The Gains from Input Trade with Heterogeneous Importers

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Online Appendix

This Online Appendix contains the following additional results and material:

1) Generalizations of equation (7),
2) Details about the identification of the Input-Output Matrix Ξ,
3) Empirical evidence on the correlates of the firm-level gains from input trade,
4) The change in consumer prices with heterogeneous export participation,
5) A description of the bootstrap procedure,
6) Empirical results on the Elasticity Bias at the sector level,
7) An extension of the welfare equation (32) to a multi-sector environment,
8) Detailed theoretical derivations of the models of Section III.D,
9) The estimation of η,
10) Details about the algorithm used to calibrate the model of Section III.D.

In this section, we consider three generalizations of equation (7), which states that the firm's unit costs is given by

$$u_i = \frac{1}{\tilde{\varphi}_i} \times (s_{Di})^{\frac{\gamma}{\varepsilon-1}} \times \left(\frac{p_D}{q_D}\right)^{\gamma} w^{1-\gamma}. \tag{O27}$$

(O27) was derived under the restrictions: (i) the production function has a constant elasticity of materials γ, (ii) domestic and foreign inputs are combined in a CES fashion with elasticity of substitution ε and (iii) foreign inputs are differentiated at the country, but not at the product level. We now relax these assumptions and derive expressions akin to (O27).

Extension 1: CES Upper Tier.. — Suppose that the production function between materials x and primary factors l is CES instead of Cobb-Douglas, i.e.

$$y = \varphi\left((1-\gamma)\, l^{\frac{\zeta-1}{\zeta}} + \gamma x^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}.$$

The rest of the environment is exactly as in Section I. Let Q denote again the price index of materials x and w denote the price of primary factors l. In this case, the firm's unit cost is given by

$$u = \frac{1}{\varphi}\left(\gamma^{\zeta} Q^{1-\zeta} + (1-\gamma)^{\zeta} w^{1-\zeta}\right)^{\frac{1}{1-\zeta}}.$$

Noting that the optimal expenditure share on materials is given by

$$s_M = \frac{\gamma^{\zeta} Q^{1-\zeta}}{\gamma^{\zeta} Q^{1-\zeta} + (1-\gamma)^{\zeta} w^{1-\zeta}}, \tag{O28}$$

we can write the firm's unit cost as

$$u = \frac{1}{\varphi} s_M^{\frac{1}{\zeta-1}} \left(\frac{1}{\gamma}\right)^{\frac{\zeta}{\zeta-1}} s_D^{\frac{1}{\varepsilon-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right) \propto s_M^{\frac{1}{\zeta-1}} s_D^{\frac{1}{\varepsilon-1}}, \tag{O29}$$

where we have substituted for Q using (4). (O29) shows that measuring the effect of input trade on the unit cost requires knowledge of the counterfactual material share in the autarky equilibrium, s_M^{Aut}.[1] Because this object is not observed in

[1] The Cobb-Douglas assumption in (1) in the main text bypasses this issue because it implies that the material share is constant and given by γ. In the non-Cobb-Douglas case, the material share endogenously reacts to changes in the import environment. A move to autarky, for example, makes materials relatively more expensive and should induce firms to substitute towards primary inputs.

the data, we can use (4) and (O28) to compute it:

$$s_M^{Aut} \quad = \quad \frac{\left(\frac{\gamma}{1-\gamma}\right)^{\zeta}\beta^{-\frac{\varepsilon}{\varepsilon-1}(1-\zeta)}\left(\frac{p_D/q_D}{w}\right)^{1-\zeta}}{1+\left(\frac{\gamma}{1-\gamma}\right)^{\zeta}\beta^{-\frac{\varepsilon}{\varepsilon-1}(1-\zeta)}\left(\frac{p_D/q_D}{w}\right)^{1-\zeta}}.$$

The firm-level gains from input trade are therefore given by

$$\text{(O30)}\quad \ln\left(\frac{u^{Aut}}{u}\right)\Bigg|_{p_D,w} \quad = \quad \ln\left(\frac{1+\left(\frac{\gamma}{1-\gamma}\right)^{\zeta}\beta^{-\frac{\varepsilon}{\varepsilon-1}(1-\zeta)}\left(\frac{p_D/q_D}{w}\right)^{1-\zeta}s_D^{\frac{1-\zeta}{\varepsilon-1}}}{1+\left(\frac{\gamma}{1-\gamma}\right)^{\zeta}\beta^{-\frac{\varepsilon}{\varepsilon-1}(1-\zeta)}\left(\frac{p_D/q_D}{w}\right)^{1-\zeta}}\right)^{\frac{1}{\zeta-1}}.$$

(O30) is the generalization of (7) for the case where the aggregator between materials and primary factors is CES. We see that, in this case, quantifying the change in the unit cost relative to autarky requires knowledge of additional parameters $[\beta, \zeta, p_D/q_D]$ to predict the material share in autarky. Under the *additional* assumption that there is no variation in β and p_D/q_D across firms, we can bypass the estimation of some of these additional parameters. In this case, all firms would feature the same material share in autarky, which is given by the material share of a domestic firm in the observed trade equilibrium, s_M^D. In this case, (O30) reduces to

$$\text{(O31)}\qquad \ln\left(\frac{u^{Aut}}{u}\right)\Bigg|_{p_D,w} = \ln\left(1-s_M^D+s_D^{\frac{1-\zeta}{\varepsilon-1}}\times s_M^D\right)^{\frac{1}{\zeta-1}},$$

so that only micro-data on domestic expenditure shares s_D and the two elasticities of substitution ε and ζ are required.[2]

Extension 2: General Production Function for Materials.. — In Section (I), we assumed that material services were a CES aggregator of a domestic variety z_D and a foreign input bundle x_I. Suppose now that the aggregator for materials is given by a general function

$$\text{(O32)}\qquad x = g\left(q_D z_D, x_I\right).$$

We continue to assume that materials x and primary factors l are combined with a Cobb-Douglas production function given in (1). Again let $A(\mathscr{S})$ be the price index of the import bundle and $Q(\mathscr{S})$ be the price index of materials. Consider any shock to the trading environment that affects $A(\mathscr{S})$. Then

$$\text{(O33)}\qquad d\ln(u)|_{p_D,w} = \gamma\times d\ln(Q)|_{p_D} = \gamma\frac{z_I A}{u}\frac{dA}{A} = \gamma(1-s_D)\,d\ln(A).$$

[2]Note that, when $\zeta\to 1$, (O31) reduces to the expression in (7):

$$\lim_{\zeta\to 1}\ln\left(\frac{u^{Aut}}{u}\right)\Bigg|_{p_D,w} = \frac{\gamma}{1-\varepsilon}\ln(s_D)$$

The optimality conditions from the cost-minimization problem imply that

$$d\ln(A) = -\frac{\left(-\frac{1}{\varepsilon_L}\right)}{1-\frac{1}{\varepsilon_L}}\frac{1}{1-s_D}d\ln(s_D),$$

where

$$-\frac{1}{\varepsilon_L} \equiv \frac{\partial\ln\left(\frac{\partial g(q_D z_D, x_I)/\partial x_D}{\partial g(q_D z_D, x_I)/\partial x_I}\right)}{\partial\ln\left(\frac{q_D z_D}{x_I}\right)}$$

is the local elasticity of substitution. Substituting this into (O33) yields

$$d\ln(u)\big|_{p_D,w} = \gamma\frac{\frac{1}{\varepsilon_L}}{1-\frac{1}{\varepsilon_L}}d\ln(s_D) = -\frac{\gamma}{1-\varepsilon_L}d\ln(s_D). \tag{O34}$$

In case the elasticity of substitution is constant, i.e. $\varepsilon_L = \varepsilon$, (O34) can be integrated to yield (7).

Extension 3: Multiple Foreign Products.. — In the main analysis, we assumed that firms source a single product from each sourcing country. In the data, firms often import multiple products from a given country. We now explore how (O27) would change in a multi-product environment. Consider first the case where the product aggregator is nested in the country aggregator, i.e. the production structure is given by (1)-(3), where

$$q_{ci}z_c \equiv \psi_{ci}\left([q_{kci}z_{kc}]_{k\in K_{ci}}\right), \tag{O35}$$

k is a product index, K_{ci} denotes the set of products that firm i sources from country c, ψ_{ci} is a constant-returns-to-scale production function and (O35) applies also to the domestic variety. As long as the number of products sourced domestically does not change when firms are forced into input-autarky, the analysis in the main text remains entirely unchanged and the firm-level gains are still given by (7).

