Online Appendix

A Elasticities of market tightness

A.1 Standard matching model

The equilibrium expression (12) for market tightness can be rewritten as

$$\frac{1-\phi}{c}(y-z) \;=\; \frac{r+s}{q(\theta)} \;+\; \phi\,\theta\,. \tag{53}$$

Implicit differentiation yields

$$\begin{aligned}\frac{d\,\theta}{d\,y} \;&=\; -\frac{\frac{1-\phi}{c}}{-\left(\frac{-q'(\theta)\,(r+s)}{q(\theta)^2} \;+\; \phi\right)} \;=\; -\frac{\left(\frac{r+s}{q(\theta)} \;+\; \phi\,\theta\right)\,\frac{1}{y-z}}{-\left(\frac{\alpha(r+s)}{\theta q(\theta)} \;+\; \phi\right)} \\ &=\; \frac{(r+s) \;+\; \phi\,\theta\,q(\theta)}{\alpha(r+s) \;+\; \phi\,\theta\,q(\theta)}\;\frac{\theta}{y-z} \;\equiv\; \Upsilon^{\mathrm{Nash}}\,\frac{\theta}{y-z}\,,\end{aligned} \tag{54}$$

where the second equality is obtained after using equation (53) to rearrange the numerator, while in the denominator, we invoke the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$; the third equality follows from multiplying and dividing by $\theta\,q(\theta)$. The elasticity of market tightness is then given by (15).

A.2 Layoff taxes under Nash product $(E-U)^{\phi}J^{1-\phi}$

Bellman equations (20) and (21) can be solved for J and E to get

$$J \;=\; \frac{y \,-\, w \,-\, \beta s(1-\phi)\tau}{1 \,-\, \beta(1-s)}\,, \tag{55}$$

$$E \;=\; \frac{w \,+\, \beta s(U-\phi\tau)}{1 \,-\, \beta(1-s)}\,. \tag{56}$$

The no-profit condition for vacancies from expression (4) and the value of an unemployed worker from expression (7) remain the same.

After equating the right sides of expressions (4) and (55) and then rearranging, we find that the equilibrium wage must satisfy

$$w \;=\; y \,-\, \frac{r+s}{q(\theta)}c \,-\, \beta s(1-\phi)\tau\,. \tag{57}$$

As compared to wage expression (5) in an economy without layoff taxes, the right side of expression (57) has an additional negative term involving the layoff tax, namely, $-\beta s(1-\phi)\tau$. However, the shared negative term $-(r+s)c/q(\theta)$ becomes less negative with a layoff tax because, as we will show, market tightness falls, increasing the probability $q(\theta)$ of filling a job. But as we shall also show, the former negative effect outweighs the latter positive one, so the equilibrium wage falls when there are layoff taxes.

To obtain another useful equation for the equilibrium wage, use expressions (55) and (56) to eliminate J and E from equation (19):

$$(1-\phi)\left\{\frac{w\,+\,\beta s(U-\phi\tau)}{1\,-\,\beta(1-s)} \,-\, U\right\} \;=\; \phi\,\frac{y\,-\,w\,-\,\beta s(1-\phi)\tau}{1\,-\,\beta(1-s)}\,. \tag{58}$$

After multiplying both sides by $(1-\beta(1-s))$ and simplifying, we find that the equilibrium wage satisfies

$$w \;=\; (1-\beta)U \,+\, \phi\big(y \,-\, (1-\beta)U\big)\,. \tag{59}$$

Regarding the annuity value of being unemployed, $(1-\beta)U$, we can follow the same steps as in Section I to arrive at expression (10), i.e.,

$$(1-\beta)U \;=\; z \,+\, \frac{\phi\,\theta\,c}{1-\phi}\,. \tag{60}$$

After using expression (60) to eliminate $(1-\beta)U$ in expression (59), and simplifying, we obtain our second wage equation:

$$w \;=\; z \,+\, \phi(y-z\,+\,\theta c)\,. \tag{61}$$

While this expression for the wage is identical to the corresponding expression (11) for a model without layoff taxes, it is now evaluated at a lower market tightness θ, and hence, the equilibrium wage rate is lower with layoff taxes. To confirm this, we equate the right sides of (57) and (61), and after rearranging, obtain the equilibrium expression for market tightness in Section IIIB:

$$y-z-\beta s\tau \;=\; \frac{r\,+\,s\,+\,\phi\,\theta\,q(\theta)}{(1-\phi)q(\theta)}\,c\,. \tag{22}$$

Since the left side of (22) is lower than the left side of (12), it follows that market tightness θ must be lower on the right side of the former expression than in the latter.

Paralleling the steps in Appendix A.1, implicit differentiation of expression (22) yields

$$\frac{d\,\theta}{d\,y} \;=\; \Upsilon^{\mathrm{Nash}}\,\frac{\theta}{y - z - \beta s \tau}\,. \tag{62}$$

The elasticity of market tightness is then given by (23).

A.3 Layoff taxes under Nash product $(E-U)^{\phi}(J+\tau)^{1-\phi}$

Under the alternative assumption that firms are liable for the layoff tax immediately upon being matched with unemployed workers regardless of whether employment relationships are eventually formed, the firm's threat point is $-\tau$ and the Nash product to be maximized is $(E-U)^{\phi}(J+\tau)^{1-\phi}$ so that the outcomes satisfy

$$E - U \;=\; \phi\big(S+\tau\big) \quad \text{and} \quad J \;=\; (1-\phi)S - \phi\tau\,, \tag{63}$$

i.e., a worker and a firm split match surpluses so that

$$(1-\phi)(E - U - \phi\tau) \;=\; \phi(J + \phi\tau)\,. \tag{64}$$

Hence, as compared to expression (2), the Bellman equation for a firm's value of a filled job is modified, but the Bellman equation for the value of an employed worker continues to be the same as expression (6), i.e.,

$$\begin{aligned} J &= y - w + \beta\big[-s\,\tau + (1-s)J\big] \\ E &= w + \beta\big[s\,U + (1-s)E\big]\,, \end{aligned}$$

where we have imposed that $V = 0$ so that vacancies break even in an equilibrium. These Bellman equations imply

$$J \;=\; \frac{y - w - \beta s\tau}{1 - \beta(1-s)} \tag{65}$$

$$E \;=\; \frac{w + \beta s U}{1 - \beta(1-s)}\,. \tag{66}$$

The no-profit condition for vacancies and the value of an unemployed worker continue to be as in expressions (4) and (7).

