Online Appendix for Escaping the Great Recession

Francesco Bianchi and Leonardo Melosi*

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A Markov-switching VAR

In this appendix, we describe the MS-VAR used in Section I.

A.1 Model setup

The variables of interest are assumed to evolve according to a Markov-switching VAR with two lags:

$$Z_t = c_{\xi_t^\Phi} + A_{\xi_t^\Phi,1} Z_{t-1} + A_{\xi_t^\Phi,2} Z_{t-2} + \Sigma^{1/2}_{\xi_t^\Sigma} \omega_t \tag{1}$$

$$\Phi_{\xi_t^\Phi} = \left[c_{\xi_t^\Phi}, A_{\xi_t^\Phi,1}, A_{\xi_t^\Phi,2} \right], \ \omega_t \sim N(0, I) \tag{2}$$

where Z_t is a $(n \times 1)$ vector of data. The unobserved states ξ_t^Σ and ξ_t^Φ can take on a finite number of values, $j^\Phi = 1, \ldots, m^\Phi$ and $j^\Sigma = 1, \ldots, m^\Sigma$, and follow two independent Markov chains. This represents a convenient way to model heteroskedasticity and to allow for the possibility of changes in the dynamics of the state variables. The probability of moving from one state to another is given by $P[\xi_t^\Phi = i | \xi_{t-1}^\Phi = j] = h_{ij}^\Phi$ and $P[\xi_t^\Sigma = i | \xi_{t-1}^\Sigma = j] = h_{ij}^\Sigma$. Given $H^\Phi = [h_{ij}^\Phi]$ and $H^\Sigma = [h_{ij}^\Sigma]$ and a prior distribution for the initial state, we can compute the likelihood of the parameters of the model, conditional on the initial observation Z_0. We impose flat priors on all parameters of the models, implying that the posterior coincides with the likelihood.

*Bianchi: Duke University, Department of Economics, 213 Social Sciences building, Box 90097, Durham, NC 27708-0097, email francesco.bianchi@duke.edu. Melosi: Federal Reserve Bank of Chicago, 230 S. LaSalle street, Chicago, IL 60604, United States, lmelosi@frbchi.org. The views in this appendix are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Chicago or any other person associated with the Federal Reserve System.

A.2 Likelihood and regime probabilities

Define the combined regime $\xi_t \equiv \left(\xi_t^{\Phi}, \xi_t^{\Sigma}\right)$, the associated transition matrix $H \equiv H^{\Phi} \otimes H^{\Sigma}$, and vector $\theta_{\xi_t} \equiv \left(\Phi_{\xi_t^{\Phi}}, \Sigma_{\xi_t^{\Sigma}}\right)$ with the corresponding set of parameters. For each draw of the parameters θ_{ξ_t} and H, we can then compute the filtered probabilities $\pi_{t|t}$, or smoothed probabilities $\pi_{t|T}$, of the regimes conditional on the model parameters. The filtered probabilities reflect the probability of a regime conditional on the data up to time t, $\pi_{t|t} = p(\xi_t|Y^t; H, \theta_{\xi_t})$, for $t = 1, ..., T$, and are part of the output obtained computing the likelihood function associated with the parameter draw H, θ_{ξ_t}. The filtered probabilities can be obtained using the following recursive algorithm:

$$\pi_{t|t} = \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}' \left(\pi_{t|t-1} \odot \eta_t\right)} \tag{3}$$

$$\pi_{t+1|t} = H\pi_{t|t} \tag{4}$$

$$p(Z_t|Z^{t-1}) = \mathbf{1}' \left(\pi_{t|t-1} \odot \eta_t\right) \tag{5}$$

where η_t is a vector whose jth element contains the conditional density $p(Z_t|\xi_t = i, Z^{t-1}; H, \theta_{\xi_t})$, the symbol $\odot$ denotes element by element multiplication, and $\mathbf{1}$ is a vector with all elements equal to 1. To initialize the recursive calculation, we need an assumption on the distribution of ξ_0. We assume that the nine regimes have equal probabilities $p(\xi_0 = i) = 1/9$ for $i = 1...m$. The likelihood for the entire data sequence Z^T is obtained multiplying the one-step-ahead conditional likelihoods $p(Z_t|Z^{t-1})$:

$$p\left(Z^T|\theta\right) = \prod_{t=1}^{T} p\left(Z_t|Z^{t-1}\right)$$

The smoothed probabilities reflect all the information that can be extracted from the whole data sample, $\pi_{t|T} = p(\xi_t|Z^T; H, \theta_{\xi_t})$. The final term $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then a recursive algorithm can be implemented to derive the other probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot \left[H' \left(\pi_{t+1|T} \left(\div\right) \pi_{t+1|t}\right)\right]$$

where $(\div)$ denotes element by element division.

Finally, it is possible to obtain the filtered and smoothed probabilities for each of the two independent chains by integrating out the other chain. For example, if we are interested in $\pi_{t|t}^{\Phi} = p(\xi_t^{\Phi}|Y^t; H, \theta_{\xi_t})$ we have:

$$\pi_{t|t}^{\Phi,i} = p(\xi_t^{\Phi} = i|Y^t; H, \theta_{\xi_t}) = \sum_{j=1}^{m} p(\xi_t = \{i, j\}|Y^t; H, \theta_{\xi_t})$$

Similarly, the smoothed probabilities are obtained as:

$$\pi_{t|T}^{\Phi,i} = p(\xi_t^{\Phi} = i|Y^T; H, \theta_{\xi_t}) = \sum_{j=1}^{m} p(\xi_t = \{i,j\}|Y^T; H, \theta_{\xi_t}).$$

A.3 Posterior Mode and Gibbs sampling algorithm

We first find the posterior mode by using a minimization algorithm on the negative of the posterior. Given that we have flat priors, our point estimates coincide with the maximum likelihood estimates. Once we have found the posterior mode, we compute the most likely regime sequence and then proceed to characterize uncertainty around the parameter values conditional on this regime sequence by using a Gibbs sampling algorithm. Alternatively, we could have imposed some identifying restrictions based on the properties of the regimes at the posterior mode, but we preferred to take this more agnostic approach. This makes the interpretation of the results more immediate because the properties of the regimes can be immediately associated with the periods during which they were in place.

