65	0.52	-0.01	1.24	0.06	0.94
EZ-BKK (II)	0.29	-0.20	0.27	0.21	3.99	0.50	-0.01	1.27	0.12	0.88
EZ-BKK (III)	0.19	-0.12	0.30	0.16	5.51	0.68	-0.03	1.80	0.28	0.72

Note: Empirical moments are computed using annual data from 1973 to 2006. All data sources are discussed in section I and appendix A. Numbers in parentheses are Newey-West adjusted standard errors. Excess returns are levered as in GarcFeijJorgensen (2010). For the EZ-BKK model, all the parameters are calibrated as in table 2 and capital vintages are homogeneous ($\phi_0 = 1$). In BKK-EZ(II), we introduce vintage capital ($\phi_0 = 0$). BKK-EZ(III) features both vintage capital and heterogeneous home bias, meaning that it is solved imposing $\lambda = 0.95$ and $\lambda_I = 0.85$, so that the total imports share remains 92%. The entries for the models are obtained by repetitions of small-sample simulations. Lowercase letters denote log units.

1) Estimation using the actual data. Using the observations in the sample Y_T of actual data, we obtain an estimate of the vector ϕ as

$$\hat{\phi}_T = \arg\min_{\phi} Q_T(Y_T, \phi),$$

where $Q_T(Y_T, \phi) = \left[m_T\left(Y_T, \phi\right)' m_T\left(Y_T, \phi\right)\right]$.

2) Simulations from the model. For a given value of the model's parameters θ, consider H simulated paths $Y^h(\theta)$, $h = \{1, ..., H\}$ based on independent drawings of ε_t.

3) Estimation using simulated data. For each simulated path, obtain an estimate of the vector of auxiliary parameters, as

$$\tilde{\phi}^h(\theta) = \arg\min_{\phi} Q(Y^h(\theta), \phi).$$

4) Estimation of the model's parameters. Obtain an indirect estimator of θ as the solution of a minimum distance problem:

$$\min_{\theta} \left[\hat{\phi}_T - \frac{1}{H}\sum_{h=1}^{H} \tilde{\phi}^h(\theta)\right]' W \left[\hat{\phi}_T - \frac{1}{H}\sum_{h=1}^{H} \tilde{\phi}^h(\theta)\right],$$

where W is a positive definite weighting matrix, which we set equal to the identity matrix, $W = I$, in all of our estimations. We denote the estimated vector as $\hat{\theta}_T^H(W)$.

Distribution of the estimated structural parameters. The estimator of the vector of structural parameters converges in distribution to

$$\sqrt{T}\left(\hat{\theta}_T^H(\Omega^*) - \theta_0\right) \xrightarrow{d} N\left[0, \Omega(H, W)\right],$$

where the covariance matrix is:

$$\Omega(H,W) = \left(1 + \frac{1}{H}\right)\left(K'J^{-1}WJ^{-1}K\right)^{-1}\left(K'J^{-1}WJ^{-1}\right)M\left(J^{-1}WJ^{-1}K\right)\left(K'J^{-1}WJ^{-1}K\right)^{-1},$$

where

$$K = \frac{\partial^2 Q}{\partial\phi\,\partial\theta'}, \quad J = -\frac{\partial^2 Q}{\partial\phi\,\partial\phi'}, \quad M = \lim_{T\to\infty} Var\left[\sqrt{T}\frac{\partial Q}{\partial\phi} - E\left(\sqrt{T}\frac{\partial Q}{\partial\phi}\right)\right].$$

Gourieroux, Monfort and Renault (1993) propose to estimate the matrix K as

$$\frac{\partial^2 Q}{\partial\phi\,\partial\theta'}\left(y^s\left(\hat{\theta}\right), \hat{\phi}\right).$$

This amounts to taking the numerical derivative of

$$\frac{\partial Q}{\partial\phi}\left(y^s\left(\hat{\theta}\right), \hat{\phi}\right)$$

with respect to the vector of structural parameters θ evaluated at $\hat{\theta}$, where $y^s\left(\hat{\theta}\right)$ is a simulated path of y based on the parameter θ (see page S113 of Gourieroux, Monfort and Renault (1993)). We average the matrices associated to each simulated path s to obtain our estimator of K.