After equating the right sides of expressions and (4) and (65) and rearranging, it follows that the equilibrium wage satisfies

$$w \;=\; y - \frac{r+s}{q(\theta)}c \;-\; \beta s \tau\,. \tag{67}$$

To obtain another equation for the equilibrium wage, use expressions (65) and (66) to eliminate J and E from equation (64),

$$(1-\phi)\left\{\frac{w \,+\, \beta s U}{1 \,-\, \beta(1-s)} \,-\, U \,-\, \phi\tau\right\} \;=\; \phi\,\frac{y \,-\, w \,-\, \beta s\tau}{1 \,-\, \beta(1-s)} \,+\, \phi\tau\,. \tag{68}$$

After multiplying both sides by $(1-\beta(1-s))$, and simplifying, we find that the equilibrium wage satisfies

$$w \;=\; (1-\beta)U \,+\, \phi\Big(y \,-\, (1-\beta)U \,+\, (1-\beta)\tau\Big)\,. \tag{69}$$

To obtain an expression for $(1-\beta)U$, we can follow steps analogous to those in Section I to eliminate $(E-U)$ and J in expression (64),

$$(1-\phi)\left\{\frac{(1-\beta)U \,-\, z}{\beta\,\theta\,q(\theta)} \,-\, \phi\tau\right\} \;=\; \phi\left\{\frac{c}{\beta\,q(\theta)} \,+\, \phi\tau\right\}, \tag{70}$$

which after simplifications yields

$$(1-\beta)U \;=\; z \,+\, \frac{\phi}{1-\phi}\big[\theta\, c \,+\, \beta\,\theta\, q(\theta)\,\tau\big]. \tag{71}$$

After using expression (71) to eliminate $(1-\beta)U$ in expression (69) and simplifying, we obtain our second equation for the equilibrium wage:

$$w \;=\; z \,+\, \phi\left\{y - z \,+\, \theta\, c \,+\, \big[1-\beta\,(1-\theta\, q(\theta))\big]\tau\right\}. \tag{72}$$

Next, we equate the right sides of (67) and (72). After moving all terms that involve θ to one side and collecting the remaining terms on the other side, we obtain the following expression for equilibrium market tightness θ:

$$\frac{1-\phi}{c}\left\{y-z-\frac{\beta(\phi r+s)}{1-\phi}\tau\right\}=\frac{r+s}{q(\theta)}+\phi\,\theta+\phi\,\beta\,\theta\,q(\theta)\,\tau/c\,. \tag{73}$$

Implicit differentiation of expression (73) yields

$$\begin{aligned}
\frac{d\,\theta}{d\,y} &= -\frac{\frac{1-\phi}{c}}{-\left(\frac{-q'(\theta)\,(r+s)}{q(\theta)^2}+\phi+\phi\,\beta\,q(\theta)\,\tau/c+\phi\,\beta\,\theta\,q'(\theta)\,\tau/c\right)} \\
&= \frac{\frac{r+s}{q(\theta)}+\phi\,\theta+\phi\,\beta\,\theta\,q(\theta)\,\tau/c}{\frac{\alpha\,(r+s)}{\theta\,q(\theta)}+\phi+(1-\alpha)\phi\,\beta\,q(\theta)\,\tau/c}\left(y-z-\frac{\beta(\phi r+s)}{1-\phi}\tau\right)^{-1} \\
&= \frac{r+s+\phi\,\theta\,q(\theta)\big[1+\beta q(\theta)\tau/c\big]}{\alpha\,(r+s)+\phi\,\theta\,q(\theta)\big[1+(1-\alpha)\beta q(\theta)\tau/c\big]}\;\frac{\theta}{y-z-\frac{\beta(\phi r+s)}{1-\phi}\tau} \\
&\equiv \Upsilon^{\mathrm{alt}}\big(\tau/c\big)\,\frac{\theta}{y-z-\frac{\beta(\phi r+s)}{1-\phi}\tau}\,,
\end{aligned} \tag{74}$$

where the second equality is obtained after using equation (73) to eliminate $(1-\phi)/c$ in the numerator, while in the denominator, we twice invoke the constant elasticity of matching with respect to unemployment, $\alpha=-q'(\theta)\,\theta/q(\theta)$; and the third equality follows from multiplying and dividing by $\theta\,q(\theta)$. We can compute the elasticity of market tightness to be

$$\eta_{\theta,y}=\Upsilon^{\mathrm{alt}}\big(\tau/c\big)\,\frac{y}{y-z-\frac{\beta(\phi r+s)}{1-\phi}\tau}\,. \tag{75}$$

The factor $\Upsilon^{\mathrm{alt}}\big(\tau/c\big)$ is bounded from above by $\max\{\alpha^{-1},(1-\alpha)^{-1}\}$. Therefore, and as before, a high elasticity of market tightness requires that the second factor appearing in expression (75) be large, i.e., that the fundamental surplus fraction be small.[29]

[29] If workers' bargaining weight ϕ is zero, the fundamental surplus in expression (75) equals that in (23), i.e., the fundamental surplus is reduced by the annuitized layoff tax, $\beta s\tau$. But if workers' bargaining weight were to be raised, the fundamental surplus would be further reduced in (75) for two reasons as detailed in footnote 9.

A.4 Fixed matching cost after bargaining

Under the assumption that a fixed matching cost H is incurred after the firm and the worker have bargained over the consummation of a match (e.g., a training cost before work commences), the match surplus S becomes

$$S = \left\{ \sum_{t=0}^{\infty} \beta^t (1-s)^t \left[y - (1-\beta)U \right] \right\} - H = \frac{y - (1-\beta)U - (1-\beta(1-s))H}{1-\beta(1-s)} . \quad (76)$$

By Nash bargaining, the firm receives S_f and the worker S_w of that match surplus, as given by

$$S_f = (1-\phi)S \qquad \text{and} \qquad S_w = \phi S. \quad (77)$$

A worker's value as unemployed can be written as

$$U = z + \beta \left[\theta q(\theta) S_w + U \right],$$

which can be rearranged, and after invoking (77), to read

$$U = \frac{z + \beta\theta q(\theta) \frac{\phi}{1-\phi} S_f}{1-\beta} . \quad (78)$$

From equations (76), (77) and (78), a firm's match surplus can be deduced as

$$S_f = (1-\phi) \frac{y - z - \beta(r+s)H}{\beta(r+s) + \beta\theta q(\theta)\phi} , \quad (79)$$

where we have used $\beta = (1+r)^{-1}$ and $1 - \beta(1-s) = \beta(r+s)$.

A firm's match surplus must also satisfy the zero profit condition in vacancy creation,

$$c = \beta q(\theta) S_f \qquad \Longrightarrow \qquad S_f = \frac{c}{\beta q(\theta)} . \quad (80)$$

The two expressions (79) and (80) for a firm's match surplus jointly determine the equilibrium value of θ:

$$\frac{1-\phi}{c} [y - z - \beta(r+s)H] = \frac{r+s}{q(\theta)} + \phi\,\theta . \quad (81)$$

Paralleling the steps of implicit differentiation in Appendix A.1, we arrive at the elasticity of market tightness with respect to productivity as given by (25).

