

## Appendix to “Do Job-to-Job Transitions Drive Wage Fluctuations Over the Business Cycle?”

### A. The “purified” job-to-job transition probability

Let  $\lambda^e$  denote the arrival probability of wage offers to the employed. Define  $G(g)$  as the distribution (density) function of wages over workers, and  $F(f)$  as the distribution (density) of wage offers. As in Burdett and Mortensen, we assume that a worker accepts any job that offers a higher wage. We also assume the arrival of wage offers is random, that is, independent of the worker's present wage. The job-to-job transition probability,  $\Lambda^e$ , is then given by

$$\Lambda^e = \lambda^e \int G(w)f(w)dw, \quad (1)$$

where  $G(w) = \Pr(\omega < w)$  is the probability that a worker's wage  $\omega$  is less than a given wage offer  $w$ . The right side of (1) has two parts. The term,  $\lambda^e$ , is the probability that an offer arrives. The integral is the acceptance probability.

To infer the offer distribution, we use the map between  $g$  and  $f$  implied by Burdett-Mortensen under *flow balance*. With respect to the wage distribution, flow balance means that the outflow of workers from any wage  $w$  equals the inflow of workers to that wage.<sup>1</sup> The outflows from  $w$  include two parts: the share of workers with wage  $w$  that separate to unemployment; and the share that contact, and migrate to, a higher-wage firm. Hence, the outflows are given by  $[\delta + \lambda^e(1 - F(w))](1 - u)g(w)$ , where  $\delta$  is the probability that a match is destroyed (after which the worker enters unemployment) and  $u$  is the unemployment rate. The inflows to wage  $w$  also consist of two parts: the share of unemployed workers that contact a firm of wage  $w$ ; and the share of employed workers with wage below  $w$  that contact a firm of wage  $w$ . The inflows, then, are  $[\lambda^u u + \lambda^e(1 - u)G(w)]f(w)$ , where  $\lambda^u$  is the probability that an offer is made to an unemployed worker.<sup>2</sup> Thus, under flow balance, these flows satisfy,

$$[\delta + \lambda^e(1 - F(w))](1 - u)g(w) = [\lambda^u u + \lambda^e(1 - u)G(w)]f(w). \quad (2)$$

The concept of flow balance also extends to the worker flows. Specifically, under flow balance, inflows into, and outflows from, unemployment are equal. This implies  $\delta(1 - u) = \lambda^u u$ , which in turn yields  $u = \delta/(\delta + \lambda^u)$ . When we use this to substitute for  $u$  in (2), we find that the density,  $f(w)$ , under flow balance satisfies,

$$f(w) = \frac{\delta + \lambda^e(1 - F(w))}{\delta + \lambda^e G(w)} g(w).$$

One may then (guess and) verify the solution to this ordinary differential equation, given known functions  $g$  and  $G$  and parameters  $\delta$  and  $\lambda^e$ . We obtain,

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<sup>1</sup> The flow-balance density is *not* the stationary (i.e., invariant) density. We return to this point a little later.

<sup>2</sup> Since no firm in equilibrium offers a wage below the reservation level, every offer to the unemployed is accepted.

$$f(w) = \frac{\delta + \lambda^e}{(\delta + \lambda^e G(w))^2} \delta g(w). \quad (3)$$

Substituting (3) into (1), one can easily confirm that the right side of (1) is zero when  $\lambda^e = 0$  and increasing in  $\lambda^e$ . Therefore, there is a single crossing, pinning down a unique solution for  $\lambda^e$ .