Consider next the case where the country aggregator is nested in the product aggregator. Suppose for example that the production structure for intermediates x is given by

$$\begin{aligned} x &= \left(\sum_{k=1}^{K}(\eta_k x_k)^{\frac{\iota-1}{\iota}}\right)^{\frac{\iota}{\iota-1}} \\ x_k &= \left(\beta_{ki}(q_{kD}z_{kD})^{\frac{\varepsilon_k-1}{\varepsilon_k}} + (1-\beta_{ki})\,x_{kI}^{\frac{\varepsilon_k-1}{\varepsilon_k}}\right)^{\frac{\varepsilon_k}{\varepsilon_k-1}} \\ x_{kI} &= h_{ki}\left([q_{kci}z_{kc}]_{c\in\mathscr{S}_{ki}}\right). \end{aligned} \tag{O36}$$

Note that the sourcing strategy is now a list of countries for each product. Letting

Q_i and Q_{ki} denote the price indices for materials x and product-specific material services x_k respectively, it can be easily shown that

$$\begin{aligned} Q_i &= \left(\sum_{k=1}^{K} (Q_{ki}/\eta_k)^{1-\iota} \right)^{\frac{1}{1-\iota}} \\ Q_{ki} &= (s_{kDi})^{\frac{1}{\varepsilon_k - 1}} \beta_{ki}^{-\frac{\varepsilon_k}{\varepsilon_k - 1}} p_{kD}/q_{kD}, \end{aligned}$$

where s_{kDi} is firm i's domestic expenditure share for product k. The firm-level gains are therefore given by

(O37) $$\ln \left(\frac{u^{Aut}}{u} \right) \Bigg|_{p_D, w} = \frac{\gamma}{\iota - 1} \times \ln \left(\sum_{k=1}^{K} \chi_{ki} (s_{kDi})^{\frac{\iota - 1}{1 - \varepsilon_k}} \right),$$

where

$$\chi_{ki} \equiv \frac{\left(\beta_{ki}^{-\frac{\varepsilon_k}{\varepsilon_k - 1}} p_{kD}/q_{kD} \right)^{1-\iota}}{\sum_{k=1}^{K} \left(\beta_{ki}^{-\frac{\varepsilon_k}{\varepsilon_k - 1}} p_{kD}/q_{kD} \right)^{1-\iota}}.$$

We see that the producer gains are akin to a weighted average of the product-specific producer gains $(s_{kDi})^{\frac{\iota-1}{1-\varepsilon_k}}$. In our empirical application, we cannot implement (O37) because we do not observe domestic shares at the product level s_{kDi} in the French data. Note that implementing (O37) also requires measuring the weights χ_{ki}. In the case where (O36) takes the Cobb-Douglas form, i.e. $\iota = 1$ as in Halpern, Koren and Szeidl (2015), (O37) simplifies to

$$\ln \left(\frac{u^{Aut}}{u} \right) \Bigg|_{p_D, w} = \sum_{k=1}^{K} \eta_k \frac{\gamma}{1 - \varepsilon_k} \ln \left(s_{Di}^{k} \right).$$

Thus, in the Cobb-Douglas case, the producer gains are a weighted average of the product-specific producer gains.

We use the French input-output tables from the OECD to discipline the demand parameters $[\alpha_s]$ and the matrix of input-output linkages Ξ. To determine Ξ, we focus on the intermediate supply from each industry j to each industry s. We abstract from any taxes and subsidies. As Ξ can be identified from expenditure shares by sourcing sector, see (19), we set

$$\zeta_j^s = \frac{\text{Intermediate supply from industry } j \text{ to industry } s}{\text{Intermediate consumption at final prices from industry } s}.$$

That is, ζ_j^s measures the importance of sector j in the production process of sector s. By construction, this ensures that $\sum_{j=1}^S \zeta_j^s = 1$ for all s. We arrange the input-output matrix so that the columns contain the distribution of expenditure for the different sectors:

$$\Xi = \begin{bmatrix} \zeta_1^1 & \zeta_1^2 & & \zeta_1^S \\ \zeta_2^1 & & & \\ & & & \\ \zeta_S^1 & & & \zeta_S^S \end{bmatrix}.$$

To determine $[\alpha_s]$, we also use the input-output tables as they contain information on the composition of final demand. Since there is no trade in final goods in the theory, we exclude any exports and imports in final goods in the data. More specifically, the input-output tables report final consumption expenditure by households on sector j, denoted by $HHFC_j$. Following (19), we hence set

$$\alpha_s = \frac{HHFC_s}{\sum_{j=1}^S HHFC_j}.$$

The OECD input-output tables report their data at the 2-digit level of ISIC Rev. 3, which gives 37 manufacturing industries. To deal with the non-manufacturing industries, we group them into a "residual" sector which we denote by S. To incorporate this sector in the analysis, we set

$$\alpha_S = 1 - \sum_{j \in M} \alpha_j, \tag{O38}$$

where M is the set of manufacturing sectors. Because in our theory this sector is not engaged in foreign sourcing[3], we set

$$\Lambda_S = 0.$$

The input-output structure of sector S can be recovered from the input-output

[3]Note that this sector may nevertheless benefit from input trade if it sources output from the manufacturing industries.

tables. In particular, we set

$$\zeta_j^S \equiv \frac{\sum_{n=1}^{NM} \text{Intermediate supply from industry } j \text{ to industry } n}{\sum_{n=1}^{NM} \text{Intermediate consumption at final prices to industry } n},$$

where NM is the number (set) of non-manufacturing sectors. To measure the materials coefficient in the production of sector S, we employ the Input-Output Matrix for the non-manufacturing sectors. As we observe value added and intermediary spending for each sector, we set

$$\gamma_S = \frac{\sum_{n=1}^{NM} X_n}{\sum_{n=1}^{NM} (X_n + VA_n)}, \tag{O39}$$

where X_n denotes total intermediary spending by sector n.

Table O3 summarizes how $[\alpha_s]$ and $[\gamma_s]$ are computed. The input-output matrix $\mathbf{\Xi}$ used in our empirical analysis is contained in Table O4.

Table O3—Measurement of α_s and γ_s

<table>
<tr><th colspan="6">Direct Data</th><th colspan="2">Aggregation</th><th></th></tr>
<tr><th>ISIC</th><th>α</th><th>Value Added</th><th>Intermediate Purchases</th><th>γ</th><th>Λ</th><th>Coarse Classif.</th><th>α</th><th>γ</th></tr>
<tr><td>1</td><td>α_1</td><td>VA_1</td><td>X_1</td><td>$\frac{X_1}{X_1+VA_1}$</td><td>0</td><td rowspan="4">Non-Manuf.</td><td rowspan="4">α_S from Eq. (O38)</td><td rowspan="4">γ_S from Eq. (O39)</td></tr>
<tr><td>2</td><td>α_2</td><td>VA_2</td><td>X_2</td><td>$\frac{X_2}{X_2+VA_2}$</td><td>0</td></tr>
<tr><td>3</td><td></td><td></td><td></td><td></td><td>0</td></tr>
<tr><td>...</td><td></td><td></td><td></td><td></td><td>0</td></tr>
<tr><td>10</td><td>α_{10}</td><td>VA_{10}</td><td>X_{10}</td><td rowspan="4">Estimate from micro-data</td><td rowspan="4">“Read off” from micro-data</td><td rowspan="4">Manuf.</td><td>α_{10}</td><td>γ_{10}</td></tr>
<tr><td>16</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>...</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>37</td><td>α_{37}</td><td>VA_{37}</td><td>X_{37}</td><td>α_{37}</td><td>γ_{37}</td></tr>
<tr><td>40</td><td>α_{40}</td><td>VA_{40}</td><td>X_{40}</td><td>$\frac{X_{40}}{X_{40}+VA_{40}}$</td><td>0</td><td rowspan="3">Non-Manuf.</td><td rowspan="3">α_S from
Eq. (O38)</td><td rowspan="3">γ_S from
Eq. (O39)</td></tr>
<tr><td>...</td><td></td><td></td><td></td><td></td><td>0</td></tr>
<tr><td>99</td><td>α_{99}</td><td>VA_{99}</td><td>X_{99}</td><td>$\frac{X_{99}}{X_{99}+VA_{99}}$</td><td>0</td></tr>
</table>

