Online Appendix for "A Model of Endogenous Loan Quality and the Collapse of the Shadow Banking System"

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In this appendix I derive the optimal contracts for three different scenarios regarding the observability of projects characteristics. A frictionless scenario in which both the ex-post outcome and the ex-ante quality of projects are observable. Then I derive the optimal contract for traditional banks, when the outcome of the project is not observable. Finally I solve the optimal contract for the shadow bank, when the outcome of the project is observable but the ex-ante quality is not.

1 Optimal Contract in the First Best Scenario

As explained in the main text, the optimal contract solves

$$\max_{k_t^{fb},\pi_t^{fb},b_{t+1}^{G,fb},b_{t+1}^{B,fb}} Q_t k_t^{fb}\left\{E_t\Lambda_{t,t+1}\left[\pi_t^{fb}\left(\bar{\theta}^G R_{t+1}^k - b_{t+1}^{G,fb}\right)+(1-\pi_t^{fb})\left(\bar{\theta}^b R_{t+1}^k - b_{t+1}^{B,fb}\right)\right]-c\left(\pi_t^{fb}\right)\right\}$$

$$Q_t k_t^{fb} - n_t^{fb} \leq E_t\Lambda_{t,t+1}\left[\pi_t^{fb} b_{t+1}^{G,fb} + (1-\pi_t^{fb}) b_{t+1}^{G,fb}\right] Q_t k_t^{fb} \quad (\mu_t)$$

$$b_{t+1}^{G,fb} \leq \bar{\theta}^G R_{t+1}^k \quad (\chi_{t+1}^g)$$

$$b_{t+1}^{B,fb} \leq \bar{\theta}^B R_{t+1}^k \quad (\chi_{t+1}^b)$$

The FOCs are

$$\pi_t^{fb}:\ c'\left(\pi_t^{fb}\right) = E_t\Lambda_{t,t+1}\left[\bar{\Delta}R_{t+1}^k - \left(b_{t+1}^{G,fb} - b_{t+1}^{G,fb}\right)\right] + \mu_t E_t\Lambda_{t,t+1}\left(b_{t+1}^g - b_{t+1}^b\right) \tag{1}$$

$$\begin{aligned} k_t^{fb} \quad : \quad & E_t\Lambda_{t,t+1}\left\{\pi_t^{fb}\left(\bar{\theta}^g R_{t+1}^k - b_{t+1}^{G,fb}\right) + (1-\pi_t^{fb})\left(\bar{\theta}^b R_{t+1}^k - b_{t+1}^{G,fb}\right)\right\} - c\left(\pi_t^{fb}\right) = \\ & \mu_t\left\{1 - E_t\Lambda_{t,t+1}\left[\pi_t^{fb} b_{t+1}^{G,fb} + (1-\pi_t^{fb}) b_{t+1}^{G,fb}\right]\right\} \end{aligned} \tag{2}$$

$$b_{t+1}^{G,fb}:\ \Lambda_{t,t+1}\pi_t^{fb}\left(\mu_t - 1\right) = \chi_{t+1}^g \tag{3}$$

$$b_{t+1}^{B,fb}:\ \Lambda_{t,t+1}(1-\pi_t^{fb})\left(\mu_t - 1\right) = \chi_{t+1}^b \tag{4}$$

From the last two equations we see that either χ_{t+1}^G and χ_{t+1}^B are both positive or they are both zero. However if they were both positive the bank would not be obtaining any payoff from funding the projects. As a result $\chi_{t+1}^g = 0$ and $\chi_{t+1}^b = 0$, so that the main equations become

$$c'\left(\pi_t^{fb}\right) = E_t\Lambda_{t,t+1}\bar{\Delta}R_{t+1}^k \tag{5}$$

$$E_t\Lambda_{t,t+1}\left\{\pi_t^{fb}\bar{\theta}^g R_{t+1}^k + (1-\pi_t^{fb})\bar{\theta}^b R_{t+1}^k\right\} - c\left(\pi_t^{fb}\right) = 1 \tag{6}$$

$$\mu_t = 1 \tag{7}$$

where I used $E_t\Lambda_{t,t+1}R_{t+1} = 1$. The first two equations are the same as in the main text, whereas the last equation simply states that in the first best the marginal value of a unit of net worth is equal to one.

