Cash-Flow Taxes in an International Setting
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Online Appendix

APPENDIX A: Derivations of various results in the main text

1. **Derivation of results around (4.11) that (1) $\frac{d(e-e^*)}{ds} < 0$; and (2) consumer prices are the same for an initial symmetric equilibrium with marginal cost pricing**:

Combining expressions (3.9') and (3.11') and then taking the derivative with respect to *s*,

(A.1) $$\frac{1}{f_1^*}(f_1 - f_1^*)(1-s^*) + (1-qf_1)(s-s^*) = 0$$

(A.2) $$\frac{1}{f_1^*}(1-s^*)\frac{df_1}{ds} - \frac{f_1}{f_1^{*2}}(1-s^*)\frac{df_1^*}{ds} + (1-qf_1) - q(s-s^*)\frac{df_1}{ds} = 0$$

With initial marginal cost pricing, $q = 1/f_1^* = 1/f_1$, so (A.2) reduces to

(A.3) $$(1-s)\frac{df_1}{ds} = (1-s^*)\frac{df_1^*}{ds}$$

Combining expressions (3.10') and (3.11') and then differentiating with respect to *s* yields,

(A.4) $$\frac{1}{f_1^*}(f_2 - f_2^*)(1-s^*) - qf_2(s-s^*) = 0$$

(A.5) $$\frac{1}{f_1^*}(1-s^*)\frac{df_2}{ds} - \frac{1}{f_1^*}(1-s^*)\frac{df_2^*}{ds} - \frac{(f_2-f_2^*)}{f_1^{*2}}(1-s^*)\frac{df_1^*}{ds} + -qf_2 - q(s-s^*)\frac{df_2}{ds} = 0$$

Using initial marginal cost pricing, $q = \frac{1}{f_1^*}$, and using (A.4) to substitute for $(f_2 - f_2^*)$ yields:

(A.6) $$(1-s^*)\frac{df_2}{ds} - (1-s^*)\frac{df_2^*}{ds} - \frac{f_2}{f_1^*}(s-s^*)\frac{df_1^*}{ds} + -f_2 - (s-s^*)\frac{df_2}{ds} = 0$$

Starting from an equilibrium in which $s = s^*$, expressions (A.3) and (A.6) reduce to:

(A.7) $$\frac{df_1}{ds} = \frac{df_1^*}{ds}$$

and

(A.8) $$\frac{df_2}{ds} - \frac{df_2^*}{ds} = \frac{f_2}{1-s}$$

Combining (A.7) and (A.8) and noting that starting from a symmetric equilibrium the second derivatives of the production functions are the same across countries, we obtain:

(A.9) $$\frac{dm^*}{ds} - \frac{dm}{ds} = -\frac{f_{11}}{D}\frac{f_2}{(1-s)} > 0$$

(A.10) $$\frac{dk^*}{ds} - \frac{dk}{ds} = \frac{f_{12}}{D}\frac{f_2}{(1-s)} > 0$$

where $D = f_{11}f_{22} - f_{12}f_{21} > 0$ is the determinant of the Hessian of the production function. Since both *m* and *k* shift abroad with an increase in *s*, so must the first stage of production.

Note that (3.9)-(3.12) imply that $p_1 h^{'} = p_1^* h^{*'}$. Since marginal utility equals the price in each country, this implies that $u^{'}\big(h(x_1)\big)h^{'}(x_1) = u^{*'}(h(x_1^*))h^{'}(x_1^*)$, where we have used the fact that the *functions* $h(.)$ and $h^{'}(.)$ are the same in the two countries. Thus, if preferences are the same in the two countries, we have $u^{'}\big(h(x_1)\big)h^{'}(x_1) = u^{'}(h(x_1^*))h^{'}(x_1^*)$. This expression is satisfied if $x_1 = x_1^*$, and the solution is unique: since both h'' and u'' are negative, the derivative of either side with respect to its argument is negative, so the equality cannot hold for $x_1 \neq x_1^*$. Hence the increase in *s* decreases domestic production but does not change relative consumption. Therefore, domestic exports fall with *s*.

Note also that, because consumption of good 1 remains the same in the two countries, so must the price of good 1, again under the assumption of common preferences, equal initial tax rates, and marginal cost pricing.

