Online Appendices: Implications of U.S. Tax Policy for House Prices, Rents, and Homeownership

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August 2017

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1 Appendix A: Solving the Model

1.1 Finding Equilibrium in the Housing and Rental Markets

Equilibrium in the housing and rental markets is formally defined by the conditions presented in Section 2.7 of the paper. In practice, the market clearing rent (ρ^*) and house price (q^*) are found by finding the (q^*, ρ^*) pair that simultaneously clear both the housing and shelter markets in a simulated economy. The market clearing conditions for a simulated cross section of N agents are

$$\sum_{i=1}^{N} h_i'(q^*, \rho^*|x) = H \tag{1}$$

$$\sum_{i=1}^{N} s_i'(q^*, \rho^*|x) = H. \tag{2}$$

The optimal housing and shelter demands for each agent are functions of the market clearing steady state prices and the agents other state variables (x). Solving for the equilibrium of the housing market is a time consuming process because it involves repeatedly re-solving the optimization problem at potential equilibrium prices and simulating data to check for market clearing until the equilibrium prices are found. The algorithm outlined in the following section exploits theoretical properties of the model such as downward sloping demand when searching for market clearing prices. Taking advantage of these properties dramatically decreases the amount of time required to find the equilibrium relative to a more naive search algorithm.

1.2 The Algorithm

Let q_k represent the kth guess of the market clearing house price, let ρ_k represent a guess of the equilibrium rent, and let $\rho_k(q_k)$ represent the rent that clears the market for housing conditional on house price q_k. The algorithm that searches for equilibrium is based on the following excess demand functions

$$ED_k^h(q_k, \rho_k) = \sum_{i=1}^{N} h_i'(q_k, \rho_k | x) - H \tag{3}$$

$$ED_k^s(q_k, \rho_k) = \sum_{i=1}^{N} s_i'(q_k, \rho_k | x) - H. \tag{4}$$

The equilibrium prices q^* and ρ^* simultaneously clear the markets for housing and shelter, so

$$ED_k^h(q^*, \rho^*) = 0 \tag{5}$$

$$ED_k^s(q^*, \rho^*) = 0. \tag{6}$$

The following algorithm is used to find the market clearing house price and rent.

1. Make an initial guess of the market clearing house price q_k.

2. Search for the rent $\rho_k(q_k)$ which clears the market for owned housing conditional on the current guess of the equilibrium house price, q_k. The problem is to find the value of $\rho_k(q_k)$ such that $ED_k^h(q_k, \rho_k(q_k)) = 0$. This step of the algorithm requires re-solving the agents' optimization problem at each trial value of $\rho_k(q_k)$, simulating data using the policy functions, and checking for market clearing in the simulated data. One useful property of the excess demand function $ED_k^h(q_k, \rho_k(q_k))$ is that conditional on q_k, it is a strictly decreasing function of ρ_k. Based on this property, $\rho_k(q_k)$ can be found efficiently using bisection.

3. Given that the *housing* market clears at prices $(q_k, \rho_k(q_k))$, check if this pair of prices also clears the market for *shelter* by evaluating $ED_k^s(q_k, \rho_k(q_k))$.

   (a) If $ED_k^s(q_k, \rho_k(q_k)) < 0$ and $k = 1$, the initial guess q_1 is too high, so set $q_{k+1} = q_k - \varepsilon$ and go to step (2). This initial house price guess q_1 is too high if $ED_k^s(q_k, \rho_k(q_k)) < 0$ because $ED_k^s(q_k, \rho_k(q_k))$ is decreasing in q_k.

   (b) If $ED_k^s(q_k, \rho_k(q_k)) > 0$ set $k = k + 1$ and $q_{k+1} = q_k + \varepsilon$ and go to step (2).

   (c) If $ED_k^s(q_k, \rho_k(q_k)) = 0$, the equilibrium prices are $q^* = q_k$, $\rho^* = \rho_k(q_k)$, so stop.

