Misallocation, Establishment Size, and Productivity

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Online Appendix

A Establishment Size Data

We describe in more detail how we construct the establishment size data for the manufacturing sector. Our standardized definition of establishment size is the number of persons engaged per establishment. Persons engaged is defined as the average number of persons working for an establishment, both paid and unpaid. A manufacturing establishment is defined as a physical location where the primary activity is manufacturing. Establishments include households who have signs posted on the property indicating commercial activity. Not all countries report persons engaged or the number of establishments, so we also use data on the number of paid employees, the number of full-time equivalent employees, and the number of firms (collections of one or more establishments under common ownership) to impute persons engaged and establishments for these countries. We explain in detail the exact procedure for these imputations but we note that imputations are only involved in about one quarter of our sample of countries.

The source data for each country is from economic censuses, as well as surveys which use comprehensive business registries to create sampling frames and as a result are representative of the population of establishments.[2] We use all publicly available data for the years 2000 through 2012.[3] In an effort to maintain consistency across countries, we do not use data unless efforts were made by a statistical agency to make the data representative of an economy's entire population of manufacturing establishments. We exclude any data collected without

[2]For some countries data is from EUROSTAT or OECD's Structural Business Statistics, but we check each country's methodology to confirm the consistency of definitions.

[3]In some cases countries have published only press releases or bulletins describing the census data. We include these countries when the data meets our criteria.

accounting for small establishments, except in cases where only establishments without paid employees are excluded. In the later case, we use U.S. data to adjust measured establishment size (this is the case for eight countries). Further, we include data for any country that excludes establishments with low revenue, as long as the revenue threshold is lower than the country's GDP per capita (this is the case for four countries). Two countries (Algeria and Honduras) do not report employment, but do report the distribution of establishments across multiple employment tranches. In these two cases we estimate total employment by using an average employment within each tranche consistent with data in comparable countries.[4] We are left with 134 countries with useable data for at least one year, with an average of six years per country.[5] Table 1 reports the total number of countries reporting each variable for at least one year, as well as the total number of poor countries and the total number of rich countries (defined as having GDP per capita below and above the median) doing the same.

Table 1: Sample of Countries

Variable	Total Number of Countries	Number of Poor Countries	Number of Rich Countries
persons engaged	101	54	47
employees	86	34	52
engaged and employees	53	21	32
full-time equivalents	25	2	23
establishments	83	45	38
firms	67	26	41
establishments and firms	16	4	12

Note: 'Poor' and 'Rich' refer to countries with GDP per capita below and above the median. Data from multiple sources, see text for details.

We construct our standardized measure of persons engaged per establishment as follows. First, the total number of persons engaged is reported for 101 countries. For the remaining 33 countries, we impute persons engaged based on each country's reported data for the number of

[4]We assume average employment within a tranche to be one third of the distance from the lower to the upper threshold. For the last open-ended tranche (for example, 200 or more employees) we assume an average employment equal to twice the lower threshold.

[5]Although size data is also available for Norfolk Island, it has been dropped for lack of any reliable measure of GDP per capita.

employees and/or the number of full-time equivalent employment. We estimate the relationship between persons engaged and employment from a regression of persons engaged on employees and/or full-time equivalent employment using country-year data for the more than 50 countries that report all these variables. We then multiply the estimated coefficients by the reported country-year data to obtain persons engaged for those countries. Hence, this first step produces a number for the total persons engaged for each country-year in our sample. Second, we compute persons engaged per establishment (83 countries) and persons engaged per firm (67 countries). This allowed us to estimate the coefficient from a regression of persons engaged per establishment on persons engaged per firm for country-years that report both and then use the estimated coefficient to impute persons engaged per establishment using the data of countries that only report persons engaged per firm.