Both the VAR coefficients and the covariance matrix can switch and the regimes are assumed to be independent. Draws for the parameters of the model can be made following the following Gibbs sampling algorithm:

1. Sampling $\Sigma_{\xi_t^\Sigma}$ given $\Phi_{\xi_t^\Phi}, \xi_t^\Phi, \xi_t^\Sigma$: Given $\Phi_{\xi_t^\Phi}$ and $\xi^{\Phi,T}$, we can compute the residuals of the MS-VAR at each point in time. Then, given ξ_t^Σ, we can group all the residuals that pertain to a particular regime. Therefore, $\Sigma_{\xi_t^\Sigma}$ can be drawn from an inverse Wishart distribution for $\xi_t^\Sigma = 1...m^\Sigma$.

2. Sampling $\Phi_{\xi_t^\Phi}$ given $\Sigma_{\xi_t^\Sigma}, \xi_t^\Phi, \xi_t^\Sigma$: When drawing the VAR coefficients, we need to take into account the heteroskedasticity implied by the switches in $\Sigma_{\xi_t^\Sigma}$. This can be done following the following steps for each $i = 1...m^\Phi$:

   (a) Based on $\xi^{\Phi,T}$, collect all the observation such that $\xi_t^\Phi = i$.

   (b) Divide the data that refer to $\xi_t^\Sigma = j$ based on $\xi^{\Sigma,T}$. We now have a series of subsamples for which VAR coefficients and covariance matrices are fixed: $\left(\xi_t^\Phi = i,\ \xi_t^\Sigma = 1\right)$, ..., $\left(\xi_t^\Phi = i,\ \xi_t^\Sigma = m^\Sigma\right)$. Denote these subsamples with $\left(y_{i,\xi_t^\Sigma},\ x_{i,\xi_t^\Sigma}\right)$, where the y_{i,ξ_t^Σ} and x_{i,ξ_t^Σ} denote left-hand-side and right-hand-side variables in the MS-VAR. Notice that some of these subsamples might be empty.

   (c) Apply recursively the formulas for the posterior of VAR coefficients conditional on a known covariance matrix. Therefore, for $j = 1...m^\Sigma$ the following formulas need to be applied recursively:

$$
\begin{aligned}
P_T^{-1} &= P_L^{-1} + \Sigma_{\xi_t^\Sigma}^{-1} \otimes (x'_{i,\xi_t^\Sigma} x_{i,\xi_t^\Sigma}) \\
B_T &= B_L + (\Sigma_{\xi_t^\Sigma}^{-1} \otimes x'_{i,\xi_t^\Sigma}) vec(y_{i,\xi_t^\Sigma}) \\
P_L^{-1} &= P_T^{-1}, B_L = B_T
\end{aligned}
$$

where the algorithm is initialized using the priors for the VAR coefficients $B_L = B_0$ and $P_L^{-1} = P_0^{-1} = \left(S_0 \otimes N_0^{-1}\right)^{-1}$. Notice that this implies that if there are not any observations for a particular regime, then the posterior will coincide with the priors. With proper priors, this is not a problem.

(d) Make a draw for the VAR coefficients $vec\left(\Phi_{\xi_t^\Phi}\right) \sim N\left(P_T B_T, P_T\right)$ with $\xi_t^\Phi = i$.

3. Sampling H^Φ and H^Σ: Given the draws for the state variables $\xi^{\Phi,T}$ and $\xi^{\Sigma,T}$, the transition probabilities are independent of Y_t and the other parameters of the model and have a Dirichlet distribution. For each column of H^Φ and H^Σ, the posterior distribution is given by:

$$H^s(:,i) \sim D(a_{ii}^s + \eta_{ii}^s, a_{ij}^s + \eta_{ij}^s),\ \ s = \Phi, \Sigma$$

where η_{ij}^Φ and η_{ij}^Σ denote respectively the numbers of transitions from state i^Φ to state j^Φ and from state i^Σ to state j^Σ.

A.4 Data

We use four observables to estimate the Markov-switching VAR: (i) inflation; (ii) real GDP growth; (iii) federal funds rate; (iv) deficit-to-debt ratio. Inflation and real output growth are defined as year-to-year first differences of the logarithm of the GDP price deflator and real GDP, respectively. Inflation, real GDP, and the federal funds rate are taken from the FRED II database of the Federal Reserve Bank of St. Louis. The primary deficit is constructed from the NIPA tables (Table 3.2. Federal Government Current Receipts and Expenditures) as detailed in Appendix C. As a measure of the fiscal stance, we consider the variable deficits over debt. The government debt series is the market value of the US government debt available on the Dallas Fed website. The sample period ranges from 1954:Q4-2014:Q1.

A.5 Volatility regimes

Table 1 reports the estimates of the covariance matrix across different volatility regimes.

$\xi_t^\Sigma = 1$	u_{ER}	u_{TY}	u_{PE}	u_{VS}
u_{ER}	1.2377 (1.1183,1.3693)			
u_{TY}	−0.2235 (−0.3623,−0.0824)	0.7015 (0.6319,0.7784)		
u_{PE}	0.0378 (−0.1003,0.1817)	−0.1778 (−0.3119,−0.0366)	0.2516 (0.2278,0.2788)	
u_{VS}	−0.4275 (−0.5414,−0.3041)	0.1983 (0.0452,0.3362)	0.2672 (0.1323,0.3996)	0.2688 (0.2425,0.2981)
$\xi_t^\Sigma = 2$	u_{ER}	u_{TY}	u_{PE}	u_{VS}
u_{ER}	3.7397 (3.2247,4.3585,)			
u_{TY}	−0.2054 (−0.4038,0.0044)	1.3657 (1.1625,1.6117)		
u_{PE}	−0.2112 (−0.4120,0.0055)	−0.1114 (−0.3284,0.1069)	0.0369 (0.2753,0.3726)	
u_{VS}	−0.2157 (−0.4057,−0.0112)	0.1558 (−0.0649,0.3668)	0.1837 (−0.0400,0.3989)	0.0279 (0.6139,0.8324)
$\xi_t^\Sigma = 3$	u_{ER}	u_{TY}	u_{PE}	u_{VS}
u_{ER}	4.4369 (3.1092,6.3281)			
u_{TY}	−0.4984 (−0.8002,−0.0732)	1.9303 (1.3216,2.7927)		
u_{PE}	−0.3404 (−0.7160,0.1282)	0.2079 (−0.3043,0.6676)	0.6111 (0.3993,0.9047)	
u_{VS}	−0.9854 (−0.9966,−0.9646)	0.5879 (0.216,0.8439)	0.241 (−0.2430,0.6507)	3.4718 (2.4454,4.9064)

Table 1: Parameter estimates for the covariance matrix. The three sets of tables contain means and 90% error bands for the posterior distribution of the parameters of the covariance matrices. The standard deviations of the shocks are on the main diagonal, whereas the correlations of the shocks are below the main diagonal.