2 Optimal Contract for the Traditional Bank

As explained in the paper, the one period contract for the traditional bank will be given by the solution of the following

$$\max_{k_t^{tb},\pi_t^{tb},b_{t+1}^{tb}} Q_t k_t^{tb} \left\{ E_t \Lambda_{t,t+1} \left[\pi_t^{tb} \bar{\theta}^G R_{t+1}^k + (1-\pi_t^{tb})\bar{\theta}^B R_{t+1}^k - b_{t+1}^{tb} \right] - c\left(\pi_t^{tb}\right) \right\}$$

$$b_{t+1}^{tb} \leq \theta_L R_{t+1}^k \text{ (IC) } (\omega_{t+1})$$

$$\left(Q_t k_t^{tb} - n_t^{tb}\right) \leq E_t \Lambda_{t,t+1} b_{t+1}^{tb} Q_t k_t^{tb} \text{ (PC) } \left(\lambda_t^{tb}\right)$$

The first order conditions with respect to $k_t^{tb}, b_{t+1}^{tb}, \pi_t^{tb}$ are

$$E_t \Lambda_{t,t+1} \left[\Theta_{t+1}(\pi_t^{tb}) R_{t+1}^k - b_{t+1}^{tb} \right] - c(\pi_t^{tb}) - \lambda_t^{tb} \left[1 - E_t \Lambda_{t+1} b_{t+1}^{tb} \right] = 0 \tag{8}$$

$$Q_t k_t^{tb} \; \Lambda_{t+1} \left[\lambda_t^{tb} - 1 \right] = \omega_{t+1} \tag{9}$$

$$c'(\pi_t^{tb}) = E_t \Lambda_{t+1} \bar{\Delta}_{t+1} R_{t+1}^k \tag{10}$$

where ω_{t+1} and λ_t^{tb} are the Lagrange multipliers on the incentive constraint and the participation constraint, and $\Theta_{t+1}(\pi_t^{tb}) = \pi_t^{tb}\bar{\theta}^G + (1-\pi_t^{tb})\bar{\theta}^B$.

The last equation directly determines the screening level for traditional banks, as reported above. In addition, from (9), we see that the SC will bind if $\lambda_t^{tb} - 1 > 0$, a condition that we assume to hold in a neighborhood of the steady state.

Then, substituting the incentive constraint into (8) this condition can be rewritten as

$$\lambda_t^{tb} = \frac{E_t \Lambda_{t,t+1} \left[\Theta_{t+1}(\pi_t^{tb}) R_{t+1}^k - \theta_L R_{t+1}^k \right] - c(\pi_t^{tb})}{1 - \theta_L E_t \Lambda_{t+1} R_{t+1}^k} > 1 \tag{11}$$

which implies

$$\frac{E_t \Lambda_{t,t+1} \left[\Theta_{t+1}(\pi_t^{tb}) R_{t+1}^k - R_{t+1} \right] - c(\pi_t^{tb})}{1 - \theta_L E_t \Lambda_{t+1} R_{t+1}^k} > 0 \tag{12}$$

that indicates how the incentive constraint for the traditional bank implies a wedge between the expected return on capital and the risk-free rate.