Table O4—Input-Output Linkages: Ξ

Sector	10-14	15-16	17-19	20	21-22	24	25	26	27
10-14	8.69	0.12	0	0.02	0.41	1.80	0.26	9.83	4.85
15-16	0.48	21.33	2.27	0.10	0.51	1.97	0.24	0.06	0.17
17-19	0.26	0.11	46.79	0.08	0.65	0.80	1.39	0.44	0.27
20	1.33	0.38	0.13	30.47	1.07	0.17	0.38	2.06	0.33
21-22	1.44	2.29	1.73	1.45	44.73	3.02	2.27	2.82	0.6
24	4.53	1.25	8.25	1.69	5.03	39.28	40.03	3.31	2.92
25	1.68	2.46	2.36	0.65	1.67	3.03	15.72	1.44	0.66
26	8.53	0.79	0.19	0.81	0.21	0.81	0.66	21.53	0.88
27	0.62	0.09	0.40	0.64	0.80	0.76	1.67	3.24	41.61
28	5.95	1.39	1.16	3.51	0.98	2.13	1.78	2.26	6.92
29	20.33	0.79	1.66	3.31	0.87	0.78	2.79	2.60	2.00
30	0	0.02	0.04	0.36	0.08	0.02	0.08	0.36	0.04
21	0.51	0.14	0.08	0.28	0.35	0.34	0.21	0.79	1.07
32	0.90	0.06	0.07	0.09	0.15	0.13	0.39	0.37	0.34
33	0.07	0.01	0.08	0.03	0.12	0.12	0.22	0.57	0.39
34	0.68	0.09	0.12	0.22	0.12	0.07	0.07	0.47	0.21
35	0.15	0.01	0.04	0.04	0.05	0.04	0.05	0.06	0.05
36-37	0.20	0.17	0.59	0.42	0.80	0.18	0.23	0.43	1.34
S	43.64	68.48	34.03	55.82	41.4	44.54	31.56	47.37	35.35

Sector	28	29	30	31	32	33	34	35	36-37	S
10-14	0.36	0.12	0.05	0.16	0.07	0.12	0	0	0.82	0.86
15-16	0.17	0.12	0.10	0.12	0.18	0.18	0.04	0.04	0.48	2.95
17-19	0.17	0.34	0.28	0.42	0.63	0.87	1.4	0.48	5.95	0.38
20	0.39	0.29	0.09	0.23	0.32	0.32	0.29	0.41	15.9	0.59
21-22	0.63	1.01	0.82	1.20	1.37	2.11	0.46	0.52	2.74	3.02
24	3.02	1.78	0.57	4.42	0.84	1.17	1.93	1.03	4.17	1.93
25	2.49	3.65	1.29	6.58	5.07	3.23	6.57	1.64	4.93	0.95
26	0.90	0.67	0.11	0.81	1.50	2.08	1.23	0.36	1.86	1.79
27	24.72	9.00	0.48	11.46	2.50	5.66	8.44	2.28	9.25	0.35
28	28.04	15.87	0.75	13.11	5.97	8.31	8.02	4.84	4.72	1.38
29	4.04	19.27	0.64	2.28	1.99	3.37	4.21	3.33	3.01	1.51
30	0.24	0.48	37.61	0.35	1.27	2.02	0	0.07	0.17	0.34
21	2.24	4.43	3.93	16.03	6.83	2.84	3.07	1.05	1.39	1.12
32	0.59	3.81	12.59	11.00	30.3	11.86	1.98	4.26	1.50	0.70
33	0.61	2.29	2.59	2.72	8.55	19.31	1.52	8.74	0.42	0.60
34	0.25	0.69	0.13	0.27	0.22	0.24	35.3	0.14	0.42	0.82
35	0.11	1.31	0.02	0.03	0.03	0.05	0.11	46.37	0.05	1.12
36-37	0.98	0.76	0.31	0.61	0.34	0.52	1.94	0.37	6.85	0.45
S	30.04	34.13	37.63	28.21	32.02	35.73	23.46	24.09	35.37	79.14

Note: The table contains the French input-output matrix used in our empirical work. We report numbers in percentage terms. Sectors are classified at the 2-digit-level according to ISIC Rev. 3. The non-manufacturing sector S is constructed as explained in the main text and Table O3.

O3. Firm-Level Heterogeneity in the Gains from Input Trade

We can measure the firm-level gains from input trade from $\frac{\gamma_s}{1-\varepsilon}\ln(s_{Dist})$ (see (7)). One can then use the micro-data to learn about the firm-level correlates of this heterogeneity. In particular, consider a regression of the firm-level gains $\frac{\gamma_s}{1-\varepsilon}\ln(s_{Dist})$ on different firm-characteristics, which according to (5) could reflect firm-variation in exogenous "import capabilities" (such as prices $[p_{ci}/q_{ci}]$ or the home bias β_i) and firms' endogenous sourcing strategies $\mathscr{S}_i$. Consider Table O5, which contains the results of regressions of the form $\frac{\gamma_s}{1-\varepsilon}\ln(s_{Dist}) = \delta_s + \delta_t + o'_{ist}\psi + u_{ist}$, where δ_s and δ_t are industry and time fixed effects and o_{ist} is a vector of firm characteristics. We find that bigger firms and exporters see higher gains. When we restrict the analysis to the sample of importers, the positive relation between firm size and the firm-level gains becomes substantially weaker. This is consistent with the pattern documented in Figure 2. When we consider the role of firms' sourcing strategies, we find a strong positive relation between firms' extensive margin (which we measure by the average number of countries that the firm sources its products from) and the firm-level gains. This is consistent with theories where import participation in foreign markets reduces firms' unit costs. The importance of other firm characteristics is substantially diminished once this extensive margin is controlled for.

O4. Accounting for Export Participation

To derive Proposition 1, we had to express firms' unobserved productivity φ in terms of value added. We did so using equation (11). This equation, however, is only correct if firms' international sales are proportional to their domestic sales. In case export participation is limited and productive firms are more likely to export, we would overestimate φ for large firms. This could be important, because export participation is correlated with both firm size and import intensity.

Fortunately, it is straight-forward to account for this effect. In particular, we directly observed domestic value added va_i^D in the micro-data. Because (11) is valid for domestic value added, we can simply evaluate Proposition 1, by using domestic value added instead of total value added. Hence, accounting for firms' export intensity reduces to a re-weighting of firms' domestic expenditure shares. In our context, we find that the gains from input trade are given by 24.4% for the manufacturing sector and 8.1% for the whole economy. These numbers are smaller than our baseline results, reflecting the negative correlation between export intensity and domestic expenditure shares.

Table O5—Correlates of the Producer Gains

	Firm-level gains $\frac{\gamma}{1-\varepsilon}\ln(s_{Di})$							
Sample:	Full sample			Importers Only				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Value Added	0.028***		0.013***	0.005***		-0.008***		-0.029***
(ln)	(0.000)		(0.000)	(0.001)		(0.001)		(0.001)
Employment		0.028***			-0.000			
(ln)		(0.000)			(0.001)			
Exporter			0.085***			0.040***		0.024***
			(0.001)			(0.002)		(0.002)
Intl. group			0.148***			0.138***		0.113***
			(0.003)			(0.003)		(0.003)
Nb. varieties		0.128***	0.144***				0.128***	0.144***
(ln)		(0.002)	(0.002)				(0.002)	(0.002)
Observations	633,240	640,610	633,240	118,799	120,344	118,799	120,344	118,799

Note: Robust standard errors in parentheses with ***, ** and * respectively denoting significance at the 1%, 5% and 10% levels. All regressions include year fixed effects and 3-digit industry fixed effects. The data corresponds to the full sample of firms between 2002 and 2006. The number of varieties is the number of countries the firm sources from (averaged across products). A firm is a member of an international group if at least one affiliate or the headquarter is located outside of France.

O5. Bootstrap Procedure

We sample firms from the empirical distribution with replacement to construct 200 replicates of our micro-data. For each of these samples, we re-calculate σ_s and re-estimate ε and $[\gamma_s]$ following the factor shares approach explained in Section III.B and then re-calculate $[\Lambda_s]$ and $\left[s_{Ds}^{Agg}\right]$. Figure O1 depicts the bootstrap distributions of these variables. For the three sector-level variables, we report the distribution of the sectoral averages, e.g. the upper right panel displays the distribution of $\frac{1}{S}\sum_{s=1}^{S}\gamma_s$. While the variation in γ and s_D^{Agg} is relatively modest, there is a quite a bit of uncertainty regarding ε. This is consistent with the non-negligible standard errors reported in Table 2. We conclude that it is the variation in ε which induces most of the variation in Λ and therefore in the consumer price gains from input trade reported in Table 4.