The matrix J can be obtained by taking the negative of the second derivative of the auxiliary model objective function and evaluate it at the observed sample and the associated estimated coefficient, i.e.,

$$\frac{\partial^2 Q}{\partial\phi\,\partial\phi'}\left(Y_T, \hat{\phi}\right).$$

Finally, using the methodology outlined by Gourieroux, Monfort and Renault (1993) (page S112) we consistently estimate the matrix M as

$$\frac{T}{H}\sum_{h=1}^{H}\left(S_h - \bar{S}\right)\left(S_h - \bar{S}\right)$$

Table D1—Indirect Inference Estimates using Tobin's Q

Panel A: GDP weights

Structural	ρ	σ_z	ρ_{srr}	ρ_{lrr}	τ
Estimates	0.98	0.08	0.27	0.97	0.02
(S.E.)	(0.00)	(0.02)	(0.25)	(0.01)	(0.02)

Auxiliary	$\beta_{NX,a}$	$\beta_{NX,z}$	$\beta_{I,a}$	$\beta_{I,z}$	$\beta_{c,a}$	$\beta_{c,z}$	β	ϱ	ς_z	ϱ_a	ϱ_z
Model	−0.04	0.87	2.04	−21.95	0.44	−0.31	0.35	0.97	0.07	0.22	0.72
Data	−0.08	0.74	5.16	−22.05	0.91	−0.66	0.01	0.91	0.12	0.12	0.77

Panel B: Market cap weights

Structural	ρ	σ_z	ρ_{srr}	ρ_{lrr}	τ
Estimates	0.98	0.08	0.28	0.97	0.02
(S.E.)	(0.00)	(0.02)	(0.17)	(0.01)	(0.02)

Auxiliary	$\beta_{NX,a}$	$\beta_{NX,z}$	$\beta_{I,a}$	$\beta_{I,z}$	$\beta_{c,a}$	$\beta_{c,z}$	β	ϱ	ς_z	ϱ_a	ϱ_z
Model	−0.04	0.90	2.05	−21.35	0.43	−0.29	0.33	0.97	0.06	0.23	0.65
Data	−0.08	0.74	5.13	−21.31	0.81	−0.22	0.01	0.93	0.12	0.62	0.79

Panel C: Equal weights

Structural	ρ	σ_z	ρ_{srr}	ρ_{lrr}	τ
Estimates	0.98	0.08	0.28	0.97	0.02
(S.E.)	(0.01)	(0.05)	(0.32)	(0.06)	(0.07)

Auxiliary	$\beta_{NX,a}$	$\beta_{NX,z}$	$\beta_{I,a}$	$\beta_{I,z}$	$\beta_{c,a}$	$\beta_{c,z}$	β	ϱ	ς_z	ϱ_a	ϱ_z
Model	−0.04	0.93	2.06	−22.16	0.43	−0.26	0.32	0.97	0.06	0.23	0.06
Data	−0.08	0.77	4.68	−22.17	0.81	−0.43	0.01	0.89	0.12	0.56	0.82

Note: Each panel refers to one of the three alternative weighting schemes that we have adopted for the Rest of the World aggregate. In each panel, the top sub-panel (Structural) reports the estimates for each element of the vector of structural parameters (θ) with the associated standard errors in parenthesis. The bottom sub-panel (Auxiliary) reports the coefficients of the auxiliary model. The row labeled "Model" shows the estimates of the auxiliary model associated to the point estimates of the structural parameters reported in the top panel. The row labeled "Data" reports the estimates of the auxiliary model obtained from actual data.

with

$$S_h = \frac{\partial Q}{\partial \theta}\left(y_h\left(\phi_{id}\right)\right), \quad \bar{S} = \frac{1}{H}\sum_h S_h,$$

where $y_h\left(\phi_{id}\right)$ is a simulation from the model based on the estimate ϕ_{id} obtained from using $W = I$.

2: Additional results

Alternative weighting schemes. In table D1, we report the estimates for our baseline specification in which Tobin's Q is used to forecast the growth rate of productivity, using alternative weighting schemes for the Rest of the World aggregate. Furthermore, we also present the complete set of parameters of the auxiliary model. The results, presented in table D1, document the strong robustness of our main findings.