2 Appendix B: SCF Data

The 1998, 2007, and 2010 waves of the Survey of Consumer Finances (SCF) are used to construct the cross-sectional moments cited in the study. The SCF is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of U.S. families. The total housing wealth is constructed as the total sum of all residential real estate owned by a household, and is taken to represent the housing wealth qh' in the model. Secured debt (i.e., debt secured by primary or other residence) is used as a model analog of the collateralized debt, m'. The model analogue of the total net worth (i.e., $d' + qh' - m'$) is constructed as the sum of household's deposits in the transaction accounts and the housing wealth (as defined above), net of the secured debt. The total household income reported in the SCF is taken to represent the total household income defined in the model as $y = w + rd' + TRI - \tau^{LL}q(h' - s)$. Data and the SAS code are available upon request, but both can also be found at the SCF website.

3 Appendix C: Sensitivity Analysis

3.1 Risk-free Interest Rate

Table 1 shows the sensitivity of our results to a fifty percent reduction in the risk-free interest rate r, from 4 percent to 2 percent. Since the mortgage interest rate r^m is determined by a constant markup, κ, over r, changes in r directly translate into change in r^m. Hence, changes in r affect both the cost of borrowing and the return on savings.

Before considering the question of how the level of the interest rate affects the predictions of the model with regard to tax reform, it is useful to examine how the interest rate affects the model under the baseline tax system. Columns 1 and 3 in Table 1 describe the steady state economies where $r = 0.04$ and $r = 0.02$, under the baseline tax system where the mortgage interest deduction (MID) is available. When r is lowered from 4 percent to 2 percent, the house price level rises by approximately 16 percent, the rent falls by 1.6 percent, and the price-to-rent ratio rises by 17 percent from its baseline value of 12.3.[1] At the same time, homeownership rate falls slightly from its baseline level of 65 percent to 64.2 percent.

[1] The simulated interest rate elasticity of house prices from the model is consistent with recent empirical evidence in Glaeser, Gottlieb and Gyourko (2012).

Table 1: The Effect of Eliminating the Mortgage Interest Tax Deduction under Alternative Level of the Interest Rate

	(A) Baseline Model ($r = 0.04$)		(B) Lower Interest Rate ($r = 0.02$)	
	(1) Baseline	(2) Eliminate MID	(3) Baseline	(4) Eliminate MID
House price	3.052	2.925	3.530	3.459
Rent	0.248	0.249	0.245	0.242
Price-rent ratio	12.320	11.715	14.399	14.281
Frac. homeowners	0.650	0.702	0.642	0.646
Fraction renter	0.350	0.297	0.357	0.354
Fraction owner-occupier	0.549	0.635	0.541	0.542
Fraction landlord	0.101	0.068	0.102	0.103
Median $\frac{house\ value}{wage}$	3.815	2.925	3.705	3.630
Fraction homeowners in debt	0.648	0.634	0.904	0.900
Average mortgage	2.815	1.931	3.739	3.514
cev^*	—	0.757%	—	0.732%

Notes: Columns (1) and (3) are the baseline tax system, under $r = 0.04$ and $r = 0.02$.
Columns (2) and (4) are the no-mortgage deduction economies, under $r = 0.04$ and $r = 0.02$.
cev^* is the ex ante consumption equivalent variation.

The mechanisms generating these equilibrium responses are discussed in detail in Sommer, Sullivan and Verbrugge (2013), which studies how interest rates affect equilibrium house prices, rents, and homeownership.[2] By way of summary, a lower interest rate reduces the cost of mortgage borrowing as well as the rate of return to the alternative investment, deposits. Both effects increase demand for owned housing, causing house prices to rise. At the same time, there is a portfolio shift: rental property investment becomes relatively more attractive as borrowing costs and the returns to deposits fall.

