Online Appendix: "Endogenous Disasters"

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January 2018*

Abstract

This appendix (for online publication only) details data sources, derivations, computation, and supplementary results for our manuscript titled "Endogenous disasters" to appear in *American Economic Review*.

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We describe in detail data sources in Section A, derivations in Section B, computational procedures in Section C, and supplementary results in Section D.

A Data

We detail the data sources used in our manuscript.

A.1 Historical Cross-country Panels

Output and Consumption

The Barro-Ursúa (2008) historical cross-country panel of real per capita output and consumption is obtained from Robert Barro's Web site. Their panel ends in 2009, and we extend it through 2013. In particular, we construct real per capita gross domestic product (GDP) growth with two series from World Development Indicators (WDI) at the World Bank (http://data.worldbank.org/data-catalog/world-development-indicators), real GDP growth (code: NY.GDP.MKTP.KD.ZG), g_{GDP}, and the population growth (code: SP.POP.GROW), g_{POP}. Both growth rate series are in annual percent. We take 2006 as the base year and set the GDP index, Y_{2006}, as 100. For year t after 2006, we calculate its GDP index as:

$$Y_t = 100 \times \frac{\prod_{2006}^{t}(1 + g_{\mathrm{GDP}}/100)}{\prod_{2006}^{t}(1 + g_{\mathrm{POP}}/100)}. \tag{A1}$$

Analogously, we construct real per capita consumption growth from two WDI series, annual percent growth of household final consumption expenditure (code: NE.CON.PRVT.KD.ZG) and the population growth (code: SP.POP.GROW). WDI revises its published data retrospectively. To be consistent, we update the output and consumption series from 2006 onward, as opposed to 2009, when the original Barro-Ursúa panel ends.

Asset Prices

We compile our cross-country panel of real stock market returns and real interest rates from Global Financial Data (GFD) and the Dimson, Marsh, and Staunton (DMS, 2002) dataset updated through 2013. We purchase the DMS dataset from Morningstar.

In constructing our cross-country panel, we search the GFD database for countries with total return indexes going back to at least early 1930s. We then supplement the GFD data with the DMS database. For a given country, the starting year of the sample for real stock returns often differs from that for real interest rates. In such cases, we take the common sample period for stocks and bills. We have only managed to verify dividend yields data for four countries from GFD, but the series are not adjusted for inflation. As such, we opt not to use the Barro-Ursúa (2009) data of asset prices. In any event, the basic asset pricing moments in our cross-country panel are relatively close to theirs.

Table A1 reports descriptive statistics of real stock and bill returns across countries. Both raw and leverage-adjusted financial moments are reported. Following Barro (2006), we

Table A1 : Properties of Asset Prices

Results are based on a cross-country panel of real stock market returns and real interest rates drawn from GFD and the DMS dataset updated through 2013. "Start" is the starting year of the sample for a given country (all country samples end in 2013). $E[\widetilde{R}]$, $\sigma_{\widetilde{R}}$, and $E[\widetilde{R} - R^f]$ are the average stock market returns, the stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[R - R^f]$ and σ_R are the equity premium and the stock market volatility, respectively, after adjusting for leverage. $E[R^f]$ is the mean interest rate, and σ_{R^f} is the interest rate volatility. All moments are in annual percent.

	Start	$E[\widetilde{R}]$	$\sigma_{\widetilde{R}}$	$E[R^f]$	σ_{R^f}	$E[\widetilde{R} - R^f]$	$E[R - R^f]$	σ_R
Australia	1876	9.81	16.68	1.24	5.28	8.57	6.08	12.34
Austria	1901	6.36	38.04	−2.44	24.55	8.79	6.24	31.19
Belgium	1901	6.34	26.21	1.34	14.67	5	3.55	20.82
Canada	1901	7.16	17.79	1.7	5.73	5.46	3.88	13.14
Denmark	1901	8.15	23.05	3.49	13.09	4.66	3.31	18.28
Finland	1901	9.57	30.65	0.32	11.47	9.25	6.57	23.01
France	1896	6.34	26.18	−1.58	10.58	7.92	5.62	19.54
Germany	1870	5.26	31.9	−1.21	16.53	6.47	4.59	25.56
Ireland	1901	7.45	25.53	1.63	12.08	5.82	4.13	20.13
Italy	1901	5.57	28.78	−0.85	12.6	6.42	4.56	21.76
Japan	1901	9.1	30.24	−0.56	13.51	9.66	6.86	23.36
Netherlands	1901	8.14	24.8	1.9	13.22	6.24	4.43	19.48
New Zealand	1901	8.32	25.74	2.42	14.76	5.9	4.19	21.21
Norway	1901	8.47	30.04	2.22	13.08	6.25	4.44	23.31
South Africa	1901	9.78	29.78	1.14	15.39	8.63	6.13	24.09
Spain	1901	6.92	28.25	2.05	18.42	4.86	3.45	23.69
Sweden	1901	8.36	21.49	1.24	6.91	7.12	5.06	15.71
Switzerland	1901	7.3	20.9	2.29	12.07	5.01	3.56	16.57
United Kingdom	1801	6.32	16.39	2.67	6.96	3.66	2.6	12.6
United States	1836	8.16	19.3	1.78	5.53	6.38	4.53	14.14
Average		7.64	25.59	1.04	12.32	6.6	4.69	20

calculate the leverage-adjusted equity premium as $E[R-R^f] = (1-\omega)E[\widetilde{R}-R^f]$, in which ω is leverage, defined as the market value of debt/(the market value of debt + the market value of equity), and $E[\widetilde{R}-R^f]$ is the equity premium without adjusting for leverage. The leverage-adjusted stock market volatility, σ_R, is the standard deviation of $(1-\omega)\widetilde{R}_t+\omega R_t^f$. We set $\omega = 0.29$, which is the mean leverage ratio across 39 countries in Fan, Titman, and Twite (2012).

It should be noted that although standard in the literature (Barro 2006, Section III; see also Kaltenbrunner and Lochstoer 2010), the leverage adjustment procedure is crude. However, this simple procedure allows one to abstract from modeling sovereign default risk, which is an important topic of its own. In any event, accounting for long-term (and defaultable) debt would reduce the equity premium in the data. As such, our procedure raises the hurdle on the model to match the equity premium. However, to the extent that bond returns are more volatile than bill rates, our procedure likely understates the leverage-adjusted volatility.

A.2 The Long U.S. Sample

Output, Profits, Prices, and Investment

To measure the profits-to-output ratio, we obtain output (gross domestic product) data from National Income and Product Accounts (NIPA) Table 1.1.6, and profits data from NIPA Table 1.12 row 13 (corporate profits with inventory valuation adjustment and capital consumption adjustment). We use the implicit price deflator of gross domestic product (NIPA Table 1.1.9) to deflate profits. The sample is annual from 1929 to 2013. To calculate the relative volatility of profits to output, we detrend annual real profits and output as HP-filtered proportional deviations from the mean with a smoothing parameter of 100.

To calculate investment moments, we obtain the series for the real U.S. gross private domestic investment from NIPA Table 1.1.6. The sample is annual from 1929 to 2013.

Unemployment and Vacancy Rates

Following Petrosky-Nadeau and Zhang (2013), we construct the series of monthly unemployment rates in the U.S. from April 1929 to December 2013 by drawing from NBER macrohistory files (chapter 8: Income and employment) and Federal Reserve Economic Data (FRED) at Federal Reserve Bank of St. Louis. We concatenate four U.S. unemployment series:

- The seasonally adjusted unemployment rates from April 1929 to February 1940 (NBER data series m08292a, FRED series id: M0892AUSM156SNBR, National Industrial Conference Board, by G. H. Moore Business Cycle Indicators, vol. II, p. 35 and p. 123).
- The seasonally adjusted unemployment rates from March 1940 to December 1946 (NBER data series m08292b, FRED series id: M0892BUSM156SNBR. U.S. Bureau of the Census, Current Population Reports, Labor Force series P-50, no. 2, 13, and 19).
- The seasonally adjusted unemployment rates from January 1947 to December 1947. To construct this series, we first obtain the monthly unemployment rates (not seasonally adjusted) from January 1947 to December 1966 (NBER data series m08292c, FRED series id: M0892CUSM156NNBR. Source: Employment and Earnings and Monthly Report on the Labor Force, vol. 13, no. 9, March 1967). We pass the monthly series from 1947 to 1966 through the X-12-ARIMA seasonal adjustment program from the U.S. Census Bureau, and take the adjusted series from January to December of 1947.
- The seasonally adjusted civilian unemployment rates from January 1948 to December 2013 from Bureau of Labor Statistics (FRED series id: UNRATE).