Each of the above regressions use all country-years which report the relevant variables. The results of the four regressions described above are;

- persons engaged $= 1.44 \cdot$ employees $- 0.40 \cdot$ full-time equivalents
- persons engaged $= 1.07 \cdot$ employees
- persons engaged $= 1.12 \cdot$ full-time equivalents
- persons engaged per establishment $= 0.89 \cdot$ persons engaged per firm

There is a small number of countries for which the data exclude non-employer establishments or that report a combination of manufacturing, extraction, and energy instead of just manufacturing. For these countries we do the following. To adjust persons engaged per establishment in countries which exclude non-employer establishments (this is the case for eight countries), we multiply these values by a factor equal to the average ratio of persons engaged per establishment to persons engaged per establishment with paid employees across all years in the U.S. data (this ratio is 0.51 in U.S. data). We similarly standardize persons engaged per establishment

for manufacturing for five countries which report statistics for a combination of manufacturing, extraction, and energy using U.S. data for the ratio of persons engaged per establishment in manufacturing relative to manufacturing, extraction and energy (this ratio is 1.14 in U.S. data). In our final dataset, the resulting measures of persons engaged per establishment are averaged over all years for each of the 134 countries.

Table 2 lists each country in the final dataset, the number of years for which data is available, and the sources from which data has been collected.

Table 2: List of Countries and Sources

Country	Code	Years	Sources
Âland Islands	ALA	9	Statistics and Research Âland: Statistical Yearbooks of Âland 2006-2010 and 2013, and www.asub.ax
Albania	ALB	8	Instituti i Statistikave: www.instat.gov.al/en/figures/statistical-databases.aspx
Algeria	DZA	1	Office National des Statistiques, Alger: Premier recensement économique -2011- Résultats définitifs
American Samoa	ASM	2	U.S. Census Bureau: 2002, 2007 County Business Patterns, and 2002, 2007 Nonemployer Statistics
Andorra	AND	12	Departament d'Estadística: 2010 Statistical Yearbook, and www.estadistica.ad
Argentina	ARG	1	Instituto Nacional de Estadística y Censos: 2005 Economic Census
Aruba	ABW	1	Central Bureau of Statistics: Business Count 2003
Australia	AUS	5	Australian Bureau of Statistics: Counts of Australian Businesses 2003-2007, Labour Force Surveys (Quarterly)
Austria	AUT	12	Statistik Austria: statcube.at, and OECD's SDBS Structural Business Statistics
Bahrain	BHR	2	Kingdom of Bahrain Central Informatics Organization: Population, Housing, Buildings, Establishments and Agriculture Census
Bangladesh	BGD	1	Bangladesh Bureau of Statistics: Economic Census 2001 & 2003
Belgium	BEL	11	Eurostat, and OECD's SDBS Structural Business Statistics
Benin	BEN	1	Institut National de la Statistique et de l'Analyse Economique: General Census of Companies, and Les Entreprises Artisanales au Benin
Bermuda	BMU	11	Department of Statistics: www.govsubportal.com
Bhutan	BTN	4	National Statistics Bureau: Statistical Yearbooks 2010-2013
Bolivia	BOL	1	Instituto Nacional de Estadística: Structural Statistics of the Manufacturing Industry, Trade and Services - 2010, and Results of the Quarterly Survey of Micro and Small Business 2010
Bosnia and Herzegovina	BIH	8	Institute for Statistics of FB&H: Statistical Yearbooks 2008-2013
Brazil	BRA	13	Brazilian Institute of Geography and Statistics: Cadastro Central de Empresas
Brunei	BRN	1	Department of Economic Planning and Development: Brunei Darussalam Statistical Yearbook 2010
Bulgaria	BGR	12	Eurostat
Cambodia	KHM	2	National Institute of Statistics: Economic Census 2011, and Establishment Listing 2009
Cameroon	CMR	1	Institut National de la Statistique du Cameroun: Recensement Général des Entreprises 2009
Canada	CAN	7	Statistics Canada: CANSIM
Cape Verde	CPV	4	Instituto Nacional de Estatística: Business Census 2007, and Annual Business Surveys 2008-2009
Columbia	COL	1	Departamento Administrativo Nacional de Estadística: Encuesta Annual Manufacturera, and www.dane.gov.co
Croatia	CRV	4	Eurostat

