Wealth Inequality, The Rate of Return on Property Ownership, and Pareto Coefficients

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Abstract

Current research on wealth inequality attributes the main reason for the long-run divergence in wealth inequality to the $r - g$ gap, where r is the rate of return on capital and g is the economy's growth rate. Nonetheless, speculations about capital and the rate of return on capital - it's definition and measurement - have raised concerns about deriving the $r - g$ gap. This paper addresses the concentration of wealth by investigating income from property ownership. Specifically, it focuses on 3 main issues: (i) I provide an alternative measure for the Piketty $r - g$ gap, by deriving the rate of return on property ownership (r_p), and show that the gap between the long-run rate of return on property ownership ($\bar{r}_p$) and the long-run growth in the economy ($\bar{g}$) explains the fast rise in wealth inequality; (ii) I show that when traditional models that focus on production only are used to capture the natural behavior of wealth inequality, wealth inequality tends to be inconclusive or explosive over the long-run; and (iii) I implement the new measure of $r_p - g$ into a simple model of wealth accumulation, that takes into account both productive and non-productive capital in generating wealth. Using the United States as a study case, I find that wealth inequality is more concentrated than suggested in the literature.

JEL Classification: D31, E01, E21

Keywords: Wealth Inequality, Wealth Distribution, The Rate of Return to Property Ownership, Pareto Coefficients, Inverse Pareto Coefficients, Dynamic Wealth-Accumulation, Inequality Gap

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1 Introduction

Wealth inequality in industrialized and developed economies, over the past five decades, have become an issue of tremendous importance both politically and intellectually. The topic is of significant importance because wealth inequality impacts the overall growth prospects of an economy and its use of its resources, it impacts inequality in other sectors of the economy, and it has tremendous global significance. According to Atkinson et al. [4], people have a sense of fairness and care about the distribution of economic resources across society. As a result, all advanced economies have set in place redistributive policies such as taxation, which effectively redistribute a significant share of National Product across income groups. Nonetheless, even with these redistributive efforts, the concentration of wealth has been on the rise and shows no sign of declining.[1] Politically, there has been growing pressures on policy makers to address the concerns of rising wealth inequality. For example, the current U.S. policy debate is centered around the issue of growing large government deficits. To eliminate these large deficits, it is required that the government raise tax revenues in the up coming years (future). However, the problem for most policy makers is, "who do you tax?" With the significant decline in middle income earners in the U.S., according to the U.S. Census Bureau at the Minnesota Population Center (IPUMS), should policy makers raise taxes on lower-income earners, middle income earners or the wealthy? Or concurrently, should higher taxes be imposed on wealth? The questions of who bears the cost of the tax incidence - whether the wealthy or everyone else - is extremely important and needs to be addressed. Also, with the constant rise in the share of wealth by the wealthy, it is important that we understand the cause of the rise in wealth concentration over time. Academically, the standard response by many economists in the past has been to dismiss the importance of the rise in wealth inequality. Nonetheless, with the recent contribution to the literature by Piketty [20] and co-authors Atkinson, Saez, Zucman etc., in the book *Capital in the Twenty-First Century*, there has been a growing importance for economic researchers to address the topic again. What is the driving force of the rise in wealth inequality over the past decades? Should there be concern for rising wealth inequality? How is economic growth affected by rising wealth inequality? These questions need to be answered, nonetheless, more needs to be done to understand the constant rise in the inequality gap over these past 5 decades. It is only then that policies can be adequately implemented to address wealth distribution, or efficiently reallocate resources in the economy.

Attempts to explain the persistent increase in wealth inequality over time have yielded

[1] See Piketty [20] for a detailed analysis of rising wealth inequality in the United States and in other developed economies.

that the long run level of wealth inequality in most developed economies is determined by the difference between the rate of return on capital (r) and the growth rate in income per capita (g). See Piketty [20, 21]. However, there have been concerns about the definition of the rate of return on capital, r. If the gap between r and g is a major determinant of the long run structure of inequality, then it is important to derive a measure of r that captures profits, rents, dividends, interest, royalties, and capital gains. My research revisits the topic of wealth inequality after the World Wars, by specifically addressing the question: "What determines the long run level of wealth inequality?" I show that the long run level of wealth inequality is dependent on income from property ownership; that the difference between the rate of return on property ownership and the growth rate in income per capita better explains the dynamics of wealth inequality. I address the period post World Wars era because the definition of the economic systems have substantially changed over time: from an agricultural society to an industrialized society, to a capitalist society, to a financial capitalist society. It is therefore important to address periods in which the structural change in the dynamics of the economy is not substantially different. Using the United States as a case study, I use after tax historical data from the U.S. National Income Accounts to derive income from property ownership and I show that the level of inequality in the U.S. economy is determined by the difference between the rate of return on income from property ownership and growth in income per capita, i.e. $r_p - g$. I find that the $r_p - g$ gap is significantly wider than Piketty's $r - g$, and that the wider gap has significant implications on wealth inequality.

The paper is organized as follows. Section 2 provides a literature review on the topic of wealth inequality. In section 3, I derive income from property ownership, and analyze trends in the rate of return on property ownership over time. In section 4, I present models of wealth inequality, and 5 offers concluding remarks.

2 Recent Developments in Wealth Inequality

The question, "how do we explain the distribution of wealth," has been on the spotlight of economic research since John Bates Clark's [8] book, "The Distribution of Wealth," the theories of Kuznets' [13] paper, "Economic Growth and Income Inequality," as well as Kaldor's [12] paper, "Capital Accumulation and Economic Growth." These papers, especially Kuznets' theory,[2] greatly influenced the literature on inequality, allowing a multitude of research papers

[2] According Kuznets' theory (Kuznets' Curve) [13], inequality first increases in response to industrialization, economic development, or technological advancement, then decreases over time, resulting in an inverted-U relationship between growth and inequality.

to empirically and theoretically attempt to capture the relationship between various forms of inequality and economic growth. Kuznets' hypothesis indicated that income or wealth inequality should follow an inverse-U relationship in response to industrialization, economic growth or technological advancement. This fostered an influx of research papers that attempted to understand the natural process and behavior of inequality overtime, but resulted in conclusions that remained ambiguous. These studies either found statistically significant evidence in support of the Kuznets' curve or refuted the hypothesis. Some even found no relationship between inequality and economic growth.[3] Although the Kuznets seminal paper vastly contributed to the inequality literature, it fell short in explaining the persistent rise in inequality over the past five decades. According to Piketty and Saez [23], today the Kuznets curve is widely held to have doubled back on itself. That has been the case for many developed economies.[4] Stiglitz [28] refers to the Kuznets theory as an attempt to explain old stylized facts that do not explain the current rise in inequality in the now developed economies. Piketty [20] indicates that the Kuznets curve theory was observed mainly for the wrong reasons, and that its empirical underpinnings were extremely fragile: the inverted-U relation covered the period from 1913 to 1948. At the beginning of this sample period, inequality is observed to be rising and then declines mainly because of the world wars. It is no surprise then that Kuznets found an inverted-U relationship. The sharp reduction in inequality is also observed in the developed countries between 1914 and 1945, affirming the fact that the decline in inequality was due to the violent economic and political shocks they entailed, and not the automatic process or course of inequality (Piketty [20]). Therefore, the "bell shape" relationship between inequality and growth that was popularized, has nothing to do with the natural or automatic process of the behavior of inequality.

New evidence provided by Thomas Piketty's book "Capital in the Twenty-First Century" and other supporting papers with co-authors, Emmanuel Saez, Gabriel Zucman and others, have completely revolutionized the literature on income and wealth inequality, with the most noticeable contribution being the new and extensive data that they use, mainly administrative tax records, to study inequality at the very top of income and wealth distribution.[5] Piketty and Saez [23] extended the methodology of Kuznets by building a homogenous series on top shares of pre-tax incomes and wages in the United States covering 1913 to 1998. Based on this, they construct annual series of top shares of salaries for the top fractile of the salary distribution.

[3] See Persson and Tabelini [19], Perotti [18], Li and Zou [14], Forbes [10], Barro [5], Banerjee and Duflo [9], and Atkinson [3] for research on inequality and growth

[4] See Piketty [20, 21] and related works by Emmanuel Sael and Grabriel Zucman

[5] See the World Top Incomes Database [1] for a complete list of countries studied

They also analyze top capital income earners. In constructing these series, the authors are able to study inequality at the top of income and wealth distribution. See Piketty [20, 21] and the World Top Income Database [1] for top income data and analysis on other countries. Piketty's [20] findings reveal the following facts about inequality: (i) the distribution of wealth, which he refers to as income from capital, is much more concentrated than the distribution of income from labor (income inequality); (ii) wealth inequality in Europe is dramatically much lower today (2010) than anytime before 1960, and is much lower than wealth inequality in the United States; (iii) wealth inequality has been rising over the past 5 to 6 decades even though the rise seems small in comparison to the decline during the World Wars. The decline in the United States was less severe, but currently it is close to the highest level of inequality attained in the first part of the twentieth century; and (iv) the rise in wealth inequality throughout history is attributable to the difference between the rates of return on capital (r) and the growth rate of income in the economy (g). This latter fact, "$r - g$ *implies inequality*", has been a topic of current debate among economist, posing the question: "Did Piketty get Inequality right or wrong?" Even though, there are shortcomings to the $r - g$ analysis, their work serves as a seminal contribution to the inequality literature, and for later developments in the literature. Their work certainly highlights the growing importance of inequality in the economy, and whether we should be concerned or not.

This paper addresses concerns and questions about the latter two stated contributions by Piketty. First, I address wealth inequality in the U.S. economy, since it serves as an anomaly to the decline in inequality during the world wars, and also because wealth inequality in the economy is much closer to the level it was during the late nineteenth century and early twentieth century. Secondly, I address the shortcomings of "$r - g$ *implies inequality*," because Piketty's definition of capital is problematic, both as a measure of capital, and a measure of wealth (Weil [29]). More specifically, the paper revisits the topic of wealth inequality in the United States, post World Wars, by addressing the question: "What determines the long-run level of wealth inequality?" In answering the question, I derive the rate of return on property ownership (r_p) for the United States economy as an alternative measure to Piketty's return on capital. I show that it is the difference between the return on property ownership (r_p) and the growth rate of income (g) that explains the fast rise in wealth inequality, and not the rate of return on capital (r) as asserted by Piketty [20]. Property ownership is more appropriate when studying wealth inequality over time. Also, I show that this alternative measure for Piketty's "r" is immune to the argument encompassing Piketty's use of capital and enforces the role played by inheritance in producing inequality. Last, the paper enforces the importance of capital gains in the U.S. economy, and that it significantly contributes both to the dynamics of property ownership and the divergence of wealth.