2. Source-based capital income taxation

As explained in Section 5.1, we model an income tax by permitting a partial deduction for the cost of capital expenditure. Specifically, instead of permitting a deduction of K, the capital income tax permits only a deduction of $(1-\mu)K$, where μK accounts for the normal return to capital. The impact of this change on the first order conditions in the source-based case are set out in Section 5.1, which discusses the incentive to marginally increase μ, offset by a fall in the tax rate, s, to maintain revenue neutrality. The condition for this to improve welfare is (5.2). In the general case, this condition can be shown to be:

$$\text{(A.11)} \quad \frac{-\beta[p_1c_1+q(e-e^*)-(1-\mu)k]-[1-\beta(1-s)]c_1\frac{dp_1}{ds}+\beta(1-s^*)c_1^*\frac{dp_1^*}{ds}}{[p_1c_1+q(e-e^*)-(1-\mu)k]+s\left(q-\frac{1}{f_1}\right)\left(\frac{d(e-e^*)}{ds}\right)+s\left(\frac{f_2dm}{f_1\,ds}\right)+s\delta\frac{dk}{ds}+sc_1\frac{dp_1}{ds}} <$$

$$\frac{-\beta sk-[1-\beta(1-s)]c_1\frac{dp_1}{d\mu}+\beta(1-s^*)c_1^*\frac{dp_1^*}{d\mu}}{sk+s\left(q-\frac{1}{f_1}\right)\left(\frac{d(e-e^*)}{d\mu}\right)+s\left(\frac{f_2dm}{f_1\,d\mu}\right)+s\delta\frac{dk}{d\mu}+sc_1\frac{dp_1}{d\mu}}$$

Where the two countries are initially in a symmetric equilibrium, with $s = s^*$ and $\beta = \beta^* = ½$, then there is no transfer pricing manipulation ($q = 1/f_1$), no exports ($e = e^*$), and the terms c_1 and dp_1/ds and $dp_1/d\delta$ are the same in both countries. In this case, the expression simplifies to:

$$\text{(A.12)} \quad \frac{-\beta[p_1c_1-(1-\mu)k]-sc_1\frac{dp_1}{ds}}{[p_1c_1-(1-\mu)k]+s\left(\frac{f_2dm}{f_1\,ds}\right)+s\delta\frac{dk}{ds}+sc_1\frac{dp_1}{ds}} < \frac{-\beta k-c_1\frac{dp_1}{d\mu}}{k+\left(\frac{f_2dm}{f_1\,d\mu}\right)+\mu\frac{dk}{d\mu}+c_1\frac{dp_1}{d\mu}}$$

As $s \to 0$, the expression simplifies to

$$\text{(A.13)} \quad -\left(\frac{f_2}{f_1}\frac{dm}{d\mu}\right)-\mu\frac{dk}{d\mu} < -c_1\frac{dp_1}{d\mu}$$

which is never satisfied because an increase in the capital income tax shifts production factors away from the home country (and hence the left-hand side is positive) and raises the production cost and hence the price of good 1 (so that the right-hand side is negative).

In analyzing a marginal switch from a source-based capital income tax to a destination-based cash-flow tax, (4.10) has some additional terms to reflect the partial deduction of capital expenditure, and becomes

$$-s\left(\frac{f_2}{f_1}\frac{dm}{ds}\right) - s\left(q - \frac{1}{f_1}\right)\left(\frac{d(e-e^*)}{ds}\right) - s\mu\frac{dk}{ds}$$

$$> \beta^*[p_1c_1 + q(e-e^*) - (1-\mu)k] + \beta(1-s^*)c_1^*\frac{dp_1^*}{ds} - \beta^*(1-s)c_1\frac{dp_1}{ds}$$

In the symmetric case, this becomes (5.2).

3. Sales apportionment

From (5.3), profits are:

(A.14) $\pi = (p_1h(x_1) + p_1^*h(f(k,m) + f(K-k, M-m) - x_1) - K)[1 - ta - t^*(1-a)]$,

where $a = \frac{p_1h(x_1)}{p_1h(x_1)+p_1^*h(f(k,m)+f(K-k,M-m)-x_1)}$.