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Cyprus	CYP	12	Eurostat
Czech Republic	CZE	10	Eurostat, and OECD's SDBS Structural Business Statistics
Denmark	DNK	12	Eurostat, and OECD's SDBS Structural Business Statistics
Ecuador	ECU	1	Instituto Nacional Estadística y Censos: National Economic Census 2010
El Salvador	SLV	1	Ministerio de Economica: Tomo I de los VII Censos Económicos Nacionales 2005
Estonia	EST	1	Statistics Estonia: Statistical Yearbooks 2011-2013, and pub.stat.ee
Ethiopia	ETH	1	Central Statistical Agency: Report on Small Scale Manufacturing Industries Survey 2005/6, Report on Large and Medium Scale Manufacturing and Electricity Industries Survey 2005/6, and Labour Force Survey 2005
Faroe Islands	FRO	12	Statistics Faroe Islands: www.hagstova.fo
Finland	FIN	1	Statistics Finland: Labour Force Survey 2013, and www.stat.fi
France	FRA	9	Institut National de la Statistique et des Études Économiques: Tableaux de l'Économie Française - Édition 2005-6, 2010-2014, L'industrie en France - édition 2007, 2008, and www.insee.fr
French Guiana	GUF	1	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
French Polynesia	PYF	13	Institut de la Statistique de la Polynésie Française: www.ispf.pf
FYR Macedonia	FYR	5	State Statistical Office: www.stat.gov.mk
Georgia	GEO	11	National Statistics Office of Georgia: Statistical Yearbooks 2009-2013, and www.geostat.ge
Germany	DEU	12	Eurostat, and OECD's SDBS Structural Business Statistics
Ghana	GHA	1	Ghana Statistical Service: National Industrial Census 2003
Greece	GRC	6	Eurostat, and OECD's SDBS Structural Business Statistics
Greenland	GRL	5	Statistics Greenland: bank.stat.gl
Guadeloupe	GLP	1	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
Guam	GUM	7	U.S. Census Bureau: 2008-2011 County Business Patterns, and 2002, 2007, 2012 Economic Census of Island Areas
Honduras	HND	1	Instituto Nacional de Estadística y Censos: Directorio de Establecimientos Económicos
Hong Kong	HKG	13	Census and Statistics Department: Annual Survey of Industrial Production, and www.statistics.gov.hk
Hungary	HUN	11	Eurostat, and OECD's SDBS Structural Business Statistics
India	IND	1	Central Statistics Office: 2005 Economic Census
Indonesia	IDN	3	Statistics Indonesia: Statistical Yearbook 2013
Iran	IRN	1	Statistical Centre of Iran: Statistical Yearbook 1382
Israel	ISR	9	Central Bureau of Statistics: www1.cbs.gov.il, Eurostat, and OECD's SDBS Structural Business Statistics
Italy	ITA	12	Eurostat, and OECD's SDBS Structural Business Statistics
Japan	JPN	3	Statistics Japan: Establishment and Enterprise Censuses 2001, 2004, 2006