B Benchmark Model

In what follows, we provide the details for the solution and estimation of the model.

B.1 System of equations

1. Linearized Euler equation:

$$\begin{aligned}\left(1+\Phi M_a^{-1}\right)\widehat{y}_t &= -\left(1-\Phi M_a^{-1}\right)\left[\widehat{R}_t - E_t\widetilde{\pi}_{t+1} - (1-\rho_d)\,d_t - \overline{d}_{\xi_t^d} + E_{\xi_t^d}\overline{d}_{\xi_{t+1}^d}\right] \\ &\quad -\left(\Phi M_a^{-1} - \rho_a\right)a_t + E_t\widehat{y}_{t+1} + \left(1-\rho_g + M_a^{-1}\Phi\right)\widetilde{g}_t \\ &\quad + M_a^{-1}\Phi\left(\widehat{y}_{t-1} - \widetilde{g}_{t-1}\right)\end{aligned}$$

where $M_a = \exp(\gamma)$ and $\overline{d}_{\xi_t^d}$ follows a Markov-switching process governed by the transition matrix H^d. Please refer to the next subsection for details about how to handle the discrete shock.

2. New Keynesian Phillips curve:

$$\begin{aligned}\widetilde{\pi}_t &= \kappa\left(\begin{array}{c}\left[\frac{1}{1-\Phi M_A^{-1}}+\frac{\alpha}{1-\alpha}\right]\widehat{y}_t-\frac{1}{1-\Phi M_A^{-1}}\widetilde{g}_t\\ -\frac{\Phi M_A^{-1}}{1-\Phi M_A^{-1}}\left(\widehat{y}_{t-1}-\widetilde{g}_{t-1}-a_t\right)\end{array}\right)\\ &\quad+\beta E_t\left[\widetilde{\pi}_{t+1}\right]+\widetilde{\mu}_t\end{aligned}$$

where we have used the rescaled markup $\widetilde{\mu}_t=\kappa\left(\frac{v}{1-v}\right)\widetilde{v}_t$

3. No arbitrage condition

$$\widetilde{R}_t=E_t\left[\widetilde{R}^m_{t,t+1}\right]$$

4. Return long term bond

$$\widetilde{R}^m_{t-1,t}=R^{-1}\rho\widetilde{P}^m_t-\widetilde{P}^m_{t-1}$$

5. Government budget constraint:

$$\begin{aligned}\widetilde{b}^m_t &= \beta^{-1}\widetilde{b}^m_{t-1}+b^m\beta^{-1}\left(\widehat{R}^m_{t-1,t}-\widehat{y}_t+\widehat{y}_{t-1}-a_t-\widetilde{\pi}_t\right)\\ &\quad-\widetilde{\tau}_t+\widetilde{tr}_t+g^{-1}\widetilde{g}_t+\widetilde{tp}_t\end{aligned}$$

6. Monetary policy rule

$$\begin{aligned}\widetilde{R}_t &= \left[1-Z_{\xi^d_t}\right]\left[\rho_{R,\xi^p_t}\widetilde{R}_{t-1}+(1-\rho_R)\left(\psi_{\pi,\xi^p_t}\widetilde{\pi}_t+\psi_{y,\xi^p_t}\left[\widehat{y}_t-\widehat{y}^*_t\right]\right)+\sigma_R\epsilon_{R,t}\right]\\ &\quad+Z_{\xi^d_t}\left[\rho_{R,Z}\widetilde{R}_{t-1}-\left(1-\rho_{R,Z}\right)\psi_Z\log\left(R\right)+\sigma_Z\epsilon_{R,t}\right]\end{aligned}$$

7. Fiscal rule

$$\widetilde{\tau}_t=\rho_{\tau,\xi^p_t}\widetilde{\tau}_{t-1}+\left(1-\rho_{\tau,\xi^p_t}\right)\left[\delta_{b,\xi^p_t}\widetilde{b}^m_{t-1}+\delta_e\left(\widetilde{tr}^*_t+g^{-1}\widetilde{g}_t\right)+\delta_y\left(\widehat{y}_t-\widehat{y}^*_t\right)\right]+\sigma_\tau\epsilon_{\tau,t}$$

8. Transfers

$$\left(\widetilde{tr}_t-\widetilde{tr}^*_t\right)=\rho_{tr}\left(\widetilde{tr}_{t-1}-\widetilde{tr}^*_t\right)+\left(1-\rho_{tr}\right)\phi_y\left(\widehat{y}_t-\widehat{y}^*_t\right)+\sigma_{tr}\epsilon_{tr,t},\ \epsilon_{tr,t}\sim N\left(0,1\right)$$

9. Long term component of transfers

$$\widetilde{tr}^*_t=\rho_{tr^*}\widetilde{tr}^*_{t-1}+\sigma_{tr^*}\epsilon_{tr^*,t},\epsilon_{tr^*,t}\sim N\left(0,1\right)$$

10. Government purchases ($\widetilde{g}_t = \ln(g_t/g)$):

$$\widetilde{g}_t = \rho_g \widetilde{g}_{t-1} + \sigma_g \epsilon_{g,t},\ \epsilon_{g,t} \sim N(0,1).$$

11. TFP growth

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}$$

12. Term premium

$$tp_t = \rho_{tp} tp_{t-1} + \sigma_{tp} \varepsilon_{tp,t}$$

13. The rescaled markup $\mu_t = \kappa \log(\aleph_t/\aleph)$, where $\aleph_t = 1/(1-\upsilon_t)$, follows an autoregressive process,

$$\mu_t = \rho_\mu \mu_{t-1} + \sigma_\mu \epsilon_{\mu,t}$$

14. Output target

$$\left[\frac{1}{1-\Phi M_a^{-1}} + \frac{\alpha}{1-\alpha}\right] \widehat{y}_t^* = \frac{1}{1-\Phi M_a^{-1}} \widetilde{g}_t + \frac{\Phi M_a^{-1}}{1-\Phi M_a^{-1}} \left(\widehat{y}_{t-1}^* - \widetilde{g}_{t-1} - a_t\right)$$

B.2 Model solution

As explained in the main text, the Markov-switching process for the discrete preference shock $\overline{d}_{\xi_t^d}$ is defined in a way that its steady state is equal to zero. In order to solve the model we implement the following steps:

1. Introduce a dummy variable $e_{\xi_t^d}$ controlling the regime that is in place for the discrete preference shock. Augment the DSGE state vector with this dummy variable.