Differentiating with respect to K yields:

(A.15) $\left(p_1^*h'(x_1^*)f_1(k^*,m^*) - 1\right)[1 - ta - t^*(1-a)] - \pi(t-t^*)\frac{da}{dK} = 0.$

But $\frac{da}{dK} = -\frac{a}{p_1c_1+p_1^*c_1^*}p_1^*h'(x_1^*)f_1(k^*,m^*)$, so (A.15) simplifies to:

(A.16) $\left[1 + \frac{a\pi^G(t-t^*)}{[1-ta-t^*(1-a)](p_1c_1+p_1^*c_1^*)}\right]p_1^*h'(x_1^*) = \frac{1}{f_1(k^*,m^*)}$

Expression (5.5) follows from the fact that there is production efficiency. A similar expression for the home country follows from the first-order condition with respect to x_1.

APPENDIX B: The impact of variations in relative country size

In the tax competition literature, a standard finding is that the optimal behavior of small and large countries differs. How would differences in country size affect our results? Intuitively, the smaller the country's relative size, the greater the responsiveness of the multinational to changes in its tax policy. But a smaller country may also own a smaller share of the multinational's shares, and so may see a greater opportunity to export taxes to foreign shareholders.

Both of these effects, which work in opposite directions, are present as a country's size falls. However, at least where the country's ownership share is proportional to its size, the effects exactly cancel and changes in relative size have no effect on the choice between source-based and destination-based taxes. This somewhat surprising result may be specific to our model, but it does illustrate that the direction of the net impact of a change in relative size is not clear.

We suppose that, rather than there being one individual with unit endowment in each country, there are α and α^*, with $\alpha + \alpha^* = 1$. Also assume that the shares of ownership in the multinational are the same, i.e., that $\beta = \alpha$ and $\beta^* = \alpha^*$. In order to scale the location-specific fixed factors to country size, let the production functions $f(\cdot)$ and $h(\cdot)$ be expressed in per capita terms, with $f(\cdot)$ the same across countries and $h(\cdot)$ the same as well when preferences are identical. For this case, it may be shown that expression (4.11) still holds, with c_1, k, and m all now interpreted in per capita rather than absolute terms. Thus, as the country's size falls, tax exporting increases and this makes keeping the source-based tax more attractive. As to the left-hand side of (4.11), note that the expression accounting for the use of M is now

(2.3') $$M = \beta m + \beta^* m^*$$

Thus, $\beta^* \frac{dm^*}{ds} + \beta \frac{dm}{ds} = 0$, so (A.9) implies that $\frac{dm}{ds} = \beta^* \frac{f_{11}}{D} \frac{f_2}{(1-s)}$, which increases in size as the home country's relative size decreases, i.e., as β^* increases. Thus, the left- and right-hand sides of (4.11) are both scaled by β^* and the effects of country size on the tax-exporting and distortion effects cancel.

APPENDIX C: The impact of local ownership of fixed factors

We have assumed that all three sources of rents accrue to multinationals. How would our results change if a greater share of these rents accrued exclusively to domestic factors, rather than to shareholders (some foreign) of the multinational? Intuition suggests that this would reduce the scope for tax exporting and make adoption of destination-based taxation more attractive, but is this actually the case?

We modify the model, assuming that rents to fixed factors accrue to domestic residents instead of to the multinational. There are two fixed factors implicit in the production functions $f(k, m)$ and $h(x_1)$. To make these explicit, we can rewrite the intermediate production function $f(\cdot)$ and the final production function $h(\cdot)$ each as having an additional argument, e.g., $f(k, m, r)$ and $h(x_1, \rho)$, with constant returns to scale and (assuming the multinational is a price-taker with respect to these fixed factors) with the corresponding competitive returns to these arguments denoted by q_r and q_ρ in the home country and likewise with an asterisk in the foreign country.