Following Owyang, Ramey, and Zubairy (2013), we adjust the pre-1948 unemployment rates. We use the monthly unemployment rates from January 1930 to December 1947 to interpolate annual unemployment rates data from Weir (1992) using the Denton (1971) proportional interpolation procedure. In addition, we scale the nine monthly unemployment rates from April to December 1929 so that their average matches the annual unemployment rate for 1929 reported in Weir. We cannot apply the Denton procedure on the nine monthly observations because the procedure requires the complete data for 12 months in a given year.

Following Petrosky-Nadeau and Zhang (2013), we construct the monthly vacancy rates in the U.S. from April 1929 to December 2013 by drawing from four series of U.S. job openings:

- The Metropolitan Life Insurance company (MetLife) help-wanted advertising index in newspaper from April 1929 to August 1960. The series (not seasonally adjusted) is obtained from the NBER macrohistory files (series id: m08082a, FRED series id: M0882AUSM349NNBR). The NBER scales the series to average 100 over the 1947–1949 period. To seasonally adjust the series, we pass the raw series through the X-12-ARIMA program from the U.S. Census Bureau.

- The help-wanted advertising index from the Conference Board, seasonally adjusted, from January 1951 to July 2006. The series is scaled to average 100 in 1987.

- The composite print and online help-wanted index from Barnichon (2010). The series, ranging from January 1995 to December 2012, is from Regis Barnichon's Web site.

- The seasonally adjusted job openings series (total nonfarm, level in thousands) from the Job Openings and Labor Turnover Survey (JOLTS) released by Bureau of Labor Statistics. The series from December 2000 to December 2013 are from FRED (id: JTSJOL).

To make the different series comparable in units, we scale the Conference Board index (multiply the index by 2.08) so that its value for January 1960 equals the MetLife index value for the same month. We use the Conference Board index until December 1994 and the Barnichon index thereafter. Because these two series have the same unit, we scale the Barnichon index in the same way as we scale the Conference Board index to concatenate with the MetLife series. We scale the JOLTS series (multiply by 0.0195) so that its value in December 2000 equals the Barnichon index value for the same month. We use the JOLTS series (scaled by $2.08 \times 0.0195 = 0.04$) after December 2000 in the overall help-wanted index.

To convert the help-wanted index into vacancy rates, we construct a labor force series. We obtain the civilian labor force over 16 years of age in thousands of persons from FRED (series id: CLF16OV). The (seasonally adjusted) series is based on Current Employment Statistics released by Bureau of Labor Statistics. The sample is from January 1948 to December 2013. To construct the labor force series for the period from April 1929 to December 1947, we obtain the annual observations of total population from the U.S. Census. We form a monthly series by linearly interpolating two adjacent annual observations across the 12 months in question. We multiply the total population estimates by the fraction of the population over 16 years of age in 1948 and the average labor force participation rate in 1948. The implicit assumption is that both rates are largely constant from 1929 to 1947. The last step in constructing the vacancy rate series is to divide the overall help-wanted index by the labor force series, while rescaling the resulting series to a known estimate of the job vacancy rate. In particular, we multiply the resulting series by 13.47 so that the series averages to 2.05% in 1965, which is the vacancy rate documented by Zagorsky (1998, Table A1).

Figure A1 : The U.S. Monthly Unemployment and Vacancy Rates, April 1929–December 2013

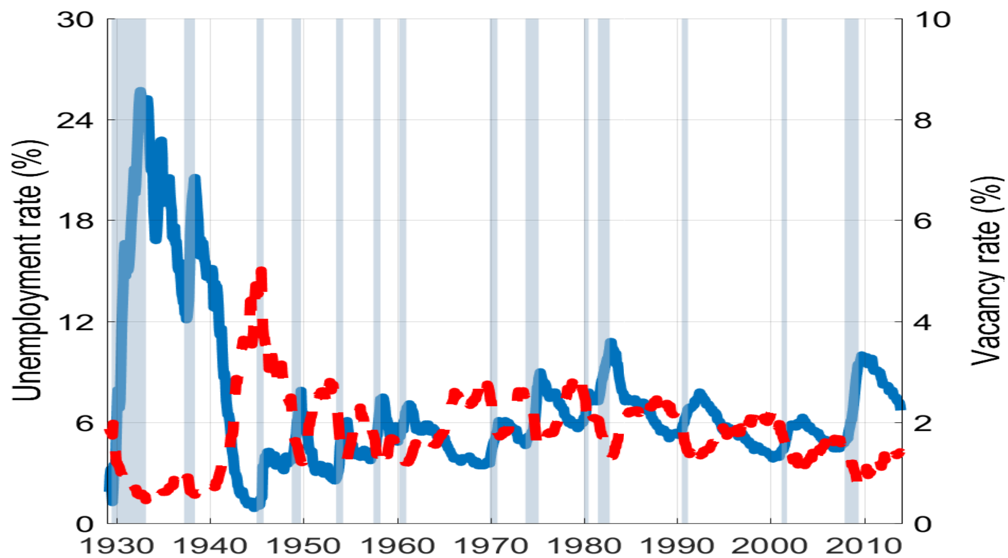


Figure A1 plots the long time series of the U.S. monthly unemployment and vacancy rates. In particular, the vacancy rate experienced large declines during the Great Depression. The vacancy rate dropped from 2% in September 1929 to 0.44% in March 1933, representing a steep decline of 78%. The vacancy rate went through another, steeper decline from the mid to late 1940s, due to the demobilization of World War II.

A.3 Postwar U.S. Data

To calculate the wage elasticity to labor productivity, we measure wages as labor share times labor productivity (real output per job) in the nonfarm business sector from Bureau of Labor Statistics, following Hagedorn and Manovskii (2008). The sample is quarterly from 1947 to 2013. As in Hagedorn and Manovskii, we detrend wages and labor productivity with log deviations from the HP-trend with a smoothing parameter of 1,600.

To calculate net payout moments in the data, we measure the net payout as net dividends of nonfinancial corporate business (Table F.102, series FA106121075.Q) minus net increase in corporate equities of nonfinancial business (Table F.101, series FA103164103.Q) from Financial Accounts of the Federal Reserve Board, following Jermann and Quadrini (2012). The sample is quarterly from the fourth quarter of 1951 to the fourth quarter of 2013. We use implicit price deflator for gross domestic product (NIPA Table 1.1.9) to deflate net payout. We detrend real net payout, gross domestic product, and consumption as HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. We do not take logs because the net payout can be negative in the data.

B Derivations

B.1 Wages

We derive equilibrium wages under recursive utility. The case with log utility is similar (and more standard). Let $\eta \in (0,1)$ denote the relative bargaining weight of the worker, J_{Nt} the marginal value of an employed worker to the representative family, J_{Ut} the marginal value of an unemployed worker to the representative family, ϕ_t the marginal utility of the representative family, S_{Nt} the marginal value of an employed worker to the representative firm, and S_{Vt} the marginal value of an unfilled vacancy to the representative firm.

A worker-firm match turns an unemployed worker into an employed worker for the representative household as well as an unfilled vacancy into a filled vacancy (an employed worker) for the firm. As such, we can define the total surplus from the Nash bargain as:

$$\Lambda_t \equiv \frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}. \tag{B1}$$

The equilibrium wage is determined via the Nash worker-firm bargain:

$$\max_{\{W_t\}} \quad \left(\frac{J_{Nt} - J_{Ut}}{\phi_t}\right)^{\eta} \left(S_{Nt} - S_{Vt}\right)^{1-\eta}, \tag{B2}$$

The outcome of maximizing equation (B2) is the surplus-sharing rule:

$$\frac{J_{Nt} - J_{Ut}}{\phi_t} = \eta\Lambda_t = \eta\left(\frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}\right). \tag{B3}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain. In what follows, we derive the equilibrium wage from the sharing rule in equation (B3).