Finally, we can combine the (IC) the (PC) in order to obtain an expression for the leverage ratio reported in the main text

$$Q_t k_t^{tb} = \frac{1}{\left[1 - \theta_L E_t \Lambda_{t+1} R_{t+1}^k\right]} n_t^{tb} = \phi_t^{tb} n_t^{tb} \tag{13}$$

In addition from the PC we can think of the face value of the debt raised by the TB as being given by

$$R_{t+1}^{tb} = \frac{b_{t+1}^{tb} Q_t k_t^{tb}}{\left(Q_t k_t^{tb} - n_t^{tb}\right)} = \theta^L R_{t+1}^k \frac{\phi_t}{\phi_t - 1} \tag{14}$$

3 Optimal Contract for the Shadow Bank

In this section I report the complet solution to the problem to the optimal contract of the shadow bank. As explained in the paper, the problem to be solved is the following:

$$\max_{k_t^{sb},\pi_t^{sb},b_{t+1}^{g,sb},b_{t+1}^{b,sb}} Q_t k_t \left\{ E_t \Lambda_{t,t+1} \left[\pi_t^{sb} \left(\bar{\theta}^G R_{t+1}^k - b_{t+1}^{G,sb} \right) + (1-\pi_t^{sb}) \left(\bar{\theta}^B R_{t+1}^k - b_{t+1}^{B,sb} \right) \right] - c\left(\pi_t^{sb}\right) \right\}$$

$$R_{t+1}\left(Q_t k_t - n_t\right) \leq \left[\pi_t^{sb} b_{t+1}^{G,sb} + (1-\pi_t^{sb}) b_{t+1}^{B,sb} \right] Q_t k_t \quad \left(\mu_{t+1}\right) \quad \text{(PC)}$$

$$c'\left(\pi_t^{sb}\right) \leq E_t\Lambda_{t,t+1}\left[\bar{\Delta}R_{t+1}^k - \left(b_{t+1}^{G,sb} - b_{t+1}^{B,sb}\right)\right] \quad (\rho_t) \quad \text{(IC)}$$

$$b_{t+1}^{G,sb} \leq \bar{\theta}^G R_{t+1}^k \quad \left(\chi_{t+1}^g\right) \ \text{(LL)}$$

$$b_{t+1}^{B,sb} \leq \bar{\theta}^B R_{t+1}^k \quad \left(\chi_{t+1}^b\right) \ \text{(LL)}$$

where $\mu_{t+1}, \rho_t, \chi_{t+1}^g$ and χ_{t+1}^b are the multipliers associated with each constraint. The implied FOCs are

$$\pi_t^{sb} : c'\left(\pi_t^{sb}\right) = E_t\Lambda_{t,t+1}\left[\bar{\Delta}R_{t+1}^k - \left(b_{t+1}^{G,sb} - b_{t+1}^{B,sb}\right)\right] + E_t\mu_{t+1}\left(b_{t+1}^{G,sb} - b_{t+1}^{B,sb}\right) - \rho_t c''\left(\pi_t^{sb}\right) \tag{15}$$

$$\begin{aligned} k_t^{sb} \quad : \quad & E_t\Lambda_{t,t+1}\left\{\pi_t^{sb}\left(\bar{\theta}^G R_{t+1}^k - b_{t+1}^g\right) + (1-\pi_t^{sb})\left(\bar{\theta}^B R_{t+1}^k - b_{t+1}^b\right)\right\} - c\left(\pi_t^{sb}\right) = \\ & E_t\mu_{t+1}\left\{R_{t+1} - \left[\pi_t^{sb} b_{t+1}^g + (1-\pi_t^{sb}) b_{t+1}^b\right]\right\} \end{aligned} \tag{16}$$

$$b_{t+1}^{G,sb} : \rho_t\Lambda_{t,t+1} = \pi_t^{sb}\left(\mu_{t+1} - \Lambda_{t,t+1}\right) - \chi_{t+1}^g \tag{17}$$

$$b_{t+1}^{B,sb} : \ \rho_t\Lambda_{t,t+1} = \chi_{t+1}^b - (1-\pi_t^{sb})\left(\mu_{t+1} - \Lambda_{t,t+1}\right) \tag{18}$$

$$\mu_{t+1} : R_{t+1}\left(\phi_t^{sb} - 1\right) = \left[\pi_t^{sb} b_{t+1}^{G,sb} + (1-\pi_t^{sb}) b_{t+1}^{B,sb}\right]\phi_t^{sb} \tag{19}$$

$$\rho_t : \ c'\left(\pi_t^{sb}\right) = E_t\Lambda_{t,t+1}\left[\bar{\Delta}R_{t+1}^k - \left(b_{t+1}^{G,sb} - b_{t+1}^{B,sb}\right)\right] \tag{20}$$