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Jordan	JOR	8	Department of Statistics: www.dos.gov.jo
Kazakhstan	KAZ	1	Committee on Statistics: www.stat.gov.kz
Korea	KOR	9	Statistics Korea: Censuses on Establishments 2007, 2009, 2011, 2012
Kosovo	UVK	6	Statistical Agency of Kosovo: Statistical Register of Business
Kuwait	KWT	10	Central Statistical Bureau: Annual Surveys of Establishments 2002-2011
Kyrgyzstan	KGZ	1	National Statistical Committee of Kyrgyz Republic: stat.kg
Laos	LAO	1	Lao Statistics Bureau: Economic Census 2006
Latvia	LVA	10	Central Statistical Bureau of Latvia: www.csb.gov.lv, and Eurostat
Libya	LBY	2	Bureau of Statistics and Census Libya: bsc.ly
Liechtenstein	LIE	6	Statistical Office: Statistical Yearbooks 2007/2008, 2009-2012
Lithuania	LTU	7	Eurostat
Luxembourg	LUX	12	Eurostat
Macau	MAC	13	Statistics and Census Service: www.dsec.gov.mo
Madagascar	MDG	1	Institut National de la Statistique: Rapport de l'enquete sur les Entreprises a Madagascar
Malawi	MWI	6	National Statistical Office: www.nsomalawi.mw
Malaysia	MYS	6	Department of Statistics Malaysia: Statistics Yearbooks 2007-2012
Maldives	MDV	1	Department of National Planning: Economic Survey 2007/2008
Malta	MLT	7	Eurostat
Martinique	MTQ	1	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
Mauritius	MUS	2	Statistics Mauritius: Censuses of Economic Activity 2002, 2007, Phases I and II
Mexico	MEX	2	Instituto Nacional de Estadstica y Geografía: Censos Economicos 2004, 2009
Moldova	MDA	8	Statistica Moldovei: www.statistica.md
Monaco	MCO	13	Monaco Statistics: Observatoire de l'Economie 2012, 2013
Mongolia	MNG	2	National Statistical Office of Mongolia: Monthly Bulletins of Statistics 2011, 2012
Montenegro	MNE	3	Statistical Office of Montenegro: www.monstat.org
Morocco	MAR	1	Haut-Commissariat au Plan du Maroc: 2001-2 Economic Census
Nepal	NPL	1	Central Bureau of Statistics: Census of Manufacturing Establishments 2006/7, Survey of Small Manufacturing 2008/9
Netherlands	NLD	11	Eurostat, Statistics Netherlands: Statistical Yearbooks 2004-2013
New Caledonia	NCL	13	Institut de la Statistique et des Etudes Economique: www.isee.nc
New Zealand	NZL	13	Statistics New Zealand: www.stats.govt.nz
Nicaragua	NIC	1	Instituto Nacional de Información de Desarrollo: Urban Economic Census
Norfolk Island	NFK	1	Australian Business Statistics: www.ausstats.abs.gov.au
Northern Mariana Islands	MNP	6	U.S. Census Bureau: 2008-2011 County Business Patterns, and 2007, 2012 Economic Census of Island Areas
Norway	NOR	8	Eurostat, and OECD's SDBS Structural Business Statistics

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Country	Code	Years	Sources
Palau	PLW	1	Office of Planning and Statistics: 2012 - 2nd, 3rd Quarters Economic Indicators
Palestinian Territories	PSE	7	Palestinian Central Bureau of Statistics: Establishment Censuses 2004, 2007, 2012, and Comparison Study on Industrial Activities 1999-2004
Panama	PAN	1	Instituto Nacional de Estadística y Censo: Preliminary Results of Economic Census 2012
Paraguay	PRY	1	Direccin General de Estadística, Encuestas y Censos: National Economic Census 2011
Peru	PER	1	Instituto Nacional de Estadística e Informática: IV Censo National Economico 2008
Philippines	PHL	2	National Statistics Office: NSO's 2012 List of Establishments, and 2003 Annual Survey of Philippine Business and Industry (ASPBI)
Poland	POL	12	Central Statistical Office of Poland: Statistical Yearbook 2011, 2012, Eurostat, and OECD's SDBS Structural Business Statistics
Portugal	PRT	11	Eurostat, and OECD's SDBS Structural Business Statistics
Puerto Rico	PRI	7	U.S. Census Bureau: 2006-2011 County Business Patterns, and 2002 Economic Census of Island Areas
Qatar	QAT	3	Ministry of Development Planning and Statistics: Establishment Censuses 2004, 2008, 2010
Réunion	REU	3	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
Romania	ROU	6	National Institute of Statistics: Statistical Yearbooks 2006-2012
Russia	RUS	3	Federal State Statistics Service: Industry of Russia 2008, 2009, 2011, and Small and Medium Businesses in Russia 2008, 2009, 2011
Rwanda	RWA	1	National Institute of Statistics of Rwanda: Establishment Census - 2011
San Marino	SMR	8	Ufficio Informatica, Tecnologia, Dati e Statistica: www.statistica.sm
São Tomé and Príncipe	STP	2	Instituto Nacional de Estatísticas de São Tomé e Príncipe: Business Statistics 2006, 2007
Saudi Arabia	SAU	1	Central Department of Statistics and Information: 2010 Economic Census
Serbia	SRB	3	Republika Srpska Institute of Statistics: Statistical Yearbook of Republika Srpska 2011, 2012, 2013
Sierra Leone	SLE	1	Statistics Sierra Leone: Report of the Census of Business Establishments 2005
Singapore	SGP	10	Department of Statistics Singapore: Census of Manufacturing Activities 2012
Slovak Republic	SVK	2	Eurostat
Slovenia	SVN	12	Eurostat, and OECD's SDBS Structural Business Statistics
South Africa	ZAF	12	Statistics South Africa: Annual Financial Statistics 2010, 2012, and Survey of Employers and the Self-Employed 2013
Spain	ESP	12	Eurostat, and OECD's SDBS Structural Business Statistics