A.5 Fixed matching cost before bargaining

Under the assumption that the firm incurs the fixed matching cost before bargaining with the worker, the match surplus S becomes

$$S \;=\; \sum_{t=0}^{\infty} \beta^t (1-s)^t \left[y - (1-\beta)U\right] \;=\; \frac{y-(1-\beta)U}{1-\beta(1-s)}\,. \tag{82}$$

Nash bargaining outcomes and a worker's value as unemployed are still given by equations (77) and (78), so we can use those expressions together with equation (82) to deduce a firm's match surplus as

$$S_f \;=\; (1-\phi)\,\frac{y-z}{\beta(r+s)+\beta\theta q(\theta)\phi}\,. \tag{83}$$

Given that the firm bears the fixed matching cost before bargaining with the worker, the zero profit condition in vacancy creation becomes

$$c \;=\; \beta q(\theta)[S_f - H] \qquad \Longrightarrow \qquad S_f \;=\; \frac{c}{\beta q(\theta)} \,+\, H\,. \tag{84}$$

The two expressions (83) and (84) for a firm's match surplus jointly determine the equilibrium value of θ:

$$\frac{1-\phi}{c}\left[y - z - \frac{\beta(r+s)}{1-\phi}\,H\right] \;=\; \frac{r+s}{q(\theta)} \,+\, \phi\,\theta \,+\, \beta\theta q(\theta)\phi H/c\,. \tag{85}$$

Paralleling the steps of implicit differentiation in Appendix A.3, we can compute the elasticity of market tightness with respect to productivity to be

$$\eta_{\theta,y} \;=\; \Upsilon^{\rm alt}\big(H/c\big)\,\frac{y}{y-z-\dfrac{\beta(r+s)}{1-\phi}\,H}\,, \tag{86}$$

where the factor $\Upsilon^{\rm alt}(\cdot)$ is defined in equation (74), and remains bounded from above by $\max\{\alpha^{-1},(1-\alpha)^{-1}\}$. The second factor, the inverse of the fundamental surplus fraction, is larger than in expression (25) for the case of a fixed matching cost *after* bargaining. Because when the fixed matching cost is now incurred by the firm *before* bargaining, it means that the cost must ex ante be financed out of the firm's match surplus and hence, the associated deduction from the fundamental surplus is amplified by a smaller share $1-\phi$ of the match surplus going to the firm.[30]

[30]The reason is the same as in item (i) of footnote 9 in the case of the firm being liable for a layoff tax immediately upon being matched with an unemployed worker.

A.6 Nash-bargained wages in the financial accelerator model

From the case of the firm incurring a fixed matching cost before bargaining in Appendix A.5, expression (83) for the firm's match surplus remains the same under Nash-bargained wages in the financial accelerator model (when assuming block-bargaining, i.e., the entrepreneur and the financier form a block when bargaining with the worker). But the zero profit conditions in vacancy creation are very different. While the fixed matching cost H in Appendix A.5 was incurred by a firm upon matching with an unemployed worker, the average search cost $K(\sigma^\star)$ for the formation of an entrepreneur-financier pair in the credit market marks the start of that pair's quest to match with a worker in the labor market. Hence, the zero profit condition becomes

$$K(\sigma^\star) \;=\; -\sum_{t=0}^{\infty} \beta^t \big[1-q(\theta)\big]^t c \;+\; \sum_{t=1}^{\infty} \beta^t \big[1-q(\theta)\big]^{t-1} q(\theta)\, S_f \;=\; \frac{-c+\beta q(\theta) S_f}{1-\beta[1-q(\theta)]}\,,$$

that is,

$$S_f \;=\; \frac{c}{\beta q(\theta)} \;+\; \frac{1-\beta[1-q(\theta)]}{\beta q(\theta)}\, K(\sigma^\star)\,. \tag{87}$$

The two expressions (83) and (87) for a firm's match surplus jointly determine the equilibrium value of θ:

$$\frac{1-\phi}{c+\beta r K(\sigma^\star)} \left[y - z - \frac{\beta(r+s)}{1-\phi}\, K(\sigma^\star) \right] \;=\; \frac{r+s}{q(\theta)} \;+\; \phi\,\theta \;+\; \beta\theta q(\theta)\phi\, \frac{K(\sigma^\star)}{c+\beta r K(\sigma^\star)}\,. \tag{88}$$

Paralleling the steps of implicit differentiation in Appendix A.3, we can compute the elasticity of market tightness with respect to productivity to be

$$\eta_{\theta,y} \;=\; \Upsilon^{\mathrm{alt}} \left(\frac{K(\sigma^\star)}{c+\beta r K(\sigma^\star)} \right) \frac{y}{y - z - \dfrac{\beta(r+s)}{1-\phi}\, K(\sigma^\star)}\,, \tag{89}$$

where the decomposition resembles expression (86) for the case of a fixed matching cost before bargaining. The earlier fixed matching cost H is replaced by the average search cost $K(\sigma^\star)$ for the formation of an entrepreneur-financier pair in the credit market. Though, there is seemingly one difference regarding the argument of the first factor $\Upsilon^{\mathrm{alt}}(\cdot)$. While the argument had been H divided by c, it is now $K(\sigma^\star)$ divided by $c+\beta r K(\sigma^\star)$. The difference in denominators can be understood when viewing them as the per period cost of a vacancy. In the case of a fixed matching cost, that per period cost is solely the vacancy posting cost c. But in the financial accelerator model, there is also a credit search cost that has to be incurred before posting a vacancy and therefore, there is an additional per period cost associated with a vacancy in the form of an interest cost for the upfront credit search cost, $\beta r K(\sigma^\star)$.

B Wage elasticity in the standard matching model

To study the determinants of the elasticity of the wage with respect to productivity, we compute the derivative of the wage rate w with respect to productivity y. First, we differentiate wage expression (11) with respect to w, y, and θ,

$$dw \;=\; \phi\, dy \,+\, \phi\, c\, d\theta$$

or

$$\frac{d\,w}{d\,y} \;=\; \phi \left[1 \,+\, c\,\frac{d\,\theta}{d\,y}\right]. \tag{90}$$