2. Use the aforementioned dummy variable to rewrite all the equations linked to the discrete preference shock. These are the linearized Euler equation and the linearized Taylor rule.

3. Solve the model using Farmer, Waggoner and Zha (2009). This returns a MS-VAR:

$$\widetilde{S}_t = \widetilde{T}(\xi_t, H, \theta)\, \widetilde{S}_{t-1} + \widetilde{R}(\xi_t, H, \theta)\, Q\varepsilon_t$$

   in the augmented state vector $\widetilde{S}_t$.

4. Extract the column corresponding to the dummy variable $e_{\xi_t^d}$ from the matrix $\widetilde{T}$ and redefine the matrices and the DSGE state vector accordingly. This will return a MS-VAR with a MS constant:

$$S_t = c(\xi_t, H, \theta) + T(\xi_t, H, \theta)\, S_{t-1} + R(\xi_t, H, \theta)\, Q\varepsilon_t$$

where Q is a diagonal matrix that contains the standard deviations of the structural shocks and S_t is a vector with all variables of the model.

Unlike other papers that have used the technique described here, our model allows for non-orthogonality between policymakers' behavior and a discrete shock. This allows us to solve a model in which agents take into account that a large preference shock leads to an immediate change in policy, the zero lower bound, and, potentially, to further changes. This proposed method is general and can be applied to other cases in which a shock induces a change in the structural parameters.

B.3 Matrices used in the counterfactual simulations

We here describe the matrices used in the simulations reported in the paper.

B.3.1 Textbook New Keynesian model: Always monetary-led

In the first counterfactual simulation, policymakers always follow the monetary-led regime when out of the zero lower bound. Furthermore, there is only one zero-lower-bound regime from which agents expect to return to the monetary-led regime. Therefore, the transition matrix used to solve this counterfactual economy is given by:

$$H^p = 1,\ H^d = \begin{bmatrix} p_{hh} & 1 - p_{ll} \\ 1 - p_{hh} & p_{ll} \end{bmatrix},\ H = H^d.$$

where p_{hh} and p_{ll} are the estimated parameter values.

B.3.2 Announcements

In the counterfactual economy with announcements, at the zero lower bound we distinguish two cases, based on the exit strategy:

1. Policymakers announce that they will move to the monetary-led regime once the economy out of the zero lower bound.

2. Policymakers announce that they will *immediately* move to the fiscally-led regime.

We assume that the probability of the first scenario is equal to the estimated probability of switching to the monetary-led regime in the benchmark model. In other words, the first scenario is more likely than the second scenario and it has a probability equal to p_{ZM}. Furthermore, their probabilities do not depend on the regime that was in place when the negative preference shock occurred. We then have a total of four regimes $\xi_t = \{[M,h],[F,h],[Z,l],[F,l]\}$ and the corresponding transition matrix is given by:

$$H = \begin{bmatrix} p_{hh}H^p & (1-p_{ll})\,H^o \\ (1-p_{hh})\,H^i & p_{ll}H^z \end{bmatrix}$$

$$H^p = \begin{bmatrix} p_{MM} & 1-p_{FF} \\ 1-p_{MM} & p_{FF} \end{bmatrix},\ H^o = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix},$$

$$H^i = \begin{bmatrix} p_{ZM} & p_{ZM} \\ 1-p_{ZM} & 1-p_{ZM} \end{bmatrix},\ H^z = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix},$$

$$H^d = \begin{bmatrix} p_{hh} & 1-p_{ll} \\ 1-p_{hh} & p_{ll} \end{bmatrix}.$$

C Estimation of the DSGE model

This appendix describes the dataset and provides details for the benchmark model.

C.1 Dataset

Real GDP, the GDP deflator, and the series for fiscal variables are obtained from the Bureau of Economic Analysis. The fiscal series are built using NIPA Table 3.2. (Federal Government Current Receipts and Expenditures). Government purchases (G) are computed as the sum of consumption expenditure (L24), gross government investment (L44), net purchases of non-produced assets (L46), minus consumption of fixed capital (L47). Transfers are given by the sum of net current transfer payments (L25-L18), subsidies (L35), and net capital transfers (L45-L41). Tax revenues are given by the difference between current receipts (L40) and current transfer receipts (L18). All variables are then expressed as a fraction of GDP. Government purchases are transformed in a way to obtain the variable g_t defined in the model. The series for the federal funds rate is obtained averaging monthly figures downloaded from the St. Louis Fed web-site.

C.2 MCMC algorithm and convergence

Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. When working with models whose posterior distribution is very complicated in shape it is very important to find the posterior mode. In a MS-DSGE model, this search can turn out to be an extremely time-consuming task, but it is a necessary

Parameter	PSRF	Parameter	PSRF	Parameter	PSRF	Parameter	PSRF
$\psi_{\pi,M}$	1.01	ψ_Z	1	ρ_d	1	$100\sigma_R$	1
$\psi_{y,M}$	1.05	κ	1.15	ρ_{tp}	1	$100\sigma_g$	1
$\rho_{R,M}$	1.01	$\delta_{b,M}$	1	ρ_μ	1.01	$100\sigma_a$	1
$\rho_{\tau,M}$	1	ρ_{tr}	1.01	100π	1	$100\sigma_\tau$	1
$\psi_{\pi,F}$	1.06	δ_y	1.03	100γ	1	$100\sigma_d$	1.04
$\psi_{y,F}$	1.05	Φ	1.04	b^m	1.01	$100\sigma_{tr}$	1
$\rho_{R,F}$	1	δ_e	1.07	g	1	$100\sigma_{tp}$	1
$\rho_{\tau,F}$	1.02	ρ_g	1	τ	1.02	$100\sigma_\mu$	1
d_l	1.01	ρ_a	1.01	ϕ_y	1.01	p_{FF}	1.02
p_{hh}	1.09	p_{ll}	1.02	p_{MM}	1.01	p_{ZM}	1.01