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Country	Code	Years	Sources
Sri Lanka	LKA	1	Department of Census and Statistics - Sri Lanka: Census of Industry 2003/4
Sudan	SDN	1	Central Bureau of Statistics: Statistical Year Book for the Year 2009
Sweden	SWE	12	Eurostat, and OECD's SDBS Structural Business Statistics
Switzerland	CHE	3	Swiss Statistics: www.bfs.admin.ch/bfs/portal/en/index.html
Syria	SYR	4	Central Bureau of Statistics: www.cbssyr.sy
Taiwan	TWN	3	National Statistics: Industry, Commerce and Service Censuses 2001, 2006, 2011
Thailand	THA	2	National Statistical Office: Industrial Censuses 2007, 2012
Tonga	TON	7	Tonga Department of Statistics: Manufacturing Output, Employment and Wages/Salaries 2000-2003, 2001-2005, 2002-2006
Trinidad and Tobago	TTO	7	Central Statistical Office: Business Establishments in T & T by Industry Economic Activity 2005-2007
Tunisia	TUN	12	Institut National de la Statistique: Statistiques Issues du Répertoire des Entreprises
Turkey	TUR	8	Eurostat, and OECD's SDBS Structural Business Statistics
Uganda	UGA	2	Uganda Bureau of Statistics: Report on the Census of Business Establishments 2010/2011, and Business Register 2001/02
Ukraine	UKR	3	State Statistics Service of Ukraine: www.ukrstat.gov.ua
United Arab Emirates	ARE	1	National Bureau of Statistics: www.uaestatistics.gov.ae
United Kingdom	GBR	12	Eurostat, and OECD's SDBS Structural Business Statistics
United States	USA	11	U.S. Census Bureau: 2002-2011 County Business Patterns, and 2002-2011 Nonemployer Statistics
Uruguay	URY	9	Instituto Nacional de Estadística: Anuario Estadístico 2000-2012
U.S. Virgin Islands	VIR	2	U.S. Census Bureau: County Business Patterns, and 2002, 2012 Economic Census of Island Areas
Venezuela	VEN	1	Instituto Nacional de Estadística: IV Censo Económico
Vietnam	VNM	3	General Statistics Office: Establishment Censuses 2002, 2007, and 2012
Yemen	YEM	2	Central Statistical Organization: Results of Economic Surveys 2005-2006

B Social Planner

We solve the social planner's problem for our model economy. In each period, the social planner chooses the number of entrants (which we denote by E), entrant productivity s_0, the growth rate of productivity for incumbents g, and labor for each producer (ℓ) to maximize the discounted present value of an infinite stream of consumption (C). Given s_0, g, and the number of firms N, the planner chooses ℓ_i for each producer i in each period to maximize;

$$C = Y \cdot (1 - I) = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \cdot (1 - I),$$

$$\text{subject to} \quad y_i = s_i z_i \ell_i \quad \text{and} \quad 1 = \int_0^N \ell_i d_i,$$

where I is the investment rate, or the fraction of aggregate output spent to finance entry, initial investment, and life-cycle investment each period. The optimal quantity of labor for each firm i is;