Two opposing forces produce the slight decline in homeownership from 65 percent when $r = 0.04$ to 64.2 percent when $r = 0.02$. On the one hand, the 200 basis point drop in r lowers the cost of borrowing for aspiring homeowners, making housing investment relatively more affordable. On the other hand, the equilibrium increase in house prices increases down payments, and the lower deposit rate makes it more difficult for prospective buyers to save up for a downpayment. Moreover, with house prices now substantially higher, larger loan amounts are needed to finance housing purchases—the size of the average mortgage rises by nearly 30 percent when r is reduced to 2 percent from 4 percent. On net, these countervailing forces associated with the reduction in r approximately offset, resulting in a homeownership rate that is roughly comparable between the two economies.

[2]This paper does not incorporate progressive taxation or study the effect of the mortgage interest rate deduction on the housing market.

Columns 2 and 4 in Table 1 depict the $r = 0.04$ and $r = 0.02$ economies when the mortgage interest rate deduction is repealed. When the deduction is repealed, the house price and the price-to-rent ratio decline in both models, but the declines are relatively less pronounced when $r = 0.02$. Whereas the house price and the price-to-rent ratio fall by approximately 4 and 5 percent when the deduction is repealed in the $r = 0.04$ economy, they decline by only 2 and 0.1 percent when $r = 0.02$. The forces driving the decline in the house price in both economies are as discussed in Section 5.1: the repeal of the mortgage interest deduction decreases house prices because, ceteris paribus, the after-tax cost of occupying a square foot of housing has risen.

Mirroring the house price effect, the response of homeownership to the repeal of the deduction when $r = 0.02$ is also somewhat muted relative to the $r = 0.04$ economy. This is the case because house prices are significantly higher in the low-interest rate economy. Even though house prices decline after the tax reform, the 2 percent drop in house prices from the significantly higher initial level is insufficient to lower down payment requirements for marginal homeowners to the point where they can afford to become homeowners, all else constant. Moreover, given the significantly lower savings rate, it also takes aspiring homeowners longer to save for a downpayment. Finally, the relative price of homeownership (as captured by the price-to-rent ratio) is almost unchanged after the tax reform when $r = 0.02$, as opposed to the nearly 5 percent decline when $r = 0.04$. As a result, when the deduction is repealed, the homeownership rate rises by 0.4 percentage points in the economy where $r = 0.02$; significantly less than the nearly 5 percentage point increase in the economy where $r = 0.04$.

Importantly, the elimination of the mortgage interest deduction improves welfare in both the high and low interest rate economies. The MID is highly distortionary, even when the interest rate is low, because of the inverse equilibrium relationship between house prices and the interest rate which causes house prices to be significantly higher when rates are low. Mortgage debt rises with house prices: Columns 1 and 3 in Table 1 show that under the baseline tax system, average mortgage debt is 33 percent higher when $r = 0.02$, and fully 90 percent of homeowners are in debt (compared to 65 percent under a 4 percent interest rate). As a result, eliminating the MID has large welfare effects at both levels of the interest rate.

It is interesting to note that the magnitude of the welfare improvement is quite similar at both interest rate levels, even though the homeownership response is much smaller when

$r = 0.02$ (columns 2 and 4 of Table 1). Why is this the case? The answer is that the main mechanism generating the welfare gain—the redistribution of housing from the wealthy to low income households—is still in effect when the interest rate is low. Specifically, under the baseline interest rate of 0.04, when the mortgage interest deduction is repealed, housing is re-allocated from high-income households (the main beneficiaries of the deduction) to low-income households, some of whom are induced to enter homeownership by the decline in the house price and the lowered price-to-rent ratio. In the alternative scenario where $r = 0.02$, the elimination of the MID leads to a similar re-allocation of housing from high to low income households. However, since house prices are much higher when $r = 0.02$, and the drop in house prices is smaller, the reallocation of housing primarily takes place on the intensive margin: relatively low income homeowners purchase larger houses, while high income households downsize.