3 Deriving Income from Property Ownership

3.1 The Role Played by r - g

It is important to note that $r - g$ is not the only or primary reason for considering changes in wealth over time, nor is it the primary reason for forecasting the path of inequality in the future (Piketty [22]). There are several factors that vastly affect wealth inequality, which Piketty refers to as wealth shock. These include family shocks, age shock, demographic shocks, shocks to the rate of return, labor market shocks, taste shocks, etc. In conjunction with these shocks, the role of $r - g$ solely explains the long run level of wealth inequality. Specifically, a higher $r - g$ gap will tend to greatly amplify the steady state inequality of wealth distribution that mainly arises out of a given mixture of shocks. (Piketty [22]). So then wealth inequality will always converge towards a finite level. However, the finite inequality level will be a steeply rising function of the $r - g$ gap.[6] Even though a strong argument is made in support of the fact that the gap $r - g$ is central to determining the long run behavior of inequality, several concerns need to be addressed: (i) Piketty's definition of capital is misleading and hence resulting in estimates of inequality that might be overstated or understated. Piketty defines capital as "the sum total of nonhuman assets that can be owned and exchanged on some market." This definition of capital ignores the value of human capital and transfer-wealth, which have grown enormously over the past years, and also ignores the possibility that wealth can be accumulated through rents. (ii) Piketty uses capital and wealth interchangeably, assuming that capital equals wealth. By assuming that all capital is used in the production process, the only means of attaining wealth is by saving (accumulating capital). Clearly, the concepts of capital and wealth are very distinct and should not be assumed to be the same. There is a vast distinction between capital, an input to production, and wealth, thought of as assets including land and the capitalized value of other rents, which give command over purchasing power. (iii) If capital equals wealth, and all capital is used in the production process, then r and g are not independent. They are rather endogenously determined. So then Piketty's *"$r - g$ implies inequality"* might not be as straight forwards as he puts it. By using the alternative measure, the rate of return on property

[6] See Piketty [20] chapter 10, Piketty [21], and Piketty and Zucman [25] section 5.4 for the theoretical evidence of the inequality amplification of $r - g$

ownership, this paper addresses the outlined flaws in the use of capital as defined by Piketty.

3.2 Definition

According to Piketty [20], the rate of return on capital measures the yield on capital over the course of a year regardless of its legal form: that is, profits, rents, dividends, interest, royalties, and capital gains expressed as a percentage of the value of capital invested. In order to measure the rate of return on capital, Piketty defines capital income as the total sum of housing capital income, corporate income, net foreign income, and capital income in the non-corporate business sector. This definition raises a couple of issues

i. Both productive capital and non-productive capital creates wealth, hence to assume that all capital is productive is misleading.

ii. Human capital is excluded from Piketty's definition of capital on the premise that it cannot be owned by another person, nor traded on a market. However, in today's world with advanced technological progress, human capital cannot be ignored.

Also, measuring capital income based on Piketty's classification can be hard to measure, especially because:

i. Home owners who contact mortgages consume financial intermediate services (FISIM) which are treated as intermediate consumption. And there is cross country heterogeneity on how FISIM is measured.

ii. The net operating surplus of the household sector only captures the income generated by household activities, but households do not own 100 percent stock of the housing stock. There are variations in the share of households owned by corporations.

iii. Rules determining whether an organization falls within the corporate or non corporate business proves to be another issue at hand.

iv. Piketty assumes that return on government capital is implicitly zero. This is mainly because public debt usually outweighs public capital. Nonetheless, public land is significantly undervalued, and lastly,

v The measurement of National Income Accounts do not all meet the same standards creating issues for comparison across countries.

Using data from the National Income and Product Accounts (NIPA), published by the Bureau of Economic Analysis (BEA), and U.S. Department of the treasury, Office of Tax Analysis and the Congressional Budget Office, I provide an alternative measure for capital income by focusing on income from property ownership. According to the National Income and Product Accounts (NIPA) income from property ownership is the sum of interest income, dividend income and rental income. Building on this definition, I define income from property ownership as

$$\begin{matrix} \textit{Income from} \\ \textit{property ownership} \end{matrix} = \begin{matrix} \textit{Interest} \\ \textit{income} \end{matrix} + \begin{matrix} \textit{Dividend} \\ \textit{income} \end{matrix} + \begin{matrix} \textit{Rental} \\ \textit{income} \end{matrix} + \begin{matrix} \textit{Capital} \\ \textit{gains} \end{matrix} \tag{1}$$

where *Interest Income* is the interest income from all sources, both monetary and imputed, that is received by individual, by private and government employee retirement plans, and by quasi-individuals. *Dividend Income* is from all sources including individuals, private and government employee retirement plans, and by quasi-individuals, non-profit institutions, and estates and trusts. Also, pension funds, some insurance reserves, and private trust funds are considered to be the property of persons, hence dividends received by these institutions are included. *Rental Income* with capital consumption adjustment is the net current-production income of persons from the rental of real property. It consists of net income from the rental of tenant occupied housing by persons, the imputed net income from the housing services of owner-occupied housing, and the royalty income of persons from patents, copyrights, and the rights to natural resources. However, the BEA definition of rental income does not include data for persons engages in the real estate business - that is, net income from the rental of tenant occupied housing by corporations or the net income of tenant occupied housing by partnerships and sole proprietors. Also, it does not include capital gains. I have adjusted my definition appropriately to account for them. Therefore, his definition captures profits, rental income, interest, dividend, royalties, capital gains, and rents more collectively than Piketty's measure of capital income. It accounts for income generated by all forms of property regardless of whether it is tangible or intangible, and if it was legally or illegally acquired.

3.3 The Rate of Return on Property Ownership (r_p)

Over the entire period, after the world wars, the rate of return on property ownership, $\hat{r}_p$, has been approximately 7.2 percent. See figure (1). The long run estimate of r_p is 2 percentage points higher than Piketty's estimate of 5 percent for r. This difference has significant implications for the divergence of wealth inequality. Since wealth inequality is a rising function of the rate of return on capital ownership, (r), a small gap between r and g can have destabilizing effects on the structure and dynamics of wealth inequality (Piketty [25]). A higher estimate of $\hat{r}_p$ implies that the concentration of wealth in the U.S. economy is projected to be narrower than projected.

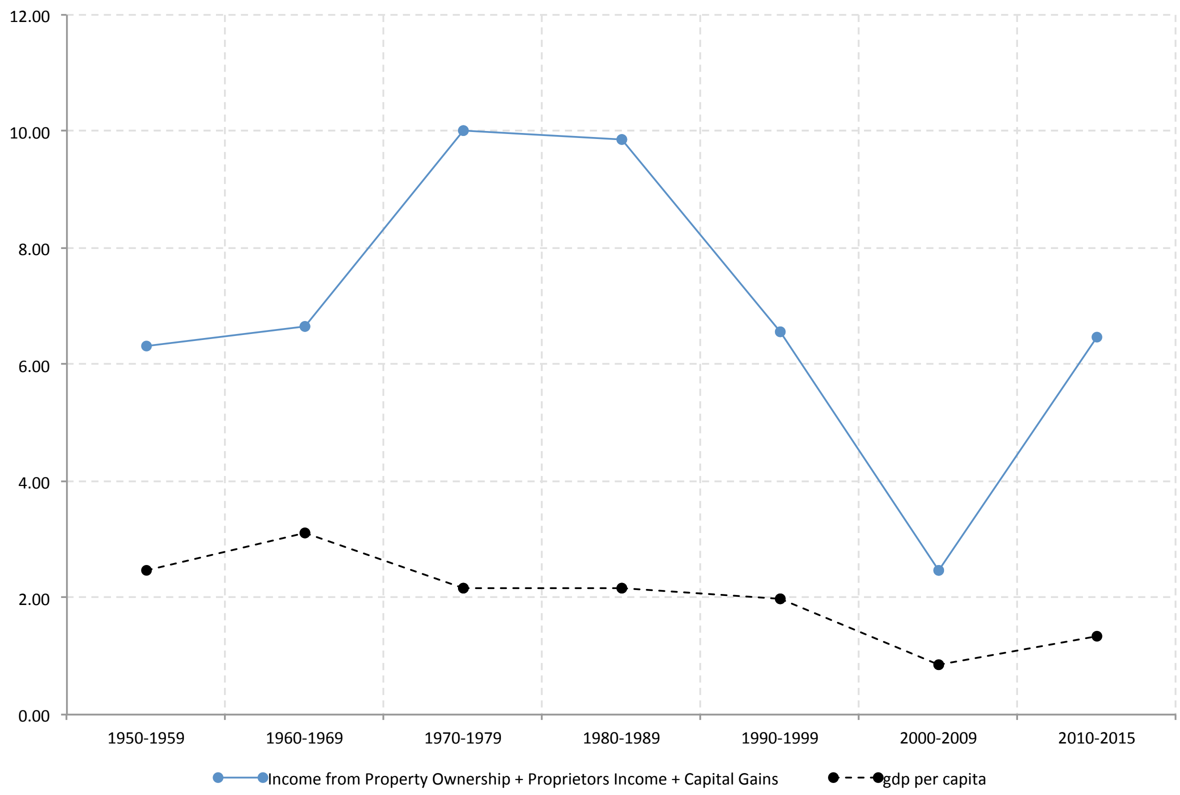

Figure 1: The $r_p - p$ Gap: Rate of Return on Property Ownership and Growth in Income per Capita in the United States, 1949 - 2015

Based on the derived income from property ownership, I find that the gap between the long run rate of return on all assets (r_p), whether used in the production process or left to accrue interest over time, and the growth in income per capita g in the U.S. economy, is approximately 5 percent. Note that r_p is the after tax rate of return on property ownership. Figure 2 shows the yearly difference between the rate of return on property ownership and the growth in income per capita. Since 1950 the difference has been approximately 5 percent amidst the fluctuation, reaching a maximum of 14 and 15 percent in 1981 and 1986, respectively the difference reached a maximum of 14 percent and 15 percent respectively. In years like 1987 and 2009, the difference reached a minimum of minus 11 percent and minus 10 percent. However, over the long run the rate of return of property ownership is much higher than the growth in the economy. Figure 1. [7]

[7]Decade averages are calculated according to the following years: 1950-59, 1960-69, 1970-79, 1980-89, 1990-99, 2000-09 and 2010-15

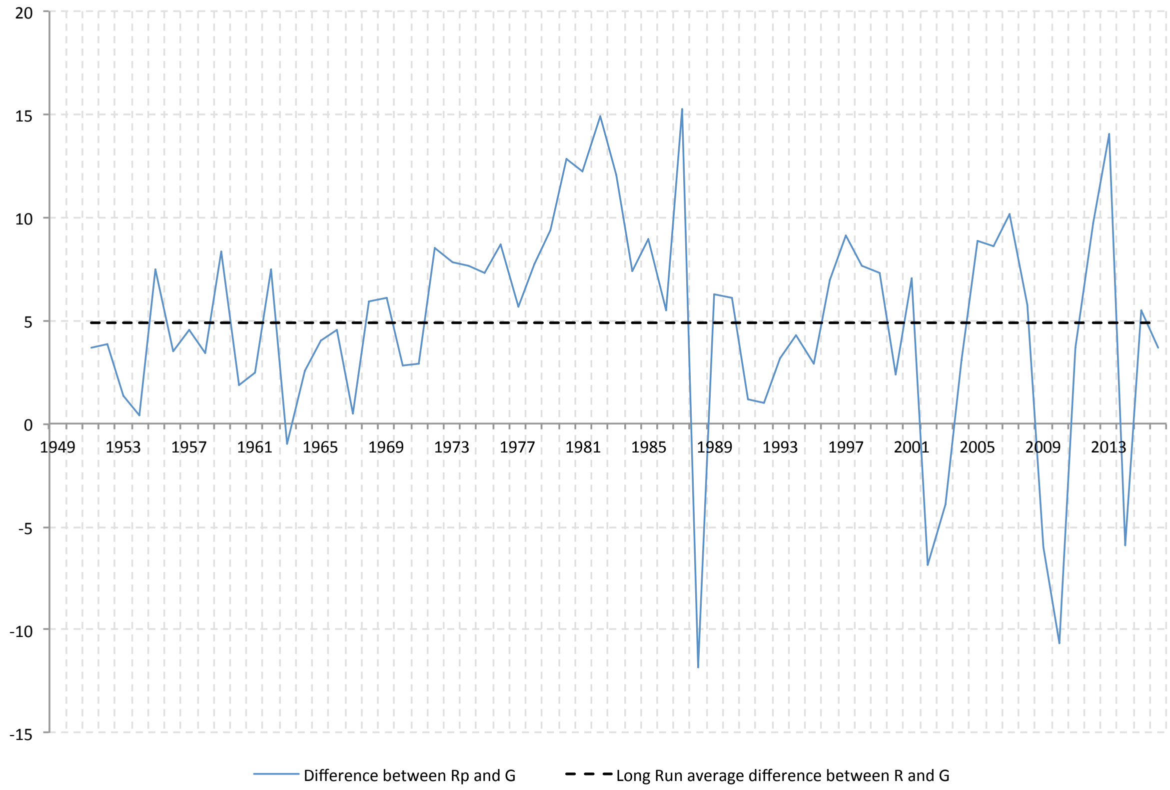

Figure 2: The Difference between the Rate of Return on Property Ownership and Growth in Income per Capita in the United States, 1949 - 2015