Together with the derivative of market tightness with respect to productivity in (54), we arrive at

$$\begin{aligned}\frac{d\,w}{d\,y} \;&=\; \phi \left[1 \,+\, c\,\Upsilon^{\text{Nash}}\,\frac{\theta}{y-z}\right] \;=\; \phi \left[1 \,+\, \frac{(1-\phi)\,q(\theta)}{r+s\,+\,\phi\,\theta\,q(\theta)}\Upsilon^{\text{Nash}}\,\theta\right] \\ &=\; \phi \left[1 \,+\, \frac{(1-\phi)\,\theta\,q(\theta)}{\alpha(r+s)\,+\,\phi\,\theta\,q(\theta)}\right] \;=\; \phi\,\frac{\alpha(r+s)\,+\,\theta\,q(\theta)}{\alpha(r+s)\,+\,\phi\,\theta\,q(\theta)}\,,\end{aligned} \tag{91}$$

where the second equality uses expression (53) to eliminate $c/(y-z)$, and the third equality invokes the definition of Υ^{Nash} in (54).[31] The derivative (91) varies from zero to one as ϕ varies from zero to one. At one extreme, we know that if workers have a zero bargaining weight, $\phi = 0$, the equilibrium wage equals the value of leisure, $w = z$, and hence, the wage does not respond to changes in productivity, $dw/dy = 0$. At the other extreme, if firms have a zero bargaining weight, $\phi = 1$, workers reap all gains from productivity, $w = y$, and hence, the wage responds one-for-one to changes in productivity, $dw/dy = 1$. But of course,

[31] Another approach to compute dw/dy (and $d\theta/dy$) is to express the total differential of wage equations (5) and (11) in matrix form,

$$\begin{bmatrix} q(\theta) & (w-y)q'(\theta) \\ 1 & -\phi\, c \end{bmatrix} \begin{bmatrix} dw \\ d\theta \end{bmatrix} \;=\; \begin{bmatrix} q(\theta)\, dy \\ \phi\, dy \end{bmatrix},$$

and then apply Cramer's rule, followed by substitutions analogous to those above. As an alternative to any one of the two wage equations, we can also use equation (53) in this calculation.

in the latter case, there would be no vacancy creation in the first place so no one would be employed.

Next, we use equation (5) to derive an expression for the wage as a fraction of productivity:

$$\begin{aligned}\frac{w}{y} &= 1 - \frac{r+s}{q(\theta)}\,\frac{c}{y} = 1 - \frac{(1-\phi)(r+s)}{r+s+\phi\,\theta\,q(\theta)}\,\frac{y-z}{y} \\ &= \frac{\big[\phi + (1-\phi)\,z/y\big](r+s) + \phi\,\theta\,q(\theta)}{r+s+\phi\,\theta\,q(\theta)}\,, \end{aligned} \tag{92}$$

where the second equality uses equation (53) to eliminate $c/q(\theta)$. In the Hagedorn-Manovskii calibration with an extremely small fundamental surplus fraction, $(y-z)/y \approx 0$, it follows immediately from the second equality of expression (92) that $w/y \approx 1$. This outcome is a necessary consequence of the fact that if workers are to be willing to work the wage rate cannot fall below z, nor can it exceed y if firms are to be motivated to post vacancies. As discussed in Section IIA, within more common calibrations of the matching model $\phi\theta q(\theta)$ is usually high relative to $(r+s)$, and hence, the third equality of expression (92) explains why w/y can still be close to one even when the value of leisure is not close to productivity.

Using expressions (91) and (92), the elasticity of the wage with respect to productivity, $\eta_{w,y} \equiv (dw/dy)/(w/y)$, becomes

$$\eta_{w,y} = \phi\,\frac{\alpha(r+s) + \theta\,q(\theta)}{\alpha(r+s) + \phi\,\theta\,q(\theta)}\;\frac{r+s+\phi\,\theta\,q(\theta)}{\big[\phi + (1-\phi)\,z/y\big](r+s) + \phi\,\theta\,q(\theta)}\,. \tag{93}$$

As just discussed, under the Hagedorn-Manovskii calibration, the second fraction in expression (93) must approximately equal one, and hence, the wage elasticity coincides with the derivative of the wage with respect to productivity in expression (91). This observation sheds light on how Hagedorn and Manovskii (2008) designed their calibration. The worker's bargaining weight ϕ in equation (91) is used to attain a target wage elasticity, while the fundamental surplus fraction, $(y-z)/y$, in equation (15) is used to attain a particular elasticity of market tightness that via equation (14) is linked to a target elasticity of unemployment. Therefore, as also argued by Hagedorn and Manovskii, the choice of wage elasticity is incidental to the outcome that unemployment is highly sensitive to productivity changes, which instead is driven by their calibration of a small fundamental surplus fraction.

As mentioned above, Hagedorn and Manovskii argue that the high wage elasticity in a common calibration of the matching model can be lowered without changing the implication that unemployment is not very sensitive to productivity changes. Such a perturbed calibration also involves modifying the wage elasticity by altering the worker's bargaining weight ϕ, though a complication is that both fractions in expression (93) are at play when $z << y$. Nevertheless, within some bounds, the workers' bargaining power can be lowered in a common calibration of the matching model to reduce the wage elasticity substantially, as argued by Hagedorn and Manovskii and illustrated by our panel B of Figure 3. And so long as the fundamental surplus fraction is kept high, the elasticity of market tightness remains low in equation (15) so that unemployment does not respond very much to productivity, as illustrated in panel A of Figure 3.

Finally, for a given workers' bargaining weight ϕ, we take the limit of the wage elasticity as y approaches z, which also implies that a worker's probability of finding a job $\theta q(\theta)$ approaches zero:

$$\lim_{y\to z,\theta q(\theta)\to 0} \eta_{w,y} \;=\; \lim_{\theta q(\theta)\to 0} \left[\phi \, \frac{\alpha(r+s) \,+\, \theta\, q(\theta)}{\alpha(r+s) \,+\, \phi\, \theta\, q(\theta)}\right] \;=\; \phi\,. \tag{94}$$

This limit is discernible in panel B of Figure 3 as productivity approaches $z = 0.6$. In Figure 2, we temporarily arrest this convergence by recalibrating three unemployment schedules for $\phi = 0.5$ to increase the efficiency parameter A in the matching function so that the unemployment rate is 5 percent at productivity 0.61, 0.63, and 0.65, respectively. This recalibration increases a worker's probability of finding a job $\theta q(\theta)$, which by the logic of our limiting calculation in expression (94) arrests the convergence to the limit described in (94) and explains why all three wage elasticities in Figure 2 are approximately 0.97 as compared to 0.83, 0.91, and 0.93 for productivity 0.61, 0.63 and 0.65, respectively, along the solid line for $\phi = 0.5$ in panel B of Figure 3.