First notice that if $\rho_t > 0$ then it can't be that $\chi_{t+1}^g = 0$ and $\chi_{t+1}^b = 0$ otherwise this would imply $\left(\mu_{t+1} - \Lambda_{t,t+1}\right) = 0$ and then $\rho_t = 0$, a contradiction, therefore at least one of the two payment has to be at the maximum. In addition, setting both payments to the maximum would not be optimal since it would imply that the bank does not receive any payoff, so that only one limited liability constraint can be binding.

In particular, by combining the first order conditions for $b_{t+1}^{G,sb}$ and $b_{t+1}^{B,sb}$ it can be seen that the only case compatible with $\rho_t > 0$ is $\chi_{t+1}^g = 0$ and $\chi_{t+1}^b > 0 \implies b_{t+1}^{B,sb} = \bar{\theta}^B R_{t+1}^k$, the intuition being that setting $b_{t+1}^{B,sb}$ to its maximum improves on the incentive constraint on monitoring, and, at the same time, it helps to satisfy the PC.

As a result, the FOCs for $b_{t+1}^{G,sb}$ implies

$$\mu_{t+1} = \Lambda_{t,t+1}\left[\rho_t \frac{1}{\pi_t^{sb}} + 1\right] \tag{21}$$

and if we substitute this relationship in the FOC for k_t^{sb} we obtain

$$\left\{E_t\Lambda_{t,t+1}\left[\Theta_{t+1}\left(\pi_t^{sb}\right)R_{t+1}^k - R_{t+1}\right] - c\left(\pi_t^{sb}\right)\right\} = \rho_t \frac{1}{\pi_t^{sb}} \frac{1}{\phi_t^{sb}} \tag{22}$$

where $\phi_t^{sb} = Q_t k_t^{sb}/n_t^{sb}$. Therefore, when the (IC) binds there will be a positive spread between the expected return on capital and the risk-free rate. In addition, if the (IC) binds then we can rewrite the FOC for π_t^{sb} and the incentive constraint as

$$\rho_t c''\left(\pi_t^{sb}\right) = E_t\mu_{t+1}\left(b_{t+1}^{G,sb} - \bar{\theta}^B R_{t+1}^k\right) \tag{23}$$

$$c'\left(\pi_t^{sb}\right) = E_t\Lambda_{t,t+1}\left[\bar{\theta}^G R_{t+1}^k - b_{t+1}^{G,sb}\right] \tag{24}$$

From the first equation we see that if $\rho_t > 0$ then $E_t\mu_{t+1}\left(b_{t+1}^{G,sb} - \bar{\theta}^b R_{t+1}^k\right) > 0$, and because of (21) this also implies $E_t\Lambda_{t,t+1}\left(b_{t+1}^{G,sb} - \bar{\theta}^B R_{t+1}^k\right) > 0$. As a result, if we rewrite the second equation as

$$c'\left(\pi_t^{sb}\right) = E_t\Lambda_{t,t+1}\bar{\Delta}_{t+1}R_{t+1}^k - E_t\Lambda_{t,t+1}\left(b_{t+1}^{G,sb} - \bar{\theta}^B R_{t+1}^k\right) \tag{25}$$

we obtain the result reported in the main text

$$c'\left(\pi_t^{sb}\right) < E_t\Lambda_{t,t+1}\bar{\Delta}_{t+1}R_{t+1}^k \tag{26}$$

This is an important relationship since it implies that the screening effort of shadow banks is lower than the one of traditional banks, that is $\pi_t^{sb} < \pi_t^{tb}$.