With these additional factors taken into account, the firm's objective is to maximize profits as given in expression (3.8) minus $(q_r r + q_\rho \rho)(1-s) + (q_r^* r^* + q_\rho^* \rho^*)(1-s^*)$, assuming that the fixed-factor rents are taxed at the same tax rate in each country as the multinational is. With this modification of its objective, the firm's first-order conditions given in (3.9')-(3.12') are unchanged, and there are four new first-order conditions for the use of each of the fixed factors:

(C.1) ρ: $p_1 h_2 = q_\rho$

(C.2) ρ^*: $p_1^* h_2^* = q_\rho^*$

(C.3) r: $p_1^* h^{*'} f_3 (1-s^*) - q f_3 (s-s^*) = q_r(1-s)$

(C.4) r^*: $p_1^* h^{*'} f_3^* = q_r^*$

where $h_2 = c_1 - h' x_1$ and $f_3 = f - f_1 k - f_2 m$ (and similarly for the foreign country). Note that by the symmetry of the set-up, it also follows that $p_1 h' f_3 = q_r$. In equilibrium, of course, the four fixed factor prices will be determined by the market clearing conditions that demand for each of the fixed factors equals its unit supply.

With this modification, consider again the issue of whether the home country will wish to shift from a source-based tax to a destination-based tax. In place of equation (4.6), the income of domestic residents is

(C.5) $$y = (1-z)[\beta\pi + \beta^* D(1-s) - \beta F(1-s^*)]$$

where π is as defined in expression (3.8), $D = q_r r + q_\rho \rho$ and $F = q_r^* r^* + q_\rho^* \rho^*$ (and each rent quantity equals 1 in equilibrium).

Based on (C.5), the change in domestic income with respect to s is now:

(C.6) $\frac{dY}{ds} = \frac{dy}{ds} - c_1 \frac{dp_1}{ds} = \begin{Bmatrix} -\beta\big(p_1 c_1 + k - q(e - e^*)\big) - \beta^* D + \\ (1-s)\left(\beta c_1 \frac{dp_1}{ds} + \beta^* \frac{dD}{ds}\right) + (1-s^*)\left(\beta c_1^* \frac{dp_1^*}{ds} - \beta \frac{dF}{ds}\right) \end{Bmatrix} - c_1 \frac{dp_1}{ds}$

where the remaining terms vanish due to the envelope theorem, from the firm's maximization of $\pi - D(1-s) - F(1-s^*)$. Adding this expression to dT/ds as defined in (4.9) yields, after some algebra:

(C.7) $-s\left(\frac{f_2}{f_1}\frac{dm}{ds}\right) - s(q - \bar{q})\left(\frac{d(e-e^*)}{ds}\right) > \quad \beta^*(p_1 c_1 + q(e - e^*) - k - D)$

$$+(1-s^*)\beta\left(\mathrm{c}_1^* \frac{\mathrm{dp}_1^*}{\mathrm{ds}} - \frac{\mathrm{dF}}{\mathrm{ds}}\right) - (1-s)\beta^*\left(c_1 \frac{dp_1}{ds} - \frac{dD}{ds}\right)$$

where $\bar{q} = 1/f_1$ is the marginal cost of the intermediate good produced at home (likewise for $\bar{q}^*$ abroad).

Once again assuming a symmetric initial equilibrium, this expression reduces to:

(C.8) $-s\left(\frac{f_2}{f_1}\frac{dm}{ds}\right) > \ \frac{1}{2}(p_1 c_1 - k - D) + \frac{(1-s)}{2}\left(\frac{dD}{ds} - \frac{\mathrm{dF}}{\mathrm{ds}}\right).$

Since, in the symmetric equilibrium, domestic and foreign fixed factor returns are profits in each country excluding returns to managerial capital (by assumption measured at true marginal cost),

(C.9) $D = p_1 c_1 - k - \bar{q} f_2 m\,; \qquad F = p_1 c_1 - k^* - \bar{q} f_2^* m^*,$

it may be also be shown (again using the envelope theorem) that

(C.10) $\frac{dD}{ds} = c_1 \frac{dp_1}{ds} - m \frac{d(\bar{q} f_2)}{ds}\,; \qquad \frac{dF}{ds} = c_1 \frac{dp_1}{ds} - m^* \frac{d(\bar{q}^* f_2^*)}{ds}$