Implementing (3) in the data is straightforward. Using CPS weekly earnings data in the outgoing rotation group files, we can measure  $G$  (and, therefore,  $g$ ) in each state in each quarter.<sup>3</sup> The density is calculated over 25 bins that are equi-distant in logs. The minimum weekly earnings level is \$100—equivalent to weekly earnings on a part-time job earning \$5 per hour—and the maximum is the top code of \$2885 (observations at the top code are dropped). Our choice of bins ensures that even small states, such as Maine, have on average 25 respondents per bin. Turning to the separation rate, we use state-level estimates of the transition probability into non-employment from the LEHD data public-use files (series “ENPersist”). This measures the share of workers who exit employment in quarter  $t$  and do not receive earnings at the start or end of the subsequent quarter. The LEHD is more attractive than the CPS for measuring state-level separation rates from employment, because state-level samples of separations in the CPS are small. Having estimates of the earnings density and separation rates, we can calculate the offer density,  $f$ , in (3). Then, substituting  $G$  and  $f$  into (1), we can solve for  $\lambda^e$ , given the LEHD estimate of  $\Lambda^e$ .

One concern with our approach is that the earnings distribution,  $G$ , that we measure using CPS data, and “feed” into (3), is not necessarily the distribution that emerges under flow balance. Hence, the internal coherence of our approach—and the quality of our measurement of  $\lambda^e$ —hinges on whether flow balance is a reasonable approximation to observed wage outcomes.<sup>4</sup> From (2), note that, under flow balance, the density of wages across workers,  $g(w)$ , can be represented as

$$g(w) = \frac{[\lambda^u u + \lambda^e (1 - u)G(w)]f(w)}{[\delta + \lambda^e (1 - F(w))](1 - u)} = \frac{\text{Share of workers that flow to } w}{\text{Share of workers that flow out of } w}. \quad (4)$$

The term on the far right can be calculated using CPS earnings data from the matched outgoing rotation group (ORGs) files. The ORGs include the one-quarter of the CPS sample that is asked about earnings (if employed). The rotating panel structure of the CPS then allows us to match one-half of the ORG sample in any given month to their responses 12 months later. We can thus track workers, including the non-employed, across the wage distribution, enabling us to measure the share of workers that move to and from a given wage. Accordingly, we can measure  $g(w)$  under flow balance. We want to compare this to the observed density of earnings. For the sake of illustration, we carry out this exercise by pooling all data across states, rather than producing densities for each state.

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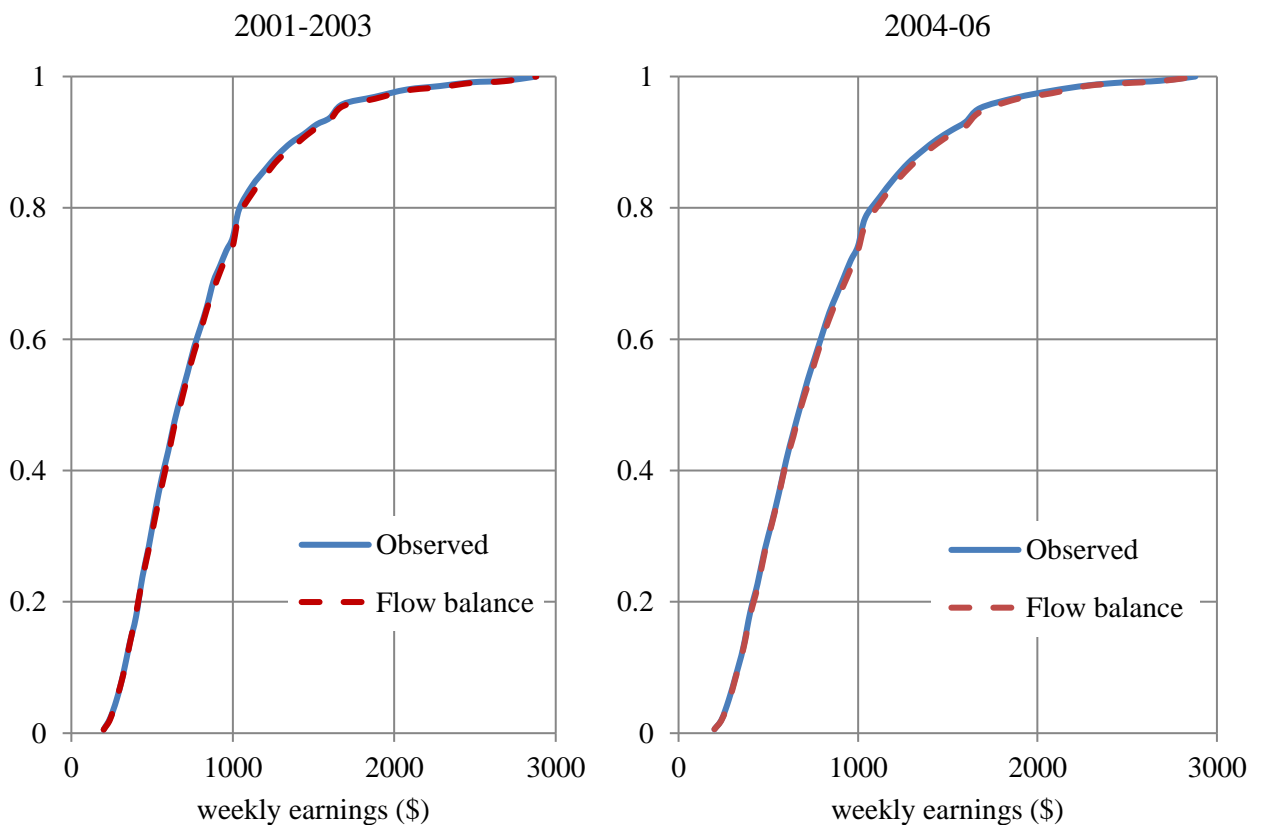
<sup>3</sup> Our use of weekly earnings is the closest analogue to the monthly earnings concept in the QWI that we use elsewhere in our analysis.