C Adjustments to Hall's (2005) parameterization

We alter Hall's value of the vacancy cost because, regrettably, his calibration implies that the job filling probability exceeds unity for all productivity levels except for the highest one.[32] By lowering the vacancy cost to $c = 0.1$, and making a corresponding adjustment of

[32] When eyeballing Hall's (2005) Figure 2, there are two ways of inferring market tightness at the lowest productivity level, for example. First, measures of vacancies and unemployment are $v \approx 0.025$ and $u \approx 0.082$. Second, the probability of finding a job is $\theta q(\theta) \approx 0.38$. Given Hall's calibration of the efficiency parameter of the matching function, $A = 0.947$, both ways imply a market tightness of around 0.30 at the lowest productivity level, which in turn implies a probability of filling a vacancy of $q(\theta) \approx 1.25$.

the efficiency parameter A of the matching function, we can preserve the same equilibrium unemployment outcomes reported by Hall.[33]

To facilitate our sensitivity analysis, we also alter Hall's model period from one month to one day because a shorter model period fosters the existence of equilibria with vacancy creation. (See footnote 6.) To accomplish our conversion from a monthly to a daily frequency, we compute a daily version $\hat{\Pi}$ of the monthly transition probability matrix Π by minimizing the sum of squared elements from the matrix operation $(\hat{\Pi}^{30} - \Pi)$ so that the monthly transition probabilities implied by $\hat{\Pi}$ are close to those of Π. Because of the high persistence in productivity, this target is approximated well with a daily value of $\hat{\rho} = 0.9996$. Other parameters that need to be converted are the efficiency parameter of the matching function and the separation rate, now with daily values of $A/30$ and $s/30$, and also the discount factor that becomes $\beta^{1/30}$ at a daily frequency.

D Fundamental surplus in complex environments

D.1 Vintage-capital growth analysis

In Figure 7, the dashed and solid lines, meant to refer to stylized versions of Europe and the U.S., respectively, are almost replicas of corresponding lines in panels A and B of Hornstein et al.'s (2007, hereafter HKV) figure 4. They depict steady-state unemployment rates

[33] As noted in Section IIIA, aside from any targets for vacancy statistics, the joint parameterization of c and A is a choice of normalization. Specifically, given a calibration c and A with an associated market tightness θ determined by equation (53), the same equilibrium unemployment rate and job finding probability can be attained with an alternative parameterization $\hat{c} = \zeta c$ and $\hat{A} = \zeta^{1-\alpha} A$, for any $\zeta \in (0, \theta^{\alpha}/A]$. To verify this claim, solve for the new market tightness $\hat{\theta}$ from an appropriate version of equation (53),

$$\frac{1-\phi}{[\zeta c]}(y-z) \;=\; \frac{r+s}{[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha}} \;+\; \phi\,\hat{\theta}\,.$$

After multiplying both sides by ζ and comparing to equilibrium expression (53) for the initial parameterization, the new solution satisfies $\hat{\theta} = \theta/\zeta$. At this new market tightness $\hat{\theta}$, the unemployment rate is unchanged because the job finding probability is unchanged, $\hat{\theta}[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha} = \theta\, A\theta^{-\alpha} = \theta\, q(\theta)$. The new probability of filling a vacancy is $[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha} = \zeta A\theta^{-\alpha} = \zeta\, q(\theta) \in (0,1]$, where the bounds follow from the range for ζ. The value of a filled job remains unchanged, as can be verified from expression (4), the no-profit condition in vacancy creation

$$\hat{J} \;=\; \frac{\hat{c}}{\beta[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha}} \;=\; \frac{\zeta c}{\beta[\zeta^{1-\alpha}A][\theta/\zeta]^{-\alpha}} \;=\; \frac{c}{\beta A\theta^{-\alpha}} \;=\; J\,,$$

where the second equality invokes the alternative parameterization $\hat{c} = \zeta c$ and the equilibrium outcome $\hat{\theta} = \theta/\zeta$.

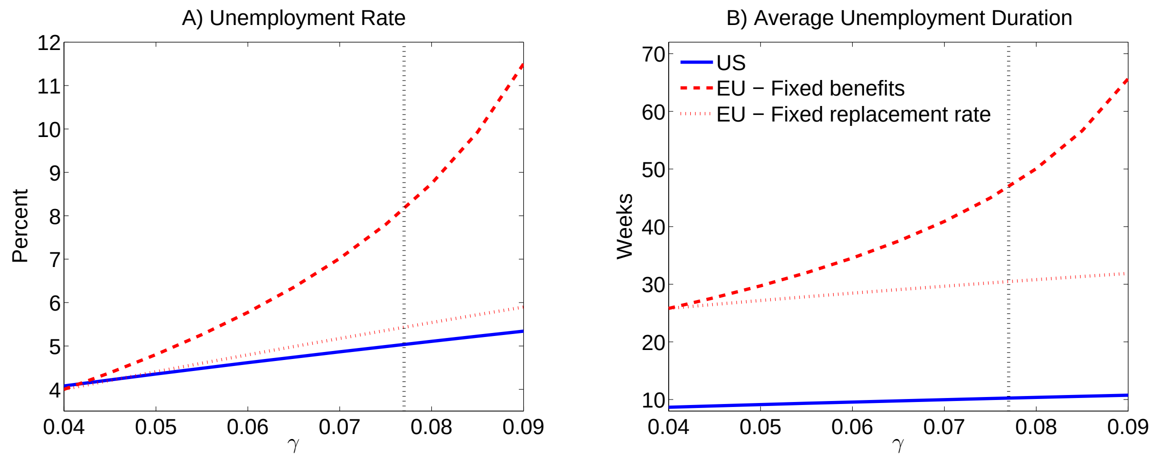

Figure 7: Unemployment rates and average durations of unemployment for different rates of capital-embodied technological change $\triangle$, where the dashed and solid lines refer to Europe and the U.S., respectively. The dotted horizontal lines depict what European outcomes would be if the replacement rate in unemployment compensation had been kept constant (rather than the quantity $\hat{b}^{\mathrm{EU}}$).

and average durations of unemployment at different rates of capital-embodied technological change $\triangle$. HKV suggest that pre-1970 and post-1990 are characterized by $\triangle = 0.04$ and $\triangle = 0.077$, respectively, i.e., the leftmost ends of the panels vis-à-vis the dotted vertical lines. Hence, starting from the same unemployment rate of 4% in pre-1970, panel A shows that the unemployment rate increases in post-1990 by over four percentage points in Europe but by just one percentage point in the U.S., with corresponding changes in average unemployment duration in panel B. These different outcomes are due to HKV's assumptions about government policies and also about exogenous separation rates that differ across Europe and the U.S. but remain fixed over time.

HKV calibrate government policies to be more active in Europe than in the U.S.: unemployment benefits $b^{\mathrm{EU}} = 0.33$ versus $b^{\mathrm{US}} = 0.05$, which correspond to replacement rates of 75% and 10% of average wages in pre-1970 (when $\triangle = 0.04$) in Europe and the U.S., respectively; a layoff tax $\tau^{\mathrm{EU}} = 0.45$ in Europe, which is equivalent to one year of average wages in pre-1970 (when $\triangle = 0.04$) versus no layoff tax in the U.S.; and a pair of European income and payroll taxes $\{24\%, 21\%\}$ versus a U.S. pair $\{17\%, 8\%\}$.[34] To attain the same 4% unemployment rate in pre-1970 (when $\triangle = 0.04$), HKV assume a significantly

[34] To study balanced growth paths, HKV assume that unemployment benefits b and layoff taxes τ (as well as the investment cost for machines) change at the economy's growth rate.