$$\ell_i = \frac{(s_i z_i)^{\sigma-1}}{\int_0^N (s_j z_j)^{\sigma-1} d_j}.$$

Let hatted variables denote variables chosen in previous or future periods. The planner chooses E, s_0, and g to maximize;

$$Y_0 \cdot \left(1 - E(c_e + c_S s_0^{\theta})\right) - Y_{-1}(1-\lambda)c_g(1+g)^{\phi}$$

$$+ \sum_{t=1}^{\infty} \left[\frac{Y_t}{(1+R)^t} \left(1 - \hat{E}(c_e + c_S \hat{s_0}^{\theta})\right) - \frac{Y_{t-1}}{(1+R)^t}(1-\lambda)c_g(1+\hat{g})^{\phi} \right].$$

A bit of rearranging results in;

$$- Y_{-1}(1-\lambda)c_g(1+g)^{\phi} \tag{B.1}$$

$$+ Y_0 \left(1 - E(c_e + c_S s_0^{\theta}) - \frac{(1-\lambda)c_g(1+\hat{g})^{\phi}}{(1+R)}\right)$$

$$+ \sum_{t=1}^{\infty} \frac{Y_t}{(1+R)^t} \left(1 - \hat{E}(c_e + c_S \hat{s_0}^{\theta}) - \frac{(1-\lambda)c_g(1+\hat{g})^{\phi}}{(1+R)}\right),$$

$$\text{where} \quad Y_{-1} = \left(\frac{\lambda N_{-1} \overline{z^{\sigma-1}} \hat{s_0}^{\sigma-1}}{1 - (1-\lambda)(1+\hat{g})^{\sigma-1}} \right)^{\frac{1}{\sigma-1}},$$

$$Y_{t\geq 0}^{\sigma-1} = \left(\frac{\lambda N_{-1}\overline{z^{\sigma-1}}\hat{s_0}^{\sigma-1}}{1-(1-\lambda)(1+\hat{g})^{\sigma-1}}\right)(1+g)^{\sigma-1}(1-\lambda)^{t+1}(1+\hat{g})^{t(\sigma-1)}$$

$$+ E\overline{z^{\sigma-1}}s_0^{\sigma-1}(1-\lambda)^t(1+\hat{g})^{t(\sigma-1)}$$

$$+\sum_{T=1}^{t}\hat{E}\overline{z^{\sigma-1}}\hat{s_0}^{\sigma-1}(1-\lambda)^T(1+\hat{g})^{T(\sigma-1)},$$

$$\text{and} \quad \overline{z^{\sigma-1}} \equiv \frac{1}{N}\int_0^N z_i^{\sigma-1}di.$$

Considering the fact that the planner's choices of E, s_0, and g are identical for each period, the first order conditions for this problem are;

$$(E): \quad Y\left(c_e + c_S s_0^{\theta}\right) = \sum_{t=0}^{\infty}\frac{\partial Y_t/\partial E}{(1+R)^t}\left(1 - c_e E - c_S s_0^{\theta}E - \frac{(1-\lambda)c_g(1+g)^{\phi}}{(1+R)}\right) \tag{B.2}$$

$$(s_0): \quad Y\theta c_S s_0^{\theta-1} = \sum_{t=0}^{\infty}\frac{\partial Y_t/\partial s_0}{(1+R)^t}\left(1 - c_e E - c_S s_0^{\theta}E - \frac{(1-\lambda)c_g(1+g)^{\phi}}{(1+R)}\right) \tag{B.3}$$

$$(g): \quad Y\frac{\phi(1-\lambda)c_g(1+g)^{\phi-1}}{\sigma} = \sum_{t=0}^{\infty}\frac{\partial Y_t/\partial g}{(1+R)^t}\left(1 - c_e E - c_S s_0^{\theta}E - \frac{(1-\lambda)c_g(1+g)^{\phi}}{(1+R)}\right) \tag{B.4}$$