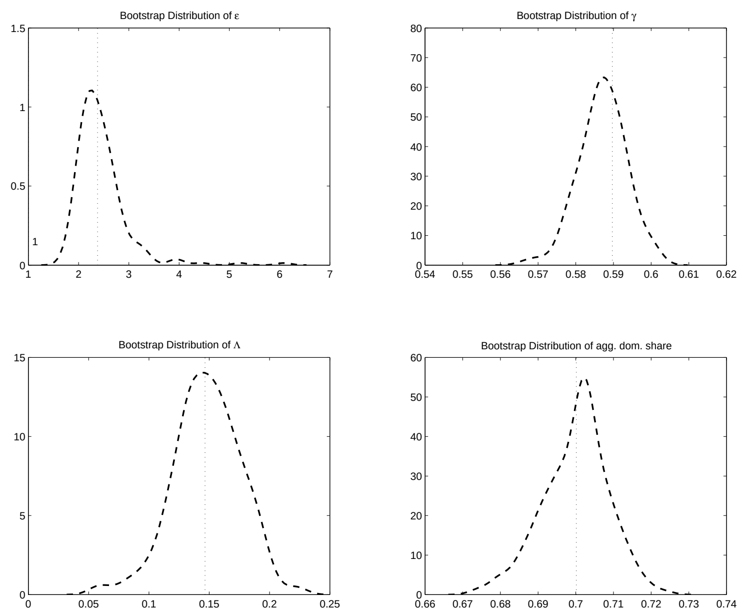

FIGURE O1. BOOTSTRAP DISTRIBUTION OF STRUCTURAL PARAMETERS AND DIRECT PRICE REDUCTIONS

Note: The upper left panel contains the bootstrap distribution of ε. The remaining panels depict the bootstrap distributions of $\frac{1}{S}\sum_{s=1}^{S}\gamma_s$, $\frac{1}{S}\sum_{s=1}^{S}\Lambda_s$ and $\frac{1}{S}\sum_{s=1}^{S}s_{Ds}^{Agg}$. The point estimates used in the main analysis are reported as vertical lines.

In Figure O2 we also show the entire distribution of the consumer price gains and the bias with respect to an aggregate approach.

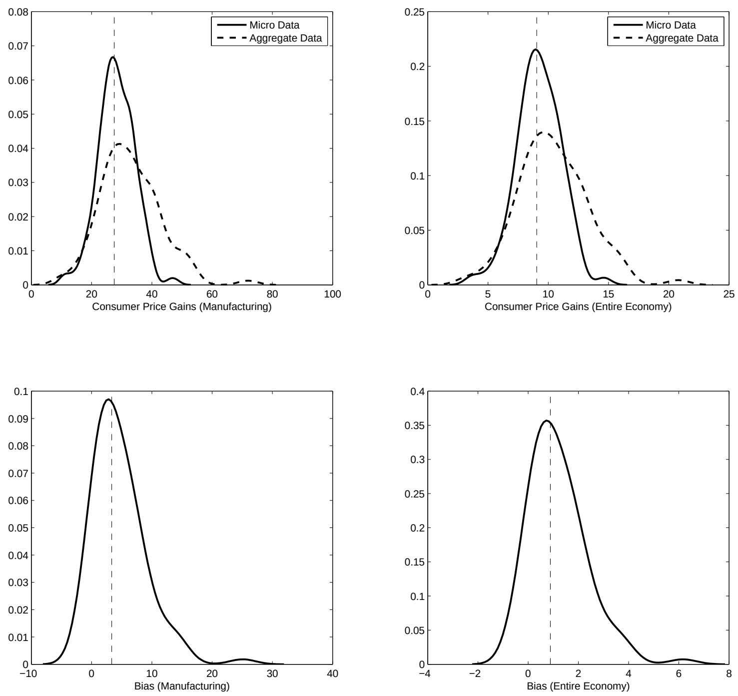

Figure O2. Sampling Variation in the Consumer Price Gains and the Bias

Note: The top panels of the figure depict the bootstrap distribution of the consumer price gains from input trade for the manufacturing sector $(P_M^{Aut}/P_M - 1) \times 100$ (left panel) and the entire economy $(P^{Aut}/P - 1) \times 100$ (right panel). These are computed according to Proposition 1. We display the gains based on the micro data, i.e. using Λ_s^{Aut}, and aggregate data, i.e. using $\Lambda_{Agg,s}^{Aut}$. The bottom panels depict the bootstrap distribution of the bias from using aggregate data, which is computed according to (17). The bootstrap procedure is described in the Online Appendix. We use 200 iterations.

O6. The Elasticity Bias

At the end of Section III.D we argued that aggregate models could be subject to an "elasticity bias". Because the mapping between the model's structural parameter ε and the implied aggregate trade elasticity depends on the particular model and the implied ε for a given trade elasticity is higher in the aggregate model (compared to the firm-based model with heterogeneous fixed costs), the implied gains from trade are downward biased. Importantly, we showed that - quantitatively - this bias can be substantial. To see that, Table O6 reports the consumer price gains from input trade for different values of the elasticity of substitution ε. Columns one and two replicate the results for our baseline estimate $\varepsilon = 2.38$. While column one reports the results based on the micro-data, column two reports the gains based on aggregate data, $\Lambda^{Aut}_{Agg,s}$. These results correspond to the ones reported in Table 4 above. In the remaining columns, we report the gains for a range of values of ε. The tables show that the gains are very sensitive to the value of ε. Table O6 shows that the economy-wide gains predicted by an aggregate approach under $\varepsilon = 5$ are about 65% lower than the gains predicted by the approach that relies on micro-data.

Table O6—The Consumer Price Gains for Different Values of ε

	Micro Data	Aggregate Data				
Value for ε:	2.38	2.38	3	4	5	6
Entire Economy	9.04	9.9	6.72	4.43	3.31	2.64
Manufacturing Sector	27.52	30.8	20.32	13.12	9.69	7.68

Note: The table reports the reduction in consumer prices for the entire economy $(P^{Aut}/P - 1) \times 100$ (first row) and the manufacturing sector $(P^{Aut}_M/P_M - 1) \times 100$ (second row) for different values of the elasticity of substitution ε. In the first two columns, we report the baseline results under $\varepsilon = 2.38$ for comparison. Column one is based on Proposition 1 where Λ_s are computed with micro data as reported in Table 4. The remaining columns contain results based on an aggregate model, i.e. they are based on Proposition 1 where the sectoral gains are measured by $\Lambda^{Aut}_{Agg,s}$ as per (16) instead of Λ^{Aut}_s. The values for $\boldsymbol{\Xi}, \gamma_s, \sigma_s$ and α_s employed for all calculations are given in Table 1.

Table O7 reports the consumer price gains from input trade for different values of the elasticity of substitution ε at the sectoral level. Columns one and two replicate the results for our baseline estimate $\varepsilon = 2.38$. While column one reports the results based on the micro-data, column two reports the gains based on aggregate data, $\Lambda^{Aut}_{Agg,s}$. As in Table O6, we find the gains from input trade are very sensitive to the value of ε.