But, using (A.7) – which implies that $\frac{d\bar{q}}{ds} = \frac{d\bar{q}^*}{ds}$ – and (A.8),

$$(1-s)\left(\frac{dD}{ds}-\frac{dF}{ds}\right)=(1-s)\left(m^*\frac{d(\bar{q}^*f_2^*)}{ds}-m\frac{d(\bar{q}f_2)}{ds}\right)$$

$$=(1-s)f_2(m^*-m)\frac{d\bar{q}}{ds}+\bar{q}\left[m^*\left(-f_2+(1-\mathrm{s})\frac{df_2}{ds}\right)-m(1-s)\frac{df_2}{ds}\right]=-\bar{q}f_2m^*,$$

so (C.8) may be rewritten

$$\text{(C.11)}\quad -s\left(\frac{f_2}{f_1}\frac{dm}{ds}\right)>\ \tfrac{1}{2}(p_1c_1-k-(p_1c_1-k-\bar{q}f_2m)-\bar{q}f_2m^*)=\tfrac{1}{2}\big(\bar{q}f_2(m-m^*)\big)=0\,.$$

Thus, unlike in the symmetric equilibrium in which all earnings go to the multinational, the home country will definitely wish to move away from the source-based tax. In this situation, with a smaller component of earnings going to the multinational and its shareholders worldwide, there are no opportunities for tax exporting because there are no domestic production or consumption rents accruing to foreigners.

Appendix D. Multinational's advantage as a public good

We have treated the multinational as possessing a firm-specific mobile factor, managerial capital, which is in fixed supply. But some firm-specific factors, such as patents and other intangible assets, might be better characterized as having at least some public good aspects, their use in one location not fully precluding their use in the other. How might this affect our results?

The answer depends on what assumptions we maintain about other factor inputs. To the extent that the firm still utilizes the factors of production assumed in our model, the addition of a public input would have little impact on the analysis, effectively reducing costs in both countries by increasing output given the levels of the other factors, but not altering the incentives. There would still be local decreasing returns to the use of capital and managerial capital, and still the same equilibrium conditions. On the other hand, if the firm had a public input but did not use managerial capital in production, the only remaining distortion would be to the internal transfer price used in the export of the firm's first-stage output from one country to the other.

APPENDIX E: A Cobb-Douglas numerical simulation

This appendix sets out a simple numerical simulation approach to illustrating the effects of transfer pricing manipulation (used in Section 3.1) and Nash equilibrium tax rates (used in Section 4.1). We assume that both countries have preferences and production characterized by Cobb-Douglas functions. The first-stage production function in each country (with "*" superscripts for the foreign country here and in the remaining equations) is:

(E.1) $f(k, m) = Ak^{\alpha}m^{\gamma}$

and that the second-stage production function is:

(E.2) $h(x_1) = Dx_1^{\delta}$

Preferences for good-1 consumption are:

(E.3) $u(c_1) = \theta c_1^{\varphi}$

and preferences for public good consumption are:

(E.4) $v(g) = Bg^{\xi}$

For the base case, we initially assume the same values of the parameters for each country, with

$$(A, \alpha, \gamma, D, \delta, \theta, \varphi, B, \xi) = (4, 0.4, 0.4, 1, 0.5, 3, 0.5, 4, 0.25).$$

In Section 3.1, we consider the impact of transfer pricing manipulation on the size and allocation of production, and the size and direction of exports. We start with the benchmark case in which the transfer price is equal to the inverse marginal product of capital, so that $q = \frac{1}{f_1} = \frac{1}{f_1^*}$. We then allow for transfer pricing manipulation of 10% of the resulting marginal product, so that $q = 0.9/f_1$ in the case of underpricing and $q = 1.1/f_1$ in the case of overpricing, if the home country exports, with the foreign country's marginal product of capital applying if the foreign country exports. We use the initial parameter values above in evaluating production and consumption based on the first order conditions (3.9')-(3.12'). To allow for a lower preference for good 1, we set either θ or θ^* = 1.

In Section 4.1, we solve for two Nash equilibria, one in which the two countries are constrained to use only source-based cash-flow taxes, and the other in which countries choose source- and destination-based cash-flow taxes simultaneously. We note in the text where we vary the assumptions about the values of the parameters.

Parameters for Table 2

	α	γ	θ	δ
Base case	0.4	0.4	3	0.5
k and m less productive	0.425	0.425	3	0.5
k more productive; m less productive	0.45	0.35	3	0.5
x more productive; lower preference for good 1	0.4	0.4	1	0.75
x less productive; higher preference for good 1	0.4	0.4	4	0.25