Workers

To derive J_{Nt} and J_{Ut}, we need to specify the details of the representative household's problem. Tradeable assets consist of risky shares and a risk-free bond. Let R_{t+1}^f denote the risk-free interest rate, which is known at the beginning of period t, Π_t the household's financial wealth, χ_t the fraction of the household's wealth invested in the risky shares, $R_{t+1}^{\Pi} \equiv \chi_t R_{t+1} + (1-\chi_t)R_{t+1}^f$ the return on wealth, and T_t the taxes raised by the government. The household's budget constraint is given by:

$$\frac{\Pi_{t+1}}{R_{t+1}^{\Pi}} = \Pi_t - C_t + W_t N_t + U_t b - T_t. \tag{B4}$$

The household's dividends income, D_t, is included in the current financial wealth, Π_t.

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (B4). The household's maximization problem is given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t\left(\frac{\Pi_{t+1}}{R_{t+1}^{\Pi}} - \Pi_t + C_t - W_tN_t - U_tb + T_t\right), \tag{B5}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta)C_t^{-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1}, \tag{B6}$$

which gives the marginal utility of consumption.

Recalling $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (B5) with respect to N_t:

$$\begin{aligned} J_{Nt} &= \phi_t W_t + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\quad \times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \end{aligned} \tag{B7}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Nt}}{\phi_t} = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \tag{B8}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned} \frac{J_{Nt}}{\phi_t} &= W_t + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\ &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \end{aligned} \tag{B9}$$

Similarly, differentiating J_t in equation (B5) with respect to U_t yields:

$$\begin{aligned} J_{Ut} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\quad \times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \end{aligned} \tag{B10}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Ut}}{\phi_t} = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \tag{B11}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}
\frac{J_{Ut}}{\phi_t} &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} \left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\
&= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right].
\end{aligned} \tag{B12}$$

The Firm

Start by rewriting the infinite-horizon value-maximization problem of the firm recursively as:

$$S_t = X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \tag{B13}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$. The first-order condition with respect to V_t says:

$$S_{Vt} = -\kappa_t + \lambda_t q(\theta_t) + E_t[M_{t+1} S_{Nt+1} q(\theta_t)] = 0. \tag{B14}$$

Equivalently,

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t[M_{t+1} S_{Nt+1}]. \tag{B15}$$

In addition, differentiating S_t with respect to N_t yields:

$$S_{Nt} = X_t - W_t + (1-s)E_t[M_{t+1} S_{Nt+1}]. \tag{B16}$$

Combining the last two equations yields the job creation condition.

The Wage Equation

From equations (B9), (B12), and (B16), the total surplus of the worker-firm relationship is:

$$\begin{aligned}
\Lambda_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] - b \\
&\quad - E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] + X_t - W_t + (1-s)E_t[M_{t+1} S_{Nt+1}] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\left(\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}} + S_{Nt+1}\right)\right] - f(\theta_t)E_t\left[M_{t+1}\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}}\right] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] - \eta f(\theta_t) E_t\left[M_{t+1}\Lambda_{t+1}\right],
\end{aligned} \tag{B17}$$

in which the last equality follows from the definition of Λ_t and the surplus sharing rule (B3).

The sharing rule implies $S_{Nt} = (1-\eta)\Lambda_t$, which, combined with equation (B16), yields:

$$(1-\eta)\Lambda_t = X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right]. \tag{B18}$$

Combining equations (B17) and (B18) yields:

$$\begin{aligned} X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] &= (1-\eta)(X_t - b) + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\ &\quad -(1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\ X_t - W_t &= (1-\eta)(X_t - b) - (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\ W_t &= \eta X_t + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right]. \end{aligned}$$

Using equations (B3) and (B15) to simplify further:

$$\begin{aligned} W_t &= \eta X_t + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}S_{Nt+1}\right] && \text{(B19)} \\ &= \eta X_t + (1-\eta)b + \eta f(\theta_t)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right]. && \text{(B20)} \end{aligned}$$

Using the Kuhn-Tucker conditions, when $V_t > 0$, then $\lambda_t = 0$, and equation (B20) reduces to the equilibrium wage equation because $f(\theta_t) = \theta_t q(\theta_t)$. On the other hand, when the nonnegativity constraint is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (B20) reduces to $W_t = \eta X_t + (1-\eta)b$. Because $\theta_t = 0$, the wage equation continues to hold.

Acceptable Wages

More generally, any wages between the workers' and firm's reservation wages would be acceptable (Hall 2005). Workers would want to work when wages are no less than their reservation wage, denoted, $\underline{W}_t$, which is the wage rate that sets the workers' surplus to zero in the worker-firm match. In particular, setting J_{Nt} in equation (B9) to J_{Ut} in equation (B12) yields:

$$\underline{W}_t = b + E_t\left[M_{t+1}(1 - f(\theta_t) - s)\frac{J_{Ut+1} - J_{Nt+1}}{\phi_{t+1}}\right]. \tag{B21}$$

On the other hand, a variable wage rate must also be lower than the highest wage that the firm is willing to pay, $\overline{W}_t$, the firm's reservation wage that sets the firm's surplus to zero. In particular, combining equations (B15) and (B16) and setting S_{Nt} to zero yield:

$$\overline{W}_t = X_t + (1-s)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right]. \tag{B22}$$

Home Production

With home production, the marginal value of an unemployed worker to the household is:

$$J_{Ut} = \frac{\partial \log(C_t)}{\partial C_{ht}} \frac{\partial C_{ht}}{\partial U_t} + \phi_t b + \beta E_t \left[f(\theta_t) J_{Nt+1} + (1 - f(\theta_t)) J_{Ut+1} \right]. \tag{B23}$$

It follows that:

$$\frac{J_{Ut}}{\phi_t} = X_h \left(\frac{1-a}{a} \right) \left(\frac{C_{mt}}{C_{ht}} \right)^{1-e} + b + \beta E_t \left[f(\theta_t) J_{Nt+1} + (1 - f(\theta_t)) J_{Ut+1} \right]. \tag{B24}$$

The wage equation under home production then follows from the same derivations in Appendix B.1, after redefining b as $X_h \left((1-a)/a\right) \left(C_{mt}/C_{ht}\right)^{1-e} + b$.

Capital

In the capital model, the derivation of wages is analogous. The firm's problem becomes:

$$S_t = Y_t - W_t N_t - I_t - \kappa_t V_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \tag{B25}$$

The marginal product of labor becomes $(1-\alpha) Y_t / N_t$, and the rest of the proof follows.

B.2 The Stock Return

We derive the stock return per Liu, Whited, and Zhang (2009). Rewrite the firm's problem:

$$S_t = \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} \left[\begin{array}{c} X_{t+\tau} N_{t+\tau} - W_{t+\tau} N_{t+\tau} - \kappa_{t+\tau} V_{t+\tau} \\ -\mu_{t+\tau} \left[N_{t+\tau+1} - (1-s) N_{t+\tau} \right. \\ \left. - V_{t+\tau} q(\theta_{t+\tau}) \right] + \lambda_{t+\tau} q(\theta_{t+\tau}) V_{t+\tau} \end{array} \right] \right], \tag{B26}$$

in which μ_t is the Lagrange multiplier on the employment accumulation equation, and λ_t is the Lagrange multiplier on the V-constraint on job creation. The first-order conditions with respect to V_t and N_{t+1} in maximizing the market value of equity are given by, respectively:

$$\mu_t = \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t, \tag{B27}$$

$$\mu_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \mu_{t+1} \right] \right]. \tag{B28}$$

Define dividends as $D_t \equiv X_t N_t - W_t N_t - \kappa_t V_t$ and the ex-dividend equity value as $P_t \equiv S_t - D_t$. Expanding S_t yields:

$$\begin{aligned} P_t + X_t N_t - W_t N_t - \kappa_t V_t = S_t = {} & X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t) V_t \\ & -\mu_t \left[N_{t+1} - (1-s) N_t - V_t q(\theta_t) \right] + E_t M_{t+1} \left[X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa_{t+1} V_{t+1} \right. \\ & \left. -\mu_{t+1} \left[N_{t+2} - (1-s) N_{t+1} - V_{t+1} q(\theta_{t+1}) \right] + \lambda_{t+1} q(\theta_{t+1}) V_{t+1} \right] + \ldots \end{aligned} \tag{B29}$$

Recursively substituting equations (B27) and (B28) yields:

$$P_t + X_tN_t - W_tN_t - \kappa_t V_t = X_tN_t - W_tN_t + \mu_t(1-s)N_t. \tag{B30}$$

Using equation (B27) to simplify further:

$$P_t = \kappa_t V_t + \mu_t(1-s)N_t = \mu_t[(1-s)N_t + q(\theta_t)V_t] + \lambda_t q(\theta_t)V_t = \mu_t N_{t+1}, \tag{B31}$$

in which the last equality follows from the Kuhn-Tucker condition.