Table O7—The Sectoral Consumer Price Gains for Different Values of ε

		Micro Data	Aggregate Data				
Value for ε:		2.38	2.38	3	4	5	6
Mining	10-14	2.96	2.50	1.72	1.14	0.86	0.68
Food, tobacco, beverages	15-16	11.06	12.62	8.53	5.61	4.18	3.33
Textiles and leather	17-19	31.14	31.87	20.99	13.55	10.00	7.92
Wood and wood products	20	8.23	9.58	6.51	4.29	3.20	2.55
Paper, printing, publishing	21-22	12.15	10.96	7.43	4.89	3.65	2.91
Chemicals	24	27.23	28.14	18.62	12.06	8.91	7.07
Rubber and plastic products	25	20.12	21.53	14.37	9.37	6.95	5.52
Non-metallic mineral products	26	13.42	13.29	8.98	5.90	4.39	3.50
Basic metals	27	21.80	28.83	19.07	12.34	9.12	7.23
Metal products	28	8.17	7.70	5.24	3.47	2.59	2.07
Machinery and equipment	29	17.64	18.23	12.23	7.99	5.94	4.72
Office and computing machinery	30	20.42	37.00	24.22	15.56	11.45	9.06
Electrical machinery	31	19.77	21.64	14.45	9.41	6.98	5.55
Radio and communication	32	21.55	22.15	14.78	9.62	7.13	5.67
Medical and optical instruments	33	17.90	15.90	10.70	7.01	5.21	4.15
Motor vehicles, trailers	34	6.24	11.23	7.61	5.01	3.73	2.98
Transport equipment	35	15.32	11.83	8.01	5.27	3.93	3.13
Recycling, nec.	36-37	12.87	14.06	9.48	6.23	4.63	3.69
Non-manufacturing		0	0	0	0	0	0

Note: The table reports the reduction in consumer prices at the sectoral level $(P_s^{Aut}/P_s - 1) \times 100$ for different values of the elasticity of substitution ε. In the first two columns, we report the baseline results under $\varepsilon = 2.38$ for comparison. Column one is based on Proposition 1 where Λ_s are computed with micro data as reported in Table 4. The remaining columns contain results based on an aggregate model, i.e. they are based on Proposition 1 where the sectoral gains are measured by $\Lambda_{Agg,s}^{Aut}$ as per (16) instead of Λ_s^{Aut}. The values for $\mathbf{\Xi}, \gamma_s, \sigma_s$ and α_s employed for all calculations are given in Table 1.

O7. General equilibrium and Welfare in the Model of Section III.D

Consider the setup of Section III.D. We now consider the aggregate allocations in this economy. An equilibrium has the usual definition:

Definition 1. *An equilibrium is a set of prices $w, [p_i]$, labor demands for production and fixed costs $\left[l_i, l_i^F\right]$, differentiated product quantities, consumption levels and foreign demands $\left[y_i, c_i, y_i^{ROW}\right]$, domestic and international input demands by local firms $[y_{vi}], [z_{ci}]$ and sourcing strategies $[n_i]$ such that:*

1) *Firms maximize profits given by* (28)-(29),

2) *Consumers maximize utility given by* (8) *subject to their budget constraint*

$$\int_i p_i c_i di = wL + \int_i \pi_i di,$$

3) *Trade is balanced* (31),

4) *Labor and good markets clear*

$$\begin{aligned} L &= \int_i \left(l_i + l_i^F\right) di \\ y_i &= c_i + y_i^{ROW} + \int_\nu y_{vi} dv. \end{aligned}$$

We fist characterize the general equilibrium in a multi-sector version of the economy of Section III.D. In particular, we consider the multi-sector structure of Section I. We derive a generalization of (32). We do not impose any assumptions on how firms' determine their extensive margin. That is, we allow for an arbitrary mapping $l_{\mathscr{S}_i}$which gives the labor resources that firm i needs to spend in order to attain the sourcing strategy $\mathscr{S}_i$. We assume that trade is balanced and that the value of exports in sector s is given by $\alpha_s^{ROW} \times IM$, where IM denotes the value of total spending on imported inputs.

Proposition 2. *Let W,I and S denote welfare, consumer income and total spending, respectively. Then, the change in welfare relative to input autarky is given by*

$$\frac{W}{W^{Aut}} = \frac{I}{I^{Aut}} \times \frac{P^{Aut}}{P},$$

where I and I^{Aut} are given by

$$\begin{aligned} I &= L + \sum_{s=1}^{S} S_s/\sigma_s - \sum_{s=1}^{S} \left(\int_0^{N_s} l_{\mathscr{S}_i} di \right), \\ I^{Aut} &= L + \sum_{s=1}^{S} S_s^{Aut}/\sigma_s, \end{aligned}$$

and $[S_j]$ *and* $\left[S_j^{Aut}\right]$ *solve*

$$\begin{aligned} S_s = \quad & \alpha_s \left(L - \sum_{j=1}^{S} \left(\int_0^{N_j} l_{\mathscr{S}_i} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) \\ & + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \, \omega_i di \end{aligned}$$

and

$$S_s^{Aut} = \alpha_s \left(L + \sum_{j=1}^{S} \frac{1 + \zeta_s^j \gamma_j (\sigma_j - 1) / \alpha_s}{\sigma_j} S_j^{Aut} \right).$$

Furthermore, $G = \frac{P^{Aut}}{P}$ is given in Proposition 1.

Proof. As labor is the only factor of production, consumer welfare is given by real income $W = I/P$, consumer income is given by

$$I = L + \sum_{s=1}^{S} \left(\int_0^{N_s} \pi_i di \right).$$

Note that L represents total labor income and π_i denotes firm i's profits. To derive π_i, recall that firms in sector s have a mark-up of $\sigma_s/(\sigma_s - 1)$ so that variable profits gross of any extensive margin resource loss are given by

$$\pi_i^V = (p_i - u_i) \, y_i = p_i y_i / \sigma_s.$$

Total revenue for firm i is given by

$$p_i y_i = \left(\frac{p_i}{P_s} \right)^{1 - \sigma_s} S_s,$$

where P_s is the consumer price index for sector s and S_s denotes total spending for sector s goods. Hence,

$$\pi_i = p_i y_i / \sigma_s - l_{\mathscr{S}_i} = \frac{1}{\sigma_s} \left(\frac{p_i}{P_s} \right)^{1 - \sigma_s} S_s - l_{\mathscr{S}_i},$$

so that

$$I \quad = \quad L + \sum_{s=1}^{S} \frac{1}{\sigma_s} S_s - \sum_{s=1}^{S} \left(\int_i^{N_s} l_{\mathscr{S}_i} di \right). \tag{O40}$$

Hence, given $[S_s]$ and $[l_{\mathscr{S}i}]$, total income I is fully determined. Now consider $[S_s]_s$. Note that

$$S_s = S_s^C + S_s^X + S_s^{ROW},$$

where S_s^C, S_s^X and S_s^{ROW} denote total spending by consumers, intermediary producers and the rest of the world, respectively. For our economy, we have that $S_c^C = \alpha_s I$ and $S_s^{ROW} = \alpha_s^{ROW} Im$ as consumers spend a fraction α_s of their income I on sector s products and balanced trade requires that total spending by the rest of the world is equal to the value of imports Im, a fraction α_s^{ROW} of which is spent on sector s products. To derive S_s^X, let total domestic intermediary purchases in sector j be given by X_j. Then

$$S_s^X = \sum_{j=1}^{S} \zeta_s^j X_j. \tag{O41}$$

Letting m_i be total material spending by firm i and s_i be total spending by firm i, we know that

$$\begin{aligned} X_j &= \int_0^{N_j} s_{Di} m_i di = \int_0^{N_j} s_{Di} \gamma_j s_i di = \int_0^{N_j} s_{Di} \gamma_j \frac{\sigma_j - 1}{\sigma_j} p_i y_i di \\ &= \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} s_{Di} \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di, \end{aligned} \tag{O42}$$

where we used that firms in sector j spend a fraction γ_j of their total input spending s_i on materials and that total spending s_i accounts for a fraction $(\sigma_j - 1)/\sigma_j$ of revenue. Hence, (O41) and (O42) imply that

$$S_s^X = \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} s_{Di} \left(\frac{p_i}{P_s} \right)^{1-\sigma_s} di. \tag{O43}$$

Similarly, total import spending is equal to

$$\begin{aligned} Im &= \sum_{j=1}^{S} Im_j = \sum_{j=1}^{S} \int_0^{N_j} (1 - s_{Di}) m_i di \\ &= \sum_{j=1}^{S} \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left(\frac{p_i}{P_s} \right)^{1-\sigma_s} di. \end{aligned} \tag{O44}$$

Hence (O43) and (O44) imply that

$$
\begin{aligned}
S_s &= \alpha_s I + \alpha_s^{ROW} \left(\sum_{j=1}^{S} \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di \right) \\
&\quad + \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} s_{Di} \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di \\
&= \alpha_s I + \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \\
&\quad + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_i^{N_j} (1 - s_{Di}) \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di.
\end{aligned}
$$

Using (O40), we get that

$$
\begin{aligned}
S_s = \quad & \alpha_s \left(L - \sum_{j=1}^{S} \left(\int_i^{N_j} l_{\mathscr{S}_i} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) \\
& + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_i^{N_j} (1 - s_{Di}) \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di.
\end{aligned}
$$

