ONLINE APPENDIX:
Ramsey strikes back:
Optimal commodity taxes and redistribution in the presence of salience effects

Hunt Allcott, Benjamin Lockwood, and Dmitry Taubinsky*

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A Mathematical Appendix

Formal definition of $\alpha(z)$

An increase dy in a consumer's (after-tax) income has the following three effects:

1. By the envelope theorem, the utility impact of an increase in net income equal to dy is $dy \cdot \frac{u_1(c_1,c_2)}{p_1(1+t_1)}$. (We denote partial derivatives using the notation $u_1 := \frac{\partial u}{\partial c_1}$, etc., throughout.)
2. This generates fiscal externalities equal to $dy \frac{dc_1}{dy} p_1 t_1 + dy \frac{dc_2}{dy} p_2 t_2$.
3. By changing consumption of c_2, this also alters the quantity of externalities (or internalities) produced, by $-dy \frac{dc_2}{dy} \chi$.

The net effect is thus

$$\alpha(z) = \frac{u_1}{p_1(1+t_1)} + \frac{dc_1}{dy} p_1 t_1 + \frac{dc_2}{dy} \left(p_2 t_2 - \chi \right).$$

Proof of Propositions 1 and 2

Let $(\hat{c}_1(z), \hat{c}_2(z))$ denote the consumption bundle that a z-earner anticipates consuming when setting labor supply, while $(c_1(z), c_2(z))$ denotes the bundle they will actually choose.

For this proof, we consider the following joint perturbation of commodity and income taxes, which preserves labor supply choices: the commodity tax t_2 is raised by dt while the income tax is reduced by $dt \cdot \theta \hat{c}_2(z) p_2$ at each income z. At the time of labor supply choice, consumers perceive the commodity tax increase to be θdt, a reform which, on its own, reduces their anticipated utility from each possible choice of z by $dt \cdot \theta p_2 \hat{c}_2(z) u_1(\hat{c}_1(z), \hat{c}_2(z))$. By construction, the income tax reduction also *raises* anticipated utility by $dt \cdot \theta p_2 \hat{c}_2(z) u_1(\hat{c}_1(z), \hat{c}_2(z))$. Together, these reforms have offsetting effects on consumers' anticipated utility from

*Allcott: New York University and NBER. hunt.allcott@nyu.edu. Lockwood: Wharton and NBER. ben.lockwood@wharton.upenn.edu. Taubinsky: UC Berkeley and NBER. dmitry.taubinsky@berkeley.edu. We thank Keith Ericson for comments. We are grateful to the Sloan Foundation for grant funding. The model and derivations of the results follow Lockwood and Taubinsky (2017); see that paper for a generalization of the results to the case in which there is correlated preference heterogeneity.

each possible choice of z, so the joint reform does not alter earnings decisions. This design therefore simplifies the characterization of the total welfare effect of this reform, which can be decomposed into the following effects for each z-earner.

1. The reform mechanically raises revenue

$$dt \cdot (c_2(z) - \theta \hat{c}_2(z))p_2 = dt \cdot (1 - \hat{\theta})c_2(z)$$

from each consumer, at a marginal value of public funds equal to λ.

2. The social value of the resulting mechanical change in income for each consumer (including the resulting fiscal externalities through changes in c_1 and c_2 due to income effects) is

$$-dt \cdot \alpha(z)(c_2(z) - \theta \hat{c}_2(z))p_2 = -dt\alpha(z)(1 - \hat{\theta}(z))c_2(z)p_2$$

.

3. The commodity tax change also generates a substitution effect from c_2 to c_1. Since the resulting change in income has mostly been compensated through the income tax reform (and the remaining true income change has been handled through the income effects in (2)) we can write this effect in terms of the compensated elasticity of demand for c_2. Specifically, the change in c_2 consumption due to a compensated tax change of dt is $d\bar{c}_2 = -dt \cdot \xi^c \frac{\bar{c}_2}{1+t_2}$. Correspondingly, the change in c_1 consumption from this readjustment is $d\bar{c}_1 = -d\bar{c}_2 \frac{p_2(1+t_2)}{p_1(1+t_1)}$. Therefore the total impact of fiscal externalities from this adjustment is equal to

$$d\bar{c}_1 p_1 t_1 + d\bar{c}_2 p_2 t_2 = dt \cdot \xi^c \bar{c}_2 p_2 \left(\frac{t_1}{1+t_1} - \frac{t_2}{1+t_2} \right),$$

weighted by the marginal value of public funds λ.