To the derive the stock return, we expand:

$$\begin{aligned}
R_{t+1} &= \frac{S_{t+1}}{S_t - D_t} = \frac{\mu_{t+1}N_{t+2} + X_{t+1}N_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1}}{\mu_t N_{t+1}} \\
&= \frac{X_{t+1} - W_{t+1} - \kappa_{t+1}V_{t+1}/N_{t+1} + \mu_{t+1}\left[(1-s) + q(\theta_{t+1})V_{t+1}/N_{t+1}\right]}{\mu_t} \\
&= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t} + \frac{\mu_{t+1}q(\theta_{t+1})V_{t+1} - \kappa_{t+1}V_{t+1}}{\mu_t N_{t+1}} \\
&= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t},
\end{aligned} \tag{B32}$$

in which the last equality follows because the Kuhn-Tucker condition implies:

$$\mu_{t+1}q(\theta_{t+1})V_{t+1} - \kappa_{t+1}V_{t+1} = -\lambda_{t+1}q(\theta_{t+1})V_{t+1} = 0. \quad \blacksquare \tag{B33}$$

C Computation

We detail our computational procedures for solving the baseline model and its extensions.

C.1 The Baseline Model

We exploit a convenient mapping from the conditional expectation function, $\mathcal{E}_t$, defined as:

$$\mathcal{E}(N_t, x_t) \equiv E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right]\right]\right], \tag{C1}$$

to policy and multiplier functions to eliminate the need to parameterize the multiplier separately. After obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0 / \left(\mathcal{E}_t - \kappa_1\right)$. If $\tilde{q}(\theta_t) < 1$, the $V_t \geq 0$ constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\theta_t)$, and $V_t = \theta_t(1-N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the V_t nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. We then perform the following set of substitutions:

$$U_t = 1 - N_t \tag{C2}$$

$$N_{t+1} = (1-s)N_t + q(\theta_t)V(N_t, x_t) \tag{C3}$$

$$
\begin{aligned}
x_{t+1} &= \rho x_t + \sigma \epsilon_{t+1} && \text{(C4)}\\
C(N_t, x_t) &= \exp(x_t) N_t - [\kappa_0 + \kappa_1 q(\theta_t)]\, V(N_t, x_t) && \text{(C5)}\\
W_t &= \eta\, [\exp(x_t) + [\kappa_0 + \kappa_1 q(\theta_t)]\, \theta_t] + (1-\eta) b && \text{(C6)}
\end{aligned}
$$

We approximate the x_t process with the discrete state space method of Rouwenhorst (1995) with 17 grid points.[1] This 17-point grid is large enough to cover the values of x_t within four unconditional standard deviations from its unconditional mean. Specifically, the Rouwenhorst grid is symmetric around the long-run mean of x_t. The grid is also even-spaced, with the distance between any two adjacent grid points, d_x, given by:

$$
d_x \equiv 2\sigma / \sqrt{(1-\rho^2)(n_x - 1)}, \tag{C7}
$$

in which ρ is the persistence, σ the conditional volatility of x_t, and $n_x = 17$. We still need to construct the transition matrix, Π, in which the (i, j) element, Π_{ij}, is the probability of $x_{t+1} = x_j$ conditional on $x_t = x_i$. To this end, we set $p = (\rho + 1)/2$, and:

$$
\Pi^{(3)} \equiv \left[\begin{array}{ccc} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-p) & p^2 + (1-p)^2 & p(1-p) \\ (1-p)^2 & 2p(1-p) & p^2 \end{array} \right], \tag{C8}
$$

which is the transition matrix for $n_x = 3$. To obtain $\Pi^{(17)}$, we use the following recursion:

$$
p \left[\begin{array}{cc} \Pi^{(n_x)} & \mathbf{0} \\ \mathbf{0}' & 0 \end{array} \right] + (1-p) \left[\begin{array}{cc} \mathbf{0} & \Pi^{(n_x)} \\ 0 & \mathbf{0}' \end{array} \right] + (1-p) \left[\begin{array}{cc} \mathbf{0}' & 0 \\ \Pi^{(n_x)} & \mathbf{0} \end{array} \right] + p \left[\begin{array}{cc} 0 & \mathbf{0}' \\ \mathbf{0} & \Pi^{(n_x)} \end{array} \right], \tag{C9}
$$

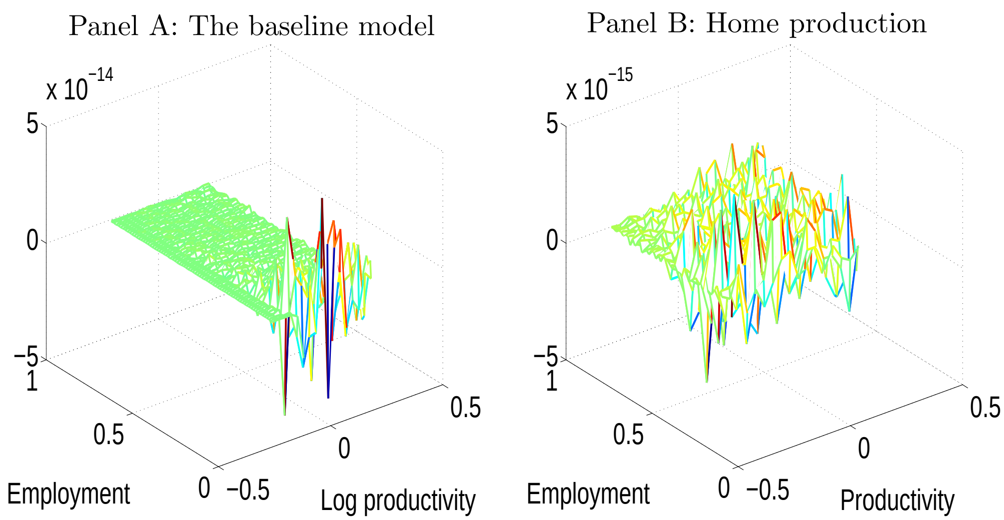
in which $\mathbf{0}$ is a $n_x \times 1$ column vector of zeros. We then divide all but the top and bottom rows by two to ensure that the conditional probabilities sum up to one in the resulting transition matrix, $\Pi^{(n_x+1)}$. See Rouwenhorst (p. 306–307 and p. 325–329) for more details.

We set the minimum value of N_t to be 0.05 and the maximum value to be 0.99. We use cubic splines with 75 basis functions on the N space to approximate $\mathcal{E}(N_t, x_t)$ on each grid point of x_t. We use extensively the approximation took kit in the CompEcon Toolbox in Matlab of Miranda and Fackler (2002). To obtain an initial guess, we use the second-order perturbation solution via Dynare. To solve the projection system of nonlinear equations, we use Matlab's `fsolve` that implements the Trust-Region-Reflective algorithm.

Solving the nonlinear model takes a lot of care, otherwise the projection algorithm would not converge. Unlike the value function, iterating on the first-order conditions is typically not a contraction mapping. We use the idea of homotopy continuation methods (Judd 1998, p. 179) extensively to ensure convergence for a wide range of parameter values. When we solve the model with a new set of parameters, we set the lower bound of N_t to be 0.4 to alleviate the burden of nonlinearity on the nonlinear solver. After obtaining the model's solution, we then apply homotopy to gradually reduce the lower bound to 0.05.

[1] Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more reliable and accurate than other methods in approximating highly persistent first-order autoregressive processes in the stochastic growth model. Petrosky-Nadeau and Zhang (2017) demonstrate similar results in the search model.

Figure C1 : $\mathcal{E}_t$ Errors, The Baseline and Home Production Models



Panel A of Figure C1 reports the error in the $\mathcal{E}$ functional equation (C1) ($\mathcal{E}_t$ minus the right-hand-side). The error, in the magnitude of 10^{-14}, is extremely small, suggesting that our algorithm does an accurate job in solving the competitive equilibrium. Petrosky-Nadeau and Zhang (2017) contain more technical details on our global algorithm.