Even though the rate of return to property ownership is seen to be higher than the growth rate in income per capita in the U.S. economy, it is also evident that the components of income from property ownership display the same relationship. That is, the long-run rate of return on interest income, dividend income, rental income (plus proprietor's income) and capital gains exceeds the growth in income per capita by at least 2 percentage points. Over the entire period, the rate of return on interest bearing assets, dividends, rental property, and capital gains were 8.01 percent, 7.96 percent, 6.2 percent, and 7.9 percent respectively. Clearly, the ownership of property yields more returns than earning a wage. What is interesting about the individual components are as follows: (i) Figures 3 shows that between the 1950s and 2015 the decade average rate of return on income from the rental of property has persistently been rising from 4 percent to approximately 8 percent. The only exception was during the 2000s where it dips to 4 percent. This was mainly due to the 2007 financial crises which resulted in the loss of rental property nationwide. (ii) From the 1950s to 1980s the average rate of return on interest bearing was approximately 11.5 percent. See figure 4. Nonetheless, there was a significant decrease in the 90s to about 2 percent, and remains relatively low, falling below the growth in income per capita. Interest bearing assets were a great means to create wealth, however, in the past 3 decades that has not been the case, compared to the other forms of property. (iii) The behavior of rate of return on income from dividends shows more volatility: increasing from 6 percent in the 1950's to 11 percent in the 1980's, then declining to almost 6 percent in the 2000's. From 2010 to 2015, the average return has increased well above 8. See figure 5. (iv) Realized capital gains present the most striking results in the U.S. economy.[8] Figure 6 shows that over the decades the return on income from realized gains is well above 10 percent, reaching 15 percent in the 1990's, and then declined to -0.1 percent in the 2000 - 2009 period. It is important to note that the decline can mainly be attributed to property owners' unwillingness to sell their assets during this period. Nonetheless, right after the 2007 recession, the return on income from realized gains increased to well over 20 percent. Paying attention to the components of income from property ownership, one can decipher that rental income, dividend income and capital gains are the main channels through which wealth can be created.

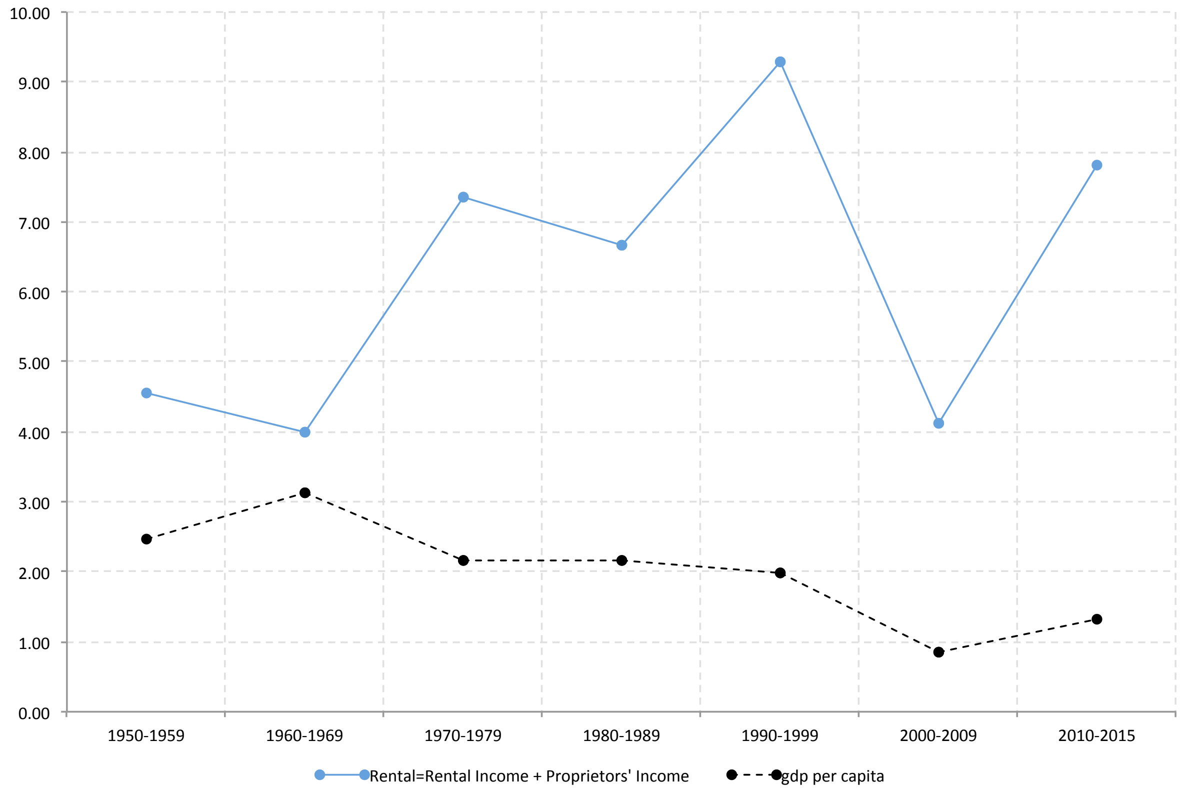

Figure 3: Rental Income and Growth in Income per Capita in the United States, 1950-2015

[8]I only used realized capital gains as data for unrealized capital is not readily available. However, capital that is not realized only appreciates in value generating wealth.

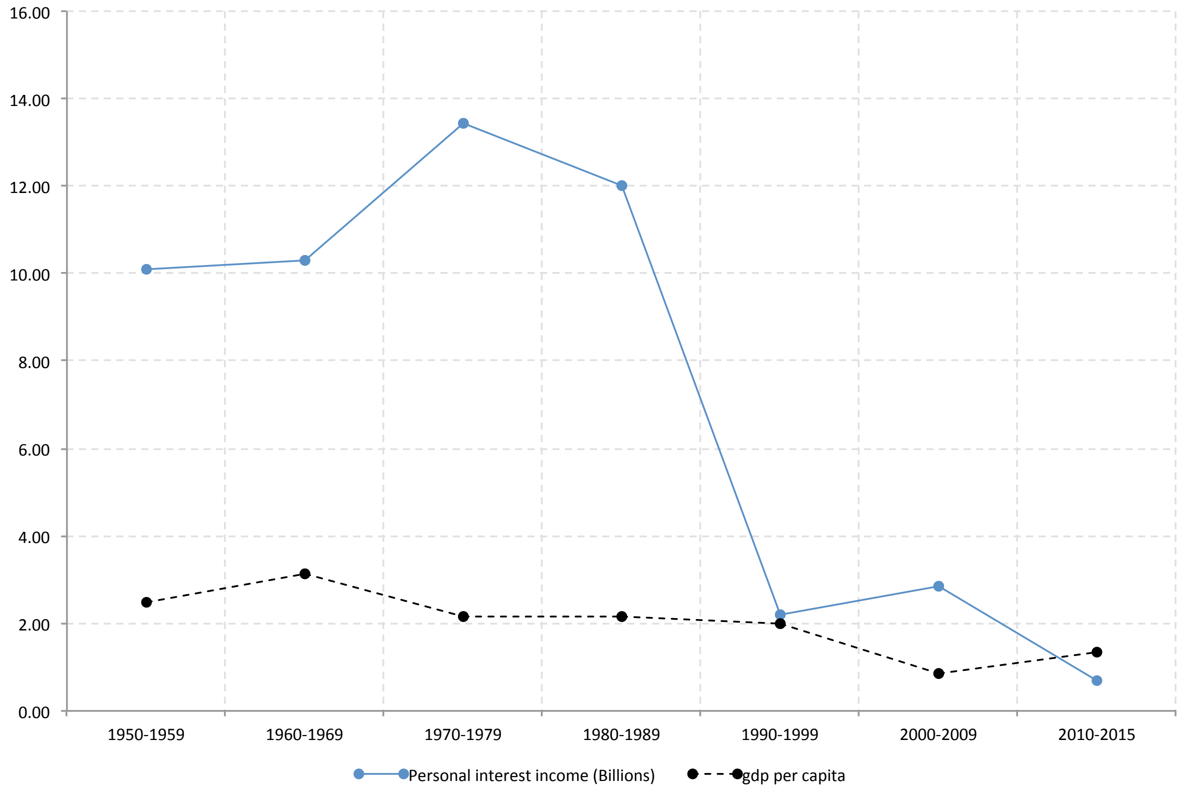

Figure 4: Interest Income and Growth in Income per Capita in the United States, 1950-2015