Table 2: The table reports the Gelman-Rubin Potential Scale Reduction Factor (PSRF) for eight chains of 540,000 draws each (1 every 200 is stored). Values below 1.2 are regarded as indicative of convergence.

step to reduce the risk of the algorithm getting stuck in a local peak. Here are the key steps of the Metropolis-Hastings algorithm:

- Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N\left(\theta_{n-1}, c\overline{\Sigma}\right)$
- Step 2: Compute $\alpha\left(\theta^m; \vartheta\right) = \min\left\{p\left(\vartheta\right)/p\left(\theta^{m-1}\right), 1\right\}$ where $p\left(\theta\right)$ is the posterior evaluated at θ.
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha\left(\theta^m; \vartheta\right)$ where $u \sim U\left([0,1]\right)$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \leq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\overline{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\overline{\theta}$. The parameter c is set to obtain an acceptance rate of around 35-percent. Table 2 reports results based on the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the five multiple chains used in the paper. The eight chains consist of $2,100,000$ draws each (1 every 3000 draws is saved). The numbers are well below the 1.2 benchmark value used as an upper bound for convergence.

C.3 Determining the time of the ZLB Regime

For tractability, we fix the sequence of Markov-switching regimes to estimate the model. To select the date at which the ZLB regime has started, we compute the posterior modes associated with a number of candidate dates. As shown in Table 3, the fourth quarter of 2008 (2008:Q4) attains the highest posterior mode and hence is selected as the date at which the ZLB regime

Starting date of ZLB Regime	Likelihood	Posterior
2008:Q1	6,428.5	6,374.2
2008:Q2	6,370.0	6,376.0
2008:Q3	6,407.4	6,415.1
2008:Q4	**6,522.4**	**6,521.1**
2009:Q1	6,496.5	6,497.7
2009:Q2	6,490.0	6,487.9
2009:Q3	6,475.8	6,476.6

Table 3: The table shows the value of the posterior and the likelihood at the posterior mode as the starting date of the ZLB regime changes. The results associated with the highest posterior mode are in bold.

has started (recall that all models only differ in terms of the starting date for the ZLB regime, so they present the same number of parameters).

D A Prototypical New Keynesian Model with a Fiscal Block

The objective of this appendix is to show that the results of Section III.B are robust when one considers models that has less bells and whistles and are more agnostic about the nature of shocks than the model we estimated in the paper. Let us a consider a prototypical New Keynesian DSGE model of the type studied in Eggertsson and Woodford (2003). This modeling framework is purposely very stylized and follows Eggertsson and Woodford (2003) in considering unanticipated shocks to the natural rate of interest as the cause of ZLB episodes.

The loglinearized equations of the model are as follows. All the variables henceforth are expressed in log-deviations from their steady-state values with the only exception of the debt-to-output ratio b_t, which is defined in deviation from its steady-state value. The IS equation reads:

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(R_t - E_t \pi_{t+1} - r_t^n \right) \tag{6}$$

where x_t denotes the gap between the actual output and its flexible-price level (henceforth, the output gap), π_t denotes inflation, R_t denotes the nominal interest rate, and r_t^n stands for the natural rate of interest, which is the real interest rate that would be realized if prices were perfectly flexible.

The New Keynesian Phillips curve is

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{7}$$

The monetary policy reaction function is:

$$R_t = \left[1 - Z_{\xi_t^d}\right]\left[\rho_R R_{t-1} + (1-\rho_R)\left(\psi_{\pi,\xi_t^p}\pi_t + \psi_x x_t\right)\right] - Z_{\xi_t^d}\ln(R) \tag{8}$$

where R is the steady-state value of the nominal interest rate R_t. Note that the monetary authority follows the Taylor rule when $Z_{\xi_t^d} = 0$ or set its (net) nominal rate equal to its *zero lower bound* when $Z_{\xi_t^d} = 1$. It should be noted that $Z_{\xi_t^d}$ is a dummy variable assuming value 0 and 1 depending on the realization of an exogenous discrete Markov-switching process ξ_t^d. As we shall discuss below, this process determines the natural rate of interest r_t^n, implying that ZLB episodes are caused by unanticipated and recurrent, exogenously-driven falls in the natural rate of interest. Furthermore, when the economy is *out of the ZLB*, the value of the policy parameter ψ_{π,ξ_t^p}, which controls how strongly the central bank adjusts the nominal interest rate to inflation, are affected by the exogenous discrete Markov-switching process ξ_t^p.

The natural rate of interest is linked to the (exogenous) dynamics of the natural output though the IS equation under flexible prices:

$$r_t^n = \sigma\left(E_t \Delta y_{t+1}^n\right) \tag{9}$$

where Δy_t^n stands for the growth rate of natural output, whose value at any time is assumed to depend on the realization of a discrete Markov-switching process ξ_t^d.

The fiscal rule that determines the primary surplus τ_t

$$\tau_t = \delta_{b,\xi_t^p} b_{t-1} + \delta_x x_t \tag{10}$$

where b_t stands for the government debt-to-output ratio. Note that the response of the primary surplus to the last period's debt-to-output ratio is given by δ_{b,ξ_t^p} whose value depends on the realization of the Markov-switching process ξ_t^p that also determines the central bank's response to inflation in the Taylor rule. Hence, the process ξ_t^p captures the monetary-fiscal policy mix *out of the zero lower bound*

The government's budget constraint is driven by

$$b_t = \beta^{-1} b_{t-1} + b\beta^{-1}\left(R_{t-1} - \pi_t - \Delta x_t - \Delta y_t^n\right) - \tau_t \tag{11}$$