C.2 Home Production

The conditional expectation function remains equation (C1). In addition to equation (C2)–(C4), we perform several substitutions that are specific to the home production model:

$$C_m(N_t, x_t) = \exp(x_t)N_t - [\kappa_0 + \kappa_1 q(\theta_t)]\, V(N_t, x_t) \tag{C10}$$

$$C_h(N_t, x_t) = X_h(1 - N_t) \tag{C11}$$

$$\begin{aligned} W_t = {} & \eta\,[\exp(x_t) + [\kappa_0 + \kappa_1 q(\theta_t)]\,\theta_t] \\ & +(1-\eta)\left[X_h\left(\frac{1-a}{a}\right)\left(\frac{C_{mt}}{C_{ht}}\right)^{1-e} + b\right] \end{aligned} \tag{C12}$$

We continue to discretize the x_t process with 17 grid points per Rouwenhorst (1995). We use cubic splines with 50 basis functions on the N_t space, $[0.35, 0.99]$. We obtain an initial guess from the second-order perturbation solution via Dynare on a smaller grid of N_t, $[0.8, 0.99]$, and then gradually reduce the minimum N_t to 0.35 via homotopy. Panel B of Figure C1 reports the errors from our projection solution to the home production model.

C.3 Capital

In the extended model with capital, the state space consists of employment, capital, and productivity, (N_t, K_t, x_t). The goal is to solve for the optimal vacancy function, $V(N_t, K_t, x_t)$, the multiplier function, $\lambda(N_t, K_t, x_t)$, and the optimal investment function, $I(N_t, K_t, x_t)$, from the following two functional equations:

$$\begin{aligned}
\frac{1}{a_2}\left(\frac{I(N_t, K_t, x_t)}{K_t}\right)^{1/\nu} &= E_t\left[M_{t+1}\left[\alpha\frac{Y(N_{t+1}, K_{t+1}, x_{t+1})}{K(N_{t+1}, K_{t+1}, x_{t+1})}\right.\right.\\
&\left.\left.+\ \frac{1}{a_2}\left(\frac{I(N_{t+1}, K_{t+1}, x_{t+1})}{K(N_{t+1}, K_{t+1}, x_{t+1})}\right)^{1/\nu}(1-\delta+a_1)+\frac{1}{\nu-1}\left(\frac{I(N_{t+1}, K_{t+1}, x_{t+1})}{K(N_{t+1}, K_{t+1}, x_{t+1})}\right)\right]\right] \qquad \text{(C13)}\\
\frac{\kappa_t}{q(\theta_t)} - \lambda(N_t, K_t, x_t) &= E_t\left[M_{t+1}\left[(1-\alpha)\frac{Y(N_{t+1}, K_{t+1}, x_{t+1})}{N_{t+1}} - W_{t+1}\right.\right.\\
&\left.\left.+\ (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda(N_{t+1}, K_{t+1}, x_{t+1})\right]\right]\right], \qquad \text{(C14)}
\end{aligned}$$

Also, $V(N_t, K_t, x_t)$ and $\lambda(N_t, K_t, x_t)$ must satisfy the Kuhn-Tucker conditions.

An advantage of the installation function is that optimal investment is always positive. When investment goes to zero, the marginal benefit of investment, $\partial\Phi(I_t, K_t)/\partial I_t = a_2(I_t/K_t)^{-1/\nu}$, goes to infinity. As such, there is no need to impose the $I_t \geq 0$ constraint, and we approximate $I(N_t, K_t, x_t)$ directly. For the intertemporal job creation condition in equation (C14), we use the Christiano-Fisher (2000) parameterized expectations method by approximating the conditional expectation (the right-hand-side).

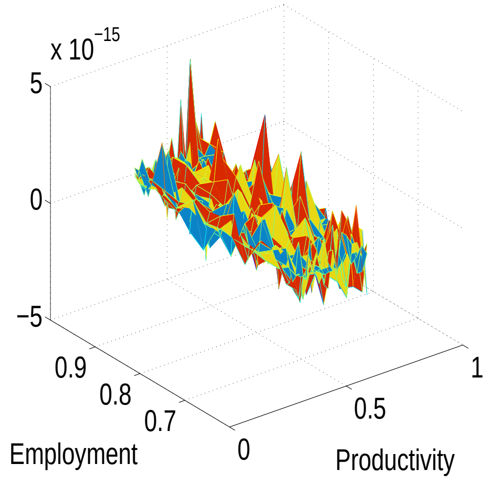
We discretize x_t via the Rouwenhorst (1995) method with 17 grid points. We use cubic splines with 10 basis functions on the N_t space, [0.65, 0.99], and 10 basis functions on the K_t space, [15, 40]. The deterministic steady state capital is 32.5. We use the tensor product of N_t and K_t, and approximate $I(N_t, K_t, x_t)$ and $\mathcal{E}(N_t, K_t, x_t)$ on each grid point of x_t. To solve the resulting system of nonlinear equations, we again use Matlab's `fsolve` that implements the Trust-Region-Reflective algorithm. Figure C2 shows that the errors of the two functional equations from the projection algorithm are extremely small. Petrosky-Nadeau and Zhang (2017) report more technical details of solving the extended model with capital.

C.4 Recursive Utility

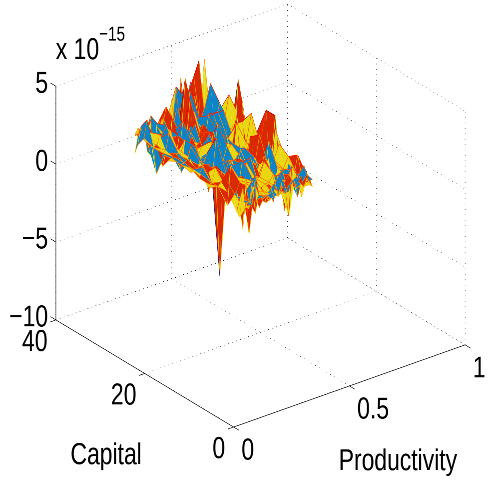
In this extended model, the state space consists of employment and productivity, (N_t, x_t). The goal is to solve for the optimal vacancy function, $V(N_t, x_t)$, the multiplier function,

Figure C2 : Errors, The Extended Model with Capital

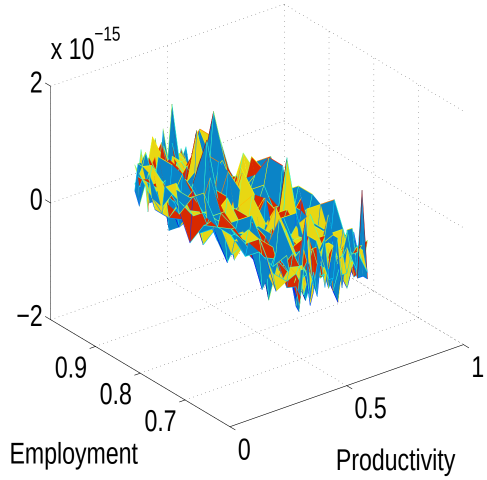
Panel A: The $\mathcal{E}_t$ error



Panel B: The $\mathcal{E}_t$ error



Panel C: The investment Euler equation error



Panel D: The investment Euler equation error

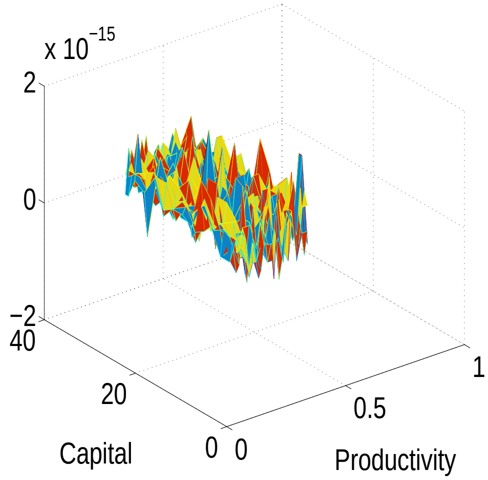
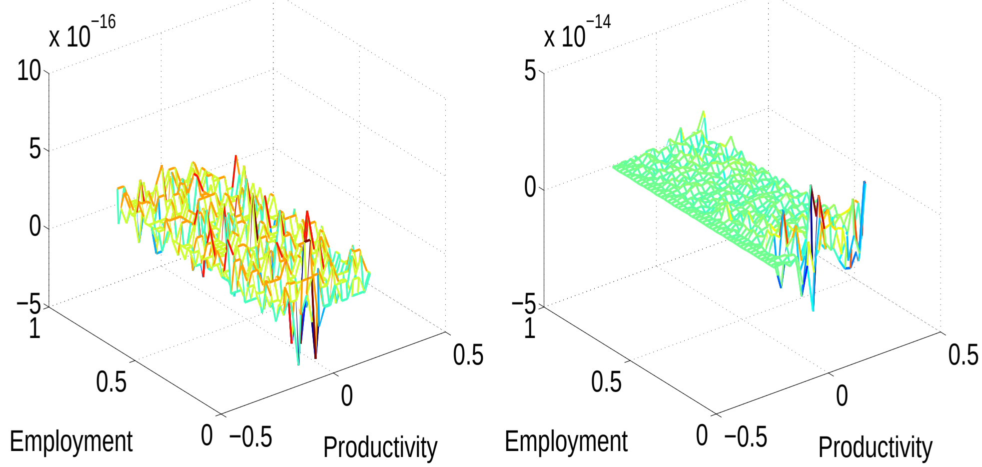