Next, from the (PC) we can obtain the payment to the bank in the good state

$$b_{t+1}^{G,sb} = \frac{1}{\pi_t^{sb}}\left[R_{t+1}\frac{\left(\phi_t^{sb}-1\right)}{\phi_t^{sb}} - (1-\pi_t^{sb})\bar{\theta}^B R_{t+1}^k\right] \tag{27}$$

and by substituting this in the (IC) we obtain the leverage constraint reported in the main text

$$\phi_t^{sb} \leq \frac{E_t\Lambda_{t,t+1}R_{t+1}}{\left\{\pi_t^{sb}c'\left(\pi_t^{sb}\right) - E_t\Lambda_{t,t+1}\left[\Theta_{t+1}\left(\pi_t^{sb}\right)R_{t+1}^k - R_{t+1}\right]\right\}} \tag{28}$$

In addition, substituting (21) and (27) in the FOC for π_t^{sb} one obtains

$$\rho_t c''(\pi_t^{sb}) = \left[\rho_t\frac{1}{\pi_t^{sb}}+1\right]E_t\Lambda_{t,t+1}\left(b_{t+1}^G - \bar{\theta}^B R_{t+1}^k\right)$$

and by using the (IC) at equality

$$\rho_t c''\left(\pi_t^{sb}\right) = \left[\rho_t\frac{1}{\pi_t^{sb}}+1\right]\left[E_t\Lambda_{t,t+1}\bar{\Delta}_{t+1}R_{t+1}^k - c'\left(\pi_t^{sb}\right)\right)\right]$$

Finally if we substitute for ρ_t^{sb} from (22) we obtain the equation determining π_t^{sb}

$$\left[E\Lambda_{t,t+1}\bar{\Delta}R_{t+1}^k - c'\left(\pi_t^{sb}\right)\right]\left\{\pi_t^{sb}c'\left(\pi_t^{sb}\right) - c\left(\pi_t^{sb}\right)\right\} = \tag{29}$$

$$\left\{E_t\Lambda_{t,t+1}\left[\Theta\left(\pi_t^{sb}\right)R_{t+1}^k - R_{t+1}\right] - c\left(\pi_t^{sb}\right)\right\}\left[\pi_t^{sb}c''(\pi_t^{sb})\right] \tag{30}$$

At this point, we can use such equation to study the determinants of π_t^{sb}. Let's define

$$\begin{aligned} g(\pi_t^{sb}, E_t\Lambda_{t,t+1}R_{t+1}^k) &= \left\{E_t\Lambda_{t,t+1}\Theta_{t+1}\left(\pi_t^{sb}\right)\right)R_{t+1}^k - 1 - c\left(\pi_t^{sb}\right)\right\}\left[\pi_t^{sb}c''\left(\pi_t^{sb}\right)\right] \\ &- \left[E\Lambda_{t,t+1}\bar{\Delta}R_{t+1}^k - c'\left(\pi_t^{sb}\right)\right]\left\{\pi_t^{sb}c'\left(\pi_t^{sb}\right) - c\left(\pi_t^{sb}\right)\right\} = 0 \end{aligned}$$

Therefore we can obtain

$$\begin{aligned} \frac{\partial g}{\partial \pi_t^{sb}} &= \left[c''\left(\pi_t^{sb}\right)\right) + \pi_t^{sb}c'''\left(\pi_t^{sb}\right)\right]\left\{E_t\Lambda_{t,t+1}\Theta\left(\pi_t^{sb}\right)R_{t+1}^k - 1 - c\left(\pi_t^{sb}\right)\right\} \\ &\quad + c''\left(\pi_t^{sb}\right)\right)\left\{\pi_t^{sb}c'\left(\pi_t^{sb}\right) - c\left(\pi_t^{sb}\right)\right\} > 0 \end{aligned}$$

In fact, given (22), and the fact that $\pi_t^{sb}c'\left(\pi_t^{sb}\right) - c\left(\pi_t^{sb}\right) \geq 0$, since this quantity is proportional to the objective of the banker when the constraint binds, then, as long as $c'''\left(\pi_t^{sb}\right) \geq 0$ (as it is implied by any quadratic cost function) we have that $\frac{\partial g}{\partial \pi_t^{sb}} > 0$.