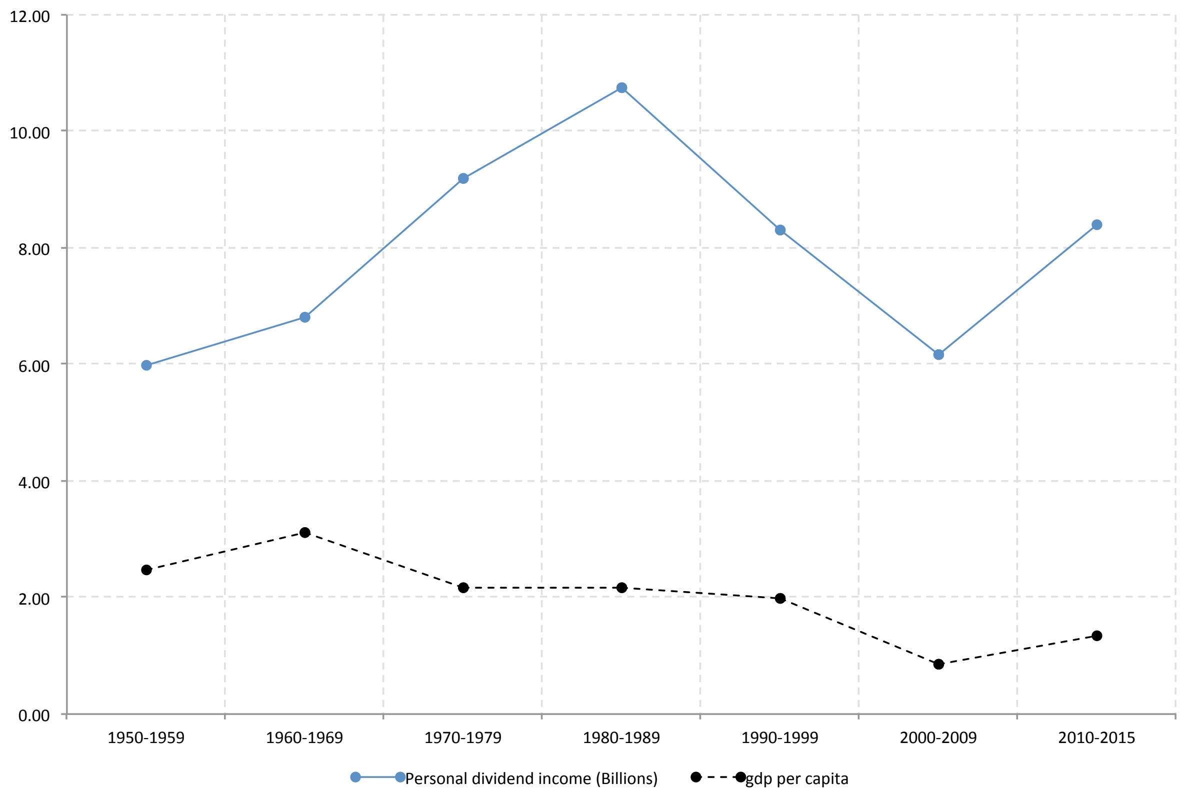

Figure 5: Dividend Income and Growth in Income per Capita in the United States, 1950-2015

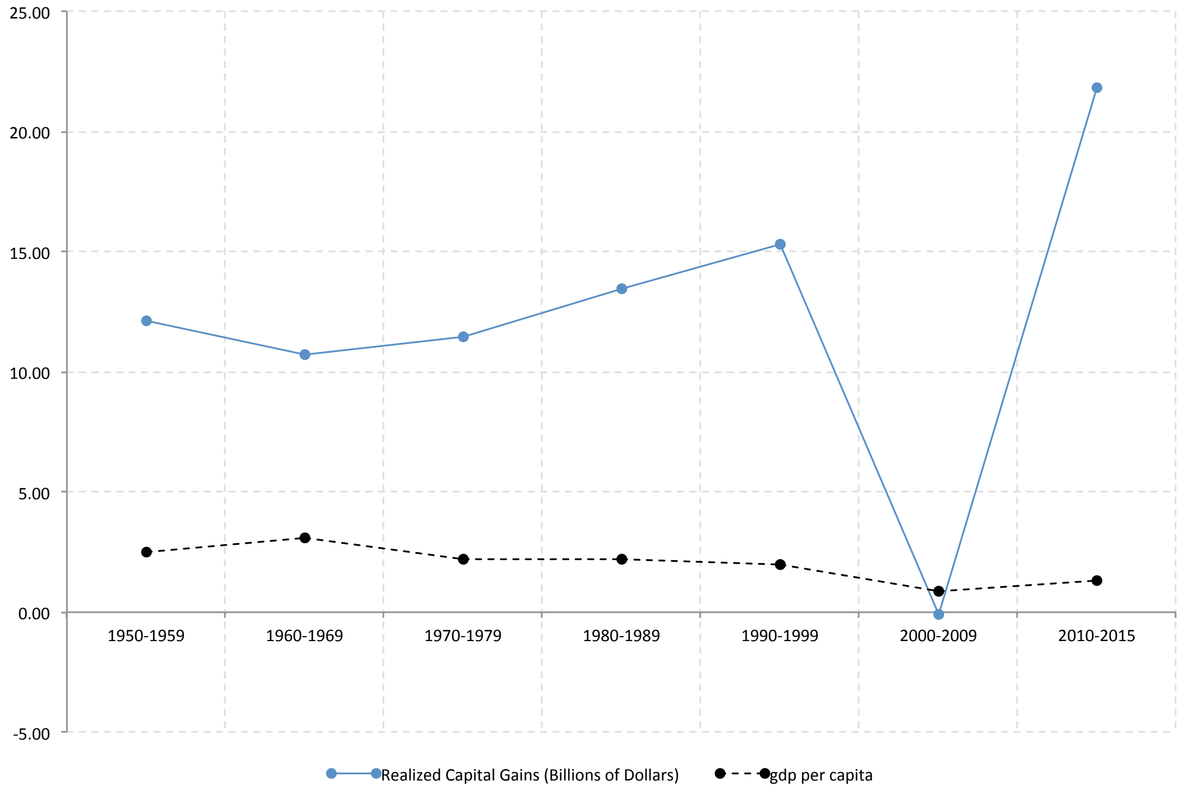

Figure 6: Capital Gains and Growth in Income per Capita in the United States, 1950-2015

4 Models of Wealth Concentration

4.1 The Piketty Model of wealth Accumulation

I employ the Piketty dynamic wealth-accumulation model with random idiosyncratic shocks [25], where I make the assumption that the return on capital (r) as defined in the model is well captured by the rate of return on property ownership (r_p). I consider a dynamic economy with a discrete set of generations $0, 1, ..., t, ...$. The model can be interpreted as an annual model, with each period lasting T=1 year, or a generational model with each period T=30 years, in which case the savings taste can be interpreted as bequest taste. Assuming a stationary population, $N_t = [0, 1]$, made of a continuum of agents of size one, aggregate variables and average variables are the same for wealth and national income. That is $W_t = w_t$ and $Y_t = y_t$. I also assume that effective labor supply $L_t = N_t \cdot h_t = h_0 \cdot (1+g)^t$ grows at an exogenous rate g. Domestic output, Y_{td}, is given by the CES production function

$$Y_{td} = F(K_t, L_t) = \left[aK_t^{\frac{\sigma-1}{\sigma}} + (1-a)L_t^{\frac{\sigma-a}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Let's assume that each individual $i \in [0,1]$ receives labor income $y_{Lt,i} = y_{Lt}$ and has some annual rate of return $r_{pt,i} = r_{pt}$. Each agent chooses c_{ti} and $w_{t+1,i}$ to maximize their utility function with wealth taste parameter, s_{ti}, given by the Cobb-Douglass utility function

$$U\left(c_{ti}, w_{ti}\right) = c_{ti}^{1-s_{ti}} w_{ti}^{s_{ti}} \tag{3}$$

and subject to the budget constraint

$$c_{ti} + w_{t+1,i} \leq y_{Lt} + (1 + r_{pt})w_{ti} \tag{4}$$

Random shocks are introduced into the model only in the form of idiosyncratic variations in the savings taste parameter, s_{ti}, which is drawn from an *iid* random process with mean $s = E(s_{ti}) < 1$. Intuitively, s cannot equal one, because people who save all their income and returns from wealth are left with nothing to consume. Utility maximization implies that

$$c_{ti} = (1 - s_{ti})\left[y_{Lt} + (1 + r_p t)w_{ti}\right] \tag{5}$$

which indicates that individuals consume a fraction of their total resources, both labor income and wealth, available at time t. By substituting equation (5) into equation (4) (the budget constraint), we have that

$$w_{t+1,i} = s_{ti}\left[y_{Lt} + (1 + r_{pt})w_{ti}\right] \tag{6}$$

which gives us the individual transition equation for wealth. At the aggregate level $y_t = y_{Lt} + r_{pt}w_t$, and $s_{ti} = s$, and by substituting into equation (6) we get that [9]

$$w_{t+1} = s\left[y_t + w_t\right] \tag{7}$$

Since we are interested in the wealth to income ratio, let's divide equation (6) by $y_{t+1} \approx (1+g)y_t$, then

[9] We also take that the aggregate variables and average variables are the same for wealth and national incomehence.

$$\frac{w_{t+1}}{y_{t+1}} = s\left[\frac{y_{Lt}}{y_t(1+g)} + \frac{(1+r_{pt})}{1+g} \cdot \frac{w_t}{y_t}\right] \tag{8}$$

By denoting the capital share as $\alpha_t = r_{pt}\beta_t$, and the labor share as $1 - \alpha_t = \frac{y_{Lt}}{y_t}$, and that the wealth-income ratio at time t is equal to β_t, by rearranging (8) this results in the transition equation for the wealth income ratio, which is given by

$$\beta_{t+1} = s\left[\frac{1-\alpha_t}{1+g} + \frac{1+r_{pt}}{1+g}\beta_t\right] \tag{9}$$

In an open economy case, $r_{pt} = r_p$ and from equation (9), it is obvious that β_t converges towards a finite limit β if and only if

$$\omega = s \cdot \frac{1+r_p}{1+g} < 1 \tag{10}$$

If $\omega > 1$ then β_t keeps increasing from generation to generation, and β_t converges to infinity (∞). In the long-run, the economy is no longer a small open economy and "r_p" will have to fall so that $\omega < 1$. This explains the fact that individuals cannot accumulate wealth forever. If $\beta_t < \beta_{t+1} < \beta_{t+2} < ...$ then eventually, r_p will fall until $\omega < 1$ at which β_t converges. On the other hand, β_t always converges, in a closed economy case, towards a finite limit. In the long-run, the rate of return on capital is equal to the marginal product of capital, and depends negatively upon β and is determined by the CES production function. However since the rate of return on capital is captured by the rate of return on property ownership, then $r_p = F_k = \alpha\beta^{-1/\sigma}$. The steady-state level of wealth-income ratio implies that $\beta_{t+1} = \beta_t$. By substituting this in equation (7) and dividing through by y_{t+1} you obtain the steady-state wealth income ratio:

$$\beta_t \to \beta = \frac{s}{g+1-s} = \frac{\tilde{s}}{g} \tag{11}$$

where $\tilde{s} = s(1+\beta) - \beta$ is the steady state savings rate expressed as a fraction of national income. Taking that normalized individual wealth (z_{ti}) is equal to w_{ti}/w_t, and by dividing both side of equation (6) by $w_{t+1} \approx (1+g)w_t$, you obtain

$$\frac{w_{t+1,i}}{w_{t+1}} = s_{ti}\left[\frac{1-\alpha}{1+g}\cdot\frac{y_t}{w_t} + \frac{1+r_{pt}}{1+g}\cdot\frac{w_{ti}}{w_t}\right] \tag{12}$$

Given that $\frac{w_t}{y_t} = \beta = \frac{s}{1+g-s}$ which implies that $\frac{y_t}{w_t} = \frac{1}{\beta}$, and that $\alpha = r_p\beta = r_p \cdot \frac{s}{1+g-s}$, then

$$z_{t+1,i} = \frac{s_{ti}}{s}\left[1 - s\frac{1+r_p}{1+g} + s\frac{1+r_p}{1+g}z_{ti}\right] \tag{13}$$

Since ω is defined as $s\frac{1+r_p}{1+g}$, the individual level transition equation for wealth can be written as

$$z_{t+1,i} = \frac{s_{ti}}{s}\left[(1-\omega) + \omega \cdot z_{ti}\right] \tag{14}$$