Figure C3 : Errors, The Recursive Utility Model

Panel A: The J_t error | Panel B: The $\mathcal{E}_t$ error



$\lambda(N_t, x_t)$, and an indirect utility function, $J(N_t, x_t)$, from two functional equations:

$$J(N_t, x_t) = \left[(1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{C15}$$

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda(N_t, x_t) = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right]\right]\right], \tag{C16}$$

in which

$$M_{t+1} = \beta\left[\frac{C(N_{t+1}, x_{t+1})}{C(N_t, x_t)}\right]^{-\frac{1}{\psi}}\left[\frac{J(N_{t+1}, x_{t+1})}{E_t[J(N_{t+1}, x_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}. \tag{C17}$$

Also, $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must satisfy the Kuhn-Tucker conditions.

We use the Christiano-Fisher (2000) parameterized expectations method by approximating the right-hand-side of equation (C16). We discretize x_t via the Rouwenhorst (1995) method with 17 grid points. We use cubic splines with 50 basis functions on the N_t space, [0.05, 0.99], to approximate $J(N_t, x_t)$ and $\mathcal{E}(N_t, x_t)$ on each grid point of x_t. To solve the system of nonlinear equations, we again use Matlab's `fsolve` that implements the Trust-Region-Reflective algorithm. Figure C3 shows that the errors from the algorithm are extremely small.

D Supplementary Results

D.1 An Alternative Calibration

Table D1 reports the detailed results from the alternative calibration, in which we rescale the volatility of productivity shocks, σ, in the baseline model and its extentions to target the real output growth volatility of 4.3% per annum in the historical U.S. sample from 1790 to 2013. All the other model parameters are unchanged. The table shows that despite the lower output volatility than 5.6% in the Barro-Ursua (2008) cross-country panel, the disaster dynamics that arise endogenously within the models remain substantial.

D.2 Additional Results from the Recursive Utility Model

Comparative Statics

Table D2 reports additional comparative statics for the recursive utility model by varying labor market parameters. We observe that labor market dynamics and disaster dynamics are tightly connected with asset pricing dynamics. Reducing the value of unemployment volatilities, b, from 0.85 to 0.825 lowers the unemployment volatility from 25.7% to 8.2%, and the equity premium from 4.5% per annum to 0.6%. The output disaster probability falls from 4.5% to 2.6%, and size from 23.9% to 15.4%.

Lowering the separation rate and removing the fixed costs of vacancy both serve to reduce the unemployment volatility, disaster probabilities and size, as well as the equity premium. Intuitively, a lower separation rate means that jobs are destructed at a lower rate, and all else equal, the economy can create enough jobs to dampen disaster dynamics, reducing the equity premium. Removing the fixed costs of vacancy weakens the downward rigidity of the marginal costs of hiring, allowing the economy to create more jobs in bad times. As such, disaster dynamics are dampened, and the equity premium lowered. Reducing the curvature of the matching function, ι, makes the labor market more frictional in matching vacancies with unemployed workers. Because job creation flows are hampered, the disaster dynamics are strengthened, and the equity premium increased. Finally, increasing the workers' bargaining power, η, from 0.04 to 0.05 raises the unemployment rate as well as the output and consumption volatilities. The disaster dynamics are also strengthened. However, the equity premium falls slightly, as the wage elasticity of productivity rises from 0.54 to 0.6.

Time-varying Risk and Risk Premiums

Table D3 reports long-horizon regressions of stock market excess returns and consumption growth on log price-to-consumption. We do not use log price-to-dividend because dividends can be negative in the model. Consistent with Beeler and Campbell (2012), stock prices forecast excess returns, but not consumption growth, both in the historical (1836–2013) annual U.S. sample and in the postwar quarterly U.S. sample.[2]

[2]Stock returns, interest rates, and S&P composite index series are from Global Financial Data. For the historical sample, the real consumption data are from Barro and Ursua (2008) extended through 2013. For the postwar sample, real consumption is real per capita nondurables plus services from NIPA Table 7.1.

Table D1 : Quantitative Results, An Alternative Calibration

In model columns, BL is the baseline model, HP the home production model, CA the capital model, and RU the recursive utility model. We rescale the volatility of productivity shocks, σ, to be 0.00925, 0.012, 0.012, and 0.00875 in BL, HP, CA, and RU, respectively, to match the output growth volatility of 4.3% per annum in the 1790–2013 U.S. sample. All the other parameters are identical to those reported in the main text. All the model results are based on 10,000 simulations. σ_Y, σ_C, and σ_I are the volatilities, S_Y, S_C, and S_I skewness, K_Y, K_C, and K_I kurtosis, and ρ_i^Y, ρ_i^C and ρ_i^I ith-order autocorrelations of log output, consumption, and investment growth, respectively. Prob_Y, Size_Y, and Dur_Y and Prob_C, Size_C, and Dur_C are the probability, size, and duration of output and consumption disasters, respectively. $E[U]$, S_U, and K_U are the mean, skewness, and kurtosis of monthly unemployment rates, respectively, and σ_U is the quarterly volatility. $\sigma_Y, \sigma_C, \sigma_I$, $E[U]$, and σ_U are in percent. $E[R - R^f]$ is the equity premium, $E[R^f]$ the risk-free rate, σ_R the stock market volatility, and σ_{R^f} the interest rate volatility, all in percent per annum.

	BL	HP	CA	RU			BL	HP	CA	RU
σ_Y	4.25	4.28	4.25	4.3		σ_C	3.61	3.71	3.06	3.68
S_Y	0.65	0.1	0.11	0.69		S_C	0.69	0.14	0.09	0.72
K_Y	10.77	4.26	4.38	12.62		K_C	12.29	4.86	5	14.26
ρ_1^Y	0.2	0.15	0.19	0.17		ρ_1^C	0.2	0.16	0.21	0.16
ρ_2^Y	−0.12	−0.13	−0.09	−0.14		ρ_2^C	−0.13	−0.13	−0.07	−0.15
ρ_3^Y	−0.12	−0.1	−0.07	−0.12		ρ_3^C	−0.12	−0.1	−0.06	−0.12
ρ_4^Y	−0.1	−0.08	−0.06	−0.09		ρ_4^C	−0.1	−0.08	−0.06	−0.09
Prob_Y	4	7.52	7.17	3.38		Prob_C	2.14	5.39	3.7	1.76
Size_Y	19.62	16.65	17.45	20.7		Size_C	22.29	16.11	16.31	24.54
Dur_Y	4.57	3.98	4.15	4.65		Dur_C	5.02	4.26	4.84	5.05
σ_I			5.88			$E[U]$	5.83	6.23	6.79	5.7
S_I			0.23			σ_U	18.71	14.81	18.77	19.41
K_I			4.94			S_U	3.29	2.18	2.58	3.46
ρ_1^I			0.17			K_U	17.75	9.05	11.3	19.92
ρ_2^I			−0.11			$E[R - R^f]$				1.78
ρ_3^I			−0.09			$E[R^f]$				2.82
ρ_4^I			−0.08			σ_R				14.32
						σ_{R^f}				1.22