Now note that

$$
\frac{va_i}{\int_0^{N_s} va_i di} = \frac{p_i y_i}{\int_0^{N_s} p_i y_i di} = \frac{(p_i/P_s)^{1-\sigma_s} S_s}{\int_0^{N_s} (p_i/P_s)^{1-\sigma_s} S_s di} = \left(\frac{p_i}{P_s} \right)^{1-\sigma_s}.
$$

Hence,

$$
\begin{aligned}
S_s = \quad & \alpha_s \left(L - \sum_{j=1}^{S} \left(\int_0^{N_j} l_{\mathscr{S}_i} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) \\
& + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \, \omega_i di, \qquad \text{(O45)}
\end{aligned}
$$

where $\omega_i = \frac{va_i}{\int_i^{N_s} va_i di}$. Given $L^{NET} = L - \sum_{j=1}^{S} \left(\int_0^{N_j} l_{\mathscr{S}_i} di \right)$, (O45) are S equations in S unknowns S_s, which we can easily solve. Now consider the case of autarky. There we have $l_{\mathscr{S}_i} = 0$ and $s_{Di} = 1$. Hence, (O45) yields

$$
S_s^{Aut} = \alpha_s \left(L + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j^{Aut} \right).
$$

In the case of a single sector (i.e. $S = 1$) it has to be the case that

$$\alpha_S = \alpha_S^{ROW} = \zeta_S^S = 1.$$

Hence,

$$S^{Aut} = L + \frac{1 + \gamma(\sigma - 1)}{\sigma} S^{Aut} = \frac{\sigma}{(1 - \gamma)(\sigma - 1)} L.$$

Substituting this in (O40) yields

$$I^{Aut} = L + \frac{1}{\sigma} S = \frac{1 + (1 - \gamma)(\sigma - 1)}{(1 - \gamma)(\sigma - 1)} L.$$

Similarly, we get from (O45) that

$$\sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j\right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \omega_i di = 0$$

so that

$$\begin{aligned} S &= \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \left(L - \left(\int_i^N l_{\mathscr{S}_i} di\right)\right) \\ I &= \frac{1 + (1 - \gamma)(\sigma - 1)}{(1 - \gamma)(\sigma - 1)} \left(L - \left(\int_i^N l_{\mathscr{S}_i} di\right)\right). \end{aligned}$$

This implies directly (32). This concludes the proof of Proposition 2. □

Because of our assumption that fixed costs do not vary by country, countries can be indexed by their quality q. We first show that the price index of the import bundle takes the power form in (27). The import price index is given by

$$\begin{aligned} A(\mathscr{S}) &= \left(\int_{q\in\mathscr{S}} \left(p(q)/q\right)^{1-\kappa} dG(q)\right)^{\frac{1}{1-\kappa}} \\ &= \left(\int_{q\in\mathscr{S}} q^{\kappa-1} dG(q)\right)^{\frac{1}{1-\kappa}}. \end{aligned} \tag{O46}$$

As quality is Pareto distributed, (O46) becomes

$$A(\mathscr{S})^{1-\kappa} = \theta q_{\min}^{\theta} \int_{q\in\mathscr{S}} q^{\kappa-1} q^{-\theta-1} dq.$$

Because fixed costs are constant across countries, the sourcing set $\mathscr{S}$ can be parametrized by a quality cutoff $\overline{q}$. In particular, the firm selects countries with high enough quality, i.e. $q \in \mathscr{S}$ as long as $q \geq \overline{q}$. It follows that

$$A(\overline{q})^{1-\kappa} = q_{\min}^{\theta} \frac{\theta}{\theta-(\kappa-1)} \overline{q}^{\kappa-1-\theta}. \tag{O47}$$

We can rewrite this expression in terms of the mass of countries sourced, n, which is given by

$$n = P(q \in \mathscr{S}) = P(q \geq \overline{q}) = q_{\min}^{\theta} \overline{q}^{-\theta}. \tag{O48}$$

Substituting (O48) into (O47) yields

$$A(n) = q_{\min}^{-1} \left(\frac{\theta}{\theta-(\kappa-1)}\right)^{\frac{1}{1-\kappa}} n^{-\left(\frac{1}{\kappa-1}\right)},$$

which is (27) in the main text where

$$\begin{aligned} z &\equiv q_{\min}^{-1} \left(\frac{\theta}{\theta-(\kappa-1)}\right)^{\frac{1}{1-\kappa}} \\ \eta &\equiv \frac{1}{\kappa-1}. \end{aligned}$$

This completes the characterization of (27). The following proposition characterizes the solution to the extensive margin problem.

Proposition 3. *Consider the setup above and suppose that*

$$\eta(\varepsilon-1) < 1 \text{ and } \eta\gamma(\sigma-1) < 1.$$

Then, the firm's profit maximization problem (28) *has a unique solution for any value of $\tilde{\varphi}$ and f. The optimal mass of countries sourced is given by a function $n(\tilde{\varphi}, f)$ and an efficiency cutoff $\overline{\varphi}(f)$ such that $n(\tilde{\varphi}, f) = 0$ for $\varphi \leq \overline{\varphi}(f)$ with $\overline{\varphi}(\cdot)$ increasing. Furthermore, $n(\varphi, f)$ is increasing in efficiency $\tilde{\varphi}$ and decreasing in the fixed costs of sourcing f.*

Proof. The firm's maximization problem follow from (28), (29) and (30) as

$$\pi = \max_{n} \left\{ \begin{array}{c} B \times \tilde{\varphi}^{(\sigma-1)} \left(\frac{p_D}{q_D}\right)^{\gamma(1-\sigma)} \left(\left(1 + \left(\frac{1-\beta}{\beta}\right)^{\varepsilon} \left(\frac{p_D}{q_D}\frac{1}{z}n^{\eta}\right)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\right) \\ -\left(nf + f_I \mathbb{I}(n>0)\right) \end{array} \right\}.$$

Conditional on importing, the optimal mass of countries is characterized by the following first order condition:

$$\begin{aligned} \left(\frac{1-\beta}{\beta}\right)^{\varepsilon\frac{\gamma(\sigma-1)}{\varepsilon-1}} &\times \left(\left(\frac{\beta}{1-\beta}\right)^{\varepsilon}\left(\frac{q_D}{p_D}\right)^{\varepsilon-1} + \left(\frac{1}{z}\right)^{\varepsilon-1} n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} \\ &\times z^{1-\varepsilon} n^{\eta(\varepsilon-1)-1} = \frac{1}{\eta\gamma(\sigma-1)}\frac{1}{B}\frac{f}{\tilde{\varphi}^{\sigma-1}}. \end{aligned} \tag{O49}$$

The second order condition is given by

$$\begin{aligned} &\left(\left(\frac{\beta}{1-\beta}\right)^{\varepsilon}\left(\frac{q_D}{p_D}\right)^{\varepsilon-1} + z^{1-\varepsilon}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} n^{\eta(\varepsilon-1)-2} \\ &\times \left\{\left(\eta(\varepsilon-1)-1\right) + \left(\gamma(\sigma-1)-\varepsilon+1\right)\eta l(n)\right\} < 0 \end{aligned} \tag{O50}$$

where

$$l(n) \equiv \frac{z^{1-\varepsilon}n^{\eta(\varepsilon-1)}}{\left(\frac{\beta}{1-\beta}\right)^{\varepsilon}\left(\frac{q_D}{p_D}\right)^{\varepsilon-1} + z^{1-\varepsilon}n^{\eta(\varepsilon-1)}} \in [0,1).$$

It follows that (O.O8) is satisfied if and only if

$$\eta(\varepsilon-1) - 1 + \left(\gamma(\sigma-1)-\varepsilon+1\right)\eta l(n) < 0. \tag{O51}$$

Because we allow for arbitrary values of φ and f, we need to verify that (O51) holds for any value of n. Sufficient conditions for this are given by

$$\eta(\varepsilon-1) < 1 \tag{O52}$$

and

$$\eta\gamma(\sigma-1) < 1. \tag{O53}$$

If (O52) is not satisfied, there exists a range of values of n close enough to zero such that (O51) is violated.[4] (O52) is therefore necessary. If $\gamma(\sigma-1)-\varepsilon+1 \leq 0$, then (O51) is satisfied for all n. If $\gamma(\sigma-1)-\varepsilon+1>0$, then (O51) holds for all n if and only if

$$\eta(\varepsilon-1)-1+(\gamma(\sigma-1)-\varepsilon+1)\eta l(1)<0. \tag{O54}$$

As $l(1)<1$, a sufficient condition for (O54) is given by (O53). This proves that, under (O52)-(O53), the optimal mass of countries conditional on importing is uniquely characterized by (O49) for any values of $\tilde{\varphi}$ and f.[5] The firm becomes an importer whenever $\pi_I \geq \pi_D$, where π_I are optimal profits conditional on importing and π_D are profits as a non-importer. It can be shown that this condition is satisfied whenever

$$\begin{aligned}\left[\left(1+\left(\frac{1-\beta}{\beta}\right)^{\varepsilon}\left(\frac{p_D}{q_D}z^{-1}n^{\eta}\right)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}-1\right](q_D/p_D)^{\gamma(\sigma-1)}\,\Gamma\tilde{\varphi}^{\sigma-1}&\\ -fn-f_I \quad > \quad 0,&\end{aligned} \tag{O55}$$

where n is the solution to (O49). It follows the firm's profit maximization problem in (28) has a unique solution for any value of φ and f.