<sup>4</sup> It is well known that flow balance in worker flows approximately holds, insofar as the implied unemployment rate,  $u = \delta/(\delta + \lambda^u)$ , very closely tracks the actual rate (Hall 2005). Relatedly, Elsby, Michaels, and Ratner (2016) calculate the analogue to (4) where the flows refer to the flows of workers across firm sizes. They find that the flow-balance firm-size density of firm size is an excellent approximation to the actual firm-size density.

Figure A presents our findings. Each panel shows the c.d.f.s implied by the flow-balance and observed densities. (Recall that we use the c.d.f.,  $G$ , in (1).) The left panel shows the distribution functions averaged over the period 2001-03, a time when the labor market was relatively slack, whereas the right panel presents the results for a period, 2004-06, when the market was tighter. The critical observation here is that the c.d.f.s in any one panel are never that far apart. The biggest discrepancy in each panel is roughly 1.25 percentage points.

Intuitively, what accounts for the closeness between the flow-balance and observed densities? A key observation from (4) is that the flow-balance density is *not* the stationary density, but rather is conditioned on the *current* realization of the flows of workers across wage levels. If these flows remained fixed at their current level, the distribution of wages would converge to its flow-balance counterpart. Hence, the observed and flow-balance densities only diverge to the extent that it takes time for the wage distribution to adjust toward flow balance. If the sizes of these flows are substantial, this adjustment takes little time. The message of Figure A is that, at an annual frequency, these flows are substantial enough to bring the actual, observed distribution nearly in line with its flow-balance counterpart.

Figure A: The observed and flow-balance cumulative distribution functions



## B. Adjusting for time aggregation

We have quarterly data on job-to-job transitions. These measurements suffer from a time aggregation bias. In this section, we illustrate a way to correct the published series for this bias.

We will take the month to be the frequency at which labor market activities unfold. In what follows, let  $\Lambda_t^e(m)$  denote the probability of a job-to-job transition in month  $m = \{1,2,3\}$  of quarter  $t$ ;  $\sigma_t(m)$  the probability of a transition from employment to non-employment; and  $f_t(m)$  the probability of a transition from non-employment to employment.