Using price-dividend ratios. In this section we perform our indirect inference estimation by replacing Tobin's Q with price-dividend ratio in the predictive regression for productivity in the auxiliary model. Specifically, we define the process for a redundant dividend in the spirit of Bansal and Yaron (2004) as

$$\begin{aligned}\Delta d_{t+1} &= \mu + \lambda \cdot z_t + \tau\,(a_t - a_t^*) + \sigma_d \cdot \varepsilon_{d,t+1},\\ \Delta d_{t+1}^* &= \mu + \lambda \cdot z_t^* + \tau\,(a_t - a_t^*) + \sigma_d \cdot \varepsilon_{d,t+1}^*,\end{aligned}$$

where ϵ_d and ϵ_d^* are *i.i.d.* shocks uncorrelated with the other shocks in the economy. We set λ to 13 and σ_d to 0.4, respectively. At the estimated values of the structural parameters reported in Panel A of table D2, this choice of parameters yields the following unconditional moments for the distribution our cash-flows: (i) an unconditional volatility relative to productivity of 14.61, (ii) an autocorrelation of 0.44, (iii) a within country correlation with consumption of 0.43, and (iv) a cross-country correlation of 0.46. These numbers are within the range of what we typically find in the data (see for example Bansal and Lundblad (2002)). We then obtain the price-dividend ratios associated to these cash flows, by using the equilibrium stochastic discount factor to solve the corresponding Euler equations:

$$PD_t = E_t[M_{t+1}^X(1 + PD_{t+1})e^{\Delta d_{t+1}}], \quad PD_t^* = E_t[M_{t+1}^Y(1 + PD_{t+1}^*)e^{\Delta d_{t+1}^*}].$$

When estimating the auxiliary model on simulated data, we replace Tobin's Q with these price-to-dividend ratios. We then apply the same empirical strategy described in both main text ad this section of the appendix. The results are reported in table D2 and they are consistent with those obtained under our benchmark estimation exercise.

Table D2—Indirect Inference Estimates using p/d ratios

Panel A: GDP weights

Structural	ρ	σ_z	ρ_{srr}	ρ_{lrr}	τ
Estimates	0.99	0.26	0.24	0.99	0.03
(S.E.)	(0.00)	(0.05)	(0.31)	(0.00)	(0.15)

Auxiliary	$\beta_{NX,a}$	$\beta_{NX,z}$	$\beta_{I,a}$	$\beta_{I,z}$	$\beta_{c,a}$	$\beta_{c,z}$	β	ϱ	ς_z	ϱ_a	ϱ_z
Model	−0.18	1.18	5.06	−22.06	0.20	−1.05	0.00	0.98	0.28	0.25	0.99
Data	−0.08	0.74	5.16	−22.05	0.91	−0.66	0.01	0.91	0.12	0.12	0.77

Panel B: Market cap weights

Structural	ρ	σ_z	ρ_{srr}	ρ_{lrr}	τ
Estimates	0.99	0.26	0.30	0.99	0.03
(S.E.)	(0.01)	(0.11)	(0.25)	(0.04)	(0.02)

Auxiliary	$\beta_{NX,a}$	$\beta_{NX,z}$	$\beta_{I,a}$	$\beta_{I,z}$	$\beta_{c,a}$	$\beta_{c,z}$	β	ϱ	ς_z	ϱ_a	ϱ_z
Model	−0.18	1.14	5.08	−21.35	0.21	−1.04	0.00	0.98	0.28	0.31	0.99
Data	−0.08	0.74	5.13	−21.31	0.81	−0.22	0.01	0.93	0.12	0.62	0.79

Panel C: Equal weights

Structural	ρ	σ_z	ρ_{srr}	ρ_{lrr}	τ
Estimates	0.99	0.26	0.34	0.99	0.03
(S.E.)	(0.01)	(0.11)	(0.69)	(0.02)	(0.14)

Auxiliary	$\beta_{NX,a}$	$\beta_{NX,z}$	$\beta_{I,a}$	$\beta_{I,z}$	$\beta_{c,a}$	$\beta_{c,z}$	β	ϱ	ς_z	ϱ_a	ϱ_z
Model	−0.16	1.19	4.81	−22.32	0.22	−1.16	0.00	0.98	0.28	0.34	0.99
Data	−0.08	0.77	4.68	−22.17	0.81	−0.43	0.01	0.89	0.12	0.56	0.82

Note: Each panel refers to one of the three alternative weighting schemes that we have adopted for the Rest of the World aggregate. In each panel, the top sub-panel (Structural) reports the estimates for each element of the vector of structural parameters (θ) with the associated standard errors in parenthesis. The bottom sub-panel (Auxiliary) reports the coefficients of the auxiliary model. The row labeled "Model" shows the estimates of the auxiliary model associated to the point estimates of the structural parameters reported in the top panel. The row labeled "Data" reports the estimates of the auxiliary model obtained from actual data.