Combined with the condition that $E = \lambda N$ in steady state, the following conditions characterize the planner's optimal allocation;

$$(E): \quad \lambda N\left(c_e + c_S s_0^{\theta}\right) = \frac{\left[1-(1-\lambda)(1+g)^{\sigma-1}\right]}{\sigma-1}\cdot\Psi\cdot(1-I) \tag{B.5}$$

$$(s_0): \quad \lambda N c_S s_0^{\theta} = \frac{\left[1-(1-\lambda)(1+g)^{\sigma-1}\right]}{\theta}\cdot\Psi\cdot(1-I) \tag{B.6}$$

$$(g): \quad \frac{(1-\lambda)c_g(1+g)^{\phi}}{\sigma} = \frac{(1-\lambda)(1+g)^{\sigma-1}}{\phi}\cdot\Psi\cdot(1-I) \tag{B.7}$$

$$\text{where} \quad (1-I) \equiv \left(1 - c_e E - c_S s_0^{\theta}E - \frac{(1-\lambda)c_g(1+g)^{\phi}}{(1+R)}\right)$$

$$\text{and} \quad \Psi \equiv \frac{1+R}{1+R-(1-\lambda)(1+g)^{\sigma-1}}.$$

The investment rate I is;

$$I = \frac{(1+R)[\phi - (\phi+1-\sigma)(1-\lambda)(1+g)^{\sigma-1}]}{(1+R)[\sigma\phi - (\phi+1-\sigma)(1-\lambda)(1+g)^{\sigma-1}] - \phi(\sigma-1)(1-\lambda)(1+g)^{\sigma-1}}, \tag{B.8}$$

which is higher than the equilibrium investment rate in an undistorted economy. The social planner chooses the same entrant productivity s_0 but allocates more resources to establishment entry and life-cycle productivity growth relative to the equilibrium allocation. This wedge between the optimal and equilibrium allocations is common in models with endogenous life-cycle growth when costs are specified in terms of goods rather than labor (e.g., Atkeson and Burstein, 2010).

C Comparative Statics

We show that entrant productivity s_0, life-cycle growth g, and average firm size $1/N$ are all decreasing in γ.

From equation (16), initial productivity is clearly decreasing in γ;

$$\frac{\partial(s_0^{\theta})}{\partial\gamma} = -\Delta \cdot \frac{\sigma\theta}{[\theta+1-\sigma(1-\gamma)]^2} < 0,$$

where Δ contains all terms that are independent of γ.

The effect of γ on firm productivity growth g can be analyzed from equation (17). We fully differentiate equation (17) and rearrange to obtain the following expression;

$$\frac{\partial(1+g)}{\partial\gamma} = \frac{-(1+g)\sigma}{\phi+1-\sigma(1-\gamma)} \cdot \left(\phi \ln(1+g)\Psi + \frac{1}{\sigma(1-\gamma)-1}\right).$$

Given $\sigma(1-\gamma) > 1$ and $\phi > \sigma(1-\gamma) - 1$, productivity growth is unambiguously decreasing in γ.

Average firm size from equation (18) is equal to;

$$1/N = \Delta \cdot [\theta + 1 - \sigma(1-\gamma)]^{-1} \cdot \left(\frac{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma)](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}} \right),$$

or

$$1/N = \Delta \cdot [\theta + 1 - \sigma(1-\gamma)]^{-1}\phi \cdot \left(\frac{1+R-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}} \right)$$

$$+\Delta \cdot \left(\frac{\sigma(1-\gamma)-1}{\theta+1-\sigma(1-\gamma)} \right) \cdot \left(\frac{(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}} \right).$$

Given g is decreasing in γ, average firm size is also decreasing in γ.

D Decomposition

We describe and solve two simplified variants of our model. The first is a model with exogenous productivity growth over a firm's life cycle, as in Fattal-Jaef (2015), with no firm investments in productivity. The second is a model with endogenous productivity growth over a firm's life cycle but without a productivity investment at entry, as in Hsieh and Klenow (2014).