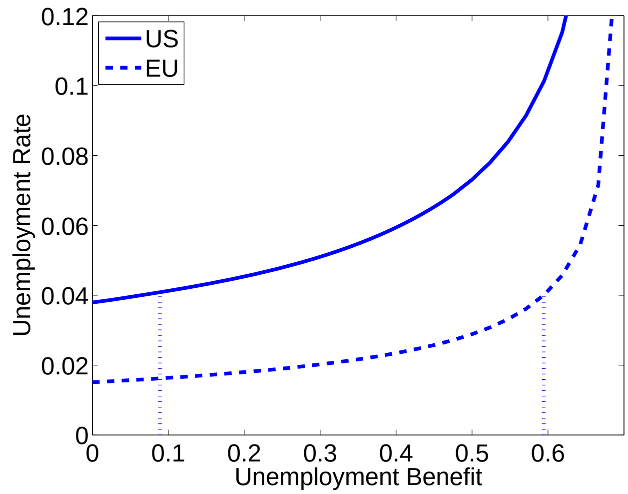

Figure 8: Unemployment rates for different unemployment benefits in pre-1970 (when $\triangle = 0.04$) in the U.S. (solid line) and Europe (dashed line).

lower exogenous separation rate in Europe than in the U.S. Based on these calibrations, the steady-state outcomes in Figure 7 emerge when varying $\triangle$. HKV (2007, p. 1110) also solve their model for Europe by activating one government policy at a time and conclude that "the technology–policy interaction is much starker when the three policies are considered together; as $\triangle$ increases, if one estimated the total role of policy by merely summing the effects of the individual policies, one would only account for less than one-third of the total technology–policy interaction predicted by the model with all policies jointly considered."

To make things more precise, we explain how we constructed Figure 7. Selecting one of HKV's possible government policies, we consider only unemployment benefits, and find that our alternative setting of $\hat{b}^{\mathrm{EU}} = 0.594$ and $\hat{b}^{\mathrm{US}} = 0.089$ can reproduce the outcomes of HKV's bundle of policies (as mentioned, the dashed and solid lines in Figure 7 are virtually the same as those of HKV's figure 4).[35] It is instructive to examine how the unemployment rate depends on pre-1970 benefits (when $\triangle = 0.04$), as depicted in Figure 8. Following HKV, we assume a much smaller exogenous separation rate in Europe than in the U.S. and hence

[35] With unemployment benefits as the sole policy, our algorithm for reproducing HKV's unemployment outcomes is as follows. First, for each value of unemployment benefits, we find an exogenous separation rate that produces HKV's targeted unemployment rate of 4% in pre-1970. Next, among all such pairs of benefits and exogenous separation rates, we select the pair that best reproduces HKV's relationship between unemployment and the exogenous rate of capital-embodied technological change (exhibited in panel A of HKV's figure 4). While this algorithm induces us to lower HKV's parameterization of the exogenous separation rate in Europe from 0.0642 to 0.0356, it leaves HKV's value for the U.S. unchanged at 0.2117.

the dashed line for Europe lies much below the solid line for the U.S. The vertical dotted lines mark benefits, $\hat{b}^{\mathrm{US}}$ and $\hat{b}^{\mathrm{EU}}$, respectively, at which the U.S. and Europe attain the same unemployment rate of 4% in pre-1970. A change to a higher $\triangle$ with its implied decline in fundamental surplus fractions is like rightward movements along the curves in Figure 8. Evidently, before 1970 Europe was poised to experience a larger increase in unemployment because a higher benefit level had decreased its fundamental surplus fraction. Thus, what matters is how far an economy is situated to the right along the curve in Figure 8 relative to where the unemployment relationship becomes much steeper. This is a way of expressing HKV's observation cited above that "the technology–policy interaction is much starker when [all] policies are considered together."

It is instructive to investigate the role of a factor contributing to the diminished post-1990 fundamental surplus fraction in HKV's theory of the outbreak of high European unemployment. Our HKV version in Figure 7 with a single policy of unemployment benefits, $\hat{b}^{\mathrm{EU}} = 0.594$, implies a replacement rate of 85% in pre-1970, while the same fixed quantity $\hat{b}^{\mathrm{EU}}$ implies a higher replacement rate of more than 93% post-1990. If instead we let $\hat{b}^{\mathrm{EU}}$ vary with $\triangle$ enough to keep the replacement rate constant at the value that prevails in pre-1970, we obtain the equilibrium relationship presented by the dotted curve in Figure 7. The resulting much smaller rise in European unemployment post-1990 suggests that HKV's explanation of higher European unemployment is not just about higher capital-embodied technological change but instead must come from their implicit assumptions that unemployment benefits became more generous and that the layoff tax became more burdensome.[36] If they had wanted to keep the unemployment benefit and the layoff tax policy constant over time in relation to the average wage, HKV would have had to calibrate an even smaller fundamental surplus fraction for pre-1970 Europe in order to generate a larger response of unemployment to higher capital-embodied technological change post-1990.

D.2 DSGE analysis

Christiano et al. (2016, hereafter CET) report on a perturbation of their Nash bargaining model by which the estimated replacement rate in unemployment insurance is cut by 58%

[36]In HKV's analysis of Europe, the unemployment benefit, $b^{\mathrm{EU}} = 0.33$, corresponds to a replacement rate of 75% (83%), and the layoff tax, $\tau^{\mathrm{EU}} = 0.45$, is equal to 1.0 (1.14) annual average wages in pre-1970 (post-1990). To keep European policy unchanged over time in terms of the average wage, post-1990 values of the unemployment benefit and the layoff tax would have to be $b^{\mathrm{EU}} = 0.295$ and $\tau^{\mathrm{EU}} = 0.40$, respectively, resulting in a post-1990 European unemployment rate of merely 5.4%. Once again, unemployment outcomes in our augmented HKV analysis are very close to those of our HKV version with only unemployment benefits.

from 0.88 to 0.37, i.e., a reduction to the estimated level of their alternating-offer bargaining (AOB) model. From the perspective of the fundamental surplus as argued in Section VIIB, we requested the authors to conduct a corresponding perturbation of their AOB model, namely, to cut by 58% both the level of unemployment insurance (from the original replacement rate of 0.37) and a firm's cost to make a counteroffer (from an original fraction 0.6 of a firm's daily revenue per worker) at the postulated daily frequency of a firm and a worker, alternatingly, making counteroffers. To keep the targeted unemployment rate at 5.5%, the vacancy posting cost is adjusted.[37]

CET generously conducted the perturbation of their AOB model to compute impulse response functions for a monetary policy shock, a neutral technology shock and an investment-specific technology shock, respectively, as reported in Figures 9, 10 and 11. When comparing these figures to the counterparts of the perturbed Nash bargaining model in CET (2016, figures 4, 5 and 6), we see that the two perturbed models generate almost identical dampened impulse responses of unemployment. Seen through the lens of the fundamental surplus, this was to be expected. It is the common channel of enlarged fundamental surplus fractions that mutes the elasticity of market tightness in both models. For example, given a large fundamental surplus fraction in the AOB model, it is immaterial that the alternative bargaining protocol of Hall and Milgrom (2008) suppresses the influence of the worker's outside value during bargaining.