Following the work of Piketty and Zucman [25], let's assume a simple 2 dimensional inequality model with Binomial Random Tastes, where the role played by taste in the model takes only two values. The shocks could come from taste or other factors such as in the primogeniture model of Stiglitz [27], or from the rates of return as seen in the models of Benhabib et al. [6, 7] and Nerei [16].[10] That is

$$s_i = \begin{cases} s_0 & \text{with probability} 1-p \\ s_1 & \text{with probability} p \end{cases}$$

Then the aggregate savings rate in the economy is given by

$$s = E(s_i) = ps_1$$

Given the transition equation for wealth and assuming that $\omega < 1 < \omega^* = \omega/p$, if $s_i = s_0 = 0$, then $z_{t+1,i} = 0$ with probability $1-p$, and if $s_i = s_1 > 0$, then $z_{t+1,i} = \frac{s_1}{s}[(1-\omega) + \omega z_{ti}]$ with probability p, which implies that $z_{t+1,i} = \frac{1-\omega}{p} + \frac{\omega}{p}\cdot z_{ti}$. The steady-state wealth distribution, $\phi(z)$, is discrete and given by

$$\begin{aligned}
&z = z_0 = 0 \quad \text{with probability} \quad 1-p \\
&z = z_1 = \frac{1-\omega}{p} = \frac{1-\omega}{p} + \frac{\omega}{p}\left(\frac{1-\omega}{p}\right) \quad \text{with probability} \quad (1-p)p \\
&z = z_2 = \frac{1-\omega}{p} + \frac{\omega}{p}z_1 = \frac{1-\omega}{p} + \frac{\omega}{p}\left(\frac{1-\omega}{p}\right) + \frac{\omega^2}{p^2}\left(\frac{1-\omega}{p}\right) \quad \text{with probability} \quad (1-p)p^2 \\
&\vdots \\
&z = z_k = \frac{1-\omega}{p} + \frac{\omega}{p}z_k = \frac{1-\omega}{p} = \frac{1-\omega}{p} + \frac{\omega}{p}\left(\frac{1-\omega}{p}\right) + \frac{\omega^2}{p^2}\left(\frac{1-\omega}{p}\right) + \cdots + \frac{\omega^k}{p^k}\left(\frac{1-\omega}{p}\right)
\end{aligned}$$

[10] From Piketty [25] this is applicable to models with multiplicative random shocks in the wealth accumulation process, whether shocks are binomial or multinomial, and whether the shocks come from tastes or other factors.

which implies that

$$z_k = \frac{1-\omega}{\omega - p}\left[\left(\frac{\omega}{p}\right)^k - 1\right] \qquad \text{with probability}(1-p)p^k \tag{15}$$

Note that as $k \to \infty$, $z_k \approx \frac{1-\omega}{\omega-p} \cdot (\frac{\omega}{p})$. Also, assuming that $\frac{\omega}{p} < 1$, then $(\frac{\omega}{p})^k - 1$ becomes negative for any k and hence $z_k = \frac{1-\omega}{p-\omega}[1 - (\frac{\omega}{p})^k]$ which has a finite upper bound $\frac{1-\omega}{p-\omega}$ as $k \to \infty$.

The cummulated distribution of wealth is given by

$$\begin{aligned} 1 - \Phi(z_k) &= Prob(z \geq z_k) \\ &= \sum_{k' \geq k} (1-p)p^{k'} \\ &= p^k \end{aligned} \tag{16}$$

as $z \to +\infty$, this implies that

$$log[1 - \Phi(z)] \approx a[log(\omega) - log(z)] \tag{17}$$

That is,

$$1 - \Phi(z) \approx \left(\frac{\lambda}{z}\right)^a \tag{18}$$

where $\lambda = \frac{1-\omega}{\omega-p}$ is a constant term and $a = \frac{log(1/p)}{log(\omega/p)}$ is the pareto coefficient. The inverted Pareto coefficient (the ratio of average wealth z^* of individuals with wealth above z to z) for the distribution is given by

$$b = \frac{a}{a-1} = \frac{log(\frac{1}{p})}{log(\frac{1}{\omega})} > 1 \tag{19}$$

For a given probability p, as $\omega \to 1$, the pareto coefficient $a = \frac{log(1/p)}{log(\omega/p)} \to 1$ and $b \to +\infty$, which implies infinite wealth inequality. That is, an increase in $\omega = s\frac{1+r_p}{1+g}$ implies a larger wealth reproduction rate ω^* for wealth holders, which provides a stronger amplification of inequality. Likewise, as $\omega \to p$, the pareto coefficient $a \to +\infty$ and the inverser pareto coefficient $b \to 1$ which implies zero wealth inequality. On the other hand, for a given level of ω, as $p \to 0$, $a \to 1$ and $b \to +\infty$, implying that a small fraction of the population gets an infinitely large stock of wealth. Conversely, as $p \to \omega$, $ato + \infty$, and $b \to 1$ which implies that the long run level on inequality approaches zero. This inherently shows that the inequality of wealth is an increasing function of r_p and g

4.2 Simulating the Piketty Model of Wealth Accumulation

By simulating a simple model for the United States economy over the 1950 - 2015 period, the dynamics of the wealth inequality profile can be produced in comparison to the findings by Piketty [21] (Chapter 10). If we assume each period to be a discrete-time model as shown above as lasting H years (with H=30 years = generation length), and if r_p and g denote instantaneous rates, then ω can be written as

$$\omega = s \cdot \frac{1 + R_p}{1 + G} = s \cdot e^{(r_p - g) \cdot H} \tag{20}$$

with $1+R_p = e^{r_p \cdot H}$ the generational rate of return on property and $1+G = e^{g \cdot H}$ the generational growth rate. Simulating the model for the entire period post World Wars, $r_p = 7.17\%$ and $g = 2.06\%$. So then the gap $r_p - g$ equals 5.09% for the United States economy. With a savings rate, $s = 8.65\%$ and $H = 65$ years, this implies that $\omega = s \cdot e^{(r_p - g) \cdot H} = 2.489$.[11] See Table 2 for results. I find that $\omega > 1$, which implies that wealth inequality is explosive in the U.S. economy. This also implies that there is no existence of a long run level of wealth concentration hence overtime ω would have to fall such that $\omega < 1$. Assuming the same estimates for growth in income per capita and savings rate, the level of r_p necessary for there to be a stable equilibrium wealth inequality in the U.S. economy, r_p has to be less than 5.83%. This corresponds to a maximum $r - g$ gap of 3.77%. By assuming that $r_p = 5$, equivalent to the long run rate of return on capital suggested by Piketty [20], for a given binomial shock structure with $p = 10\%$, $\omega = 0.6080$, the pareto coefficient $a =$ equals 1.2757, and the inverse pareto coefficient b equals 4.627. This estimate corresponds to an economy with high wealth inequality where the top 1 percent wealth share is around 50-60 percent. Significantly higher than the estimates of wealth inequality suggested by Piketty. Table 1 shows the estimates of the pareto coefficients (a) and the inverse pareto coefficients (b) over the decades. Table 2 shows the estimates over a 30-year period and for the entire 1950-2015 period. Both Table 1 and Table 2 assume that the probability of a wealth distribution shock (p) occurring is 0.1 (10 percent). For Table 3, I simulate the model assuming that the probability of a shock occurring is 0.0015 (0.15 percent).

[11] I assume a savings rate equal to the United States personal savings rate expressed as a percentage of disposable income (NIPA Table 2.1) over the period 1950 - 2014.

Table 1: Results from Simulating the Piketty Model: $H = 10$ years

Time	Savings Rate s	Return on Property Ownerhsip r_p	Growth per capita g	omega $\omega = s \cdot e^{H \cdot (r_p - g)}$	Pareto Coefficient $a = \frac{log(1/p)}{log(\omega/p)}$	Inverse Pareto Coefficient $b = \frac{a}{a-1}$
1950-1959	0.1063	0.0632	0.0248	0.156	5.18	1.24
1960-1969	0.1113	0.0666	0.0312	0.1586	4.99	1.25
1970-1979	0.1183	0.1002	0.0217	0.0929	negative	-
1980-1989	0.0931	0.0987	0.0218	0.2009	3.3	1.43
1990-1999	0.0672	0.0657	0.0199	0.1062	38	1.03
2000-2009	0.0426	0.0246	0.0085	0.05	negative	-
2010-2015	0.0567	0.0646	0.0133	0.0947	negative	-

Calculations by Author. The probability of a shock occuring in the United States economy (p) is set to 0.1

Table 2: Results from Simulating the Piketty Model: $H = 30$ years or more

Time	Savings Rate s	Return on Property Ownerhsip r_p	Growth per capita g	omega $\omega = s \cdot e^{H \cdot (r_p - g)}$	Pareto Coefficient $a = \frac{log(1/p)}{log(\omega/p)}$	Inverse Pareto Coefficient $b = \frac{a}{a-1}$
1956-1985	0.1119	0.084	0.0234	0.689	1.196	6.102
1986-2015	0.0583	0.0484	0.0156	0.156	5.181	1.239
1950-2015	0.0865	0.0715	0.0206	2.489	-	-

Calculations by Author. The probability of a shock occuring in the United States economy (p) is set to 0.1

Table 3: Results from Simulating the Piketty Model: $p = 0.0015$

Time	Savings Rate s	Return on Property Ownerhsip r_p	Growth per capita g	omega $\omega = s \cdot e^{H\cdot(r_p - g)}$	Pareto Coefficient $a = \frac{log(1/p)}{log(\omega/p)}$	Inverse Pareto Coefficient $b = \frac{a}{a-1}$
1950-1959	0.1063	0.0632	0.0248	0.156	1.40	3.5
1960-1969	0.1113	0.0666	0.0312	0.1586	1.39	3.56
1970-1979	0.1183	0.1002	0.0217	0.0929	1.58	2.72
1980-1989	0.0931	0.0987	0.0218	0.2009	1.33	4.03
1990-1999	0.0672	0.0657	0.0199	0.1062	1.52	2.92
2000-2009	0.0426	0.0246	0.0085	0.05	1.86	2.16
2010-2015	0.0567	0.0646	0.0133	0.0947	1.57	2.75
1956-1985	0.1119	0.084	0.0234	0.689	1.06	17.67
1986-2015	0.0583	0.0484	0.0156	0.156	1.40	3.5
1950-2015	0.0865	0.0715	0.0206	2.489	-	-

Calculations by Author. The probability of a shock occuring in the United States economy (p) is set to 0.0015.

The main findings of this paper is that, the Piketty Random Shock model does not explain wealth inequality in the United States when considering the return on property ownership. This is mainly due to the restrictions that are placed on the model: $\omega = s \cdot \frac{1+r_p}{1+g} < 1$ and $\omega < p$. These conditions are necessary for an equilibrium level of wealth inequality to exist. Intuitively, people don't decide to accumulate wealth because of these conditions. The results shown in Table 1 indicate that the inverse pareto coefficient (b) ranges between 1.0 and 1.5. This shows that the U.S. economy is an extremely egalitarian society with the top 1 percent owning between 0 to 10 percent of the nations wealth. For certain decades, 1970-1979, 2000-2009, and 2010-2015, $\omega < p$ resulting in a negative pareto coefficient.[12] Since a must always be greater than zero, there cannot exist an equilibrium level of wealth inequality when $a < 0$. The reason why Piketty [20] suggests that the U.S. economy hasn't reached levels of wealth inequality previously experienced by most European economies before the World Wars is because, enough time hasn't elapsed for wealth to be accumulated. Perhaps, this is true and evident when simulating the dynamic wealth-accumulation model with random idiosyncratic shocks over decades. That is, the long run effect of $r_p - g$ on wealth is not made evident within the decade. Next, I evaluate

[12] Calculation are made given a binomial shock structure $p = 0.1$. This is what is used in Piketty and Zucman [25].

the model over 30 year periods and over the entire period of study, 1950 to 2015. See table 2 for results.