D.1 Exogenous Life-Cycle Growth

From equation (15), the expected operating profits of an entrant are equal to;

$$\mathbb{E}[\pi_0] = \frac{Y(1-\overline{\tau})}{\sigma\lambda N} \cdot \xi_{US},$$

$$\xi_{US} \equiv 1 - (1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1},$$

where g_{US} is the exogenous growth rate of firm productivity. With no investments in productivity, free entry requires that the present value of expected life-time profits be equal to the cost of entry;

$$c_e = \frac{1-\overline{\tau}}{\sigma\lambda N} \cdot \xi_{US} \cdot \Psi_{US},$$

$$\Psi_{US} \equiv \sum_{t=0}^{\infty} \left(\frac{(1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}{1+R} \right)^t = \frac{1+R}{1+R-(1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}.$$

The above free entry condition can be rearranged to express average firm size $1/N$ as;

$$N^{-1} = \frac{\sigma\lambda c_e}{1-\overline{\tau}} \cdot (\xi_{US} \cdot \Psi_{US})^{-1}.$$

As in Fattal-Jaef (2015) an increase in γ when life-cycle growth is exogenous leads to an increase in N, and therefore a decrease in firm size. Given that aggregate output is increasing in N, this partially offsets the effect of misallocation on output through factor misallocation.

D.2 Endogenous Life-Cycle Growth

We extend the model of exogenous life-cycle productivity growth to allow for investment in productivity in each period after a firm enters (but not at entry). From equation (11), the expected discounted value of life-time operating profits for a firm net of investments in life-cycle productivity is;

$$\mathbb{E}[\pi_0] \cdot \phi \cdot \Theta,$$

$$\Theta \equiv \frac{1+R}{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma)](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}},$$

where $\mathbb{E}[\pi_0]$ is defined as above in Section D.1. With no initial investment in entrant productivity, free entry requires that the above net profits be equal to the cost of entry;

$$c_e = \frac{\phi(1-\overline{\tau})}{\sigma\lambda N} \cdot \xi \cdot \Theta.$$

Average firm size can now be expressed as;

$$N^{-1} = \frac{\sigma\lambda c_e}{\phi(1-\overline{\tau})} \cdot (\xi \cdot \Theta)^{-1}.$$

Equation (9) shows that g is decreasing in γ. To prove that average size $1/N$ is decreasing in γ, we therefore show that average size is decreasing in γ given g, and increasing in g given γ.

$$\frac{\partial(N^{-1})}{\partial\gamma} = \frac{\partial}{\partial\gamma}\left(\Delta\frac{\phi(1+R)-[\phi+1-\sigma(1-\gamma)-1](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}\right)$$

$$= -\Delta\frac{\sigma(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{\left[1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}\right]^2}\cdot\left[\xi+\ln(1+g)\left(\phi(1+R)-[\phi+1-\sigma(1-\gamma)]\right)\right].$$

Given $\xi > 0$, $\ln(1+g) > 0$, and $\gamma < (\sigma-1)/\sigma$, the above derivative is indeed negative. And the following expression shows that average firm size is indeed increasing in g, given γ;

$$\frac{\partial(N^{-1})}{\partial g} = \Delta\frac{[\sigma(1-\gamma)-1](1-\lambda)(1+g)^{\sigma(1-\gamma)-2}}{\xi^2}\cdot(\phi(1+R)-[\phi+1-\sigma(1-\gamma)]) > 0.$$

We now prove that (as we discuss in Section II.D) a decrease in life-cycle growth g from an increase in γ dampens the effect of factor misallocation on aggregate output by compressing the productivity distribution. Using equation (21), the percentage decrease in Y through factor misallocation from a small increase in γ is;

$$\frac{\partial Y_{FM}}{\partial\gamma}Y_{FM}^{-1} = \Delta\cdot\frac{-\left[(1+g)^{(\sigma-1)(1-\gamma)}-(1+g)^{\sigma(1-\gamma)-1}\right]}{\left(1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}\right)\left(1-(1-\lambda)(1+g)^{(\sigma-1)(1-\gamma)}\right)}\cdot(1-\lambda)\sigma\ln(1+g)$$

where Δ is independent of g. The magnitude of the decrease in Y through factor misallocation from an increase in γ is clearly higher when g is higher. It follows that the lower g induced by γ dampens the impact of factor misallocation.

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