[37]Initially, we sought to adjust the multiplicative efficiency parameter of the matching function to keep the targeted unemployment rate at 5.5%, but the computation algorithm broke down because of poor numerical properties when the resulting vacancy filling rate fell too close to zero. In any case, as shown in footnote 33, it is just a question of normalization whether we adjust the vacancy posting cost or the efficiency parameter of the matching function to target a particular unemployment rate.

For the record, regarding CET's own perturbation of the Nash bargaining model, they informed us to have used adjustments of bargaining weights to keep the targeted unemployment rate at 5.5%.

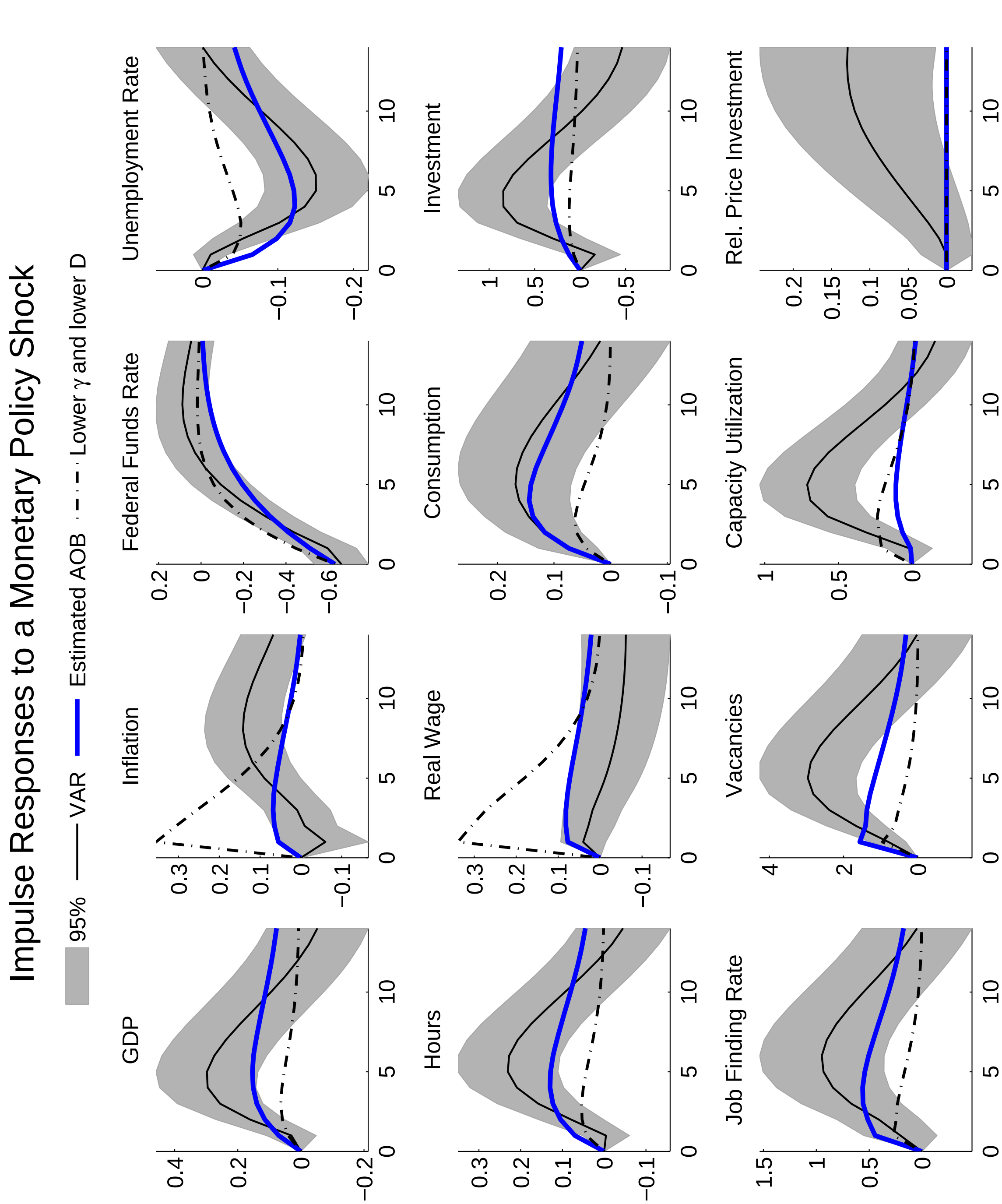

Figure 9: Impulse responses to a monetary policy shock in the AOB model. The dashed lines refer to the perturbed model in which the replacement rate in unemployment insurance and a firm's cost to make a counteroffer are both cut by roughly one half.