There are two exogenous Markov-switching processes: ξ_t^p and ξ_t^d. The former captures monetary and fiscal authority's response to their targets *out of the zero lower bound*. More specifically we assume that there are two monetary and fiscal policy mix: a monetary-led regime ($\xi_t^p = M$) and a Fiscally-led regime ($\xi_t^p = F$). Under the monetary-led regime the monetary authority responds strongly to inflation $\psi_{\pi,\xi_t^p} > 1$ and the fiscal authority promptly adjusts the primary surplus to changes in the debt-to-output ratio $\delta_{b,\xi_t^p} > \left(\beta^{-1} - 1\right)$.[1] Under the fiscally-led policy regime the monetary authority adjusts the nominal interest rate R_t less vigorously to inflation $\psi_{\pi,\xi_t^p} \leq 1$ and the fiscal authority pays less attention to the dynamics of its debt-to-output ratio $\delta_{b,\xi_t^p} \leq \left(\beta^{-1} - 1\right)$. The transition matrix driving the policy regime out of the zero lower bound ξ_t^p is given by the following matrix

$$H^p = \begin{bmatrix} p_{MM} & 1 - p_{FF} \\ 1 - p_{MM} & p_{FF} \end{bmatrix}$$

The non-Gaussian process ξ_t^d determines the growth rate of natural output and hence the natural interest rate through equation (9). The growth rate of natural output $\Delta y_t^n \in \left\{\Delta y_H^n, \Delta y_L^n\right\}$, where $\Delta y_H^n > \Delta y_L^n$, and these two states evolve according to the transition matrix:

$$H^d = \begin{bmatrix} p_{hh} & 1 - p_{ll} \\ 1 - p_{hh} & p_{ll} \end{bmatrix}$$

When the growth rate of natural output is low, the natural rate is low, and the policymakers are assumed to engage in the *ZLB policy regime*, which is characterized by a nominal interest rate set to zero and no adjustment of primary surplus to changes in the debt-to-output ratio.

In summary, the joint evolution of policymakers' behavior and the shock to the natural rate is captured by the regime obtained combining the two chains $\xi_t = \left[\xi_t^p, \xi_t^d\right]$. The combined chain can assume three values: $\xi_t = \{[M, h], [F, h], Z, l\}$. The corresponding transition matrix H is obtained by combining the transition matrix H^d, which describes the evolution of the preference shock; the transition matrix H^p, which describes policymakers' behavior out of the zero lower bound, and the parameter p_{ZM} that controls the probability of moving to the monetary-led regime once the negative preference shock is reabsorbed:

$$H = \begin{bmatrix} p_{hh} H^p & (1 - p_{ll}) \begin{bmatrix} p_{ZM} \\ 1 - p_{ZM} \end{bmatrix} \\ (1 - p_{hh}) \cdot [1, 1] & p_{ll} \end{bmatrix}$$

Table 4 reports the parameter values we will use to study the property of this stylized model. The parameters π, and b denote the steady-state inflation and the steady-state value of the government debt-to-output ratio.

The exogenous drop in the growth rate of natural output is chosen so that to induce an annualized natural rate of -20-percent during the ZLB periods. In the benchmark calibration, we set the probability of moving to the monetary-led policy mix after the ZLB episode equal to

[1] See Leeper (1991) for the derivation of this cut-off values for the policy parameters defining the monetary-led and the fiscally-led policy regimes.

Parameters	Values	Parameters	Values
$\psi_{\pi,M}$	2.00	p_{hh}	0.98
$\delta_{b,M}$	0.03	p_{ll}	0.95
$\psi_{\pi,F}$	0.80	p_{MM}	0.99
$\delta_{b,F}$	0.00	p_{FF}	0.99
$\delta_{b,Z}$	0.00	p_{ZM}	0.50
κ	0.03	100π	0.5
σ	1.00	b	0.30
ψ_x	0.10	β	0.995
ρ_i	0.85	$\Delta y_t^n \left(\xi_t^d = h\right)$	5.30
δ_x	0.5	$\Delta y_t^n \left(\xi_t^d = l\right)$	-21.33

Table 4: Parameters used for the prototypical New Keynesian model.

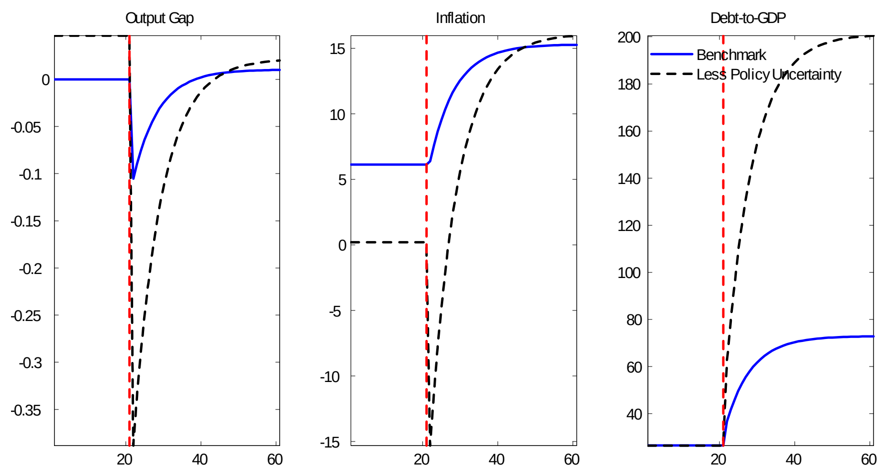

Figure 1: **Prototypical New Keynesian model.** The figure reports the impulse responses to a discrete shock to the natural interest rate. In the Benchmark model there is high policy uncertainty, while in the counterfactual economy agents think that they are more likely to move to the Monetary led regime.

$p_{ZM} = 50$ percent so as to capture a situation of sizable uncertainty about the policymakers' behaviors when the economy will exit the ZLB.

Figure 1 shows the dynamics of the output gap, inflation, and debt-to-GDP ratio in the aftermath of a discrete shock to the natural rate. We consider the benchmark case with parameter values reported in Table 4 and a counterfactual case in which agents are much more certain that the policy mix out of the ZLB will be monetary-led ($p_{ZM} = 85$ percent). Both economies are hit by a negative shock to the natural rate at time 6.[2]

It should be observed that larger policy uncertainty causes absence of deflation in presence

[2]Both economies are assumed to be at their respective out-of-ZLB steady-state equilibrium. However, the starting level of the debt-to-GDP ratio in the counterfactual economy is set to be equal to that in the benchmark so as to ease the comparison.

of a negative output gap as the economy hits the ZLB. Furthermore, policy uncertainty about policymakers' future behavior largely mitigates the output gap. These results are qualitatively in line with the ones obtained from our estimated model in Section III.B. The exercise made in this section makes it clear that the results analyzed in the paper are not driven by the type of shock we chose to trigger the ZLB episode or by the more articulated nature of the model used for estimation.