There are two main contradictions from simulating the longer period effect of $r_p - g$ on wealth inequality, assuming a probability shock of 10 percent ($p = 0.1$). First, between 1956 and 1985, $a = 1.196$ and $b = 6.102$ which implies a level of wealth inequality where the top 1 percent own approximately 70 - 80 percent of total wealth in the U.S. economy. However, we FIND that from 1986 to 2015, there is a decline in the level of wealth inequality in the U.S. economy. That is, the inverse pareto coefficient, b, is equal to 1.239 which is equivalent to an economy where the top 1 percent own approximately 5 percent of total wealth. This contradicts Piketty's [20] findings as wealth inequality after the World Wars has been gradually rising. By taking the average of the two wealth inequality estimates over the two periods, we get an estimate close to Piketty's prediction of wealth inequality in the U.S. economy, where the top 1 percent own about 40 percent of wealth. The second contradiction comes by ways of estimating the model over the entire period, 1950 - 2015. From Table 2 we see that $\omega = s \cdot e^{(r_p - g)\cdot H} = 2.489$. If ω is greater than 1, then we have explosive inequality. Therefore, based on the model, there is explosive wealth inequality in the U.S. economy. The level of $r_p - g$, given the estimate for s in the U.S. economy over the 66 year period, needed to ensure that there exists an equilibrium level of inequality ($\omega < 1$) implies that $r_p - g < 0.037$ (3.7 percent). Piketty [20] assuming an $r - g$ gap of 3 percent certainly ensures that an equilibrium level does exist. However, by looking at income from property ownership we see that inequality is explosive in the U.S. economy.

To ensure a level of inequality where the top 1 percent own approximately 35 percent of wealth as in Piketty [23, 24], where the pareto coefficient (b) is equal to 3.5, this implies that, the probability of a binomial random shock occurring has to equal 0.0015 (0.15 percent).[13] This implies that the probability of a shock occurring, whether from taste, primogeniture, the number of children, or from the rates of return, which Piketty [20] mentions, has been practically non-existent since the 1950's. Table 3 shows the results for the simulated model for decades, starting from the 1950s, for 30 year periods, and for the entire period assuming that $p = 0.0015$.

4.3 A Simple Model of Wealth Accumulation

A lot of ideology went into the design and display of the U.S. National Income and Product Accounts (NIPA). The NIPA propagates the notion that national income is identical to the production of goods and services. Hence, the only way of generating income is through the

[13] Calculation are made with the assumption that $\frac{a}{a-1} = 3.5$ hence $a = 1.4$, and the current estimates of the savings rate (s), the rate of return on property ownership (r_p), and the growth rate of GDP per capita (g) in the U.S. economy.

production of goods and services valued by the people. Likewise, it also corroborates the notion that wealth is built through production, which is in line with Piketty's claim that all capital is used in the production process only. However, as shown in section 3.1 wealth can be created without the production of goods and services taking place. The central role of Piketty's $r - g$ in generating wealth has given rise to some confusion. According to Jones [11], This confusion is rooted in the fact that, the relationship between Piketty's $r - g$ and wealth inequality is not obviously clear in the Neoclassical Growth model. Jones [11] indicates that the relationship between $r - g$ and inequality is much more easily appreciated in models that explicitly generate pareto wealth inequality. The key link between the data and the theory is the pareto distribution.

In this section, I examine a simple model of wealth accumulation that gives rise to Pareto Distribution, and considers the economic forces that influence top inequality over time for the U.S. economy. Pareto inequality is generated through a mechanism that is characterized by the power law: *an exponential growth that occurs over an exponentially distributed amount of time results in a pareto distribution.*[14] My contribution here is that I introduce the rate of return on property ownership into the model.

Following the work of Jones [11], let's assume an economy with heterogenous people where there is no labor income, individuals consume a constant fraction α of their wealth, wealth is subject to tax, τ, and that the average wealth per person (capital per person) grows at an exogenous rate g. So then, let a denonte an individuals wealth which accumulates over time according to

$$\dot{a} = r_p a - \tau a - c \tag{21}$$

where r_p is the rate of return on property ownnership (assets), τ is a wealth tax rate, and c is the individuals consumption.[15] Assuming that consumption is a constant fraction of wealth α, then

$$\dot{a} = (r_p - \tau - \alpha) \cdot a \tag{22}$$

which is the law of motion for wealth accumulation. The wealth of an individual of age x at date t is then given by

$$a_t(x) = a_{t-x}(0) \cdot e^{(r_p - \tau - \alpha) \cdot x} \tag{23}$$

where $a_{t-x}(0)$ is the initial wealth of a new born at date $t - x$. Note that capital is not equal to

[14]The defining characteristic of a Pareto Distribution is $Pr[x > y] = y^{-\frac{1}{\eta}}$, which implies that the probability that x is greater than y is proportional to y raised to some power. See Pareto [17]

[15]Other models that promote a simple model of wealth accumulation include Wold and Whittle [30], Stiglitz [27], Quadrini [26], Benhabib and Bisin [6], Nirei [16], Moll [15], and Piketty and Zucman [20]

property. That is, the exponential growth of wealth is fundamentally affected by the return on property ownership, r_p. Therefore, in the asset accumulation equation, the return on property ownership is the key determinant of an individuals wealth. To obtain a variable that exhibits a stationary distribution, the individual's wealth is normalized by average wealth per person or income per person in the economy.

Same as Jones [11], I introduce heterogeneity into the model through a birth-death process, where each new born in the economy inherits the same amount of wealth, and the aggregate inherittance is simply equal to the aggregate wealth of the people who die each period. The number of people born at date t is give by

$$B_t = B_0 e^{\bar{n}t} \tag{24}$$

Death follows a poisson process with the arrival rate $\bar{d}$. Hence the stationary distribution of the birth-death process is an exponential process, given by

$$Pr[Age > x] = e^{-(\bar{n}+\bar{d})x} \tag{25}$$

See Appendix A for more details.

The model can be analyzed in either a general equilibrium framework or in a partial equilibrium framework.[16] For comparison with the Piketty model, I focus on a partial equilibrium model. The idea with wealth distribution in a partial equilibrium framework is that newborns inherit the wealth of the people who die in the economy, hence

$$\begin{aligned} a_t(0) &= \frac{\bar{d}K_t}{(\bar{n}+\bar{d})N_t} \\ &= \bar{a}k_t \end{aligned} \tag{26}$$

where $\bar{a} \equiv \frac{\bar{d}}{\bar{n}+\bar{d}}$ and $k_t \equiv \frac{K_t}{N_t}$ is capital per person in the economy.[17] Even though I am using capital in the model, the assumption is that capital is not equal to wealth. If an economy is in a steady state, then the capital per person grows at a constant and exogenous rate, g, over time. That is

$$k_t = k_0 e^{gt} \tag{27}$$

[16]Note that in a partial equilbrium model, the growth rate of normalized wealth is $r_p - g - \tau - \alpha$. In a general equilibrium model, the key source of heterogeneity is population growth. Newborns in such an economy inherit wealth of the people who die. However, since there are more newborns than people who die, newborns inherit less than the average amount of wealth per capita.

[17]$\bar{d}K_t$ equals the aggreate wealth of the people who die, and $(\bar{n}+\bar{d})N_t$ is the number of new borns. Because of population growth, new borns inherit less than the average amount of capital per person in the economy, and this fraction is $\bar{a}$.

The assumption here is that the growth of income per person determines the initial wealth of a person, therefore the amount of wealth that a person of age x at date t inherited when they were born at date $t-x$ is given by

$$a_{t-x}(0) = \bar{a}k_{t-x} = \bar{a}k_t e^{-gx} \tag{28}$$

Substituting this into the equation for wealth accumulation, we get that

$$a_t(x) = \bar{a}k_t e^{(r_p-g-\tau-\alpha)x} \tag{29}$$

The above equation is the exponential growth process that is central to the pareto distribution of normalized wealth, and it is obvious that $r_p - g$ plays an important role as shown by Piketty [20]. However, another process central to the process is the exponential age distribution (population growth) providing heterogeneity in the model. We now have an exponential growth process occuring over an exponentially distributed amount of time. By inverting $a_t(x)$ we have that

$$x(a) = \frac{1}{r_p - g - \tau - \alpha} log(\frac{a}{\bar{a}k_t}) \tag{30}$$

which gives the age at which a person achieves wealth a. The wealth distribution is then given by

$$\begin{aligned} Pr[wealth > a] &= Pr[Age > x(a)] \\ &= e^{-(\bar{n}+\bar{d})x(a))} \\ &= (\frac{a}{\bar{a}k_t})^{(-\frac{\bar{n}+\bar{d}}{r_p-g-\tau-\alpha})} \end{aligned} \tag{31}$$

Since the pareto inequality is measured by the inverse of the exponent, we get a steady state distribution that is pareto wealth, such that

$$\eta_{wealth} = \frac{r_p - g - \tau - \alpha}{\bar{n} + \bar{d}} \tag{32}$$

The above equation is at the basis of wealth creation and emphasises the Piketty $r - g$ relation as well. Note that as the gap $r_p - g$ increases so does wealth inequality, all other things being equal. Likewise, a lower wealth tax will increase wealth inequality and vice versa. It is also very important to address $\bar{n} + \bar{d}$ in the pareto wealth equation. In a society where $\bar{n} + \bar{d}$ is low either because of a decine in birth rate($\bar{n}$) or a decline in death rate ($\bar{d}$) - people who are part of a long lived dynasty - greater stocks of wealth will be accumulated. So then, low population growth will aid wealth creation by minimizing the distribution of wealth leading to high inequality and vice versa.

4.4 Simulating the Simple Model of Wealth Accumulation

Simulating the simple model of wealth accumulation for the United States economy over the period 1950 - 2015, I compare the results to the model in section 4.1 and 4.2, and also to the findings by Piketty [20]. The analysis is based on the simple pareto wealth accumulation, where

$$\eta_{wealth} = \frac{r_p - g - \tau - \alpha}{\bar{n} + \bar{d}} \tag{33}$$

I simulate the model for the entire period post World Wars where $r_p = 7.17\%$ and $g = 2.06\%$, implying that the long-run gap between r_p and g is 5.09%. I also simulate the model over 30-year periods, starting 1956, and over 10-year periods starting in 1950. The results are show in Table 4. In column (1) of Table 4, I evaluate the model based on the criteria that, data for income from property ownership already accounts for taxes, consumption and depreciation. However, in columns (2) and (3), I evaluate the model considering different tax growth rates (progressive taxes). Based on the Pareto wealth inequality (Inverse pareto coefficient), three (3) main facts are evident: First and foremost, After the World Wars, wealth inequality starts rising as suggested by Piketty [20]. In the 1950's, it is observed that the average wealthy individual owns approximately 3.6 times as much wealth as the average person. An increase is observed through the 1960's and 1970's until the 1980's where the average wealthy individual owns approximately 10 times as much wealth as the average individual in the U.S. economy. This pattern is also observed when different levels of tax growth rates are examined. Nonetheless, the level of inequality throughout this period far exceeds the level suggested by Piketty. Based on the the results in column (1) of Table 4, it is observed that the top 1 percent own approximately 35 percent of wealth in the 1950's. This rise in inequality rises until the 1980's where the top 1 percent own about 90 percent of wealth in the economy. When the model is evaluated assuming higher levels of tax growth rates, I find that from 1950 to the 1980's, the top 1 percent go from owning between 30 percent and 25 percent to owning between 80 and 70 percent of wealth respectively. This is shown in columns (2) and (3) of Table 4.