Note that, under (O52)-(O53), the left hand side of (O49) is decreasing in n. Therefore, the optimal mass of countries sourced is weakly increasing in φ and weakly decreasing in f. Holding n fixed, an increase in φ tends to increase the left hand side of (O55). Additionally, π_I is increasing in φ. It follows that $\pi_I-\pi_D$ is increasing in φ for a given f. This proves that $n=0$ if and only if $\varphi \leq \overline{\varphi}(f)$ where $\overline{\varphi}(\cdot)$ is implicitly defined as the value of φ that makes the left hand side of (O55) equal to zero. The fact that $\overline{\varphi}(f)$ is increasing in f follows from the fact that $\pi_I-\pi_D$ is decreasing in f for a given φ.[6] This proves Proposition 3. □

To solve for the aggregate allocations, we have to consider the general equilibrium of the economy. The formal derivation and the analytical characterization is contained in Section O.O10 in the Online Appendix.

[4]This follows from the fact that $l(n)$ is continuous and strictly increasing.

[5]When the solution to (O49) exceeds unity, the solution is given by $n=1$. Clearly, $n=0$ cannot be a solution as the firm always prefers to be a non-importer and avoid payment of f_I. Note that our calibrated and estimated parameters satisfy (O52)-(O53) - see Table 1.

[6]To see why this is the case, note that the left hand side of (O55) is decreasing in f given φ and n. Additionally, π_I is decreasing in f.

O9. *Estimation of* η

To solve for firms' optimal domestic shares in the heterogeneous fixed cost model, we require an estimate for η. To do so, we need to take a stand on what the counterpart of the number varieties, n, is in the data. We focus on the number of countries the firm sources its products from, i.e. the number of foreign varieties.[7] Using this assumption we can estimate η from the cross-sectional relationship between firms' domestic expenditure share and the number of sourcing countries, because the theory predicts a log-linear relationship between n and $\frac{1-s_D}{s_D}$ (see (30)). Hence, we estimate η from the following regression:

$$\ln\left(\frac{1-s_{Dist}}{s_{Dist}}\right) = \delta_s + \delta_t + \delta_{NK} + \eta\left(\varepsilon - 1\right)\ln\left(n_{ist}\right) + u_{ist}, \tag{O56}$$

where n_{ist} denotes firm i's average number of countries per product sourced, δ_{NK} contains a set of fixed effects for the number of products sourced and δ_s and δ_t are sector and year fixed effects. Hence, we identify η from firms sourcing the same number of products from a different number of supplier countries. We measure products at the 8-digit level.

Table O8 contains the results of estimating (O56). Columns one and two show that it is important to control for the number of products sourced as import-intensive firms source both more varieties per-product and more products on international markets - without the product fixed effects, the estimated η increases substantially.[8] Columns three and four show that the estimate of η is virtually unaffected by additional firm-level controls that can affect firms' import behavior conditional on the number of varieties sourced. In column five, we focus on a subsample of firm-product pairs that source their respective products from at least two supplier countries. In this case, the estimated η decreases as the single-variety importers have very high domestic shares in the data. For our quantitative analysis, we take column five as the benchmark.[9] The implied value of η is 0.382 and it is precisely estimated.

[7]This notion of "varieties" is widely used in the literature - see e.g. Broda and Weinstein (2006) and Goldberg et al. (2010). Moreover, the choice of the number of products sourced may be determined to a large degree by technological considerations, while the demand for multiple supplier countries within a given product category may plausibly stem from love-for-variety effects, which are at the heart of the mechanism stressed by our theory. However, we note that the analysis that follows can be done under alternative interpretations of n.

[8]Recall that η is a combination of different structural parameters of the economy. While η is sufficient to characterize the welfare gains from trade, one might be interested in decomposing the returns to international sourcing into the the elasticity of substitution across varieties κ, the dispersion in input quality θ, and the elasticity of input prices with respect to quality ν. To do so, we need two additional pieces of information: import prices (to identify ν) and data on firms' expenditure shares across trading partners (to identify θ).

[9]We are concerned that the single-variety observations may not help identify the extensive margin channel emphasized by our theory but rather pick-up other variation across firms. Additionally, a non-parametric regression shows that the log linear relation between n and $(1-s_D)/s_D$ in (O56) fits the data better in the sample with at least two varieties than in the full sample (results available upon request).

Table O8—Estimating η

Dep. Variable:	$\ln(\frac{1-s_D}{s_D})$					
					> 1	> 2
		All Importers			Variety	Varieties
Nb.of varieties	1.308***	0.707***	0.733***	0.739***	0.526***	0.463***
(ln)	(0.009)	(0.010)	(0.010)	(0.010)	(0.011)	(0.019)
Capital / Emp.				-0.070***		
(ln)				(0.006)		
Exporter			-0.395***	-0.388***	-0.254***	-0.198***
dummy			(0.013)	(0.013)	(0.017)	(0.029)
International			0.150***	0.174***	0.216***	0.223***
group			(0.016)	(0.016)	(0.016)	(0.019)
Implied η	0.950***	0.513***	0.532***	0.536***	0.382***	0.336***
	(0.260)	(0.142)	(0.147)	(0.148)	(0.106)	(0.096)
Control for						
nb. of products	No	Yes	Yes	Yes	Yes	Yes
Observations	120,344	120,344	120,344	120,344	73,651	35,751

Note: Robust standard errors in parentheses with ***, **, and * respectively denoting significance at the 1%, 5% and 10% levels. All regressions include year fixed effects and 3-digit industry fixed effects. The number of varieties is the average number of countries a firm sources its foreign products from. To back out the value for η, we use our benchmark value for $\varepsilon = 2.378$ from Section III.

We adopt a solution algorithm that allows us to bypass the computation of the general equilibrium variables within the calibration. Intuitively, we work with a normalized version of fixed costs, where these are scaled by an appropriate transformation of the general equilibrium variables. Because the equilibrium variables depend on firms' import behavior only through the domestic shares, which are itself a calibration target, we can compute them *after* the calibration. That is, we can first ensure that the moments of the joint distribution of value added and domestic shares are matched[10], and then back out the underlying general equilibrium variables required to compute welfare. We also show that the parameter z is not required for the calibration.

We first start with three aggregate variables, which are determined in equilibrium. In the single-sector version of the model, characterized in Section O.O7 in the Online Appendix, we have that aggregate spending S and the price level (which is also equal to the price of domestic varieties) is given by

$$S = \frac{\sigma}{(1-\gamma)(\sigma-1)}\left(L-\left(\int_i^N l_{\mathscr{S}_i} di\right)\right) \tag{O57}$$

$$P = \left(\frac{\sigma}{\sigma-1}\left(\frac{1}{\gamma}\right)^{\gamma}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{q_D}\right)^{\gamma}\Upsilon\right)^{\frac{1}{1-\gamma}}, \tag{O58}$$

where

$$\Upsilon = \left(\int_{i=0}^N \left(\frac{1}{\tilde{\varphi}_i}(s_{D,i})^{\gamma/(\varepsilon-1)}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}. \tag{O59}$$

We start by noting that the firm's optimality conditions from the profit maximization problem, contained in Section O.O8, can be expressed in terms of s_D instead of n. To see this, note that (30) and (27) imply

$$n^{\eta(\varepsilon-1)} = \left(\frac{1-s_D}{s_D}\right)\left(\frac{\beta}{1-\beta}\right)^{\varepsilon} z^{\varepsilon-1}\left(\frac{q_D}{p_D}\right)^{\varepsilon-1}. \tag{O60}$$