E Model without the fiscal block

In what follows, we provide the details for the model that removes the fiscal block. As explained in the main text, this model is nested in the benchmark model and it does not feature any uncertainty about the way debt will be financed. For this reason, debt and non-distortionary taxation become irrelevant for macroeconomic dynamics.

E.1 System of equations

1. Linearized Euler equation:

$$\begin{aligned}\left(1+\Phi M_a^{-1}\right)\widehat{y}_t &= -\left(1-\Phi M_a^{-1}\right)\left[\widehat{R}_t - E_t\widetilde{\pi}_{t+1} - (1-\rho_d)\,d_t - \overline{d}_{\xi_t^d} + E_{\xi_t^d}\overline{d}_{\xi_{t+1}^d}\right] \\ &\quad - \left(\Phi M_a^{-1} - \rho_a\right)a_t + E_t\widehat{y}_{t+1} + \left(1-\rho_g + M_a^{-1}\Phi\right)\widetilde{g}_t + M_a^{-1}\Phi\left(\widehat{y}_{t-1} - \widetilde{g}_{t-1}\right)\end{aligned}$$

where $M_a = \exp(\gamma)$ and $\overline{d}_{\xi_t^d}$ follows a Markov-switching process governed by the transition matrix H^d. Please refer to the next subsection for details about how to handle the discrete shock.

2. New Keynesian Phillips curve:

$$\begin{aligned}\widetilde{\pi}_t &= \kappa\left(\left[\frac{1}{1-\Phi M_A^{-1}} + \frac{\alpha}{1-\alpha}\right]\widehat{y}_t - \frac{1}{1-\Phi M_A^{-1}}\widetilde{g}_t - \frac{\Phi M_A^{-1}}{1-\Phi M_A^{-1}}\left(\widehat{y}_{t-1} - \widetilde{g}_{t-1} - a_t\right)\right) \\ &\quad + \beta E_t\left[\widetilde{\pi}_{t+1}\right] + \widetilde{\mu}_t\end{aligned}$$

where we have used the rescaled markup $\widetilde{\mu}_t = \kappa\left(\frac{\upsilon}{1-\upsilon}\right)\widetilde{\upsilon}_t$

3. No arbitrage condition

$$\widetilde{R}_t = E_t\left[\widetilde{R}_{t,t+1}^m\right]$$

4. Return long term bond

$$\widetilde{R}_{t-1,t}^m = R^{-1}\rho\widetilde{P}_t^m - \widetilde{P}_{t-1}^m$$

5. Monetary policy rule

$$\begin{aligned}\widetilde{R}_t &= \left[1 - Z_{\xi_t^d}\right] \left[\rho_{R,\xi_t^p} \widetilde{R}_{t-1} + (1-\rho_R)\left(\psi_{\pi,\xi_t^p}\widetilde{\pi}_t + \psi_{y,\xi_t^p}\left[\widehat{y}_t - \widehat{y}_t^*\right]\right) + \sigma_R \epsilon_{R,t}\right] \\ &\quad + Z_{\xi_t^d}\left[\rho_{R,Z}\widetilde{R}_{t-1} - \left(1-\rho_{R,Z}\right)\psi_Z \log(R) + \sigma_Z \epsilon_{R,t}\right]\end{aligned}$$

6. Government purchases ($\widetilde{g}_t = \ln(g_t/g)$):

$$\widetilde{g}_t = \rho_g \widetilde{g}_{t-1} + \sigma_g \epsilon_{g,t}, \;\; \epsilon_{g,t} \sim N(0,1).$$

7. TFP growth

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}$$

8. The rescaled markup $\mu_t = \kappa \log(\aleph_t/\aleph)$, where $\aleph_t = 1/(1-\upsilon_t)$, follows an autoregressive process,

$$\mu_t = \rho_\mu \mu_{t-1} + \sigma_\mu \epsilon_{\mu,t}$$

9. Output target

$$\left[\frac{1}{1-\Phi M_a^{-1}} + \frac{\alpha}{1-\alpha}\right]\widehat{y}_t^* = \frac{1}{1-\Phi M_a^{-1}}\widetilde{g}_t + \frac{\Phi M_a^{-1}}{1-\Phi M_a^{-1}}\left(\widehat{y}_{t-1}^* - \widetilde{g}_{t-1} - a_t\right)$$

E.2 Parameter estimates

Table 5 reports the parameter estimates for the model that excludes the fiscal block. As observables, we use four of the seven series used to estimate the benchmark model: GDP growth, inflation, federal funds rate, and government expenditure. Note that including the remaining fiscal series would be irrelevant for the dynamics of the macroeconomy because Ricardian equivalence applies when imposing that fiscal policy is always passive.

E.3 Dynamics at the zero lower bound

Figure 2 shows that the model without the fiscal block needs to use a combination of shocks in order to explain the absence of deflation during the zero lower bound. The figure reports the dynamics of inflation and output starting from 2008:Q4 in response to *two* shocks. The discrete preference shock and a large negative TFP shock. To ease the comparison with the results reported in Section IV, we also report the 90-percent error bands for the impulse response to the discrete preference shock only. While the discrete preference shock accounts for the bulk of the decline of inflation, the fall in output growth is mostly explained by the negative preference