Secondly, in the 1990's and 2000's, I observe a drastic decline in wealth inequality to where the average wealthy individual owns only approximately 2.5 times more than the average individual in the economy. In the model where different tax rates are observed, the decline is as low as the average wealthy individual owning the same amount of wealth as the average individual. See column (2) and (3). The drop in wealth inequality is evident with the technology boom in the 1990 and the recessions in the 2000's. Basically, with the technology boom of the 1990's, different channels were present for individuals to attain property though copyrights and patents, which had nothing to do with physical property. Likewise, during the 2000's, the great recession led to a loss of value in housing prices which could have translated to a loss in wealth for many. Not only that, but the stock market crush also led to a loss of wealth for many. Third but not the least, after 2010, I observe that inequality is getting back to the level where the average wealthy individual owned 10 times more wealth than the average individual. Based on the model simulation, after 2010 the average wealthy individual owned 6 times more than the average individual in the U.S. economy which corresponds to the top 1 percent owning more than 50 percent of wealth in the economy.

Table 4: Results from Simulating the Simple Model of Pareto Wealth Accumulation

Time	Return on Property Ownerhsip	Population Growth	(1) $\frac{r_p-g-\tau-\alpha}{\bar{n}+\bar{d}}$	(2) $\frac{r_p-g-\tau(2)-\alpha}{\bar{n}+\bar{d}}$	(3) $\frac{r_p-g-\tau(3)-\alpha}{\bar{n}+\bar{d}}$
1950-1959	6.32	1.73	3.65	3.07	2.49
1960-1969	6.66	1.36	4.90	4.16	3.43
1970-1979	10.02	1.05	9.53	8.58	7.62
1980-1989	9.87	0.95	10.41	9.35	8.30
1990-1999	6.57	1.22	5.38	4.56	3.74
2000-2009	2.46	0.96	2.57	1.53	0.48
2010-2015	6.46	0.74	8.71	7.36	6.01
1956-1985	8.67	1.23	7.05	6.24	5.42
1986-2015	5.20	1.00	5.22	4.21	3.21
1950-2015	7.17	1.17	6.13	5.28	4.42

Calculations by Author.

When the model is evalutate over 30 year periods, we see that wealth inequality remains high after the World Wars with the average wealthy individual owning 7 times more wealth than the average individual. There is a slight drop in the pareto wealth inequality coefficient after 1985. However, it still remains high with the average wealthy individual owning 5 times more wealth than the average individual.

5 Conclusion

Antonio [2], on his review of Piketty's work states that, "Piketty's rentier thesis taps public fears that the new normal of declining opportunity, mobility, fairness, and political efficacy is here to stay. The impressive array of comparative data that he deploys to make his case illuminates the enormous scale of economic inequality, radical rupture from the postwar Trent Glorieuses, and prospect of a much more unequal, undemocratic future, should divergence continue unopposed." Many are skeptical to the findings that wealth inequality in developed economies is rising, which contradicts the popular equitable portrayal of these economies politically. Nonetheless, the evidence cannot be refuted that there is growing concerns of inequality in our society, which needs to be addressed by researchers. This paper, adds to the literature by specifically addressing wealth inequality, and by focusing on property ownership. For the first part of the paper, I use after tax historical data from the U.S. National Income and Product Accounts to derive income from property ownership, and show that the level of wealth inequality in the U.S. economy is much higher than suggested in the Piketty literature. I find that when you account for property ownership, the inequality gap $r_p - g$ is wider than Piketty's $r - g$, which has significant implications for wealth concentration. Also, I implement the $r_p - g$ gap into different models of wealth inequality: the Piketty model and a simple model of wealth accumulation. The Piketty model assumes that wealth is created through production, where as the latter model accounts specifically for wealth creation without using the Neo-Classical Growth framework.

In the Piketty model, I find that there is no convergence of wealth inequality over the entire period when you account for income from property ownership. That is, inequality is explosive in the U.S. economy. The model is highly dependent on fact that *omega*, the difference between the exponential growth between the return on property ownership and growth in income per capita, over time has to be less than 1. That is, $\omega = s \cdot \frac{1+R_p}{1+G} = s.e^{(r_p - g)\cdot H} < 1$. Nonetheless, when $\omega > 1$ this implies that wealth inequality is explosive. Surprisingly, evaluating the model over shorter periods, 10 years, reveals that wealth ownership has been more equitable in the U.S. economy. Certainly, this contradicts the fact that when evaluated over the entire period, wealth is explosive. The varying results can be attributed to the fact that wealth accumulates over time. However, because the Piketty model emphasizes on wealth creation based on production, inconsistencies are created when wealth created without production is accounted for. That is, wealth creation cannot be restricted by the fact that ω has to be less than 1. To replicate the economy described by Piketty, I had to evaluate the model with the assumption that the probability of a shock affecting wealth accumulation is 0.015. I found that wealth inequality declined when the model was evaluated over two 30 year period: 1954-1985 and 1986-2015. However, even with a low probability of a disruptive shock occurring, wealth inequality is explosive over the entire period.

Nonetheless, the main problem with the Piketty model is that it is rooted in the Neoclassical Growth framework, meaning wealth must be created through production. As indicated by Jones [11], the confusion with the central role of Piketty's $r - g$, is that the relationship between $r - g$ and wealth inequality is not obviously clear in the Neoclassical Growth model. I use a simple model of wealth accumulation for the U.S. economy, accounting for income from property ownership. 3 main facts are obvious. (i) After the World Wars, there is a spike in wealth inequality in the U.S. economy, reaching a peak in the 1980's were the average wealthy individual owns 10 times as much wealth as the average person. (ii) There is a decline in wealth inequality in the 1990's and 2000's, however an increase in wealth in equality is observed again after 2010. (iii) Overall, the level of wealth inequality found in the simple model of wealth accumulation is much higher than levels found in the Piketty literature. I find that with $r_p - g = 5\%$, the top 1 percent of wealth owners, own approximately 50 percent of wealth in the economy.

A Heterogeneity through a birth-death process

The birth-death process here follows the demography literature. The number of people born at date t is given by

$$B_t = B_0 e^{\bar{n}t} \tag{34}$$

Death is a Poisson process with arrival rate $\bar{d}$. To find the stationary distribution for the birth-death process, let $G(x,t) = Pr[Age > x]$ denote the age distribution at time t. Given that the population growth rate is $\bar{n}$ and the death rate is $\bar{d}$, the distribution evolves over a time interval Δt as

$$G(x, t + \Delta t) = \frac{1 - \bar{d}\Delta t}{1 + \bar{n}\Delta t} \cdot G(x,t) + G(x - \Delta x, t) - G(x,t) \tag{35}$$

where $\frac{1-\bar{d}\Delta t}{1+\bar{n}\Delta t} \cdot G(x,t)$ captures the change from deaths and population growth whiles the $G(x - \Delta x, t) - G(x,t)$ captures the inflow of younger people ito higher ages. Using the taylor expansion but ignoring higher order terms, this implies that

$$\frac{G(x, t + \Delta t) - G(x.t)}{\Delta t} = -(\bar{n} + \bar{d})G(x,t) - fracG(x,t) - G(x - \Delta x, t)\Delta x \tag{36}$$

where $\Delta x = \Delta t$. Taking the limit as $\Delta t \to 0$ implies that

$$\frac{\partial G(x.t)}{\partial t} = -(\bar{n} + \bar{t})G(x,t) - \frac{\partial G(x,t)}{\partial x} \tag{37}$$

By setting the time derivative to zero and solving the equation, the result is the stationary distribution for the birth-death process which is exponential:

$$G(x) = Pr[Age > x] = e^{-(\bar{n}+\bar{d})x} \tag{38}$$

Table 5: Summary Statistics

		Share of Personal Income					Share of Income from Property Ownership			
Year	Personal income	Dividend Income	Interest Income	Rental Income	Capital Gains	Property Income	Dividend Income	Interest Income	Rental Income	Capital Gains
1949	211.20	3.41	5.07	20.12	2.18	65.01	11.07	16.46	65.37	7.10
1950	233.90	3.76	5.04	19.79	2.12	71.87	12.24	16.42	64.43	6.91
1951	264.50	3.25	4.88	19.77	2.02	79.14	10.87	16.30	66.08	6.75
1952	282.70	3.04	4.92	19.03	2.03	82.05	10.48	16.94	65.57	7.01
1953	299.60	2.97	5.24	18.02	2.07	84.79	10.50	18.52	63.69	7.30
1954	302.60	3.07	5.72	18.31	2.37	89.16	10.43	19.40	62.14	8.03
1955	324.60	3.23	5.82	17.78	3.04	96.98	10.83	19.49	59.50	10.19
1956	348.40	3.24	6.08	17.08	2.78	101.68	11.11	20.85	58.52	9.52
1957	368.50	3.18	6.46	16.80	2.20	105.51	11.09	22.56	58.67	7.69
1958	379.50	3.06	6.80	17.13	2.49	111.84	10.37	23.07	58.12	8.44
1959	403.20	3.13	6.94	16.34	3.26	119.64	10.53	23.40	55.08	10.98
1960	422.50	3.17	7.34	15.88	2.78	123.25	10.87	25.15	54.44	9.53
1961	441.10	3.15	7.53	15.96	3.63	133.50	10.41	24.87	52.73	11.99
1962	469.10	3.20	7.80	15.60	2.87	138.25	10.85	26.47	52.95	9.73
1963	492.80	3.29	8.08	15.24	2.96	145.68	11.12	27.32	51.55	10.01
1964	528.40	3.44	8.38	14.74	3.30	157.83	11.53	28.07	49.36	11.04
1965	570.80	3.54	8.50	14.54	3.76	173.18	11.66	28.00	47.93	12.41
1966	620.60	3.34	8.62	14.15	3.44	183.35	11.29	29.18	47.89	11.64
1967	665.70	3.23	8.76	13.49	4.14	197.14	10.91	29.57	45.55	13.97
1968	730.70	3.22	8.73	12.85	4.87	216.81	10.84	29.43	43.31	16.42
1969	800.30	3.02	9.30	12.17	3.93	227.44	10.64	32.71	42.82	13.82
1970	864.60	2.81	10.21	11.39	2.41	231.95	10.48	38.07	42.47	8.99
1971	932.10	2.68	10.44	11.35	3.04	256.44	9.75	37.94	41.26	11.05
1972	1023.60	2.62	10.40	11.52	3.50	287.07	9.34	37.10	41.07	12.49
1973	1138.50	2.63	10.61	11.93	3.14	322.26	9.28	37.49	42.14	11.10
1974	1249.30	2.66	11.38	10.85	2.42	341.12	9.73	41.69	39.72	8.86
1975	1366.90	2.41	11.88	10.29	2.26	366.90	8.97	44.26	38.35	8.42
1976	1498.50	2.60	11.58	10.12	2.64	403.69	9.66	43.00	37.55	9.78
1977	1654.60	2.70	12.01	9.71	2.74	449.44	9.95	44.21	35.76	10.09
1978	1859.70	2.73	12.22	9.83	2.72	511.43	9.91	44.44	35.76	9.88
1979	2078.20	2.76	12.55	9.43	3.53	587.74	9.77	44.39	33.35	12.50
1980	2317.50	2.76	13.89	8.25	3.20	651.43	9.82	49.43	29.37	11.38
1981	2596.50	2.83	15.37	7.90	3.12	758.84	9.70	52.61	27.03	10.67
1982	2779.50	2.79	16.68	7.10	3.24	828.75	9.36	55.95	23.81	10.88
1983	2970.30	2.80	16.87	7.20	4.13	920.87	9.05	54.40	23.22	13.33
1984	3281.80	2.76	17.50	7.80	4.28	1061.60	8.53	54.11	24.12	13.23
1985	3516.30	2.77	17.50	7.70	4.89	1155.39	8.43	53.26	23.42	14.89
1986	3725.70	2.85	17.43	7.47	8.80	1361.43	7.79	47.69	20.45	24.07
1987	3955.90	2.84	16.88	7.75	3.75	1235.05	9.08	54.06	24.83	12.02
1988	4276.30	3.03	16.59	8.21	3.80	1352.79	9.59	52.45	25.94	12.02
1989	4619.90	3.42	17.19	7.93	3.33	1472.34	10.72	53.94	24.88	10.46
1990	4906.40	3.44	16.76	7.86	2.52	1500.78	11.25	54.80	25.71	8.25
1991	5073.40	3.55	15.87	7.84	2.20	1495.09	12.05	53.86	26.62	7.46
1992	5413.00	3.49	14.65	8.62	2.34	1575.59	12.00	50.34	29.62	8.04
1993	5649.00	3.62	13.88	9.28	2.70	1664.96	12.29	47.08	31.48	9.14
1994	5937.30	3.96	13.38	9.72	2.57	1759.13	13.37	45.15	32.80	8.68
1995	6281.00	4.11	13.63	9.77	2.87	1907.83	13.52	44.87	32.17	9.44
1996	6667.00	4.53	13.11	10.42	3.91	2131.60	14.18	41.02	32.58	12.23
1997	7080.70	4.77	13.00	10.45	5.15	2363.13	14.30	38.95	31.31	15.44
1998	7593.70	4.68	13.00	10.72	5.99	2611.82	13.61	37.79	31.17	17.43
1999	7988.40	4.34	12.31	11.06	6.92	2765.81	12.52	35.55	31.94	19.98
2000	8637.10	4.44	12.39	10.95	7.46	3043.29	12.59	35.17	31.07	21.17
2001	8991.60	4.11	11.92	11.61	3.89	2835.14	13.02	37.82	36.83	12.33
2002	9153.90	4.35	10.83	11.89	2.93	2747.32	14.51	36.10	39.61	9.78
2003	9491.10	4.55	10.41	11.99	3.41	2881.91	15.00	34.29	39.49	11.22
2004	10052.90	5.59	9.37	12.11	4.97	3220.45	17.45	29.24	37.81	15.50
2005	10614.00	5.45	10.25	11.47	6.50	3573.95	16.18	30.45	34.06	19.31
2006	11393.90	6.35	10.66	11.07	7.01	3997.81	18.10	30.38	31.55	19.97
2007	12000.20	6.80	11.25	9.74	7.70	4259.36	19.17	31.70	27.44	21.70
2008	12502.20	6.44	10.89	10.31	3.98	3953.44	20.37	34.44	32.59	12.59
2009	12094.80	4.58	10.45	10.80	2.18	3388.16	16.34	37.32	38.57	7.78
2010	12477.10	4.36	9.58	11.51	3.16	3569.33	15.26	33.48	40.22	11.04
2011	13254.50	5.15	9.29	12.29	3.05	3947.14	17.28	31.20	41.27	10.24
2012	13915.10	6.00	9.26	12.70	4.81	4559.96	18.31	28.26	38.74	14.68
2013	14068.40	5.61	9.04	13.14	2.96	4325.27	18.24	29.39	42.74	9.63
2014	14694.20	5.55	8.86	13.32	3.82	4635.64	17.59	28.09	42.23	12.09
2015	15357.40	5.65	8.55	13.32	4.25	4880.13	17.79	26.90	41.93	13.39

Table 6: Capital Gains and Taxes Paid on Capital Gains in the United States, 1954 - 2009 Dollar amounts in millions

Tax Year	Total Realized Capital Gains	Taxes Paid on Capital Gains	Average Effective Tax Rate	Realized Gains as a Percentage of GDP	Maximum Tax Rate on Long-Term Gains
1954	7,157	1,010	14.1	1.88	25
1955	9,881	1,465	14.8	2.38	25
1956	9,683	1,402	14.5	2.21	25
1957	8,110	1,115	13.7	1.76	25
1958	9,440	1,309	13.9	2.02	25
1959	13,137	1,920	14.6	2.59	25
1960	11,747	1,687	14.4	2.23	25
1961	16,001	2,481	15.5	2.94	25
1962	13,451	1,954	14.5	2.3	25
1963	14,579	2,143	14.7	2.36	25
1964	17,431	2,482	14.2	2.63	25
1965	21,484	3,003	14	2.99	25
1966	21,348	2,905	13.6	2.71	25
1967	27,535	4,112	14.9	3.31	25
1968	35,607	5,943	16.7	3.91	26.9
1969	31,439	5,275	16.8	3.19	27.5
1970	20,848	3,161	15.2	2.01	32.21
1971	28,341	4,350	15.3	2.52	34.25
1972	35,869	5,708	15.9	2.9	36.5
1973	35,757	5,366	15	2.59	36.5
1974	30,217	4,253	14.1	2.02	36.5
1975	30,903	4,534	14.7	1.89	36.5
1976	39,492	6,621	16.8	2.16	39.875
1977	45,338	8,232	18.2	2.23	39.875
1978	50,526	9,104	18	2.2	39.875/33.85
1979	73,443	11,753	16	2.87	28
1980	74,132	12,459	16.8	2.66	28
1981	80,938	12,852	15.9	2.59	28.00/20.00
1982	90,153	12,900	14.3	2.77	20
1983	122,773	18,700	15.2	3.47	20
1984	140,500	21,453	15.3	3.57	20
1985	171,985	26,460	15.4	4.08	20
1986	327,725	52,914	16.1	7.35	20
1987	148,449	33,714	22.7	3.13	28
1988	162,592	38,866	23.9	3.19	28
1989	154,040	35,258	22.9	2.81	28
1990	123,783	27,829	22.5	2.13	28
1991	111,592	24,903	22.3	1.86	28.93
1992	126,692	28,983	22.9	2	28.93
1993	152,259	36,112	23.7	2.28	29.19
1994	152,727	36,243	23.7	2.16	29.19
1995	180,130	44,254	24.6	2.43	29.19
1996	260,696	66,396	25.5	3.33	29.19
1997	364,829	79,305	21.7	4.38	29.19/21.19
1998	455,223	89,069	19.6	5.18	21.19
1999	552,608	111,821	20.2	5.91	21.19
2000	644,285	127,297	19.8	6.47	21.19
2001	349,441	65,668	18.8	3.4	21.17
2002	268,615	49,122	18.3	2.52	21.16
2003	323,306	51,340	15.9	2.9	21.05/16.05
2004	499,154	73,213	14.7	4.21	16.05
2005	690,152	102,174	14.8	5.47	16.05
2006	798,214	117,793	14.8	5.97	15.7
2007	924,164	137,141	14.8	6.59	15.7
2008	497,841	68,791	13.8	3.48	15.35
2009	263,460	36,686	13.9	1.89	15.35

[1] Source of Data: Department of the Treasury, Office of Tax Analysis, June 8, 2012.
[2] Data include returns with positive total net capital gains, both short and long-term. Data for each year include some late-filed prior year returns. The maximum rate is the effective rate applying to high-income taxpayers, including effects of provisions that alter effective rates for significant amounts of gains. Maximum rates include the effects exclusions (1954-1986), alternative tax rates (1954-1986, 1991-1997), the minimum tax (1970-1978), the alternative minimum tax (1979 -), income tax surchages (1968-1970), and phaseouts of itemized deductions (3% 1991-2005, 2% 2006-2007 and 1% 2008-2009). The maximum statutory rate on long-term gains was 28% starting 1991, 20% starting May 1997 and 15% starting May 2003. The 2009 maximum rate included the effect of the 1% itemized deduction phaseout, computed as $15.35=15+0.01\times35$. Starting 1997, gains on collectibles and certain depreciation recapture are taxed at ordinary rates, up to maximum rates of 28% on collectibles and 25% on recapture. Midyear rate changes occurred in 1978, 1981, 1997 and 2003. Estimates are subject to revision.

Table 7: Actual and Projected Capital Gains Realizations and Tax Receipts, 1995 - 2014

	Capital Gains Realizations[a]		**Capital Gains Tax Receipts**[b]	
	(Billions of dollars)	(Percentage of GDP)	(Billions of dollars)	(Percentage of individual income tax receipts)
		Actual		
1995	180.13	2.43	39.84795	6.75
1996	260.695619	3.33	54.2179	8.26
1997	364.829	4.38	72.20505	9.79
1998	455.223	5.18	83.6988	10.10
1999	552.608	5.91	99.3074	11.29
2000	644.285	6.47	118.7852	11.83
2001	349.441	3.40	99.56395	10.01
2002	268.615	2.52	58.2223	6.78
2003	323.306	2.90	50.1201	6.31
2004	499.1537	4.21	61.18285	7.58
2005	690.1521	5.47	86.24545	9.26
2006	798.214	5.97	109.20255	10.46
2007	924.164	6.59	126.4996	10.87
2008	497.8407	3.48	106.3835	9.29
2009	263.460082	1.89	54.34375	5.94
2010	394.229541	2.6	44.88635	5.00
2011	404.3443	2.55	54.5958	5.00
2012	669.5572	4.12	67.548	5.97
2013	416.473373	2.48	104.21964	7.92
		Projected		
2014	590.4547194	3.50	89.75894659	6.63

[1] Source: Congressional Buget Office
[2] Notes: Capital gains realizations are the sum of net capital gains from tax returns reporting a net gain. Data for realizations after 2011 and data for tax receipts in all years are estimated or projected by the CBO. Data on realization before 2012 are estimated by the Treasury Department
[a] Calenda year basis
[b] Fiscal year basis. This measure is CBO's estimate of when tax liabilities resulting from capital gains are paid to the Treasury.

Table 8: Regular and Capital Gains Tax Rates for 2015

Ordinary Income Rate	Long-term Capital Gain Rate	Short-term Capital Gain Rate	Long-term Gain on Commercial Buildings	Long-term gains rate (Collectibles)	Long-term Gains on Certain Small Business Stock
10%	0%	10%	10%	10%	10%
15%	0%	15%	15%	15%	15%
25%	15%	25%	25%	25%	25%
28%	15%	28%	25%	28%	28%
33%	15%	33%	25%	28%	28%
35%	15%	35%	25%	28%	28%
39.60%	20%	39.60%	25%	28%	28%

[1] Source of Data: http://www.irs.gov/pub/irs-drop/rp-08-66.pdf

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