Table D2 : Additional Comparative Statics, Recursive Utility

RU is the benchmark calibration. The remaining columns report six comparative statics: (i, ii) $b = 0.825$ and $b = 0.4$ are for the value of unemployment activities set to 0.825 and 0.4, respectively; (iii) $s = 0.035$ is for the job separation rate set to 0.035; (iv) $\kappa_t = 0.7$ is for the proportional unit costs of vacancy $\kappa_0 = 0.7$ and the fixed unit costs $\kappa_1 = 0$; (v) $\iota = 1.1$ is for the elasticity of the matching function set to 1.1; and (vi) $\eta = 0.05$ is for the workers' bargaining weight set to 0.05. In each experiment, all the other parameters are identical to those in the benchmark calibration. The results are based on 10,000 simulated samples. σ_Y and σ_C denote the volatilities, S_Y and S_C skewness, K_Y and K_C kurtosis, and ρ_i^Y and ρ_i^C the ith-order autocorrelations of log output and consumption growth, respectively. Prob_Y, Size_Y, and Dur_Y are the probability, size, and duration of output disasters, respectively, and Prob_C, Size_C, and Dur_C are analogously defined for consumption disasters. $E[U]$, S_U, and K_U are the mean, skewness, and kurtosis of monthly unemployment rates, respectively, and σ_U is the quarterly unemployment volatility. $\sigma_Y, \sigma_C, \sigma_I$, $E[U]$, and σ_U are in percent. $E[R - R^f]$ is the equity premium, $E[R^f]$ the risk-free rate, σ_R the stock market volatility, and σ_{R^f} the interest rate volatility, all in percent per annum.

	RU	b 0.825	b 0.4	s 0.035	κ_t 0.7	ι 1.1	η 0.05
σ_Y	5.67	2.98	2.47	4.67	3.77	5.77	5.86
S_Y	0.87	0.17	0	0.8	0.48	0.88	0.84
K_Y	15.47	5.53	3.36	14.07	9.85	15.2	14.85
ρ_1^Y	0.21	0.14	0.14	0.17	0.14	0.21	0.23
ρ_2^Y	−0.14	−0.13	−0.12	−0.15	−0.14	−0.14	−0.13
ρ_3^Y	−0.13	−0.1	−0.1	−0.12	−0.11	−0.13	−0.13
ρ_4^Y	−0.1	−0.08	−0.08	−0.09	−0.08	−0.1	−0.11
Prob_Y	4.49	2.58	1.82	3.68	3.14	4.83	4.95
Size_Y	23.92	15.39	13.35	21.48	18.19	23.71	23.83
Dur_Y	4.46	4.8	4.93	4.6	4.68	4.42	4.41
$E[U]$	6.26	4.86	3.97	5.06	5.39	6.7	6.66
σ_U	25.67	8.16	0.12	23.24	13.8	24.99	25.81
S_U	3.66	2.63	0.3	3.76	3.12	3.48	3.47
K_U	20.71	14.58	2.84	22.91	18.99	19.03	18.65

	RU	b 0.825	b 0.4	s 0.035	κ_t 0.7	ι 1.1	η 0.05
σ_C	5.05	2.24	2.04	4.08	3.01	5.14	5.34
S_C	0.88	0.2	0	0.8	0.51	0.89	0.85
K_C	17.09	6.25	3.36	15.52	10.94	16.83	16
ρ_1^C	0.19	0.14	0.14	0.16	0.13	0.2	0.22
ρ_2^C	−0.15	−0.13	−0.12	−0.15	−0.15	−0.15	−0.14
ρ_3^C	−0.13	−0.1	−0.1	−0.12	−0.11	−0.13	−0.13
ρ_4^C	−0.1	−0.08	−0.08	−0.09	−0.08	−0.1	−0.11
Prob_C	2.51	1.06	1.03	1.94	1.35	2.73	3.13
Size_C	28.86	15.32	12.14	25.72	20.84	28.21	27.32
Dur_C	4.84	5.45	5.35	5.02	5.26	4.81	4.74
$E[R - R^f]$	4.45	0.64	0.23	2.67	1.32	4.57	4.33
$E[R^f]$	2.58	2.88	2.86	2.75	2.87	2.59	2.5
σ_R	15.79	13.66	4.68	15.94	16.97	15.61	14.59
σ_{R^f}	1.64	0.63	0.16	1.3	0.98	1.67	1.65

Table D3 : Predicting Excess Returns and Consumption growth with the Log Price-to-Consumption Ratio

The long-horizon predictive regression of excess returns is $\sum_{h=1}^{H}\log(1+R_{t+h})-\log(1+R^f_{t+h}) = \alpha_R+\beta_R\log(P_t/C_t)+\epsilon^R_{t+h}$, in which H is the forecast horizon, R_t the real stock market return, R^f the real interest rate, P_t the real stock market index, and C_t real consumption. Excess returns in the data are adjusted for financial leverage. The long-horizon predictive regression of log consumption growth is $\sum_{h=1}^{H}\log(C_{t+h}/C_t) = \alpha_C+\beta_C\log(P_t/C_t)+\epsilon^C_{t+h}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a variance of unity. H ranges from one year (1y) to five years (5y), and from one quarter (1q) to 20 quarters (20q). The t-statistics are Newey-West (1987) adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and the R^2s are in percent. The model moments are averaged across 10,000 simulations.

H	1y	2y	3y	4y	5y	1q	4q	8q	12q	16q	20q
	U.S. annual data, 1836–2013					U.S. quarterly data, 1947q2–2013q4					
β_R	−3.68	−7.64	−10.43	−13.66	−16.65	−0.74	−3.39	−6.78	−9.69	−12.18	−15.03
t_R	−4.54	−5.49	−5.2	−5.57	−6.43	−2.31	−2.75	−2.94	−3.24	−3.36	−3.68
R^2_R	8.1	17.09	23.65	31.31	38.97	1.75	8.06	16.64	24.56	30.67	36.69
β_C	0.46	0.21	0.05	−0.10	−0.15	0.06	0.07	−0.05	−0.16	−0.24	−0.34
t_C	1.69	0.43	0.07	−0.10	−0.13	1.66	0.40	−0.13	−0.27	−0.31	−0.34
R^2_C	1.48	0.15	0.01	0.02	0.03	1.23	0.27	0.05	0.31	0.51	0.77
	Recursive utility					Recursive utility					
β_R	−1.91	−3.44	−4.71	−5.75	−6.62	−0.74	−2.72	−4.89	−6.68	−8.20	−9.49
t_R	−1.83	−2.17	−2.24	−2.28	−2.31	−1.65	−2.04	−2.19	−2.38	−2.60	−2.82
R^2_R	2.04	3.49	4.65	5.59	6.35	1.17	4.14	7.35	9.96	12.14	13.97
β_C	−0.67	−1.68	−2.69	−3.62	−4.42	−0.00	−0.56	−1.53	−2.51	−3.41	−4.19
t_C	−2.19	−2.82	−3.37	−3.83	−4.19	−1.11	−2.14	−2.98	−3.65	−4.24	−4.76
R^2_C	7.62	11.95	16.28	20.1	23.32	2.56	8.54	16.09	22.43	27.63	31.82

In particular, in the historical sample, the negative slopes for the log price-to-consumption ratio across 1- to 5-year horizon are all more than 4.5 standard errors from zero, and the R^2 rises monotonically from 8.1% to 39%. The postwar evidence is similar, although the t-statistics for the slopes are somewhat smaller. In contrast, the consumption growth is largely unpredictable. None of the slopes are significant at the 5% level. The R^2 starts at 1.5% in the 1-year horizon, but quickly fades to zero afterward. The postwar evidence is similar.

The model predicts time-varying risk premiums. The price-to-consumption slopes are all negatively, and mostly significant, but the amount of predictability is weaker in the model than in the data. In the annual frequency, the R^2 rises from 2% to only 6.4% (39% in the data), and in the quarterly frequency, the R^2 goes from 1.2% to 14% (36.7% in the data). More important, the model overstates the predictability of consumption growth. The slopes are all negative, and mostly significant. The annual R^2 ranges from 7.6% to 23.3%, and the quarterly R^2 from 2.6% to 31.8%. In contrast, R^2 never exceeds 2% in the data.