In addition,

$$\begin{aligned} \frac{\partial g}{\partial E_t\Lambda_{t,t+1}R_{t+1}^k} &= \pi_t^{sb}c''\left(\pi_t^{sb}\right)E_t\Theta_{t+1}\left(\pi_t^{sb}\right) - E_t\bar{\Delta}_{t+1}\left\{\pi_t^{sb}c'\left(\pi_t^{sb}\right) - c\left(\pi_t^{sb}\right)\right\} \\ &= \pi_t^{sb}c''\left(\pi_t^{sb}\right)E_t\bar{\theta}_{t+1}^B + E_t\bar{\Delta}_{t+1}c\left(\pi_t^{sb}\right) + \pi_t^{sb}E_t\bar{\Delta}_{t+1}\left[\pi_t^{sb}c''\left(\pi_t^{sb}\right) - c'\left(\pi_t^{sb}\right)\right] > 0 \end{aligned}$$

where the term in the square brackets is positive for the class of cost functions that we consider.

At this point, if we employ the implicit function theorem we will have

$$\frac{\partial \pi_t^{sb}}{\partial E_t\Lambda_{t,t+1}\bar{\Delta}R_{t+1}^k} = -\frac{\partial g/\partial E_t\Lambda_{t,t+1}\bar{\Delta}R_{t+1}^k}{\partial g/\partial \pi_t^{sb}} < 0 \tag{31}$$

so that the monitoring intensity of shadow banks will be decreasing in the expected return on capital.

Finally, if we use the cost function $c(\pi_t) = \frac{\tau}{2}\left(\pi_t^2 + \iota\pi_t\right)$ equation (30) implies

$$\pi_t^{sb} = 2\frac{E_t\Lambda_{t,t+1}\left[R_{t+1} - \theta^B R_{t+1}^k\right]}{\left[E\Lambda_{t,t+1}\bar{\Delta}_{t+1}R_{t+1}^k - \frac{\tau}{2}\iota\right]} \tag{32}$$

4 Cross-Sectional Volatilities of Equity Returns

Given the binomial structure of bank payoffs, the standard deviation for traditional banks' return per unit of net worth, $\tilde{\sigma}_t^{tb}$, at time t is simply[1]

$$\tilde{\sigma}_t^{tb} = \left\{\left[(\pi_{t-1}^{tb} + \left(1 - \pi_{t-1}^{tb}\right)p_t^B\right]\left[\left(1 - \pi_{t-1}^{tb}\right)\left(1 - p_t^B\right)\right]\right\}^{1/2}\phi_{t-1}^{tb}\left(\theta_H - \theta_L\right)R_t^k \tag{33}$$

On the other hand, for shadow banks, the same quantity will be given by

$$\tilde{\sigma}_t^{sb} = \left[\pi_{t-1}^{sb}(1 - \pi_{t-1}^{sb})\right]^{1/2}\phi_{t-1}^{sb}\left(\bar{\theta}^G R_t^k - b_t^{G,sb}\right)$$

and by using the formula for $b_t^{G,sb}$ from the main text we obtain

$$\tilde{\sigma}_t^{sb} = \left[\frac{\left(1 - \pi_{t-1}^{sb}\right)}{\pi_{t-1}^{sb}}\right]^{.5}\left[\phi_{t-1}^{sb}\left(\Theta\left(\pi_{t-1}^{sb}\right)R_t^k - R_t\right) + R_t\right] \tag{34}$$

[1] In the formula below I am using the normalization $p^G = 1$.