There are several scenarios that lead to a measured job-to-job transition within a quarter. We detail these below. Note that, to simplify the algebra slightly, we shall assume (realistically) that the probability of multiple job-to-job transitions in a quarter is negligible.

i] A worker makes a job-to-job transition in the first month of the quarter. This happens with probability  $\Lambda_t^e(1)$ . The worker may transit out of employment later in the quarter, but this makes no difference to the LEHD measurement of job-to-job transitions: the quarterly records will still (rightly) show such a transition.

ii] A worker remains with her employer in the first month but moves to a new employer in the second month. This event happens with probability  $(1 - \sigma_t(1))(1 - \Lambda_t^e(1))\Lambda_t^e(2)$ . Again, if the probability of a second job-to-job transition in the quarter is negligible, any subsequent transitions to (and/or from) non-employment do not affect the LEHD measurement of job-to-job transitions.

iii] A worker remains with her employer in the first *and* second months but transits to a new employer in the third month. This event happens with probability

$$(1 - \sigma_t(1))(1 - \Lambda_t^e(1))(1 - \sigma_t(2))(1 - \Lambda_t^e(2))\Lambda_t^e(3).$$

iv] A worker exits employment in the first month but matches with at least one new employer *within the quarter*. This event occurs with probability

$$\sigma_t(1)f_t(2)\Lambda_t^e(3) + \sigma_t(1)f_t(2)(1 - \Lambda_t^e(3)) + \sigma_t(1)(1 - f_t(2))f_t(3).$$

Note that the first term here involves a genuine job-to-job transition: a worker returns to employment in the second month and transits directly (with no intervening spell of non-employment) to a new employer in the third. The second and third terms, however, do not involve a job-to-job move but would be measured as such by the LEHD because the worker is observed to switch employers within the quarter.

v] Lastly, a worker remains with her employer in the first month; exits employment in the second month; and matches with a new employer in the third month. This event occurs with probability

$$(1 - \sigma_t(1))(1 - \Lambda_t^e(1))\sigma_t(2)f_t(3).$$

It follows that the measured quarterly job-to-job transition probability,  $\Lambda_t^e$ , is

$$\begin{aligned}
& \Lambda_t^e \\
& = \Lambda_t^e(1) + (1 - \sigma_t(1))(1 - \Lambda_t^e(1))\Lambda_t^e(2) \\
& + \{(1 - \sigma_t(1))(1 - \Lambda_t^e(1))(1 - \sigma_t(2))(1 - \Lambda_t^e(2)) + \sigma_t(1)f_t(2)\}\Lambda_t^e(3) \\
& + \sigma_t(1)f_t(2)(1 - \Lambda_t^e(3)) + \sigma_t(1)(1 - f_t(2))f_t(3) \\
& + (1 - \sigma_t(1))(1 - \Lambda_t^e(1))\sigma_t(2)f_t(3)
\end{aligned} \tag{5}$$

The top two lines involve genuine job-to-job transitions. The bottom two lines involve spurious job-to-job transitions. To estimate the true quarterly job-to-job transition probability, denoted by  $\Lambda_t^e$ , we net off these spurious flows from the published figure,

$$\begin{aligned}
\Lambda_t^e \equiv & \Lambda_t^e - \sigma_t(1)f_t(2)(1 - \Lambda_t^e(3)) - \sigma_t(1)(1 - f_t(2))f_t(3) \\
& - (1 - \sigma_t(1))(1 - \Lambda_t^e(1))\sigma_t(2)f_t(3).
\end{aligned} \tag{6}$$

To calculate  $\Lambda_t^e$ , we need a few inputs. For starters, we require measurements of the monthly inflow rate into non-employment,  $\sigma_t(m)$ , as well as the monthly outflow rate from non-employment,  $f_t(m)$ . We again use the estimates published by Fallick and Fleischman and derived from the CPS. In addition, we need an estimate of the *monthly* job-to-job transition probability,  $\Lambda_t^e(m)$ , which is of course unavailable in the LEHD.<sup>5</sup> In the absence of a better alternative, we take  $\Lambda_t^e/3$  as an approximation to  $\Lambda_t^e(m)$  for any  $m = \{1,2,3\}$ .<sup>6</sup> Plugging this into (6) and rearranging, we solve for  $\Lambda_t^e$ :