Table D4 reports long-horizon regressions of volatilities of excess returns and consump-

Table D4 : Predicting Volatilities of Stock Market Excess Returns and Consumption Growth with the Log Price-to-Consumption Ratio

For a given H, the excess return volatility is $\sigma^R_{t,t+H-1} = \sum_{h=0}^{H-1} |\epsilon^R_{t+h}|$, in which ϵ^R_{t+h} is the h-period-ahead residual from the first-order autoregression on excess returns, $\log(1 + R_{t+1}) - \log(1 + R^f_{t+1})$. Excess returns in the data are adjusted for financial leverage. The long-horizon predictive regression of the excess return volatility is $\log \sigma^R_{t+1,t+H} = \alpha_R + \beta_R \log(P_t/C_t) + u^R_{t+h}$. The consumption growth volatility is $\sigma^C_{t,t+H-1} = \sum_{h=0}^{H-1} |\epsilon^C_{t+h}|$, in which ϵ^C_{t+h} is the h-period-ahead residual from the first-order autoregression on consumption growth, $\log(C_{t+1}/C_t)$. The long-horizon predictive regression of the consumption growth volatility is $\log \sigma^C_{t+1,t+H} = \alpha_C + \beta_C \log(P_t/C_t) + u^C_{t+h}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a variance of unity. For annual data, H ranges from one (1y) to five years (5y), and for quarterly data, from one (1q) to 20 quarters (20q). The t-statistics are Newey-West (1987) adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and the R^2s are in percent. The model moments are averaged across 10,000 simulations.

H	1y	2y	3y	4y	5y	1q	4q	8q	12q	16q	20q
	U.S. annual data, 1836–2013					U.S. quarterly data, 1947q2–2013q4					
β_R	−8.36	0.09	−1.12	−1.31	−0.13	−10.79	−1.23	3.19	5.08	5.55	5.56
t_R	−0.96	0.01	−0.24	−0.31	−0.03	−1.74	−0.21	0.53	0.92	1.06	1.11
R^2_R	0.44	0.00	0.04	0.07	0.00	0.93	0.05	0.55	2.07	3.45	4.47
β_C	−6.48	−7.85	−6.88	−5.75	−4.79	−15.87	−14.10	−13.04	−12.41	−11.25	−10.17
t_C	−0.72	−0.96	−0.82	−0.63	−0.50	−2.51	−2.16	−1.68	−1.50	−1.35	−1.24
R^2_C	0.32	0.86	0.98	0.88	0.70	2.18	6.19	8.33	9.33	9.49	9.83
	Recursive utility					Recursive utility					
β_R	−3.05	−3.28	−3.10	−2.86	−2.63	−2.16	−3.52	−3.55	−3.27	−2.94	−2.62
t_R	−0.40	−0.57	−0.61	−0.61	−0.59	−0.27	−0.43	−0.54	−0.58	−0.58	−0.57
R^2_R	0.61	1.02	1.56	2.06	2.46	0.87	1.26	2.60	4.14	5.27	6.09
β_C	−34.78	−33.27	−31.51	−29.80	−28.16	−29.42	−24.86	−22.45	−20.75	−19.15	−17.66
t_C	−3.52	−4.01	−3.71	−3.51	−3.34	−2.33	−1.81	−2.19	−2.41	−2.47	−2.47
R^2_C	8.05	14.85	18.24	19.56	19.70	3.11	5.35	11.00	16.07	18.21	18.74

tion growth on the log price-to-consumption ratio. Neither volatility appears predictable in the historical data, but the consumption volatility seems at least somewhat predictable in the postwar data. The slopes are all negative, and significant in short horizons up to four quarters. The R^2 rises from 2.2% at the 1-quarter to 9.8% at the 20-quarter horizon.

The model is largely consistent with the lack of predictability for the excess return volatility. However, the model overstates the predictability of the consumption volatility. In the quarterly frequency, the slopes are all negative, and mostly significant. The R^2 rises from 3.1% at the 1-quarter to 18.7% at the 20-quarter horizon (9.8% in the data).

Finally, we have also calculated the conditional equity premium and stock market volatilities across normal and disaster states in the model. The equity premium is 6.5% in output disasters, 7.5% in consumption disasters, and 3.7% in normal times. These moments are largely consistent with predictive regressions, which show that the model underestimates the amount of return predictability in the data. In addition, the stock market volatility is 16% in both output and consumption disasters, and 15.2% in normal times. These moments are also consistent with the lack of predictability of the stock market volatility in the model.

D.3 Leisure in the Utility Function

In this subsection, we modify the baseline model by incorporating leisure into the utility function. In particular, the household's utility becomes $\log(C_t+hU_t)$, in which $h > 0$ is a constant. This utility is the logarithmic form of the general Greenwood, Hercowitz, and Huffman (1988) preferences. The stochastic discount factor becomes $M_{t+1} = \beta((C_t + hU_t)/(C_{t+1} + hU_{t+1}))$. Going through similar steps as in Section B.1 yields the equilibrium wage:

$$W_t = \eta(X_t + \kappa_t\theta_t) + (1 - \eta)z, \tag{D1}$$

in which $z \equiv b+h$ is the flow value of unemployment activities. The rest of the leisure model, including the intertemporal job creation condition, remains identical to the baseline model.

We set $b = 0.5$ and $h = 0.35$, or $z = 0.85$, to ease comparison with the baseline model. We also rescale the volatility of productivity shocks, σ, to 0.0095 to align the output volatility in the leisure model, 5.5% per annum, with that in the data, 5.6%, as well as that in the baseline model, 5.3%. Table D5 shows that the results from the leisure model are largely comparable with those from the baseline model. The output disaster probability is 4.4%, which is somewhat lower than 5% in the baseline model, but the average size and duration are both close. For consumption disasters, the probability is 2.4%, which is somewhat lower than 2.9% in the baseline model, but the average size is 27%, which is slightly higher than 25.6% in the baseline model. We also report two comparative statics, (i) $b = 0.45$ and $h = 0.4$; as well as (ii) $b = 0.4$ and $h = 0.45$. The results seem insensitive to the parameter changes.

Table D5 : Quantitative Results, Leisure in the Utility

BL is the baseline model. The other columns report the results from the leisure model, in which we rescale the volatility of productivity shocks, σ, to be 0.0095. We vary b from 0.5, 0.45, to 0.4, and correspondingly h from 0.35, 0.4, to 0.45. All the other parameters are identical to those in the baseline model. σ_Y and σ_C are the volatilities, S_Y and S_C, K_Y and K_C kurtosis, and ρ_i^Y and ρ_i^C ith-order autocorrelations of log output and consumption growth, respectively. Prob_Y, Size_Y, and Dur_Y and Prob_C, Size_C, and Dur_C are the probability, size, and duration of output and consumption disasters, respectively. $E[U]$, S_U, and K_U are the mean, skewness, and kurtosis of monthly unemployment rates, respectively, and σ_U is the quarterly volatility. $\sigma_Y, \sigma_C, \sigma_I$, $E[U]$, and σ_U are in percent. The results are based on 10,000 simulations.

	BL	Leisure				BL	Leisure		
b	0.85	0.5	0.45	0.4	b	0.85	0.5	0.45	0.4
h	0	0.35	0.4	0.45	h	0	0.35	0.4	0.45
σ	0.01	0.0095	0.0095	0.0095	σ	0.01	0.0095	0.0095	0.0095
σ_Y	5.31	5.5	5.53	5.68	σ_C	4.65	4.91	4.95	5.11
S_Y	0.85	0.85	0.83	0.81	S_C	0.91	0.86	0.85	0.82
K_Y	12.8	15.83	16	16.61	K_C	14.4	17.02	17.31	17.81
ρ_1^Y	0.24	0.18	0.17	0.16	ρ_1^C	0.23	0.16	0.16	0.15
ρ_2^Y	-0.11	-0.16	-0.16	-0.16	ρ_2^C	-0.12	-0.18	-0.17	-0.17
ρ_3^Y	-0.12	-0.13	-0.13	-0.13	ρ_3^C	-0.12	-0.13	-0.13	-0.13
ρ_4^Y	-0.11	-0.1	-0.09	-0.09	ρ_4^C	-0.11	-0.09	-0.09	-0.09
Prob_Y	5.04	4.37	4.38	4.39	Prob_C	2.86	2.41	2.42	2.43
Size_Y	22.22	22.7	22.98	23.18	Size_C	25.64	27.01	27.51	27.83
Dur_Y	4.44	4.47	4.46	4.45	Dur_C	4.91	4.88	4.86	4.84
$E[U]$	6.28	6	5.97	6	S_U	3.52	3.63	3.66	3.7
σ_U	23.41	24.15	24.51	25.38	K_U	19.18	21.09	21.59	22.01

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