4. Finally, this substitution from c_2 to c_1 also alters externalities. This generates a welfare change of $-d\bar{c}_2\chi = dt \cdot \xi^c \frac{\bar{c}_2}{1+t_2}\chi$.

Under the optimal policy, the sum of first-order effects (1)–(4) must equal zero:

$$\lambda \xi^c \bar{c}_2 p_2 \left(\frac{t_1}{1+t_1} - \frac{t_2}{1+t_2} \right) + \frac{\bar{c}_2 \xi^c}{(1+t_2)}\chi + (1-\theta)p_2 E[(1-\hat{\theta}(z))(\lambda - \alpha(z))c_2(z)] = 0$$

and thus

$$t_2 - t_1 \left(\frac{1+t_2}{1+t_1} \right) = \frac{\chi}{\lambda p_2} + \frac{1+t_2}{\lambda \xi^c} \frac{E[(1-\hat{\theta}(z))(\lambda - \alpha(z))c_2(z)]}{\bar{c}_2}$$

at the optimal policy.

To complete the proof, we show that if an optimum can be implemented with some pair of commodity tax rates (t_1^*, t_2^*), then it can also be implemented with the pair of tax rates (t_1^{**}, t_2^{**}) satisfying $t_1^{**} = 0$. To that end, let T^* be the optimal income tax given the tax rates (t_1^*, t_2^*). Now consider $t_1^{**} = 0$, $t_2^{**} = \frac{t_2^* - t_1^*}{1+t_2^*}$, and $T^{**}(z) = \frac{t_1^* z}{1+t_1^*} + \frac{T^*(z)}{1+t_1^*}$. Note that $p_2(1+t_2^{**})(1+t_1^*) = p_2(1+t_2^*)$ and $(z - T^{**}(z))(1+t_1^*) = z - T^*(z)$. Thus, compared to the (t_1^*, t_2^*, T^*) regime, the $(t_1^{**}, t_2^{**}, T^{**})$ regime simply multiplies both the after-tax prices and consumers' after-tax incomes by $\frac{1}{1+t_1^*}$. Because both of the after-tax prices are multiplied by the constant

$A = \frac{1}{1+t_1^*}$, the perceived after-tax price of c_2 is multipled by this constant as well, as the perceived price will be given by

$$\theta A p_2(1+t_2) + (1-\theta) A p_1(1+t_{\cdot})\hat{r} = A\left[\theta p_2(1+t_2) + (1-\theta) p_1(1+t_1)\hat{r}\right]$$

Thus, the set of consumption bundles available to a z-earner is identical under the two different tax regimes, and thus the equilibrium allocation will be identical under the two tax regimes.

Writing anticipated consumption in terms of actual consumption

The optimal tax depends on (mistaken) anticipated consumption $\hat{c}_2(z)$, as is evident in the preceding proof. However, since the difference between anticipated consumption $\hat{c}_2(z)$ and actual consumption $c_2(z)$ is effectively driven by an unanticipated price change from $(1-\theta)\hat{p}_2 + \theta p_2(1+t_2)$ to $p_2(1+t_2)$—a difference of $(1-\theta)(p_2(1+t_2) - \hat{p}_2)$. Thus the resulting change in c_2 consumption (written in terms of the demand elasticity) as a share of actual c_2 consumption, satisfies the following expression:

$$\frac{c_2(z) - \hat{c}_2(z)}{c_2(z)} \approx (1-\theta)\left(1 - \frac{\hat{p}_2}{p_2(1+t_2)}\right)\xi(z),$$

where $\xi(z)$ is the income-conditional (uncompensated) demand elasticity for c_2. (This is an approximation because this effective price change need not be infinitesimal change.) Thus

$$1 - \hat{\theta}(z) \approx (1-\theta) + \theta(1-\theta)\left(1 - \frac{\hat{p}_2}{p_2(1+t_2)}\right)\xi(z).$$