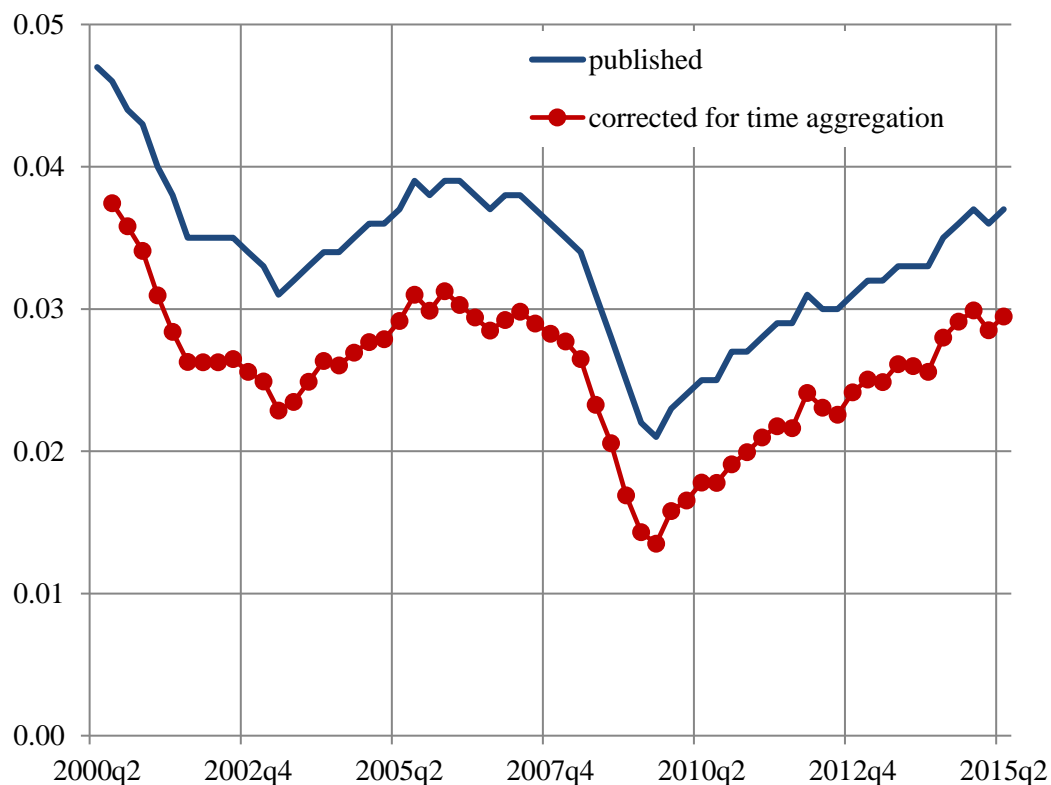
$$\begin{aligned}
\Lambda_t^e \equiv & \left\{ 1 - \frac{\sigma_t(1)f_t(2)}{3} - \frac{(1 - \sigma_t(1))\sigma_t(2)f_t(3)}{3} \right\}^{-1} \\
& \times [\Lambda_t^e - \sigma_t(1)f_t(2) - \sigma_t(1)(1 - f_t(2))f_t(3) - (1 - \sigma_t(1))\sigma_t(2)f_t(3)]
\end{aligned}$$

Figure B plots  $\Lambda_t^e$  and  $\Lambda_t^e$  for the U.S. Here, we use the *EEHire* series from the LEHD, which restricts job-to-job transitions to *within* transitions. The results do not depend on this but the time aggregation adjustment above is most applicable to the *EEHire* series. The figure indicates that the corrected estimate runs parallel to the published series, but is shifted below it. In other words, the correction does not affect the variance or cyclical of the job-to-job measure. One reason for this is that *products* of transition probabilities, such as  $\sigma_t(1)f_t(2)$ , are small relative to the published series,  $\Lambda_t^e$ . In addition,  $\sigma_t$  and  $f_t$  move in opposite directions in recessions: the outflow rate from non-employment falls, but the inflow rate into non-employment rises. This tends to render the product of the two,  $\sigma_t(1)f_t(2)$ , relatively more acyclical.