	Mean	5%	95%	Type	Mean	Std
$\psi_{\pi,1}$	2.2157	1.7523	2.6568	N	2.5	0.3
$\psi_{y,1}$	0.3334	0.1542	0.5421	G	0.4	0.2
$\rho_{R,1}$	0.8641	0.8170	0.9118	B	0.5	0.2
$\psi_{\pi,2}$	1.1032	0.8242	1.3961	G	0.8	0.3
$\psi_{y,2}$	0.2678	0.1388	0.4414	G	0.15	0.1
$\rho_{R,2}$	0.8361	0.7811	0.8874	B	0.5	0.2
$\bar{d}_l$	−0.2592	−0.4124	−0.1272	N	−0.3	0.1
p_{hh}	0.9610	0.9204	0.9886	D	0.96	0.03
p_{ll}	0.8958	0.7792	0.9711	D	0.83	0.10
p_{MM}	0.9613	0.9072	0.9923	D	0.96	0.03
p_{FF}	0.9595	0.9075	0.9914	D	0.96	0.03
p_{ZM}	0.5009	0.1478	0.8451	D	0.50	0.22
ψ_Z	0.9698	0.9608	0.9781	B	0.95	0.02
κ	0.0707	0.0467	0.1025	G	0.3	0.15
Φ	0.8601	0.8147	0.8996	B	0.5	0.2
ρ_g	0.9919	0.9847	0.9971	B	0.5	0.2
ρ_a	0.1608	0.0583	0.2786	B	0.5	0.2
ρ_d	0.9358	0.9083	0.9599	B	0.5	0.2
ρ_μ	0.3291	0.1082	0.6432	B	0.5	0.2
$100\sigma_R$	0.2118	0.1958	0.2295	IG	0.5	0.5
$100\sigma_g$	0.2775	0.2568	0.2997	IG	1.00	1.00
$100\sigma_a$	1.4785	1.3037	1.6729	IG	1.00	1.00
$100\sigma_d$	13.8890	9.8559	19.2040	IG	10.00	2.00
$100\sigma_\mu$	0.2274	0.1965	0.2715	IG	1.00	1.00
100π	0.5678	0.4800	0.6607	G	0.5	0.05
100γ	0.4100	0.3352	0.4912	G	0.4	0.05
g	1.0758	1.0604	1.0942	N	1.06	0.04

Table 5: Posterior means, 90% posterior error bands and priors of thel parameters for the model that exludes the fiscal block. For the structural parameters, the suffix denotes the regime. The letters in the column "Type" indicate the prior density function: N, G, B, D, and IG stand for Normal, Gamma, Beta, Dirichlet, and Inverse Gamma, respectively.

shock. As explained in the paper, the result shows that fiscal uncertainty plays a key role in explaining the joint dynamics of inflation and output. Once the fiscal block in removed, a combination of shocks is necessary to explain the joint dynamics of output and inflation.

Figure 3 examines the properties of the traditional New Keynesian model to match the joint behavior of output growth and inflation from a different angle. This exercise is based on the estimates for the benchmark model, but removing the fiscally-led regime. We chose the estimated benchmark model as a starting point because the size of the discrete shock is in fact able to generate a realistic contraction in real activity. We then ask what slope of the Phillips curve can deliver a behavior of inflation and output growth in line with what observed in the data. The solid blue line corresponds to the case in which the slope of the Phillips curve is divided by two, implying that in average the slope is around .0036. Clearly in this case the model can generate a sizeable recession, but a the cost of generating deflation. Dividing the estimates slope by four, things slightly improve, but inflation is still too low. Finally, with a

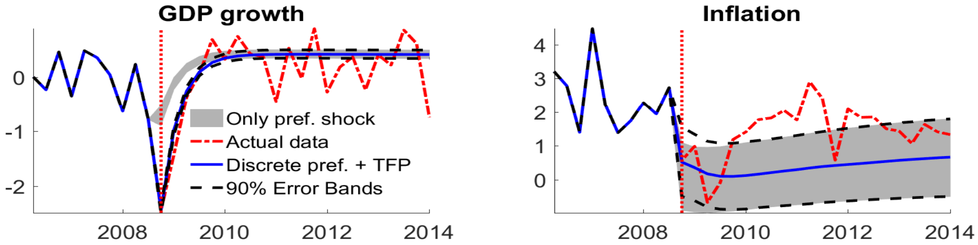

Figure 2: **Macroeconomic dynamics at the zero lower bound without fiscal block: The role of TFP shocks.** Response of GDP growth and inflation to a discrete negative preference shock and a contemporaneous negative TFP shock based on a model that excludes the fiscal block. The red dashed line reports actual data, while the shaded areas report the 90% error bands when only the discrete prefence shock occurs.

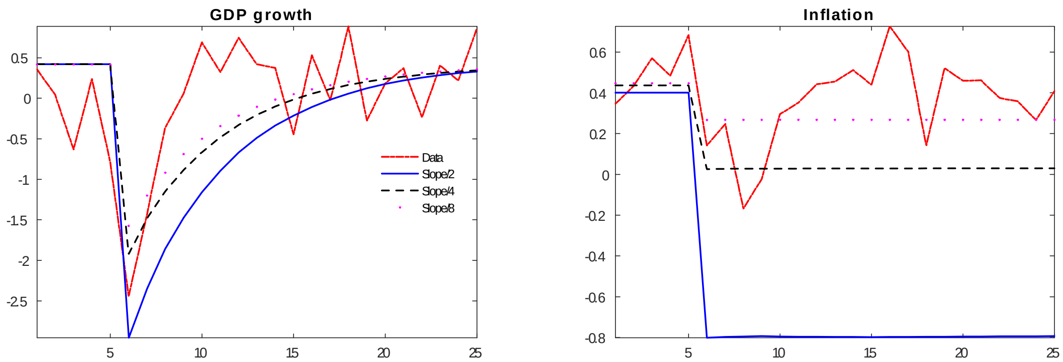

Figure 3: **Macroeconomic dynamics at the zero lower bound without fiscal block: The role of nominal rigidities.** Response of GDP growth and inflation to a large discrete negative preference shock for different values of the slope of the Phillips curve, κ.

mean of the slope around .0009 and ranging from .0006 to .0024 we can obtain a behavior of inflation more in line of the data, at the cost of a smaller recession.

Summarizing, we can highlight three conclusions based on the analysis of a model that excludes the fiscal block. First, in order to rationalize the joint dynamics of inflation and output, a very large level of nominal rigidities are necessary. Second, when using both data before and at the zero lower bound, this high level of nominal rigidities is rejected by the estimates. Instead, the model explains the zero lower bound dynamics as a result of two combined shocks: A discrete preference shock and contemporaneous negative TFP shock. This is because we do not ask the model to simply match the zero lower bound events, but also what happened before this event. Finally, the standard New Keynesian model cannot generate the drop followed by the slight upward of inflation observed in the data. In the model this is caused by the fact that as more time is spent at the zero lower bound, the fiscal burden increases, generating inflationary pressure.

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