3.2 Risk Aversion Parameter

The baseline calibration assumes that the risk aversion parameter, σ, equals 2.5. Table 2 shows how setting the risk aversion coefficient to a lower value ($\sigma = 1.5$) affects the model. Specifically, the table shows steady state summary statistics for the baseline tax system and the counterfactual tax system under the two different levels of risk aversion.

The combination of non-convex transaction costs and idiosyncratic income risk in the model implies that when the risk aversion coefficient is lowered, the demand for housing rises, all else equal. Comparing columns 1 and 3 of Table 2 shows that the increase in demand causes house prices to rise (by 3.5 percent) and induces households to increase borrowing (the average mortgage rises by over 20 percent). At the same time, higher house prices are passed through into rents by landlords, so the price-rent ratio remains essentially unchanged. Given that house prices are higher and the relative cost of owning is unchanged when $\sigma = 1.5$, the homeownership rate falls by 1.3 percentage points, from 65 percent in the baseline model to 63.7 percent in the model where $\sigma = 1.5$.

When the mortgage interest deduction is repealed, house prices and the price-to-rent ratio fall under both levels of risk aversion (columns 2 and 4). Under the baseline level of risk aversion, the repeal of the MID increases homeownership from 65 percent to 70 percent (columns 1 and 2). Under the lower level of risk aversion, the quantitative response of

Table 2: The Effect of Eliminating the Mortgage Interest Tax Deduction under Alternative Level of Risk Aversion

	(A) Baseline Model ($\sigma = 2.5$)		(B) Lower Risk Aversion ($\sigma = 1.5$)	
	(1) Baseline	(2) Eliminate MID	(3) Baseline	(4) Eliminate MID
House price	3.052	2.925	3.161	3.066
Rent	0.248	0.249	0.257	0.261
Price-rent ratio	12.320	11.715	12.297	11.756
Frac. homeowners	0.650	0.702	0.637	0.703
Fraction renter	0.350	0.297	0.362	0.297
Fraction owner-occupier	0.549	0.635	0.536	0.623
Fraction landlord	0.101	0.068	0.101	0.080
Median $\frac{\textit{house value}}{\textit{wage}}$	3.815	2.925	3.952	3.218
Fraction homeowners in debt	0.648	0.634	0.723	0.698
Average mortgage	2.815	1.931	3.414	2.534
cev^*	—	0.757%	—	0.928%

Notes: Columns (2) and (4) are the no-mortgage deduction economy.
cev^* is the ex ante consumption equivalent variation.

homeownership to the repeal is slightly larger, as homeownership increases from 64 percent to 70 percent (columns 3 and 4). The reform improves welfare ($cev^* > 0$) under both levels of risk aversion. Mirroring the responses of homeownership, the welfare gain is somewhat larger in the lower risk aversion economy.

3.3 Distribution of Housing

Given the discreteness of the housing choice, and the role of down payment constraints in our model, it is interesting to explore the distribution of renters and homeowners across house sizes. Specifically, one might be interested to know whether renters are disproportionately clustered at the smallest rental unit that is not available for purchase, $\underline{s}$. As the first column in Table 3 illustrates, about 45 percent of renters in the model occupy the smallest size unit that is not available for ownership, $\underline{s}$, while an additional 53 percent of renters occupy the next house size, $h(1)$, which is available for purchase. The remaining 2 percent of renters are dispersed across the larger house sizes.

Turning to homeowners, the second column in Table 3 shows the distribution of homeowners across shelter sizes. While over 90 percent of homeowners occupy house sizes $h(1)$–$h(4)$, nearly 10 percent of homeowners occupy larger housing units.

Table 3: The Distribution of Shelter Consumption for Renters and Homeowners

	Homeownership Status	
Size	(1) Renter ($h' = 0$)	(2) Owner ($h' > 0$)
$\underline{s}$	45.1	0.0
$h(1)$	53.5	17.6
$h(2)$	0.5	54.6
$h(3)$	0.01	13.3
$h(4)$	0.00	5.6
$\geq h(5)$	0.9	8.9

Notes: Entries are percentages (%).

References

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