Substituting (O60) into the firm's first order condition (O49), we obtain

$$s_D^{\frac{1-\gamma(\sigma-1)\eta}{(\varepsilon-1)\eta}}(1-s_D)^{1-\frac{1}{\eta(\varepsilon-1)}} = \left(\frac{\beta}{1-\beta}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\eta}}\frac{\tilde{f}}{\tilde{\varphi}^{\sigma-1}}, \tag{O61}$$

where

$$\tilde{f} \equiv f \times (zq_D)^{1/\eta}\frac{1}{\eta\gamma(\sigma-1)}\frac{1}{\Theta}\times\frac{1}{P^{1/\eta}\Gamma}, \tag{O62}$$

[10]For this step, it is important that the dispersion and correlation moments are in logs. See below.

where

$$\Theta = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma}\left(\frac{1}{q_D}\right)\right)^{\gamma}\right)^{1-\sigma}, \tag{O63}$$

$$\Gamma = \frac{S}{P^{(1-\gamma)(1-\sigma)}}. \tag{O64}$$

Similarly, (O60) and the import status condition (O55) imply that the firm is an importer as long as

$$\begin{aligned} &\left[s_D^{-\frac{\gamma(\sigma-1)}{\varepsilon-1}}-1\right]\tilde{\varphi}^{\sigma-1}-\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta(\varepsilon-1)}}\gamma(\sigma-1)\,\eta\left(\frac{\beta}{1-\beta}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\eta}}\tilde{f} \\ &\qquad -\tilde{f}_I > 0, \end{aligned} \tag{O65}$$

where

$$\tilde{f}_I \equiv \frac{1}{\Gamma}\frac{1}{\Theta}\times f_I. \tag{O66}$$

(O61) and (O65) show that we can solve for firms' optimal domestic share and import status with knowledge of $\tilde{\varphi}^{\sigma-1}, \tilde{f}$ and $\tilde{f}_I$ only. Thus, we can work with the joint distribution of $\left(\varphi, \tilde{f}\right)$ to match the moments of the joint distribution of domestic shares and value added. We can then back out the exogenous component of fixed costs f_I and f from $\tilde{f}_I$ and $\tilde{f}$ using the equilibrium variables S and P and (O64).

To solve for S, we require the aggregate resource loss of fixed costs (see (O57)). To do so, note that

$$\begin{aligned} l_{\mathscr{S}_i} &= l_i(s_{Di}) \\ &= f_i \times \left(\frac{s_{Di}}{1-s_{Di}}\right)^{\frac{1}{\eta(1-\varepsilon)}}\left(\frac{1}{P}\right)^{1/\eta}(q_D z)^{1/\eta}\left(\frac{\beta_i}{1-\beta_i}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\eta}} + f_I \\ &= \Gamma\Theta\left\{\eta\gamma(\sigma-1)\times\tilde{f}_i\times\left(\frac{s_{Di}}{1-s_{Di}}\right)^{\frac{1}{\eta(1-\varepsilon)}}\left(\frac{\beta_i}{1-\beta_i}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\eta}} + \tilde{f}_I\right\}. \end{aligned}$$

Hence,

$$\int_i^N l_{\mathscr{S}_i} di = \Gamma\Theta\left\{\begin{array}{c} \eta\gamma(\sigma-1)\times\int_i^N \tilde{f}_i\left(\frac{s_{Di}}{1-s_{Di}}\right)^{\frac{1}{\eta(1-\varepsilon)}}\left(\frac{\beta_i}{1-\beta_i}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\eta}} di \\ +\int_i^N \tilde{f}_I 1\left[s_{Di}\right] di \end{array}\right\}. \tag{O67}$$

The key is now to argue that Γ is known given the calibration. If so, we can calculate $\int_i^N l_{\mathscr{S}_i} di$ from (O67) given the calibrated $\tilde{f}$ and $\tilde{f}_I$ and parameters, as

$$\int_i^N l_{\mathscr{S}_i} di = \Gamma\times\Theta\times\Delta,$$

where

$$\Delta \equiv \quad \eta\gamma(\sigma-1)\times\int_i^N \tilde{f}_i\left(\frac{s_{Di}}{1-s_{Di}}\right)^{\frac{1}{\eta(1-\varepsilon)}}\left(\frac{\beta_i}{1-\beta_i}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\eta}}di$$
$$+\int_i^N \tilde{f}_I 1\left[s_{Di}\right]di. \tag{O68}$$

Recall that (O64) and (O57) imply that

$$\begin{aligned}\Gamma &= \frac{S}{P^{(1-\gamma)(1-\sigma)}}=\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}\left(L-\left(\int_i^N l_{\mathscr{S}_i}di\right)\right)\\ &= \frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}L-\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}\Gamma\Theta\Delta.\end{aligned}$$

Solving for Γ yields

$$\Gamma=\frac{\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}}{1+\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}\Theta\Delta}L. \tag{O69}$$

As L is a normalization (see below), (O69) shows that Γ is fully determined as P can be evaluated from the calibrated data on domestic shares (see (O58) and (O59)). Hence,

$$\int_i^N l_{\mathscr{S}_i}di=\Gamma\Theta\Delta=\frac{\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}\Theta\Delta}{1+\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}\Theta\Delta}L.$$

This implies that

$$\frac{L-\int_i^N l_{\mathscr{S}_i}di}{L}=\frac{1}{1+\frac{1}{P^{(1-\gamma)(1-\sigma)}}\frac{\sigma}{(1-\gamma)(\sigma-1)}\Theta\Delta}, \tag{O70}$$

so that L is indeed a normalization. Finally we only have to show that (O70) does not depend on q_D, even though Θ does (see (O63)). However, it can easily be shown that

$$\Theta P^{(1-\gamma)(\sigma-1)} \quad = \quad \Upsilon^{\sigma-1}\frac{1}{\sigma}.$$

Hence, the quality of domestic varieties q_D and the foreign price level z can be normalized for the calibration.

The five models we consider fit in this framework as follows:

1) The *aggregate model* assumes that $\beta_i=\beta$ and $f_i=f_I=0$. Hence, $\int_i^N l_{\mathscr{S}_i}di=0$ and $s_{Di}=s_D$ can be solved from (O60) using that $n=1$ (as all firms are importers and import from every country). The level of β is chosen to match the aggregate domestic share. The dispersion in produc-

tivity σ_φ is chosen to match the dispersion in value added.

2) The *homogeneous bias model* assumes that $\beta_i = \beta$ and $f_i = 0 < f_I$. Hence, conditional on importing, we have that $s_{Di} = s_D$, which can be solved from (O60) using that $n = 1$. The required level $\tilde{f}_I$ in (O65) is chosen to match the share of importers. Given a distribution of productivity $[\tilde{\varphi}_i]_i$ we can then calculate Δ from (O68), P from (O58) and (O59) and hence Γ from (O69). This is sufficient to calculate welfare using (O70) and P^{Aut}/P.

3) The *heterogeneous bias model* assumes that β_i varies across firms and $f_i = 0 < f_I$. As for the case with fixed costs, it is useful to consider a scaled version of the home-bias $\tilde{\beta}_i = \frac{\beta_i}{1-\beta_i}$. In particular, (O60) shows that s_D only depends on $\beta^* = \left(\tilde{\beta}\right)^{\varepsilon} \left(\frac{1}{p}\right)^{\varepsilon-1}$ (again, we have $n = 1$ as there are no fixed costs per country). Hence, we draw $(\tilde{\varphi}, \beta^*)$ from a joint log-normal distribution. Using (O60), this generates a joint distribution of $(\tilde{\varphi}_i, s_{Di})$. We can then calibrate $\tilde{f}_I$ from (O65) to match the share of importers. Like for the case of the homogeneous bias model, we can then use (O68), P and (O69) to compute all equilibrium objects.

4) For the *heterogeneous fixed cost model,* we draw $\left(\tilde{\varphi}_i.\tilde{f}_i\right)$ from a joint log-normal distribution. Using the (O61), this implies a joint distribution of $(\tilde{\varphi}_i, s_{Di})$. We can then calibrate $\tilde{f}_I$ from (O65) to match the share of importers. As above, we can then use (O68), P and (O69) to compute all equilibrium objects.

5) The *homogeneous fixed cost model,* is a special case of the heterogeneous fixed cost model where $\tilde{f}_i = \tilde{f}$. Hence, the procedure is exactly the same given a marginal distribution for $\tilde{\varphi}_i$.