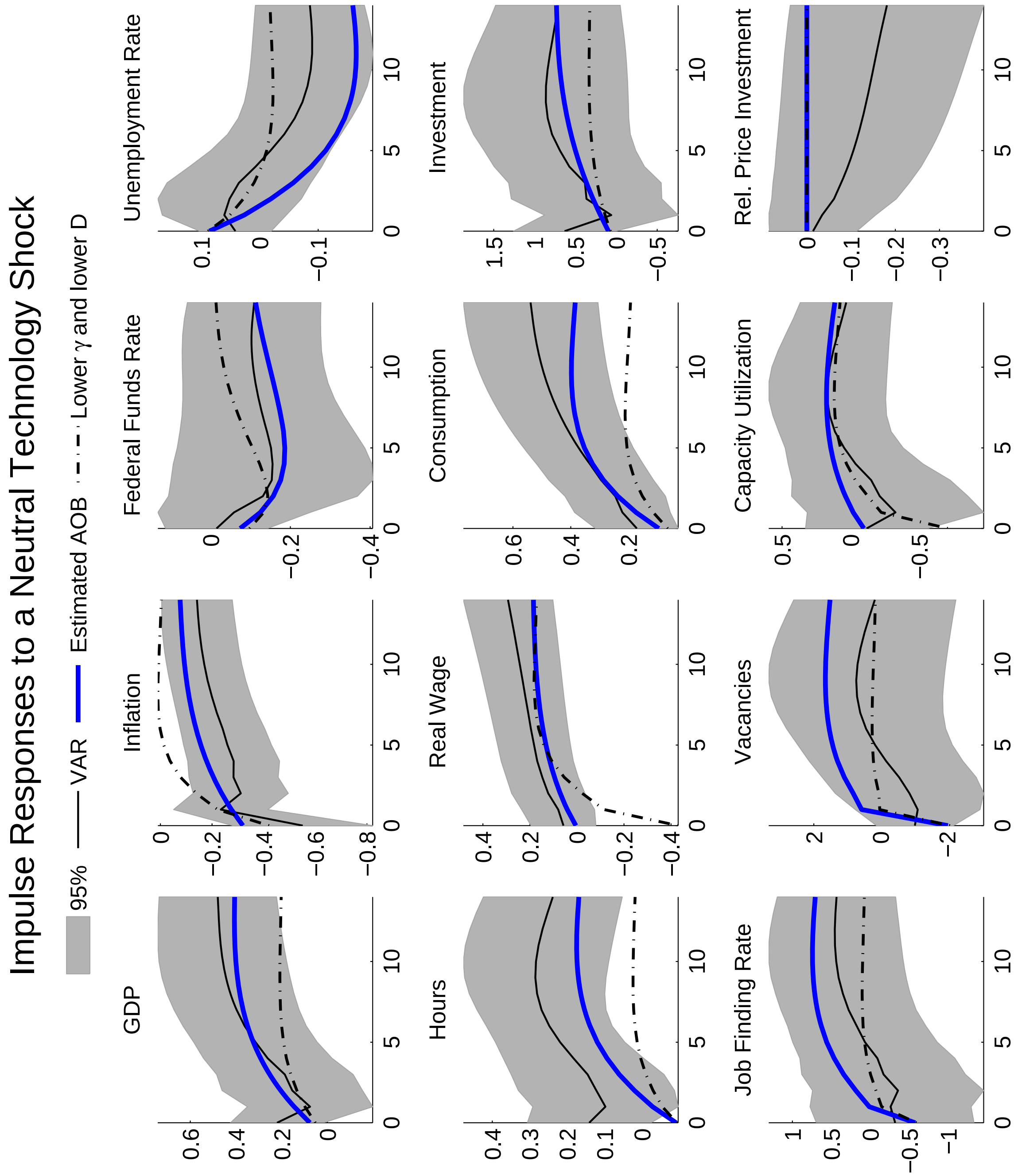

Figure 10: Impulse responses to a neutral technology shock in the AOB model. The dashed lines refer to the perturbed model in which the replacement rate in unemployment insurance and a firm's cost to make a counteroffer are both cut by roughly one half.

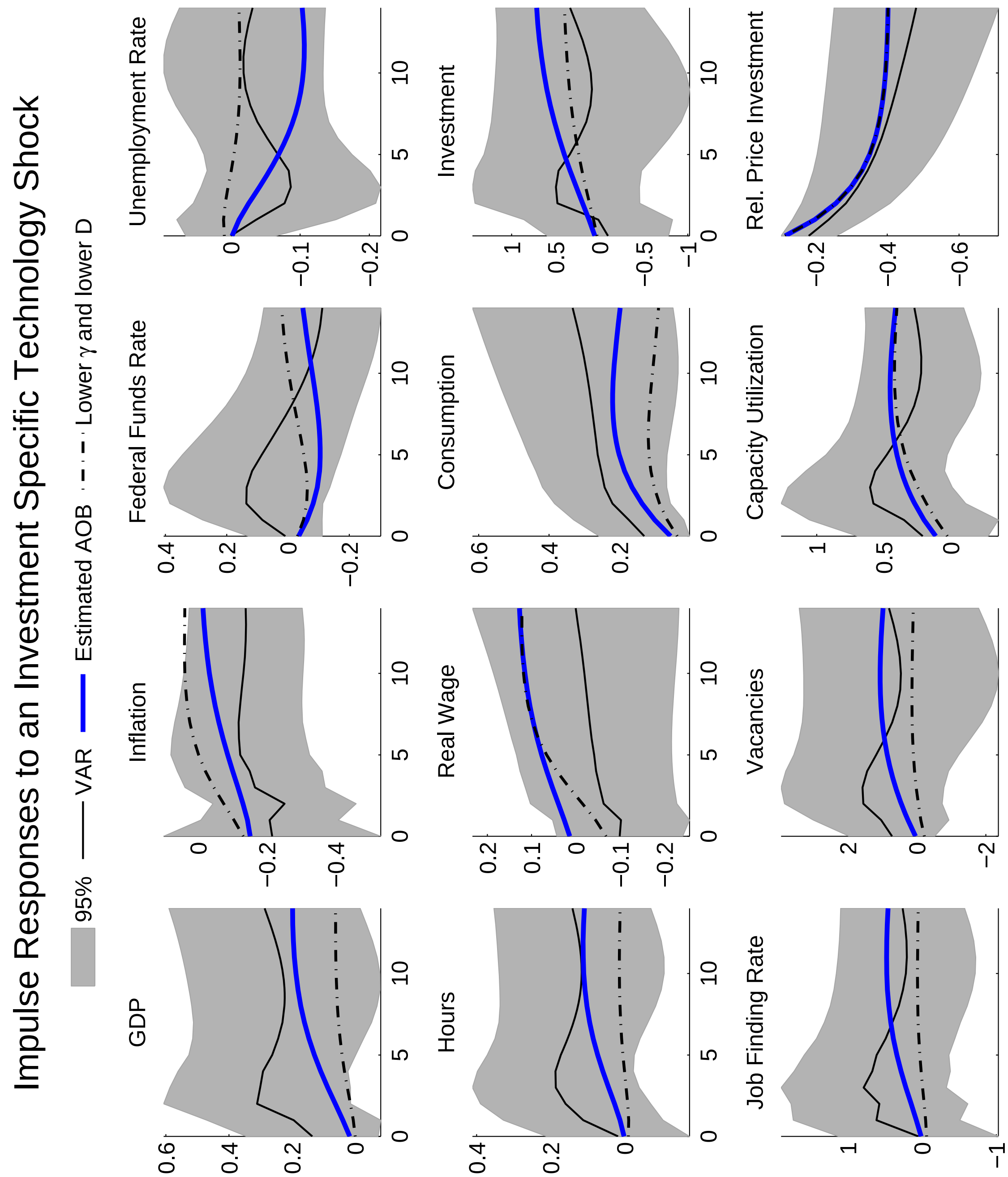

Figure 11: Impulse responses to an investment-specific technology shock in the AOB model. The dashed lines refer to the perturbed model in which the replacement rate in unemployment insurance and a firm's cost to make a counteroffer are both cut by roughly one half.