<sup>5</sup> Fallick and Fleischman also report an estimate of the employer-to-employer transition probability based on CPS data. However, we have opted not to use the CPS estimates in this paper because of concern over measurement error of this object specifically (see Moscarini and Postel-Vinay, 2016). That said, we have confirmed that Figure B is virtually the same if we substitute in the CPS estimates for  $\Lambda_t^e(m)$ .

<sup>6</sup> This is only exact if  $1 - \sigma_t(m) = 1$  and  $\sigma_t(m)f_t(n) = 0$  for any months  $m, n$ . Then equation (6) collapses to  $\Lambda_t^e = 3\Lambda_t^e(m) - 3\Lambda_t^e(m)^2 + \Lambda_t^e(m)^3$ . The higher order terms will be nearly zero, such that  $\Lambda_t^e \approx 3\Lambda_t^e(m) = \Lambda_t^e$ . Since  $\sigma_t(m)$  is in fact very small, our approximation should perform quite well.

Figure B: The quarterly job-to-job transition probability in the U.S.



Though Figure B pertains to the aggregate series, rather than to the state-specific series used in our regression, it strongly suggests that time aggregation is unlikely to play a material role in our regression analysis. Estimates of regression (1) in the main text that use our bias-corrected estimate,  $\lambda_t^e$ , can be made available on request.

### C. Additional regression results

The final section of the appendix reports two additional sets of regression results. First, we present results from a regression using aggregate earnings and worker flows. The outcome is average monthly earnings from the QWI corresponding to “stable” workers and deflated using the Personal Consumption Expenditure (PCE) price index.<sup>7</sup> The regressors are the aggregate time series for the (headline) job-to-job

<sup>7</sup> Deflating is arguably more consistent with our approach in the main text than using nominal earnings. Since we remove the aggregate component of earnings growth using quarter fixed effects in our state-level regressions, our identification strategy relies on variation in *relative* earnings growth across states (and its comovement with relative worker flows). This has the effect of controlling for the common component of price inflation.

transition probability from the LEHD,  $\Lambda_t^e$ , and the unemployment-to-employment transition probability from the CPS,  $\Lambda_t^u$ . A linear time trend is included all regressions, consistent with the specifications used in the main text. The time series is quarterly and begins in 2000q3.

Table C.1: Aggregate results

Regressor	Outcome:	
	Avg. monthly earnings in 2009 dollars	
$\Lambda_t^u$	0.330*** [0.072]	0.141 [0.222]
$\Lambda_t^e$		1.047 [0.976]
No. of obs.	60	60

NOTES: [1] Newey-West standard errors are presented.  
[2] \*\*\* indicates significance at  $p < 0.01$  level.

The results in Table C.1 echo the theme of the main text. In particular, the job-finding probability,  $\Lambda_t^u$ , enters significantly on its own, but diminishes in importance when the job-to-job transition probability,  $\Lambda_t^e$ , is included. However,  $\Lambda_t^e$  is lacking in statistical significance, owing to the fact that the aggregate time series offers us a very small sample. This indicates the value of using state-level variation.

The second set of regression results illustrates the robustness of our findings in the main text to multiple measures of state-level price deflators. Table C.2 presents results using two deflators. One is the retail price index constructed by Beraja et al (2016) using scanner data. These data are available from 2006q1-2011q4. The other source is the Bureau of Economic Analysis' estimates of state-level GDP price deflators. These data are available from 2000 through 2015, but only at an annual frequency.

We re-estimate equation (1) in the main text using a measure of real earnings as the outcome variable. We focus here on earnings of "stable" workers in order to conserve on space. The results conform quite closely with the baseline analysis.

Table C.2: Different state-level price deflators

Regressor	Deflator:	
	GDP price index	Retail price index
$\Lambda_t^u$	0.013 [0.013]	-0.002 [0.005]
$\Lambda_t^e$	2.288*** [0.382]	2.214*** [0.602]
No. of obs.	690	1044

NOTES: [1] Standard errors are clustered at the state level. [2] Both columns include state-specific time trends. [3] \*\*\* indicates significance at  $p < 0.01$  level.

## References

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